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Further Examinations on the Thermodynamic Stability of the Mie-Grüneisen Equation of State

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The Mie-Grüneisen equation of state (EOS) is further examined for restrictions upon the form of the Grüneisen parameter. Employing basic thermodynamic rules and applying them to the Mie-Grüneisen EOS, a more restrictive condition is offered for the Grüneisen parameter, which extends previous work on the subject.

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TABLE OF CONTENTS

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1. BACKGROUND

In previous work, Segletes (1991a, 1991b) examined entropy changes along various thermodynamic paths of simple single-phased materials and established criteria governing three modes of thermodynamic stability of the Mie-Grüneisen equation of state (EOS). Since these criteria are used as a starting point for the current analysis, they will be briefly summarized here.

The Mode I criterion codifies the basic notion that any point on the Hugoniot in p,v space must rise more steeply (with decreasing v) than the Rayleigh line connecting the Hugoniot "foot" to that point. Expressed in the more familiar p, μ space, where μ is the compression, and $\mu = \rho/\rho_o - 1$, the Mode I criterion becomes:

$$\frac{dp_h/d\mu}{p_h} > \frac{1}{\mu(1+\mu)} \quad ,$$

where p_h is the Hugoniot pressure, expressed as a function of compression, and the Hugoniot "foot" is taken to be $p_o = 0$.

The Mode II criterion makes use of the fact that thermodynamic stability requires that the isentrope through an arbitrary point on a Hugoniot curve lie between the Hugoniot curve itself and the Rayleigh line originating from the Hugoniot "foot" (see Figure 1). As a result of enforcing this condition upon the Mie-Grüneisen EOS, a stability relation was established which restricts the permissible values of the Grüneisen coefficient to fall in the range

$$0 < \Gamma < \frac{2}{\mu} \quad .$$

The Mode III criterion provides an inequality involving Grüneisen terms, applicable off as well as on the Hugoniot, which, if satisfied, guarantees that the slope of the isentrope through that point is of the right sign (i.e., pressure increases with increasing compression). It is given by:

$$\left(\frac{\Gamma+1}{1+\mu} + \frac{1}{\Gamma}\frac{d\Gamma}{d\mu}\right)(p - p_h) + \frac{dp_h}{d\mu}\left(1 - \frac{\Gamma\mu}{2}\right) + \frac{(\Gamma/2)}{1+\mu}(p_h - p_o) > 0 \quad .$$

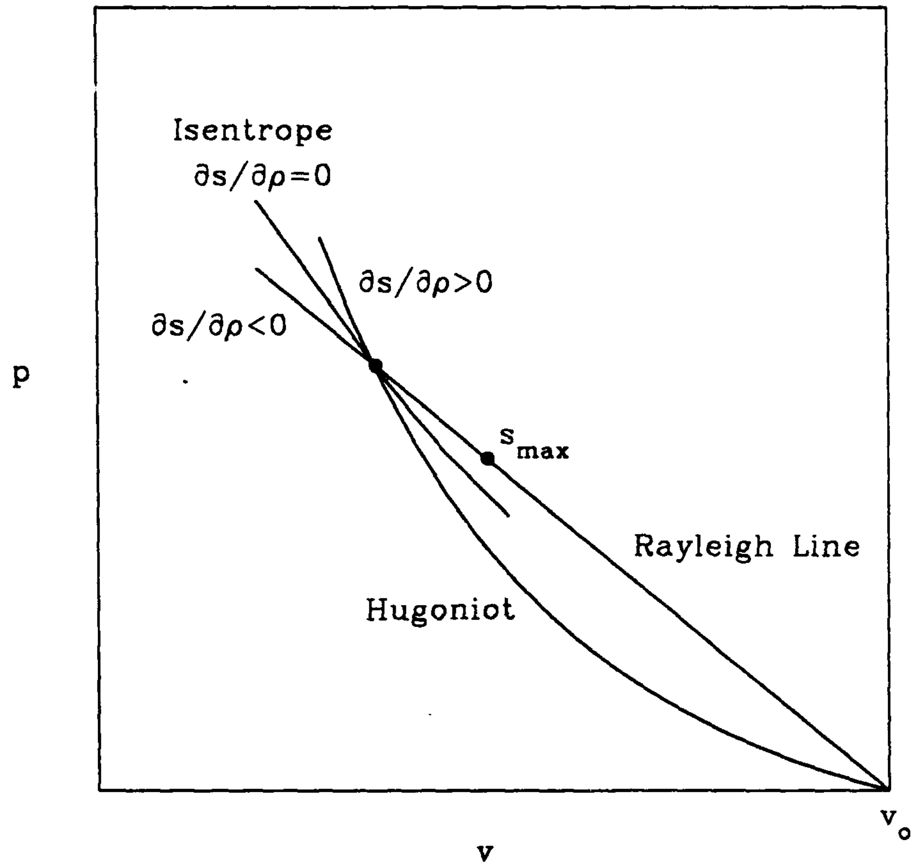

Figure 1. The Mode II stability criterion is based upon the realization that the slope of an isentrope through a point must lie between that of a Hugoniot through the point and that of the Rayleigh line through the point and the Hugoniot "foot."

These criteria establish new guidelines for EOS implementation in numerical "hydrocodes." These hydrocodes often use the Mie-Grüneisen EOS to model high pressure impacts, involving large deformations and, as such, can be sensitive to the choice of functional form on the Grüneisen relation. Segletes (1991a, 1991b) shows that polynomial Hugoniot fits in the literature (Kohn 1969) are prone to Mode I stability violations and that the Grüneisen relations existing in many of the popular hydrocodes have generally been of an ad hoc nature and are susceptible to Modes II and III violation (e.g., EPIC [Johnson et al. 1978; Johnson & Stryk 1992], DYNA [Whirley et al. 1992], JOY [Couch 1983], MESA [Cagliostro et al. 1990], and to a lesser extent, HULL [Matuska & Osborn 1987]).

An alternative Grüneisen relation was proposed by Segletes:

$$\Gamma = \frac{\Gamma_o}{1 + \beta\mu} .$$

The term β is a constant parameter which must be chosen greater than or equal to

$$\frac{\Gamma_o - 2/\mu_x}{2} ,$$

where Γ_o is the Grüneisen value at the Hugoniot origin, and μ_x is the terminal value of compression (that compression where the Hugoniot pressure becomes unbounded). This alternative form is guaranteed to satisfy the Mode II criterion at all valid compression states and was found to greatly diminish susceptibility to Mode III violation.

For general materials, these rules do not necessarily hold in all locations in p-v thermodynamic space. First of all, because the current discussion is limited to the classical case where the Grüneisen coefficient is taken as a functional of volume only. Furthermore, phase changes and the like can cause unusual local behavior, for example, tensile shocks and Mode I violations. However, these unusual regions constitute the exception and not the rule over the full thermodynamic range of material behavior. In a sense, then, the rules developed are applicable, for Grüneisen materials, in a piece-wise linear fashion, valid between anomalous thermodynamic state regions. Furthermore, numerical implementations of equations of state in wave propagation codes are almost universally Grüneisen type and typically continuous and continuously differentiable, so that anomalies of the type discussed (e.g., phase changes) are not modeled anyway. Thus, these stability criteria take on added importance when considering numerical aspects of "production" equation of state modeling, where characterization of material behavior is required over a wide range of pressures and volumes.

2. THEORY

In the current effort, a fundamental thermodynamic rule is again examined to see what influence, if any, is exerted upon the Mie-Grüneisen EOS. The rule examined is simply that the slope of a Rayleigh line $-(\partial p/\partial v)|_R$, between the Hugoniot origin and any point on the Hugoniot, must be positive, with increasing compression. Put another way, the rule implies that the pressure at a shocked state must exceed the preshocked pressure. This analysis generalizes the Mode III criterion, in that all valid preshock states will be considered, rather than just those arising from the infinitesimal shock (i.e., isentropic) condition.

In typical numerical implementations of the Mie-Grüneisen EOS, arbitrary thermodynamic states of a material are related back to those states along a specified reference curve at the same specific volume:

$$p - p_{ref} = \frac{1}{\psi}(E - E_{ref}) \quad ,$$

where p and E are pressure and specific internal energy, p_{ref} and E_{ref} are the pressure and specific energy along the reference curve, and $\psi = v/\Gamma$, where v is the specific volume. In hydrocodes, the reference curve is typically chosen as the Hugoniot curve, so that p_{ref} and E_{ref} become p_h and E_h, respectively. In addition to the Mie-Grüneisen relation, the following discussions will also make use of the shock energy equation, which relates the specific internal energies before and after a shock front to the pressures and specific volumes:

$$E_2 - E_1 = \frac{1}{2}(p_1 + p_2)(v_1 - v_2) \quad .$$

Consider a shock from an arbitrary thermodynamic state "1" to state "2," as described in Figure 2. Note that the states corresponding to (with the same specific volume as) "1" and "2," but along the reference Hugoniot curve, are given as states "h_1" and "h_2." Also, the "foot" of the reference Hugoniot, which is almost universally the ambient state, is given as state "o." Of course, $v_1 > v_2$ must hold to permit shock waves into simple solids, and $v_o \geq v_1$ must hold in order to appropriately apply the Mie-Grüneisen EOS with Hugoniot reference. Thus, the relation, $v_o \geq v_1 > v_2$, will hold throughout this analysis.

For the current analysis, the reference state "o" will be assumed to be the ambient condition, so that pressure and internal energy, p_o and E_o, will both be set to zero. The ambient state is almost universally chosen as the "foot" of experimental Hugoniots, so that this choice of the Hugoniot reference state

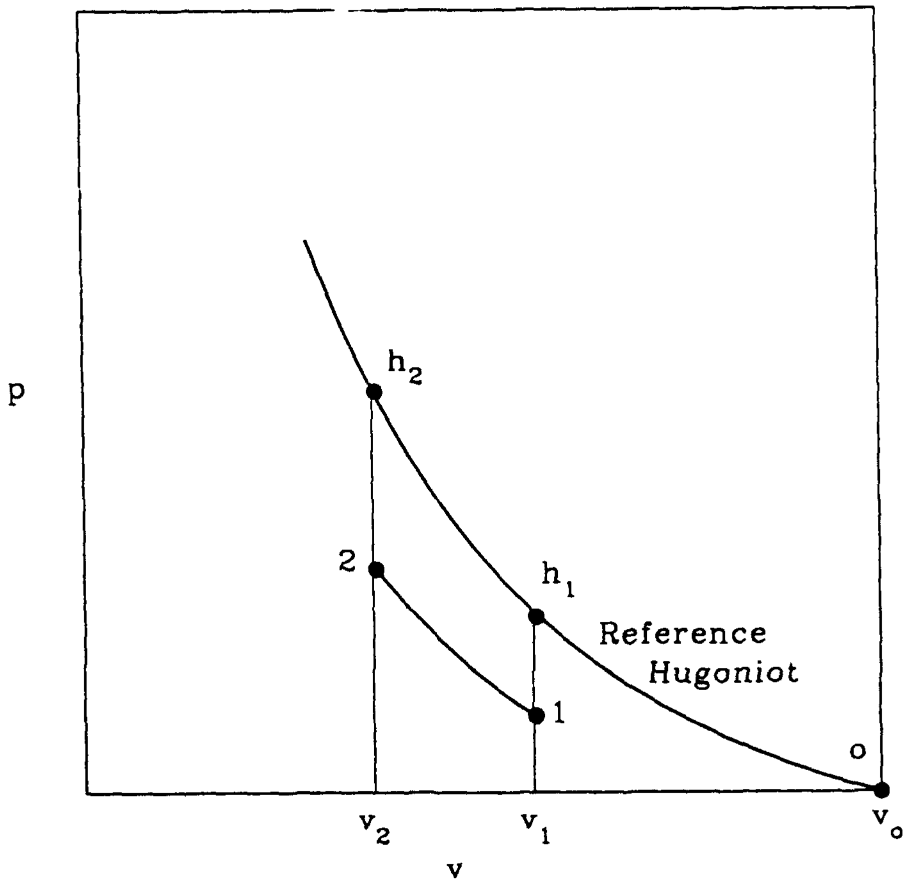

Figure 2. A depiction of the material states of i[illegible]rest: state "o" is the reference Hugoniot "foot"; states "1" and "2" are the pre- and post-shocked states under consideration; states "h_1" and "h_2" are those states on the reference Hugoniot corresponding to the specific volumes of states "1" and "2."

provides the most fruitful assumption for study. The following five equations will now be considered: the Mie-Grüneisen EOS at states "1" and "2," and the shock energy equation between states "o" and "1," "o" and "2," and between "1" and "2," respectively:

$$p_2 - p_{h_2} = \frac{1}{\psi_2}(E_2 - E_{h_2})$$

$$p_1 - p_{h_1} = \frac{1}{\psi_1}(E_1 - E_{h_1})$$

$$E_{h_1} = \frac{1}{2} p_{h_1}(v_o - v_1)$$

$$E_{h_2} = \frac{1}{2} p_{h_2}(v_o - v_2)$$

$$E_2 - E_1 = \frac{1}{2}(p_1 + p_2)(v_1 - v_2) .$$

It is desired to combine these equations in a way which eliminates all energy terms (E_1, E_2, E_{h_1}, and E_{h_2}). Such a combination may be obtained:

$$p_2\left[\psi_2 - \frac{1}{2}(v_1 - v_2)\right] - p_{h_2}\left[\psi_2 - \frac{1}{2}(v_o - v_2)\right] = p_1\left[\psi_1 + \frac{1}{2}(v_1 - v_2)\right] - p_{h_1}\left[\psi_1 - \frac{1}{2}(v_o - v_1)\right] .$$

If this equation is used to express p_1 in terms of p_2, then the positive Rayleigh slope condition being considered (namely, $(p_2 - p_1) > 0$), is given as

$$p_2 - p_1 = \frac{p_{h_2}\left[\psi_2 - \frac{1}{2}(v_o - v_2)\right] - p_{h_1}\left[\psi_1 - \frac{1}{2}(v_o - v_1)\right] + p_2\left[\psi_1 - \psi_2 + (v_1 - v_2)\right]}{\left[\psi_1 + \frac{1}{2}(v_1 - v_2)\right]} > 0 .$$

The denominator of this expression is always positive and may thus be eliminated from the inequality without changing its sense. This resulting expression may be used to isolate the terms $(p_{h_2} - p_2)$ in the following form:

$$(p_{h_2} - p_2)(\psi_1 - \psi_2 + v_1 - v_2) < (p_{h_2} - p_{h_1})\left[\psi_1 - \frac{1}{2}(v_o - v_1)\right] + p_{h_2}\left[\frac{1}{2}(v_1 - v_2)\right] .$$

Though this paper will continue to derive its results in the convenient ψ, v space, results may, as an alternative, be expressed in the more familiar Γ, μ space as follows:

$$(p_{h_2} - p_2)[\Gamma_2(1 + \mu_2) - \Gamma_1(1 + \mu_1) - \Gamma_1\Gamma_2(\mu_2 - \mu_1)] < (p_{h_2} - p_{h_1})\Gamma_2(1 + \mu_2)\left(1 - \frac{\Gamma_1\mu_1}{2}\right) + \frac{p_{h_2}}{2}\Gamma_1\Gamma_2(\mu_2 - \mu_1) .$$

This inequality (in either form) defines *the p, v (or p, µ) thermodynamic* space where $p_2 > p_1$ holds true. Those state pairs "1" and "2" which fail this metric imply that the thermodynamic path of shocking from state "1" to "2" is not thermodynamically admissible. Considering the right-hand side of the inequality, it is clear that terms like p_{h_2}, $(p_{h_2} - p_{h_1})$ and $(v_1 - v_2)$ are all positive, for a thermodynamically stable condition. The remaining term,

$$\psi_1 - \frac{1}{2}(v_o - v_1) ,$$

when converted to Γ, μ space, is expressed as

$$\frac{v_1}{\Gamma_1}\left(1 - \frac{\Gamma_1\mu_1}{2}\right) .$$

From the Mode II criterion, the term must be positive. Since Γ and v are always positive, this term and, in fact, the complete right-hand side of the inequality will always be positive.

The first term on the left-hand side of the stability metric, $(p_{h_2} - p_2)$, will be positive for pressure states below the reference Hugoniot and negative for pressure states above the reference Hugoniot. The second term on the left, $(\psi_1 - \psi_2 + v_1 - v_2)$, is a function of specific volume only. If it is negative, then, for any fixed specific volume, pressure states far enough above the reference Hugoniot will eventually violate this stability metric. If, on the other hand, this term is positive for a given specific volume, then pressure states far enough below the reference Hugoniot will eventually violate the metric. The consequences seem startling, since they imply, unless the second left-hand term is precisely zero, that the Mie-Grüneisen EOS will necessarily violate Rayleigh slope stability considerations in some pressure range. In reality, however, valid thermodynamic states are bounded from below by the "cold curve": that p, v curve associated with the absolute zero isotherm, below which physically plausible states do not exist. There are no such physical upper limitations on p, v states. Therefore, it becomes clear that the only possible state of the stability metric which can make physical sense is to have the second left-hand term be nonnegative, so that any inadmissable paths lie below the reference curve—indeed, far enough below the reference Hugoniot, it is hoped, so as to lie below the cold curve.

3. MODE IV STABILITY CRITERION

This stability metric forms the basis for a new constraint on the Grüneisen relationship. Adopting the nomenclature of Segletes (1991a, 1991b), this relationship might be added to his existing list and thus be termed the Mode IV criterion for EOS stability:

$$(\psi_1 - \psi_2) + (v_1 - v_2) \geq 0 \ .$$

Expressed in Γ, μ space, the Mode IV criterion becomes:

$$\Gamma_2(1 + \mu_2) - \Gamma_1(1 + \mu_1) + \Gamma_1 \, \Gamma_2 \, (\mu_2 - \mu_1) \geq 0 \ .$$

Several special cases of this criterion are examined for their specific functional behavior. For example, if the pre-shocked volume is the ambient specific volume, such that $v_1 = v_o$ (or correspondingly, $\mu_1 = 0$), then the resulting constraint on the Grüneisen relationship becomes

$$\Gamma \geq \frac{\Gamma_o}{1 + (1 + \Gamma_o)\mu} \ .$$

Note that this criterion provides a restriction more constraining than that of the Mode II criterion, $0 < \Gamma < 2/\mu$. It is also interesting to find that the form of this criterion exactly matches the functional form of the Grüneisen relation proposed by Segletes (1991a, 1991b), namely:

$$\Gamma = \frac{\Gamma_o}{1 + \beta\mu} \quad ,$$

where β is a constant which must, in the present case, be smaller than $(1 + \Gamma_o)$, in order to satisfy the inequality.

As a result, the second special case tested does not fix a value for the pre-shocked specific volume, v_1, but rather assumes merely that the functional form on Γ is of the form, involving β, proposed above. The result in this case, which surprisingly is independent of both pre- and post-shock specific volume, reveals again that

$$\beta \le (1 + \Gamma_o)$$

is required to satisfy the Mode IV criterion. On the other hand, the Mode II criterion shows, when this form for Grüneisen relation is adopted, that the acceptable range of β is given by

$$\beta > \frac{\Gamma_o - (2/\mu_x)}{2} \quad .$$

Recall that μ_x is the compression at which the Hugoniot reference function becomes unbounded. Thus, to adopt the form of Grüneisen relation proposed by Segletes, the constraint on β, by combining both the Mode II and IV criteria, becomes

$$\frac{\Gamma_o - (2/\mu_x)}{2} < \beta \le (1 + \Gamma_o) \quad .$$

Note that the proposed form of the Grüneisen function, with β so constrained, will be guaranteed not to violate either the Mode II or IV criteria.

The final special case investigated for the Mode IV criterion is that of the infinitesimal shock, where the preshock specific volume, v_1, approaches the post-shock condition, v_2. This special case turns out to be the strictest application of the Mode IV criterion, as will be subsequently shown, and is thus the most useful form. The differential Mode IV criterion, in this case, becomes

$$\frac{d\psi}{dv} \geq -1 \quad .$$

In Γ, μ space, the cr ion is expressed as

$$\frac{d\Gamma}{d\mu} \geq -\frac{\Gamma(1+\Gamma)}{(1+\mu)} \quad .$$

As $d\Gamma/d\mu$ identically equals the right-hand side of the inequality, Segletes' form can be recovered through integration, with β equaling $(1 + \Gamma_o)$. The true advantage to this differential form of the Mode IV criterion is that any proposed Grüneisen function may be evaluated at any given volume v (or compression μ), rather than requiring both pre- and post-shocked volumes (v_1 and v_2) in order to ascertain stability, as in the earlier, nondifferential form of the criterion.

It was stated that this differential form of the criterion is the strictest form. That is to say, it fully encompasses all cases covered in the original nondifferential criterion. To prove this assertion, one must show that it is not possible for a functional form to satisfy the differential criterion, while simultaneously failing the nondifferential criterion. It may seem intuitive, since the differential form constrains the instantaneous rate of change of the Grüneisen function, rather than merely limiting its finite change over a finite specific volume interval. However, a more rigorous proof shall be employed in this regard.

One may devise a Grüneisen function which satisfies the differential criterion by creating a perturbation from that criterion:

$$\frac{d\psi}{dv} = -1 + g(v); \qquad g(v) \geq 0 \quad ,$$

where g(v) is some arbitrary, nonnegative perturbation function. Since the term g(v) is always nonnegative, this functional form will satisfy the differential criterion always. The solution of this differential equation, which represents (through ψ) all plausible, stable Grüneisen functions, is:

$$\psi = G(v) + C_1 - v$$

where C_1 is a constant of integration and G(v) is the integral of g(v) with respect to v. Of course, this equation is subject to the condition that $(G(v) + C_1) > v$, in order to keep ψ (thus Γ) positive. Since g(v) is always nonnegative, G(v) will be a monotonically increasing function of v. Substituting this ψ function back into the nondifferential criterion produces the result

$$G(v_1) - G(v_2) \geq 0 \quad .$$

Since G(v) is monotonically increasing and $v_1 > v_2$, this result shows that the nondifferential criterion will always be satisfied by any arbitrary Grüneisen function which also satisfies the differential criterion. Therefore, the differential criterion is also a rigorous statement of Mode IV stability.

Figure 3 depicts the consequences of the Mode IV criterion, by illustrating several curves-of-maximum-descent. These curves represent the steepest possible gradient of the Grüneisen function, short of violating the Mode IV criterion.

4. ALTERNATIVE MODE III STABILITY

The primary consequence of Mode IV stability is that any and all unstable thermodynamic paths are forced to occur below the Hugoniot reference curve of the Mie-Grüneisen EOS. Figure 4 depicts a stable reference Hugoniot and two unstable derivative Hugoniots that can occur if Mode IV stability is violated. The stable reference Hugoniot (for aluminum) is based upon the common assumption of a linear relationship between the shock velocity, U_s, and the particle velocity behind the shock, u_p. The mathematical form is

$$U_s = C_o + S\, u_p \quad ,$$

and the Hugoniot form which results from it is given by

$$(p_h - p_o) = \frac{\rho_o C_o^2 \mu (1 + \mu)}{[1 - (S - 1)\mu]^2} \quad .$$

Note that the unstable Hugoniots lie above the reference Hugoniot, in thermodynamic space where Rayleigh slope violations should not occur. The derivative Hugoniots were generated from the reference by assuming a Grüneisen function equal to $\Gamma_o/[1 + (3 + \Gamma_o)\mu]$, which was shown in the previous section to be clearly unstable.

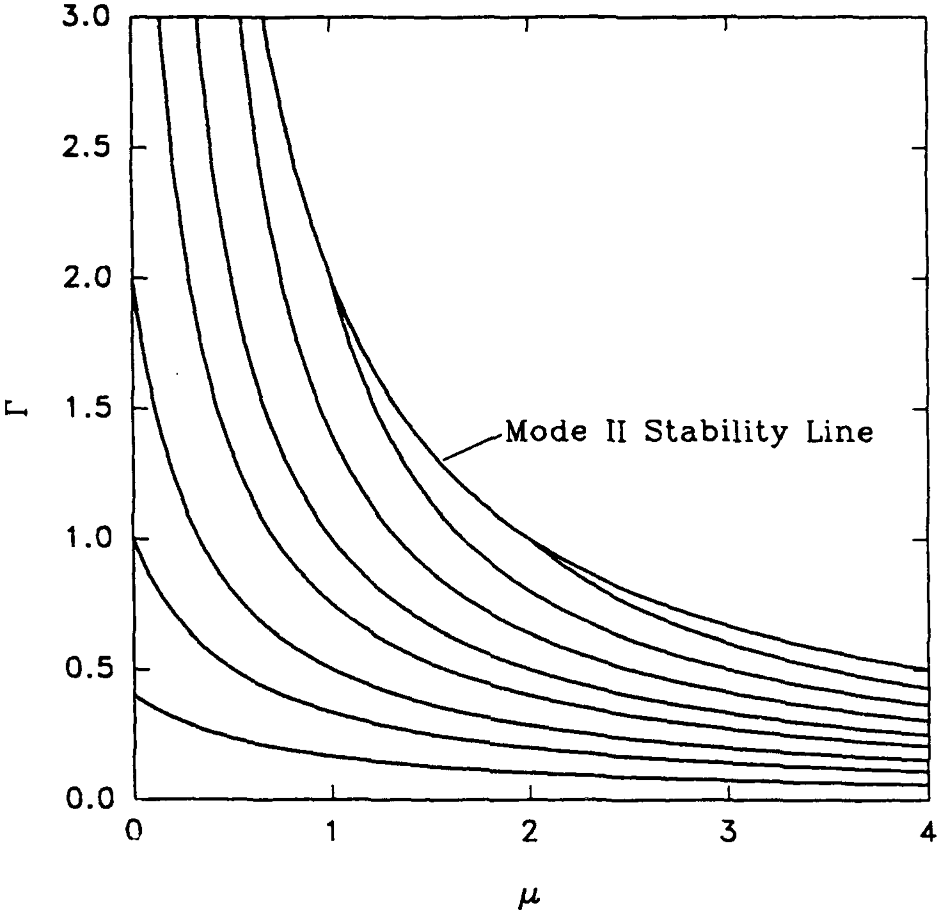

Figure 3. The Mode IV criterion defines the maximum allowable rate-of-descent of the Grüneisen function. Several such curves are plotted in Γ, μ space. In any case, the Grüneisen value must remain below a value of (2/μ), as dictated by the Mode II criterion.

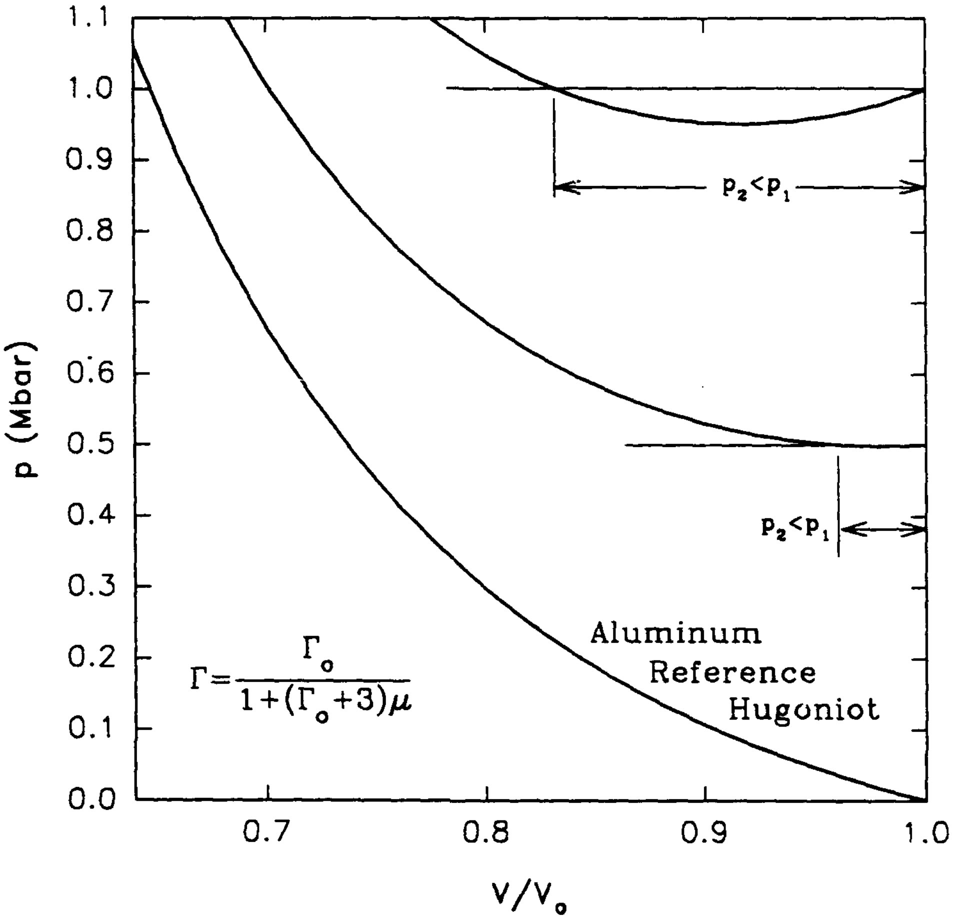

Figure 4. A depiction of unstable derivative Hugoniots which can arise from a Mode IV violation. The stable reference Hugoniot is that of aluminum, with $\rho_o = 2.7$, $U_s = 5{,}440$ m/s + 1.33 u_p, and a value for Γ_o equal to 2.09. The Grüneisen form is that of Segletes (1991a, 1991b), with the value for β set to $(\Gamma_o + 3)$, illustratively chosen to lie beyond the stable limiting value of $\Gamma_o + 1)$.

The Mie-Grüneisen EOS form under consideration is defined only for volumes where the reference function is defined. Since the Hugoniot reference function is only defined for positive compressions, discussions throughout this section are limited to compressed material states (at specific volumes less than the ambient specific volume).

Though satisfying the Mode IV criterion will guarantee that thermodynamically unstable regions will lie below the reference Hugoniot, the question arises as to how far below? In particular, it is desired to have any unstable regions lie below the cold curve (i.e., the zero-degree isotherm), where they would pose no problems. Gases, for example, have their zero-degree isotherm identically at $p = 0$. Solids, on the other hand, have a cold curve which monotonically increases in pressure with compression and, except for a small region near ambient density, will always lie above $p = 0$.

Thus, if it can be shown that all thermodynamic instabilities lie below the cold curve, while satisfying the Mode IV criterion, then one may conclude that such instabilities are no longer relevant, since they lie in a nonachievable range of the EOS range.

Recall the relation required to keep the Rayleigh slope positive:

$$p_{h_2}\left[\psi_2 - \frac{1}{2}(v_o - v_2)\right] - p_{h_1}\left[\psi_1 - \frac{1}{2}(v_o - v_1)\right] + p_2\left[\psi_1 - \psi_2 + v_1 - v_2\right] > 0 \quad ,$$

where $v_1 > v_2$. The differential form of this equation follows as:

$$-\frac{d}{dv}\left\{p_h\left[\psi - \frac{1}{2}(v_o - v)\right]\right\} + p\left(\frac{d\psi}{dv} + 1\right) > 0 \quad .$$

The limiting case for the stability of this equation is when p is at a minimum, namely, along the cold curve, p_c. Though not shown here, a similar analysis may be employed to that derived previously, which shows the differential criterion to fully encompass the nondifferential criterion. If one now takes ψ times the Mie Grüneisen equation of state, with the ambient Hugoniot as its reference, where $E_h = 1/2\ p_h(v_o - v)$, one obtains:

$$p\psi - E = p_h\left[\psi - \frac{1}{2}\left(v_o - v\right)\right] \ .$$

The latest two terms may be combined by taking the derivative of this latest expression and substituting into the previous expression, eliminating the term

$$d/dv\left\{p_h\left[\psi - \frac{1}{2}\left(v_0 - v\right)\right]\right\} \ .$$

The result is

$$-\left(p_c\frac{d\psi}{dv} + \psi\frac{dp_c}{dv} - \frac{dE_c}{dv}\right) + p_c\left(\frac{d\psi}{dv} + 1\right) > 0 \ .$$

Utilizing the facts that $p_c = -dE_c/dv$, and that ψ is positive, this equation ultimately simplifies to

$$\frac{dp_c}{dv} < 0 \ .$$

One may thus conclude that a monotonic cold pressure curve is sufficient to keep the Rayleigh slope stable.

Recall however, that the differential Rayleigh slope (i.e., for infinitesimal shocks) coincides with the isentropic and Hugoniot slopes. This Rayleigh stability condition is thus equivalent to an isentrope stability condition, which was the basis of Segletes' Mode III criterion. The difference lies in the fact that the original criterion was expressed, not in terms of cold curve behavior, but rather in terms of Hugoniot behavior. Nonetheless, the two criteria are alternatives, given that the Hugoniot and cold curve functions may be derived from each other when the Grüneisen function is known.

5. SUMMARY

The Grüneisen parameter within the Mie Grüneisen EOS is often treated as an independent function of specific volume, judging from forms employed in the hydrocode literature. This report and its predecessor (Segletes 1991a, 1991b) strive to demonstrate the coupling which exists among EOS parameters (including the Grüneisen parameter), and in so doing, set forth criteria which govern various aspects of thermodynamic stability. The types of EOS behavior which may be expected from violations of the various criteria have also been depicted.

In this report, an inequality was established which quantifies the interrelation of thermodynamic quantities necessary to prevent the occurrence of a Rayleigh slope instability (whereby pre-shock pressure erroneously exceeds post-shock pressure). This inequality was expressed in the numerically convenient ψ, v space, as well as the more popular Γ, μ space (see Table 1).

By making use of this relation, two criteria were developed (denoted Mode IV and alternate Mode III). Satisfaction of the Mode IV criterion will guarantee that Rayleigh slope violations can only possibly occur below (and not above) the reference Hugoniot curve (see Table 2). Such a condition is necessary if there is any hope of keeping stability violations below the thermodynamic cold curve in a thermodynamically unachievable range.

The alternate Mode III criterion establishes a relation which will keep differential Rayleigh slope instabilities (thus, isentropic instabilities below the cold curve (see Table 3). Simply stated, the cold curve must monotonically decrease with increasing volume. This criterion offers an alternate form over Segletes' original Mode III criterion which expresses isentropic stability in terms of Hugoniot rather than cold curve behavior.

Table 1. Criterion to Avoid Rayleigh Slope Instabilities

(ψ, v space) $$(p_{h_2} - p_2)(\psi_1 - \psi_2 + v_1 - v_2) < (p_{h_2} - p_{h_1})\left[\psi_1 - \frac{1}{2}(v_o - v_1)\right] + p_{h_2}\left[\frac{1}{2}(v_1 - v_2)\right]$$
(Γ, μ space) $$(p_{h_2} - p_2)[\Gamma_2(1+\mu_2) - \Gamma_1(1+\mu_1) - \Gamma_1\Gamma_2(\mu_2 - \mu_1)] < (p_{h_2} - p_{h_1})\Gamma_2(1+\mu_2)\left[1 - \frac{\Gamma_1\mu_1}{2}\right] + \frac{p_{h_2}}{2}\Gamma_1\Gamma_2(\mu_2 - \mu_1)$$

Table 2. Mode IV Criterion to Keep Possible Rayleigh Slope Instabilities Below Hugoniot

nondifferential form (ψ, v space) $$(\psi_1 - \psi_2) + (v_1 - v_2) \geq 0$$
nondifferential form (Γ, μ space) $$\Gamma_2(1 + \mu_2) - \Gamma_1(1 + \mu_1) + \Gamma_1 \Gamma_2 (\mu_2 - \mu_1) \geq 0$$
differential form (ψ, v space) $$\frac{d\psi}{dv} \geq -1$$
differential form (Γ, μ space) $$\frac{d\Gamma}{d\mu} \geq - \frac{\Gamma(1 + \Gamma)}{(1 + \mu)}$$

Table 3. Alternate Mode III Criterion to Eliminate Rayleigh and Isentropic Slope Instabilities Above Cold Curve

(ψ, v space) $$\frac{dp_c}{dv} < 0$$
(Γ, μ space) $$\frac{dp_c}{d\mu} > 0$$

The stability criteria resulting from this work provide guidance to those researchers who wish to make use of the Mie-Grüneisen EOS, in hydrocodes or other media. Adherence to the Modes III and IV criteria remove the threat of Rayleigh slope instability over the plausible equation of state domain. Finally, a better understanding of the interdependence of the Mie-Grüneisen equation of state parameters may be obtained through the understanding of the thermodynamic stability criteria described in this report.

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6. REFERENCES

Cagliostro, D. J., D. A. Mandell, L. A. Schwalbe, T. A. Adams, and E. J. Chapyak. "MESA-3D Calculations of Armor Penetration by Projectiles With Combined Obliquity and Yaw." International Journal of Impact Engineering, vol. 10, pp. 81–92, 1990.

Couch, R., E. Albright, and N. Alexander. The JOY Computer Code. Lawrence Livermore Laboratory Report UCID-19688, 1983.

Johnson, G. R., D. D. Colby, and D. J. Vavrick. "Further Development of the EPIC-3 Computer Program for Three-Dimensional Analysis of Intense Impulsive Loading." AFATL-TR-78-81, Air Force Armament Laboratory, 1978.

Johnson, G. R., and R. A. Stryk. "User Instructions for the 1991 Version of the EPIC Research Code." WL/MN-TR-91-53, Wright Laboratory Report, 1992.

Kohn, B. J. "Compilation of Hugoniot Equations of State." AFWL-TR-69-38, Air Force Weapons Laboratory, 1969.

Matuska, D. A., and J. J. Osborn. HULL documentation. Orlando Technology, Inc. Report., 1987.

Segletes, S. B. "Thermodynamic Stability of the Mie-Grüneisen Equation of State and its Relevance to Hydrocode Computations." Journal of Applied Physics, vol. 70 no. 5, pp. 2489–2499; Erratum: vol. 71 no. 2, p. 1074, 1991a.

Segletes, S. B. "An Analysis on the Stability of the Mie-Grüneisen Equation of State for Describing the Behavior of Shock-Loaded Materials." BRL-TR-3214, U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD, 1991b.

Whirley, R. G., B. E. Engelmann, and J. O. Hallquist. "DYNA2D, A Nonlinear, Explicit, Two-Dimensional Finite Element Code for Solid Mechanics User Manual." UCRL-MA-110630, Lawrence Livermore National Laboratory, 1992.

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