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DELAY ESTIMATION ON CONGESTED WATERWAYS

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DELAY ESTIMATION ON CONGESTED WATERWAYS

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EXECUTIVE SUMMARY

A waterway simulation model has been developed to estimate tow delays at series of locks. This model can also estimate tow travel time along waterways, the means and variances of interarrival and interdeparture times at each lock, and the coal inventory levels and expected stock-out amounts. This is a stochastic, microscopic and event-scanning model. Each simulation run takes from several seconds to several minutes on a PS/2 personal computer depending on system complexity.

This model can handle any distributions for trip generation, lock service times and tow sizes. These distributions can be specified with tables to best represent reality or with equations to represent standard statistical distributions. Currently, travel speeds are assumed to be normally distributed while general distributions based on empirical observations are used for trip generation, lock service times and tow sizes. The service discipline in this model is FIFO. The model can accommodate parallel servers with unequal service rates. In addition, it can evaluate stall effects explicitly.

Validation results show that the model works satisfactorily. To check the logic of this model, it is compared against the well established M/G/1 queues over a wide range of volume to capacity ratios (0.04 to 0.89). The average absolute error is 0.54%. This simulation results are also compared with the observed data at five lock sites on the Mississippi River. The validation criteria include average waiting times, chamber volumes, and cut volumes. The validation results show that the simulation model represents the real system quite well.

Although this simulation model requires only a few seconds to a few minutes for each lock and each run on a PS/2 personal computer, that is still hardly affordable for direct application in large combinatorial network investment problems.

A numerical method has been developed for estimating delays through a series of G/G/1 queues with inflows and outflows occurring only at end nodes. This numerical method is an approximation of the simulation model. It can quickly evaluate large combinatorial investment problems and thus helps identify the best combinations of investment alternatives for further analysis (e.g., by simulation).

This numerical method was originally developed for systems with bi-directional servers. With a few simplifications, this method can be adapted for the more generally encountered systems with one-directional servers. The two-way algorithm employs an iterative alternating direction scanning procedure to estimate the interarrival and interdeparture time distributions lock by lock until the interdeparture time variances for successive iterations converge. The performance of this two-way algorithm is tested with satisfactory results. The one-way algorithm only scans the interarrival time and interdeparture time distributions from the first to the last lock without any iteration and should, theoretically, be less subject to interdependence errors.

Both the two-way and one-way algorithms rely on several metamodels estimated from the results of a previously developed simulation model These metamodels provide the following valuable results for series of G/G/1 queues.

1. The delay metamodel (Eq. 41) indicates how the V/C ratios, interarrival time distributions, interdeparture time distributions and service distributions affect the average waiting times for G/G/1 queues. This delay function is an exact solution based on Marshall's formula for the variance of interdeparture times.

2. The relations among the coefficients of variation of interdeparture times, interarrival times, service times, and the V/C ratio are formulated in the departures metamodel. The structure of the departure function (Eq. 36) is based on functions for the squared coefficients of variation of interdeparture times. By applying Laplace transforms, these functions (i.e., Eqs. 31 and 32) are derived theoretically in this study. Statistical estimation of the parameters yields a very good fit. The function's standard error of 0.0058 is extremely tight compared to its mean of 0.8311. The parameters also have very tight standard errors. In addition, this departure function is consistent with Burke's Theorem. The results show that the metamodeling approach combining queuing theory and statistical estimation based on simulation outputs is quite successful. This approach for approximating departure processes snould be very useful for analyzing networks of queues.

3. The arrivals module provides the relation between the variance of interarrival times and the variance of interdeparture times from the adjacent queue stations when speed variations change the

headway distributions between successive queue stations.

It should be noted that the metamodel parameters have been estimated from data bases representing conditions on waterways. However, results to date indicate that such good metamodels can be extended to other applications.

The applications of the numerical method are currently limited to series of G/G/1 queues in waterways with inflows and outflows occurring at end nodes. The following extensions are desirable to increase its applicability.

1. A function should be developed to estimate the variance of interarrival times when inflows and outflows occur between queuing stations. Such an arrival function should reflect the complex superposition and splitting of flows in general networks of queues, including tree and grid networks. A proposed approach would compute the variance of overall arrival rates as the sum of the arrival rate variances from all inflows, assuming individual inflows are independent of each other, and then develop the relation between the variance of interarrival times and the variance of arrival rates.

2. The effects of random failures (i.e., stalls) might be incorporated by treating stalls as a second class of users with its own arrival and service time distributions. The present method can already incorporate stalls as part of the exogenously specified service time distribution, to the extent that stalls are related to traffic volumes. However, it is desirable to have a method which can evaluate stall effects explicitly.

3. If possible, the numerical method should be extended to locks with two or more dissimilar chambers. This is rather difficult for chambers with different characteristics, because the chamber assignment process affects lock capacity.

4. The ability to handle unequal directional trip rates would be desirable for many applications, although waterways usually operate with roughly equal directional flows.

5. It would be desirable to model queues with limited storage space, which are highly unusual on waterways but fairly common in dense road

networks, computers, and other queuing network applications.

Additional statistical and computational tests are also desirable to further validate this method and to extend its applicability.

The approach developed in the numerical method may be applied not only to a lock queuing system, but also to some other systems of queues. More comprehensive simulation experiments are desirable for developing the arrival and departure process modules for networks of G/G/k queues. These simulation experiments may include wider ranges of variables, and other special distributions for interarrival and service time distributions. Further research in estimating interarrival time distributions with multiple unequal inflows would be necessary for extending the numerical method to general networks of queues which may have inflows and outflows at any node.

The final methodology for estimating waterway delays may combine simulation and numerical method. The numerical method may be used in analyzing large combinatorial problems, that may be encountered in investment scheduling or real time traffic control. The numerical method may be used to screen alternatives so that only a few of the most promising ones are examined more thoroughly with the simulation model. Guidelines should be developed for switching between the numerical method and simulation in applications requiring intermediate accuracy.

The feasibility of approximating simulation models for complex queuing systems with simple metamodels should be intensively explored. If feasible, it would have important applications in combinatorial optimization and real-time control for transportation and communication networks, computers, and manufacturing systems.

CHAPTER 1
INTRODUCTION

Background

Inland waterway transportation is quite important in the U.S. and other countries, especially for heavy or bulky commodities, since it is inexpensive, energy efficient, and safe. Most U.S. waterways consist of stepped navigable pools formed by dams across natural rivers. The lock structures used to raise or lower vessels between adjacent pools constitute the major bottlenecks in the U.S. waterway network [27] and generate extensive queues. The resulting delays and variability of service times have very substantial economic implications.

Some locks have only one chamber, while others may have two parallel chambers whose characteristics may differ. The most common chamber sizes are 110′ x 1200′ (i.e., 110 feet wide and 1200 feet long) and 110′ x 600′. Each chamber size can accommodate a limited number of barges at one time. For example, a 110′ x 1200′ chamber can accommodate at most 17 standard barges plus a towboat, while a 110′ x 600′ chamber can accommodate at most 8 standard barges plus a towboat. If a tow has more barges than the chamber can accommodate, it must be disassembled into several pieces (called "cuts") to move through the chamber and must later be reassembled. Therefore, the service time distributions depend on chamber-size and tow-size distributions. Sometimes, chambers will be out of service (i.e., "stalled") for various reasons such as freezing, accidents, and mechanical failures.

Figure 1 shows a simple diagram of a lock queuing system. Locks are the servers and tows are customers waiting to be served by locks. In the lock queuing system, tows from both directions, upstream and downstream, share the same lock servers, while in most other queuing systems the servers are exclusively one-directional. In this work, the term "two-way traffic operations" characterizes the lock queuing system while "one-way traffic operations" characterizes queuing systems in which servers are exclusively one-directional.

Arrival-time and service-time distributions at locks are fairly complex. Carroll [5] and Desai [14] found that service times are not exponentially distributed, and arrivals are not Poisson distributed. Other standard distributions have been tested for the present study, without consistent success. Locks with a single chamber may be modeled as G/G/1 queuing systems. (The notation means "generally distributed arrivals/ generally distributed

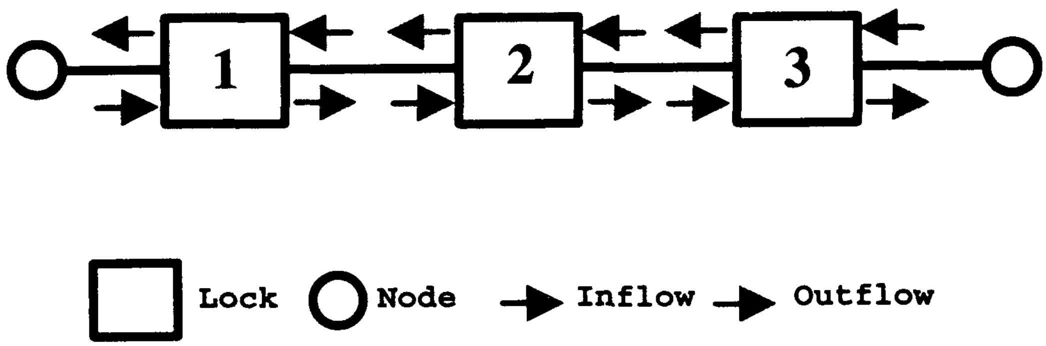

FIGURE 1. LOCK QUEUING SYSTEM

service times/ one server".) However, locks with two parallel chambers may not be treated simply as G/G/2 queuing systems unless the parallel chambers are identical.

The lock service-time distributions are affected by the chamber assignment discipline at locks with two dissimilar chambers. There the "main" chamber is larger than the "auxiliary" chamber and can accommodate without disassembly large tows that might require several cuts and far larger service times to move through the auxiliary chamber. However, if the same number of cuts is required through either chamber, a shorter auxiliary chamber may provide faster service since entry and exit times may be reduced. Therefore, lock service-time distributions are dynamic and depend on the chamber assignment discipline.

Considerable interdependence may exist among locks in a series. The departure distributions differ from the arrival distributions since the service-time distributions change the tow headways. The departures from one lock usually affect the arrivals at the next lock. Therefore, it is improper to assume that the locks are independent. The interdependence among locks increases the difficulty in estimating delays for the lock queuing system since it is necessary at each lock to identify the interarrival-time distributions of flows from adjacent locks.

Two-way traffic operation through common servers complicates the interdependence of lock delays and precludes the use of some otherwise interesting queuing models. Delays are determined by the arrival distributions and service-time distributions. It is much more difficult to identify the arrival distributions for two-way traffic systems than for one-way traffic systems. The arrival distribution at one lock is affected by departures from both upstream and downstream locks, while departures from this lock also affect the arrivals at upstream and downstream locks. For example, in Figure 1 the arrivals at Lock 2 would be affected by the departures from Locks 1 and 3. The departures from Locks 1 and 3 toward Lock 2 are highly correlated with the arrivals at 1 and 3 from 2. Some of the arrivals at 1 and 3 represent departures from 2. Hence, the arrival distributions of these three locks are interdependent. Thus, two-way traffic operation complicates the estimation of the two arrival distributions at each lock. The arrival distributions depend not only on the departure distributions from the adjacent locks, but also on speed variations and distances between adjacent locks.

Random failures, which in inland waterways are called stalls, contribute significantly to the difficulties in estimating delays. Stalls, which interrupt lock operations and thereby increase delays and service time variances, are relatively rare compared to other events, however their occurrence is very difficult to predict.

Problem Statement

A reliable and efficient method for estimating delays is essential for evaluating and scheduling waterway investments. When many investment proposals are considered, their selection and scheduling becomes a large combinatorial problem, since waterway investments are often interdependent.

The purpose of this research is to develop an efficient and reliable method to estimate delays through a series of waterway locks. The difficulties in estimating delays for such a lock queuing system are summarized as follows:

1. Arrival-time and service-time distributions are generally distributed (i.e., they cannot be represented by any standard statistical distribution).

2. The service rates of parallel chambers may be very different.

3. Service-time distributions are affected by the chamber assignment discipline.

4. Considerable interdependence exists among locks in series.

5. Two-way traffic operation through bi-directional chambers complicates the analysis.

6. The arrival distributions depend not only on the departures from previous locks but also on the distances and speed distributions between locks.

7. Stalls increase the means and variances of delays.

Even neglecting the special complexities associated with waterways, the available analytic solutions for estimating delays through a series of G/G/1 queues are quite inadequate. These analytic solutions are approximations or are subject to some limitations. They will be discussed in greater detail in the literature review.

Scope

This research seeks to estimate delays along a series of locks in waterways. The arrivals and service times at these locks are generally distributed. Some locks may have two parallel unequal chambers. All chambers must handle upstream and downstream traffic. Also, these locks are subject to interruptions of operation due to stalls.

This research also seeks to explore the feasibility of substituting single equations for simulations of complex systems of queues. If feasible, such substitutes for simulation would have important applications in combinatorial optimization and real-time control for various transportation and communication networks, computers, and manufacturing systems.

CHAPTER 2
LITERATURE REVIEW

The literature reviewed in this study concentrates on lock delay models, lock interdependence, waterway simulation models, single queues, and networks of queues which are the subjects of greatest relevance to this work. Of these, the work on waterway simulation models and on networks of queues represent the closest parallels to the models developed for this study.

Lock Delay Models

Two models based on the application of queuing theory have been found for estimating lock delays. DeSalvo and Lave [13] represent the lock operation as a simple single-server queuing process with Poisson distributed arrivals and exponentially distributed service times. Wilson [37] extends the previous model by treating the service processes as general distributions. Both models are designed for analyzing single-lock delays.

Lave and DeSalvo [13] proposed that approximate lock delays could be estimated with an M/M/1 queuing model (Poisson arrivals/Exponential service times/1 server). Unfortunately, their assumptions about the arrival and service-time distributions do not satisfactorily fit the physical system of locks on waterways. In particular, Carroll and Wilson [7] found that the exponentially distributed service times do not correspond well with empirical evidence. Therefore, it is not proper to assume that the service times are exponentially distributed.

The assumption of Poisson distributed arrivals does not fit every lock in the waterway system. Carroll and Desai [5,14] studied the arrival processes for 40 locks on the Illinois, Mississippi, and Ohio River systems, utilizing 1968 data. The results of Chi-square tests show that 13 out of 40 locks had non-Poisson arrivals at the 5% significance level. Therefore, the assumption of Poisson distributed arrivals is questionable.

Wilson [37] proposed an M/G/1 model (Poisson arrivals/general service times/1 server) to analyze lock delays. Unlike DeSalvo and Lave [13], Wilson represented the service processes as generally distributed rather than exponentially distributed, which is far more realistic [7]. However, even the Poisson arrivals assumption is not realistic for all locks. Another deficiency in Wilson's model is that no exact queuing results are available for locks with two chambers in parallel. Thus, Wilson's model can only be applied to locks with Poisson distributed arrivals and a single chamber.

Two other deficiencies exist in both of the above models. First, neither of these models accounts for stalls. Stalls cause service interruptions at locks, thus reducing lock capacities or increasing delays. Their occurrence is very difficult to predict. Thus, Kelejian's efforts to model stalls and stall durations have not yet yielded strong results despite the rigorous statistical methods employed [19]. The second deficiency is that both models were developed to analyze delays at a single lock. Since the delays at adjacent locks may be highly related, it is desirable to analyze lock delays for entire systems.

Based on the above discussion, it seems desirable to develop a model that can represent the physical conditions satisfactorily and analyze systems of locks. The physical conditions that should be represented include generally distributed arrivals and service times, and service interruptions due to stalls.

Lock Interdependence

The departure process of a queue in a network is of special interest because it is likely to determine the arrival process at the following queue in the network. This produces interdependence among system elements. In other words, lock interdependence can be expressed in terms of the relations among the arrival, service, and departure processes at one lock. If the distributions of tow arrivals and departures at one lock are identical, it can be shown [11] that this lock is independent of other locks.

Burke's theorem [11] states that the steady-state output of a stable M/M/m queue with input parameter λ and service time parameter μ for each of the m channels is in fact a Poisson process with the same rate λ. Therefore, for two locks in series, if the arrival process is Poisson distributed and the service process is exponentially distributed at the first lock, the second lock will be independent of the first lock. In addition, the second lock will have the same arrival distribution as the first lock.

Burke's theorem established that an M/M/m queue is independent of the other processes in a network of queues. Hence, the decomposition technique can be applied to analyze the delays of this network (if all queues in it are M/M/m). While this is a powerful result, the delays at inland waterway locks cannot be analyzed independently since the service processes are observed to be non-exponentially distributed [7].

Carroll and Desai [5,14] studied tow arrival processes at locks on the inland waterway system. They assumed that, if the tow arrivals at a lock follow a Poisson distribution, the queuing processes at this lock are independent of what has occurred at the preceding lock. They employed a waterway-system simulation model to analyze operations through a series of locks. The goodness-of-fit of the tow interarrival distributions to a negative exponential distribution was then tested at all locks, using the Chi-Square test. The results showed that 13 of 40 locks had non-Poisson arrivals at the 5% significance level. Hence, Poisson distributed arrivals may not be assumed for the inland waterway system.

Waterway Simulation Models

The system simulation models developed to analyze lock delays and tow travel times originated mainly from Howe's microscopic model [17]. In that model service times were based on empirically-determined frequency distributions. To avoid some troublesome problems and errors associated with the requirement to balance long-run flows in Howe's model, Carroll and Bronzini [6] developed another waterway-system simulation model. It provided detailed outputs on such variables as tow traffic volumes, delays, processing times, transit times, averages and standard deviations of delay and transit times, queue lengths, and lock utilization ratios.

Both of the above models simulate waterway operations in detail but require considerable amounts of data and computer time, which limit their applicability for problems with large networks and numerous combinations of improvement alternatives. Both models assume Poisson distributions for tow-trip generation, which is not always realistic. More importantly for reliability analyses, neither of these models explicitly accounts for stalls, which are very different in frequency and duration from other events and have significant effects on overall transit-time reliability.

Hence a waterway simulation model that explicitly accounts for stalls is desirable for evaluating and scheduling lock improvement projects.

Single Queues

Queuing theory deals with the stochastic behavior of service systems. Historically, queuing theory was first developed in the context of telephone traffic. Erlang, in particular, made many important contributions to the subject in the early part of this century. Telephone traffic remained one of the principal applications until about 1950.

Recently, queuing theory has been applied to many other fields including communications, computer networks, transportation, and manufacturing.

Elementary queuing theory is based on pure Markov queues. In a Markov process, the state of the queuing system at any time depends only on the current state and not on any previous states. A very important special class of Markov processes is the birth-death process in which state transitions take place between neighboring states only. The application of the birth-death process has helped in developing exact equilibrium solutions for queues with Poisson distributed arrival processes and exponentially distributed service times.

G/M/1 or M/G/1 queues are non-Markovian stochastic processes that may be analyzed with one of the following four approaches. First, the method of imbedded Markov chains focuses on the number of customers present in a queuing system immediately following a departure [21]. Second, the method of stages, proposed by Erlang, has the disadvantage that it merely gives a procedure to find the solution but does not show the solution as an explicit expression [21]. Third, G/G/1 systems are solved using Lindley's integral equation. Therefore, this approach can be applied to the special cases of G/M/1 or M/G/1 [21]. Fourth, the method of supplementary variables uses the state vector $[N(t), X_0(t)]$ where $N(t)$ is the number of customers in the system at time t and $X_0(t)$ is the service time already experienced by the customer served at time t [21].

G/G/1 queues are difficult cases in queuing theory and the available techniques for handling them are incomplete. As mentioned above, Lindley's integral equation is applicable for G/G/1 systems. The waiting-time distribution for customers W(y) in the system G/G/1 can be written as

$$W(y) = \begin{cases} \int_{-\infty}^{y} W(y-x)\,dC(x) & \text{if } y \geq 0 \\ 0 & \text{if } y < 0 \end{cases} \quad \textbf{(1)}$$

where

y = waiting time
x = the difference between the service time for one customer and the interarrival time between this customer and the next.
$C(x)$ = the probability distribution function (PDF) of u

In addition, the mean waiting time is

$$W = \frac{\sigma_A^2 + \sigma_S^2 + E[t_A]^2(1-\rho)^2}{2E[t_A](1-\rho)} - \frac{E[I^2]}{2E[I]} \quad \textbf{(2)}$$

where

σ_A^2: variance of the interarrival times
σ_S^2: variance of service times
t_A: interarrival time
ρ: volume to capacity ratio
I = idle period

Unfortunately, it is very difficult to solve the two moments ($E[I^2]$ and $E[I]$) of the idle period since this period depends on the particular way in which the previous busy period terminated.

Solving G/G/m queues is even more difficult than solving G/G/1 queues. The methods of approximations and bounds have been proposed to solve G/G/m queues [28,29]. These are accurate and efficient under heavy-traffic conditions.

Bertsimas [4] proposed a methodology for analyzing the waiting times of G/G/m queues with First-In-First-Out (FIFO) service discipline and mixed generalized Erlang distributed arrivals and service times. This method could be applied to more realistic situations than Poisson arrivals and exponential service times. However, without a departure function, this result is difficult to extend to a series of locks.

When traffic is heavy, and thus the ratio ρ approaches 1, the average waiting time for G/G/1 queues can be written as

$$W \approx \frac{(\sigma_A^2 + \sigma_S^2)}{2(1 - \rho)E[t_A]} \tag{3}$$

There are two types of approximation methods for G/G/m queues [28,29]. First, the fluid approximation uses the mean values to represent the stochastic processes. Second, the diffusion approximation improves the fluid approximation by describing the processes with means given by the fluid approximation but with a normal distribution describing the fluctuations about that mean. The important assumption behind these two approximation methods is that the queue never empties, which is consistent with the heavy traffic assumption.

Wolff [38] compared the delays under different orders of service (service disciplines) and concluded that the order of service would not change the mean of waiting times, but the service discipline Last-In-First-Out (LIFO) has a larger variance than the First-In-First-Out (FIFO) discipline.

Networks of Queues

Modeling networks of queues is very useful for analyzing the performance of complex systems, such as communication, computer, transportation, and manufacturing networks. Exact solutions are only available for the Markovian networks of queues [2]. For more realistic networks of queues whose arrivals are not Poisson and whose service times are not exponential, approximation methods are usually employed for system performance analysis.

In a Markovian network of queues, all arrival processes in the network are Poisson distributed and all service times are exponentially distributed. The queue-length distribution of individual queues in the Markovian networks is the same as that of a queue with Poisson arrival and departure processes [2]. In addition, the joint queue length distribution is the product of the individual queue-length distributions [3,8,9,16,18,23]. That is, the joint queue-length distribution has the product form. Therefore, the decomposition technique can be applied to the Markovian networks of queues and each individual queue is analyzed independently.

The concept of decomposition can also be applied to more general (non-Markovian and one-way) networks of queues. The general networks of queues are decomposed approximately into individual queues. These individual queues are analyzed independently and then the results are recombined for evaluating an entire system. The Queuing Network Analyzer (QNA) [35], based on decomposition, is the one most relevant to this study. The decomposition technique is applied in other examples [10,22,30,31].

QNA is a comprehensive software package for approximating the congestion measures of open networks of queues with multiple servers. The service discipline in QNA is FIFO and the network capacity is unlimited. The most important feature of QNA is that the arrivals need not be Poisson distributed and the service times need not be exponentially distributed. QNA employs two parameters, namely the mean and the coefficient of variation, to approximately characterize the arrival processes and then analyzes the individual queuing nodes independently. The decomposition procedures in QNA include three stages of analyses: superposition of arrival streams, departure processes, and splitting of departure streams. The superposition of arrival processes in QNA is a variation of the hybrid method in Albin [1]. The departure processes in QNA are identified by the stationary-interval method [35].

To approximate the superposition of arrival processes, Albin [1] and Whitt [34] suggested the use of the hybrid method, which is a combination of the stationary-interval method and asymptotic method. The hybrid method is mainly used for estimating the coefficient of variation. The hybrid coefficient of variation of intervals between two consecutive arrivals, C_h^2, is a convex combination of the squared coefficients of variation obtained from the stationary-interval method and asymptotic method:

$$C_h^2 = wC_1^2 + (1-w)C_2^2 \quad \textbf{(4)}$$

where

- C_1^2: squared coefficient of variation obtained from the asymptotic method [34]
- C_2^2: squared coefficient of variation obtained from the stationary-interval method [34]
- w: weighting function, $0 \le w \le 1$, $w \equiv w(\rho, n^*)$
- ρ: volume to capacity ratio of the queue
- n^*: effective number of component processes
- n^*: $[(\lambda_1^2 + \ldots + \lambda_n^2)/\lambda^2]^{-1}$
- λ_i: the rate of the ith component process
- λ: $\lambda = \lambda_1 + \ldots + \lambda_n$

Albin [1] proposed that the weighting function should approach 1 as ρ approaches 1 and approach 0 as n^* approaches infinity. Based on such a conjecture, Albin created a list of candidate weighting functions and used simulation to identify the best one. Thus, the weighting function was developed empirically. She also developed a different weighting function for each operating characteristic to be approximated. For example, she suggested using $w=[1+6(1-\rho)^{2.2}n^*]^{-1}$ for approximating the expected number in the system; $w=[1+8.3(1-\rho)^{2.1}n^*]^{-1}$ for approximating the standard deviation of the number in the system; and $w=0$ for approximating the probability of delay.

It should be noted that Albin's study [1] mainly estimated the coefficient of variation for the superposed arrival processes to queues. The weighting function in her study is a function of the congestion level (ρ) and varies for evaluating different operating characteristics (the expected number in the system, the standard deviation of the number in system, and the probability of delay). However, theoretically, for a single queuing station, the coefficient of variation of the superposed arrival processes should be independent of the operating characteristics of queues. Thus, the coefficient of variation should not be affected by ρ and should not vary for different evaluation purposes.

To understand why ρ is influential in Albin's weighting function we need to know how she obtained the results. In fact, the weighting function in Albin's study was not developed directly based on the coefficient of variation but based on the approximating operating characteristics [1]. She obtained a weighting function that can produce close agreement with the approximating operating characteristics. Therefore, her results may be affected by the approximating operating characteristics and not represent the true coefficient of variation. Since the superposition arrival processes in QNA is a variation of Albin's hybrid method, QNA also has the deficiency described above.

Departure processes are very important for networks of queues since they often become the inputs for other queues. Whitt [36] tried four methods, the asymptotic method, the stationary-interval method, the lag-1 correlation, and hybrid method, for approximating the departure processes by characterizing two parameters or the first two moments.

The asymptotic method tries to approximate the parameters of departure processes by matching their long-run behavior. The departure process approximated by the asymptotic method is just the arrival process. Therefore, the result obtained by the asymptotic method may not be useful for capturing the interdependence between queues.

The potential drawback of the stationary-interval method is that it does not take account of the dependence among successive intervals. Whitt [36], based on Marshall's expression for variances of departure processes [26], characterized the coefficient of variation of departures in terms of the waiting time as follows:

$$C_D^2 = C_A^2 + 2\rho^2 C_S^2 - 2\lambda(1-\rho)W \tag{5}$$

where

- C_D^2: squared coefficient of variation of interdeparture times
- C_A^2: squared coefficient of variation of interarrival times
- C_S^2: squared coefficient of variation of service times
- ρ: volume to capacity ratio
- λ: average arrival rate
- W: average waiting time

However, the exact average waiting time for G/G/1 queues is not available. Kraemer and Langenbach-Belz [35] developed the following approximation formula:

$$W = \rho(C_A^2 + C_S^2)g/2\mu(1 - \rho) \tag{6}$$

where

μ: the average service rate
$g \equiv g(\rho, C_A^2, C_S^2)$

$$g(\rho, C_A^2, C_S^2) = \exp[-2(1-\rho)(1-C_A^2)^2/3\rho(C_A^2 + C_S^2)], \quad C_A^2 < 1$$
$$= 1, \quad C_A^2 \geq 1$$

Whitt [36] combines the above approximation while g is assumed to be one. The expression of C_D^2 is as follows:

$$C_D^2 = \rho^2 C_S^2 + (1 - \rho^2) C_A^2 \tag{7}$$

Therefore, the coefficient of variation obtained with the stationary-interval method by Whitt is based on two successive approximations and may generate errors when C_A^2 is less than 1, which is very common in waterway queuing networks.

The lag-1 correlation provides a compromise between the stationary-interval method and the asymptotic method. It tries to match the local behavior of a departure process and also to partially account for the dependence among successive intervals. However, Whitt's results did not show any improvement when the lag-1 correlation method was used.

The hybrid method for the departure process is also a convex combination of the parameters from the asymptotic method and the stationary-interval method. However, this approach shows no improvement either compared to the stationary-interval method [36].

Whitt [36] concluded that the stationary-interval method has the best performance among these four methods. Therefore, the departure process in QNA uses the stationary-interval approach. However, Whitt's approach [36] approximates the departure process twice and may generate significant error when C_A^2 is less than one. Therefore, it is desirable to develop a method to approximate the departure process more precisely.

Albin [2] also tried to approximate the departure process for a single server with exponentially distributed service times by using a hybrid approach. The building blocks in her hybrid method are the asymptotic method and the Poisson method. The Poisson method approximates the departure process as a Poisson process. The asymptotic method approximates the departure process as the arrival process. Therefore, the C_D^2 in Albin [2] becomes:

$$C_D^2 = wC_A^2 + (1 - w) \tag{8}$$

where

- w: weighting function, $w \equiv w(n^*, \rho_1, \rho_2)$
- n^*: the effective number of component arrival processes
- ρ_i: the volume to capacity ratio at queuing station i

It is notable that Albin's approach for approximating departure processes has the same deficiency associated with superposition arrival processes since the weighting function depends on the volume to capacity ratio at the second queuing station. Theoretically, the departure processes at the first queuing station should not be affected by the operation at the second queuing station unless there is a spillback from the second queuing station that blocks the operation of the first queuing station. Also, the departure processes should not vary for different evaluation purposes. In addition, Albin's results are only applicable to queues with exponential service times. Therefore, it is desirable to develop a methodology to accurately estimate departure processes for generally distributed arrivals and service times.

CHAPTER 3
MICROSCOPIC SIMULATION MODEL

A simulation model has been developed to analyze tow operations along waterways. It may be used to determine the relations among delays, tow trips, distributions of generated tow trips, lock operations, lock service-time distributions, travel times, coal consumption, and coal inventories. The simulation model can take into account stochastic effects and seasonal variations. This model enables the following to be estimated: tow delays at each lock, interarrival and interdeparture-time distributions for each lock and for each direction, tow travel times along the waterway, inventory levels and expected stock-out amounts for coal delivered by waterway, and many other variables of interest to waterway users and managers. The estimation of tow delays, tow travel times, inventory levels, and expected stock-out amounts is useful for estimating economic benefits of lock improvements. Moreover, the interarrival and interdeparture time distributions and the delays estimated with this model should be useful for analyzing series of queues with generally-distributed interarrival and service-time distributions.

Data Base

The simulation model is developed on the basis of PMS (lock Performance Monitoring System) data collected since 1975. This data base includes very detailed information on traffic through the locks as well as physical aspects of lockage [15]. It is very useful for understanding and quantifying waterway characteristics, such as lock operations, arrival distributions, service-time distributions, tow-size distributions, and stalls.

Features of Simulation Model

The simulation model developed in the early stages of the study focuses on the relations among delays, trip generations, distributions of tow speeds, arrivals, departures and service times, and coal inventories. The output of this model includes delays, means and variances of interarrival-time and interdeparture-time distributions, and inventory level. These are useful for estimating inventory costs, stock-out costs, and expected benefits due to lock rehabilitation or lock construction. The relations among delays, interarrival, interdeparture and service-time distributions provide results for the analysis of series or networks of queues.

The simulation model is microscopic. It traces the movement of each individual tow and records its characteristics, including its number of barges, commodity types, speed, origin and destination, travel direction, and arrival time at various points. In addition, the model determines cumulative deliveries, cumulative consumption, and actual inventories at various plants.

The simulation model is an event-scanning model where the system status is updated by events. The system status includes the simulation clock, lock operating condition, movement of tows, and inventory level at each utility plant. The heart of the simulation model is the scheduler routine. This routine provides the control for the entire length of the simulation period. The scheduler, at appropriate times, invokes all other operational routines necessary to process the simulation.

This model can handle any distributions for trip generation, travel speeds, lock service times and tow sizes. These distributions can be specified for each interval in tables or by standard statistical distributions. Currently, travel speeds are assumed to be normally distributed, while general distributions based on empirical observations are used for other input variables. Tows are allowed to overtake other tows. A FIFO (First-In-First-Out) service discipline is currently employed. This model simulates two-way traffic through common servers and accounts for stalls.

There are five types of events in this model. First, tow trips are generated stochastically based on the actually observed traffic distributions. Currently, the model uses a table to represent the trip-generation pattern and is, therefore, not limited to standard mathematical probability distributions.

Second, the tow entrance in a lock is determined by tow arrival time at that lock, the times when chambers become available, and the chamber assignment discipline. If a tow arrives before the lock is available, it needs to wait in the queue storage area. Otherwise, it is served according to the chamber assignment discipline that will be discussed later. In general, the lock service is presently First-In-First-Out, subject to the chamber assignment procedure.

Third, whenever a coal tow arrives at its destination, the cumulative delivery at the destination will be increased by the amount of coal that tow is carrying. The cumulative consumption and inventory at the destination will also be updated then.

Fourth, the status of cumulative consumptions, inventories, and consumption rates for all coal destinations will be updated in every unit of time. This provides detailed information on inventory levels for all coal destinations.

Fifth, whenever a stall occurs at a chamber, the chamber becomes unavailable until the end of the stall.

The size of waterway systems that the model can handle is limited by the computer capacity and the storage capacity of the Fortran compiler or linker. The simulation model has been developed with "dynamic dimensioning" to the degree allowed by the computer system available. Parameter statements are used so that the dimensions, and hence capacities, of the model components may readily be modified. This allows the maximum flexibility of waterway system design and the most efficient computer utilization. Thus, the dynamic dimensioning programming technique allows flexibility in the number of locks, chambers, cuts, waterway links, tows, utility plants, origin-destination (O-D) pairs and simulation time periods. Currently this model can simulate two-way operation on a mainline waterway.

This simulation model is programmed in Fortran-77, which allows us to simulate relatively complex operations. The following is a more detailed description of how tow trip generation, tow travel times, and coal inventory levels are computed in this model. The overall structure of the simulation model is displayed in Figure 2. A logic flow chart is provided in Figure 3.

Assumptions of Simulation Model

The simulation model was developed based on the following assumptions:

1. The time interval between successive tow trip generations, service times, and tow sizes are generally distributed. These distributions are represented by probability distribution tables which are exogenous inputs. Therefore, the simulation model may be applied to any systems of lock systems.

2. The tow maintains a constant size through the entire trip.

3. The service discipline is First-In-First-Out (FIFO). This assumption is consistent with waterway operations on the Mississippi and Ohio Rivers.

INPUT:

Link & Lock Characteristics
Traffic Demand
Probability Distributions
Inventories and Consumption

PROCESS:

Origin Nodes: generating tow trips
Destination Nodes: updating: cumulative deliveries
cumulative consumption
inventory levels
Locks: assigning chamber
determining number of cuts
determining lock service times
calculating queuing times
Links: determining traveling times
determining arrival times to next locks or destinations

OUTPUT:

Average waiting time at each lock
Average waiting time at each lock for each O-D pair
Total waiting time for each lock
Means and variances of interarrival and interdeparture times at each lock
Cumulative deliveries, cumulative consumption, inventory levels
Average tow speed
Total number of tow trip for each O-D pair
Total lock service time for each lock and chamber
Total tow travel times and distances

FIGURE 2. STRUCTURE AND ELEMENTS OF SIMULATION MODEL

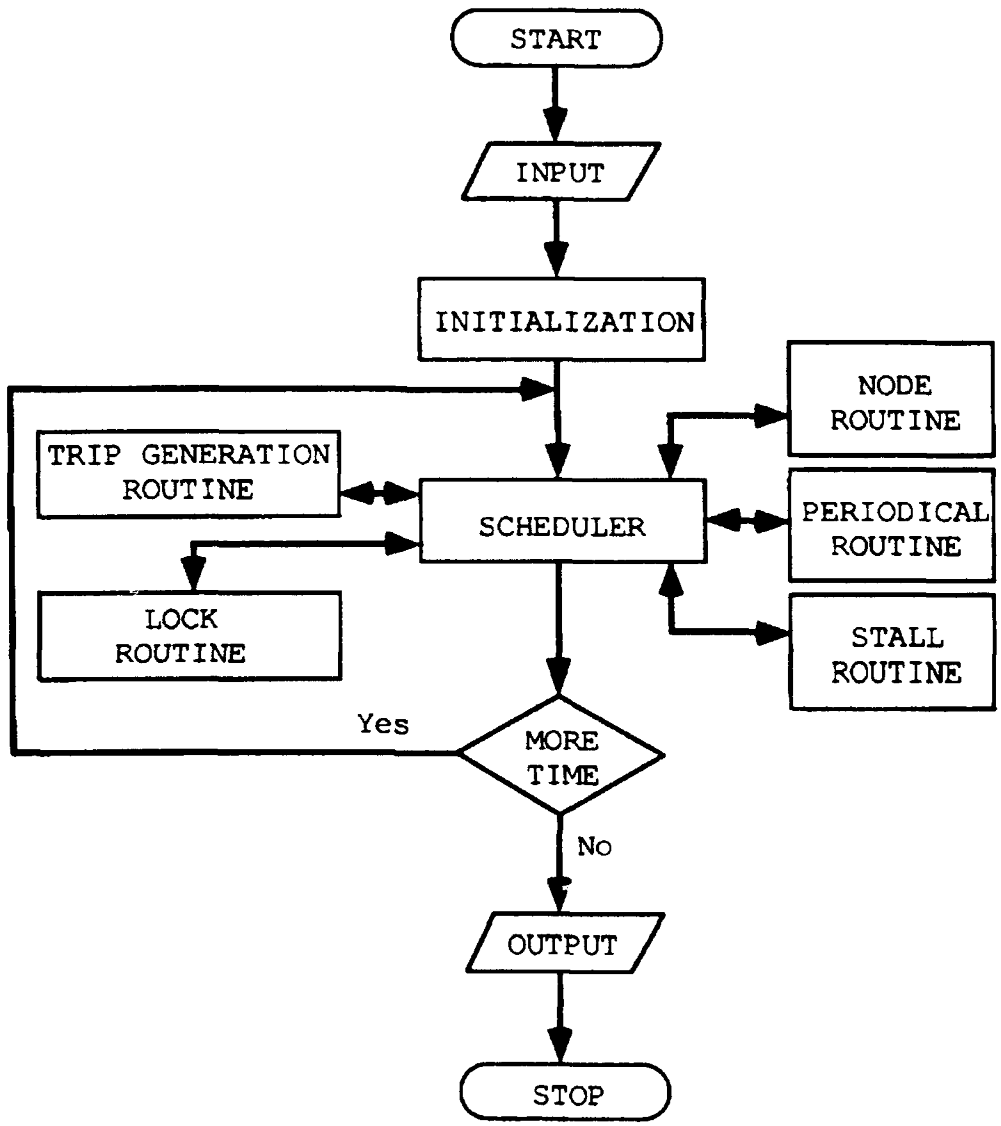

FIGURE 3. FLOW CHART OF SIMULATION MODEL

4. The main chamber is preferred for chamber assignment. This reflects the wish of lock operators to avoid the additional work and delays due to disassembling and re-assembling extra cuts through an auxiliary chamber.

5. The queue storage area is unlimited. Storage space is not a significant problem on waterways. However, the assumption requires modification when the simulation model or the numerical method (developed based on the results of the simulation model) is applied to other systems.

6. Tow speeds are normally distributed. For lack of better data, this assumption is based on waterway statistics of vessel performance [32].

7. Each tow maintains a constant speed between its origin and destination.

8. The average backhaul speed is a constant ratio of the average linehaul speed. The assumption reflects the effects of currents and barge loads.

9. The time intervals between two successive stalls and the durations of stalls are exponentially distributed.

10. The barge payload is the same for all coal barges and remains constant throughout a trip. This implies that the industry has a standard barge size and loads each barge to capacity.

11. There is an exogenously specified fraction of barges carrying coal on a coal tow. That is, a coal tow also carries other commodities. This assumption is also consistent with the barge industry operations.

12. The consumption rates at each utility plant are uniformly distributed within specified (and arbitrarily short) intervals.

Simulation Routines

The simulation model consists of five operation routines and one scheduler routine. The operation routines are associated with the five types of events: (1) tow trip generation (trip generation routine), (2) tow arrival at a lock (lock routine), (3) coal tow arrival at its destination (node routine), (4) periodical inventory and consumption updates (periodical routine), and (5) stall occurrence (stall routine). These operation routines are invoked by the scheduler routine.

Trip Generation Routine

Tow trips are generated stochastically, but the mean of their generating distribution is constant for each origin-destination (O-D) pair over each simulation time period. Each O-D pair has its own distribution for trip generation which may be represented in this model by a probability distribution table or a standard statistical distribution. The probability distribution tables represent cumulative distribution curves, where the abscissas represent cumulative frequency and the ordinates represent the ratio of the tabulated variable to its mean. The trip generation tables can be easily changed since they are specified explicitly in the input data.

Whenever a tow trip is generated, its size (numbers of barges per tow) and speed are also generated. This model assumes that each tow will maintain its size and speed throughout its entire trip. As in trip generation, the long run average tow size is constant for each O-D pair, but the size of each individual tow is generated stochastically. The distribution of tow sizes is represented by a probability distribution table and each O-D pair has its own distribution table. The tow size distribution table is an exogenous input data. Therefore, it flexibly accommodates any type of tow size distribution.

Tow speeds are specified as an input to the model in the form of a probability distribution. For lack of better data, the distribution of speeds is currently assumed to be normal [32]. This model currently assumes that tows maintain constant speeds between origins and destinations and that backhaul speeds are a constant ratio of linehaul speeds.

To avoid generating extreme speed values, a speed range is specified. Currently, if the generated speeds are lower than the 2.5 percentile speed or zero, or higher than the 97.5 percentile speed, these speeds must be regenerated.

Tow traffic is divided into coal traffic and non-coal traffic. Therefore, for the same O-D trips, there may be two different O-D pairs with different trip rates for coal traffic and for non-coal traffic, respectively. It is presently assumed that only a specified fraction of the barges on a coal tow are carrying coal. In addition, the randomly generated tow sizes for coal traffic are restricted by upper and lower limits to preclude extremely small or extremely large tows. The tow size, the fraction of barges carrying coal, and the barge payload determine the amount of coal delivered by a coal tow.

After the characteristics associated with the newly generated tow trip are determined, the trip generation routine also determines the next lock the tow must pass through and the tow's arrival time at this lock. The tow's arrival time at this lock is determined by its trip generation time and its travel time over the distance between the origin and this lock. Meanwhile, the trip generation routine also determines the arrival sequence at this lock for the tow and updates the next tow's arrival time at this lock. The information about next tow arrival time at this lock, the tow arrival sequence at this lock, and the associated tow characteristics (size, speed, travel direction) is recorded for reference by other routines.

Lock Routine

The lock routine is the most complex one in the simulation model. It performs the following tasks:

1. chamber assignment;
2. determination of tow entrance time;
3. computation of tow waiting time;
4. determination of chamber available time (tow departure time) for next tow;
5. computation of interarrival time between two successive tows;
6. computation of interdeparture time between two successive tows;
7. determination of next stop (a lock or the destination node) for the tow;
8. computation of travel time between this lock and the next stop; and,
9. determination of arrival sequence of the tow at the next stop.

The lock service discipline is currently First-In-First-Out in the lock routine. The lock routine serves the first tow based on the arrival sequence. Such an assumption simplifies the simulation model and is consistent with waterway operations on the Mississippi and Ohio Rivers.

The chamber assignment is based on the chamber available time, the expected chamber service time, and the lock selection bias factor. The chamber available time is either the departure time of previous tow or the stall ending time. The expected chamber service time depends on the number of cuts required for the tow moving through chamber. The lock selection bias factor reflects in equivalent time units the advantage of utilizing the main chamber.

The number of cuts is determined by cut sizes and tow sizes. The cut size is exogenous. Each chamber has an upper limit on cut size (number of barges that can be handled simultaneously), which determines the number of cuts required. A tow may be disassembled into different numbers of cuts at different lock chambers. Therefore, the lock routine needs to determine the required number of cuts at each chamber for each tow.

It is noted that the expected chamber service time used in chamber assignment is the average value of service time for the required cut numbers. This reflects the reality of operation. In fact, the lock operator would not know in advance the actual service time for the tow. The operator could only roughly estimate the average service time for the tow based on its size.

If a lock has dissimilar chambers in parallel, (main and auxiliary chambers are usually provided), it is currently assumed that the main chamber will be preferred, unless the additional wait time it requires (compared to the auxiliary chamber) exceeds a specified level. This lock selection bias factor reflects the additional work and delays required to break tows into more (and smaller) cuts, move them separately through an auxiliary chamber and then reassemble them. Bias factors have been estimated separately for various locks from empirical data. They can be easily modified since they are specified explicitly in the input data.

Once the chamber is assigned, the tow entrance time is also determined. The tow entrance time is either the tow arrival time or the available chamber time, whichever is greater. If the tow arrives before the chamber is available, the tow must wait in the queue storage area. Otherwise, the tow enters its assigned chamber immediately. The queue storage area is currently assumed to have unlimited capacity and to be adjacent to the lock.

For most applications the tow waiting time is the most important output of the simulation model. It is defined as the difference between a tow's arrival time and its entry time into the lock. Such tow waiting times may occur well before traffic levels approach lock capacity since tow arrivals and lock service times are not constant.

The available chamber time for the next tow is the tow departure time which depends on the entrance time and the lock service time of the current tow and on the possible occurrence of a stall. In general, without a stall occurrence, the tow departure time is defined as the sum of the tow entrance time and the required lock service time. If a stall occurs before the service is completed, the tow would be detained until the end of the stall. Therefore, the tow departure time is equal to the sum of the tow entrance time, the required lock service time, and stall duration if there is a stall.

Lock service times may be generated from a specified distribution table or a standard statistical distribution. The distribution table can directly reflect actually observed service times. Therefore, the model can be applied to any types of locks. Lock service times will be affected by lock improvements which are represented by smaller average lock service times or reduced service time variances. The average lock service times vary for different locks, chambers, and numbers of cuts.

Whenever a stall occurs before lock service is completed, the lock routine must update the stall event at this lock. That includes information on the stall occurrence time and duration.

The interarrival time between two successive tows is defined as the time interval between the current tow's arrival time and the arrival time of a previous tow. Similarly, the interdeparture time is defined as the time interval between the current tow's departure time and the departure time of a previous tow.

The lock routine also determines the next stop for the tow and the tow arrival time at the next stop. The next stop could be a lock or the destination of the tow. The tow arrival time at the next stop is determined by the tow departure time and tow travel time over the distance between this lock and the next stop. If the next stop is a destination and the tow is not a coal tow, the lock routine will let the tow leave the system; otherwise, the lock routine will determine the tow's arrival sequence at the next stop and update the next tow's arrival time at this stop in the meantime. The information about the next tow's arrival time at this stop, the tow arrival sequence at this lock, and the associated tow characteristics (size, speed, and travel direction) is recorded for reference by other routines.

Node Routine

Whenever a coal tow arrives at its destination, i.e., at a utility plant, the node routine updates the inventory status there. The node routine computes the cumulative deliveries, cumulative consumption, inventory level, and stock-out amounts and durations.

Cumulative deliveries are determined from initial inventory levels, delivery amounts, and arrival time at the destination. The initial inventory level is an exogenous parameter specified for each destination. The node routine can estimate several different cumulative deliveries associated with different initial inventory levels simultaneously. The delivered amount is determined from the barge payload and the number of arriving coal barges. Currently, the barge payload is assumed to be constant for all loaded barges and the number of coal barges is currently assumed to be a constant fraction of tow size but these assumptions are easy to modify. The coal barge fractions vary for different origin-destination pairs. Although coal barge fractions are constant throughout the simulated period, the amount delivered from each tow is not constant since tow sizes are randomly generated, as mentioned previously.

Cumulative consumption is a function of consumption rate and time. The mean consumption rate is constant for each utility plant during each simulation period. However, in the short run the consumption rate fluctuates randomly around its mean. Currently, the consumption rate is assumed to be uniformly distributed. The actual distribution is yet to be determined. In addition, this model assumes that the consumption rate stays constant during the unit time interval which is specified exogenously. The consumption rate is updated every time unit by the periodical routine. Therefore, cumulative consumption is a step-wise linear function over time whose slopes represent consumption rates.

Inventory levels are represented in this model by the difference between cumulative deliveries and cumulative consumption. Whenever inventory levels drop to negative values, the node routine computes stock-out amounts and durations for the analysis of total costs.

Periodical Routine

This model assumes that the consumption rate stays constant during the unit time interval which is specified exogenously. The periodical routine updates the consumption rate every time unit, and also updates the cumulative consumption, the inventory status, and the stock-out amount and duration for each utility plant in every time unit. There are three functional differences between the node routine and the periodical routine.

1. The periodical routine does not update the cumulative deliveries.

2. The periodical routine updates the cumulative consumption and the status of inventory level for each utility plant. However, the node routine updates those for only one utility plant at a time.

3. The periodical routine can update the consumption rate based on a seasonal factor. The node routine can only update that when a coal tow arrives its destination.

With the periodical routine, consumption and inventories are estimated more precisely than without it.

Stall Routine

Stalls are random failures during which chambers are not available to serve tows. Thus stalls tend to increase delays and service time variability. Stall characteristics differ among chambers and are defined in terms of durations and frequencies which depend on weather conditions and physical conditions at each chamber. Stalls are relatively rare compared to other events. Their occurrence is very difficult to predict. Currently, the model assumes that the time intervals between two successive stalls are exponentially distributed.

The stall routine updates the available chamber time and the next stall occurrence time and duration at this chamber. The available chamber time is defined as the sum of the stall occurrence time and its duration.

Scheduler Routine

The scheduler routine controls the simulation clock and invokes the five operation routines to process the necessary function in the simulation model. The procedures in the scheduler routine are as follows:

1. updating the next event occurrence time for each event type;

2. determining the next event occurrence time and event type among the five types of events;

3. moving the simulation clock further to the occurrence time of next event; and,

4. invoking the appropriate operation routine.

Input Requirements

Generally, four types of inputs are required to operate the simulation model:

1. those related to link and lock characteristics;

2. those related to traffic demand between origins and destinations;

3. those related to probability distributions; and,

4. those related to inventories and consumption.

Link and Lock Characteristics

The following kinds of information are needed for each link:

1. end nodes;

2. link length;

3. distances between the end nodes and the lock;

4. number of chambers;

5. average frequencies and durations of stalls;

6. maximum number of cuts at each chamber;

7. average service time for each number of cuts at each chamber;

8. maximum number of barges for each cut size at each chamber;

9. bias time for each auxiliary chamber; and,

10. random number seeds for lock service times.

Traffic Demand

Traffic demand in tows per day is expressed in the form of origin-destination (O-D) matrices by time periods. The lengths of time periods may be different and need to be specified. The required input data for traffic demand are as follows:

1. origin and destination nodes for each O-D pair;
2. average trip rate in tows per day for each O-D pair, each direction and each time period;
3. average number of barges per tow for each O-D pair;
4. fraction of coal barges in a tow for each coal O-D pair;
5. maximum and minimum limits of tow size for each coal O-D pair;
6. barge payload in short-tons;
7. speed distribution (mean and standard deviation);
8. ratio of backhaul speed to linehaul speed (full/empty or upstream/downstream); and,
9. durations of time periods.

Probability Distributions

Probability distributions are specified in this model for the following:

1. lock service times
2. trip generation
3. tow composition (barges per tow); and,
4. coal consumption at power plants.

The probability distribution tables represent cumulative distribution curves, where the abscissas represent cumulative frequency, and the ordinates represent the ratio of the tabulated variable to its mean. To reduce the input complexity, a specified number of equal intervals is currently used for any cumulative frequency distribution. Therefore, only the values on the ordinates associated with frequency intervals are required.

Inventories and Consumption

Initial inventory levels in short-tons for different nodes (utility plants) must be specified. In addition, consumption rates in short-tons per day are expressed in the form of node matrices by time period. The information on cumulative deliveries, cumulative consumption, and inventory levels, is provided for intervals whose duration in days must be specified.

Model Output

This model provides the following results:

1. mean and standard deviation of waiting time at each lock;
2. mean and standard deviation of the interarrival time distribution for each lock;
3. mean and standard deviation of interarrival time distribution for each lock and direction;
4. mean and standard deviation of interdeparture time distribution for each lock;
5. mean and standard deviation of interdeparture time distribution for each lock and direction;
6. total tow travel time (not including the queuing time, lockage time, and dwell time) in days;
7. mean and standard deviation of travel time in hours per tow between any lock and any adjacent node;
8. total tow travel distances in 1,000 miles;
9. average travel speed in miles per day;
10. average lock service time for each chamber and cut;
11. total number of tow trips for each O-D pair;
12. monthly cumulative deliveries, cumulative consumption, and inventory levels tables in 1,000 short-tons for different power plants;
13. cumulative deliveries, cumulative consumption, and inventory levels tables for specified intervals in 1,000 short-tons for different power plants;

14. graphs of cumulative deliveries and cumulative consumption by specified time intervals for different power plants; and,

15. graphs of inventory level by specified time interval for different power plants.

Validation

To check the logic of this simulation model, its results are first compared to theoretical (but very well established) results from queuing theory. This also checks the ability of the model to represent general stochastic distributions. The results of the model are then compared with observed data to demonstrate how closely the model represents real systems and verify its ability to simulate the special features of waterways.

The predicted waiting times by the simulation model at a single lock are compared with those obtained from queuing theory when arrivals are Poisson distributed and service times are generally distributed. The validation is conducted for a variety of volume/capacity (V/C) ratios ranging from 0.0471 to 0.8934. To reduce the variance of the output each result is obtained by averaging the output from 30 independent simulation runs. To insure results are compared for a steady state, each simulation run discards the first 10,000 tow waiting times and collects the next 12,000 values for computing the average waiting time. The results are shown in Table 1. They confirm that the simulated and theoretical average waiting times are extremely close. Such results verify that the overall mechanism of the simulation model is correct. They also show that generally distributed service times are generated satisfactorily in the simulation model. That is reassuring since the same logic is also used to generate generally distributed interarrival times for G/G/1 queues and, ultimately to develop metamodels for series of G/G/1 queues.

The simulation results are then compared with the observed data (from January 1987) at Locks 22, 24, 25, 26, and 27 on the Mississippi River. These particular five locks were selected mainly because they were considered especially critical to the entire network by the Corps of Engineers. Hence extensive data analysis and performance evaluations had already been conducted for these five locks.

At that time, Locks 22, 24, and 25 had single 600 ft long chambers. Locks 26 and 27 had two chambers (600 ft and 360 ft long at Lock 26, 1200 ft and 600 ft at Lock 27). It is noted that the simulation results for the five lock system were obtained simultaneously. The validation results

TABLE 1. COMPARISON OF THEORETICAL AND SIMULATED RESULTS FOR A SINGLE LOCK QUEUE (M/G/1)

V/C	T_A *1		T_S *2		W_{sim} *3	W_t *4	Devia. *5
	Avg (hr)	Var (hr^2)	Avg (hr)	Var (hr^2)	(hr)	(hr)	(%)
0.893	0.888	0.789	0.793	0.319	4.9516	5.0059	-1.09
0.755	0.888	0.789	0.670	0.227	1.5575	1.5522	0.34
0.566	0.888	0.789	0.503	0.128	0.4926	0.4935	-0.19
0.330	0.888	0.789	0.293	0.044	0.1082	0.1087	-0.46
0.047	0.888	0.789	0.042	0.001	0.00155	0.00156	-0.64

*1 T_A : interarrival times
*2 T_S : service times
*3 W_{sim} : average waiting times from simulation
*4 W_t : average waiting times from queuing theory
*5 Devia. : deviation which is defined as $(W_{sim}-W_t)/W_t*100\%$

TABLE 2. COMPARISON OF SIMULATED AND OBSERVED AVERAGE WAITING TIMES

Lock	W_{sim} *1 (min)	W_{obs} *2 (min)	Difference (min)	95% Confidence Interval
22	4.09	3.73	0.36	3.49
24	6.12	6.36	0.24	6.72
25	4.49	10.94	6.45	- *3
26	119.40	130.99	11.59	60.73
27	36.49	34.43	2.06	23.92

*1 W_{sim} : simulated average waiting times
*2 W_{obs} : observed average waiting times
*3 The comparison is not appropriate.

TABLE 3. COMPARISON OF CHAMBER VOLUMES

Lock	Chamber	Vol_{sim} *1 (tows/day)	Vol_{obs} *2 (tows/day)	Difference (tows/day)	95%C.I. *3
22	1	44.61	45	0.39	1.88
24	1	56.00	56	0.00	2.24
25	1	51.40	52	0.60	2.13
26	1	275.20	265	10.20	2.83
26	4	155.96	167	11.04	3.81
27	1	390.95	389	1.95	10.25
27	4	306.73	306	0.7	10.62

*1 Vol_{sim} : simulated volumes
*2 Vol_{obs} : observed volumes
*3 95%C.I.: 95% confidence interval based on t test

TABLE 4. COMPARISON OF CUT VOLUMES

Lock	Cham *1	Cuts *2	Vol_{sim} *3 (tows/day)	Vol_{obs} *4 (tows/day)	Difference (tows/day)	95% C.I. *5
22	1	1	30.35	31	0.65	1.47
22	1	≥2	14.26	14	0.26	-
24	1	1	39.76	40	0.24	1.76
24	1	≥2	16.24	16	0.24	-
25	1	1	35.88	37	1.12	1.65
25	1	≥2	15.52	15	0.52	0.97
26	1	1	75.46	74	1.46	2.04
26	1	≥2	199.74	191	8.74	3.25
26	4	1	137.25	147	9.75	2.87
26	4	≥2	18.71	20	1.29	-
27	1	1	390.95	389	1.95	10.25
27	4	1	269.05	265	4.05	5.59
27	4	≥2	37.67	41	3.33	5.76

*1 Cham : chamber
*2 "Cuts" : are subsets of barges into which tows are subdivided for passage through lock chambers
*3 Vol_{sim} : simulated volumes
*4 Vol_{obs} : observed volumes
*5 95% C.I. : 95% confidence interval based on t test.

are summarized in Tables 2, 3, and 4. Each result is averaged from 80 independent simulation runs. The initial condition for simulation is assumed to be an empty system,which is consistent with the observed condition for this system in winter.

Table 2 shows that the difference between the simulated and observed average waiting times for each lock are within the 95% confidence interval based on the t test, except at Lock 25. The observed data also show that tows sometimes were kept waiting at Lock 25 even when the chamber was idle. Such operation is somewhat unusual. Therefore, no direct comparison of average waiting times at Lock 25 is appropriate. Tables 3 and 4 also show that the simulation model represents the real system quite well.

Each simulation run takes from a few seconds to a few minutes on a personal computer, depending on traffic volumes, duration of simulation periods, network size, and other factors. Despite that, simulation time becomes expensive for evaluating large combinatorial investment scheduling problems. For example, when there are n=20 possible investment projects, it is necessary to simulate 2^{20} combinations to make the best decision. $30*2^{20}$ separate simulation runs are then required if each performance measure is based on the average over 30 independent replications. Furthermore, the project combinations may have to be evaluated over several time periods. Therefore, as n increases, direct evaluation by simulation becomes very expensive.

A metamodeling approach is proposed to overcome the computational requirements of simulation. A simulation model can then be treated as a function with unknown explicit form that turns input parameters into output performance measures. The metamodeling approach provides a method to develop simple formulas to approximate this function.

CHAPTER 4
NUMERICAL METHOD

Methodology

In this study, a numerical method has been developed for estimating delays through a series of queues. This method was originally developed for systems with bi-directional servers. With a few simplifications, this method can be adapted for the more generally encountered systems with one-directional servers.

The numerical method decomposes large systems into single lock queuing stations and then analyzes the interarrival and interdeparture-time distributions at theses single locks one by one. Although the numerical method is a decomposition procedure, each lock is not analyzed independently. The interarrival-time distribution at one lock is affected by the interdeparture-time distributions at adjacent locks.

The method consists of three major modules: arrival processes, departure processes, and delay functions. Arrival processes at a particular lock depend on the interdeparture time distributions from the upstream and downstream locks. The departure processes depend on the interactions among the interarrival-time distributions and service-time distributions at one lock. The delay functions define the relations among waiting times, interarrival-time distributions, interdeparture-time distributions and service-time distributions. The basic concept of this method is to identify two parameters, namely the mean and coefficient of variation, of the interarrival and interdeparture-time distributions for each lock, and then estimate the implied waiting times. Currently, the following assumptions are used in the numerical method.

1. Interarrival times and service times are generally (i.e., arbitrarily) distributed.

2. Each lock has one chamber.

3. Inflows and outflows occur only at the two end-nodes of a series of locks.

4. The average upstream volumes are equal to the downstream volumes.

5. The volume-to-capacity ratio (ρ) is less than 1.0 at every lock.

It should be noted that Assumptions 2, 3, 4, and 5 are only applicable to the numerical method. The simulation model is not limited by those assumptions. The numerical method can provide a quick and inexpensive approach for the analysis of lock delays. However, Assumptions 2, 3, 4, and 5 limit the applicability of the currently developed numerical method and necessitate the substitution of the simulation model when significant deviations from those assumptions must be considered. With some extensions to the numerical method expected in the near future, Assumptions 2 and 3 may be eliminated. Assumption 4 could be relaxed even though it is usually realistic for waterways.

Each module of the numerical method consists of one or more metamodels that are functional relations whose parameters are statistically estimated from simulation results. Thus, simulation is not just used to validate these metamodels; it is the basis for their development.

Two simulation experiments were conducted in this study. Experiment 1 served two goals here, namely the development of metamodels and the validation of these metamodels. A split-sample analysis was conducted to achieve both of the above goals in a single experiment. The split-sample analysis proceeded as follows. First, the collected data points from Experiment 1 were grouped in strata based on one or more important categorical variables (This method is called a stratum-specific split-sample scheme [20]). Second, all data points within a stratum were randomly assigned to one of two groups, namely the training group (used to develop metamodels) and the holdout group (used to validate metamodels and to test their reliability). Such assignment was done separately for each stratum. The goal of stratum-specific random assignment is to insure that the two groups of data (training and holdout) are equally representative of the parent population [20]. In this study, a computer randomly assigned 2/3 of data to the training group and 1/3 of data to the holdout group. It is noted that the above two steps were conducted before any data were analyzed.

Experiment 2 aimed to validate the numerical method against the simulation model. The ranges of variables in these experiments were carefully chosen to reflect the ranges encountered in waterway systems.

The procedures used in developing each metamodel are summarized below.

1. Queuing theory was applied to identify the input (independent) variables which will affect the output measures (dependent variables), and to propose

variables and the output measures with appropriate structure form.

2. The relevant output measures (dependent variables) were plotted versus the input (independent) variables. This step helped confirm the relations between the output measures and the input parameters.

3. Pearson correlation coefficients were computed. This step confirmed correlations between selected dependent variables and the independent variables and helped avoid multicollinearity problems among the independent variables.

4. The parameters were estimated for the proposed functional relations ("metamodels"). The selection of the preferred metamodel was based mainly on the sample squared multiple correlation (R^2), the standard error of residuals, and the residual analysis. The R^2 defines the explanatory power of the alternative metamodels. In general, the metamodel with R^2 closest to 1 is preferred since it best accounts for the variation of dependent variables. It is also important to examine if the independent variable is significant in explaining the variation of dependent variable.

5. Residual analysis was performed to detect outliers and to check whether any metamodel violated certain regression assumptions, such as normality and homoscedasticity. The residual analysis should include the property-analysis and the graphical analysis of residuals. The basic residual properties to be examined include the mean, variance, skewness, and kurtosis. The residual mean should equal 0. The variance is the residual mean square. Skewness indicates the degree of asymmetry of a distribution; it should be close to 0 to avoid violating the normality assumption. Kurtosis indicates the heaviness of the tails relative to the middle of the distribution; it should be close to 3 to avoid violating the normality assumption. (The commercial statistical software usually deducts 3 from the value of kurtosis. Therefore, the kurtosis should be close to 0 if it is obtained from commercial statistical software.) However, skewness and kurtosis statistics are highly variable in small samples and hence are often difficult to interpret [20]. The graphical analysis is the most direct and revealing way to examine a set of residuals. The residuals

can be displayed in one or two dimensions. The useful one-dimensional plot of residuals includes histograms (or stem-and-leaf), schematic plots, and normal probability plots [20]. The two-dimensional plot examines the relationships of the residuals to either dependent or independent variables and is sometimes useful for identifying violation of regression assumptions [20].

Structure of Numerical Method

Basically, the numerical method is a decomposition model. It decomposes systems of queues into separate queuing stations. The analysis of each queuing station is decomposed into three steps, namely arrival processes, departure processes and delays.

The arrival processes module serves two functions. First, for each direction at each lock it identifies the interarrival-time distribution based on the interdeparture-time distribution from the previous lock and the intervening speed distribution. Second, the overall interarrival-time distribution at each lock is estimated by combining the interarrival-time distributions from the two adjacent locks. The arrival processes module is very important in identifying and taking into account the interdependence among locks, even when the system is decomposed into individual lock queuing stations.

The departure processes module also serves two functions. First, the overall interdeparture-time distribution at each lock is estimated based on the overall interarrival-time distribution and the lock's service-time distribution. Second, the overall interdeparture-time distribution at each lock is split into directional interdeparture-time distributions. Thus, at each lock, the departure processes module estimates two output distributions (interdeparture times for two directions) based on three input distributions (interarrival times for two directions and bi-directional service times).

The delay module estimates average waiting time at each lock. The inputs for the delay function are the variances of the interarrival times, interdeparture times, and service times. While the service-time distributions at each lock are exogenously specified inputs, the parameters of the interarrival-time and interdeparture-time distributions are obtained from the arrival processes module and the departure processes module, respectively.

A scanning procedure is employed in the numerical method to analyze successive locks. Interrelations among the locks in a system are accounted for since the departure distribution from one lock influences the arrival distribution at the next lock. In the initial scan along one direction, no endogenous estimate is yet available of arrival distributions from the opposing direction (unless there is only one-way traffic). If there is opposing traffic, some initializing estimates must be provided for the variance of interdeparture times from the opposing direction. The scans are then repeated in an iterative algorithm which alternates scanning direction. After the first iteration, the initializing estimates are replaced with endogenous estimates from previous iterations. The iterative scans continue until convergence is achieved in values of variables such as wait times or interdeparture-time variances.

Sampling Plan

The data points in Experiment 1 were collected from 40 different simulated lock systems. Each system consisted of three locks. The trip rate differed for each lock system and remained constant throughout the data collection period. Therefore, the data points in Experiment 1 represent the characteristics of 40 different trip rates. These 40 different trip rates were generated uniformly in the range between 0.0417 tows/hr (1 tow/day) and 1.6667 tows/hr (40 tows/day). The range was selected to cover all possible levels empirically found on waterways.

The three locks within any system differed from each other, in terms of their Volume to Capacity (V/C) ratio and the squared coefficient of variation of service times. Therefore, the data points in Experiment 1 represent the characteristics of 120 V/C ratios and 120 squared coefficients of variation of lock service times. The V/C ratios were generated uniformly between 0.04 to 0.97, covering all observed waterway operations. The squared coefficients of variation of lock service times were generated uniformly between 0.34 to 0.81, covering almost all waterway operations.

The distances between locks and the tow speed distributions were also different in each lock system. The distances were generated uniformly between 0 to 156 miles. The average tow speeds were uniformly distributed between 72 to 295 miles/day. The standard deviations of tow speeds were uniformly distributed between 2.22 to 136.3 miles/day. The ranges of distances and speed distributions were selected to cover most waterway situations of practical interest.

At each lock, the collected characteristics for its corresponding data point include the directional interarrival-time distributions, the directional interdeparture-time distributions, the overall interarrival-time distribution, the overall interdeparture-time distribution, and the average waiting time. The corresponding trip rate, V/C ratio, lock service time distribution, distances, and tow speed distribution were also recorded. Therefore, the database in Experiment 1 contains 120 different average waiting times, 120 different overall interarrival-time distributions, 120 different overall interdeparture-time distributions, 240 different directional interarrival-time distributions, and 240 different directional interdeparture-time distributions.

To obtain the above data, each lock system was simulated with 30 independent replications. To ensure that the results were for a steady state, each simulation run discarded the first 10,000 observations and collected the next 12,000 values for computing the data characteristics. 10,000 observations were discarded based on moving average analysis [33]. In addition, the random number seeds were all 200,000 apart, which ensured the random number streams would be independent since each independent replication discarded 10,000 observations and collected the next 12,000 values. Therefore, the average waiting time at each lock was obtained by the following steps.

1. Thirty independent replications of the simulation were made for each lock system, each generating an output sequence of 22,000 tows. Hence, a set of observations were obtained:

   W_{mn}= n-th member of the output (tow waiting time) sequence from the m-th independent replication of the simulation, m=1,... 30, n=1,... 22,000.

2. The sample mean of the average waiting time was obtained from each replication [33].

$$W_m = \frac{1}{12,000} \sum_{n=10,001}^{22,000} W_{mn} \tag{9}$$

3. The unbiased estimate of the average waiting time W is estimated as follows [33]:

$$W = \frac{1}{30} \sum_{m=1}^{30} W_m \tag{10}$$

The variances of overall interarrival-time distributions were obtained by the following five steps.

1. A set of observations were obtained from 30 independent simulation replications:

   $t_{Amn}=$ n-th member of the overall interarrival-time sequence from the m-th independent replication of the simulation, m=1,...30, n=1,...22,000.

2. The sample mean of the overall interarrival times was obtained on each replication [33]:

$$t_{Am} = \frac{1}{12,000} \sum_{n=10,001}^{22,000} t_{Amn} \tag{11}$$

3. The estimate of the average interarrival time was obtained as follows [33]:

$$t_A = \frac{1}{30} \sum_{m=1}^{30} t_{Am} \tag{12}$$

4. The variance estimate was obtained for each replication [33]:

$$s_{Am}{}^2 = \frac{1}{12,000} \sum_{n=10,001}^{22,000} (t_{Amn} - t_A)^2 \tag{13}$$

5. The variance of overall interarrival times was obtained as follows [33]:

$$\sigma_A{}^2 = \frac{1}{30} \sum_{m=1}^{30} s_{Am}{}^2 \tag{14}$$

The variances of overall interdeparture-time distributions, directional interarrival-time distributions, and directional interdeparture-time distributions also were obtained following the above procedures.

Arrival Processes

The following two steps were used for estimating the mean and standard deviations of interarrival times.

Step 1

Estimate the means and standard deviations of directional interarrival times at a particular lock from the directional interdeparture-time distributions of the adjacent upstream and downstream locks.

If flows are conserved between locks and if the V/C ratio is less than 1, the average directional arrival rates at one lock should be equal to the average directional departure rates from adjacent upstream and downstream locks. Therefore, the average directional interarrival times at that lock should also be equal to the average directional interdeparture times from adjacent upstream and downstream locks. Such relations are represented by:

$$\bar{t}_{aji} = \bar{t}_{djk} \quad \begin{cases} k=i-1, \text{if } j=1 \\ k=i+1, \text{if } j=2 \end{cases} \tag{15}$$

where

$\bar{t}_{aji}$: the average interarrival time for Direction j and Lock i

$\bar{t}_{djk}$: the average interdeparture time for Direction j and Lock k

j : direction index (1 = downstream, 2 = upstream)

If each tow moves at the same speed, the directional interarrival time distributions at one lock will be identical to the directional interdeparture time distributions at the preceding lock. However, speed variations change headway distributions along the distance between locks.

A metamodel is developed to estimate the standard deviation of directional interarrival times at one lock. Before developing the metamodel, the database obtained in Experiment 1 was split into two groups of data (training and holdout). The stratum-specific split-sample scheme was employed for data assignment. For developing the metamodel, 4 data points were available from each of the 40 lock systems (downstream: the relation between Locks 1 and 2, and the relation between Locks 2 and 3; upstream: the relation between Locks 3 and 2, and the relation between Locks 2 and 1). Therefore, a total of 160 data points was available from Experiment 1 for metamodel development. The split ratio between the training and holdout groups is approximately 2:1. Therefore, 107 observations were included in the training group and 53 observations were included in the holdout group.

The following metamodel was estimated with data from the training group is as follows:

$$\sigma_{aji} = \sigma_{djk} + \underset{(0.002)}{0.0251} \ln\left(1 + \frac{D_{ik}\sigma_{vik}}{\mu_{vik}}\right) \quad \begin{cases} k = i-1, \text{ if } j=1 \\ k = i+1, \text{ if } j=2 \end{cases} \tag{16}$$

$R^2 = 0.999954$ $n = 107$ $s_e = 0.0586$ $\mu_y = 5.168561$

where

σ_{aji} : standard deviation of interarrival times for Direction j and Lock i

σ_{djk} : standard deviation of interdeparture times for Direction j and Lock k

D_{ik} : distance between Locks i and k

μ_{vik} : average tow speed between Locks i and k

σ_{vik} : standard deviation of tow speeds between Locks i and k

j : direction index (1 = downstream, 2 = upstream)

S_e : standard error of dependent variable

μ_y : mean of dependent variable

The numbers shown in the parentheses are the standard errors of the estimated parameters directly above.

Currently, there is less theoretical basis for this metamodel than for the other metamodels developed in this study. This metamodel was developed largely by empirical analysis. The dependent variable (standard deviation of directional interarrival-time distribution) was plotted versus possible influential factors that include the standard deviation of directional interdeparture-time distribution, distance between two locks, average tow speed, and standard deviation of tow speeds. The correlation coefficients between the dependent variable and influential factors were also computed. The plot and the correlation coefficient show that the standard deviation of the directional interarrival-time distribution is highly correlated to the standard deviation of the directional interdeparture-time distribution. However, the plot and the correlation coefficient also show that the standard deviation of the directional interarrival-time distribution is also affected by the factors of distance, average tow speed, and standard deviation of tow speeds. Therefore, various structural forms were considered for this metamodel, and two were intensively pursued.

The first metamodel for σ_a includes only one independent variable (the standard deviation of the directional interdeparture-time distribution). The second metamodel for σ_a also considers the distance, average tow speed, and standard deviation of tow speeds as influential factors. The R^2 for the first metamodel is 0.999885. The descriptive statistics and test results for the first metamodel are included in Table 5.

The structural form of the second metamodel was preferred since it satisfies an important logical constraint. The standard deviation of the directional interarrival times should be equal to the standard deviation of the directional interdeparture times from the preceding lock when the distance between these two locks is 0 or when the standard deviation of tow speeds is 0. The chosen second type metamodel (Eq. 16) does satisfy those logical constraints.

It is noteworthy that in Eq. 16, the coefficient for the standard deviation of directional interdeparture times from the preceding lock is equal to 1 and the intercept is 0. In fact, in the first version of this metamodel, the intercept was 0.00526 with a standard error of 0.021 which indicated that the intercept was not significant. The coefficient for the standard deviation of directional interdeparture times was 0.999, or approximately equal to 1. This suggested that, theoretically, the standard deviation of directional interarrival times should be equal to the standard deviation of directional interdeparture times plus an adjustment factor depending on the speed distribution and distance. Therefore, this metamodel was redeveloped under two restrictions: (1) the intercept should be 0 and (2) the coefficient for the standard deviation of directional interdeparture times should be 1. The descriptive statistics and test results for the second type metamodel are listed in Table 6.

The comparison between Tables 5 and 6 shows that the second metamodel does predict better the standard deviation of directional interarrival-time distribution. The R^2 of the second metamodel is slightly higher. However, the statistics of ε and $\varepsilon\%$ of the second metamodel are much better. The mean of ε is 0.04 for the second and 0.07 for the first. The mean of $\varepsilon\%$ is 1.68 for the second and

TABLE 5. DESCRIPTIVE STATISTICS AND TEST RESULTS FOR THE TRAINING GROUP, FIRST METAMODEL FOR σ_a

Variable	Mean	S.D.*	Min	Max
σ_{aji}	5.17	8.61	1.17	49.57
σ_{djk}	5.10	8.67	1.03	49.57
ε**	0.07	0.07	0.00041	0.29
ε%***	3.17	3.02	0.0016	12.66
n^{+} = 107, R_1^{2++} = 0.999885				

*S.D. : standard deviation
**ε : absolute error of σ_{aji} between simulation and metamodel
***ε% : percentage error of σ_{aji} between simulation and metamodel
+n : number of observations
++R_1^2 : the coefficient of determination

TABLE 6. DESCRIPTIVE STATISTICS AND TEST RESULTS FOR THE TRAINING GROUP, SECOND METAMODEL FOR σ_a

Variable	Mean	S.D.*	Min	Max
σ_{aji}	5.17	8.61	1.17	49.57
σ_{djk}	5.10	8.67	1.03	49.57
D_{ik}	77.83	46.65	0.00	156.00
μ_{vik}	169.00	56.25	72.00	295.00
σ_{vik}	47.54	32.30	2.22	136.30
ε**	0.04	0.04	0.00046	0.21
ε%***	1.68	1.57	0.0016	7.60
n^{+} = 107, R_1^{2++} = 0.999954				

*S.D. : standard deviation
**ε : absolute error of σ_{aji} between simulation and metamodel
***ε% : percentage error of σ_{aji} between simulation and metamodel
+n : number of observations
++R_1^2 : the coefficient of determination

3.17 for the first. The maximum ε is 0.21 for the second and 0.29 for the first. The maximum ε% is 7.6 for the second and 12.66 for the first. Therefore, the second metamodel was chosen for predicting the standard deviation of the directional interarrival times.

The holdout group data was used to test the reliability of this metamodel, which is used to predict the standard deviation of directional interarrival times. It is assumed in this test that the values obtained from the simulation model are exact. Therefore, the deviations between the values obtained from simulation and metamodel are considered errors. The percentage error is defined as the error divided by the value obtained from simulation. In this test, the absolute values are used to compute errors and percentage errors. The use of this metamodel is to predict the standard deviation of directional interarrival times for the holdout group constitutes cross-validation. The descriptive statistics and test results for the holdout group are listed in Table 7.

The shrinkage on cross-validation, defined as $R_1^2 - R_2^2$, indicates the reliability of metamodels. In general, shrinkage values less than 0.10 are indicative of a reliable model [20]. Therefore this metamodel is reliable since the shrinkage is 0.000001. In addition, the absolute error (mean=0.04 and maximum value=0.17) and absolute percentage error (mean=1.74 and maximum value=7.76) in Table 7 also show this metamodel predicts the standard deviation of directional interarrival times well. However, the maximum ε% is 7.6 for the training group and 7.76 for the holdout group, which indicates that the directional arrivals metamodel sometimes generates considerable errors and leaves room for improvement.

Step 2

The mean and coefficient of variation of the overall interarrival-time distribution for this lock are estimated based on the coefficient of variation of directional interarrival times. The coefficient of variation of directional interarrival times could be obtained by dividing the standard deviation of directional interarrival-time distribution with its mean.

$$\bar{t}_{Ai} = \frac{\bar{t}_{a1i} * \bar{t}_{a2i}}{\bar{t}_{a1i} + \bar{t}_{a2i}} \qquad \textbf{(17)}$$

$$\underset{(0.027)\quad(0.014)}{C_{Ai}^2 = 0.179+0.41(C_{a1i}^2 + C_{a2i}^2)} \qquad \textbf{(18)}$$

$R^2 = 0.9188$ $\quad$ $n = 79$ $\quad$ $s_e = 0.0059$ $\quad$ $\mu_y = 0.988$

where

$\bar{t}_{Ai}$: the average interarrival time at Lock i

c_{Ai}^2 : squared coefficient of variation of interarrival times at Lock i

c_{aji}^2 : squared coefficient of variation of directional interarrival times for Direction j and Lock i

The meaning of Eq. 17 would be clearer if viewed in terms of the average trip rates rather than the average interarrival times. Eq. 17 implies that the overall arrival rate at certain lock is the sum of the average directional arrival rates from upstream and downstream.

Before developing the metamodel of Eq. 18, the stratum-specific split-sample scheme was also employed to split the database of Experiment 1 into the two groups (training and holdout). For developing the metamodel, there were 120 observations available from the database of Experiment 1. Each observation was obtained from one individual lock and included the characteristics of overall interarrival-time distribution and corresponding directional interarrival-time distributions at one lock. The split ratio between the training and holdout groups was approximately 2:1. Therefore, the computer randomly assigned 79 observations to the training group and 41 observations to the holdout group.

In Eq. 18, the squared coefficients of variations of upstream and downstream interarrival times carry the same weight in estimating the overall variance of interarrival times, since directional trip rates are equal according to Assumption 4. Eq. 18 should be reestimated when applied to a more general network of queues with imbalanced flows.

The data in the holdout group was also used to validate the reliability of this metamodel and the cross-validation test was again conducted. The descriptive statistics and test results for the training group and the holdout group are listed in Tables 8 and 9.

The shrinkage (R_1^2-R_2^2) in this metamodel is 0.0282 which indicates that this metamodel is fairly reliable. This becomes more convincing after examining the absolute error and absolute percentage error in the cross-validation test. The absolute error ranges from 0.00022 to 0.03 with a mean of 0.007. The absolute percentage error ranges from 0.022 to 3.26 with a mean of 0.70.

TABLE 7. DESCRIPTIVE STATISTICS AND TEST RESULTS FOR THE HOLDOUT GROUP (σ_a)

Variable	Mean	S.D.*	Min	Max
σ_{aji}	5.04	7.82	1.23	49.10
σ_{djk}	4.98	7.84	1.08	49.10
D_{ik}	78.34	45.64	0.00	152.00
μ_{vik}	171.56	60.09	79.20	295.00
σ_{vik}	45.78	34.54	2.22	136.30
ε^{**}	0.04	0.04	0.00004	0.17
$\varepsilon\%^{***}$	1.74	1.81	0.0027	7.76
$n^{+} = 53$, $R_2^{2++} = 0.999953$				

*S.D. : standard deviation
**ε : absolute error of σ_{aji} between simulation and metamodel
***$\varepsilon\%$: percentage error of σ_{aji} between simulation and metamodel
+n : number of observations
++R_2^2 : the coefficient of determination

TABLE 8. DESCRIPTIVE STATISTICS AND TEST RESULTS FOR THE TRAINING GROUP (C_A^2)

Variable	Mean	S.D.*	Min	Max
C_{Ai}^2	0.99	0.02	0.89	1.02
C_{a1i}^2	0.98	0.04	0.74	1.02
C_{a2i}^2	0.99	0.03	0.84	1.02
ε^{**}	0.004	0.004	0.00007	0.02
$\varepsilon\%^{***}$	0.46	0.40	0.0066	1.79
$n^{+} = 79$, $R_1^{2++} = 0.9188$				

*S.D. : standard deviation
**ε : absolute error of C_{Ai}^2 between simulation and metamodel
***$\varepsilon\%$: percentage error of C_{Ai}^2 between simulation and metamodel
+n : number of observations
++R_1^2 : the coefficient of determination

TABLE 9. DESCRIPTIVE STATISTICS AND TEST RESULTS FOR THE HOLDOUT GROUP (C_A^2)

Variable	Mean	S.D.*	Min	Max
C_{Ai}^2	0.98	0.03	0.90	1.02
C_{a1i}^2	0.98	0.04	0.83	1.02
C_{a2i}^2	0.98	0.04	0.79	1.02
ε**	0.007	0.007	0.00022	0.03
ε%***	0.70	0.74	0.022	3.26
$n^+ = 41$, $R_2^{2++} = 0.8906$				

*S.D. : standard deviation
**ε : absolute error of C_{Ai}^2 between simulation and metamodel
***ε% : percentage error of C_{Ai}^2 between simulation and metamodel
+n : number of observations
++R_2^2 : the coefficient of determination

Departure Processes

The departure-processes module estimates the mean and squared coefficient of variation of interdeparture times. Based on the flow conservation law, if the V/C ratio is less than 1, the mean outflow rate should be equal to the mean inflow rate. Therefore, the average directional interdeparture time can be determined from the corresponding interarrival time:

$$\bar{t}_{dji} = \bar{t}_{aji} \tag{19}$$

The coefficient of variation of interdeparture times is estimated in two steps.

__Step 1__

The coefficient of variation of interdeparture times is first estimated with the two directions combined. Departure processes with generally distributed arrivals and service times are analyzed by using Laplace transforms. The use of Laplace transforms for derivations in queuing theory (which is quite frequent) is presented in texts such as Kleinrock [21]. Some analytic relations obtained in this dissertation are shown below using the following notation:

Let:

$\bar{t}_A, \sigma_A^2$: mean and variance of interarrival times

$\bar{t}_S, \sigma_S^2$: mean and variance of lock service times

$\bar{t}_D, \sigma_D^2$: mean and variance of interdeparture times

$\bar{t}_I, \sigma_I^2$: mean and variance of lock idle times

ρ : V/C ratio

C_A^2, C_S^2, C_D^2 : squared coefficients of variation for interarrival times, service times, and interdeparture times

$f_A(t)$, $f_S(t)$, $f_D(t)$, $f_I(t)$: probability density functions (pdf) for interarrival times, lock service times, interdeparture times, and lock idle times

$F_A^*(z)$, $F_S^*(z)$, $F_D^*(z)$, $F_I^*(z)$: Laplace transforms for $f_A(t)$, $f_S(t)$, $f_D(t)$, $f_I(t)$

For example, for interarrival times, the Laplace transform is expressed as

$$F_A^*(z) = \int_0^\infty f_A(t) e^{-zt} \, dt \qquad \textbf{(20)}$$

The departure process in a queuing station may be analyzed for two different conditions: with and without a queue. The interdeparture-time distribution would be equal to the service-time distribution while there are queues waiting for service. However, the interdeparture time would be equal to the sum of the idle time and the service time while there is no queue. Therefore, the Laplace transforms for the interdeparture time distributions can be represented as follows:

$$F_D^*(z)\big|_{with\ queue} = F_S^*(z) \qquad \textbf{(21)}$$

$$F_D^*(z)\big|_{without\ queue} = F_I^*(z) F_S^*(z) \qquad \textbf{(22)}$$

The probability of having a queue is given by the volume/capacity ratio ρ [21]. Then, the probability of not having a queue is $(1-\rho)$. Therefore, the Laplace transform for the interarrival-time distribution can be represented by Eq. 23:

$$F_D^*(z) = (1-\rho)\,F_D^*(z)\,|_{without\ queue} + \rho F_D^*(z)\,|_{with\ queue} \\ = (1-\rho)\,F_I^*(z)\,F_S^*(z) + \rho F_S^*(z) \qquad \textbf{(23)}$$

The mean of a distribution can be represented by the negative value of the first derivative of its Laplace transform when z equals 0. Therefore, the average interarrival time, service time, interdeparture time, and idle time can be represented by Eqs. 24a to 24d:

$$\bar{t}_A = -\frac{\partial F_A^*(z)}{\partial z}\Big|_{z=0} \qquad \textbf{(24a)}$$

$$\bar{t}_S = -\frac{\partial F_S^*(z)}{\partial z}\Big|_{z=0} \qquad \textbf{(24b)}$$

$$\bar{t}_D = -\frac{\partial F_D^*(z)}{\partial z}\Big|_{z=0} \qquad \textbf{(24c)}$$

$$\bar{t}_I = -\frac{\partial F_I^*(z)}{\partial z}\Big|_{z=0} \qquad \textbf{(24d)}$$

The variance can be expressed as the difference between the second derivative of the Laplace transform and the squared mean. Eqs. 25a-25d express such relations for interarrival time, service time, interdeparture time, and idle time distributions:

$$\sigma_A^2 = \frac{\partial^2 F_A^*(z)}{\partial z^2}\Big|_{z=0} - \bar{t}_A^{\,2} \qquad \textbf{(25a)}$$

$$\sigma_S^2 = \frac{\partial^2 F_S^*(z)}{\partial Z^2}\Big|_{z=0} - \bar{t}_S^{\,2} \qquad \textbf{(25b)}$$

$$\sigma_D^2 = \frac{\partial^2 F_D^*(z)}{\partial z^2}\Big|_{z=0} - \bar{t}_D^{\,2} \qquad \textbf{(25c)}$$

$$\sigma_I^2 = \frac{\partial^2 F_I^*(z)}{\partial z^2}\Big|_{z=0} - \bar{t}_I^{\,2} \qquad \textbf{(25d)}$$

When z equals 0, the Laplace transform is equal to 1, producing the following relations for interarrival-time, service-time, interdeparture-time, and idle-time distributions:

$$F_A^*(0)=1 \tag{26a}$$

$$F_S^*(0)=1 \tag{26b}$$

$$F_D^*(0)=1 \tag{26c}$$

$$F_I^*(0)=1 \tag{26d}$$

Combining Eqs. 23, 24b, 24c, 24d, 26b, and 26d yields

$$\overline{t}_D = -\frac{\partial F_D^*(z)}{\partial z}\Big|_{z=0} = -((1-\rho)(-\overline{t}_I-\overline{t}_S)+\rho(-\overline{t}_S)) \tag{27}$$

Due to flow conservation, if the V/C ratio is less than 1, then the average interdeparture time would be equal to the average interarrival time:

$$\overline{t}_D = \overline{t}_A \tag{28}$$

Therefore, Eqs. 27 and 28 can be combined:

$$\overline{t}_D = (1-\rho)(\overline{t}_I+\overline{t}_S)+\rho\overline{t}_S = \overline{t}_A \tag{29}$$

Since $\rho = t_S/t_A$, Eq. 29 yields

$$\overline{t}_I = \overline{t}_A \tag{30}$$

Combining Eqs. 23, 24b, 24d, 25b, 25c, 25d, 26b, 26c, 26d, and 30 yields

$$\sigma_D^2 = \frac{\partial^2 F_D^*(z)}{\partial z^2}\Big|_{z=0}-\overline{t}_D^2 = (1-\rho)\sigma_I^2+\sigma_S^2+(\overline{t}_A-\overline{t}_S)\overline{t}_S \tag{31}$$

Dividing Eq. 31 by t_D^2, we can obtain the following relation for the squared coefficient of variation C_D^2:

$$C_D^2 = (1-\rho)\frac{\sigma_I^2}{\overline{t}_A^2}+\rho+C_S^2\rho^2-\rho^2 \tag{32}$$

In the special case where the arrival process is Poisson distributed and the service times are exponentially distributed, then due to the memoryless property of the

Poisson distribution, the variance of idle times would be equal to the variance of interarrival times. Since the interarrival times for a Poisson process are exponentially distributed and since the mean and standard deviation of an exponential distribution are equal, we can state the following:

$$\sigma_I^2 = \sigma_A^2 = \overline{t}_A^2 \tag{33}$$

$$\sigma_S^2 = \overline{t}_S^2 \tag{34}$$

Therefore, in this special case, Eq. 31 can be simplified to

$$\sigma_D^2 = \sigma_A^2 \tag{35}$$

which is consistent with Burke's theorem [11]. In that theorem Burke proved that, when the arrivals are Poisson distributed and the service times are exponentially distributed, then the departures must be Poisson distributed with the same mean and variance as the arrivals.

The main difficulty in estimating the squared coefficient of variation of interdeparture times (Eq. 32) when arrivals and service times are generally distributed is determining the variance of the lock idle times. These depend on the way in which the previous busy period terminated. This problem may be bypassed by developing a metamodel for directly estimating the squared coefficient of variation of interdeparture times.

Before developing the metamodel for estimating the coefficient of variation of overall interdeparture-time distribution, the stratum-specific split-sample scheme was also employed to split the database of Experiment 1 into the two groups (training and holdout). For developing the metamodel, there were 120 observations available from the database of Experiment 1. Each observation was obtained from one individual lock and included the characteristics of overall interdeparture-time distribution and corresponding overall interarrival-time, service-time distributions, and V/C ratio at one lock. The split ratio between the training and holdout groups is approximately 2:1. Therefore, the computer randomly assigned 79 observations to the training group and 41 observations to the holdout group.

Following the approach outlined in Section 4.1, the following metamodel is developed:

$R^2 = 0.9984$ $n = 79$ $s_e = 0.0058$ $\mu_Y = 0.83116$

$$C_D^2 = 0.207 + 0.795\,(C_A^2(1-\rho)+\rho) + 1.001\,(C_S^2\rho^2-\rho^2) \qquad \textbf{(36)}$$
$$\qquad (0.065) \quad (0.066) \qquad\qquad (0.0046)$$

$R^2 = 0.9984$ $\quad$ $n = 79$ $\quad$ $s_e = 0.0058$ $\quad$ $\mu_Y = 0.83116$

The metamodel of Eq. 36 was developed based on the structural form of Eq. 32. Eq. 36 was originally developed by using four separate variables: (1) $C_A^2(1-\rho)$, (2) ρ, (3) $C_S^2\rho^2$, and (4) ρ^2. However very high correlations were observed between Variables 1 and 2 and Variables 3 and 4. The correlation coefficient between Variables 1 and 2 is -0.9993 and the correlation coefficient between Variables 3 and 4 is 0.9222. Moreover, the coefficients of Variables 1 and 2 were almost equal. Variables 3 and 4 also had nearly equal coefficients. To avoid multicollinearity problems, Variables 1 and 2 were combined into a single variable while Variables 3 and 4 were combined into a second variable to develop Eq. 36. The correlation coefficient between the combined variables (1 and 2) and (3 and 4) is -0.1775, which indicates the new combined variables are not highly correlated. It is noteworthy that the sum of the intercept (0.207) and the parameter of $C_A^2(1-\rho)+\rho$ (0.795) is approximately equal to 1. In addition, the dependent variable has a standard error of 0.0058, which is only 0.7% of its mean.

The holdout group data was used to validate the reliability of this metamodel and the cross-validation test was again conducted. The descriptive statistics and test results for the training group and the holdout group are listed in Tables 10 and 11.

This metamodel is very reliable since the shrinkage is 0.0029. It performs especially well in the cross-validation test, where its absolute error ranges from 0.00014 to 0.03, with a mean of 0.007, and its absolute percentage error ranges from 0.02 to 3.34, with a mean of 0.84.

Since the mean and standard deviation of an exponential distribution must be equal, its coefficient of variation must be 1.0. Thus, for the special case of an M/M/1 queue:

$$C_A^2 = 1 \qquad \textbf{(37)}$$

$$C_S^2 = 1 \qquad \textbf{(38)}$$

TABLE 10. DESCRIPTIVE STATISTICS AND TEST RESULTS FOR THE TRAINING GROUP (C_D^2)

Variable	Mean	S.D.*	Min	Max
C_D^2	0.83	0.15	0.39	1.02
C_A^2	0.99	0.02	0.89	1.02
C_S^2	0.53	0.14	0.34	0.81
ρ	0.54	0.27	0.04	0.97
ε**	0.004	0.004	0.00009	0.02
ε%***	0.54	0.46	0.02	2.30
$n^+ = 79$, $R_1^{2++} = 0.9984$				

*S.D. : standard deviation
**ε : absolute error of C_D^2 between simulation and metamodel
***ε% : percentage error of C_D^2 between simulation and metamodel
+n : number of observations
++R_1^2 : the coefficient of determination

TABLE 11. DESCRIPTIVE STATISTICS AND TEST RESULTS FOR THE HOLDOUT GROUP (C_D^2)

Variable	Mean	S.D.*	Min	Max
C_D^2	0.82	0.17	0.46	1.02
C_A^2	0.98	0.03	0.90	1.02
C_S^2	0.53	0.13	0.35	0.79
ρ	0.54	0.27	0.04	0.94
ε**	0.007	0.009	0.00014	0.03
ε%***	0.84	0.96	0.02	3.34
$n^+ = 41$, $R_2^{2++} = 0.9955$				

*S.D. : standard deviation
**ε : absolute error of C_D^2 between simulation and metamodel
***ε% : percentage error of C_D^2 between simulation and metamodel
+n : number of observations
++R_2^2 : the coefficient of determination

Substituting Eqs. 37 and 38 into Eq. 36, the latter may be simplified as follows:

$$C_D^2 = 0.207+0.795(1-\rho+\rho)+(\rho^2-\rho^2) = 0.207+0.795 = 1.002 \approx 1.0 \tag{39}$$

This result is also consistent with Burke's Theorem [11].

The parameters and structural form of this metamodel are similar to those of Eq. 32, which was analytically derived for G/G/1 queues. In addition, its standard error is extremely tight ($S_e/\mu_y = 0.007$) and it satisfies Burke's Theorem very closely when applied to the special M/M/1 case. It may be concluded that the good performance of the metamodel is due to its development approach. Its structural form was based on queuing theory while its coefficients were estimated statistically from simulation results. Such a metamodel should be very useful for predicting interdeparture time distributions from G/G/1 queues embedded in larger systems, such as series and networks.

Step 2

The coefficients of variation of directional interdeparture-times for upstream and downstream traffic must be estimated. For this purpose the following metamodel was developed:

$$C_{dji}^2 = \underset{(0.0056)}{0.518} + \underset{(0.0068)}{0.491} C_{aji}^2 \, C_{Di}^2 \tag{40}$$

$R^2 = 0.9710$ $\quad$ $n = 158$ $\quad$ $s_e = 0.013$ $\quad$ $\mu_y = 0.9164$

where

C_{dji}^2: squared coefficient of variation of directional interdeparture times for Direction j and Lock i

C_{aji}^2: squared coefficient of variation of directional interarrival times for Direction j and Lock i

C_{Di}^2: squared coefficient of variation of interdeparture times for Direction j and Lock i

Before developing the above metamodel (Eq. 40), the stratum-specific split-sample scheme was also employed to split the database of Experiment 1 into the two groups (training and holdout). For developing this metamodel, 240 observations were available from the database of Experiment 1. Two observations (upstream and downstream directional interdeparture-time distributions) could be obtained from one individual lock. Each observation included the directional interdeparture-time distribution and corresponding V/C ratio, the overall interdeparture-time, and the directional interarrival-time and service-time distributions at one lock. The split ratio between the training and holdout groups was approximately 2:1. Therefore, the computer randomly assigned 158 observations to the training group and 82 observations to the holdout group.

The development of this particular metamodel (Eq. 40) was based on empirical analysis. The Pearson correlation coefficients were computed and the dependent variable (squared coefficient of variation of directional interdeparture-time distribution) was plotted versus each possible influential factor. The possible influential factors included the following:

1. C_D^2 (squared coefficient of variation of overall interdeparture-time distribution),

2. C_a^2 (squared coefficient of variation of directional interarrival-time distribution),

3. C_A^2 (squared coefficient of variation of overall interarrival-time distribution),

4. C_S^2 (squared coefficient of variation of service times),

5. V/C ratio,

6. $\mathrm{Ln}(C_D^2)$,

7. $C_a^2 C_D^2$,

8. $(C_D^2)^{0.5}$,

9. $(C_a^2)^2$,

10. $(C_a^2)^{0.5}$,

11. $C_a^2 \mathrm{Ln}(C_D^2)$, and

12. $C_D^2 \mathrm{Ln}(C_a^2)$

The seventh factor in the above list ($C_a^2 C_D^2$) was selected because it yielded the highest R^2 and the smallest s_e.

The holdout group data was used to validate the reliability of this metamodel. The cross-validation test was again performed. The descriptive statistics and test results for the training group and the holdout group are shown in Tables 12 and 13.

The shrinkage (0.0007) indicates that the reliability of this metamodel is high. In addition, the results of the cross-validation test further support its reliability since the absolute error ranges from 0.00004 to 0.05, with a mean of 0.009, and the absolute percentage error ranges from 0.005 to 5.41, with a mean of 0.96. However, the maximum absolute percentage error is 7% in the training group and 5.41 in the holdout group. Therefore, this metamodel does not always predict precisely the coefficient of variation of directional interdeparture-time distribution and could stand further improvement.

Delay Function

The delay function is intended to estimate the average waiting time at a lock. Marshall [26] has tried to express the variance of interdeparture times in terms of the average waiting time. In this study, the average waiting time is expressed in terms of the variance of interdeparture times by applying Marshall's formula. An exact solution for the average waiting time is obtained as follows:

$$W = \frac{\sigma_A^2 + 2\sigma_S^2 - \sigma_D^2}{2\overline{t_A}(1-\rho)} \quad \textbf{(41)}$$

where

- W : the average waiting time
- σ_A^2 : variance of interarrival times
- σ_S^2 : variance of service times
- σ_D^2 : variance of interdeparture times
- t_A : average interarrival time
- ρ : volume to capacity ratio

TABLE 12. DESCRIPTIVE STATISTICS AND TEST RESULTS FOR THE TRAINING GROUP (C_d^2)

Variable	Mean	S.D.*	Min	Max
C_{dji}^2	0.92	0.08	0.69	1.02
C_{aji}^2	0.98	0.04	0.74	1.02
C_{Di}^2	0.83	0.15	0.39	1.02
ε**	0.01	0.009	0.00005	0.05
ε%***	1.08	1.08	0.006	7.00
$n^+ = 158$, $R_1^{2++} = 0.9710$				

*S.D. : standard deviation
**ε : absolute error of C_{dji}^2 between simulation and metamodel
***ε% : percentage error of C_{dji}^2 between simulation and metamodel
+n : number of observations
++R_1^2 : the coefficient of determination

TABLE 13. DESCRIPTIVE STATISTICS AND TEST RESULTS FOR THE HOLDOUT GROUP (C_d^2)

Variable	Mean	S.D.*	Min	Max
C_{dji}^2	0.92	0.08	0.70	1.02
C_{aji}^2	0.98	0.03	0.79	1.02
C_{Di}^2	0.83	0.15	0.41	1.02
ε**	0.009	0.0096	0.00004	0.05
ε%***	0.96	1.03	0.005	5.41
$n^+ = 82$, $R_2^{2++} = 0.9703$				

*S.D. : standard deviation
**ε : absolute error of C_{dji}^2 between simulation and metamodel
***ε% : percentage error of C_{dji}^2 between simulation and metamodel
+n : number of observations
++R_2^2 : the coefficient of determination

For the general queuing system the difficulty with Eq. 41 is that only the variances of interarrival and service times are known, while the variance of interdeparture times is unknown. This difficulty can now be overcome by using the metamodel developed in this study to estimate the coefficient of variation of interdeparture times (Eq. 36).

In this delay function, the average waiting time increases as the variance of interarrival and service times increase and decreases as the variance of interdeparture times increases. The average waiting time approaches infinity as the volume/capacity ratio ρ approaches 1.0.

Algorithm for Two-Way Traffic Systems

Delays depend on the interarrival, interdeparture, and service-time distributions. Therefore, to estimate delays, we need to know in advance the means and variances of the interarrival-time, interdeparture-time, and service-time distributions. For two-way traffic systems with series of G/G/1 queues and bi-directional servers, a difficulty arises in identifying the variances of interarrival and interdeparture times. The variance of interarrival times at a certain lock labeled k is affected by the interdeparture-time distributions from adjacent upstream and downstream locks. The interdeparture-time distributions at adjacent upstream and downstream locks depend on their interarrival-time distributions, which are affected by the interdeparture-time distributions from Lock k. Hence, the variances of interarrival times at adjacent locks depend upon each other. Therefore, the variances of interarrival times cannot be determined from a single one-directional scan along a series of queues. For example, if we tried to estimate delays by scanning from upstream toward downstream, we could determine at Lock k the variance of interarrival times from upstream, but the variance of interarrival times from downstream would be unknown and would be affected by the interdeparture-time distribution from this Lock k. To overcome such complex interdependence, an iterative algorithm is proposed. It starts scanning in one direction while using some initialized assumed values for the variances of interdeparture times from the opposite direction. It can thus sequentially estimate the interarrival and interdeparture-time distributions for each lock. Then, the scanning direction is reversed and the process is repeated, using the interdeparture-time distributions for the opposite direction estimated in the previous scan. Alternating directions, the scanning process continues until the specified convergence criteria (usually the squared coefficient of variation or the variances of interdeparture times) computed in successive iterations converge. Then the algorithm stops reestimating the arrival distributions and proceeds to estimate delays.

The following algorithm is designed to apply the metamodels in estimating single-chamber lock delays. The notation used in this algorithm is defined as follows:

$\bar{t}_{aji}$	:	average interarrival time for Direction j and Lock i
$\bar{t}_{djk}$	:	average interdeparture time for Direction j and Lock k
σ_{ajin}^2	:	variance of interarrival times for Direction j, Lock i and Iteration n
σ_{djkn}^2	:	variance of interdeparture times for Direction j, Lock k and Iteration n
σ_{d10}^2	:	variance of interdeparture times from origin node
σ_{d2L+1}^2	:	variance of interdeparture times from destination node
D_{ik}	:	distance between Locks i and k
μ_{vik}	:	average tow speed between Locks i and k
σ_{vik}	:	standard deviation of tow speeds between Locks i and k
i	:	lock index
j	:	direction index (1 = downstream, 2 = upstream)
n	:	iteration
$\bar{t}_{Ai}$	:	average interarrival time at Lock i
σ_{Ain}^2	:	variance of interarrival times at Lock i and Iteration n
$\bar{t}_{Di}$	:	average interdeparture time at Lock i
σ_{Din}^2	:	variance of interdeparture times at Lock i and Iteration n
σ_{Si}^2	:	variance of lock service times at Lock i
ρ_i	:	volume/capacity (=V/C) ratio at Lock i
λ_i	:	average inflow rate at Lock i

C_{ajin} : coefficient of variation of directional interarrival times for Lock i, Direction j, and Iteration n

C_{djin} : coefficient of variation of directional interdeparture times for Lock i, Direction j, and Iteration n

C_{Si} : coefficient of variation of service time at Lock i

C_{Ain}, C_{Din} : coefficients of variation for interarrival times and interdeparture times, respectively, at Lock i and Iteration n

c_i : constant (assumed value)

L : total number of Locks

W_i : average waiting time at Lock i

m : indicator of scanning direction (1= from origin to destination, 2= from destination to origin)

The required inputs of this algorithm include the means and coefficients of variation for service-time distributions and inflow distributions, distances between locks, and the means and standard deviations of speed distributions. This algorithm estimates delays as well as means and coefficients of variation for interarrival and interdeparture-time distributions. The algorithm consists of the following steps:

1. Compute the average directional interarrival and interdeparture times for each lock:

$$\bar{t}_{dji} = \bar{t}_{aji} = \bar{t}_{djk} \quad \begin{cases} k=i-1, \text{ if } j=1 \\ k=i+1, \text{ if } j=2 \end{cases} \qquad \textbf{(15a)}$$

2. Compute the average interarrival time for each lock:

$$\bar{t}_{Ai} = \frac{\bar{t}_{a1i} * \bar{t}_{a2i}}{\bar{t}_{a1i} + \bar{t}_{a2i}} \quad i=1, \ldots M \qquad \textbf{(17a)}$$

3. Estimate the coefficients of variation for interarrival and interdeparture times at each lock.

3.1 Set n=1, m=1

3.2 Assume initial values for the standard deviations of interdeparture times in Direction 2, at Locks 2 through L-1

$$\sigma_{d2i0} = c_i, \quad i=2, \ldots L-1 \tag{42}$$

3.3 Starting from Lock 1, let i = 1

3.4 Compute the standard deviations of directional interarrival times at Lock i using the metamodel expressed in Eq. 16:

3.4.1 if j=1 and m=1

$$\sigma_{a1in}=\sigma_{d1i-1n}+0.0251\ \ln(1+\frac{D_{ii-1}\sigma_{vii-1}}{\mu_{vii-1}}) \tag{16a}$$

3.4.2 if j=2 and m=1

$$\sigma_{a2in}=\sigma_{d2i+1n-1}+0.0251\ \ln(1+\frac{D_{ii+1}\sigma_{vii+1}}{\mu_{vii+1}}) \tag{16b}$$

3.4.3 if j=1 and m=2

$$\sigma_{a1in}=\sigma_{d1i-1n-1}+0.0251\ \ln(1+\frac{D_{ii-1}\sigma_{vii-1}}{\mu_{vii-1}}) \tag{16c}$$

3.4.4 if j=2 and m=2

$$\sigma_{a2in}=\sigma_{d2i+1n}+0.0251\ \ln(1+\frac{D_{ii+1}\sigma_{vii+1}}{\mu_{vii+1}}) \tag{16d}$$

3.5 Compute the squared coefficients of variation of directional interarrival times at Lock i:

$$C_{ajin}{}^2 = (\frac{\sigma_{ajin}}{\bar{t}_{aji}})^2 \tag{43}$$

3.6 Compute the squared coefficient of variation of combined interarrival times at Lock i

$$C_{Ain}{}^2 = 0.179+0.41(C_{a1in}{}^2 + C_{a2in}{}^2) \tag{18a}$$

3.7 Estimate the squared coefficient of variation for interdeparture times at Lock i using the metamodel expressed in Eq. 36:

$$C_{Din}{}^2 = 0.207+0.795(C_{Ain}{}^2(1-\rho_i)+\rho_i)+1.001(C_{Si}{}^2\rho_i^2-\rho_i^2) \tag{36a}$$

3.8 Compute the variance of interarrival times at Lock i

$$\sigma_{Ain}{}^2 = C_{Ain}{}^2\bar{t}_{Ai}{}^2 \tag{44}$$

3.9 Compute the variance of interdeparture times at Lock i

$$\sigma_{Din}^2 = C_{Din}^2 \bar{t}_{Di}^2 \quad \textbf{(45)}$$

3.10 Compute the squared coefficients of variation of directional interdeparture times at Lock i, using the relation developed in Eq. 40:

$$C_{djin}^2 = 0.518 + 0.491 C_{ajin}^2 \; C_{Din}^2 \quad \textbf{(40a)}$$

3.11 Compute the standard deviations of directional interdeparture times at Lock i:

$$\sigma_{djin} = C_{djin} \, \bar{t}_{dji} \quad \textbf{(46)}$$

3.12 Repeat Steps 3.4 - 3.11 for i = 2, ...L

3.13 Set n = n+1, m=2

3.14 Starting from Lock L, let i = L, and repeat Steps 3.4 - 3.12 for i = L, (L-1), ...1

3.15 Set n = n+1, m=1

3.16 Repeat Steps 3.4 - 3.14

3.17 If the following condition is satisfied, then go to Step 4. Otherwise, go to Step 3.15

$$\frac{\left| C_{djin}^2 - C_{djin-1}^2 \right|}{C_{djin-1}^2} \le 0.001 \quad i=1, \ldots L, \quad j=1,2 \quad \textbf{(47)}$$

4. Estimate the average waiting times using the formula expressed in Eq. 41:

$$W_i = \frac{\sigma_{Ain}^2 + 2\sigma_{Si}^2 - \sigma_{Din}^2}{2\bar{t}_{Ai}(1-\rho_i)} \quad \textbf{(41a)}$$

Algorithm for One-Way Traffic Systems

Although the numerical method was originally developed for two-way traffic systems, with a few simplifications this method can be adapted for the more generally encountered systems with one-directional servers. The one-directional systems may be treated as a special case of the two-directional systems. The one-directional algorithm should perform better since the interarrival time distributions will be affected by the interdeparture time distributions from upstream only and are not subject to circular interdependence. Therefore, the interarrival-time distributions may be determined in a single one-directional scan without any iteration. The notation used in this algorithm for one-way traffic is defined as follows:

D_{ik} : distance between Locks i and k

μ_{vik} : average tow speed between Locks i and k

σ_{vik} : standard deviation of tow speeds between Locks i and k

$\bar{t}_{Ai}$: the average interarrival time at Lock i

σ_{Ai}^2 : variance of interarrival times at Lock i

$\bar{t}_{Di}$: average interdeparture time at Lock i

σ_{Di}^2 : variance of interdeparture times at Lock i

σ_{d0}^2 : variance of interdeparture times from origin node

σ_{Si}^2 : variance of lock service times at Lock i

ρ_i : V/C ratio at Lock i

λ_i : average inflow rate at Lock i

C_{Ai}^2, C_{Di}^2, C_{Si}^2 : squared coefficients of variation for interarrival times, interdeparture times, and service times respectively, at Lock i.

L : total number of locks

W_i : the average waiting time at Lock i

This algorithm has the same inputs and outputs as the algorithm for two-way systems. The algorithm consists of the following steps:

1. Compute the average interarrival time and interdeparture time for each lock:

$$\bar{t}_{Di} = \bar{t}_{Ai} = \bar{t}_{Di-1} \quad i=1, \ldots L \tag{15b}$$

2. Estimate the squared coefficients of variation and, from them, the variances of interarrival and interdeparture time distributions for each lock.

2.1 Starting from Lock 1, let i=1

2.2 Compute the standard deviation of interarrival times at Lock i using the metamodel developed for two-way traffic (Eq. 16):

$$\sigma_{Ai}=\sigma_{Di-1}+0.0251\ln\left(1+\frac{D_{ii-1}\sigma_{vii-1}}{\mu_{vii-1}}\right) \qquad \textbf{(16e)}$$

2.3 Compute the squared coefficient of variation for interarrival times at Lock i

$$C_{Ai}{}^2 = \left(\frac{\sigma_{Ai}}{\overline{t}_{Ai}}\right)^2 \qquad \textbf{(48)}$$

2.4 Estimate the squared coefficient of variation for interdeparture times at Lock i using the metamodel developed for two-way traffic (Eq. 36):

$$C_{Di}{}^2 = 0.207+0.795(C_{Ai}{}^2(1-\rho_i)+\rho_i)+1.001(C_{Si}{}^2\rho_i^2-\rho_i^2) \qquad \textbf{(36b)}$$

2.5 Compute the variance of interdeparture times at Lock i

$$\sigma_{Di}{}^2 = C_{Di}{}^2\overline{t}_{Di}{}^2 \qquad \textbf{(49)}$$

2.6 Repeat steps 2.2 - 2.5 for i = 2, ...L

3. Estimate average waiting times using the relation developed for two-way traffic (Eq. 41):

$$W_i = \frac{\sigma_{Ai}{}^2+2\sigma_{Si}{}^2-\sigma_{Di}{}^2}{2\overline{t}_{Ai}(1-\rho_i)} \qquad \textbf{(41b)}$$

CHAPTER 5
VALIDATION OF NUMERICAL METHOD

Comparison of Numerical and Simulated Results

An experiment is conducted to test how well the numerical method duplicates the results of simulations. Eight three-lock, two-directional systems are tested. The controlled variables in this experiment include the V/C ratio ρ, the variance of lock service times, inflow rate, distance between locks, and tow speed. Values for these variables are generated randomly and uniformly between specified bounds which cover the ranges of values where waterway delays can be significant. In this test the ranges of the controlled variables are 0.01 to 0.89 for the V/C ratio, 0.0007 to 0.3332 for the variance of lock service times, 6.0 to 28.5 tows per day for the inflow rate, 5 to 60 miles for the distance between locks, 108 to 325 miles per day for the average tow speed, and 33.84 to 101.52 miles per day for the standard deviation of tow speeds. Table 14 lists the values of controlled variables for each system in this experiment. For this system the variances of the interdeparture times converged within 0.1% for every lock and direction in no more than 6 iterations. The results are shown in Tables 15 and 16.

The performance of the numerical method could be discussed in terms of the scanning process and the average waiting time. The performance of scanning processes is the overall performance of arrival, departure and iterative scanning processes. The performance indicators are $\mathcal{E}_A$ and $\mathcal{E}_D$, which directly measure the variance deviations of interarrival times and interdeparture times, respectively, between numerical and simulated results. The average absolute $\mathcal{E}_A$ is 3.57% for these 8 systems. For 17 of 24 locks in these 8 three-lock systems $\mathcal{E}_A$ is less than 4%. The average absolute $\mathcal{E}_D$ is 1.80% for these 8 systems. For 23 of the 24 locks $\mathcal{E}_D$ is less than 4%. The results show that the scanning process closely approximates the simulation estimates for the variances of interarrival and interdeparture times.

The deviation of average waiting times are due to the scanning process and the delay function. To clearly define the deviation of average waiting times, the following notations are used:

W_{sim}: simulated waiting times

W_{num}: waiting times estimated with numerical method

W_{num2}: waiting times estimated by Eq. 41 with the simulated variances of interarrival and interdeparture times

ΔW: $W_{num}-W_{sim}$, representing the total deviation between the numerical method and simulation

$\Delta W2$: $W_{num2}-W_{sim}$, representing the deviation due to the delay function

$\Delta W1$: $\Delta W-\Delta W2$, representing the deviation due to the scanning process

ε_W: $\Delta W/W_{sim}*100\%$, representing the total percentage deviation between numerical and simulated results

ε_{W2}: $\Delta W2/W_{sim}*100\%$, representing the percentage deviation due to the delay function

ε_{W1}: $\varepsilon_W-\varepsilon_{W2}$, representing the percentage deviation due to the scanning process

A question arises about the deviation between W_{sim} and W_{num2} (or $\Delta W2$). The W_{num2} is obtained by applying Eq. 41 while the input variances of interarrival and interdeparture times are simulated results. Since the inputs are simulated results and Eq. 41 is an exact formula for average waiting times, why is there some deviation between W_{sim} and W_{num2}? This test assumed the simulated results exactly represent the true values. However, simulation is a kind of experiment and thus its results are always subject to certain errors. More replications and longer simulation periods can reduce, but not completely eliminate, the errors. That is why there is some deviation between W_{sim} and W_{num2}. Table 15 shows that some locks have quite high ε_{W2}, but Table 16 shows that for 21 of the 24 locks absolute $\Delta W2$ values are below 0.1 hr, and for 10 locks the absolute $\Delta W2$ values are below 0.01 hr. It may be concluded that the delay function produces results consistent with simulated ones.

$\Delta W1$ and ε_{W1} indicate the deviation and percentage deviation of average waiting times from the scanning process. Table 15 shows that 10 of the 24 locks have ε_{W1} values above 10%. However, Table 16 shows that 21 of the 24 locks have absolute $\Delta W1$ values below 0.1 hr, and 8 locks have values below 0.01 hr. Therefore, the deviations attributable to the scanning process are quite minor in most cases.

The total percentage deviation of waiting times ε_W in Table 15 shows that 8 of the 24 locks have absolute values above 10%. The total deviation of waiting times ΔW in Table 16 shows that 21 of the 24 locks have absolute values below 0.1 hr, and 11 locks have values below 0.01 hr. Therefore, the numerical method works satisfactorily. In general, these results indicate that the numerical method may be used to screen alternatives and greatly reduce the number of lock improvement combinations that have to be evaluated by the more detailed microscopic simulation model.

Three types of errors arise in the numerical method: from simulation, from the metamodeling procedure, and from the iterative scanning process. The errors from simulation could be reduced by increasing the number of replications and duration of simulated periods. The errors from the metamodeling procedure could be reduced by collecting more data (that is, increasing the data base). Such improvements could increase the accuracy of the numerical method.

To apply the numerical method to more general networks of queues, it is desirable to build a new data base for redeveloping or validating metamodels since the applicable ranges of the current one are mainly appropriate for waterways.

TABLE 14. VALUES OF CONTROLLED VARIABLES

System	Lock	λ*1 tows/day	V/C	Dist mi	Speed mi/day Mean	S.D.*3	σ_S^2*2
1	1	6.0	0.01	5	270	85	0.0007
	2	6.0	0.07	5	270	85	0.0360
	3	6.0	0.17	5	270	85	0.1897
2	1	12.0	0.15	5	325	102	0.0309
	2	12.0	0.34	5	325	102	0.1620
	3	12.0	0.25	5	325	102	0.0915
3	1	18.0	0.22	5	108	34	0.0309
	2	18.0	0.03	5	108	34	0.0006
	3	18.0	0.50	5	108	34	0.1618
4	1	24.0	0.50	5	162	51	0.1883
	2	24.0	0.29	5	162	51	0.0646
	3	24.0	0.67	5	162	51	0.3330
5	1	27.0	0.75	10	108	34	0.2271
	2	27.0	0.57	10	108	34	0.1279
	3	27.0	0.89	10	108	34	0.3167
6	1	27.0	0.75	20	216	68	0.1616
	2	27.0	0.57	20	216	68	0.0909
	3	27.0	0.89	20	216	68	0.2259
7	1	28.5	0.60	5	325	102	0.1557
	2	28.5	0.05	5	325	102	0.0011
	3	28.5	0.80	5	325	102	0.2738
8	1	28.5	0.35	60	162	51	0.0645
	2	28.5	0.60	60	162	51	0.1882
	3	28.5	0.80	60	162	51	0.3332

*1 λ : two-way flow rate
*2 σ_S^2 : variance of service times
*3 S.D.: standard deviation

TABLE 15. COMPARISON OF NUMERICAL AND SIMULATED RESULTS (1)

Sys*1	Lock	W_{sim}*2 hr	ΔW*3 hr	ε_W*4 %	ε_{W1}*5 %	ε_{W2}*6 %	ε_A*7 %	ε_D*8 %
1	1	0.0003	.0015	484.95	584.95	-100.00	1.20	1.09
	2	0.0153	.0054	35.57	67.94	-32.37	1.90	1.42
	3	0.0989	.0042	4.22	8.17	-3.95	0.95	0.63
2	1	0.0334	-.0010	-3.08	47.43	-50.51	3.38	1.98
	2	0.2316	.0047	2.04	14.87	-12.83	3.81	1.55
	3	0.1139	.0009	0.81	25.39	-24.58	3.20	0.99
3	1	0.0542	.0030	5.53	12.45	-6.92	3.20	2.45
	2	0.0008	-.0008	-100.00	0.00	-100.00	3.74	3.94
	3	0.4621	.0155	3.34	4.63	-1.29	3.14	1.82
4	1	0.4355	.0155	3.55	6.40	-2.85	2.57	-0.23
	2	0.0962	.0094	9.79	28.73	-18.94	5.75	1.59
	3	1.2028	.0016	0.13	4.18	-4.05	2.51	-0.91
5	1	1.3926	.1155	8.30	-0.41	8.71	-0.78	-0.62
	2	0.3901	.0413	10.59	-0.59	11.18	-0.15	0.11
	3	4.9837	-.0868	-1.74	0.27	-2.01	-0.15	-0.79
6	1	1.2203	.1339	10.97	-0.67	11.64	-0.50	-0.07
	2	0.3286	.0371	11.29	-4.61	15.90	-0.53	1.36
	3	4.4608	-.0278	-0.62	1.09	-1.71	0.73	-0.93
7	1	0.5430	.0615	11.32	10.57	0.75	6.59	0.85
	2	0.0012	-.0012	-100.00	0.00	-100.00	10.73	10.82
	3	2.0874	.0391	1.87	5.42	-3.55	5.14	-0.68
8	1	0.1372	.0227	16.57	16.11	0.46	6.51	3.23
	2	0.6381	.0722	11.32	10.30	1.02	9.07	3.03
	3	2.3165	.1134	4.89	6.79	-1.90	9.46	2.18

*1 Sys : System
*2 W_{sim} : simulated waiting time
*3 ΔW : W_{num}-W_{sim}, where W_{num} is waiting times estimated with numerical method
*4 ε_W : $\Delta W/W_{sim}*100\%$
*5 ε_{W1} : $\varepsilon_W-\varepsilon_{W2}$
*6 ε_{W2} : $(W_{num2}-W_{sim})/W_{sim}*100\%$, where W_{num2} is waiting times estimated by Eq. 41 with simulated variances of interarrival and interdeparture times
*7 ε_A : $(\sigma_A{}^2{}_{num}-\sigma_A{}^2{}_{sim})/\sigma_A{}^2{}_{sim}*100\%$, where $\sigma_A{}^2{}_{num}$ is variance of interarrival times estimated by numerical method; $\sigma_A{}^2{}_{sim}$ is simulated variance of interarrival times
*8 ε_D : $(\sigma_D{}^2{}_{num}-\sigma_D{}^2{}_{sim})/\sigma_D{}^2{}_{sim}*100\%$, where $\sigma_D{}^2{}_{num}$ is variance of interarrival times estimated by numerical method; $\sigma_D{}^2{}_{sim}$ is simulated variance of interarrival times

TABLE 16. COMPARISON OF NUMERICAL AND SIMULATED RESULTS (2)

Sys*1	Lock	W_{sim}*2 hr	ΔW*3 hr	ΔW1*4 hr	ΔW2*5 hr
1	1	0.0003	.0015	0.0018	-0.0003
	2	0.0153	.0054	0.0104	-0.0050
	3	0.0989	.0042	0.0081	-0.0039
2	1	0.0334	-.0010	0.0159	-0.0169
	2	0.2316	.0047	0.0344	-0.0297
	3	0.1139	.0009	0.0289	-0.0280
3	1	0.0542	.0030	0.0067	-0.0037
	2	0.0008	-.0008	0.0000	-0.0008
	3	0.4621	.0155	0.0214	-0.0059
4	1	0.4355	.0155	0.0279	-0.0124
	2	0.0962	.0094	0.0276	-0.0182
	3	1.2028	.0016	0.0504	-0.0488
5	1	1.3926	.1155	-0.0058	0.1213
	2	0.3901	.0413	-0.0023	0.0436
	3	4.9837	-.0868	0.0135	-0.1003
6	1	1.2203	.1339	-0.0081	0.1420
	2	0.3286	.0371	-0.0151	0.0522
	3	4.4608	-.0278	0.0487	-0.0765
7	1	0.5430	.0615	0.0574	0.0041
	2	0.0012	-.0012	0.0000	-0.0012
	3	2.0874	.0391	0.1132	-0.0741
8	1	0.1372	.0227	0.0221	0.0006
	2	0.6381	.0722	0.0657	0.0065
	3	2.3165	.1134	0.1573	-0.0439

*1 Sys : System
*2 W_{sim} : simulated waiting time
*3 ΔW : $W_{num}-W_{sim}$, where W_{num} is waiting times estimated with numerical method
*4 ΔW1 : ΔW-ΔW2
*5 ΔW2 : $W_{num2}-W_{sim}$, where W_{num2} is waiting times estimated by Eq. 41 with simulated variances of interarrival and interdeparture times

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APPENDICES

APPENDIX A

List of Variables

Variable — Description

ANB(i): Average number of barges for O-D pair i, i=1,NP

BIAS(i): Chamber assignment policy for favoring the use of main chamber at lock i, i=1,NL, (min)

DIST(i): Link length, i=1,NL, (miles)

DISTL(i): Distance between in-node to lock on link i, i=1,NL, (miles)

DTEX(i): Mean of dwell time for O-D pair i, i=1,NP, (days)

DTSTD(i): Standard deviation of dwell time for O-D pair i, i=1,NP, (days)

EXS: Mean of tow speed, (miles/day)

FRB(i,j): The lower bound of interval j in the tow size distribution table for O-D pair i, i=1,NP, j=1,NBI+1

FRI(i): The lower bound of interval i for trip generation table, i=1,NII+1

FRS(i,j,k,l): The lower bound of interval l for lock service time table for lock i, chamber j, cut k, i=1,NL, j=1,NPL(i), k=1,ML(i,j), L=1,NSI+1

FSTA(i,j): Average stall frequency at lock i, chamber j, (#/yr)

ID(i): Index of destination for O-D pair i, i=1,NP

IDBG: =1, to actuate the debugging function

IDBG1: =1, to debug the sequence of time advance and event type

IDBG2: =1, to debug the lock event

IDBG3: =1, to debug the lock event only at certain locks

ILBG(i): =1, to actuate the debugging function at lock i, i=1,NL

INS: Specifying the starting inventory level INS while printing detail inventory information

IO(i): Index of origin for O-D pair i, i=1,NP

IOF: =1, to plot inventory level

IOI: =1, to print input data

IOM: =1, to provide the inventory information

IOU: =1, to plot cumulative delivery and consumption

IP: Indicator for printing wait time data
=0, no printing
=1, by lock and direction in different files
=2, in one file

IXB(i,j): Random number seed of tow size (number of barges) for O-D pair i, direction j, i=1,NP, j=1,2

IXC(i): Random number seed for coal consumption at node i, i=1,NN

IXL(i,j,k): Random number seed for lock service time of k cuts at lock i, chamber j, i=1,NL, j=1,NPL(i), k=1,ML(i,j)

IXS: Random number seed for tow speed

IXSD(i,j): Random number seed for stall duration at lock i, chamber j, i=1,NL, j=1,NPL(i)

IXST(i,j): Random number seed for stall frequency at lock i, chamber j, i=1,NL, j=1,NPL(i)

IXT(i,j): Random number seed for inter-trip-generation-time for O-D pair i, direction j, i=1,NP, j=1,2

KSG: Indicator of trip generation distribution
=1, empirical distribution
=2, Poisson distribution

KST: Indicator of lock service time distribution
=1, empirical distribution
=2, uniform distribution
=3, exponential distribution

ML(i,j): Types of cuts at lock i, chamber j, i=1,NL, j=1,NPL(i)

NBI: Total number of intervals in the distribution table for the number of barges

NCB(i,j,k): Upper limit on cut size k, at lock i, chamber j, i=1,NL, j=1,NPL(i), k=1,ML(i,j)

NC: O-D pairs 1-NC are coal traffic

NCTL(i): Lower bound of number of coal barges per coal tow for coal O-D pair i, i=1,NC

NCTU(i): Upper bound of number of coal barges per coal tow for coal O-D pair I, I=1,NC

NI(i): Index of in node for link i, i=1,NL

NII: Total number of intervals in the trip generation table

NIS: Number of different starting inventory levels

NL: Total number of locks/links

NN: Total number of nodes

NO(i): Index of out node for link i, i=1,NL

NP: Total number of O-D pairs

NPL(i): Total number of chambers in lock i, i=1,NL

NRAN: Index of random number set

NSI: Total number of intervals in lock service time distribution table

NTP: Total number of simulation time periods

NTR: Index of trip rate set

PLOAD: Barge payload, (short-tons)

RCON(i,j): Average consumption rate for simulation time period i, at node j, i=1,NTP, j=1,NN, (short-tons/day)

RCT(i): Coal barge fraction of a tow for O-D pair i, i=1,NC

RCTL(i): Lower bound ratio of RCT(i), i=1,NC

RCTU(i): Upper bound ratio of RCT(i), i=1,NC

RL(i): Lower bound ratio of average consumption rate at node i, i=1,NN

RR(i): Upper bound ratio of average consumption rate at node i, i=1,NN

RSTA(i,j): Stall duration at lock i, chamber j, i=1,NL, j=1,NPL(i)

RSTD(i,j,k): Ratio to tighten the standard deviation for lock service time distribution, i=1,NL, j=1,NPL(i), k=1,ML(i,j)

RV: Ratio of backhaul speeds to linehaul speeds

SFST(i,j): jth starting inventory level at node i, i=1,NN, j=1,NIS

SRL(i): Lower bound ratio of TLO(i), i=1,NL

SRR(i): Upper bound ratio of TLO(i), i=1,NL

STDS: Standard deviation of tow speeds

TLO(i,j,k): Average lock service time for cut type k, at lock i, chamber j, i=1,NL, j=1,NPL(i), k=1,ML(i,j) (days/tow)

TN(i,j): Trip rate for O-D pair i, simulation time period j, i=1,NP, j=1,NTP, (tow trips/day)

TSTOP(i): End of simulation time period i, i=1,NTP, (day)

TUT: Specified time interval for printing inventory levels, (days)

TWT: Specific time period to provide queue length data, (day)

Input Format

Model Input

The model input is divided into 13 sets and is described below:

1. Basic model parameters: (1 line)
   NN,NL,NP,NTP,NC,NIS,KST,KSG,IP (12I6)

2. Links/Locks characteristics: ((4+3*NPL(i))*NL lines)
   (1) NI(i),NO(i),NPL(i) (12I6)
   (2) DIST(i),DISTL(i) (10F7.2)
   (3) FSTA(i,j),j=1,NPL(i) (10F7.2)
   (4) RSTA(i,j),j=1,NPL(i) (10F7.2)
   (5) Chamber characteristics: (3*NPL(i) lines)
      a. ML(i,j) (12I6)
      b. TLO(i,j,k),k=1,ML(i,j) (8F8.5)
      c. NCB(i,j,k),k=1,ML(i,j) (12I6)
         j=1,NPL(i)
         i=1,NL

3. Traffic demand: (3*NP lines)
   (1) IO(i),ID(i) (12I6)
   (2) DTEX(i),DTSTD(i) (10F7.2)
   (3) ANB(i) (10F7.2)
      i=1,NP

4. Coal tow characteristics: (3 lines, if NC>0)
   (1) RCT(i),i=1,NC (10F7.2)
   (2) NCTL(i),i=1,NC (12I6)
   (3) NCTU(i),i=1,NC (12I6)

5. Speeds: (2 lines)
   (1) EXS,STDS (10F7.2)
   (2) RV (10F7.2)

6. Simulation time periods (NTP lines)
   TSTOP(i) (10F7.2)
   i=1,NTP

7. Lock service time distributions:
   (1) if KST=1 (NL*NPL(i)*(NSI+1)+1 lines)
       a. NSI (12I6)
       b. FRS(i,j,k,l),k=1,ML(i,j) (6F12.4)
          l=1,NSI+1
          j=1,NPL(i)
          i=1,NL
   (2) if KST=2 (NL lines)
       a. SRL(i),SRR(i) (10F7.2)
          i=1,NL

8. Trip generation distributions:
   (1) if KSG=1 (NII+2)
       a. NII (12I6)
       b. FRI(i) (6F12.4)
          i=1,NII+1

9. Tow size distributions: (NP*(NBI+1)+1 lines)
   (1) NBI (12I6)
   (4) FRB(i,j) (6F12.4)
       j=1,NBI+1
       i=1,NP

10. Inventories: (2*NN+NTP+3)
   (1) if NC>0
       a. SFST(i,j), j=1,NIS (7F10.1)
          i=1,NN
       b. PLOAD (2F10.1)
       c. RCON(i,j),j=1,NN (10F7.1)
          i=1,NTP
       d. RL(i),RR(i) (10F7.2)
          i=1,NN
       e. INS (12I6)
   (2) TUT (10F7.2)

11. BIAS(i),i=1,NL (10F7.2) (1 line)

12. Debugging function: (2 or 3 lines)
   (1) IOI,IOM,IOU,IOF (12I6)
   (2) IDBG,IDBG1,IDBG2,IDBG3,IDBG4,IDBG5,IDBG6
       (12I6)
   (3) if IDBG3=1
       a. ILBG(i),i=1,NL (12I6)

13. if KST=1:
   (1) RSTD(i,j,k),k=1,ML(i,j) (6F12.4)
       j=1,NPL(i,j)
       i=1,NL

Random Number Seeds Input

1. NRAN (7I11) (1 line)

2. ((NPL(i)+2)*NL lines)
   (1) IXST(i,j),j=1,NPL(i) (7I11)
   (2) IXSD(i,j),j=1,NPL(i) (7I11)
   (3) IXL(i,j,k),k=1,ML(i,j) (7I11)
   j=1,NPL(i)
   i=1,NL

3. (3*NP lines)
   (1) IXDT(i) (7I11)
   (2) IXT(i,j),IXB(i,j) (7I11)
   j=1,2
   i=1,NP

4. IXS (7I11)

5. if NC>0 (NN lines)
   (1) IXC(i) (7I11)

6. TWT (10F7.2) (1 line)

Trip Volume Input

1. NTR (12I6)

2. TN(i,j),j=1,NTP (12F6.2)
   i=1,NP

APPENDIX B

Residual Analysis of Metamodels

Residual analysis is intended to check whether the residuals of metamodels are identically and independently distributed (IID), which is an important assumption of linear regression analysis.

The residual analysis in the study was performed with the SAS commercial statistical software package. The ARIMA process in SAS was employed to test whether the correlation of the residuals is within the 95% confidence interval. The results are summarized as follows:

Metamodel	Correlation	95% Confidence Interval
σ_a	0.17	0.19
C_A^2	0.14	0.22
C_D^2	0.20	0.22
C_d^2	0.02	0.15

The results show that the correlations are all within the 95% confidence intervals, indicating that the residuals of each metamodel are identically and independently distributed. Therefore, the metamodels developed in the study do not violate the IID assumption of linear regression analysis.

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13. ABSTRACT *(Maximum 200 words)* A simulation model has been developed for estimating delays on congested waterways and a numerical method has been developed to approximate the results of the simulation model. The structural relations in the numerical method were largely based on exact analytic results from queueing theory while their unknown parameters were estimated statistically with results from the simulation model. The numerical method is valuable for screening improvement alternatives, while the simulation model is valuable for very detailed and accurate analysis.

The simulation model is microscopic and event-scanning. It can accommodate generally distributed trip generations and service times, unequal parallel servers, and random failure effects. Its outputs include tow travel times along waterways, tow delays at each lock, inventory levels, expected stock-out amounts for commodities transported on waterways, and variances of interarrival and interdeparture times at each lock. The simulation model has been validated in comparisons with established theoretical results and empirical data from the U.S. inlands waterways.

14. SUBJECT TERMS Lock delay models, microscopic simulation model, numerical method, networks of queues, delay function, algorithm for one-way and two traffic systems variables and input format.

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