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EXCITATION OF EARTH-IONOSPHERE WAVEGUIDE IN THE ELF AND LOWER VLF BANDS BY MODULATED IONOSPHERIC CURRENT

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In this report we use the principal of reciprocity in conjunction with a full-wave propagation code to calculate ground-level fields excited by ionospheric currents modulated at frequencies between 50 and 100 Hz with HF heaters. Our results show the dependence on source orientation, altitude, and dimension and therefore pertain to experiments using the HIPAS or HAARP ionospheric heaters.

In the end-fire mode, the waveguide excitation efficiency of an ELF HED in the ionosphere is up to 20 dB greater than for a ground-based antenna, provided its altitude does not exceed 80-to-90 km. The highest efficiency occurs for a source altitude of around 70 km; if that altitude is raised to 100 km, the efficiency drops by about 20 dB in the daytime and 10 dB at night.

That efficiency does not account for the greater conductivity modulation that might be achieved at altitudes greater than 70 km, however. The trade-off between the altitude dependencies of the excitation efficiency and maximum achievable modulation depends on the ERP of the HF heater, the optimum altitude increasing with increasing ERP. For HIPAS the best modulation altitude is around 70 km, whereas for HAARP there might be marginal value in modulating at altitudes as high as 100 km.

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TABLE OF CONTENTS

LIST OF FIGURES

Figure **Page**

SUMMARY

In this report we use the principal of reciprocity in conjunction with a full-wave propagation code to calculate ground-level fields excited by ionospheric currents modulated at frequencies between 50 and 100 Hz with HF heaters. Our results show the dependence on source orientation, altitude, and dimension and therefore pertain to experiments using the HIPAS or HAARP ionospheric heaters.

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That efficiency does not account for the greater conductivity modulation that might be achieved at altitudes greater than 70 km, however. The trade-off between the altitude dependencies of the excitation efficiency and maximum achievable modulation depends on the ERP of the HF heater, the optimum altitude increasing with increasing ERP. For HIPAS the best modulation altitude is around 70 km, whereas for HAARP there might be marginal value in modulating at altitudes as high as 100 km.

Our results show that the often used lumped dipole approximation is always valid at night, but is invalid in the daytime for frequencies that exceed about 100 Hz. An additional restriction is that the lateral scale, Leff, of the ionospheric current distribution must be smaller than the reduced, free-space wavelength, $\lambda_0/2\pi$, of the radiated ELF/VLF signal. When the lumped dipole approximation is not valid, it is necessary to use a full-fledged Green's function solution.

Section 1
INTRODUCTION

It has been a decade since powerful high frequency (HF) radio waves were first used to modulate ionospheric current systems and produce extremely low frequency (ELF) and very low frequency (VLF) radiation that was observed on the earth's surface (e.g., Barr et al., 1984; Ferraro, et al., 1984). In some cases this longwave radiation was detected at great distances from the source [Lunnen et al., 1985]. The physical mechanism is modulated heating of the lower ionosphere which, in turn, causes modulation of the electron collision frequency and ionospheric conductivity. These modulated currents act as elevated virtual transmitters that excite the transverse electromagnetic (TEM) mode in the earth-ionosphere waveguide.

Although there are other important phenomena induced by ionospheric heaters, the prospect of using ambient ionospheric currents to radiate ELF/VLF signals provided prime motivation for developing the HF Active Auroral Research Program (HAARP). The HAARP experiment facility, scheduled for construction in Alaska, will be a powerful, high-gain ionospheric heater that will radiate in the lower HF band. It is therefore important to predict the ELF/VLF radiation that HAARP will generate, given various choices of carrier and modulation frequency. This report describes a fully computerized method that we have developed for making such predictions and presents some results that illustrate the effects of certain parameter trade-offs.

It is mandatory that calculations of longwave waveguide excitation by ionospheric currents use full-wave methods [Pitteway, 1965; Budden, 1955; Clemmow and Heading, 1954] because the wavelengths exceed scale heights for charged-particle densities and collision frequencies thereby violating conditions for application of eikonal methods. Approximate treatments that include only height-integrated absorption in propagation calculations [Tripathi, et al., 1982] ignore the important phenomenon of gradient reflection.

There are two possible full-wave approaches to calculating the longwave waveguide excitation by sources in the ionosphere. The first is the so-called forward approach, which starts with the actual elevated source and calculates the radiation into the anistropic, ionospheric plasma and subsequent downward propagation through the lower ionosphere into the waveguide. The second approach starts with a calculation of waveguide excitation and leakage into the ionosphere from a ground-based source--and then uses the principle of reciprocity to exchange the ground-based source for the desired ionospheric source [Galejs, 1973]. In a classic paper, Pappert [1973] carried out detailed numerical calculations to demonstrate that the forward and reciprocal approaches gave identical results, as predicted by the reciprocity theorem.

In principle, there is little to choose between the forward and reciprocal approaches; both are extremely complicated. In practice, however, the reciprocal approach is much easier to implement because sophisticated computer codes have already been developed to calculate ELF/VLF earth-ionosphere waveguide excitation by ground-based antennas. Most of the work therefore has already been done. We simply use an existing waveguide code to calculate ionospheric fields produced by ground-based sources, then apply a simple transformation to exchange source and receiver, and hence calculate the ground-based field generated by an ionospheric source. The core of this procedure is nothing more than calculation of waveguide height-gain functions as presented, for example, by Ferguson and Hitney [1987] or Field, et al., [1986]. We will use reciprocity in conjunction with a well established waveguide propagation code to calculate the results given in this report. Because we consider only a single waveguide mode, our formulation applies to frequencies below 5-to-10 kHz, at higher frequencies, higher order modes must be included.

We are not the first to apply reciprocity to calculate ground-level ELF/VLF fields excited by modulated ionospheric currents. Barr and co-workers [1984a, 1984b, 1987, 1988] calculated fields generated by modulating the TROMSO HF heater at frequencies between 100 Hz and 1 kHz. Carroll [1986] performed similar calculations for frequencies between 1 and 5 kHz. Our approach is similar in concept, but uses a somewhat more advanced waveguide code and an ionospheric model that accounts for heavy ions as well as electrons and allows for arbitrary orientation of the geomagnetic field. An important extension of the earlier analysis mentioned above is our inclusion of multiple dipole sources which allows us to assess how radiation from different height-regions of a distributed ionospheric current system will interfere.

Section 2 gives numerical results for waveguide excitation frequencies between 50 Hz and 1 kHz. The graphs show the effects of changing dipole orientation and altitude, and we use them to assess the suggestion of Papadopoulos, et al., [1990] that conductivity modulation should be carried out at an altitude of 100 km rather than at lower altitudes. We also challenge the routine use of the "lumped-current" approximation, wherein the actual height-distributed current system is represented by a current sheet at its centroid [Barr and Stubbe, 1984].

Section 3 gives our conclusions; Section 4 is bibliography; the Appendix describes our mathematical procedure.

Section 2
RESULTS

Throughout this section we present graphical results that illustrate the effectiveness of various electric dipole configurations in exciting the earth-ionosphere waveguide. (The full-wave computational method is described in the Appendix.)

MODEL IONOSPHERES

Figure 1 shows the model daytime and nighttime electron- and ion-density profiles used for our calculations. These profiles are typical of nominal models in current use. We assume a high-latitude electron gyrofrequency of 1.1 MHz and an average ion mass of 29 AMU. The assumed electron and ion collision frequencies ν_e and ν_i are:

$$\nu_e = 1.8 \times 10^{11} e^{-1.5z}$$

$$\nu_i = 0.025 \nu_e$$

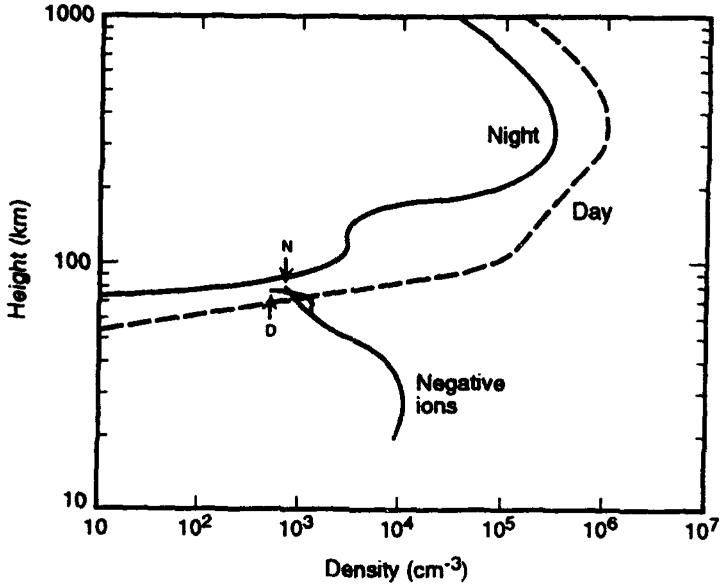

Figure 1. Nominal electron and ion densities in the ionosphere.

WAVEGUIDE EXCITATION EFFICIENCY

We consider modulation frequencies below 1 kHz; therefore we only need to consider excitation for the TEM mode because all other modes are below cut-off and do not propagate. This single-mode situation makes it possible to express results in terms of a single excitation efficiency that is independent of lateral distance from the source. At higher frequencies where many modes propagate it is necessary to calculate an excitation efficiency for each mode, a procedure that greatly complicates the analysis.

The efficiency $\varepsilon_{\alpha\beta}$ is defined as the ratio of TEM mode field radiated by an elevated electric dipole to the field that would be radiated by a ground-based electric dipole. An electric dipole can be either horizontal or vertical, and we denote those two orientations, respectively, by the subscripts H and V. Our convention is to use the first subscript (α) to denote the polarization of the ionospheric dipole and the second subscript (β) to denote the polarization of the ground-based reference dipole. For example ε_{HH} denotes the efficiency of an elevated horizontal electric dipole relative to a horizontal ground-based electric dipole, whereas ε_{HV} denotes the efficiency of an elevated horizontal dipole relative to a vertical ground-based dipole.

We will present results for three frequencies: 50 Hz, 150 Hz, and 1 kHz. The two lower frequencies span the band normally considered for the shore-to-submarine ELF communications. Because ground-based ELF transmitting antennas are horizontally oriented, we normalize our results at 50 and 150 Hz to a ground-based HED and calculate the efficiency ε_{HH}. On the other hand, a 1 kHz ground-based transmitter (if one existed) would more likely be vertical than horizontal, so we normalize our 1 kHz calculation to a ground-based VED and present graphs for ε_{HV} and ε_{VV} . Although useful for comparing the effectiveness of elevated versus ground-based sources, the particular normalization convention is actually of little consequence for our purpose, which is to compare the relative efficiencies of modulated heating at various ionospheric heights.

Figures 2 through 5 show how the excitation efficiencies of HED and VED antennas vary as the altitude of the antenna is increase from 50 km to 150 km. These altitudes span the ones at which ionospheric conductivity might be modulated by a ground-based HF heater. The results were calculated for nearly vertical (77 deg dip angle) geomagnetic field lines and apply reasonably well to any high geomagnetic latitude. The assumed ground conductivity σ_g of 3 x 10^{-4} s/m was chosen to represent conditions at the Navy's Wisconsin/Michigan ELF sites. That choice is important for the HED efficiencies ε_{HH} and ε_{HV}, but the VED efficiencies ε_{VV} are virtually independent of ground conductivity at the frequencies considered.

Figures 2 and 3 show the HED efficiencies at frequencies of 50 and 150 Hz for daytime and nighttime conditions, respectively. As in the case for ground-based ELF transmitters, the end-fire mode is far superior to the broadside mode in the lowest ionosphere; specifically, modulation altitudes below about 80-to-90 km. However, this distinction between end-fire and broadside modes vanishes once the HED is higher than 90 km because the waves at such high altitudes become circularly rather than linearly polarized.

In the end-fire mode the ionospheric horizontal antenna is an order of magnitude more efficient than a ground-based ELF antenna, provided its altitude does not exceed 80-to-90 km. That improved efficiency occurs because destructive interference from return currents in the ground is diminished as the antenna becomes less proximate to the ground. That increase efficiency is lost, however, as the source is raised to altitudes above 80-to-90 km because absorption and gradient reflection in the lower ionosphere become severe if the elevation is too great. In the daytime, the highest efficiency occurs at altitudes around 70 km; if the HED source is raised from 70 km to 100 km, the efficiency drops by an order of magnitude which is equivalent to a 20 dB drop in useful radiated power. Figure 3 shows, as would be expected, that the drop in efficiency suffered by increasing the source altitude above 70 km is less severe at night than in the daytime, but it is still noticeable.

Figures 4 and 5 show efficiencies verses altitude for a frequency of 1 kHz. Note that these efficiencies are normalized to a ground-based VED rather than an HED, and we have considered vertical as well as horizontal modulated ionospheric currents. As expected, the VED performs better than the HED at the lower altitudes, where it is aligned with the electric vector of the TEM mode. However, the HED is far more efficient than the VED at altitudes greater than about 80 km, where the source feels the full effect of the ionosphere. That behavior confirms Galejs' conclusion that electric dipoles aligned with the geomagnetic field in the ionosphere are "particularly ineffective" [Galejs, 1972]. We therefore concentrate on horizontal electric sources.

With regard to the important issue of optimum modulation altitude, Figures 4 and 5 indicate the same conclusions as can be made from Figures 2 and 3--namely, raising the source altitude from 70 km to 100 km will reduce the waveguide excitation efficiency by about 20 dB in the daytime and 10 dB at night. As discussed below, those reduced

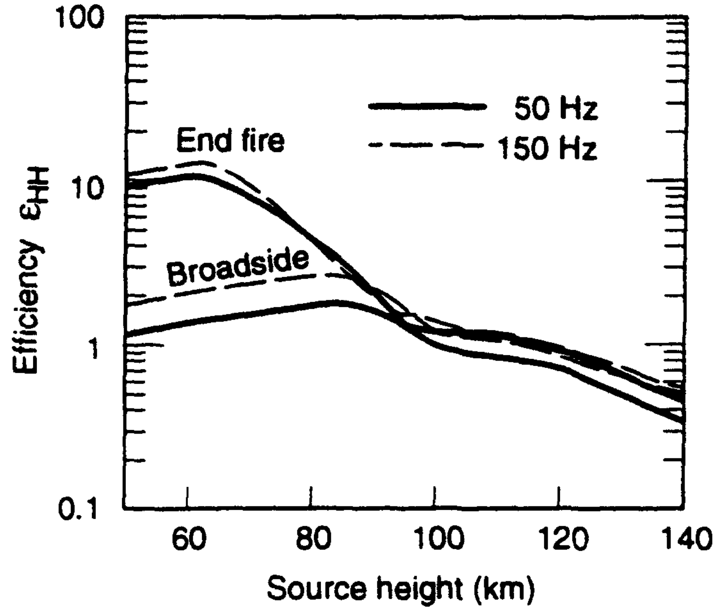

Figure 2. Daytime efficiency versus height of an elevated HED relative to an end-fire, ground-based HED; ground conductivity = 3 x 10^{-4} s/m.

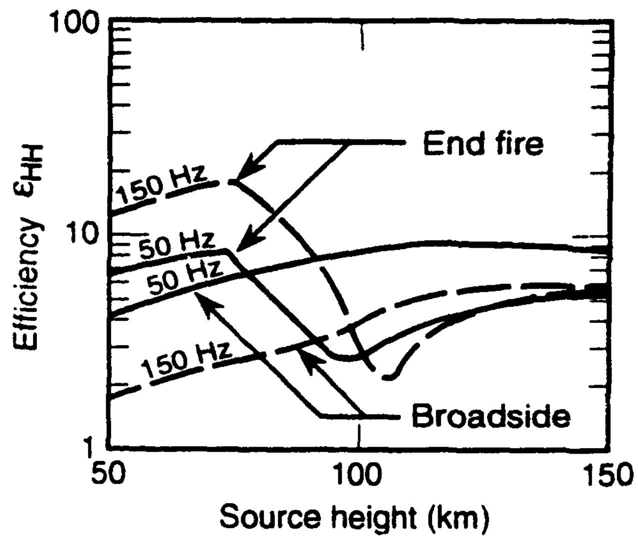

Figure 3. Nighttime efficiency versus height of an elevated high-latitude HED relative to an end-fire, ground-based HED; ground conductivity = 3 x 10^{-4} s/m.

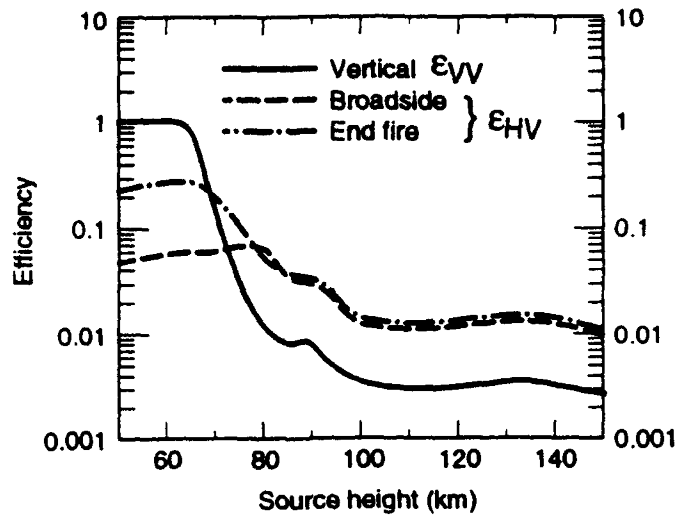

Figure 4. Daytime efficiency versus height of high latitude VED and HED sources relative to a ground-based VED; frequency = 1 kHz; ground conductivity = 3 x 10^{-4} s/m.

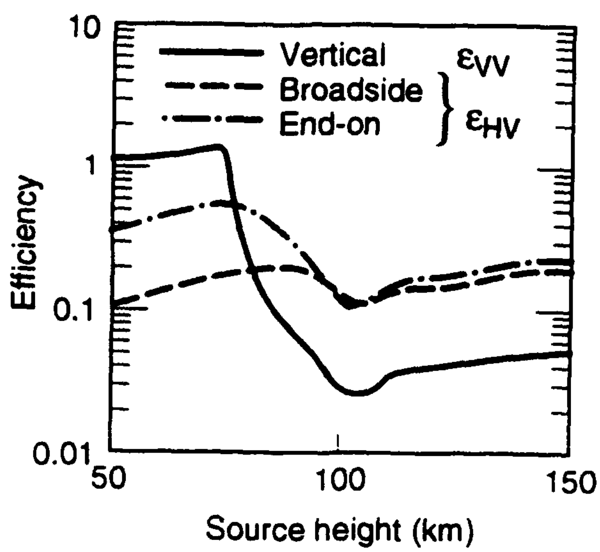

Figure 5. Nighttime efficiency versus height of high-latitude VED and HED sources relative to a ground-based VED; frequency = 1 kHz; ground conductivity = 3 x 10^{-4} s/m.

efficiencies must be traded off against the possibly stronger dipoles that can be created at increased altitudes.

OPTIMUM MODULATION ALTITUDE

Questions regarding the optimum altitude at which to modulate ionospheric currents were raised by Papadopolous et al. [1990], who showed that very powerful heaters produced much greater conductivity modulation $\Delta\sigma$ at an altitude of 100 km than at the more customary altitude of 70 km. That effect occurs because the electron distribution function becomes strongly non-Maxwellian at 100 km, whereas it remains essentially Maxwellian at the collision-dominated altitude of 70 km. This tendency for greater modulation to be possible at greater altitude competes with the tendency of the waveguide excitation efficiency ε to decrease at greater altitude as cited earlier. Recall that those efficiencies depend only on the state of the ionosphere, being calculated for dipole moments that are invariant with altitude.

Figure 5 presents results calculated by Papadopolous et al. [1990] and shows how the conductivity modulation at altitudes of 70 km and 100 km depends on the flux S incident at the altitude in question. Several comments must be made regarding Figure 6 prior to making conclusions. First, the electron density N_e was assumed to be 10^5 el/m^3 regardless of altitude--an imprecise assumption that is somewhat representative of daytime conditions, but not nighttime conditions. Second, the calculation assumed that the heater carrier frequency was 2.8 MHz, but did not account for losses suffered by this modifying wave in propagating upward from 70-to-100 km. Third, smaller modulation would occur at the specified incident flux levels if the carrier frequency were raised above 2.8 MHz. Fourth, and finally, Papadopolous et al. neglected anomalous self-absorption of the heating wave caused by increased temperature and, hence, increased electron-neutral collision frequency. Despite these inaccuracies, the curves in Figure 6 do show a trend that must be considered in designing future ionospheric modulation experiments.

Figure 6 shows that at an altitude of 70 km the conductivity modulation $\Delta\sigma$ saturates at an incident flux of about 3×10^{-3} W/m^2 ; further increases in flux do not produce corresponding increases in $\Delta\sigma$ and are therefore wasted. At an altitude of 100 km, however, the conductivity modulation increases strongly with increasing flux all the way up to flux values of 1 W/m^2 . This behavior is of little consequence for fluxes below 10^{-3} W/m^2 where there is little to choose between the two altitudes considered. However, for fluxes that exceed about 2×10^{-2} W/m^2 , the conductivity modulation at 100 km is 30-to-100 times stronger than at 70 km; i.e., it is theoretically possible to gain 30-to-40 dB in ionospheric source strength by modulating currents at an altitude of 100 km rather than 70 km.

We next ask whether the enhanced high-altitude modulation just discussed can be achieved with practical ground-based heaters--and to what extent these modulation gains are canceled by absorption and reflection losses in the lower ionosphere. In order to connect Figure 6 to realistic facilities, we have indicated order-of-magnitude incident

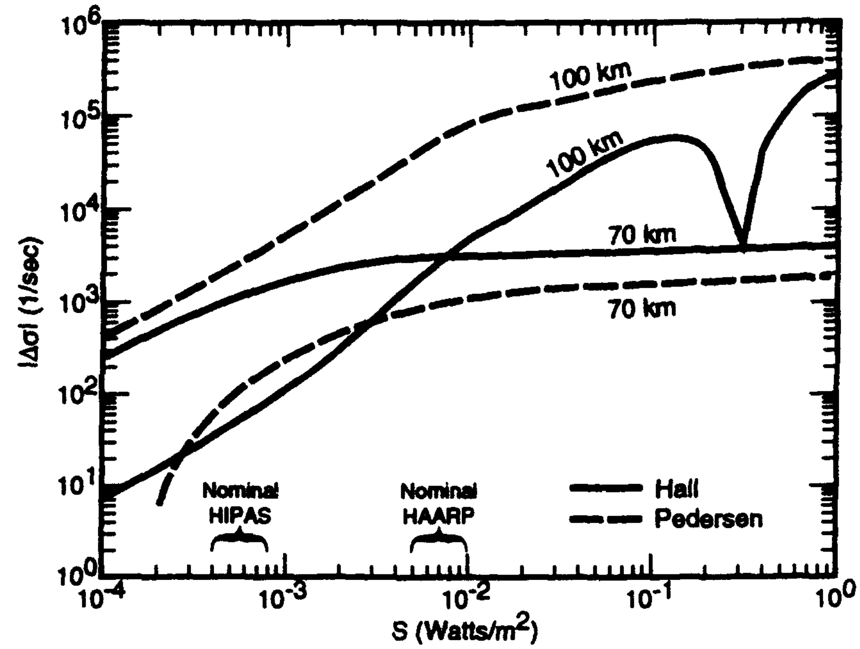

Figure 6. Conductivity modulation versus incident power flux [Papadopolous et al., 1990]; $N_e = 10^5$ el/cm^3.

fluxes produced by the existing HIPAS and developmental HAARP Alaskan ionospheric heaters. HIPAS can produce lower-ionospheric fluxes of many tenths of a milliwatt, whereas HAARP is expected to have an ERP that is roughly an order of magnitude greater. Please note that the HIPAS/HAARP fluxes indicated on Figure 6 depend on frequency and state-of-the-ionosphere and are thus nominal values intended for discussion purposes only.

In order to evaluate the trade-off between enhanced modulation and reduced waveguide excitation efficiency, compare Figure 6 to Figures 2 and 4, which show daytime values of ε_{HH} and ε_{HV} versus altitude. As discussed earlier, the excitation efficiency is about an order of magnitude (20 dB) worse at an altitude of 100 km than at 70 km. Figure 6 shows that for fluxes S that can be delivered by HIPAS, there is virtually no difference between the modulation $\Delta\sigma$ at 70 km and 100 km, so the net effect of modulating at 100 km would be to suffer a nearly 20 dB loss in waveguide excitation. There is clearly a disadvantage in trying to increase the modulation altitude of HIPAS.

The competition between excitation efficiency and modulation strength is more even for HAARP than HIPAS. Figure 6 shows that at fluxes that could be delivered by HAARP, about 20 times (26 dB) greater modulation can be achieved at 100 km than at 70 km, which gives a net 6 dB gain in waveguide excitation. However, that 6 dB gain could prove elusive because, for reasons cited above, it is more difficult for a heater to deliver flux to the greater altitude.

We conclude that modulating the current at 100 km versus 70 km altitude would be deleterious for HIPAS and, at best, of marginal value for HAARP. It could be advantageous for facilities much more powerful than HAARP, however.

A CRITIQUE OF THE "LUMPED DIPOLE" APPROXIMATION

In the "lumped current" approximation, the actual distribution current system is represented by a single elemental dipole located at the centroid of the actual system. The current in this effective dipole is give by

$$I_{eff} = \int_0^\infty \tilde{I}(z)dz \quad \text{amperes} \tag{1}$$

where $\tilde{I}(z)$ is the current density in a thin layer at height z. The strength P_{eff} of the effective dipole is simply

$$P_{eff} = I_{eff}L_{eff} \quad \text{ampere-meters} \quad , \tag{2}$$

where L_{eff} denotes the lateral dimension of the ionospheric current system. The location of the effective dipole is generally taken to be at the altitude where the current distribution peaks. The point of the lumped dipole approach is, of course, to substitute a single point source for a complicated distributed source, thereby avoiding the need for a Green's function solution. Unfortunately, as demonstrated below, the lumped dipole approximation is often invalid.

In order to illustrate the application (or misapplication) of the lumped current approximation, we will use height-profiles of Hall and Pedersen currents in the polar electrojet, as calculated by Barr and Stubbe [1984] for a "quiet daytime" ionosphere and reproduced in Figure 7. Those graphs show that the Hall current has a single peak between 75 and 80 km, whereas the Pedersen current has two peaks: one at about 72 km, the other at about 80 km. The magnitudes of the Hall and Pedersen current peaks are remarkably similar, differing by less than a factor of two. The phase of the Hall current is nearly constant over the altitude range shown; however, as indicated on the graph, the Pedersen current undergoes a nearly 180 deg phase change at about 75 km, so the current in the upper peak is almost equal and opposite to the current in the low peak. The height integral Eq. (1) of the Hall current therefore gives essentially the area under the curves, whereas the height integral of the Pedersen current is nearly zero.

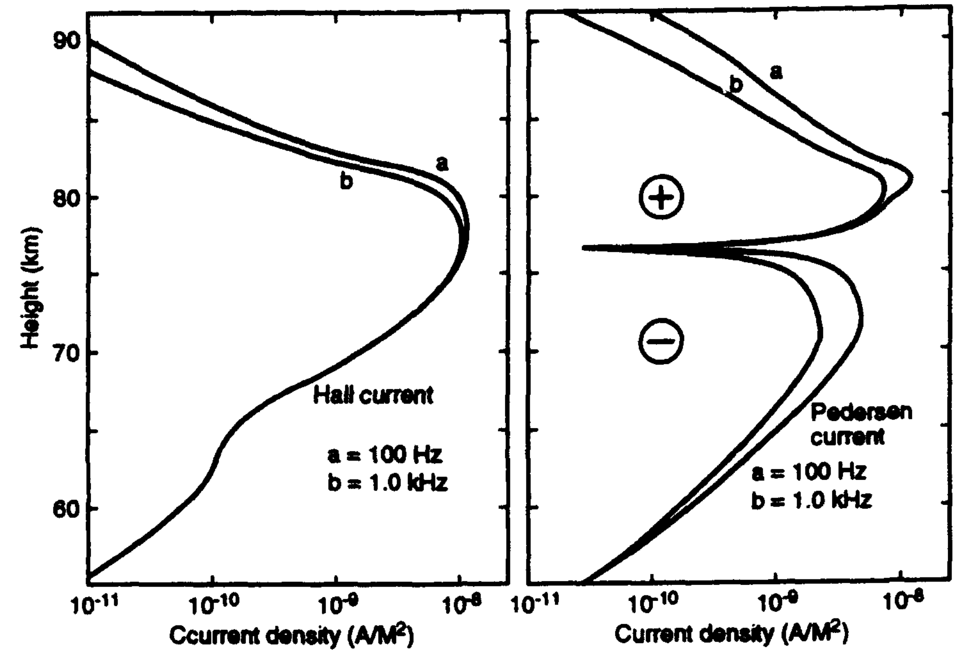

Figure 7. Magnitude of Pedersen and Hall current densities [Barr and Stubbe, 1984] for modulation frequencies of 100 Hz and 1 kHz.

Barr and Stubbe [1984] used the above reasoning to ignore the contribution of the Pedersen current--its effective moment given by Eqs. (1) and (2) is small--and represent polar electroject current by an elemental Hall-current-dipole centered at 78 km for the ionospheric model on which Figure 7 is based. For other mode ionospheres, they centered the Hall-current dipole at altitudes between 60 and 80 km as appropriate. The problem with that approach is that the currents should be weighted by the excitation efficiencies ε_{HH} and ε_{HV} prior to integration, and the integral in Eq. (1) is therefore valid only if the amplitude and phase of the excitation efficiencies are nearly constant over the 65-to-90 km altitude range. Such is not necessarily the case, particularly at frequencies around a kilohertz where local wavelengths in the ionosphere are short and substantial phase differences can occur over vertical distances of only a few kilometers.

Figure 8 shows the geometry that we use to test the validity of the lumped dipole approximation of Barr and Stubbe. We consider a primary horizontal dipole P_0 at an altitude of 70 km and a secondary dipole P_1 that has a moment equal to P_1, but is aligned either parallel or antiparallel to P_0. The secondary dipole is separated from the primary one by an altitude Δh and a range Δr. We calculate the remote ground-level vertical electric field E_0 and E_1 produced, respectively by P_0 and P_1 and sum those fields to obtain the total field $E_0 \pm E_1$, where the minus sign pertains to the anti-parallel dipole. In this manner we determine a critical separation Δh_0, Δr_0 within which $E_0 + E_1 \approx 2E_0$ and

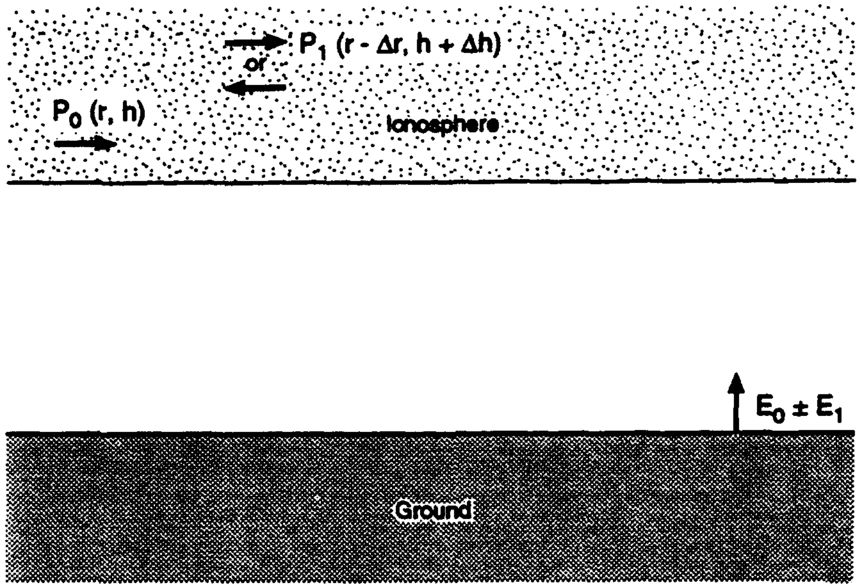

Figure 8. Schematic showing in-phase and out-of-phase sources separated by Δh in height and Δr in range.

$E_0 - E_1 << E_0$. The lumped dipole approximation may be safely used for current distributions whose dimensions do not exceed Δh_0, Δr_0; it should not be used for distributions whose dimensions exceed Δh_0, Δr_0.

Figure 9 shows the normalized superposition of ground-level fields

$$\tilde{E}_+ = \frac{E_0 + E_1}{2E_0} \tag{3}$$

from in-phase dipoles separated in altitude, but not range, and therefore pertains to the Hall current distribution. The lower dipole is assumed to be at an altitude of 70 km. The lumped-dipole approximation is valid for separations $\Delta h = h_1 - 70$, small enough that the normalized superposition is nearly unity. If we insist on an error no greater than, say, 30 percent, then the vertical separation must be small enough so $\tilde{E} > 0.7$. Figure 9 shows that this condition is satisfied provided that the upper HED is below: 83 km at a frequency of 50 Hz; 80 km at 150 Hz; and 76 km at 1 kHz. Figure 7 shows that the Hall current distribution spans roughly the 70-to-85 km altitude range. We conclude, therefore, that the modulated Hall currents can be represented by a lumped dipole, provided the frequency does not substantially exceed 100 kHz. However, application of that approximation to frequencies of 1 kHz or higher would lead to serious overestimate of the total field, so a full-fledged Green's function approach must be used at such frequencies.

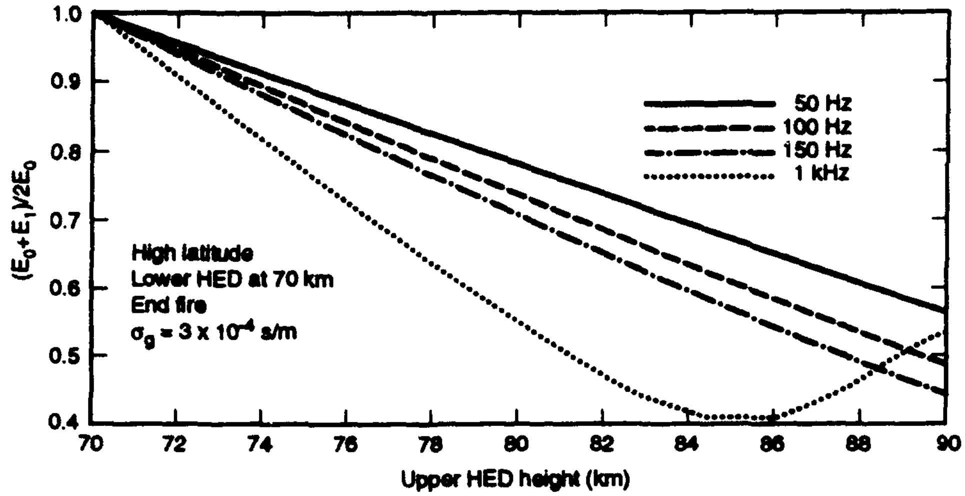

Figure 9. Daytime normalized field of in-phase HEDs separated in altitude. The lumped dipole approximation is valid when the normalized field is close to unity.

According to Figure 7, the Pedersen current distribution can be approximated roughly by a current sheet centered at about 70 km and an equal but anti-parallel (i.e., out of phase) sheet centered at about 80 km. As mentioned above, Barr and Stubbe argued that these two sources will cancel one another, so the Pedersen currents can be neglected in comparison to the Hall currents. In order to test that assertion, we calculate the quantity

$$\tilde{E}_- = \frac{E_0 - E_1}{2E_0} \tag{4}$$

versus the altitude of the upper dipole, holding the altitude of P_0 constant at 70 km. The lumped dipole approximation is valid only when $\tilde{E}_- << 1$. For the realistic case where the upper dipole is at 80 km, the results show that: $\tilde{E}_- = 0.2$ at a frequency of 50 Hz; 0.3 at 150 Hz; and 0.5 at 1 kHz. It follows, therefore, that the lumped dipole approximation is only marginally valid at the lowest ELF communications frequencies and is grossly inaccurate at frequencies above a few hundred hertz. It also follows that the Pedersen current cannot be neglected under daytime conditions.

We have also calculated $\tilde{E}_+$ and $\tilde{E}_-$ versus vertical separation for the nighttime ionosphere shown in Figure 1 and found that $\tilde{E}_+ \approx 1$ and $\tilde{E}_- << 1$ throughout the 70-to-90 km altitude range. The detailed graphs are uninteresting and therefore are not reproduced here. That result is expected because the lower ionosphere is nearly transparent at night. The lumped dipole approximation may safely be applied under nighttime ionospheric conditions.

Figure 11 shows the behavior of the total field as the second dipole is separated laterally from the first for a frequency of 1 kHz, the most stringent case considered in this report. As

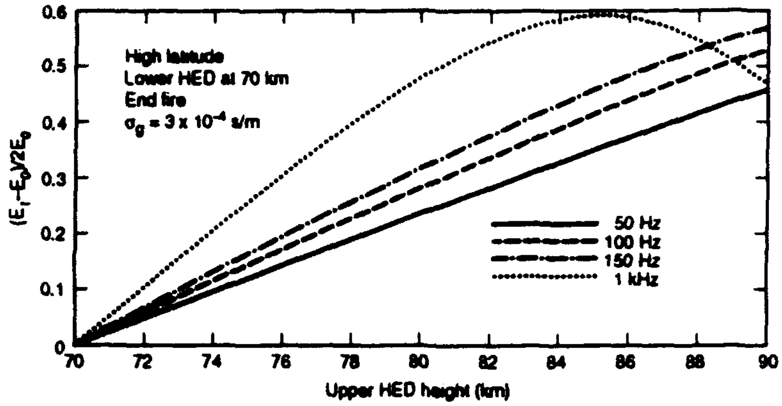

Figure 10. Daytime normalized field of out-of-phase HEDs separated in altitude. The lumped-dipole approximation is valid when the normalized field is much smaller than unity.

expected, the field is quasi-periodic with the spatial period being roughly equal to a free-space wavelength λ_0. The average field increases with increasing separations simply because in this example distance between the second dipole and the receiver is decreasing. The results show that

$$\begin{aligned} E_+ &\approx 1 \\ & \qquad \text{if } \Delta r < \frac{\lambda_0}{2\pi} \qquad (5) \\ E_- &\ll 1 \end{aligned}$$

which gives the upper limit on the lateral separation for which the lumped dipole approximation can be used.

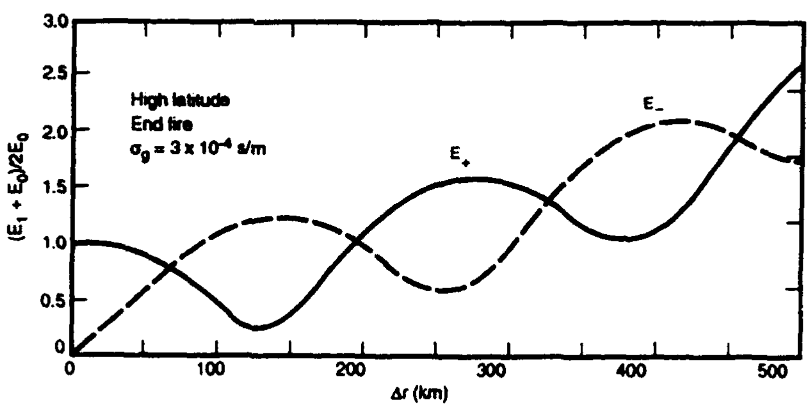

Figure 11. Daytime normalized fields, $\tilde{E}_+$ and $\tilde{E}_-$, of in-phase and out-of-phase HEDs at an altitude of 70 km and separated laterally by Δr. Range to most distant source 2 Mm; frequency = 1 kHz.

Section 3
CONCLUSIONS

In the end-fire mode, the waveguide excitation efficiency of an ELF HED in the ionosphere is up to 20 dB greater than for a ground-based antenna, provided its altitude does not exceed 80-to-90 km. The highest efficiency occurs for a source altitude of around 70 km; if that altitude is raised to 100 km, the efficiency drops by about 20 dB in the daytime and 10 dB at night.

The efficiency is calculated for an altitude-invariant dipole moment and does not account for the greater conductivity modulation that might be achieved at altitudes greater than 70 km. This trade-off between the altitude dependencies of the excitation efficiency and maximum achievable modulation depends on the ERP of the HF heater, the optimum altitude increasing with increasing ERP. For HIPAS the best modulation altitude is around 70 km, whereas for HAARP there might be marginal value in modulating at altitudes as high as 100 km. An unresolved issue that needs future work is whether unacceptable self-absorption is incurred by powerful heater-waves at altitudes above 70 km.

The often used lumped dipole approximation is always valid at night, but is invalid in the daytime for frequencies that exceed about 100 Hz. An additional restriction is that the lateral scale, L_{eff}, of the ionospheric current distribution must be smaller than the reduced, free-space wavelength, $\lambda_0/2\pi$, of the radiated ELF/VLF signal. When the lumped dipole approximation is not valid, it is necessary to use a full-fledged Green's function solution.

Section 4
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Appendix
EXCITATION OF EARTH-IONOSPHERE WAVEGUIDE BY ELEVATED SOURCES

We seek to calculate the dependence of ELF/VLF ground fields upon the elevation of ionospheric point dipole sources. Of particular practical interest is the comparison between the field at the receiving point due to a distant ground source and that from an elevated source the same lateral distance away. The ratio of these quantities, which varies with source height, will be called the "efficiency" of the elevated source with respect to a specific ground-based source. It is proportional to the so-called "height-gain" of a variable-altitude source as measured on the ground.

We begin by discussing briefly the mode-sum method of calculating fields radiated by sources in the earth-ionosphere waveguide, and then show how a general reciprocity principle can be used to convert those fields to ones radiated by sources in the ionosphere.

FULL-WAVE ELF/VLF PROPAGATION

In the spherical geometry of the earth-ionosphere waveguide (or, equivalently, a flat, Cartesian geometry with a modified refractive index) the wave equation separates into a lateral (x) dependence (as a Hankel function, with S, the sine of the complex modal angle as a parameter) multiplied by a complicated height-dependence, which must be calculated numerically for a given ionospheric profile, geomagnetic field, and propagation path. This height dependence is usually normalized to equal one on the ground and is called the "height gain." (Actually, the mode constant, S, is a by-product of iterating the height gain calculation; see Budden [1966]). The complete field is then the product of these spatial dependencies and a factor which accounts for the magnitude of the source dipole moment and for its orientation--the "excitation factor." Pappert and Bickel [1970] present the equations for this "mode sum approach," and many computer codes have been developed to calculate long wave signals in this fashion. The present appendix shows how this considerable existing computational capability can be easily applied to ionospheric sources.

In typical applications of the mode-sum approach, of course, the sources are located on the ground. With elemental sources in free space on the ground, the excitation factors that determine the far field radiated by these sources are particularly simple. Such is not the case when sources are located within the anisotropic ionospheric medium. Pappert [1973] shows how the ground-level fields from such an elevated source can be calculated directly. That approach decomposes the source region fields into Fourier spectra of downgoing magnetionic components; the sources are assumed to be embedded at the bottom of a semi-infinite, homogenous, but anisotropic half-space. Each Fourier component thus determined is continued downward by a so-called "full wave" solution for the height dependence of the wave equation. This "forward" approach to the problem of ionospheric sources is very complicated, mostly because the requisite computer codes have not been developed to the same extent as the mode-sum codes for ground-based sources.

The difficulties that inhere in this so-called "forward" approach to finding the height dependence of wave fields in the guide are avoided altogether by a second method, which we adapt from Galejs [1972]. A generalized theorem of reciprocity for anisotropic media allows us to solve a "reciprocal" problem in which a fictitious source (corresponding to the actual receiver polarization) is located on the ground, in free space. The full wave height gains for the fields are thus integrated to obtain ionospheric fields. The excitation factors for the fictitious elemental source on the ground are particularly simple (see Pappert and Bickel [1970]). The reciprocity method allows us to easily use existing mode-sum codes and take combinations of current moments and magnetic moments in the ionosphere and find the fields at some point on the ground, at any lateral distance not too close to the source or antipode. Closely related to this variation is what we shall call the "coupling efficiency" of the ionospheric source. This is defined as the ratio between some field quantity produced by the elevated source at the receiver--often E_z-- and the field produced by a source located on the ground directly below. Since the fields produced by both the elevated and comparison sources have identical lateral (x) dependence, it is clear that the coupling efficiencies are equal to the height gain functions of the elevated source, normalized to a particular value on the ground.

RECIPROCITY

As indicated, the problem of wave field excitation from a source within the ionospheric plasma is complicated. However, we may use a generalized principle of reciprocity and recast the problem into one involving fictitious ground-based sources and their corresponding excitations--which problem, in principle, is already solved. The reciprocity relation connects the actual fields from the given elevated source to what are called "reciprocal fields" due to the fictitious ground sources. (The choice of these "dummy" ground sources is wholly arbitrary; this degree of freedom will be exploited to greatly simplify the problem.) The reciprocal fields are calculated in what we shall call the "reciprocal geometry." In the reciprocal geometry, Maxwell's equations and all subsidiary relations are rewritten with the earth's geomagnetic field vector reversed. In addition, all relations between sending and receiving point are interchanged. This will be made explicit in what follows.

The well-known reciprocity between transmitter and receiver in a linear, isotropic medium has a generalization to the anisotropic ionospheric medium. Galejs [1972], citing Ginzburg [1970], states it thus for electric dipoles:

$$\bar{p}_1 \cdot \bar{E}_2(\bar{r}_1;\bar{p}_2;+H_0;\alpha) = \bar{p}_2 \cdot \bar{E}_1(\bar{r}_2;\bar{p}_1;-H_0;\alpha+180) \tag{A1}$$

and

$$\bar{p}_1 \cdot \bar{E}_3(\bar{r}_1;\, \bar{p}_3;\, +H_0;\, \alpha) = \bar{p}_3 \cdot \bar{E}_1(\bar{r}_3;\, \bar{p}_1;\, -H_0;\, \alpha+180) \quad . \tag{A2}$$

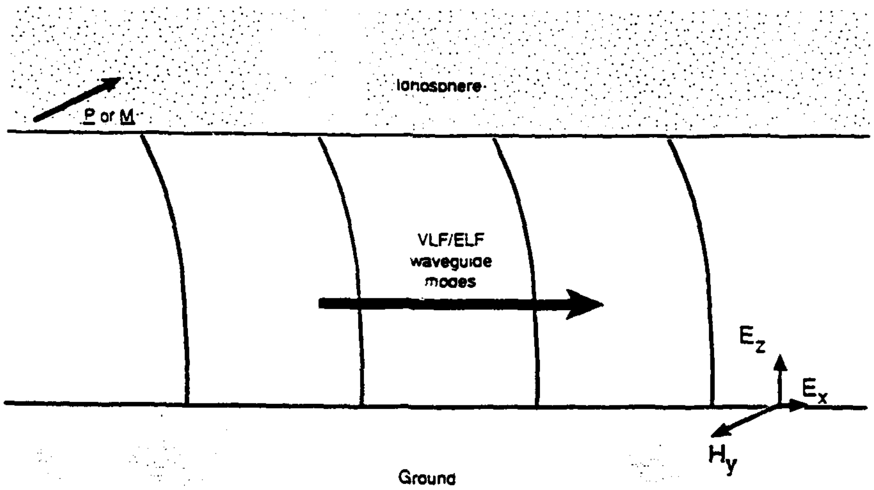

Figure A1. Schematic of the propagation, showing ionsopheric sources and primary TEM/TM waveguide mode field components.

$\bar{p}_2$ is the electric dipole moment of the given ionospheric source.

$\bar{E}_2(\bar{r}_1; \bar{p}_2; ...)$ is the electric field on the ground at the receiver location ("1") produced by source $\bar{p}_2$.

$\bar{p}_1$ is the dummy source at the receiver (at point $\bar{r}_1$), which may be picked to simplify the calculation.

$\bar{E}_1(\bar{r}_2; \bar{p}_1; ...)$ is the field at location $\bar{r}_2$ produced by the dummy source $\bar{p}_1$ at $\bar{r}_1$, calculated in the reciprocal geometry--we call it the reciprocal field and it is calculated using waveguide computation plus full-wave integration. Geomagnetic field and azimuth of propagation are reversed in the calculation.

$\bar{p}_3$ is a comparison source on ground below $\bar{p}_2$ to be used for the efficiency calculation.

$\bar{E}_3(\bar{r}_1; \bar{p}_3; ...)$ is the field at the receiver $\bar{r}_1$ produced by $\bar{p}_3$ to be used for efficiency calculation.

$\bar{H}_0$ is the static geomagnetic field vector.

α is the azimuth east of magnetic north for the path from $\bar{r}_2$ to $\bar{r}_1$.

At once we see that by choosing $p_1 = \hat{e}_x, \hat{e}_y$, or $\hat{e}_z$, we obtain

$$E_{z2}(\bar{r}_1; \bar{p}_2; +H_0; \alpha) = \bar{p}_2 \cdot \bar{E}_1(\bar{r}_2; \hat{e}_z; -H_0; \alpha+180)$$
$$E_{y2}(\bar{r}_1; \bar{p}_2; ...) = \bar{p}_2 \cdot \bar{E}_1(\bar{r}_2; \hat{e}_y; ...)$$
$$E_{x2}(\bar{r}_1; \bar{p}_2; ...) = \bar{p}_2 \cdot \bar{E}_1(\bar{r}_2; \hat{e}_x; ...)$$

We will always consider reciprocal fields excited by the dummy source $\bar{p}_1$ at $\bar{r}_1$ equal to the unit VED $\hat{e}_z$. Thus the problem assumes the simplified form

$$E_{z2}(\bar{r}_1; \bar{p}_2) = \bar{p}_2 \cdot \bar{E}_1(\bar{r}_2; \hat{e}_z; -H_0; \alpha+180) \quad ,$$

or in words:

$$\begin{matrix}\text{Vertical electric field on ground} \\ \text{produced by elevated source } p_2\end{matrix} = \bar{p}_2 \begin{bmatrix}\text{Reciprocal field at point} \\ \bar{r}_2 \text{ by unit VED on ground} \\ \text{at point } \bar{r}_1 .\end{bmatrix}$$

(where it is always understood that the reciprocal field is calculated with respect to reversed azimuth E of North, reversed geomagnetic field $-H_0$.) Figure A2 diagrams the geometry of these sources.

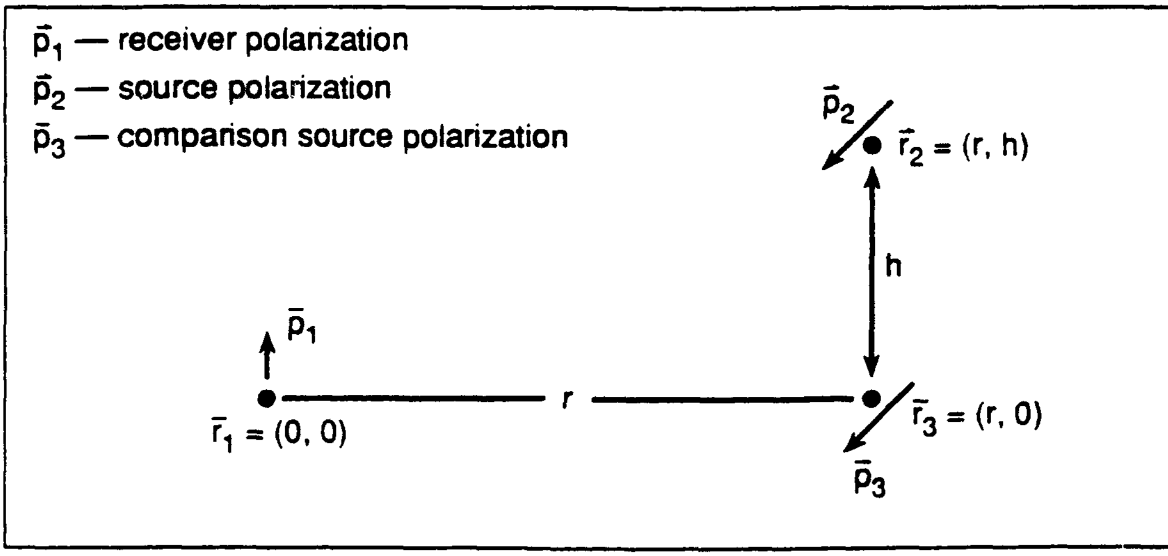

Figure A2. Diagram of reciprocity geometry.

Magnetic field components are easily obtained from electric fields on the ground from the relations

$$H_x(0) = q_g E_y(0) \quad ,$$
$$H_y(0) = -E_z(0)/S \quad ,$$

and

$$H_z(0) = S E_y(0) \quad ,$$

where $q_g^2 = n_g^2 - S^2$, n_g is the refractive index of the ground, and S is the sine of the complex modal angle. S is computed using widely available mode-sum computer codes.

The reciprocity relations (A1) and (A2) are further generalized to include magnetic fields and magnetic dipoles as

$$\overline{p}_1 \cdot E_2(1) - \overline{m}_1 \cdot \overline{H}_2(1) = \overline{p}_2 \cdot \overline{E}_1(2) - \overline{m}_2 \cdot \overline{H}_1(2) \tag{A3}$$

and

$$\overline{p}_1 \cdot \overline{E}_3(1) - \overline{m}_1 \cdot \overline{H}_3(1) = \overline{p}_3 \cdot \overline{E}_1(3) - \overline{m}_3 \cdot \overline{H}_1(3) \quad , \tag{A4}$$

where it is understood that the subscript refers to the source, the argument refers to the receiver position, and that fields having subscript 1 are calculated in the reciprocal geometry.

These reciprocal fields and their corresponding mode constants are calculated with both the geomagnetic field and the azimuthal bearing east of magnetic north reversed; the ground conductivity assumes the value at the receiver, as does the geomagnetic dip angle.

A simple calculation shows that the reversal of both the azimuthal bearing and the geomagnetic field vector amounts to replacing the direction numbers (l, m, n) of the geomagnetic field with the numbers (l, m, -n).

In summary, the problem of determining the ground field dependence on the altitude of an ionospheric source is transformed into that of determining the altitude variation of fields from a fictitious ground transmitter in the reciprocal geometry, which is handled easily using available computer codes.

Relative Efficiency Of Ionospheric Sources

By taking the ratio of Eqs. (A3) and (A4) we obtain

$$\frac{E_{2z}(1)}{E_{3z}(1)} = \frac{\underline{p}_2 \cdot \underline{E}_1(2) - \underline{M}_2 \cdot \underline{H}_1(2)}{\underline{p}_3 \cdot \underline{E}_1(3) - \underline{M}_3 \cdot \underline{H}_1(3)} \quad , \tag{A5}$$

which is the ratio for the fields produced, respectively, by ionospheric and ground-based sources at the same lateral range, r, calculated in the reciprocal geometry. Galejs [1972] showed that this relation applies individually to each waveguide mode as well as to the total field. Moreover, the range dependence is the same between numerator and denominator in each ratio. Therefore, the efficiency for a given mode will be proportional only to the height-gain of the ionospheric or reciprocal source and does not depend on range, r.

In practice we'll want efficiencies that compare sources in the ionosphere to standard antennas on the ground. There are two standard ground antennas

a) Endfire HED: $\underline{p}_3 = p_3\hat{e}_x$; $m_3 = 0$ [Standard ELF Communication]

b) VED: $\underline{p}_3 = p_3\hat{e}_z$; $\underline{M}_3 = 0$ [Standard VLF Communication]

Results comparing ionospheric sources to these standard antennas are as follows:

A) VED in ionosphere vs. that VED on ground

$|p_3| = 1\hat{e}_z$; $\underline{p}_2 = \hat{e}_z$; $\underline{M}_2 = 0$; $\underline{M}_3 = 0$

$$\frac{E_{2z}(1)}{E_{3z}(1)} = \frac{E_{1z}(r,h)}{E_{1z}(r,0)}(\text{due to } p_1 = \hat{e}_z) = \varepsilon_{VV}(h) \tag{A6}$$

Because E_{1z}(r,h) and E_{1z}(r,0) have the same r-dependence, ε_{VV} is simply the height gain of E_z. Note: to get absolute field $E_{2z}(1)$, calculate $E_{3z}(1)$ the old fashioned way and multiply by ε_{VV}(h) (height gain).

B) HED in ionosphere vs. HED on ground

$|p_3| = 1\hat{e}_x$; $\underline{p}_2 = \hat{e}_x$; $\underline{M}_2 = 0$; $\underline{M}_3 = 0$

$$\frac{E_{2z}(1)}{E_{3z}(1)} = \frac{E_{1x}(r,h)}{E_{2x}(r,0)} = \varepsilon_{HH} \tag{A7}$$

C) VED in ionosphere vs. HED on ground

$|p_3| = 1\hat{e}_x$; $p_2 = 1\hat{e}_3$; $\underline{M}_2 = \underline{M}_3 = 0$

$$\frac{E_{2z}(1)}{E_{3z}(1)} = \frac{E_{1z}(r,h)}{E_{1x}(r,0)} = \frac{E_{1z}(r,h)}{E_{1z}(r,0)}\frac{E_{1z}(r,0)}{E_{1x}(r,0)} = \varepsilon_{VH}(h) \tag{A8}$$

where

$$\varepsilon_{VH} = (\varepsilon_{VV})\left(\frac{E_{1z}(r,0)}{E_{1x}(r,0)}\right)$$

and $\dfrac{E_{1z}(r,0)}{E_{1x}(r,0)}$ is the so-called wave-tilt and is known from boundary conditions at ground.

D) HMD in ionosphere vs. HMD on ground

$P_3 = p_2 = 0$; $\underline{M}_3 = \underline{M}_2 = \hat{e}_y$

$$\frac{E_{2z}(1)}{E_{3z}(1)} = \frac{H_{1y}(r,h)}{H_{1y}(r,0)} = \mathcal{E}^{m}_{HH}(h)$$

and $\mathcal{E}^{m}_{HH}$ is simply the height gain function of H(r,h), the magnetic field radiated by an HMD at ground-level. These results can be related to the more typical case of a ground-based VED by noting that a ground-based HMD of moment M behaves like a VED of moment $2\pi M/\lambda$.

PROPAGATION THEORY

What follows is framed in the reciprocal geometry. Thus, both the geomagnetic field and the azimuthal bearing are reversed; the ground conductivity assumes the value at the receiver, as does the geomagnetic dip angle. As usual, the x-axis is taken to be in the direction of the propagation.

The propagation of long waves is most simply treated by the earth-ionosphere waveguide modal decomposition. In the case of ELF/VLF propagation the lowest few are so strongly attenuated as to be of no practical interest. Thus, a great simplification is achieved. The reciprocal field components, E_x, E_y, E_z, are everywhere in (x,z)-space represented by a product of a function of height (z), normalized to be on the ground, called the height-gain, the lateral dependence of each mode, which is

$$\frac{e^{-ikS_n x}}{\sqrt{a \cdot \sin(x/a))}} \quad ,$$

where S_n is the sine of the n^{th} complex modal angle, k the free space wave number, a the radius of the earth, and constants depending on the transmitter (excitation factors).

The modes are calculated, for a given ionospheric profile and propagation path (ground conductivity, geomagnetic dip and path azimuth east of geomagnetic north) using a well tested mode-finding program [Morfitt and Shellman, 1976]. The modal constant C is used in the differential equations that determine the height-dependence of the fields. Following Budden [1966]; Carroll [1986]; Pitteway [1965]; and Ferguson [1987], we integrate numerically a set of four complex differential equations for the height gains e_x, e_y, h_x, h_y :

$$\frac{d\bar{e}}{dz} = -ikT\bar{e} \quad , \qquad \bar{e} = \left(e_x, -e_y, h_x, h_y\right);$$

T is the (4 x 4) matrix in Budden [1966], Eq. (18.17).

The quantities e_z and h_z are computed secondarily, using:

$$e_z = -\left(S \cdot h_y + m_{31}e_x - m_{33}e_y\right)/\left(1+m_{33}\right)$$
$$h_z = S \cdot e_y$$

where m is the (3 x 3) susceptibility matrix [Budden, 1966, Eq. (3.24)].

The integration yields what is known as the full-wave solution for the height-dependence of the field quantities in the anisotropic medium. After these functions are fully determined, they are normalized to one on the ground and then combined as a product with the above mentioned lateral dependence term. This completely describes the spatial dependence of the wave fields.

For each reciprocal field component, E_x, E_y, E_z, there are three transmitter factors which correspond to each of the three elementary dipoles (see Pappert and Bickel [1970]). These factors are the so-called excitation factors for the particular waveguide mode. They depend explicitly on the modal parameter S and the reflection coefficients (which are a by-product of the computation of S).

As indicated above, the fictitious, ground-based transmitter will always be the vertical electric dipole (VED), insofar as it corresponds (by the reciprocity principle) to the measurement of the E_z component of the field at the receiving point. Thus the reciprocal fields we calculate are always normalized by the excitation factor for the elementary dipole in the vertical direction.