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13. ABSTRACT (Maximum 200 words) Differential algebraic equations arise in many applications including constrained mechanical systems. This project has investigated both their numerical solution and analytic properties. Among the many contributions has been the creation, analysis, and partial development of the first numerical method for general nonstructured solvable DAEs. 94 2 08 132 94-04432 DTIC QUALITY INSPECTED 5

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## FINAL REPORT

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1. S. L. Campbell, N. K. Nichols, and W. J. Terrell, *Duality, observability, and controllability for linear time varying descriptor systems*, Circuits Systems & Signal Processing, 10 (1991), 455-470.
2. S. L. Campbell, *Descriptor Systems in the 90's*, Proc. 29th IEEE Conference on Decision and Control, 1990, 442-447.
3. S. L. Campbell, *Least squares completions of nonlinear index three Hessenberg DAEs*, Proc. 1991 IMACS World Congress on Scientific Computation, Dublin, 1991, Vol. 3, 1145-1148.
4. S. L. Campbell, *2-D (Differential-Delay) implicit systems*, Proc. 1991 IMACS World Congress on Scientific Computation, Dublin, 1828-1829.
5. K. D. Clark, *Decomposition of Hessenberg DAE systems to state space form*, Linear Algebra & Its Applications, 172 (1992), 33-55.
6. S. L. Campbell and E. Moore, *Progress on a general numerical method for nonlinear higher index DAEs*, Circuits Systems & Signal Processing, (to appear).
7. S. L. Campbell and M. Rakowski, *Explicit formulae for completions of linear time varying singular systems of differential equations*, Circuits Systems & Signal Processing, (to appear).
8. S. L. Campbell and E. Griepentrog, *Solvability of general differential algebraic equations*, NCSU Math. Tech. Report NA.0292011, 1992.
9. S. L. Campbell, *Uniqueness of completions for linear time varying differential algebraic equations*, Linear Algebra & Its Applications, 161 (1992), 55-67.
10. K. D. Clark, *A framework for arbitrary index differential-algebraic equations in Hessenberg form*, Preprint, 1992.

11. S. L. Campbell, *Least squares completions for nonlinear differential algebraic equations*, Numerische Mathematik, 65 (1993), 77-94.
12. S. L. Campbell, E. Moore, and Y. Zhong, *Utilization of automatic differentiation in control algorithms*, Proc. 31st IEEE Conference on Decision & Control, pp. 122-127, 1992.
13. S. L. Campbell, *Nonregular descriptor systems with delays*, Proc. SINS '92, International Symposium on Implicit and Nonlinear Systems, Automation and Robotics Research Institute, Ft. Worth, Texas, pp. 275-281, 1992.
14. S. L. Campbell, *A survey of time varying and nonlinear descriptor control systems*, Proc. SINS '92, International Symposium on Implicit and Nonlinear Systems, Automation and Robotics Research Institute, Ft. Worth, Texas, pp. 356-363, 1992.
15. S. L. Campbell and C. W. Gear, *The index of general nonlinear DAEs*, Preprint, 1993.
16. S. L. Campbell and Y. Shtessel, *Sliding mode control and differential algebraic equations*, NCSU Math. Tech. Report NA.0292061, 1992.
17. S. L. Campbell, *Numerical methods for unstructured higher index DAEs*, Annals of Numerical Mathematics, (to appear).
18. S. L. Campbell and E. Moore, *Constraint preserving integrators for general nonlinear higher index DAEs*, Preprint, 1993.
19. S. L. Campbell, *Linearization of DAEs along trajectories*, Preprint, 1993.
20. S. L. Campbell, *High index differential algebraic equations*, Preprint, 1993.

#### 8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS REPORTING PERIOD.

Dr. Stephen L. Campbell received .5 month summer salary support during each summer of this project.

Mr. Edward Moore was hired as a Research Assistant in September of 1991. Mr. Moore is working on a Ph.D. in applied mathematics under the direction of Drs. Clark and Campbell. Mr. Moore is working on the computation of the solutions of nonlinear higher index differential algebraic equations. His thesis is titled *Constraint Preserving Integrators for Differential Algebraic Equations* and will complete in the fall of 1993. Mr. Moore will graduate in the Spring of 1994.

## 9. REPORT OF INVENTIONS (BY TITLE ONLY):

There were no inventions during this period.

### BRIEF OUTLINE OF RESEARCH FINDINGS

Differential algebraic equations (DAEs) are systems of differential and algebraic equations which have not been reworked into an explicit format. Many engineering problems are initially modeled as DAEs. Reduction to explicit models can be parameter dependent, time consuming, and require model simplification. Small design changes can require a complete rederivation of an explicit model. This project is to develop numerical methods for working directly with the original implicit equations.

Previously developed numerical methods for DAEs such as backward differentiation (BDF) and implicit Runge-Kutta (IRK) methods experience numerical difficulties with the higher index problems that arise in such areas as constrained mechanics, for example with models for vehicular systems. Even when these numerical methods can be used it is important, and often difficult, to determine sufficiently accurate initial conditions for all variables.

In 1988, one the principal investigators outlined a more general numerical method for nonlinear DAEs. While computationally more intensive than BDF and IRK methods, it had several potential advantages. It will work when the other methods do not. It can also be used to compute consistent initial conditions for use with either this method or for BDF and IRK integrators when they are appropriate. This general method involved symbolic differentiations of the original equations followed by a singular Newton's method which was carried out numerically.

One emphasis of this project has been to develop the theoretical basis of the general method. This has proven to be highly nontrivial but substantial progress has been made. In [3] we showed that nonlinear index three Hessenberg systems can solved by this method. This result is important since many mechanics and control problems have this form.

A closely related aspect of our research has been to carefully examine the information present in the derivative arrays which occur in the general method [1,3,8,9,11,15,18,19]. This information will eventually be used to develop nonlinear boundary value (BVP) codes along the lines suggested by Clark and Petzold. However, we have initially applied our results to determining control theoretic information for implicit control problems [1]. As shown in our earlier work, this will have direct application to such problems as tracking or path control for mechanical systems. These results are also relevant for classical methods since portions of the derivative arrays are often used to reformulate higher index DAEs.

In other related research, we have developed an algorithm for computing the state space decomposition of linear time varying Hessenberg systems of arbitrary index [5,10]. This

algorithm does not involve derivative array computations but does require symbolic differentiation of the coefficients and input functions. In principle it can be used to compute exactly consistent initial conditions for linear Hessenberg systems, and consequently to determine complementary conditions for the unique solvability of two-point boundary value problems for these systems. An analogous algorithm can be used to analyze the errors in BDF methods for arbitrary index Hessenberg DAEs to establish convergence under suitable smoothness assumptions on the starting values. These are the first results are linear Hessenberg systems without any restrictions on the index and resolve several open questions in the literature.

A second thrust of our research has been to develop numerical methods and guidelines for their use. Mr. Moore, the research assistant, is currently developing a software package that will carry out the entire general procedure for both integrating DAEs and for finding initial conditions. The user will need only to write down the equations and specified desired quantities. The software will then symbolically differentiate the equations several times, generate Jacobians of the corresponding derivative arrays, generate FORTRAN code for these arrays and Jacobians, and finally carry out the numerical calculations in FORTRAN. One advantage of this software is that the symbolic part will need to be done only once. Design and model parameters can then be varied in the numerical routines. [6] is the first report on this prototype code. Numerical results reported in [6] on the shuttle reentry problem of Brenan show that contrary to some prior expectations, the growth of symbolic expressions under repeated differentiations will not be such a major factor with the method thanks to the new optimize feature in the fortran command in MAPLE V. Rather, the dominant factor will usually be the dimension of the problem. Indications are that the prototype code will be able to handle moderate sized nonlinear DAEs (10-20 variables) of higher index (1-6) in reasonable time.

In many mechanical and control systems there are delays present. There has been essentially no prior work none on either the numerical or analytic solution of differential-delay DAEs. Initial results on this problem are in the invited [4,13].

Regardless of the integrator being used for general DAEs some type of differentiation is needed. While we have shown that this can be done for many problems with reasonable speed, it is highly desirable to be able to speedup this part of the computation. This will permit faster integration and the consideration of significantly larger problems. One software technology that looks promising in this regard is automatic differentiation. In conjunction with Mr. Zhong, a Ph.D. student of one of the PI's, we investigated the use of automatic differentiation with DAEs. [12] reports on our experience with the ADOL-C code of Dr. Griewank of Argonne National Laboratory. We found that automatic differentiation can result in a significant speed up of the computation, and a reduction in storage, under some operating conditions. In other scenarios automatic differentiation was actually more time consuming.

The general approach described in previous reports has several advantages. However, it has the disadvantage that it does not reimpose the explicit and implicit constraints that are present in a DAE. Thus there can be drift off the constraint manifold. This is often undesirable, particularly in simulations of vehicular and other systems. Mr. Moore has examined the use of the ideas from the general approach to design an integrator which will work for unstructured higher index DAEs and also conserve constraints. In [18] we outline the first general higher index DAE integrator which preserves constraints. The method is performing well in tests and has already integrated several DAEs that other methods cannot integrate.

Most of the prior work on DAEs has assumed that the system was solvable, that is that there is a well defined family of solutions. However, in applications there can occur situations where the system is not solvable. In mechanics this occurs, for example, when the object is in a singular configuration and there is a sudden change in the degrees of freedom. For a complex system it may not be obvious when this occurs. Recently there has been some work in the mathematical literature by Rheinboldt, Rabier, and Reich on proving solvability for nonlinear DAEs. This work, while important, is based on a recursive nonlinear version of the arguments used for the linear time invariant case. It can be very difficult to actually carry out these procedures. In [8] we go a long way toward rectifying this problem. We give direct characterizations of solvability that are not recursive in nature. These characterizations are under weaker assumptions than any other characterizations in the literature. We also show how one can actually prove solvability on a computer using a mixture of readily available symbolic and numeric software. An example is given from mechanics consisting of the path control of a robotic arm with a flexible joint. The standard graph theoretic approaches give an incorrect answer on this problem but our approach easily handles it.

Many problems, particularly in mechanics, possess a structure which is known as the Hessenberg structure. Index two and index three versions of this structure have been utilized throughout the numerical literature on DAEs. However, mechanical and other systems can have even higher index. In [5] we have established, for the first time, the necessary mathematical machinery for analyzing problems with Hessenberg systems of arbitrary index. In [10] we apply this mathematical machinery to analyzing the behavior of the numerical BDF (Backward Differentiation Formulas) methods on these systems. These results provide the definitive result on these systems which have been studied for the last decade in special situations. The results should also prove useful for other problems such as the control of Hessenberg type systems.

Dr. Marek Rakowski visited NCSU. Jointly he and one of the PIs examined whether his results on rational matrix functions could be useful for DAEs. In [7] they showed that this theory could be used to find *explicit* completions of linear time varying DAEs with rational matrix coefficients. While the general usefulness of this work is not clear, it will have at

least one important benefit. It will make possible the derivation of exact solutions for very complex linear time varying DAEs. This will be very useful for deriving test problems for testing more general numerical approaches on.

Building on the previous papers two additional important mathematical contributions have been made to the understanding of DAEs in the last year. There has been a perception in the literature that most of the concepts of index were equivalent. Spurred by the need to consider more complex systems we investigated this carefully in [15]. There we showed that the different types of index could be very different in value for fully implicit DAEs. We also provided an unifying theory and related these ideas to the numerical results in [8]. In addition, it is standard practice in engineering to linearize equations. Little work had been done on the linearizations of higher index DAEs. In [19] we showed that the usual time invariant linearizations about trajectories were inappropriate but that time varying linearizations could be appropriate for some applications.

In order for the research on DAEs to be useful it is important for the results to be brought to the attention of scientists and engineers, many of whom do not read the mathematical journals where the results are initially published. [2,14,17], which are primarily expository, are part of our effort to inform this larger audience of the current successes, and long term prospects, of the current research on implicit problems and their numerical solution. There is a misconception in some scientific communities that higher index DAEs are not common. In [20] we show that they can frequently occur, particularly when more complex models incorporating actuator and other control affects are included.

### POTENTIAL TECHNOLOGICAL APPLICATIONS

The early work on index one DAEs lead to the development of such numerical production codes as LSODI and DASSL, and more recent codes such as DASSLRT and RADUAIla. These codes are being used daily at laboratories for problems from chemical engineering to open loop mechanical system simulation. It is expected that the research of this project, combined with that going on elsewhere in the United States and in Europe, will lead to the development of production nonlinear higher index codes within the next few years. These codes will have immediate application in several areas. One such area is the design and simulation of complex multibody dynamical systems such as moving multibody vehicular systems and telerobots.

### CONTACTS WITH ARMY LABORATORY PERSONNEL

The PIs made a two day visit in 1991 to the US Army Tank-Automotive Command (TACOM) on May 1 & 2. During this visit they talked with Dr. R. A. Wehage, Dr. N. A. Rose, and Mr. J. L. Overman. The visit was to familiarize the PIs with the personnel and research interests of this group at TACOM. Particular topics of discussion were the

numerical simulation of mechanical systems (Wehage), nonlinear control of the Stewart Platform Simulator (Overholt), and a numerical problem arising in optics (Rose).

Dr. Rose had a problem that he thought might be handled better symbolically. One of the PI's (S. Campbell) subsequently held two telephone conversations with Dr. Rose and ran the symbolic calculations using the symbolic language MAPLE on a workstation at NCSU. The expressions turned out to be so complicated, however, that it was decided that it would be better to just evaluate them by standard approaches.

Since the numerical solution of DAEs is of direct interest to the Army for such purposes as vehicular simulation, a timely transfer of results and problems between laboratory personnel and university researchers is important. Drs. Clark and Campbell, in coordination with Dr. Linda Petsold of the University of Minnesota, organized an ARO workshop on Modelling and Analysis for Mechanical Systems and Algorithms for Real Time Simulation. The workshop was held November 5-7, 1992, at the Brownstone Hotel in Raleigh, NC. There were several Army Laboratory personnel involved in this workshop as well as University personnel and some researchers from laboratories in Europe. All indications are that the workshop was viewed as highly successful.

In addition Dr. Clark will be editing a special issue of the Journal of Mechanics of Structures and Machines which will consist of papers presented at, or arising out of, this workshop. Dr. Roger Wehage of TACOM may be assisting Dr. Clark in the editing.

#### ABSTRACTS OF RESEARCH MANUSCRIPTS

1. S. L. Campbell, N. K. Nichols, and W. J. Terrell, *Duality, observability, and controllability for linear time varying descriptor systems*

A characterization of observability for linear time varying descriptor systems  $E(t)x'(t) + F(t)x(t) = B(t)u(t)$ ,  $y(t) = C(t)x(t)$ , was recently developed. Neither  $E$  nor  $C$  were required to have constant rank. This paper defines a dual system, and a type of controllability so that observability of the original system is equivalent to controllability of the dual system. Criteria for observability and controllability are given in terms of arrays of derivatives of the original coefficients. In addition, the duality results of this paper lead to an improvement on a previous fundamental structure result for solvable systems of the form  $E(t)x'(t) + F(t)x(t) = f(t)$ .

2. S. L. Campbell, *Descriptor Systems in the 90's*

After briefly summarizing the growth in the theory of descriptor, or differential algebraic equations (DAEs), over the last two decades, the current challenges and potential successes will be discussed. This paper was the introductory paper for a special session on DAEs at the 29th IEEE Conference on Decision and Control, Dec. 1990.

3. S. L. Campbell, *Least squares completions of nonlinear index three Hessenberg DAEs*

Many control problems are most easily initially modeled as a nonlinear implicit system of differential and algebraic equations (DAEs),  $F(y'(t), y(t), u(t), t) = 0$ . Recently a numerical method has been proposed that can be used when classical methods cannot be used. This invited paper continues the examination of how general that method is. In particular, it is shown that nonlinear index three Hessenberg DAEs can be solved by this method.

4. S. L. Campbell, *2-D (Differential Delay) Implicit systems,*

This invited paper discusses what is known about linear time invariant differential-delay descriptor systems. The literature is surveyed and several observations are made.

5. K. D. Clark, *Decomposition of Hessenberg DAE systems to state space form*

An algorithm is given for symbolically decoupling the solutions of a linear, time dependent DAE,  $Ez' = A(t)z + f(t)$ ,  $z(t) \in \mathbb{R}^n$  in Hessenberg form into state and algebraic components. The state variables are the solutions to an ordinary differential equation with initial conditions restricted to a subspace of  $\mathbb{R}^n$  while the algebraic components are linear functions of the state variables which involve derivatives of the coefficients and input functions up to order  $r - 1$  where  $r$  is the index of the system. The implications of the algorithm for computing consistent initial conditions, for certain singular optimal control problems, and for numerical solutions are briefly discussed.

6. S. L. Campbell and E. Moore, *Progress on a general numerical method for nonlinear higher index DAEs*

A method has been proposed for numerically solving lower dimensional, nonlinear, higher index differential algebraic equations for which more classical methods such as backward differentiation or implicit Runge-Kutta may not be appropriate. This method is based on solving nonlinear DAE derivative arrays. This paper discusses progress on the implementation of this method, resolves some of the issues involved, and lists some remaining problems. Computational experience showing that the approach should prove practical for many applications is presented.

7. S. L. Campbell and M. Rakowski, *Explicit formulae for completions of linear time varying singular systems of differential equations*

Completions of linear time varying singular systems of the form  $E(t)x'(t) + F(t)x(t) = f(t)$  are explicitly computed using recent results on rational matrix functions. The

algorithm and the theory behind it are carefully described. Computational issues are discussed.

8. S. L. Campbell and E. Griepentrog *Solvability of general differential algebraic equations*

In the last few years there has been considerable research on differential algebraic equations (DAEs)  $f(t, x, x') = 0$  where  $f_{x'}$  is identically singular. Most of this effort has focused on computing a solution that is assumed to exist. More recently there have been existence results developed using differential geometry in a recursive manner that follows a pattern similar to that used in the linear time invariant case. In this paper we shall establish the existence of solutions using more direct characterizations. Verification of these conditions using readily available numerical and symbolic software is discussed. An example from robotics where classical graph theoretical approaches give an incorrect answer is worked to illustrate the usefulness of the characterization.

9. S. L. Campbell, *Uniqueness of Completions for linear time varying differential algebraic equations*

The extent to which a completion  $x' = G(t)x + \sum_{i=0}^r R_i(t)f^{(i)}(t)$  of a linear time varying differential equation  $E(t)x'(t) + F(t)x(t) = f(t)$  is unique is carefully examined. The implications for numerical methods for solving DAEs based on differentiated equations are discussed.

10. K. D. Clark, *A framework for arbitrary index differential-algebraic equations in Hessenberg form*

This paper presents an algebraic framework, related to a state space decomposition algorithm, which can be used to analyze the convergence behavior of backward differentiation formulas for the solution of differential-algebraic equations in Hessenberg form. The system considered is assumed to be linear time varying but is allowed to have arbitrary fixed index. In this preliminary report, the index two and three cases are discussed in detail to illustrate both the usefulness of the approach developed, and also the increase in complexity of the analysis as the index increases. However, the growth of complexity is regular in such a manner as to allow for the general case analysis. In the final version of this paper, the analysis will be extended to the arbitrary index linear case. The nonlinear case will be given in a subsequent paper.

11. S. L. Campbell, *Least squares completions for nonlinear differential algebraic equations*

A method has been proposed for numerically solving lower dimensional, nonlinear, higher index differential algebraic equations for which more classical methods such as backward differentiation or implicit Runge-Kutta may not be appropriate. This method is based on solving nonlinear DAE derivative arrays using nonlinear singular least squares methods. The theoretical foundations, generality, and limitations of this approach remain to be determined. This paper carefully examines several key aspects of this approach. The emphasis is on general results rather than specific results based on the structure of various applications.

12. S. L. Campbell, E. Moore, and Y. Zhong, *Utilization of automatic differentiation in control algorithms*

Symbolic languages are increasingly being used in the analysis and implementation of control algorithms. Many of these control procedures involve some type of differentiation or Jacobian formulation. Automatic differentiation provides an alternative means of computing this information which is rarely considered in the control literature. This paper will discuss the use of automatic differentiation in control problems. Numerical results comparing automatic and symbolic approaches on a nonlinear control system will be given.

13. S. L. Campbell, *Nonregular descriptor systems with delays*

This paper examines descriptor systems with delays which are in the form  $Bx'(t) + Dx(t) + Cx(t - \alpha) = Eu(t)$ . Previous papers have examined the case when  $\lambda B + D$  is a regular pencil. In this paper we examine the case when  $\lambda B + D$  is a singular pencil but  $\det(\lambda B + D + \omega C) \neq 0$ . Behavior not previously reported for descriptor systems is observed. Several basic issues are discussed.

14. S. L. Campbell, *A survey of time varying and nonlinear descriptor control systems*

The mathematical results on the structure of matrix pencils are well known in the control community. This theory has been heavily exploited by control theorists working with linear time invariant control problems in descriptor form. The structure and properties of time varying and nonlinear descriptor systems are less well known in the systems and control community since most of these results has been developed in recent years and published outside of the usual control literature by analysts and numerical analysts. This nonlinear theory has taken on a very different appearance from that of the linear time invariant theory. This survey paper discusses what is known about nonlinear and time varying descriptor systems and the ways in which they are different from linear time invariant descriptor systems. Both analytical results and numerical methods will be mentioned. Hopefully this will give the listener

some idea of what tools are available for attacking control problems and what types of assumptions may be necessary in order to apply these results.

15. S. L. Campbell and C. W. Gear, *The index of general nonlinear DAEs*

In the last few years there has been considerable research on differential algebraic equations (DAEs)  $F(t, y, y') = 0$  where  $F_{y'}$  is identically singular. Most of this effort has focused on computing a solution that is assumed to exist. More recently there has been some discussion of solvability of DAEs. There has historically been some imprecision in the use of the two key concepts of solvability and index for DAEs. The consideration of increasingly complex fully implicit nonlinear DAEs makes a clear and correct development necessary. This paper will try to clarify several points concerning solvability and the index. After establishing some new and more precise terminology that we need, some inaccuracies in the literature will be corrected. The two types of indices most frequently used, the differentiation index and the perturbation index, are defined with respect to solutions of unperturbed problems. Examples are given to show that these indices can be very different for the same problem. We define new "maximum indices," which are the maxima of earlier indices in a neighborhood of the solution over a set of perturbations and show that these indices are simply related to each other. These indices are also related to an index defined in terms of Jacobians of the problem.

16. S. L. Campbell and Y. Shtessel, *Sliding mode control and differential algebraic equations*

Sliding mode controllers can be advantageous in stabilization because of increased robustness under small disturbances. For some systems, implementation of a sliding mode controller can involve the solution of higher index differential algebraic equations (DAEs). The relationship between DAEs and sliding mode control will be examined in this paper. A voltage stabilizer example is carefully considered.

17. S. L. Campbell, *Numerical methods for unstructured higher index DAEs*

Many physical problems are initially modelled as systems of differential and algebraic equations (DAEs). The widespread occurrence of DAEs has led to an intensive examination of the numerical solution of DAE initial value problems. Several numerical methods have been developed for DAEs. Some of these methods are suited either for what are called low index problems or systems with special structure. There is a need for numerical methods for nonlinear higher index DAEs which do not have any of the obvious structure to exploit. This paper will survey what is currently known about general DAEs and methods for their numerical solution.

18. S. L. Campbell and E. Moore, *Constraint preserving integrators for general nonlinear higher index DAEs*

In the last few years there has been considerable research on numerical methods for differential algebraic equations (DAEs)  $f(x', x, t) = 0$  where  $f_{x'}$  is identically singular. The index provides one measure of the singularity of a DAE. Most of the numerical analysis literature on DAEs to date has dealt with DAEs with indices no larger than three. Even in this case, the systems were often assumed to have a special structure. Recently a numerical method was proposed that could, in principle, be used to integrate general unstructured higher index solvable DAEs. However, that method did not preserve constraints. This paper will discuss a modification of that approach which can be used to design constraint preserving integrators for general nonlinear higher index DAEs.

19. S. L. Campbell, *Linearization of DAEs along trajectories*

Over the last decade there has been a considerable amount of research on numerical and analytic aspects of linear and nonlinear differential algebraic equations (DAEs)  $F(x', x, t, u) = 0$ . Many of these papers have either considered linear equations or based their analysis on linear equations. However, until very recently there has been little rigorous investigation of the relationship between the linearization of a DAE and the original equations. In this paper we carefully examine several aspects of this relationship. Positive results for time varying linearization and counter examples for time invariant linearization are given.

20. S. L. Campbell, *High index differential algebraic equations*

In the last few years there has been considerable research on differential algebraic equations (DAEs)  $f(x', x, t) = 0$  where  $f_{x'}$  is identically singular. The index provides one measure of the singularity of a DAE. Most of the numerical analysis literature on DAEs to date has dealt with DAEs with indices no larger than three because of technical difficulties and because many basic applications including constrained mechanical systems have this index. This paper will discuss several situations where DAEs of index higher than three naturally occur. It will also discuss the relationship between certain concepts in nonlinear control theory such as relative degree and zero dynamics, the index, and constrained mechanical systems.

#### ORAL PRESENTATIONS OF RESEARCH RESULTS

1. Invited talk at Humboldt University, Berlin, June, 1993.

2. **Talk at Conference on Scientific Computation and Differential Equations, Auckland, January, 1993.**
3. **Invited talk at Oberwolfach Workshop on Simulation, June, 1993.**
4. **Invited talk at World Congress of Nonlinear Analysts, Tampa, 1992**
5. **Two invited talks at 1992 SIAM Summer Meeting, Los Angeles**
6. **Talk at SIAM Control Meeting, Minneapolis, 1992.**
7. **Presented two papers at the Symposium on Implicit and Nonlinear System at the Robotics Institute of the University of Texas at Arlington, 1992.**
8. **Presented paper at the 31 IEEE Conference on Decision and Control, Tucson, 1992.**
9. **Gave talk at the ARO Workshop held in Raleigh, 1992.**
10. **Invited talk at University of Umea, Sweden, 1991**
11. **Two invited talks at IMACS World Congress on Scientific Computation, Dublin, Ireland, 1991**
12. **Invited talk at Inst. Nat. Res. Info. Auto. (INRIA), Rocquencourt, France, 1991**
13. **Invited talk at International Institute of Applied System Analysis (IIASA), Laxenburg, Austria, 1991**
14. **Invited talk at Applied Linear Algebra Meeting, Northern Illinois University, 1991**
15. **European Control Conference, Grenoble, France, 1991 (panel discussion)**
16. **Talk at 29 IEEE Conference on Decision and Control, Honolulu, 1990.**

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