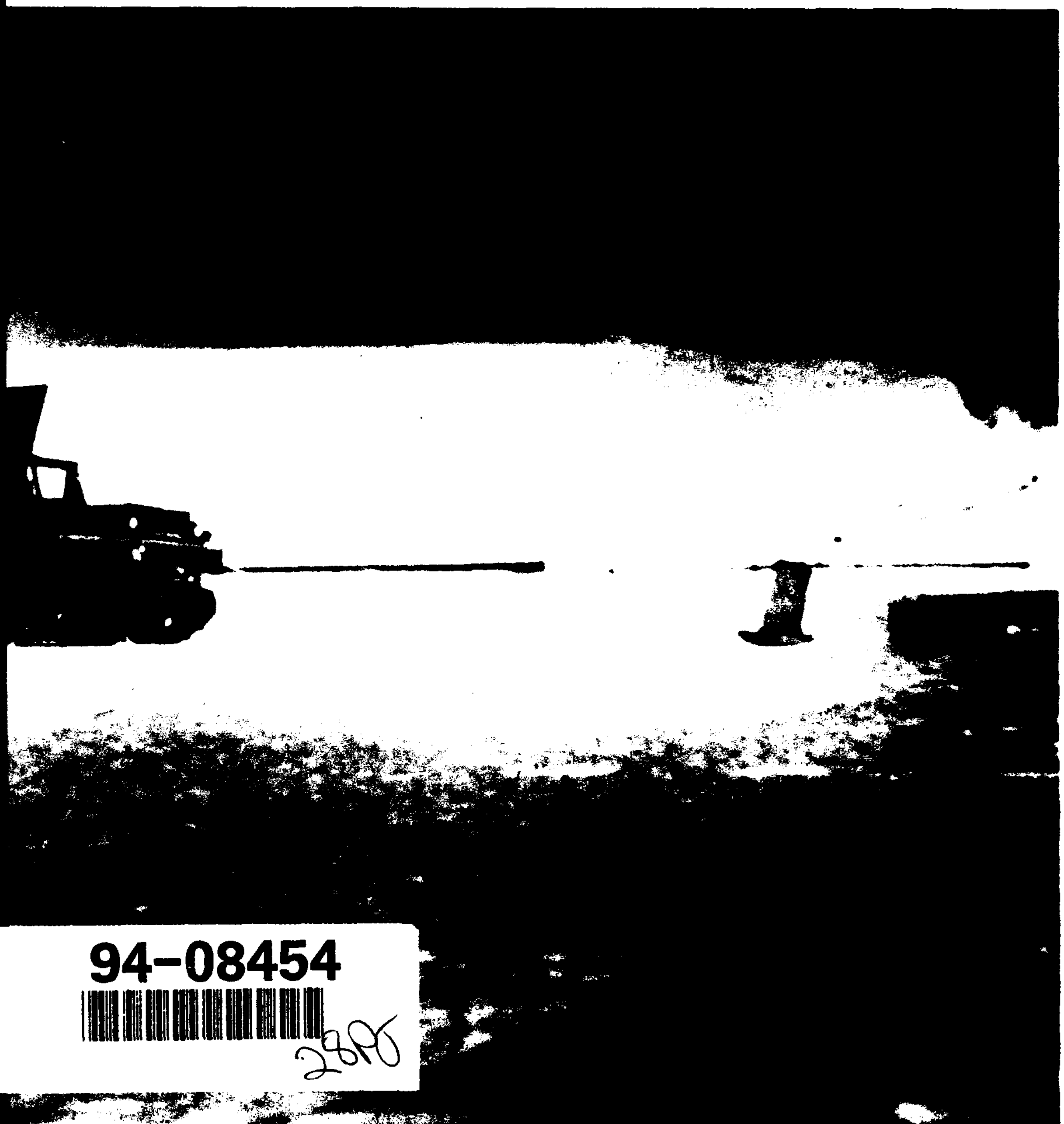
CRREL REPORT 93-26

A Reassessment of the In-Situ Dielectric Constant of Polar Firn

Austin Kovacs, Anthony J. Gow and Rexford M. Morey December 1993



Abstract

The success in using VHF and UHF frequency systems for sounding polar ice sheets has been tempered by an uncertainty in the in-situ dielectric constant ε', which controls the effective velocity V_e of an electromagnetic wave propagating in an air–ice mixture. An empirical equation for determining ε' vs. density (specific gravity, ρ) was proposed in 1968 by Robin *et al.* where $\varepsilon' = (1 + 0.851\rho)^2$. However, this expression has met with uncertainty because wide-angle radar refraction sounding techniques have produced values of ε' that are lower than Robin's equation predicts. This report discusses radar soundings made on the McMurdo Ice Shelf, Antarctica, and compares the resulting ε' values with Robin's equation, laboratory measurements on firn and ice and other expressions given in the literature for determining ε' vs. the specific gravity of dry firn and ice. Our findings indicate that the form of Robin's equation is valid. However, our analysis also indicates the expression could be slightly improved to read $\varepsilon' = (1 + 0.845\rho)^2$. Reasons are suggested as to why previous wide-angle radar sounding studies did not reproduce Robin's findings.

Cover: *Radar transceiver antenna being towed from the front of a boom mounted to a tracked vehicle. Behind the antenna box is a disk and support leg used to elevate the boom above the snow surface. This antenna arrangement was used in 1974 for profiling the brine layer in the McMurdo Ice Shelf and for crevasse detection during a transverse in the Pensacola Mountains, Antaractia, in search of blue ice runway sites. The radar console and recording equipment were located in the vehicle. (Photo by A. Kovacs.)*

For conversion of SI metric units to U.S./British customary units of measurement consult ASTM Standard E380-89a, *Standard Practice for Use of the International System of Units,* published by the American Society for Testing and Materials, 1916 Race St., Philadelphia, Pa. 19103.

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US Army Corps
of Engineers
Cold Regions Research &
Engineering Laboratory

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DIVISION OF POLAR PROGRAMS
NATIONAL SCIENCE FOUNDATION

PREFACE

This report was prepared by Austin Kovacs, Research Civil Engineer, of the Applied Research Branch, Experimental Engineering Division, Dr. Anthony J. Gow, Geologist, of the Snow and Ice Branch, Research Division, U.S. Army Cold Regions Research and Engineering Laboratory, and Rexford M. Morey of Morey Research Company. Funding for this research was provided by the National Science Foundation, Division of Polar Programs, under grants NSF-DPP 74-23654 and NSF-DPP 8004221.

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A Reassessment of the In-Situ Dielectric Constant of Polar Firn

AUSTIN KOVACS, ANTHONY J. GOW AND REXFORD M. MOREY

INTRODUCTION

Since 1948, when Steenson (1951) first used a 100-MHz pulse radio sounding system to estimate ice thickness on the Seward Glacier in Alaska, "radioglaciology" has been extensively used for measuring the depth of polar ice sheets. Success in using sounding systems operating at VHF and UHF frequencies stems in part from the generally very favorable electromagnetic properties of polar firn and ice, which allow for deep penetration at these frequencies.

The relative complex permittivity ε_r^* of an air-ice mixture can be described by the Debye expression

$$\varepsilon_r^* = \varepsilon_r' - j\varepsilon_r'' = \varepsilon_r' + \frac{\varepsilon_{rs} - \varepsilon_r'}{1 + j\omega\tau} \tag{1}$$

where ε_r' = the relative real part of ε^* (or the dielectric constant)
ε_r'' = the loss factor or imaginary part of ε^*
ε_{rs} = the relative static dielectric constant
τ = the relaxation time
ω = the angular frequency.

In this report the variation in the dielectric constant vs. firn density or specific gravity is discussed, as this value governs the propagation speed of an electromagnetic wave transmitted from VHF and UHF sounding systems into polar firn and ice. The loss tangent, tan δ, is represented by $\varepsilon_r''/\varepsilon_r'$ and for polar firn and ice is typically >> 0.1

An electromagnetic wave propagates through firn and ice with an effective phase velocity V_e of

$$V_e = \omega/\beta$$

where β is the phase constant determined by

$$\beta = \omega \left(\frac{\mu'\varepsilon_e'}{2}\right)^{1/2} \left[\left(1 + \frac{\sigma_e^2}{\omega^2 (\varepsilon_e')^2}\right)^{1/2} + 1\right]^{1/2}$$

and μ' = real part of the magnetic permeability
ε_e' = real effective (bulk) dielectric constant
σ_e = effective (bulk) conductivity.

Since the conductivity of dry polar firn and ice is extremely low, the effective phase velocity of high-frequency electromagnetic waves can be determined from

$$V_e = c/\sqrt{\varepsilon_e'} \tag{2}$$

where c is the velocity of an electromagnetic wave in a vacuum, 0.3 m/ns.

If the two-way vertical travel time t of a transmitted wavelet traveling from the surface to the ice bottom (or from some internal horizon) and back is measured, then this time may be used to estimate the distance D to the reflective boundary from

$$D = \frac{tc}{2\sqrt{\varepsilon_e'}} = \frac{t}{2} V_e \, . \tag{3}$$

To accurately make this distance determination, ε_e' for the site must be known. ε_e' can be determined when t and D are known as follows:

$$\varepsilon_e' = \left|\frac{tc}{2D}\right|^2 . \tag{4}$$

Since D is seldom known, this equation is of limited use. Where firn and ice density profile data are available for the site, it is possible to estimate ε_e' for a given depth increment based on the mean interval density and a laboratory-determined ε_e' value for this mean density.

To circumvent these requirements, borehole interferometry and wide angle radar reflection techniques have been used to determine D or ε_e'. These methods have been widely discussed by Bentley (1979), Jezek and Roeloffs (1983), Bogorodsky et al. (1985) and others. However, Jezek and Roeloffs (1983) state that "collectively, these experiments are inconclusive." It has been found that V_e determined from wide-angle reflection sounding measurements tends to be too high, and the resulting low value for ε'_r is not in agreement with laboratory determinations of ε'_r made on firn and ice of the same density. System timing error and curved ray paths are two of the possible reasons given for the observed discrepancies (Jezek and Roeloffs 1983).

The lack of agreement between laboratory-determined ε'_r values and the various radar field results has brought into question the validity of Robin's (Robin et al. 1969, Robin 1975) borehole radar interferometry measurements, from which he proposed an empirical equation relating the index of refraction n to the firn-ice density, which is henceforth referenced as the specific gravity ρ for the purpose of making his and subsequent equations dimensionally correct:

$$n = \frac{c}{V_e} = 1 + 0.851\rho.$$

Since $n^2 = \varepsilon'_e$ in the radio-radar frequency range of interest, then

$$\varepsilon'_e = (1 + 0.851\rho)^2. \quad (5)$$

The form of this exponential expression is hereafter referred to as a refraction type equation.

During their study of the brine infiltration zone in the McMurdo Ice Shelf, Antarctica (Fig. 1), Kovacs and Gow (1975) and Kovacs et al. (1981, 1982) used two impulse radar sounding methods to profile the depth variations, lateral continuity and "inland" boundary of seawater infiltration into the ice shelf

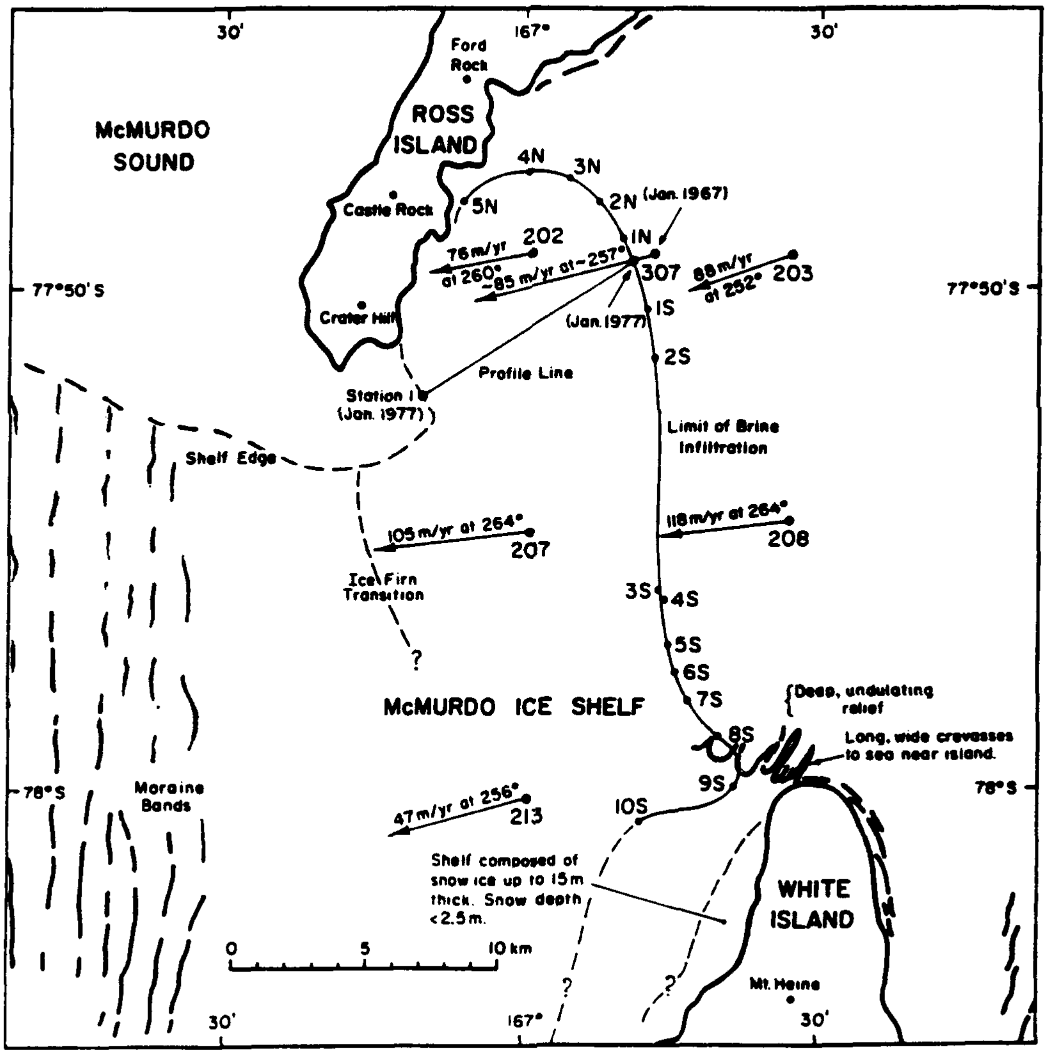

Figure 1. Map of the McMurdo Ice Shelf area. The ice/firn transition controls the western limit of brine infiltration. The eastern limit of brine infiltration is controlled by ice porosity and westward ice shelf movement. Very close to the western side of White Island snow accumulation can exceed 2 m/yr and the ice shelf is composed of recently infiltrated snow ice. In the crevassed area at the north end of White Island is a small population of land-locked seals (from Kovacs et al. 1981).

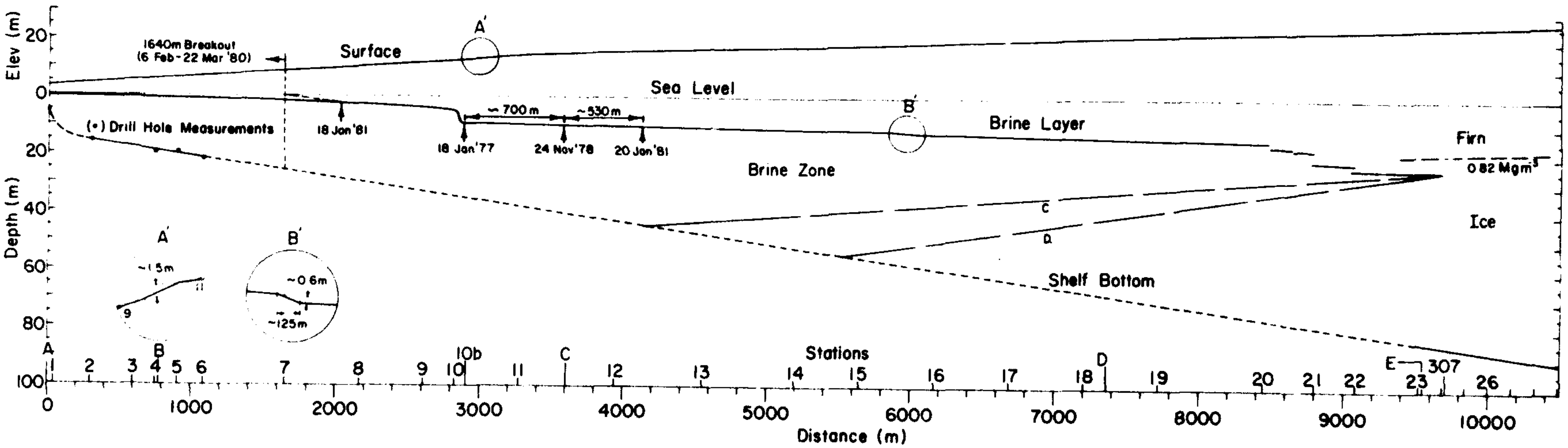

Figure 2. Cross section of the McMurdo Ice Shelf along the subsurface radar sounding profile line shown in Figure 1. The height of the ice shelf face varies in time and space and is a function of ice-shelf movement rate, break-out dates and magnitude and the rate of snow accumuation and bottom melting (from Kovacs et al. 1981).

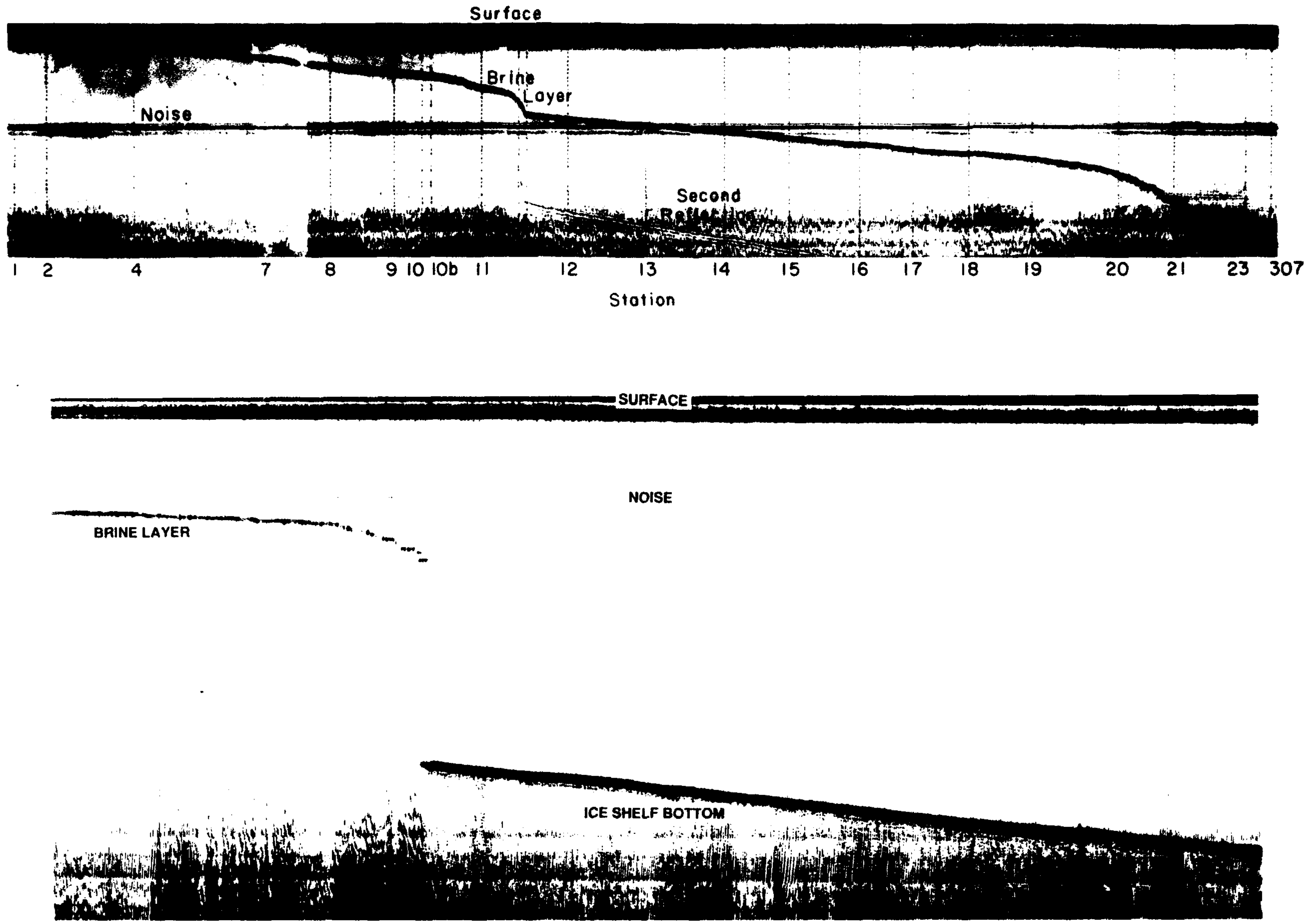

Figure 3. Radar records of McMurdo Ice Shelf along sounding line. Top graphic record shows 300-MHz radar profile of the brine layer along the sounding line shown in Figure 1. Steps in the brine layer are the result of ice shelf breakout events that led to a new zone of seawater-infiltrated firn above the older brine layer. Note the internal firn layering, especially on the right side of the graphic record. The bottom record shows the terminus of the brine zone and the sloping bottom of the McMurdo Ice Shelf. (From Kovacs et al. 1981.)

(Fig. 2). One method previously discussed (Kovacs et al. 1982) used a fixed separation, dual-antenna configuration in which one antenna was a transceiver (colocated transmitter-receiver antenna) and the second antenna was a receiver only. The dual-antenna methodology used is discussed in Kovacs and Morey (1979). With this radar sounding method, V_e, D and ε'_e were determined for the firn between the snow surface and the brine infiltration layer. The depth of the brine layer was determined by direct borehole measurements one year after the radar measurements were made. After the new snow accumulation was taken into consideration, the agreement between the radar- and borehole-determined depths was typically within 2%. These results indicate that, for the shelf firn layer, where the most significant depth variation in density occurs, the dual-antenna radar sounding method provided a satisfactory means for determining ε'_e and V_e as well as the depths to internal layers, which were clearly revealed in the radar records (Fig. 3 and 4).

While Kovacs et al. (1982) showed good agreement between their radar-determined depth to the brine layer and the drill-hole-measured depth and gave estimates of ε'_e and V_e, they did not provide information for an assessment of the Robin equation. Nor did they attempt "to resolve the question of the possible discrepancy between laboratory and field dielectric constant measurements" for firn and ice as did Jezek and Roeloffs (1983). In this report, data obtained from the vertical two-way impulse radar sounding measurements on the McMurdo Ice Shelf are presented. The findings are discussed in relation to the Robin equation and other field, laboratory, and theoretical determinations of the dielectric constant of an air-ice mixture.

FIELD MEASUREMENTS

A Geophysical Survey Systems impulse radar system was used in the field study. The transceiver antenna, when snow-coupled, transmitted a broadband wavelet with a center frequency of about 100 MHz. The vertical two-way travel time of the wavelet from the snow surface to various subsurface annual accumulation layers and the top of the brine infiltration zone in the McMurdo Ice Shelf was recorded in real time on a graphic recorder (Fig. 4) and on a magnetic tape recorder for later analysis on a frequency analyzer. The two-way flight time was measured from where the transmit wavelet first crossed the zero voltage line to where the relected wavelet of interest first crossed the zero line as depicted in Figure 5.

Two-way travel time measurements were made at six sites in 1978 where cores were obtained for determining the depth to the brine layer and the depth-density profile. The measured distance from the edge of the McMurdo Ice Shelf, brine layer depth, average firn density and the two-way travel time are listed in Table 1 as are the calculated ε'_e and V_e values for each radar sounding site. Also given is the location and the two-way travel time to the bottom of the ice shelf at station 307, which was located immediately beyond the 1978 brine infiltration limit (Figs. 2 and 3). The depth of the ice shelf at station 24, located just 40 m before station 307, was estimated by Kovacs et al. (1982). Using the 1977 dual-antenna radar sounding data, they first determined the depth to the brine layer at station 24. The two-way flight time from the brine layer to the bottom of the ice shelf was then used, along with an assumed value of $\varepsilon'_e = 2.95$ for this depth interval. The thickness of the ice shelf was then estimated (using eq 2

Table 1. 1978 McMurdo Ice Shelf station data along traverse from shelf edge to ice shelf movement marker 307.

Station no.	*Distance inland (m)*	*Brine depth* (m)*	*Average specific gravity* ρ	*Wavelet travel time* t *(ns)*	*Effective bulk dielectric constant* ε'_e	*Effective velocity* V_e *(m/ns)*
A	5	2.5	0.395	22	1.74	0.227
B	770	8.85	0.509	83	1.98	0.213
10b	2900	13.5	0.540	130	2.08	0.208
C	3590	9.45	0.565	190	2.15	0.204
D	7370	33.8	0.624	344	2.33	0.196
E	9560	50.4	0.686	529	2.48	0.190
307	9700	—	—	1263	—	—

*Drillhole measurements

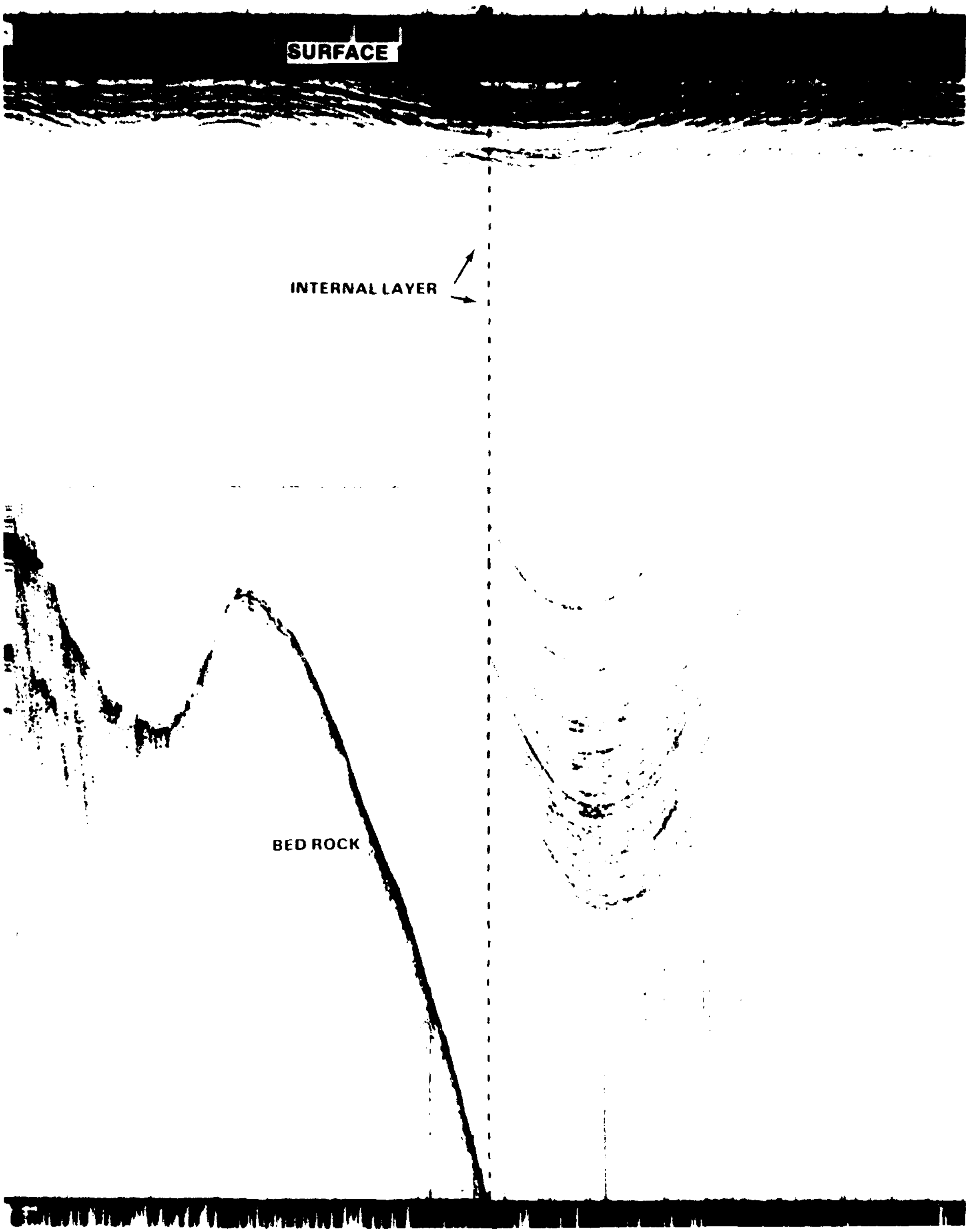

Figure 4. Graphic record of 300-MHz radar sounding data showing internal layers in the snow field below Castle Rock (Fig. 1). The dotted line marks the site where a 20-m firn core was obtained. Core analysis revealed the internal layers were summer stratigraphy horizons.

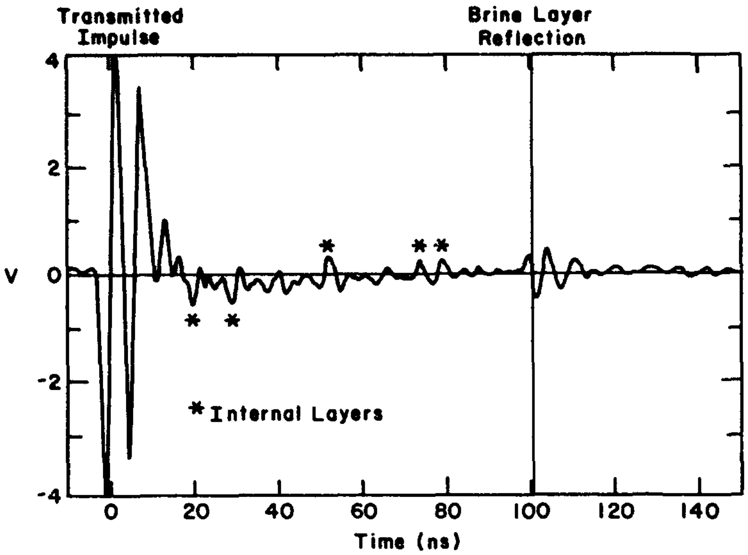

Figure 5. Example of received radio echo wavelet. In the graphic record many horizons or internal layers were observed in the ice shelf firn. These layers could be tracked for many kilometers. Their voltage amplitude signature is shown above (from Kovacs et al. 1982).

and 3) to be 113 m. This thickness estimate will be compared with the estimated thickness for station 307 as determined from the 1978 radar two-way travel time data obtained at this site.

The brine layer depths and two-way travel time differences between stations A–B, B–C, C–D and D–E are listed in Table 2 along with the average firn densities for the depth increments. Also listed are the calculated ε'_e and V_e values for the depth intervals. A plot of the ε'_e vs. ρ values given in Tables 1 and 2 is shown in Figure 2.

Table 2. McMurdo Ice Shelf depth increment data.

Station increment	*Depth increment (m)*	*Average specific gravity* ρ	*Wavelet travel time* t_i *(ns)*	*Effective bulk dielectric constant* ε'_e	*Effective velocity* V_e *(m/ns)*
A–B	6.35	0.504	61	2.08	0.208
B–C	10.6	0.633	107	2.29	0.199
C–D	14.35	0.726	154	2.59	0.186
D–E	16.6	0.808	185	2.79	0.180

Numerous investigators have determined that, at ~ –10°C and a ρ of 0.917, the dielectric constant of pure polycrystalline ice, at the frequencies of interest, is well represented by a value of 3.15 (e.g., Cummings 1952, Evans 1965, Ulaby et al. 1986, Koh 1992). Koh (1992) also reaffirmed that ε'_r is high-frequency-independent. Fujita et al. (1992) reconfirmed that ε'_r for polycrystalline ice is also insensitive to temperature, varying at about 0.0009/C°. Therefore, in the subsequent discussion and equations, an ε'_r value of 3.15 will be used for pure bubble-free ice at a specific gravity of 0.917. The intercept of these values is shown by a cross in Figure 6 and in subsequent figures.

It has been found that when glacial ice is under stress, as in a flow regime, an anisotropy develops in which the c-axes of the ice crystals become aligned. Johari and Chasette (1975) found that in the infrared region a dielectric constant change on the order of 0.004 could arise if the electric field was parallel rather than perpendicular to the crystal c-axis. Similarly, Fujita et al. (1992) have shown experimentally that the change could be as much as 0.04 at 9.7 GHz. Thin-section studies of the crystalline structure of the cores obtained from the McMurdo Ice Shelf revealed that the ice grains had a random c-axis fabric that existed from the surface to the bottom of our 50-m-deep drill hole near station 307 (Fig. 2). This random fabric orientation therefore eliminates ice crystal anisotropy effects as a contributing factor in our radar measurements.

Three curves are also shown in Figure 6. One curve represents Robin's equation (eq 5). Another is a refraction type curve statistically fitted through the data. The equation for this curve,

$$\varepsilon'_r = (0.992 + 0.848\rho)^2 \qquad (6)$$

fits the data with an $r^2 = 0.989$ and a standard error of 0.035. The third curve shown was determined as follows. The four-parameter volume fraction type equation

$$\varepsilon'_r = \left(v_a\sqrt{\varepsilon_a} + v_i\sqrt{\varepsilon'_{ri}}\right)^2 \qquad (7)$$

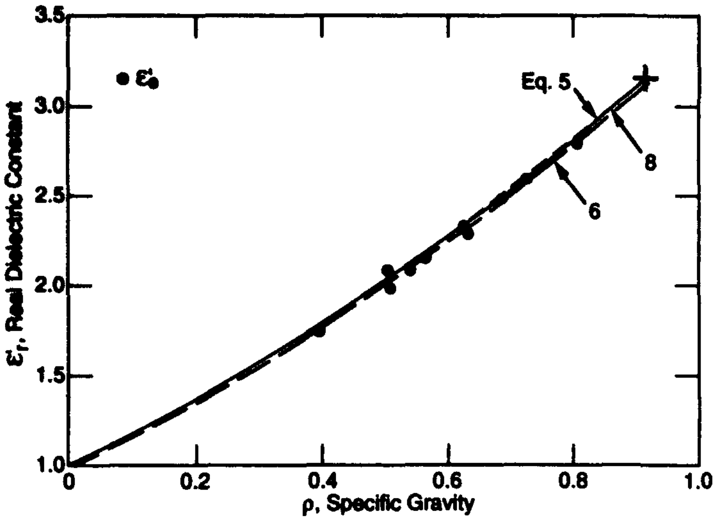

Figure 6. Field-determined in-situ effective dielectric constant ε'_e vs. the bulk specific gravity of McMurdo Ice Shelf firn. The curves passing through the field data • were derived from the indicated equations which are described in this report. The + represents the dielectric constant (3.15) of pure ice (ρ = 0.917).

where ε_a is the dielectric constant of air (= 1) and ε'_r is the dielectric constant of pure ice, v_a and v_i, the volume fractions of air and ice, respectively, were used to calculate the dielectric constant of firn and ice over the specific gravity range from 0 to 0.917 in increments of 0.5. A refraction type expression was then statistically fitted through this data. From this procedure it was found that eq 7 is well represented, at a cross-correlation of r^2 = 1.000, by the expression

$$\varepsilon'_r = (1 + 0.845\rho)^2. \tag{8}$$

It is apparent from Figure 6 that both Robin's empirical equation (eq 5) and the simplified volume fraction expression (eq 8) fit the McMurdo Ice Shelf dielectric constant vs. specific gravity field data very well.

Numerous other empirical and theoretical expressions have been used to describe the dielectric constant of an air-ice mixture. A number of these expressions will be compared with the McMurdo Ice Shelf data and, where statistically permissible, the expressions will be simplified to a refraction equation, following the procedure used to obtain eq 8.

Looyenga (1965) proposed a mixing formula based on spherical inclusions

$$\varepsilon'_r = \left[\left(\varepsilon'^{1/3}_{ri} - \varepsilon_a^{1/3}\right) v_i + \varepsilon_a^{1/3}\right]^3 \tag{9}$$

and because ε_a = 1 then

$$\varepsilon'_r = \left[\left(\varepsilon'^{1/3}_{ri} - 1\right) v_i + 1\right]^3.$$

This equation was simplified by Stiles and Ulaby (1981) to read

$$\varepsilon'_r = (1 + 0.508\rho)^3 \tag{10}$$

Equation 9 can also be well represented by a refraction type equation, with an r^2 = 0.999, as

$$\varepsilon'_r = (0.988 + 0.845\rho)^2 \tag{11}$$

which is in very near agreement with eq 8.

Wiener (1910) presented a two-phase mixing formula containing an empirical structure factor called the Formzahl μ. This function varies with the shape and orientation of the inclusions. In firn and ice the Formzahl may vary if the medium is nonisotropic. The form of the equation for an air-ice mixture is

$$\frac{\varepsilon'_r - 1}{\varepsilon'_r + \mu} = v_i\left(\frac{\varepsilon'_{ri} - 1}{\varepsilon'_{ri} + \mu}\right) + (1 - v_i)\left(\frac{\varepsilon_a - 1}{\varepsilon_a + \mu}\right). \tag{12}$$

Since ε_a is one, the second term on the right is zero. It was found through statistical regression that eq 12 fits the McMurdo Ice Shelf firn dielectric constant vs. specific gravity data best when μ = 5.5. This value is in good agreement with the values of 5 given in Bogorodsky et al. (1985) and 7.5 given in Ambach and Denoth (1972) for vertically propagating electromagnetic waves in firn and ice. But it is not in agreement with Shabtaie and Bentley (1982), who found that μ = 0 provided the best fit for their common reflection point measurements at Dome C, Antarctica.

The results from this measurement technique have caused the most concern regarding the validity

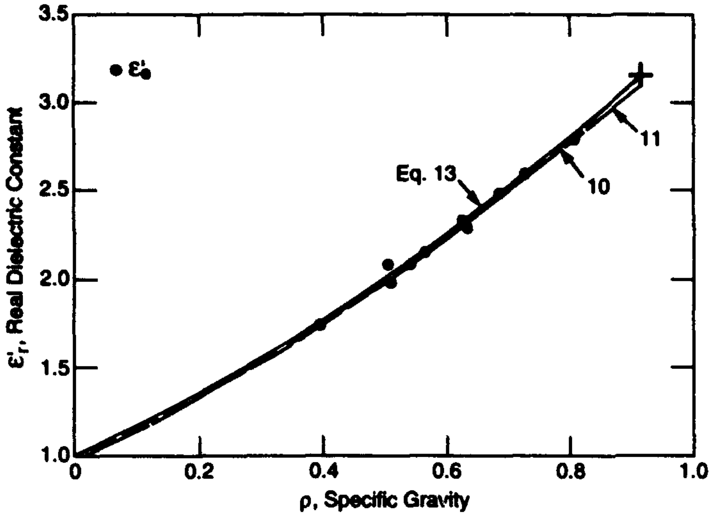

Figure 7. Field-determined in-situ effective dielectric constant ε'_e vs. the bulk specific gravity of McMurdo Ice Shelf firn. The curves passing through the field data were derived from the indicated equations which are described in this report.

of the Robin equation. It is generally expected that for polar firn and ice μ should be above 2 (e.g., Evans 1965). A Formzahl in the range of ~ 5–7 suggests that the horizontal layering in polar firn has a weak effect on ε'_e. Equation 12 can be simplified to

$$\varepsilon'_r = (1 + 0.840\rho)^2 \tag{13}$$

which cross-correlates extremely well with the Wiener equation, when μ = 5.5, at $r^2 = 1.000$.

The curves for eq 10, 11 and 13 and the McMurdo Ice Shelf ε'_e vs. ρ data are shown in Figure 7. Here again the curves for the simplified equations are seen to fit the field data very well.

A mixing formula by deLoor (1956), when applied to an air–ice medium containing spherical inclusions, is

$$\varepsilon'_r = \Big[3(1-\phi)(\varepsilon'_{ri}-1) - \varepsilon'_{ri} + 2 \pm \sqrt{\left[3(1-\phi)(\varepsilon'_{ri}-1) - \varepsilon'_{ri} + 2\right]^2 + 8\varepsilon'_{ri}}\,\Big]\Big/4 \tag{14}$$

where ϕ is the porosity. This equation, when simplified to a refraction type expression, becomes

$$\varepsilon'_r = (0.983 + 0.858\rho)^2 \tag{15}$$

with a cross-correlation of $r^2 = 0.999$.

The Böttcher (1933) and Polder and van Santen (1946) mixing formula for a medium composed of spherical inclusions is

$$\frac{\varepsilon'_r - 1}{3\varepsilon'_r} = v_a\left(\frac{\varepsilon_a - 1}{\varepsilon_a + 2\varepsilon'_r}\right) + v_i\left(\frac{\varepsilon'_{ri} - 1}{\varepsilon'_{ri} + 2\varepsilon'_r}\right).$$

Since $\varepsilon_a = 1$, the first term on the right side is zero, and the equation for an air–ice mixture becomes

$$\frac{\varepsilon'_r - 1}{3\varepsilon'_r} = v_i\left(\frac{\varepsilon'_{ri} - 1}{\varepsilon'_{ri} + 2\varepsilon'_r}\right) \tag{16}$$

which in turn is well represented at a cross-correlation of $r^2 = 0.999$ by

$$\varepsilon'_r = (0.988 + 0.850\rho)^2. \tag{17}$$

For disk-shaped particles the Polder and van Santen (1946) equation is

$$\varepsilon'_r = \frac{3\varepsilon'_{ri} + 2\varepsilon'_{ri}(\varepsilon'_{ri}-1)(1-\phi)}{\varepsilon'_{ri} - (\varepsilon'_{ri}-1)(1-\phi)}$$

which can be simplified to

$$\varepsilon'_r + (1.006 + 0.839\rho)^2 \tag{18}$$

with a cross-correlation of $r^2 = 1.000$.

The Tinga et al. (1973) mixing formula for an air–ice medium containing spherical inclusions is

$$\varepsilon'_r = 1 + \frac{3v_i\left(3\varepsilon'_{ri} - 1\right)}{\left(2 + 3\varepsilon'_{ri}\right) - v_i\left(\varepsilon'_{ri} - 1\right)} \tag{19}$$

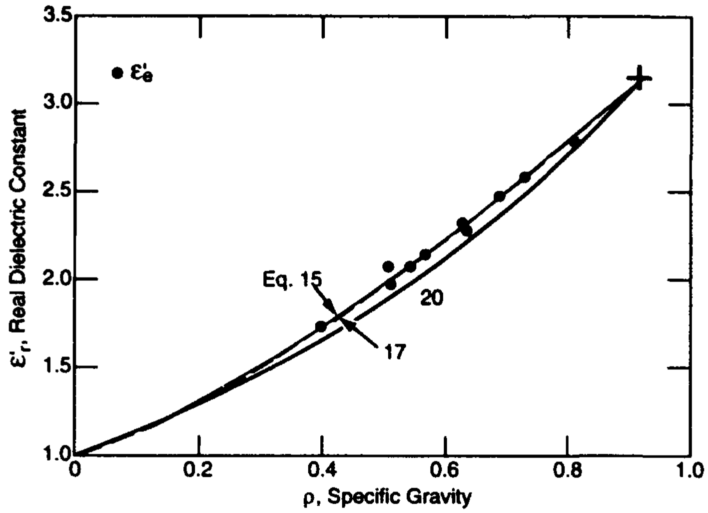

Figure 8. Field-determined in-situ effective dielectric constant ε'_e vs. the bulk specific gravity of McMurdo Ice Shelf firn. The curves passing through the field data were derived from the indicated equations which are described in this report.

which when simplified becomes

$$\varepsilon'_r = (1 + 0.910\rho)/(1 - 0.455\rho) \tag{20}$$

with a cross-correlation of $r^2 = 1.000$. Equation 19 does not cross-correlate very well with a refraction type expression.

The curves respresenting eq 15, 17 and 20 are shown in Figure 8, along with the ε'_e vs. ρ data from the McMurdo Ice Shelf. Here we see that the curves representing eq 15 and 17 pass nicely through the field data but the curve representing eq 20 does not. From Figure 8 it appears that the Tinga et al. equation should not be used. Sihvola et al. (1985) also found that the Tinga et al. expression (eq 19) is not appropriate for determining ε'_r for an air–ice mixture.

There are other expressions that also do not fit the McMurdo Ice Shelf field data well. Three of these equations are given below. The first is a volume average expression (Lang 1983):

$$\varepsilon'_r = \phi\varepsilon_a + (1 - \phi)\,\varepsilon'_{ri} \tag{21}$$

which can be further simplified to

$$\varepsilon'_r = 1 + 2.345\rho \tag{22}$$

with a cross-correlation of $r^2 = 1.000$.

Ackley and Keliher (1979) proposed the use of a mixture formula by van Beek to determine ε'_r for polar ice containing spherical inclusions:

$$\varepsilon'_r = \varepsilon'_{ri}\left(1 + \frac{3\phi(\varepsilon_a - \varepsilon'_{ri})}{\varepsilon'_{ri} + \varepsilon_a}\right) \tag{23}$$

which can be simplified to

$$\varepsilon'_r = 0.367 + 3.035\rho \tag{24}$$

with a cross-correlation of $r^2 = 1.000$.

The linear model proposed by Hufford (1991) for an air-ice mixture is

$$\varepsilon'_r = 1 + 3v_i\left(\frac{\varepsilon'_{ri} - 1}{\varepsilon'_{ri} + 2}\right) \tag{25}$$

which can be reduced to

$$\varepsilon'_r = 1 + 1.366\rho \tag{26}$$

with a cross-correlation of $r^2 = 1.000$.

Equations 22, 24 and 26 are graphically shown in Figure 9, along with eq 6 and the McMurdo Ice Shelf ε'_e vs. ρ field data. Clearly, eq 22 does not track through the field results and should not be used. Equations 24 and 26 appear to have a limited range of use; however, they are not considered acceptable in light of how well, for example, eq 6 fits the data.

Other expressions for determining the dielectric constant of an air–ice mixture vs. specific gravity are listed in Appendix A.

To further show that a simple refraction type equation can well represent the dielectric constant of a mixture of air and ice, the 10 McMurdo Ice Shelf dielectric constant vs. specific gravity data from Tables 1 and 2 were combined with 15 laboratory determinations of Cummings (1952) at 9.4 GHz, 17 field measurements made at 20 MHz on "old" alpine snow by Denoth (1978) and 47 laboratory measurements by Nyfors (1982) made at UHF and SHF

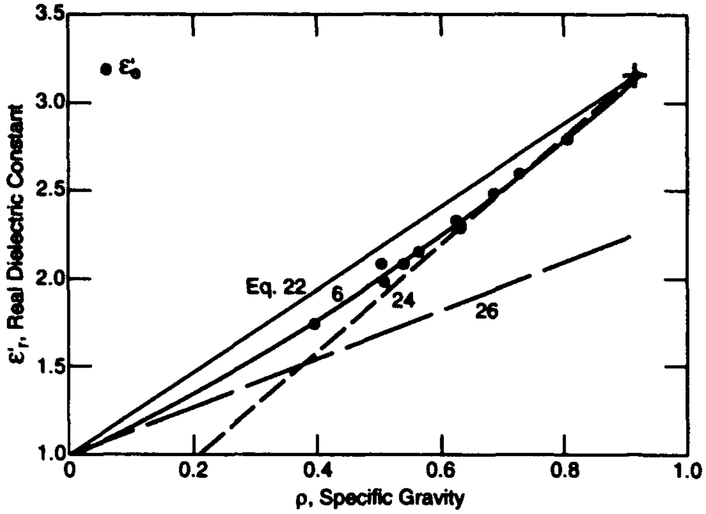

Figure 9. Field-determined in-situ effective dielectric constant ε'_e vs. the bulk specific gravity of McMurdo Ice Shelf firn. The curves passing through the field data were derived from the indicated equations which are described in this report.

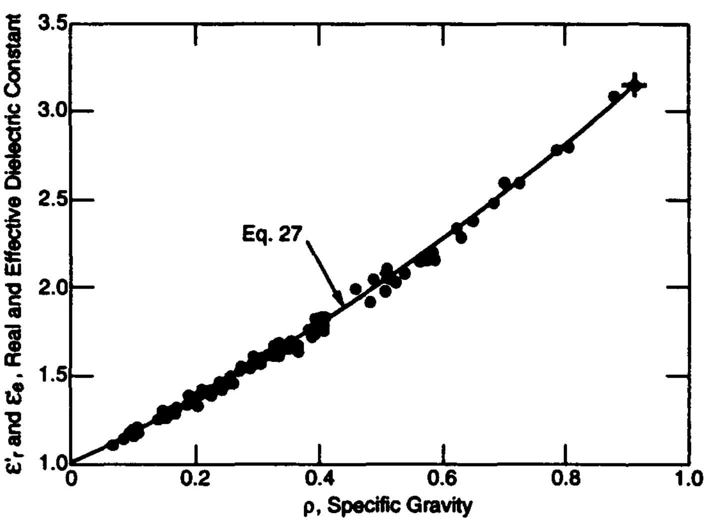

Figure 10. Field and laboratory-determined values of the dielectric constant of firn vs. specific gravity. The regression curve passing through the data is represented by equation 27 as discussed in this report.

frequencies on a variety of snow types (e.g., fine- and coarse-grained snow). A plot of these data along with a weighted point of $\varepsilon'_{ri} = 3.15$ at $\rho = 0.917$ is given in Figure 10. The resulting regression curve passing through the data is derived from

$$\varepsilon'_r = (1.000 + 0.845\rho)^2 \tag{27}$$

This curve fits the data with a correlation of $r^2 = 0.999$ and a standard error of $\pm$ 0.031. It should be noted that eq 27 is also the simplified form of the volume fraction expression (eq 8).

The justification for adding a weighted dielectric constant of 3.15 at a specific gravity of 0.917 to the data in Figure 7 is based on our earlier determination that this dielectric constant is a representative value for pure ice. In addition, the weighted value was added because of the paucity of dielectric constant test data at the higher specific gravities. Without the weighted point, the regression equation would read $\varepsilon'_{ri} = (1.002 + 0.836\,\rho)^2$ with an r^2 of 0.994 and a standard error of 0.035. This equation gives an ε'_r of 3.13 at a ρ of 0.917, the same value obtained by using eq 6.

It was previously stated that Jezek and Roeloffs (1983) made radar sounding measurements, in Greenland and Antarctica, for the purpose of evaluating the validity of the Robin equation. In these measurements, a metal target was lowered down a borehole and the travel time of a transmitted wave from the surface to the target and back was recorded. Using the known depth to the target and the density of the firn and ice above the target, they calculated the effective radar wave velocity in the medium for different target depths. The average V_e data for each depth at the Dye 3 Greenland site are

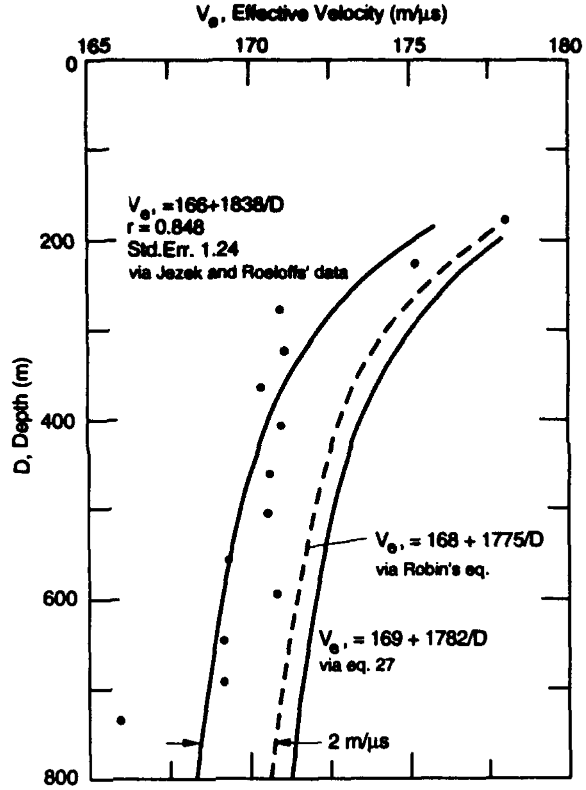

Figure 11. Effective radar velocity vs. depth at DYE-3 Greenland. The data points are Jezek and Roeloffs' average values, the Robin curve is from their analysis and the right curve is based on eq 17 in this report.

shown in Figure 11. Also shown is the V_e vs. depth curve they calculated using Robin's equation, the depth–density profile data for the site and eq 3. The regression curve we fitted to their velocity data was selected because it paralleled the calculated curve using Robin's equation.

As indicated in Figure 11, the regression curve through Jezek and Roeloffs' data gives an apparent 2 m/µs slower velocity gradient than the velocity vs. depth curve calculated with use of Robin's equation.

The third curve shown in Figure 11 was calculated with the use of eq 27, the site depth–density data and eq 3. As one would expect, this curve gives a slightly higher velocity vs. depth profile. Nevertheless, the agreement between the three curves is considered to be very good.

We believe that the close similarity between eq 5 and eq 6 and 27 reaffirms that the empirical assessment of Robin et al. (1969) and Robin (1975) for describing the dielectric constant of polar firn and ice vs. their specific gravity is valid. It may be of interest to note that the form of this refraction expression may also be used to describe the dielectric constant of soil vs. its moisture content, as shown in Appendix B.

DISCUSSION

The borehole-measured depths to the brine layer in the McMurdo Ice Shelf were plotted vs. the radar wavelet two-way flight time measurements in Figure 12. The two-way flight time measured to the bottom of the ice shelf in 1978 at station 307 was 1263 ns (Table 1). From the regression equation given in Figure 12, this time would represent a depth of 116 m. As previously stated, Kovacs et al. (1982) gave an estimated thickness of 113 m for the shelf at station 24 in 1977. Since station 307 was 40 m east of station 307, one could assume that the ice is ~ 0.3 m thicker at station 307 than at station 24 because the ice shelf increases in thickness to the east of this station (Fig. 2) at about 8 m/km. In any event, the depth difference between the two determinations (113 m and 116 m) is about a 2 1/2% variance. Clearly the two assessments of ice shelf thickness are in close agreement, given the snow accumulation (0.9 m between surveys), bottom melting and shelf movement dynamics, which can cause short- as well as long-term ice shelf thickness variations at a location that is not fixed in space. In addition, the 1982 estimate was based on an assumed bulk ice density for the shelf below the brine layer depth, a density that now appears to be a bit high.

Another interesting plot is that of the dielectric constant values vs. depth, as shown in Figure 13. From the regression equation for the curve in Figure 13, ε'_e for the ~116-m-thick ice shelf at station 307 can be estimated to be ~ 2.74. Jericek and Bentley (1971) gave an ε'_e of 3.01 ± 0.03 for the 510-m-thick Ross Ice Shelf near Roosevelt Island, and Jezek et al. (1978) estimated an ε'_e value of 2.99 for the 480-m-thick Ross Ice Shelf at station N19. The equation in Figure 13 gives an ε'_e of 3.02 and 3.01 for these two ice shelf thicknesses, respectively. The close agreement between the field and calculated values are surprising when one considers that the depth–density profile of the firn layer can be expected to vary with ice shelf locations. Nevertheless, this variation may not be enough to significantly influence the bulk dielectric constant of the firn layer and in turn that of the ice shelf. If so, then the results from the McMurdo Ice Shelf, a lobe of the Ross Ice Shelf, may be applicable to other areas of the Ross Ice Shelf. In any event, Figure 12 may be used for estimating the

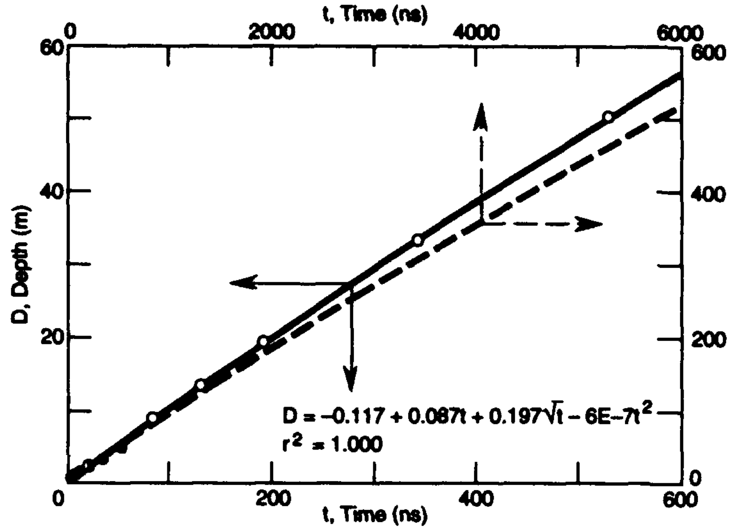

Figure 12. Horizon depth within the McMurdo Ice Shelf vs. radar wavelet two-way flight time.

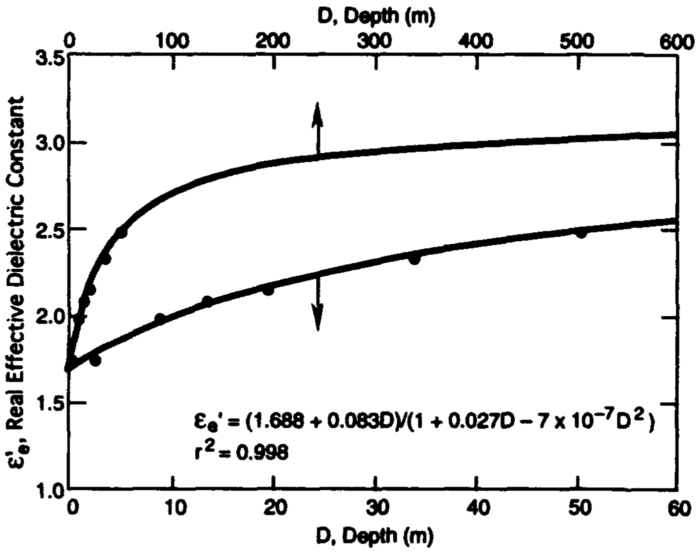

Figure 13. Effective dielectric constant of the firn-ice in the McMurdo Ice Shelf vs. depth increment.

depth of these ice shelves from the two-way travel times obtained from vertical radar sounding measurements.

However, a note of caution is in order. For example, the Ross Ice Shelf at Site J-9 is known to have a ~6-m-thick layer of saline ice accreted to the bottom (Zotikov 1979, Zotikov et al. 1980, Morey and Kovacs 1982). Because this ice attenuates electromagnetic energy at the frequencies often used in radar sounding (as does the brine-soaked firn), and given the spreading losses which occur with distance, a reflection from the saline ice/seawater interface may not be obtained. Nevertheless, a reflection may be received from the fresh ice/saline ice interface, and therefore the depth to this interface could be estimated using a two-way travel time measurement and Figure 12. Should a reflection also be obtained from the saline ice/seawater interface, then the two-way travel time measured in this saline ice layer may be used to estimate the layer thickness T from

$$T = -0.708 + 0.088\,t \qquad (28)$$

and the layer effective dielectric constant from

$$\varepsilon_e' = 5.983\,\exp(-0.477T) + 3.172 \qquad (29)$$

Equations 28 and 29 are from the results of impulse radar two-way time of flight measurements made at ~ 100 MHz on thick sea ice by Kovacs and Morey (1990). For saline ice growth over 10 m thick on the bottom of an ice shelf, a value of 3.25 ± 0.05 for ε_e' may be reasonable for estimating T using eq 3 wherein T is substituted for D.

It seems that many distributions in nature tend to follow exponential-like trends. The above results for the variation in the dielectric constant of firn and ice vs. specific gravity as well as the results for the dielectric constant vs. the moisture content of soil (App. B) appear to represent two of them.

CONCLUSIONS

The findings of this report reaffirm that a simple refraction type equation can be used to describe extremely well the dielectric constant vs. specific gravity of firn and ice. It was further verified that the so-called Robin expression [$\varepsilon_r' = (1 + 0.851\ \rho)^2$] is very suitable for determining the dielectric constant vs. specific gravity of dry polar firn and ice, but that it may be slightly improved by changing the constant 0.851 in the equation to 0.845, especially if one accepts 3.15 for the dielectric constant of pure ice as measured by Cummings (1952) again reaffirmed by Koh (1992). Why previous investigators' field data did not correlate well with the Robin equation is debatable. Jezek and Roeloffs' (1983) explanation of a timing error is plausible. However, wide-angle reflection measurements have been made with radar sounding systems other than the ones they state to have the timing error and still a discrepancy exists in the D and ε_e' determinations. We suggest that curved ray path effects associated with Snell's law and the standard wide-angle reflection analysis using a time^2–distance^2 plot are a cause of the discrepancy. Indeed, the findings of Hughes (1985) and Morey and Kovacs (1985) show that a standard wide-angle reflection measurement analysis gives lower ε_e' and higher D determinations than are appropriate.

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APPENDIX A: SUPPLEMENT EQUATIONS FOR DETERMINING THE DIELECTRIC CONSTANT OF AN AIR-ICE MIXTURE.

Additional formulations are presented in this section for determining the ε_r' vs. ρ for firn and ice. All equations have been offered by the indicated authors as a means of determining the ε_r' of an air–ice mixture. Below each equation is a table giving example air–ice mixture specific gravities and the calculated ε_r' value as determined by eq 27 and by the new expressions presented.

Tiuri et al. (1984) made measurements in the 0.85–12.6 GHz range on snow. Their empirical equation for ε_r' is

$$\varepsilon_r' = 1 + 1.7\,\rho + 0.7\,\rho^2 \tag{A1}$$

which they state for practical purposes can be simplified to

$$\varepsilon_r' = 1 + 2\,\rho. \tag{A2}$$

ρ	ε_r' (eq 27)	ε_r' (eq A1)	ε_r' (eq A2)
0.0	1.0	1.0	1.0
0.2	1.37	1.37	1.40
0.4	1.79	1.79	1.80
0.6	2.27	2.27	2.20
0.8	2.81	2.81	2.60
0.917	3.15	3.15	2.83

Ambach and Denoth (1980) give the following empirical equation for their dry snow measurements at 20 MHz:

$$\varepsilon_r' = 1 + 2.2\,\rho. \tag{A3}$$

ρ	ε_r' (eq 27)	ε_r' (eq A3)
0.0	1.0	1.0
0.2	1.37	1.44
0.4	1.79	1.88
0.6	2.27	2.32
0.8	2.81	2.76
0.917	3.15	3.02

Hallikainen et al. (1982) made measurements at 4–18 GHz on dry snow and found empirically that

$$\varepsilon_r' = 1.91\,\rho. \tag{A4}$$

ρ	ε_r' (eq 27)	ε_r' (eq A4)
0.0	1.0	1.1
0.2	1.37	1.38
0.4	1.79	1.76
0.6	2.27	2.15
0.8	2.81	2.53
0.917	3.15	2.75

Burns et al. (1985) made measurements at 100 MHz on dry snow and found empirically that

$$\varepsilon_r' = 1.1 + 2.2\rho. \tag{A5}$$

ρ	ε_r' (eq 27)	ε_r' (eq A5)
0.0	1.0	1.1
0.2	1.37	1.54
0.4	1.79	1.98
0.6	2.27	2.42
0.8	2.81	2.86
0.917	3.15	3.12

Fujita et al. (1992) made measurements at 9.7 GHz on polycrystalline ice and found from their results that

$$\varepsilon_r' = 3.08\rho + 0.41. \tag{A6}$$

ρ	ε_r' (eq 27)	ε_r' (eq A6)
0.0	1.0	0.41
0.2	1.37	1.03
0.4	1.79	1.64
0.6	2.27	2.26
0.8	2.81	2.87
0.917	3.15	3.23

Pearce and Walker (1967) proposed the following expression based on theoretical considerations:

$$\varepsilon_r' = 3.16\rho + 0.41 \tag{A7}$$

where $0.535 \leq \rho \leq 0.920$.

ρ	ε'_r (eq 27)	ε'_r (eq A7)
0.6	2.27	2.31
0.7	2.53	2.62
0.8	2.81	2.94
0.917	3.15	3.31

Sihvola et al. (1985) proposed a mixing formula by Taylor (1965) for an air–ice mixture with randomly oriented disk-shaped inclusions of the form

$$\varepsilon'_r = \frac{1 - \frac{2}{3}\phi\left(1 - \varepsilon'_{ri}\right)}{1 + \frac{\phi}{3\varepsilon'_{ri}}\left(1 - 3\varepsilon'_{ri}\right)} \tag{A8}$$

which can be simplified to

$$\varepsilon'_r = (1.007 + 0.838\rho)^2 \tag{A9}$$

with a cross-correlation of $r^2 = 1.000$. Sihvola et al. showed that eq A8 fitted their data, obtained at 0.8-13 GHz, better than another one by Taylor for spherical inclusions.

ρ	ε'_r (eq 27)	ε'_r (eq A9)
0.0	1.00	1.01
0.2	1.37	1.38
0.4	1.79	1.80
0.6	2.27	2.28
0.8	2.81	2.81
0.917	3.15	3.15

In an important paper on dielectric mixing formulas, Sihvola (1989) presents a self-consistent general mixing formula for determining the complex permittivity of a homogeneous material containing spherical inclusions. As applied to determining the dielectric constant of dry firn and ice, his expression is

$$\varepsilon'_r = \frac{\sqrt{b^2 + 4\sigma c} - b}{2\sigma} \tag{A10}$$

where

$$b = \varepsilon'_{ri} + 2\varepsilon_a - 2\sigma\varepsilon_a - f_i\left(\varepsilon'_{ri} - \varepsilon_a\right)(1 + \sigma)$$

$$c = \varepsilon_a\left[\varepsilon'_{ri} + (2 - \sigma)\varepsilon_a + f_i\,(2 - \sigma)\left(\varepsilon'_{ri} - \varepsilon_a\right)\right]$$

and parameter σ is a positive integer and parameter f_i is the ice density fraction.

Sihvola showed the interrelationship between his mixing formula and that of other authors. His results indicated that when $\sigma = 3$, eq A10 gave ε_r' values for firn and ice in close agreement with eq A1. Equation A10, with $\sigma = 3$, can be simplified to

$$\varepsilon_r' = (0.988 + 0.859\rho)^2 \tag{A11}$$

with a cross-correlation of $r^2 = 1.000$.

ρ	ε_r' (eq 27)	ε_r' (eq A11)
0.0	1.00	0.98
0.2	1.37	1.35
0.4	1.79	1.77
0.6	2.27	2.26
0.8	2.81	2.81
0.917	3.15	3.15

Sihvola also showed that an expression by Sen et al. (1981) for a medium containing spherical inclusions:

$$\frac{\varepsilon_r' - \varepsilon_a}{\varepsilon_{ri}' - \varepsilon_a}\left(\frac{\varepsilon_{ri}'}{\varepsilon_r'}\right)^{1/3} = \phi \tag{A12}$$

when applied to firn and ice gave ε_r' values in good agreement with equation A1. When eq A12 is simplified to a refraction type expression we obtain

$$\varepsilon_r' = (0.995 + 0.848\rho)^2 \tag{A13}$$

with a cross-correlation of $r^2 = 1.000$.

ρ	ε_r' (eq 27)	ε_r' (eq A13)
0.0	1.0	0.99
0.2	1.37	1.36
0.4	1.79	1.78
0.6	2.27	2.26
0.8	2.81	2.80
0.917	3.15	3.14

The interested reader is directed to Sihvola's paper for additional mixing formulas that may be applied to estimating the dielectric constant of firn and ice.

Of the above expressions, it appears that eq A1, A9, A11 and A13 correlate best with eq 27.

APPENDIX B: THE DIELECTRIC CONSTANT OF A SOIL–WATER MIXTURE.

The dielectric constant of dry soil ε'_{rsd} typically lies between 2 and 4, while the relative dielectric constant of water ε'_{rw} is 81 at 20°C. This value will vary with temperature from about 88 at 0°C to lower values as the temperature climbs. The value for ε'_{rw} is also frequency dependent. Since ε'_{rw} is such a large value it has a major influence on the relative dielectric constant of the soil–moisture mixture ε'_{rm}. Here we show the variation of ε'_{rm} vs. soil moisture content m_v at a fixed frequency of 1.074 GHz and at room temperature. The soil moisture content is the ratio of the weight of water per unit volume of soil.

Presented in Figure B1 are the results of 403 laboratory ε'_{rm} vs. m_v tests by Lundien (1971). His measurements were made on sand, silt, silt-loam and clay soils using an L-band interferometer. Also shown is a refraction equation which well describes ε'_{rw} vs. m_v for the commingled soils. The regression curve in Figure B1 fits the data with an $r^2 = 0.970$ and a standard error of ± 2.02. When the regression curve is forced through an ε'_{rw} for water of 81, the equation for the curve becomes as shown in Figure B2. The data lying below the regression curve tends to be those for clay type soil while the data above the curve is that for the coarser grained soils. Representative regression curves passed through the sand and clay material and forced through an ε'_{rw} of 81 at $m_v = 1$ are shown in Figure B3.

The test results shown in Figure B1 will vary with temperature as previously mentioned. However, Lundien (1971) found that over a temperature range of 0° to 65°C the change in ε'_{rm} appears to be uniform, varying at about 1/2% per °C.

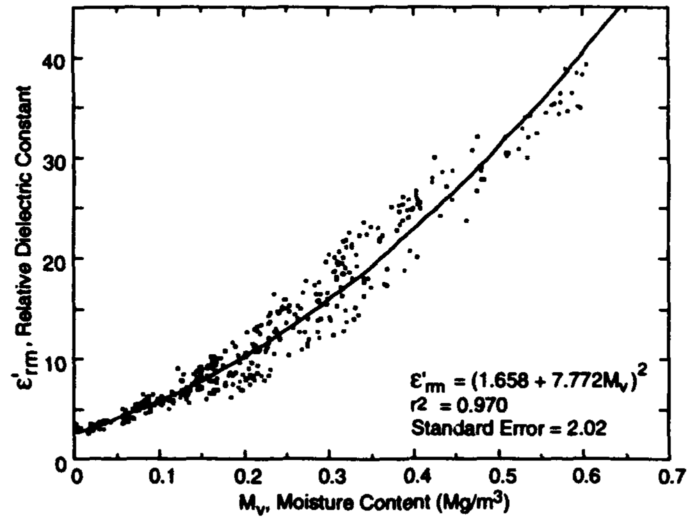

Figure B1. Dielectric constant of soil vs. moisture content.

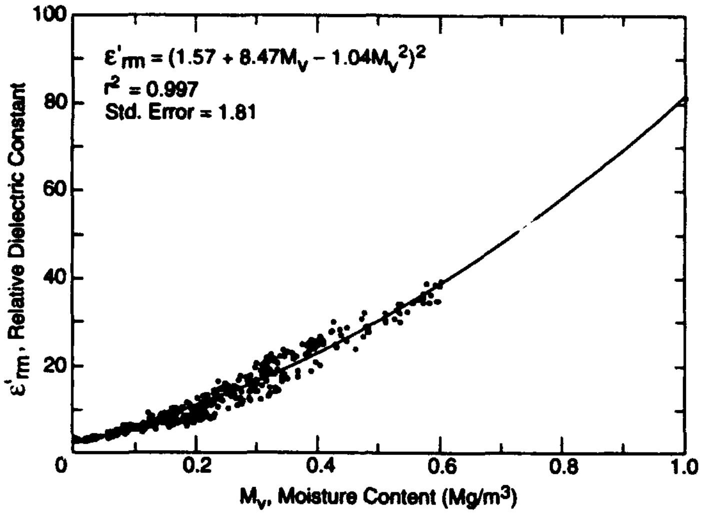

Figure B2. Dielectric constant of soil vs. moisture content where the regression curve was forced through a dielectric constant for water of 81.

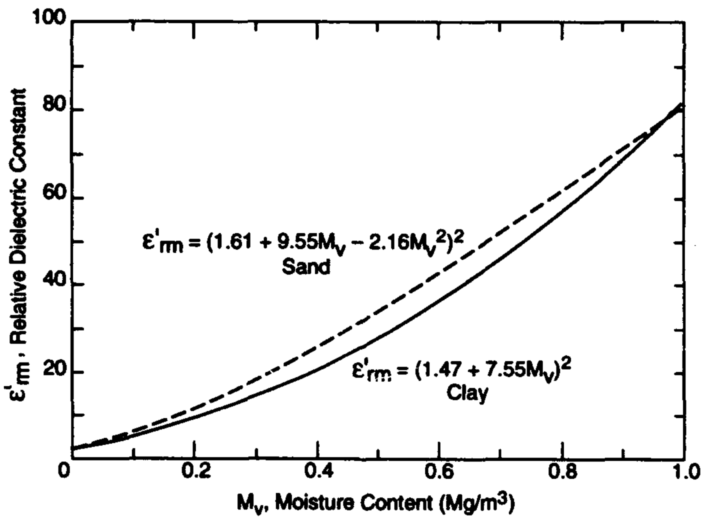

Figure B3. Representative curves for the dielectric constant of sand and clay soil vs. moisture where the respective curves were forced through a dielectric constant for water of 81.

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13. ABSTRACT (*Maximum 200 words*)

The success in using VHF and UHF frequency systems for sounding polar ice sheets has been tempered by an uncertainty in the in-situ dielectric constant ε', which controls the effective velocity V_e of an electromagnetic wave propagating in an air–ice mixture. An empirical equation for determining ε' vs. density (specific gravity, ρ) was proposed in 1968 by Robin et al. where $\varepsilon' = (1 + 0.851\ \rho)^2$. However, this expression has met with uncertainty because wide-angle radar refraction sounding techniques have produced values of ε' that are lower than Robin's equation predicts. This report discusses radar soundings made on the McMurdo Ice Shelf, Antarctica, and compares the resulting ε' values with Robin's equation, laboratory measurements on firn and ice and other expressions given in the literature for determining ε' vs. the specific gravity of dry firn and ice. Our findings indicate that the form of Robin's equation is valid. However, our analysis also indicates the expression could be slightly improved to read $\varepsilon' = (1 + 0.845\ \rho)^2$. Reasons are suggested as to why previous wide-angle radar sounding studies did not reproduce Robin's findings.

14. SUBJECT TERMS

Antarctica	Firn	Glaciology
Dielectric constant	Glacial sounding	Ice shelves
		Radio echo sounding

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INFORMATION

ERRATA
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Throughout the report, ε'_e should read ε_e and be defined as the effective (bulk) dielectric constant.

Inside cover

Abstract, line 5, "1968" should read "1969"

Cover caption, the words "traverse" and "Antarctica" were misspelled

p. 1, column 1, paragraph 2, last line: ">>" should read "<<"

p. 9, column 1, paragraph 4, line 1, "Böttcher (1933)" should read "Böttcher (1952)"
column 2, equation 18 should read as follows:

$$\varepsilon'_r = (1.006 + 0.839\rho)^2$$

column 2, equation 19 should read as follows:

$$\varepsilon'_r = 1 + \frac{3\nu_i(\varepsilon'_{ri} - 1)}{(2 + 3\varepsilon'_{ri}) - \nu_i(\varepsilon'_{ri} - 1)}$$

p. 14 *Literature Cited*, Add or correct the following:

Burns, B.A., R.W. Larson and R.G. Onstott (1985) 100-MHz dielectric constant measurements of snow cover: Dependence on environmental and snow pack parameters. *Proceedings, International Geoscience and Remote Sensing Symposium (IGARSS '85) Amherst, Massachusetts*, **2**: 829–834. IEEE cat. no. 85CH2162-6.

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Hallikainen, M., F.T. Ulaby and M. Abdel-Razik (1982) Measurement of the dielectric properties of snow in the 4–18 GHz frequency range. *Proceedings 12th European Microwave Conference, Kent, England*, Microwave Exhibition and Publishers Ltd., London, p. 151–156.

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