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ADDITIVE TURBULENT DECOMPOSITION OF THE INCOMPRESSIBLE AND COMPRESSIBLE NAVIER-STOKES EQUATIONS

U. S. Air Force Office of Scientific Research Grant #F49620-92-J-0113

(Final Report, 1991-93)

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ABSTRACT

Research results summarizing the past two years' efforts are presented. These include analysis of both the large- and small-scale equations of additive turbulent decomposition (ATD) for the 2-D incompressible Navier-Stokes equations. Both of these solution procedures are complete for Cartesian coordinates and check out is essentially finished for generalized coordinates. We discuss our method used to filter solutions to the large-scale equations, and the domain decomposition methods we are employing to recouple local small-scale solutions to produce global solutions on the small scale. We also present results obtained from our large-scale equations/chaotic map turbulence modeling approach for intermittent flow.

1. INTRODUCTION

This report summarizes research carried out under AFOSR Grant #F49620-92-J-0113 "Additive Turbulent Decomposition of the Incompressible and Compressible Navier-Stokes Equations," during the period 15 Nov 1991 to 14 Nov 1993. The work originally proposed for this grant included continuing studies of additive turbulent decomposition (ATD) in the context of two-dimensional (2-D) incompressible flows in generalized coordinates, and initial studies of 2-D compressible turbulent flows. Due to a significant reduction in funding from the proposed amount, the second of these tasks has not been carried out under this grant. We note, however, that this work has begun under a separate AFOSR Grant #F49620-92-J-0488, effective 1 Sep 1992, and will be reported at the appropriate time.

Studies completed during the first year of this project have been discussed in fair detail in the Annual Report for 1991-1992 (McDonough et al., 1992), so this work will mainly only be summarized in the present report for the sake of completeness. This research included the analyses required for the construction of ATD in generalized coordinates and discussion of the projection method algorithm employed to solve the large-scale equations. Details of the small-scale equations and their solutions were presented, and the complete ATD solution algorithm was discussed briefly. Two key parts of this were treatment of aliasing of the coarsely resolved solutions of the large-scale equations; and parallelization of ATD. We will include considerably more regarding both of these topics in the present report, as well as on new domain decomposition approaches for coupling the local small-scale solutions.

A second major topic was also presented in the referenced Annual Report, namely turbulence models based on the large-scale equations of ATD. We provided background information and motivation for this approach, and we discussed the basic approaches being employed to correlate the chaotic maps to be used as turbulence models to date. We then gave some preliminary results for pipe flow. In the present report we will supply results from the continuation of these studies. Considerable progress has been made, both in terms of completed

results and with respect to analytical studies to put this approach on a firm theoretical foundation.

A final topic reported in the Annual Report involved the effects of Reynolds averaging. These studies have been concluded for the incompressible N.-S. equations, and a paper has been submitted to the *Physics of Fluids A* (now *Physics of Fluids*.)

2. COMPLETE ADDITIVE TURBULENT DECOMPOSITION

One of the main problems proposed for study under the current grant was implementation of ATD for the incompressible N.-S. equations in generalized coordinates. The method requires no averaging, or modeling at any level due to additive two-scale decomposition of governing equations. Thus, like direct numerical simulation, it is completely consistent with the original unaveraged equations; but required arithmetic is significantly reduced via consistent linking of large-scale and small-scale phenomena, resulting in the ability to focus on local regions. For completeness, we will summarize the main idea of ATD in Sec. 2.1. Next, the progress on the large-scale equations, mainly focusing on filtering of the aliasing error which arises from not being able to resolve all wavenumber components in the coarse large-scale grid, is reported in Sec. 2.2. In Sec 2.3, we summarize the extensive investigation of the small-scale equations, most of which had already been completed at the time of the Annual Report, and in Sec 2.4 we discuss our recent efforts pertaining to construction of global small-scale solutions by applying domain decomposition-like algorithms to the collection of local small-scale results.

2.1 Theoretical Formulation of ATD

The equations we are studying are the 2-D incompressible N.-S. equations given as

$$\nabla \cdot \mathbf{U} = 0, \tag{1}$$

$$\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{U} = -\nabla P + \frac{1}{Re} \Delta \mathbf{U}. \tag{2}$$

Here $\mathbf{U} = (U,V)^T$ is the velocity field, and P is pressure(divided by constant density); Δ is the 2-D Laplacian. Using additive decomposition we first decompose dependent variables in the usual manner:

$$U = u + u^*, \quad V = v + v^*, \quad P = p + p^*, \tag{3}$$

where u, v and p are large-scale quantities, and " * " denotes the small-scale part. It should be remarked that large-scale quantities are not averages, and should be interpreted as the first few terms of a Fourier representation; the small-scale part then corresponds to the series remainder. Substituting Eqs. (3) into Eq. (1), we get the large- and small-scale continuity equations,

$$u_x + v_y = +\delta, \tag{4a}$$

$$u^*_x + v^*_y = -\delta, \tag{4b}$$

where δ is the decomposition divergence, which here we set to zero. We note that the choice $\delta = 0$ is not necessary within the ATD formalism; but it is convenient, and it allows ATD to more nearly conform with classical approaches. Applying ATD, as analyzed by McDonough et al. (1989), we obtain the coupled system of differential equations for **u** and **u***:

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla\cdot(\mathbf{uu}) + \nabla\cdot(\mathbf{u^*u}) = -\nabla p + \frac{1}{Re}\Delta\mathbf{u}, \tag{5a}$$

$$\frac{\partial \mathbf{u^*}}{\partial t} + \nabla\cdot(\mathbf{uu^*}) + \nabla\cdot(\mathbf{u^*u^*}) = -\nabla p^* + \frac{1}{Re}\Delta\mathbf{u^*}. \tag{5b}$$

We note that Eqs. (4, 5) comprise six equations for the six unknowns; hence there is no closure problem, and no need for modeling. Equation (5a) is the equation for the large-scale velocities, and it can be readily solved by standard numerical methods, as described in the next section.

One of the main features of the overall ATD algorithm as originally proposed by McDonough et al. (1984a, 1984b) is the spatial decomposition of the small-scale equation (5b)

into local systems defined on non-overlapping subdomains containing each large-scale discretization point. This is shown schematically in Fig. 1. The local equations can be solved independently, providing complete parallelization, and recoupled to obtain the global small-scale solution via domain decomposition techniques (cf. Glowinski et al., 1988). We remark that although there are opportunities for parallelization in most algorithms associated with the standard methods, these usually lie at the numerical analytic level. However, ATD provides a high degree of natural parallelizability in its basic structure, completely independent of specific numerical methods used in the solution process (and, of course, the numerical methods still can be parallelized). Thus, ATD will provide advantages on massive parallel processors (MPPs) that are not available in standard approaches to turbulence simulation.

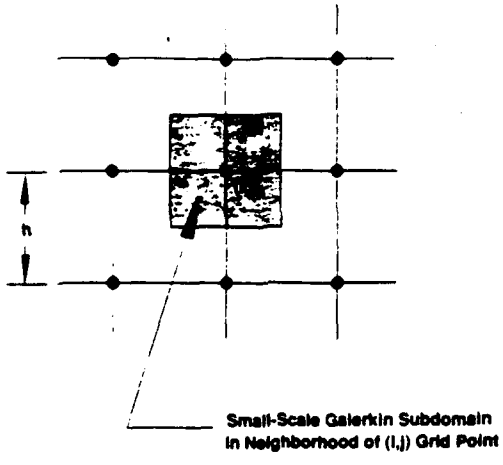

Figure 1. Local Galerkin Region

2.2 Large-Scale Calculations

The large-scale equations have been coded in the generalized coordinates based on Gresho's projection method (1990). The equations are discretized using centered differencing in space and generalized trapezoidal integration in time. The details of implementation can be found in the Annual Report by McDonough et al. (1992).

It is well known that the reason for filtering the N.-S. equations in large eddy simulation

(LES) is to produce a system of equations whose solutions can be completely resolved with a reasonable discretization. The price that is paid for this is introduction of an eddy viscosity that leads to non-physical dissipation terms in the differential equations, and the need for subgrid-scale (SGS) modeling for constructing this eddy viscosity. In the ATD algorithm we have deliberately avoided averaging or filtering the governing equations, but this means that we must find a way to deal with aliasing errors that arise from not being able to resolve all wavenumber components present in the actual solution.

As mentioned earlier, the finite-difference method is used to solve the large-scale equations, because it is simple to implement and also the most commonly used method in current application codes. It is well known that oscillatory behavior may occur in the centered finite difference solution when the cell Reynolds number is greater than two (Roach, 1972). The cell Reynolds number is defined as

$$Re_{cell} = \frac{uh}{\mu}$$

where u is local velocity; μ is dynamic viscosity, and h is grid spacing. This results in the loss of diagonal dominance and positivity of the associated tridiagonal matrices, and consequent oscillatory fundamental solutions to the difference equations. The oscillations can be eliminated by reducing the grid size and therefore the local value of Re_{cell}. But this is not always possible due to core memory and CPU time requirements. The spurious oscillations can also be controlled by using schemes that are intrinsically dissipative, such as the upwind differencing, or by explicitly adding artificial viscosity to damp out numerical oscillation. However, upwind differencing alters the difference equations, and in extreme cases (such as simple first-order upwinding) results in qualitatively incorrect solutions; and addition of artificial dissipation formally changes both the differential and difference equations.

The other possible approach is to use filters to suppress the numerical oscillations, as has been successfully done in shock capturing (Engquist et al., 1989 and Shyy et al., 1992). As

suggested by Shyy et al. (1992), one may choose to extract the "useful", i.e., physically realizable, information from oscillatory solutions obtained using unsatisfactory numerical schemes. The idea is to eliminate undesirable portions of the solution while retaining only the desired, physically realizable ones. This approach is fundamentally different from LES where a filter is used to filter the differential equations rather than the solutions. As a consequence, there is no inherent closure problem, and hence, no required modeling when solutions are filtered.

Motivated by the work of Shuman (1957) and Shapiro (1970), we choose to use a family of Shuman-like filters designed as a postprocessor to damp the information that is not resolvable on the large-scale. The 2-D filters are in the form

$$\bar{f}_{i,j} = \frac{f_{i+1,j} + f_{i-1,j} + f_{i,j+1} + f_{i,j-1} + mf_{i,j}}{4+m} \tag{6}$$

These filters are second-order accurate with a truncation error $(h_x^2 u_{xx} + h_y^2 u_{yy})/(4 + m)$. In general, m can take on any value greater than four. However, the optimal choice is governed by the minimum amount of numerical viscosity required to suppress the aliasing error growth. As pointed out by Khosla and Rubin (1980) there may be no general way of arriving at an optimal value of m analytically; it must be obtained by numerical experiments. The analysis of above family of filters shows the following properties: i) they provide good results on reasonably fine grids, and ii) they have little effect on the low wavelength components of the solutions.

We have conducted a number of numerical experiments to aid in our characterization of the behavior of the filters represented by Eq. (6). These have included studies of both steady and time-dependent 1-D Burgers' equations and in conjunction with time-accurate projection methods for the lid-driven cavity problem. Figure 2 shows the filtered and unfiltered solution for a steady Burgers' equation problem with the exact solution given as

$$u = \frac{1}{2}(1 - \tanh\frac{x}{4\nu}).$$

For these results, the grid spacing and the pseudo-time step size are both 0.2. The diffusion coefficient is taken to be 10^{-6}. Hence, the cell-Reynolds number restriction is grossly violated. The smooth solution is obtained using a filter with m = 3.4 in Eq. (6), which is almost identical to the exact solution. Without filtering, the solution diverges. The oscillatory solution shown is obtained using a filter with m = 100, which corresponds to a very weak filter. We observe that with an appropriate filter, oscillations characteristic of large cell Reynolds numbers can be eliminated, i.e., the aliasing error growth can be controlled. The fact that the filtered solution is almost identical to the exact solution demonstrates that the filters are able to distinguish the aliased solution from the unaliased solution.

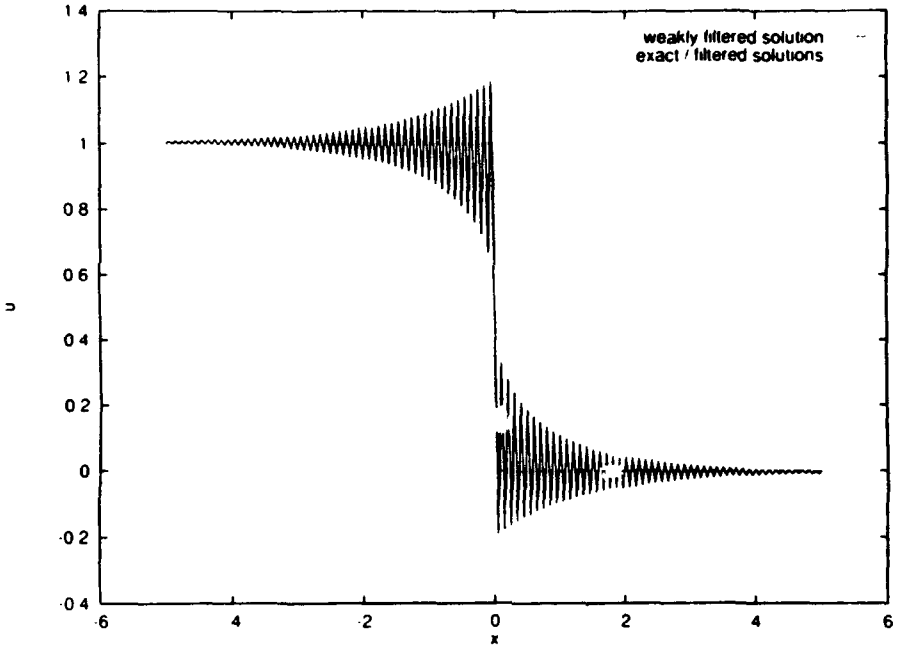

Figure 2. Comparison of Solution Profiles with Different Filters

The above analysis is confined to the stationary solution of Burgers' equation. The next problem considered was a time-dependent Burgers' equation with forcing term such that the exact solution becomes $u = x^t$. For short time, no oscillation is found because local the cell Reynolds number is small. When we integrate Burgers' equation for a longer time with no filter, zigzag-type oscillation appears in the solution. Figure 3 shows part of the filtered and unfiltered

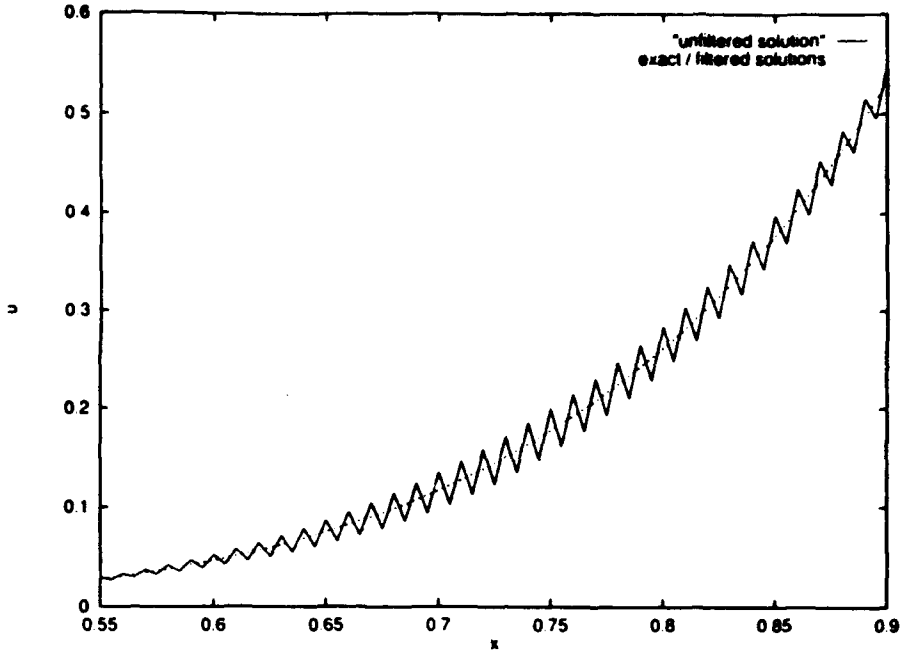

Figure 3. Comparison of Solution Profiles with and without Filtering

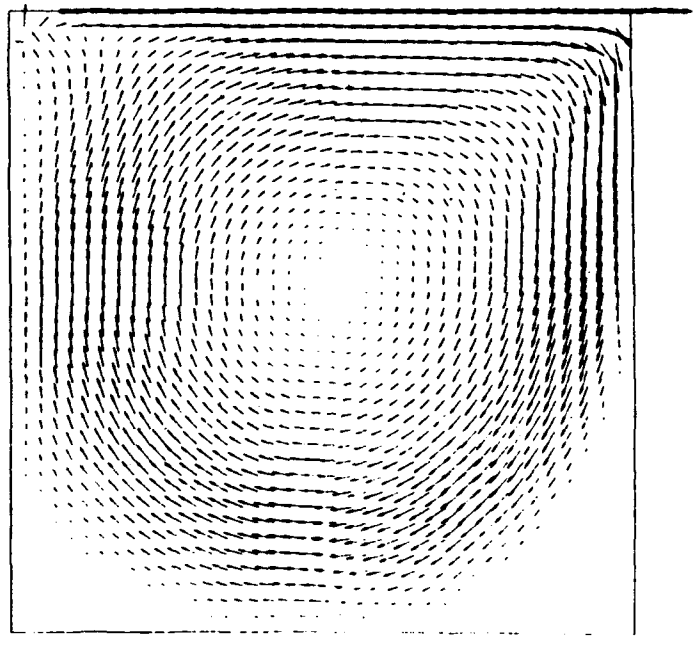

Figure 4. Filtered Velocity Field of Lid-Driven Cavity at Re = 5000

numerical solution as well as the exact solution at t =6.0 . The solutions are obtained with both grid spacing and time step being 0.005. Once more it is demonstrated that filtering can suppress the growth of errors arising from aliasing.

Numerical calculations were also performed for the 2-D lid-driven square enclosure flow with and without filters. Figure 4 presents velocity vector plots for Re = 5000 on a 41×41 grid. Without filters the solution diverges due to the aliasing error growth. This plot agrees qualitatively with published results, showing the primary central vortex and secondary vortices in both of the two lower corners.

The complete report on the study of the linear filters can be found in a forthcoming paper by Yang and McDonough (1994). We would like to conclude the discussions presented here by making the following remarks concerning the numerical results given in this section. First, quite satisfactory solutions have been obtained from otherwise oscillatory ones via the filtering procedure. Second, for the steady state problems, we need to add a pseudo-time derivative in the equation to be solved, and use a time-stepping method to get the steady-state solution. This idea is similar to the multistage smoothing in the multigrid method. As implied by Majda et al. (1978), and also pointed out by Shyy et al. (1992), it is too late just to filter the final steady-state solution (if one is obtained) because the characteristic wavelength of the oscillations is too long to be affected by a low-pass filter. From the discussion in the previous section, we know in this case the filter can do little to improve the solution accuracy. On the other hand, the high-frequency, short wavelength oscillations may lead to nonlinear instabilities, and divergence, before a steady state can be achieved if the filter is not applied at every time step. Finally, we should note that the use of post-processing filters is particularly attractive in the context of the ATD algorithm because if the filter can be accurately characterized so that it is known how much of the large-scale information has been removed, it is possible to add this back in during the small-scale calculations. Thus, it may be of value to employ ATD-like algorithms even for laminar flow calculations as a means of treating cell Reynolds number problems.

We have made steady progress toward completion of the large-scale calculation procedure. The current code is undergoing thorough investigation for lid-driven cavity flow in generalized coordinates, and generation of the grid for flow over a NACA 0012 airfoil is finished. The proper implementation of outflow, inflow and solid boundary conditions is now in progress, and more efficient Poisson solvers based on the strongly implicit procedure (SIP) are being implemented and compared with SOR and ADI. We expect this will greatly reduce the CPU time required for each time step since most CPU time is spent on solving the Poisson equation resulting from use of projection methods.

2.3 Small-Scale Calculations

The complete investigation of small-scale performance and the large- to small-scale transfers of data is reported in Yang and McDonough (1992a). Unlike the large-scale calculation, the small-scale equations are solved through constructing local Galerkin approximations in the neighborhood of each finite difference grid point, as indicated in Fig. 1.

We assume the solution can be represented by the truncated Fourier series defined as follows on a local domain containing the grid point (x_i,y_j):

$$u_{ij}^*(x,y,t) = \sum_{k,m=1}^{K} a_{km}^{(i,j)}(t)\cos\alpha_k x^* \sin\alpha_m y^*, \tag{7a}$$

$$v_{ij}^*(x,y,t) = \sum_{k,m=1}^{K} b_{km}^{(i,j)}(t)\sin\alpha_k x^* \cos\alpha_m y^*, \tag{7b}$$

$$p_{ij}^*(x,y,t) = \sum_{k,m=1}^{K} c_{km}^{(i,j)}(t)\sin\alpha_k x^* \sin\alpha_m y^*, \tag{7c}$$

where

$$x^* = x - (x_i - h/2), \qquad y^* = y - (y_j - h/2),$$

with $(x,y) \in [x_i-h/2, x_i+h/2]\times[y_j-h/2, y_j+h/2]$, and $\alpha_m = m\pi/h$. Here h is the finite difference grid spacing for the large-scale calculation.

By substituting Eqs. (7) into (4b) and (5b) and constructing the Galerkin inner products, we obtain a system of ordinary differential equations for the time-dependent Fourier coefficients. Details can be found in Yang and McDonough (1992a,b). The local large-scale velocity data are directly input to the small-scale equations and appear as coefficients of the convolution terms. This allows computation of the small-scale solution without explicitly representing all modes of the large-scale as well. This is the critical difference between small-scale ATD and usual DNS, and it renders ATD far more efficient for local calculations than would be DNS. At the same time we believe it is capable of providing more faithful simulations than does "minimal" DNS proposed by Moin (e.g., Moin,1992), specifically because of this large-scale information.

The resulting system of ordinary differential equations are integrated using a second-order Runge-Kutta method (Heun's method) with constant time step. Complete details of choice of initial conditions, etc. can be found in Yang and McDonough (1992a,b). Figure 5 displays a typical time series corresponding to a chaotic solution. The value of Re at which this calculation has been performed ($Re = 10^6$) is several orders of magnitude higher than has previously been possible with standard methods. This is accomplished by performing the calculations on very small length scales, and by supplying large-scale input to these scales in a consistent manner. Nevertheless, it is important to check the resolution adequacy of these results. Figure 6 provides an indication of the degree of convergence of the small-scale solution by depicting the rate of decay of Fourier coefficient time-averaged amplitudes as a function of wavenumber. In addition we comment here that a comparison of our overall effective resolution (large-scale, based on the small-scale subinterval length, plus small-scale modal representation) with predictions by Foias and Treve (1981) regarding required number of modes as a function of Reynolds number shows that our calculations should be fairly reliable.

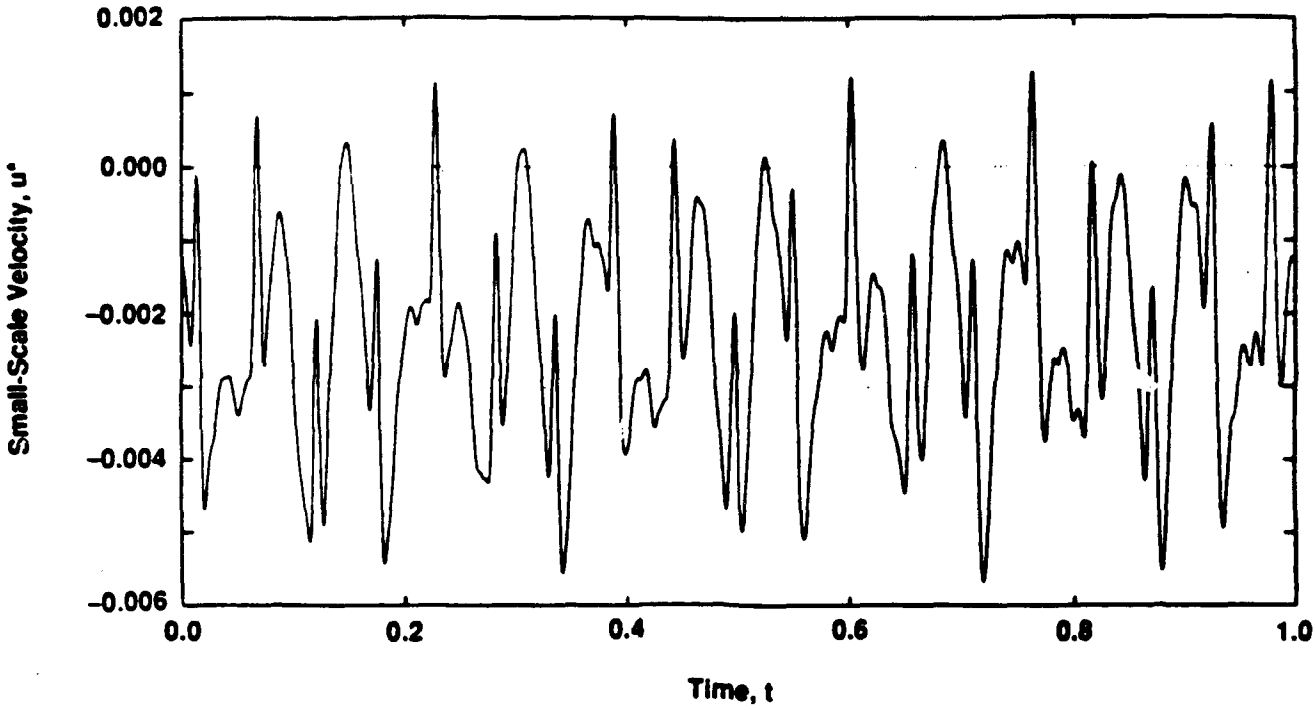

Figure 5. Small-Scale Fluctuating Velocity, Re = 10^6

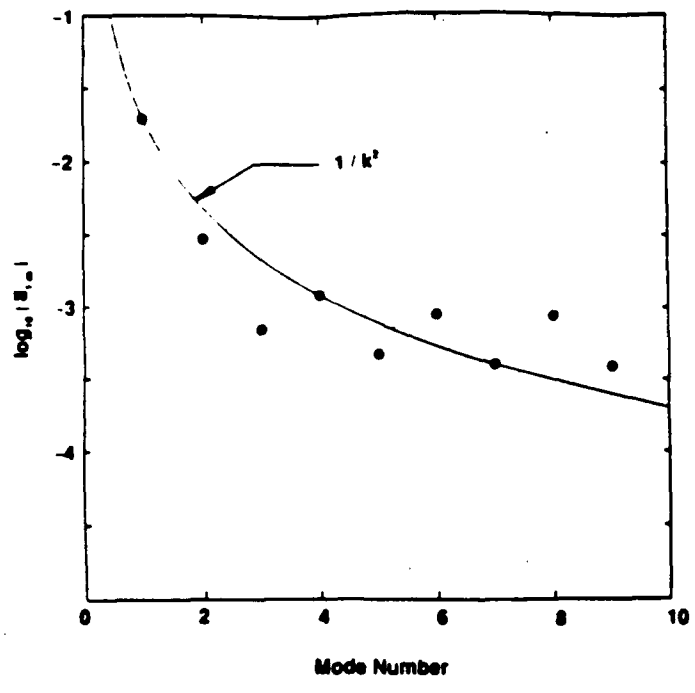

Figure 6. Convergence of Small-Scale velocity Fourier Representation,Re = 10^6

Our experience working with the small-scale equations has shown that the discrete convolution evaluations consume a tremendous fraction of computing time, as would be expected, since this requires $O(K^2)$ arithmetic per time step for each of the K ODEs. We also note that the number of modes typically used in the small-scale calculation is not large enough for a FFT to provide much improvement. The remedy is to take advantage of rapid development of parallel computers. Our preliminary parallel processing studies using 6-processor IBM 3090-600Js demonstrate that speedup is linear with number of available processors. At the same time, we expect to achieve significant, but less than linear, speedup by parallelizing ODE solvers on each small-scale local domain. This work is scheduled to start very soon. Fulfillment of this task will enable us to simulate turbulent flows at high Reynolds numbers that are not achievable by LES and DNS.

2.4 Domain Decomposition

We have as yet not implemented a *specific domain decomposition procedure* for the small-scale equations, but we have been studying two different approaches. Results due to Chan and Mathew, reported in McDonough et al. (1989) suggest that it may not be necessary to employ a traditional domain decomposition algorithm in the context of ATD because the small-scale solutions are needed only at large-scale grid points, which happen to lie well within the interior of the small-scale subdomains, and because the effects of boundary errors (the thing that must be corrected by domain decomposition iterations) are not very large in the interior of the subdomains.

Our first approach to recoupling the local small-scale calculations to obtain the global small-scale solution makes direct use of this. This method is, in a sense, a predictor-corrector procedure requiring two separate solutions of the small-scale equations. The prediction is made using overlapping subdomains of size 2h, where h is the large-scale grid spacing, and Fourier representations containing K+1 modes. These calculations can be performed in an

"embarrassingly" parallel fashion with no iteration, and there are essentially the same number of such subdomains (in 1-D) as occur for the fully-resolved h-subdomain small-scale calculations. It can be seen from Fig. 7 that the center of each of the 2h-subdomains (where errors are smallest) coincides with the boundary between two h-subdomains. Thus, the 2h-subdomain calculation should provide a reasonably accurate prediction of boundary values for the h-subdomain calculations. The latter are then performed, again non-iteratively, and their results—taken from the center of the h-subdomains—are used to construct the small-scale solutions for use in the large-scale equations. It should be observed that at each stage of this process, results that are actually used are taken from the part of the subinterval at which errors should be a minimum. This predictor-corrector form of domain decomposition is currently being tested in the context of 1-D model problems.

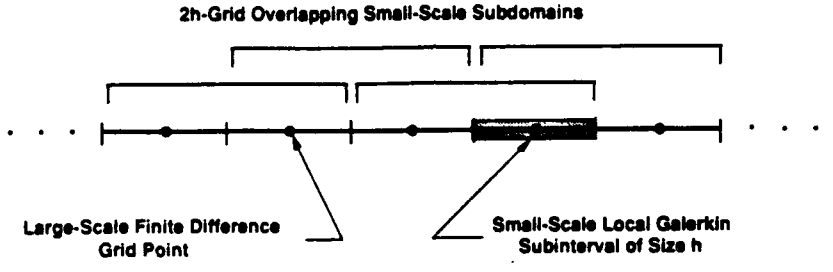

Figure 7. Domain Decomposition for Recoupling Small-Scale Solutions

The second method that we are considering for recoupling the small-scale solutions makes use of the well-known fact from domain decomposition theory that the fine-grid recoupling iterations converge faster if there is an underlying coarse grid calculation. (This fact is used implicitly in the first approach also.) In particular, after the small-scale solutions have been computed on all of the individual subdomains independently, they can be combined to form a global solution in the following way. We discretize the small-scale equations (5b) on the large-scale time step. Let n+1 be the desired advanced large-scale time level, and suppose N^* is the small-scale time step corresponding to this. Let

$$u^{n+1} = u^n + \frac{k}{2}\left[L_h\left(u^{n+1^*}\right) + L_h\left(u^n\right)\right]$$

be a generic representation of Eq. (5b). Then in terms of the above notation we have

$$u^{*,n+1} = u^{*,N^*-1} + \frac{k_s}{2}\left[L_h\left(u^{*,N^*}\right) + L_h\left(u^{*,N^*-1}\right)\right] \tag{8}$$

where k_s is the small-scale time step, and h is the large-scale space step. We observe that Eq. (8) directly leads to nearest-neighbor coupling of the small-scale solutions, but in the context of projection methods should probably be preceded by a mass-conservation correction of both u^{*,N^*} and u^{*,N^*-1}. Thus, completely global coupling will be achieved through the pressure Poisson equations. In particular, Eq. (8) could be used iteratively to set boundary conditions for the next set of small-scale calculations, but it seems unlikely that this would be necessary.

It is this second procedure that we plan to employ first in the construction of our complete ATD algorithm due to its relative simplicity for multi-dimensional calculations. We also note in closing this section that for high-Re, fully developed turbulence, it may be unnecessary to expend much effort in recoupling the small-scale solutions because spatial correlation lengths may be far smaller than the large-scale grid spacings.

The final theoretical area that has received attention by the PI concerns using very local (only a few grid points in each direction) finite difference (or finite volume) discretization of the small-scale equations. These are, of course, nonlinear algebraic maps, but not a great deal is yet known regarding their properties. Such a formalism lies between the large-scale ATD/chaotic map model approach and complete ATD. It is clearly consistent with the N.-S. equations, and it is embarrassingly parallel. At the same time, it is far more efficient than complete ATD (but would converge to complete ATD as the small-scale domain sizes approach the large-scale grid spacing, or vice versa), and unlike complete ATD it would be fairly straightforward to implement in the context of adaptive, multi-level (more than two levels) unstructured grids. We

have so far been able to demonstrate only what would be expected on intuitive grounds; namely, the accuracy of the large-scale grid spacing can be maintained, and qualitatively realistic small-scale solutions can be computed to this same level of accuracy.

3. LARGE-SCALE ATD WITH CHAOTIC MAP TURBULENCE MODELS

In this section we will discuss an aspect of ATD that was new to the current grant, namely use of the large-scale equations (4a, 5a) alone with fluctuating quantities modeled with chaotic algebraic maps. We have provided considerable information on this approach in the 1992 Annual Report (McDonough et al., 1992), so the present discussion will focus mainly on results obtained during 1993. Nevertheless, we will include some basic background information in this report for the sake of completeness. Following this background information will be a subsection in which we present further results obtained by applying this approach to pipe flow. Then in a final subsection we will discuss some theoretical developments associated with method.

3.1 Background

We first remark that in light of recent results showing that Reynolds-averaged approaches cannot be consistent with the Navier-Stokes equations (McDonough, 1993), and that this lack of consistency results from having to model terms that arose, in the first place, as a consequence of averaging, it would seem reasonable to investigate the possibility of developing turbulence models based on unaveraged equations. ATD provides a quite natural setting in which this can be done. In particular, we can see from Eq. (5a) that if u* can be modeled in such a way as to approach zero as the grid spacing goes to zero, then this approach would be consistent with the N.-S. equations. It is interesting to note that the Reynolds stresses in LES typically have this property, but there remains a realizability problem in LES, at least when dynamic SGS models are used, that is completely absent in ATD. In fact. Galilean invariance and realizability are all but automatic in ATD because it is the fluctuating quantities themselves that are modeled. This is a major advantage of this approach.

3.2 Intermittent Turbulent Pipe Flow

As we have reported in the 1992 Annual Report, we have been studying large-scale ATD with chaotic map models for intermittent turbulence in an axisymmetric pipe. Although it is not entirely realistic for turbulent flow, we have retained the axial symmetry in our formulation. The basic algorithm proceeds in the following way for each time step:

1. Compute an estimate of **u** from (4a, 5a) using **u*** from time level n.
2. Use these large-scale results as parameters in chaotic maps to calculate $\mathbf{u}^{*,n+1}$ at each grid point.
3. Correct $\mathbf{u}^{n+1}$ by solving (4a, 5a) with $\mathbf{u}^{*,n+1}$ in the crossterms.
4 Construct the complete solution at time level n+1:

$$\mathbf{u}^{n+1} = \mathbf{u}^{n+1} + \mathbf{u}^{*,n+1}, \quad p^{n+1} = p^{n+1} + p^{*,n+1}.$$

It should be observed that in our current algorithm we apply a mass conservation calculation to the small-scale velocity fluctuations created from the algebraic map in order to guarantee that $\nabla\cdot\mathbf{u}^* = 0$ (and thus $\nabla\cdot\mathbf{U} = 0$), and to simultaneously generate $p^{*,n+1}$ for calculating p^{n+1}. We do not, however, guarantee that the small-scale momentum equations (5b) are satisfied, either locally or globally. We have discussed in McDonough et al. (1992) the various criteria we feel must be met to assure that the chaotic map models behave in a realistic way, but even if all such criteria are satisfied we cannot be guaranteed that **u*** is truly close to a solution to (5b). Thus, the procedure as we have implemented it to date is indeed a model, and although it is guaranteed to be consistent with the N.-S. equations as discretization step sizes approach zero, little can be guaranteed a priori for the large step sizes one might typically use in solving practical engineering problems.

From a physical viewpoint we see that Eq. (5a) accounts for transport of small-scale quantities by the large-scale velocity field, but not vice versa. It is possible that in many flow situations this is reasonably accurate, and for such flows this modeling procedure may produce

quite accurate results. But in the contrary case it may not. It should already be clear, however, that there is an easily implemented, relatively inexpensive improvement that can be made to cover this problem. Namely, construct a global small-scale solution in the same manner as discussed in Sec. 2.4, but now start with results from chaotic maps rather than local solutions to the N.-S. equations. We have, as yet, not implemented this promising addition, so the results we now present were obtained from our original formulation.

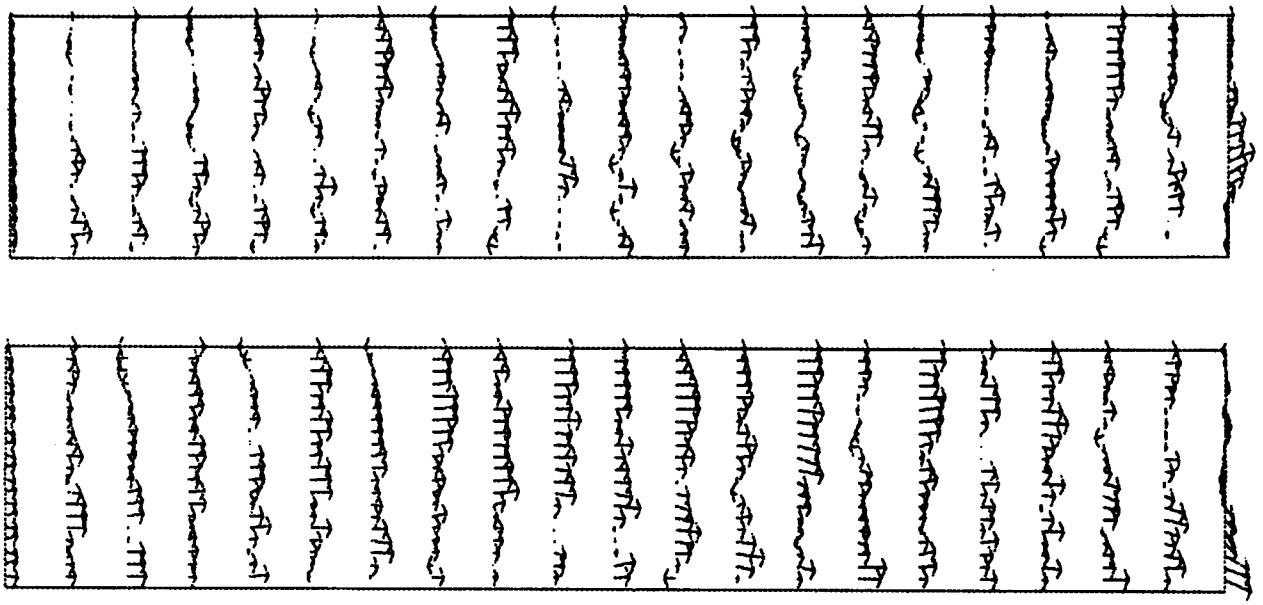

Figure 8. Velocity Fluctuation Distribution from Chaotic Map Model
for Iteration step = 1500, 3000

These calculations were performed on a quite coarse 41×21 grid for a pipe with L/D = 20 and Re = 2000. Thus, it can be expected that the flow will not be fully developed at the downstream end of the pipe. With the uniform grid spacing used in the radial direction, we expect to have no more than one grid point in the log layer near the pipe wall, but the goal in the present study is more to explore the potential of the method rather than to produce highly accurate results. Figure 8 displays the spatial distribution of fluctuating velocities at two different times during the calculations. The chaotic map being employed here is simple the logistics map. It should be noted that the velocity vectors are plotted on a much larger scale than that used in the calculations. In particular, $\mathbf{u}^* \sim 0.1\mathbf{u}$ is rather typical. Close examination of Fig. 8 reveals the

spatial and temporal chaos that would be expected in a turbulent flow, as well as indications of intermittency.

Figure 9 displays the distribution of large-scale velocities at the same times as in Fig. 8. We note that if the fluctuating velocities were identically zero , the large-scale velocities should be very regular (although not fully developed for Re = 2000). But the effects of the chaotic fluctuations are clearly evident. Finally, in Fig. 10 we present time averaged velocity profile along the pipe. This has been obtained by time averaging the results computed for the complete velocities at every time step. It can be seen that the flow is still developing, and that the velocity profile is distinctly different from the laminar case. However, the distribution of turbulent kinetic energy is not in complete agreement with what is expected for turbulent pipe flow, so evidently additional work needs to be put into development of the chaotic maps. More details regarding these calculations will appear in a forthcoming paper, McDonough and Zhong (1994).

3.3 Theoretical Studies

It is not difficult to see that there are numerous theoretical aspects of the large-scale ATD/chaotic map modeling approach that should be dealt with. We will mention a few here.

First, we would expect on intuitive grounds that the amplitudes of the small-scale fluctuations should depend, in some way, on local large-scale flow properties, but the question of how to quantify this dependence needs to be addressed. Moreover, as we have already indicated, we must require that these amplitudes also depend on the (local) discretization step sizes, and in particular approach zero with the discretization step size. But these are all rather vague, qualitative requirements. Work has recently been underway by the PI and students funded by AFOSR Grant #49620-92-J-0488 to provide quantitative results along these lines. Details will be reported in Hylin et al. (1994) and Weatherly et al. (1994). These studies are quite general and are being done in the context of 2- and 3-D compressible N.-S. equations. Some of the results, however, are quite generic and apply also to incompressible flows. In particular, we have shown

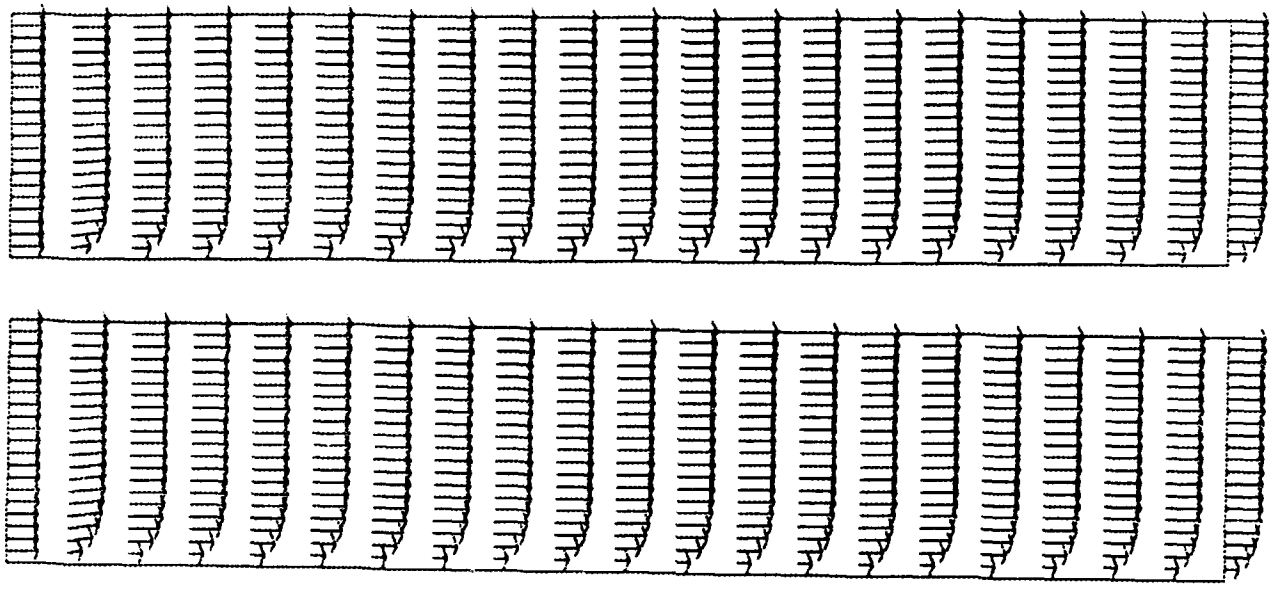

Figure 9. Large-Scale Velocity Distribution for Turbulent Pipe Flow
for Iteration step = 1500, 3000

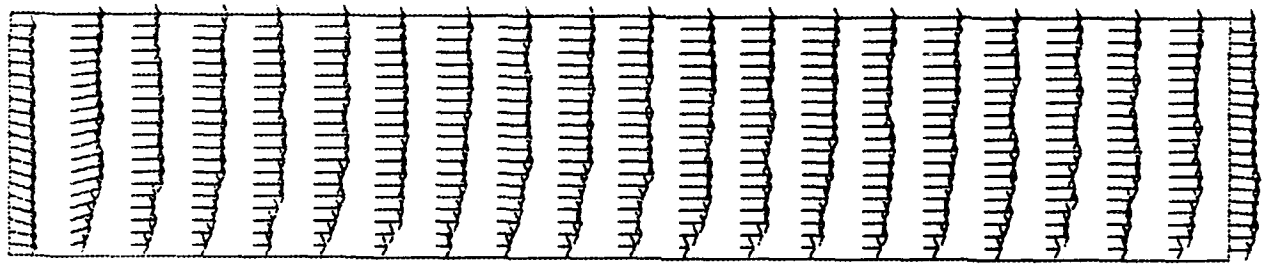

Figure 10. Averaged Total Velocity Distribution for Turbulent Pipe Flow

that velocity amplitudes obtained from chaotic maps should scale according to the following formula:

$$A_u = C_u \nu^{1/3} h^{1/3} \|\nabla u\|^{2/3}$$

where h is the large-scale grid spacing; ν is kinematic viscosity, and C_u is a constant that must be determined from data for particular classes of flows.

A second area of study involves details of the structure of the algorithm for implementing

large-scale ATD with chaotic maps. There are various alternatives to the one provided in the preceding subsection, distinguished primarily by the details of maintaining the divergence-free constraint. As noted earlier, it is not necessary that this be satisfied on either the large- or small-scale separately (but if it is satisfied on one, it must be on both), and this suggests a potentially more efficient procedure since most of the computational work on both scales is devoted to maintaining the divergence-free condition. As an alternative, we might separately calculate $\mathbf{u}$ and $\mathbf{u}^*$ without mass conservation, and then find P such that $\nabla \cdot \mathbf{U} = 0$. Once P is known, the complete equations can be solved for $\mathbf{U}$. This would eliminate half of the work involving pressure Poisson equation solves.

4. SUMMARY

We begin this summary section by observing that although we have not quite attained all of the goals set forth in the proposal for this grant, we have nevertheless advanced the development of ATD quite significantly in both of the two main areas under investigation: i) complete (large-scale plus small-scale ATD) in 2-D generalized coordinates for the N.-S. equations, and ii) large-scale ATD/chaotic map turbulence models. The list of major accomplishments include the following.

1. Completion of Cartesian coordinate large-scale equation solution algorithm, and near completion in generalized coordinates.
2. Thorough analysis of post-processing filtering techniques for treating the cell-Re problem arising in coarse-grid large-scale calculations, and demonstration that this is a very appropriate approach in the context of ATD
3. Completion of the generalized coordinate local Galerkin small-scale algorithm, including initial studies of parallelization.
4. Analysis of the domain decomposition techniques required to obtain global small-scale solutions.
5. Setting criteria and procedures to be used in developing chaotic map small-scale turbulence

models.

6. Construction and testing of code for modeling intermittent axisymmetric pipe flow.

7. Theoretical analysis of the small-scale turbulence models.

The results of these accomplishments have been, or will be, reported in the list of publications, conference presentations and invited talks given below.

Conference and Journal Papers

Hylin, E. C., McDonough, J. M. and Weatherly, D. C., 1994, "Modeling the Subgrid Scale Flow with a Chaotic Map," to be submitted to *J. Fluid Mech.*

McDonough, J. M., 1993, "On the Effects of Modeling Errors in Turbulence Closures for Reynolds-Averaged Navier-Stokes Equations," submitted to *Phys. Fluids A.*

McDonough, J. M. and Zhong, X., 1994, "Intermittent Pipe Flow Computation Via Additive Decomposition of the Navier-Stokes Equations with Chaotic Map Models," submitted to for ASME Winter Annual Meeting, 1994.

Weatherly, D. C., Hylin, E. C. and McDonough, J. M., 1994, "Additive Turbulent Decomposition with Subgrid Scale Chaotic Maps for Compressible Turbulence Simulation," to be submitted to *J. Fluid Mech.*

Yang, Y. and McDonough, J. M., 1992a, "Bifurcation Studies of Navier-Stokes Equations via Additive Turbulent Decomposition," in *Bifurcation Phenomena and Chaos in Thermal Convection,* Bau et al. (eds.), HTD-Vol. 214, ASME, New York.

Yang, Y. and McDonough, J. M., 1994, "Studies of Linear Filters For Treating Cell-Re Problems in Finite Difference Schemes," to be submitted to *Int. J. Comput. Fluid Dynamics.*

Conference Presentations and Invited Talk

Hylin, E. C., McDonough, J. M. and Weatherly, D. C., 1993, "Modeling the Subgrid Scale Flow with a Chaotic Map," *Bull. Amer. Phys. Soc. 38,* 2304

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