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D. G. Grabowska
P. Kavanagh

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Final Technical Report

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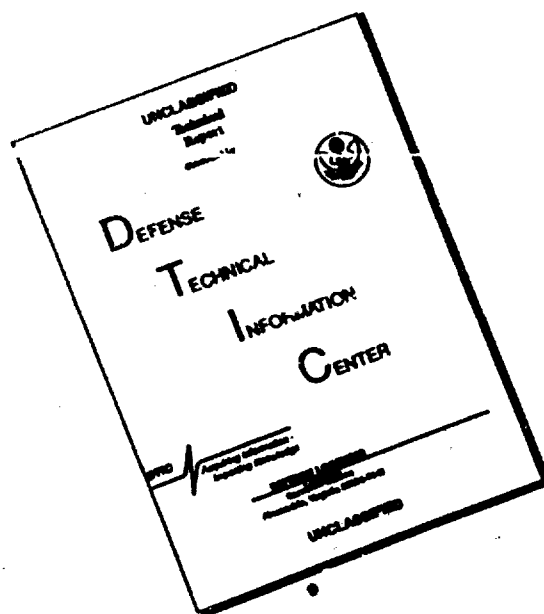
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Incorporating Radial Mixing in Axisymmetric Streamline Curvature Through-flow Analysis

Final Technical Report

D. G. Grabowska
P. Kavanagh

Department of Mechanical Engineering
Iowa State University

September 1993

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ABSTRACT

Based on the original development of radial mixing effects by Gallimore, the development of the governing equations for hub-to-tip streamline curvature solutions in tubomachinery passages and blade rows and incorporating viscous radial mixing is presented. Arbitrary passage geometries and quasi-orthogonal descriptions are accounted for. FORTRAN program coding is carried out for the special case of unbladed, axial-flow passages. Flow solutions are obtained by iteration based on fixed streamline patterns in the meridional plane followed by further iterations involving readjustment of the streamlines until overall convergence is obtained. Simple examples are presented to illustrate the solution technique. This work was carried out under AFOSR Grant F49620-92-J-0026.

SYMBOLS AND NOTATION

A, B	Parameter used to integrate the momentum equation
$\hat{B}$	unit vector tangent to the mean blade surface and which lies in the q - θ plane at an angle ϵ from the q - o direction
E_n, E_θ, E_m	viscous body force components
f_n, f_θ, f_m	distributed blade body force components
f_p	force vector normal to the blade surface
g	gravitational acceleration
H	stagnation enthalpy
h	static enthalpy
i	quasi-orthogonal index
j	streamline index
k	heat transfer coefficient
m	meridional direction
$\dot{m}$	mass flow rate
n	normal direction
p	static pressure
r	radial direction
T°	stagnation pressure
T	static temperature
z	axial direction
V_z, V_m, V_θ, V_r	velocities
Sc	Schmidt number $Sc = \mu/(\rho\epsilon)$
Pr	Prandtl number $Pr = \mu c_p/k$
$\hat{e}_n, \hat{e}_\theta, \hat{e}_m$	unit vectors
$e_{nn}, e_{\theta\theta}, e_{mm}$	normal strain rates
$e_{nm}, e_{n\theta}, e_{m\theta}$	shear strain rates
r_c	radius of curvature
q	heat flux
s	entropy
GREEK	
β	angle measured from the meridional direction to a unit vector tangent to the turbomachine blade (see Fig. A1)
γ	quasi-orthogonal lean angle measured from the radial direction

v

ϵ	mixing coefficient
θ	tangential direction
μ	viscosity
ρ	density
$\sigma_{nn}, \sigma_{\theta\theta}, \sigma_{mm}$	stress tensor components
ϕ	angle measured from the axial direction to a line tangent to a streamline
Φ	viscous dissipation function

1. INTRODUCTION

For many years, axisymmetric solutions obtained from streamline curvature analyses have been a standard approach in analyzing axial compressor or turbine design and performance. In this approach, the flow calculations are reduced as a practical consideration to a mathematically two-dimensional problem in which the through-flow is assumed to be steady, inviscid, and confined to concentric stream surfaces. As a result, no transport of mass, momentum, or energy takes place across these concentric surfaces, and thus no accounting of essentially three-dimensional effects is made due to secondary flows, blade or endwall boundary layer flows, or tip leakage flows. In particular, comparisons with detailed interstage traverses have shown that calculated results do not accurately model radial profiles of stagnation temperature, pressure, and flow angle as the profiles develop through the machine. Thus inclusion of spanwise mixing terms in the through-flow calculations would serve as important function in improving the modeling of radial profiles and allow more realistic modeling of losses. Streamline curvature calculations are carried out in an iterative fashion to satisfy momentum and continuity requirements along quasi-orthogonals established in the two-dimensional field. These calculations have proved to be acceptable in terms of stability and convergence, at least in the case of hub-to-tip solutions conducted in the meridional plane. This quasi 3-D approach is very likely to continue as the standard method of aerodynamic analysis in turbomachinery for some years to come, although fully 3-D codes are used on occasion to further refine details of the flow.

Practical approaches to account for secondary flow effects have had a considerably heuristic goal in devising reasonably simple but adequate models. Wennerstrom (1991), in reviewing modeling ideas that have been proposed over the last 20 or 25 years, points out that an approach based on transport phenomena has been used to superimpose semi-empirical models of secondary flow features as corrections to otherwise classical axisymmetric solutions. Wennerstrom has termed these modeling attempts as hybrid solutions since they are not truly two-dimensional, nor are they fully three-dimensional. By necessity, the objective is not to deal with the three-dimensional details of the flow, but to deal with the overall computation for arbitrary geometries. The major contributors to the modeling efforts in recent years have been Adkins and Smith (1982), who proposed that secondary flows were responsible for spanwise mixing, and Gallimore and Cumpsty (1986), who viewed turbulent diffusion, rather than secondary flow, to be the dominant mechanism of spanwise mixing. Leylek and Wisler (1991) have, through extensive numerical analysis and experiments, largely resolved the issue of the dominant mixing mechanism. They concluded that the relative importance of secondary flow and turbulent diffusion depends upon stage loading, solidity, and aspect ratio. Thus, it would appear that both mechanisms should be incorporated in the performance calculation process. It is worth noting that if one is able to predict with reasonable assurance the performance of a given machine, then one has the most powerful design tools available. Howard and Gallimore (1992) extended the turbulent diffusion model of Gallimore and Cumpsty (1986) to include a viscous wall.

The objective of the research described in this work was to develop the governing equations for hub-to-tip flow analysis in intrinsic coordinates using the streamline curvature throughflow method incorporating the turbulent diffusion model of spanwise mixing developed by Gallimore and Cumpsty.

A special case of the governing equations for unbladed, axial-flow passages was then used to create a FORTRAN computer program to demonstrate the iterative solution technique.

2. GOVERNING EQUATIONS FOR HUB-TO-TIP FLOW ANALYSIS

2.1 The Momentum Equations

Turbomachinery flow is viscous, compressible, unsteady, and highly three-dimensional. One method often used in turbomachinery design and analysis is to assume steady, axisymmetric flow through the annular passage, allowing the flow to be treated in terms of a so-called hub-to-tip analysis in the meridional plane. Thus the flow analysis is reduced to one involving only two space dimensions. In the case of bladed regions, these flow simplifications are equivalent to assuming an infinite number of infinitely thin blades.

Figure 2.1 shows the meridional plane; a cylindrical coordinate system (r, θ, z) is indicated with the z axis aligned along the axis of the machine, and r pointing in the radial direction. The θ or tangential direction points out of the paper normal to the meridional plane. Also shown in Figure 2.1 is an intrinsic coordinate system (n, θ, m) with the m and n coordinates lying in the plane. The m coordinate corresponds to the meridional streamlines in the flow, and the n coordinate is orthogonal to them. Thus the flow velocity can be expressed as

$$\vec{V} = \vec{V}_m + \vec{V}_\theta \quad (2.1)$$

where V_m is the meridional velocity, and V_θ is the tangential velocity. The angle ϕ is the slope of the streamlines measured from the z -axis, and $\hat{e}_n$ and $\hat{e}_m$ are unit vectors along the

m and n coordinate directions. The (n, θ, m) coordinates will be used in the development of the analysis which follows.

In Figure 2.2, the meridional plane is shown again with representative hub and tip boundaries of the annulus, and with rotor and stator blade rows included. In intrinsic coordinates, the normal, tangential, and meridional momentum equations may be written (Tsien, 1958) as

$$\begin{aligned} \rho \left[\frac{V_\theta^2}{r} \cos\phi - V_m^2 \frac{\partial\phi}{\partial m} \right] &= \frac{\partial\sigma_{mn}}{\partial m} + \frac{\partial\sigma_{nn}}{\partial n} + \sigma_{mn} \left(\frac{\sin\phi}{r} + \frac{\partial\phi}{\partial n} \right) \\ &+ \sigma_{nm} \frac{\partial\phi}{\partial n} + \sigma_{nn} \left(\frac{\cos\phi}{r} - \frac{\partial\phi}{\partial m} \right) \\ &- \sigma_{\theta\theta} \frac{\cos\phi}{r} - \sigma_{mm} \frac{\partial\phi}{\partial m} + \rho f_n \end{aligned} \quad (2.2)$$

$$\begin{aligned} \rho \frac{V_m}{r} \frac{\partial(r V_\theta)}{\partial m} &= \frac{\partial\sigma_{m\theta}}{\partial m} + \frac{\partial\sigma_{n\theta}}{\partial n} + \sigma_{m\theta} \left(\frac{\sin\phi}{r} + \frac{\partial\phi}{\partial n} \right) + \sigma_{\theta m} \frac{\sin\phi}{r} \\ &+ \sigma_{n\theta} \left(\frac{\cos\phi}{r} - \frac{\partial\phi}{\partial m} \right) + \sigma_{\theta n} \frac{\cos\phi}{r} + \rho f_\theta \end{aligned} \quad (2.3)$$

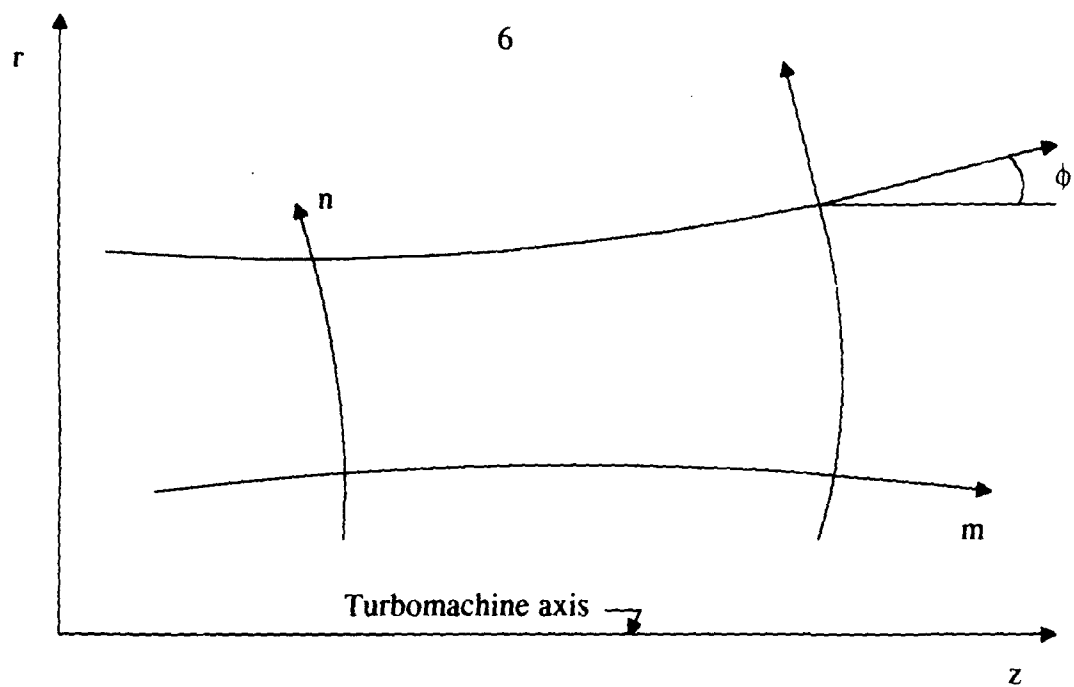


Figure 2.1 Meridional plane with intrinsic coordinate system

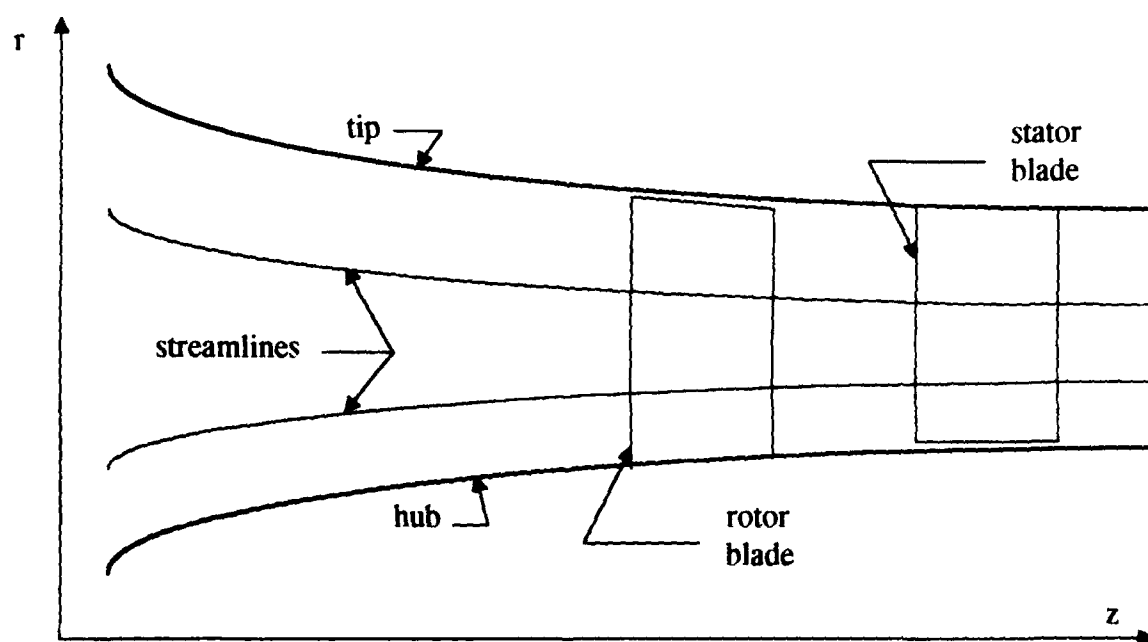


Figure 2.2 Turbomachine geometry

$$\begin{aligned}
\rho \left[V_m \frac{\partial V_m}{\partial m} - \frac{V_\theta^2}{r} \sin \phi \right] &= \frac{\partial \sigma_{mm}}{\partial m} + \frac{\partial \sigma_{nm}}{\partial n} + \sigma_{mm} \left(\frac{\sin \phi}{r} + \frac{\partial \phi}{\partial n} \right) \\
&+ \sigma_{nm} \left(\frac{\cos \phi}{r} - \frac{\partial \phi}{\partial m} \right) + \sigma_{mn} \frac{\partial \phi}{\partial m} - \sigma_{nn} \frac{\partial \phi}{\partial n} \\
&- \sigma_{\theta\theta} \frac{\sin \phi}{r} + \rho f_m
\end{aligned} \tag{2.4}$$

where ρ is density, the various σ terms are the stress tensor components, the f terms are distributed blade body forces, and V_m , V_θ , ϕ , and r are as previously defined. See Appendix A for the derivation of the distributed blade body force components.

The stress-strain rate relationships for a compressible fluid may be found in White (1991). Using thermodynamic pressure and viscosity, the stress-strain rate relationships written in terms of the intrinsic coordinates (n , θ , m) are:

$$\begin{aligned}
\sigma_{nn} &= -p + 2\mu e_{nn} \\
\sigma_{\theta\theta} &= -p + 2\mu e_{\theta\theta} \\
\sigma_{mm} &= -p + 2\mu e_{mm} \\
\sigma_{nm} &= \sigma_{mn} = 2\mu e_{nm} \\
\sigma_{n\theta} &= \sigma_{\theta n} = 2\mu e_{n\theta} \\
\sigma_{m\theta} &= \sigma_{\theta m} = 2\mu e_{m\theta}
\end{aligned} \tag{2.5}$$

The strain rates may be expressed as functions of the velocity gradients (White, 1991). Substituting for the strain rates in Eqn. (2.5), the stress-strain rate equations take the form:

$$\begin{aligned}
\sigma_{nn} &= -p + 2\mu V_m \frac{\partial \phi}{\partial n} \\
\sigma_{\theta\theta} &= -p + 2\mu \frac{V_m}{r} \sin\phi \\
\sigma_{mm} &= -p + 2\mu \frac{\partial V_m}{\partial m} \\
\sigma_{nm} &= \sigma_{mn} = \mu \left(\frac{\partial V_m}{\partial n} - V_m \frac{\partial \phi}{\partial m} \right) \\
\sigma_{n\theta} &= \sigma_{\theta n} = \mu \left(\frac{\partial V_\theta}{\partial n} - \frac{V_\theta}{r} \cos\phi \right) \\
\sigma_{m\theta} &= \sigma_{\theta m} = \mu \left(\frac{\partial V_\theta}{\partial m} - \frac{V_\theta}{r} \sin\phi \right)
\end{aligned} \tag{2.6}$$

These expressions for the stresses may be substituted into Eqns. (2.2) - (2.4), leaving the equations of motion as functions of p , ρ , ϕ , μ , V_m and V_θ , and f_n , f_θ , and f_m .

As an approach to viscous analysis of hub-to-tip flows, Gallimore (1985) proposed a spanwise mixing model in cylindrical coordinates of the velocity and temperature fields by retaining in the momentum equations the normal stresses, σ_{nn} , $\sigma_{\theta\theta}$, σ_{zz} , and only the shear stresses acting on the inner and outer cylindrical surfaces of the fluid element, $\sigma_{r\theta}$ and σ_{rz} . He argued also that, because mixing is assumed to occur across the cylindrical surfaces, the normal strain rates in the axial and tangential directions (e_{zz} and $e_{\theta\theta}$) could be neglected. Heat transfer due to mixing would be modeled by retaining only radial temperature gradients in the energy equation.

A spanwise mixing model for intrinsic coordinates may be derived in a similar fashion with mixing assumed to occur across the meridional stream surfaces. Therefore,

the shear stresses $\sigma_{n\theta}$ and σ_{nm} are retained while the remaining shear stresses are neglected and the normal stresses, σ_{nn} , σ_{mm} , and $\sigma_{\theta\theta}$, are retained with the strain rates e_{mm} and $e_{\theta\theta}$ neglected. Heat transfer due to mixing is modeled by retaining only normal temperature gradients in the energy equation. Figure 2.3 shows a fluid element in intrinsic coordinates with the stress terms retained by the spanwise mixing model. The momentum equations obtained from Eqns. (2.2) - (2.4) and (2.6) incorporating the spanwise mixing model in intrinsic coordinates are:

$$\rho \left[\frac{V_\theta^2}{r} \cos\phi - V_m^2 \frac{\partial\phi}{\partial m} \right] = - \frac{\partial p}{\partial n} + 2 \frac{\partial \left(\mu V_m \frac{\partial\phi}{\partial n} \right)}{\partial n} + \mu \frac{\partial\phi}{\partial n} \left(\frac{\partial V_m}{\partial n} - V_m \frac{\partial\phi}{\partial m} \right) \quad (2.7)$$

$$+ 2 \mu V_m \frac{\partial\phi}{\partial n} \left(\frac{\cos\phi}{r} - \frac{\partial\phi}{\partial m} \right) + \rho f_n$$

$$\rho \frac{V_m}{r} \frac{\partial(rV_\theta)}{\partial m} = \frac{\partial \left(\mu r \frac{\partial}{\partial n} \left(\frac{V_\theta}{r} \right) \right)}{\partial m} + \mu r \frac{\partial}{\partial n} \left(\frac{V_\theta}{r} \right) \left(\frac{\cos\phi}{r} - \frac{\partial\phi}{\partial m} \right) + \rho f_\theta \quad (2.8)$$

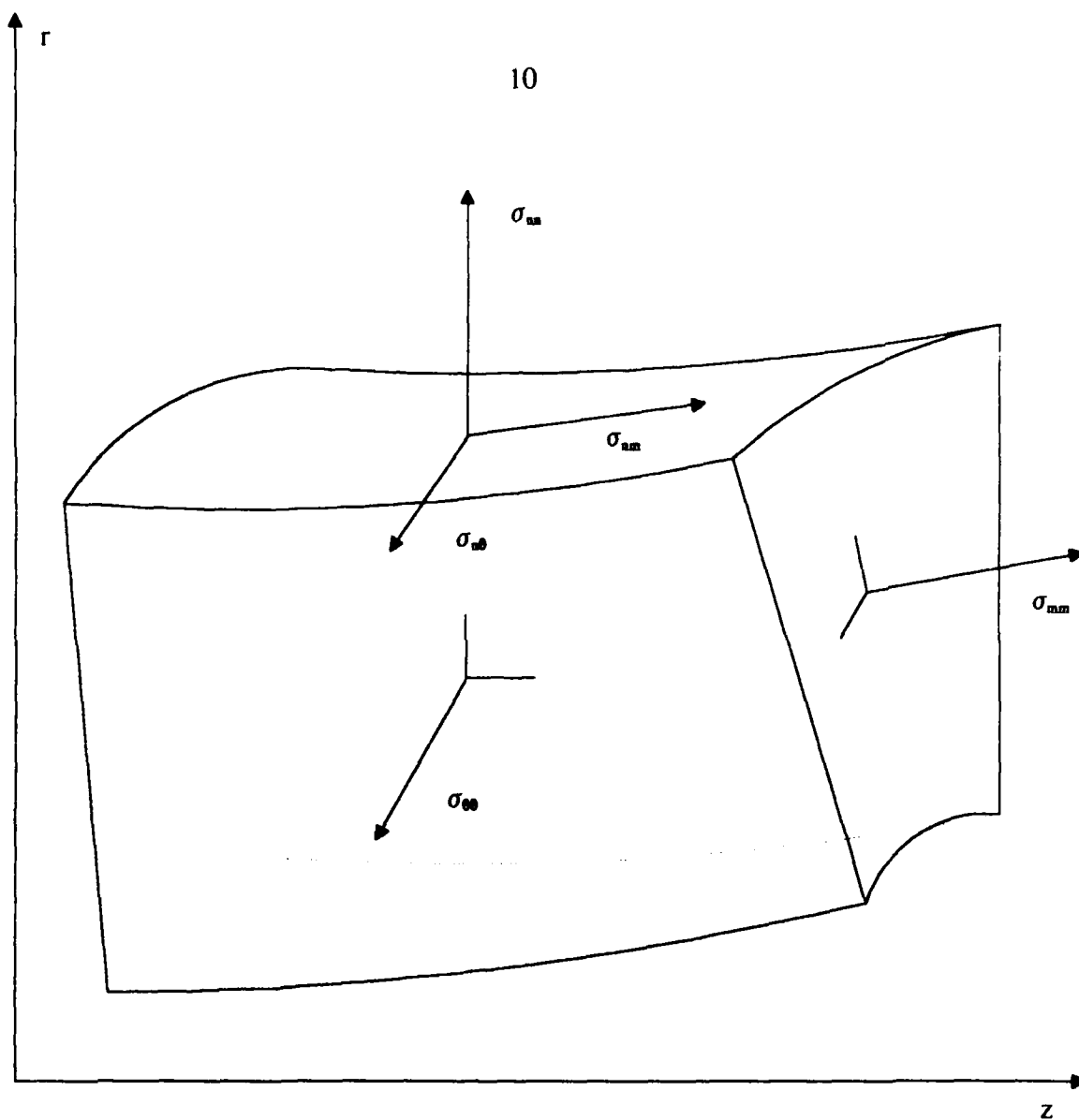


Figure 2.3. Fluid element for mixing model

$$\begin{aligned}
\rho \left[V_m \frac{\partial V_m}{\partial m} - \frac{V_\theta^2}{r} \sin \phi \right] = & - \frac{\partial p}{\partial m} + \frac{\partial \left[\mu \left(\frac{\partial V_m}{\partial n} - V_m \frac{\partial \phi}{\partial m} \right) \right]}{\partial n} \\
& + \mu \left(\frac{\partial V_m}{\partial n} - V_m \frac{\partial \phi}{\partial m} \right) \left(\frac{\cos \phi}{r} - \frac{\partial \phi}{\partial m} \right) \\
& - 2 \mu V_m \left(\frac{\partial \phi}{\partial n} \right)^2 + \rho f_m
\end{aligned} \tag{2.9}$$

The notation in these equations may be simplified to

$$V_m^2 \frac{\partial \phi}{\partial m} - \frac{V_\theta^2}{r} \cos \phi = - \frac{1}{\rho} \frac{\partial p}{\partial n} + E_n + f_n \tag{2.10}$$

$$\frac{V_m}{r} \frac{\partial(r V_\theta)}{\partial m} = E_\theta + f_\theta \tag{2.11}$$

$$V_m \frac{\partial V_m}{\partial m} - \frac{V_\theta^2}{r} \sin \phi = - \frac{1}{\rho} \frac{\partial p}{\partial m} + E_m + f_m \tag{2.12}$$

where the E_n , E_θ , and E_m terms represent viscous body forces per unit mass and are written as:

$$\begin{aligned}
E_n &= \frac{1}{\rho} \left[2 \frac{\partial \left(\mu V_m \frac{\partial \phi}{\partial n} \right)}{\partial n} + \mu \frac{\partial \phi}{\partial n} \left(\frac{\partial V_m}{\partial n} - V_m \frac{\partial \phi}{\partial m} \right) + 2 \mu V_m \frac{\partial \phi}{\partial n} \left(\frac{\cos \phi}{r} - \frac{\partial \phi}{\partial m} \right) \right] \\
E_\theta &= \frac{1}{\rho} \left[\frac{\partial \left(\mu r \frac{\partial \left(\frac{V_\theta}{r} \right)}{\partial n} \right)}{\partial n} + \mu r \frac{\partial \left(\frac{V_\theta}{r} \right)}{\partial n} \left(\frac{\cos \phi}{r} - \frac{\partial \phi}{\partial m} \right) \right] \\
E_m &= \frac{1}{\rho} \left[\frac{\partial \left(\mu \left(\frac{\partial V_m}{\partial n} - V_m \frac{\partial \phi}{\partial m} \right) \right)}{\partial n} + \mu \left(\frac{\partial V_m}{\partial n} - V_m \frac{\partial \phi}{\partial m} \right) \left(\frac{\cos \phi}{r} - \frac{\partial \phi}{\partial m} \right) \right. \\
&\quad \left. - \left(\frac{\partial \phi}{\partial n} \right)^2 2 \mu V_m \right]
\end{aligned} \tag{2.13}$$

2.2 The Radial Equilibrium Equation

The determination of flow variables in the meridional plane is accomplished by the iterative solution of the uncoupled momentum and continuity equations. The solution of the normal and meridional momentum equations is complicated by the fact that both the n and m directions change throughout the flow field as the computation approaches convergence. The flow field solution may be simplified by constructing fixed quasi-orthogonal ($q-o$) coordinates approximately perpendicular to the streamlines. The n direction may then be eliminated by transforming the momentum equations to the m and $q-o$ directions. A flow field grid is then formed by the streamlines and the quasi-orthogonals. Figure 2.4 shows the coordinate systems in the meridional plane, including

the defined $q-o$. The angle, γ , between the r and $q-o$ directions is taken to be positive when measured as shown.

The chain rule for a function of two variables may be written to express the directional derivative along the $q-o$ as:

$$\frac{\partial(\quad)}{\partial q} = \frac{\partial(\quad)}{\partial n} \frac{dn}{dq} + \frac{\partial(\quad)}{\partial m} \frac{dm}{dq} \quad (2.14)$$

In addition, the following geometric relationships can be found from Figure 2.4:

$$\frac{\partial n}{\partial q} = \cos(\phi + \gamma) \quad \frac{\partial m}{\partial q} = \sin(\phi + \gamma) \quad (2.15)$$

Combining Eqns. (2.14) and (2.15) and solving for $\partial(\quad)/\partial n$ yields

$$\frac{\partial(\quad)}{\partial n} = \sec(\phi + \gamma) \frac{\partial(\quad)}{\partial q} - \tan(\phi + \gamma) \frac{\partial(\quad)}{\partial m} \quad (2.16)$$

This relationship is used in the development below to transform the momentum component equations to the (m, q) coordinate system.

From Gibbs' Tds relationship we can write

$$\frac{dp}{\rho} = -Tds + dH - VdV \quad (2.17)$$

where H is the stagnation enthalpy and V is the flow velocity. The pressure gradients in

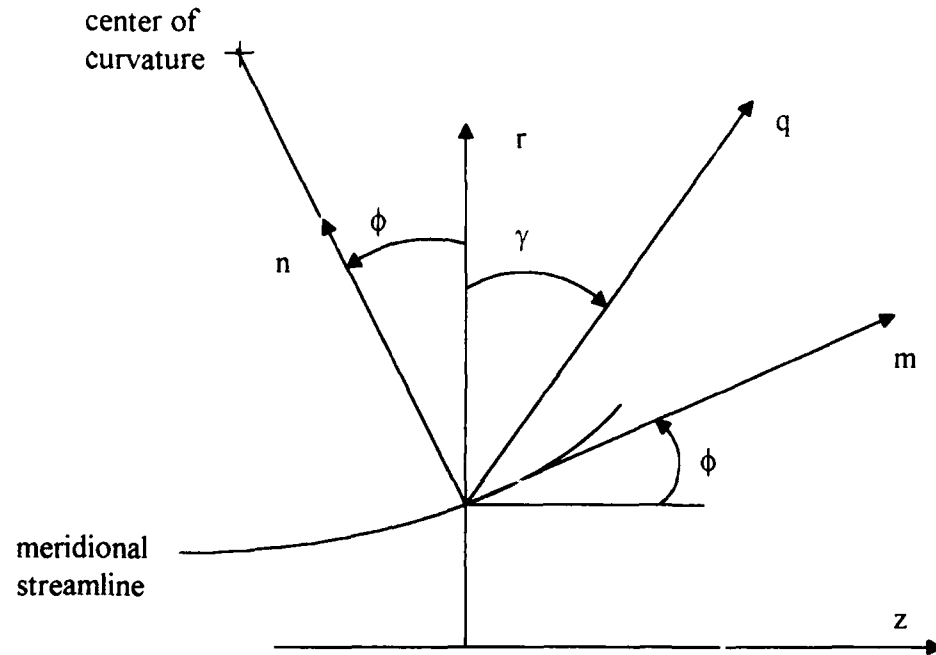


Figure 2.4. The (m, n) , (r, z) , and (m, q) coordinate systems in the meridional plane

the m and n directions therefore are

$$\begin{aligned}\frac{1}{\rho} \frac{\partial p}{\partial m} &= -T \frac{\partial s}{\partial m} + \frac{\partial H}{\partial m} - V \frac{\partial V}{\partial m} \\ \frac{1}{\rho} \frac{\partial p}{\partial n} &= -T \frac{\partial s}{\partial n} + \frac{\partial H}{\partial n} - V \frac{\partial V}{\partial n}\end{aligned}\quad (2.18)$$

Substituting Eqns (2.18) into the normal and meridional momentum equations, Eqns.

(2.10) and (2.12), the momentum equations become:

$$\begin{aligned}V_m \frac{\partial V_m}{\partial m} - \frac{V_\theta^2}{r} \sin \phi &= -\frac{\partial H}{\partial m} + V_m \frac{\partial V_m}{\partial m} + V_\theta \frac{\partial V_\theta}{\partial m} + T \frac{\partial s}{\partial m} + E_m + f_m \\ V_m^2 \frac{\partial \phi}{\partial m} - \frac{V_\theta^2}{r} \cos \phi &= -\frac{\partial H}{\partial n} + V_m \frac{\partial V_m}{\partial n} + V_\theta \frac{\partial V_\theta}{\partial n} + T \frac{\partial s}{\partial n} + E_n + f_n\end{aligned}\quad (2.19)$$

Using Eqn. (2.16) to replace the n directional gradients in the normal momentum equation with gradients in the m and q directions, we obtain

$$\begin{aligned}V_m^2 \frac{\partial \phi}{\partial m} - \frac{V_\theta^2}{r} \cos \phi &= -\sec(\phi + \gamma) \frac{\partial H}{\partial q} + \tan(\phi + \gamma) \frac{\partial H}{\partial m} \\ &+ T \sec(\phi + \gamma) \frac{\partial s}{\partial q} + T \tan(\phi + \gamma) \frac{\partial s}{\partial m} \\ &+ V_m \sec(\phi + \gamma) \frac{\partial V_m}{\partial q} + V_m \tan(\phi + \gamma) \frac{\partial V_m}{\partial m} \\ &+ V_\theta \sec(\phi + \gamma) \frac{\partial V_\theta}{\partial q} + V_\theta \tan(\phi + \gamma) \frac{\partial V_\theta}{\partial m} \\ &+ E_n + f_n\end{aligned}\quad (2.20)$$

The m and n momentum equations may now be combined by multiplying the m momentum equation by $\tan(\phi + \gamma)$ and adding the result to the n momentum equation.

Further rearrangement gives

$$\begin{aligned} \tan(\phi + \gamma) \left[V_m \frac{\partial V_m}{\partial m} - \frac{V_\theta^2}{r} \sin\phi - E_m - f_m \right] + V_m^2 \frac{\partial \phi}{\partial m} - \frac{V_\theta^2}{r} \cos\phi = \\ -\sec(\phi + \gamma) \frac{\partial H}{\partial q} + \sec(\phi + \gamma) T \frac{\partial s}{\partial q} + \sec(\phi + \gamma) V_\theta \frac{\partial V_\theta}{\partial q} + \\ E_n + f_n + \sec(\phi + \gamma) V_m \frac{\partial V_m}{\partial q} \end{aligned} \quad (2.21)$$

Multiplying Eqn. (2.21) by $\cos(\phi + \gamma)$ and rearranging gives:

$$\begin{aligned} V_m \frac{\partial V_m}{\partial q} = \sin(\phi + \gamma) V_m \frac{\partial V_m}{\partial m} + \cos(\phi + \gamma) V_m^2 \frac{\partial \phi}{\partial m} + \frac{\partial H}{\partial q} - T \frac{\partial s}{\partial q} \\ - \sin(\phi + \gamma)(E_m + f_m) - \cos(\phi + \gamma)(E_n + f_n) \\ - \cos(\phi + \gamma) \cos\phi \frac{V_\theta^2}{r} - \sin(\phi + \gamma) \sin\phi \frac{V_\theta^2}{r} - V_\theta \frac{\partial V_\theta}{\partial q} \end{aligned} \quad (2.22)$$

The radius of curvature, r_c , of the meridional streamline is defined by:

$$\frac{1}{r_c} = \frac{\partial \phi}{\partial m} \quad (2.23)$$

Using this result in Eqn (2.22) along with further rearrangement, and recognizing that

$\partial r / \partial q = \cos \gamma$, we get the final form of radial equilibrium equation

$$\begin{aligned}
V_m \frac{\partial V_m}{\partial q} = & \sin(\phi + \gamma) V_m \frac{\partial V_m}{\partial m} + \cos(\phi + \gamma) \frac{V_m^2}{r_c} + \frac{\partial H}{\partial q} - T \frac{\partial s}{\partial q} \\
& - \sin(\phi + \gamma) [E_m + f_m] - \cos(\phi + \gamma) [E_n + f_n] - \frac{V_\theta}{r} \frac{\partial(r V_\theta)}{\partial q}
\end{aligned} \tag{2.24}$$

Equation (2.16) may be used to eliminate the n directional gradient terms in the normal, tangential, and meridional viscous body force equations, leaving

$$\begin{aligned}
E_n = & \frac{1}{\rho} \left\{ 2 \sec(\phi + \gamma) \frac{\partial}{\partial q} \left[\mu V_m \left(\sec(\phi + \gamma) \frac{\partial \phi}{\partial q} - \tan(\phi + \gamma) \frac{\partial \phi}{\partial m} \right) \right] \right. \\
& \left. - 2 \tan(\phi + \gamma) \frac{\partial}{\partial m} \left[\mu V_m \left(\sec(\phi + \gamma) \frac{\partial \phi}{\partial q} - \tan(\phi + \gamma) \frac{\partial \phi}{\partial m} \right) \right] + \right. \\
& \mu \left(\sec(\phi + \gamma) \frac{\partial V_m}{\partial q} - \tan(\phi + \gamma) \frac{\partial V_m}{\partial m} - V_m \frac{\partial \phi}{\partial m} \right) \left(\sec(\phi + \gamma) \frac{\partial \phi}{\partial q} - \tan(\phi + \gamma) \frac{\partial \phi}{\partial m} \right) \\
& \left. + 2 \mu V_m \frac{\partial \phi}{\partial m} \left(\frac{\cos \phi}{r} - \frac{\partial \phi}{\partial m} \right) \right\}
\end{aligned} \tag{2.25}$$

$$\begin{aligned}
E_{\theta} = & \frac{1}{\rho} \left\{ \sec(\phi + \gamma) \frac{\partial}{\partial q} \left[\mu r \left(\sec(\phi + \gamma) \frac{\partial}{\partial q} \left(\frac{V_{\theta}}{r} \right) - \tan(\phi + \gamma) \frac{\partial}{\partial m} \left(\frac{V_{\theta}}{r} \right) \right) \right] \right. \\
& - \tan(\phi + \gamma) \frac{\partial}{\partial m} \left[\mu r \left(\sec(\phi + \gamma) \frac{\partial}{\partial q} \left(\frac{V_{\theta}}{r} \right) - \tan(\phi + \gamma) \frac{\partial}{\partial m} \left(\frac{V_{\theta}}{r} \right) \right) \right] \\
& \left. + \mu r \left(\frac{\cos \phi}{r} - \frac{\partial \phi}{\partial m} \right) \left(\sec(\phi + \gamma) \frac{\partial}{\partial q} \left(\frac{V_{\theta}}{r} \right) - \tan(\phi + \gamma) \frac{\partial}{\partial m} \left(\frac{V_{\theta}}{r} \right) \right) \right\} \quad (2.26)
\end{aligned}$$

$$\begin{aligned}
E_m = & \frac{1}{\rho} \left\{ \sec(\phi + \gamma) \frac{\partial}{\partial q} \left[\mu \left(\sec(\phi + \gamma) \frac{\partial V_m}{\partial q} - \tan(\phi + \gamma) \frac{\partial V_m}{\partial m} - V_m \frac{\partial \phi}{\partial m} \right) \right] \right. \\
& - \tan(\phi + \gamma) \frac{\partial}{\partial m} \left[\mu \left(\sec(\phi + \gamma) \frac{\partial V_m}{\partial q} - \tan(\phi + \gamma) \frac{\partial V_m}{\partial m} - V_m \frac{\partial \phi}{\partial m} \right) \right] + \\
& + \mu \left(\frac{\cos \phi}{r} - \frac{\partial \phi}{\partial m} \right) \left(\sec(\phi + \gamma) \frac{\partial V_m}{\partial q} - \tan(\phi + \gamma) \frac{\partial V_m}{\partial m} - V_m \frac{\partial \phi}{\partial m} \right) \\
& \left. - 2 \mu V_m \left(\sec(\phi + \gamma) \frac{\partial \phi}{\partial q} - \tan(\phi + \gamma) \frac{\partial \phi}{\partial m} \right)^2 \right\} \quad (2.27)
\end{aligned}$$

Because momentum and temperature mixing are assumed to occur only in the normal direction, the terms in the viscous body force equations which contain velocity gradients in the meridional direction will be neglected, leaving

$$\begin{aligned}
E_n = \frac{1}{\rho} & \left\{ 2 \sec(\phi + \gamma) \frac{\partial}{\partial q} \left[\mu V_m \left(\sec(\phi + \gamma) \frac{\partial \phi}{\partial q} - \tan(\phi + \gamma) \frac{\partial \phi}{\partial m} \right) \right] \right. \\
& - 2 \tan(\phi + \gamma) \frac{\partial}{\partial m} \left[\mu V_m \left(\sec(\phi + \gamma) \frac{\partial \phi}{\partial q} - \tan(\phi + \gamma) \frac{\partial \phi}{\partial m} \right) \right] + \\
& \mu \left(\sec(\phi + \gamma) \frac{\partial V_m}{\partial q} - V_m \frac{\partial \phi}{\partial m} \right) \left(\sec(\phi + \gamma) \frac{\partial \phi}{\partial q} - \tan(\phi + \gamma) \frac{\partial \phi}{\partial m} \right) \\
& \left. + 2 \mu V_m \frac{\partial \phi}{\partial m} \left(\frac{\cos \phi}{r} - \frac{\partial \phi}{\partial m} \right) \right\}
\end{aligned} \tag{2.28}$$

$$\begin{aligned}
E_\theta = \frac{1}{\rho} & \left\{ \sec(\phi + \gamma) \frac{\partial}{\partial q} \left[\mu r \left(\sec(\phi + \gamma) \frac{\partial}{\partial q} \left(\frac{V_\theta}{r} \right) - \frac{V_\theta}{r^2} \tan(\phi + \gamma) \frac{\partial r}{\partial m} \right) \right] \right. \\
& - \tan(\phi + \gamma) \frac{\partial}{\partial m} \left[\mu r \left(\sec(\phi + \gamma) \frac{\partial}{\partial q} \left(\frac{V_\theta}{r} \right) - \frac{V_\theta}{r^2} \tan(\phi + \gamma) \frac{\partial r}{\partial m} \right) \right] \\
& \left. + \mu r \left(\frac{\cos \phi}{r} - \frac{\partial \phi}{\partial m} \right) \left(\sec(\phi + \gamma) \frac{\partial}{\partial q} \left(\frac{V_\theta}{r} \right) - \frac{V_\theta}{r^2} \tan(\phi + \gamma) \frac{\partial r}{\partial m} \right) \right\}
\end{aligned} \tag{2.29}$$

$$\begin{aligned}
E_m = \frac{1}{\rho} & \left\{ \sec(\phi + \gamma) \frac{\partial}{\partial q} \left[\mu \left(\sec(\phi + \gamma) \frac{\partial V_m}{\partial q} - V_m \frac{\partial \phi}{\partial m} \right) \right] \right. \\
& - \tan(\phi + \gamma) \frac{\partial}{\partial m} \left[\mu \left(\sec(\phi + \gamma) \frac{\partial V_m}{\partial q} - V_m \frac{\partial \phi}{\partial m} \right) \right] \\
& + \mu \left(\frac{\cos \phi}{r} - \frac{\partial \phi}{\partial m} \right) \left(\sec(\phi + \gamma) \frac{\partial V_m}{\partial q} - V_m \frac{\partial \phi}{\partial m} \right) \\
& \left. - 2 \mu V_m \left(\sec(\phi + \gamma) \frac{\partial \phi}{\partial q} - \tan(\phi + \gamma) \frac{\partial \phi}{\partial m} \right)^2 \right\}
\end{aligned} \tag{2.30}$$

In the iterative solution of the radial equilibrium equation, the first two terms on the right hand side of Eqn. (2.24) use values of V_m , $\phi + \gamma$, and r_c from a previous solution of the flow field. In order to close the system, equations are needed for the entropy and stagnation enthalpy gradients. These equations are derived in the following sections.

2.3 The Energy Equation

Expressed in terms of enthalpy, the energy equation may be written:

$$\rho \frac{Dh}{Dt} = \frac{Dp}{Dt} - \nabla \cdot \vec{q} + \Phi \quad (2.31)$$

where $\vec{q}$ is heat flux and Φ is the dissipation function. Gibbs' Tds relationship, Eqn. (2.17), may be used to eliminate the pressure derivative in the energy equation yielding:

$$\rho T \frac{Ds}{Dt} = -\nabla \cdot \vec{q} + \Phi \quad (2.32)$$

The left hand side of this equation may be expanded by using the definition of the substantial derivative, which, due to the assumption of steady flow, yields

$$\rho T \frac{Ds}{Dt} = \rho T V_m \frac{\partial s}{\partial m} = -\nabla \cdot \vec{q} + \Phi \quad (2.33)$$

The intrinsic coordinate forms of the del operator and the divergence are needed in

order to expand the heat transfer term. The del operator is defined as

$$\nabla = \hat{e}_m \frac{\partial}{\partial m} + \hat{e}_n \frac{\partial}{\partial n} \quad (2.34)$$

Retaining only the n directional temperature gradients, ∇T gives

$$\nabla T = \hat{e}_n \frac{\partial T}{\partial n} \quad (2.35)$$

Assuming that conduction is modeled by Fourier's Law , the gradient of the heat flux may expanded to give

$$\nabla \cdot (-k) \nabla T = \nabla(-k) \cdot \nabla T + (-k) \nabla \cdot \nabla T \quad (2.36)$$

where k is the coefficient of thermal conductivity. The coordinate system geometry gives the following relationships for the derivatives of the unit vectors

$$\frac{\partial \hat{e}_n}{\partial m} = -\frac{\partial \phi}{\partial m} \hat{e}_m \quad \frac{\partial \hat{e}_n}{\partial n} = -\frac{\partial \phi}{\partial n} \hat{e}_m \quad (2.37)$$

Combining Eqn. (2.34) - (2.37) gives

$$-\nabla \cdot \vec{q} = -\nabla \cdot (-k \nabla T) = -k \frac{\partial \phi}{\partial m} \frac{\partial T}{\partial n} + \frac{\partial}{\partial n} \left(k \frac{\partial T}{\partial n} \right) \quad (2.38)$$

The second term on the right hand side of the energy equation, Eqn. (2.31), is the dissipation function, which, written in terms of strain rates (Hughes and Gaylord, 1964)

for the intrinsic coordinates, (n, θ, m) is

$$\Phi = \mu \{ 2(e_{nn}^2 + e_{\theta\theta}^2 + e_{mm}^2) + (2e_{n\theta})^2 + (2e_{nm})^2 + (2e_{\theta m})^2 \} \quad (2.39)$$

The dissipation function may be simplified by using the mixing model assumption in which the normal strain rates and $\partial V/\partial m$ may be neglected. With these assumptions, the dissipation function becomes:

$$\Phi = \mu \left\{ \left[\frac{\partial V_\theta}{\partial n} - \frac{V_\theta}{r} \cos\phi \right]^2 + \left[\frac{\partial V_m}{\partial n} - V_m \frac{\partial \phi}{\partial m} \right]^2 + \left[\frac{V_\theta}{r} \sin\phi \right]^2 \right\} \quad (2.40)$$

Substituting Eqns. (2.38) and (2.40), the energy equation, Eqn. (2.31) may be written:

$$\frac{\partial s}{\partial m} = -\frac{k}{\rho TV_m} \frac{\partial \phi}{\partial m} \frac{\partial T}{\partial n} + \frac{1}{\rho TV_m} \frac{\partial}{\partial n} \left(k \frac{\partial T}{\partial n} \right) + \frac{\Phi}{\rho TV_m} + \frac{\partial S_e}{\partial m} \quad (2.41)$$

where $\partial S_e/\partial m$ is an empirically based entropy gradient which accounts for losses generated by flow over the blades.

Equation (2.16) is used to transform Eqn. (2.38) from the (n, m) to the (m, q) coordinate system yielding

$$\begin{aligned}
\frac{\partial s}{\partial m} = & -\frac{k}{\rho T V_m} \frac{\partial \phi}{\partial m} \left\{ \sec(\phi + \gamma) \frac{\partial T}{\partial q} \right\} + \frac{\sec(\phi + \gamma)}{\rho T V_m} \frac{\partial}{\partial q} \left\{ k \sec(\phi + \gamma) \frac{\partial T}{\partial q} \right\} \\
& - \frac{\tan(\phi + \gamma)}{\rho T V_m} \frac{\partial}{\partial m} \left\{ k \sec(\phi + \gamma) \frac{\partial T}{\partial q} \right\} \\
& + \frac{\mu}{\rho T V_m} \left\{ \left[\sec(\phi + \gamma) \frac{\partial V_\theta}{\partial q} - \frac{V_\theta}{r} \sin \phi \right]^2 + \left[\sec(\phi + \gamma) \frac{\partial V_m}{\partial q} - V_m \frac{\partial \phi}{\partial m} \right]^2 \right. \\
& \left. + \left[\frac{V_\theta}{r} \sin \phi \right]^2 \right\} + \frac{\partial S_e}{\partial m}
\end{aligned} \tag{2.42}$$

2.4 The Stagnation Enthalpy Equation

The definition of stagnation enthalpy may be combined with Gibbs' Tds relationship to give

$$\frac{1}{\rho} \frac{\partial p}{\partial m} = \frac{\partial H}{\partial m} - V_m \frac{\partial V_m}{\partial m} - V_\theta \frac{\partial V_\theta}{\partial m} - T \frac{\partial s}{\partial m} \tag{2.43}$$

Substituting Eqn. (2.43) into the meridional momentum equation, Eqn. (2.12), yields

$$\frac{V_\theta^2}{r} \sin \phi = \frac{\partial H}{\partial m} - T \frac{\partial s}{\partial m} - V_\theta \frac{\partial V_\theta}{\partial m} + E_m + f_m \tag{2.44}$$

Solving Eqn. (2.44) for $\partial H / \partial m$ and recognizing that $\sin \phi = \partial r / \partial m$ gives

$$\frac{\partial H}{\partial m} = T \frac{\partial s}{\partial m} + \frac{1}{2r^2} \frac{\partial (r V_\theta)^2}{\partial m} + E_m + f_m \quad (2.45)$$

2.5 The Angular Momentum Equation

Multiplying the tangential momentum equation, Eqn. (2.11), by V_θ/V_m and rearranging yields a form of the angular momentum equation convenient for use with the stagnation enthalpy, Eqn. (2.44), and radial equilibrium, Eqn. (2.24) equations:

$$\frac{1}{2r^2} \frac{\partial (r V_\theta)^2}{\partial m} = \frac{V_\theta}{V_m} (E_\theta + f_\theta) \quad (2.46)$$

2.6 Closure of the System of Equations

In the preceding sections, seven equations have been developed from the three component momentum equations and the energy equation. These equations are:

E_n viscous body force	Eqn. (2.28)
E_θ viscous body force	Eqn. (2.29)
E_m viscous body force	Eqn. (2.30)
Radial equilibrium	Eqn. (2.24)
Energy/entropy	Eqn. (2.42)
Meridional momentum/enthalpy	Eqn. (2.45)
Angular momentum	Eqn. (2.46)

Additional equations are needed to close the system of equations.

An equation for density may be derived by integrating Gibbs' Tds relationship to give

$$s = c_p \ln T - R \ln p \quad (2.47)$$

where s is the entropy, R is the gas constant, and c_p is the specific heat at constant pressure. An arbitrary reference state of $s = 0$ at $T = 1$ and $p = 1$ was used for integration of the Tds relationship. Combining Eqn (2.47) with the ideal gas equation of state yields

$$\rho = \frac{T^{\frac{c_p - R}{R}} e^{-\frac{s}{R}}}{R} \quad (2.48)$$

The definition of stagnation enthalpy may be rearranged to give the temperature equation

$$T = \frac{H}{c_p} - \frac{V_m^2 + V_\theta^2}{2c_p} \quad (2.49)$$

The streamline slope, ϕ , and the q - o angle, γ , are determined from the streamlines and input quasi-orthogonal geometry (see Fig 2.4) by

$$\begin{aligned}\phi &= \tan^{-1} \frac{\partial r}{\partial z} \bigg|_m \\ \gamma &= \tan^{-1} \frac{\partial z}{\partial r} \bigg|_q\end{aligned}\tag{2.50}$$

Through specification of the Prandtl number, Pr , the Schmidt number, Sc , and the mixing coefficient, ϵ , the viscosity and heat transfer coefficient may be determined by

$$\begin{aligned}\mu &= Sc \rho \epsilon \\ k &= \frac{\mu c_p}{Pr}\end{aligned}\tag{2.51}$$

The mixing coefficient is a function of the blade and annulus geometry and the turbomachine operating conditions. Those interested in more details on the calculation of ϵ are referred to Gallimore (1985) and Schlichting (1979).

The distributed blade body forces, f_n , f_θ , and f_m , required in Eqns. (2.24), (2.45), and (2.46), depend on blade geometry for their calculation (See Appendix A).

For a given streamline pattern, the seven equations listed above combined with Eqns. (2.48) - (2.51) constitute a complete set of equations for streamline curvature analysis. The input variables required for this system are the Schmidt and Prandtl number, the mixing coefficient, the constant pressure specific heat, the gas constant, and the empirical entropy gradient as determined from blade geometry and flow conditions.

2.7 Inlet Conditions

A consistent set of initial conditions is needed to start the iterative solution of the flow field. The following assumptions are made at the inlet to the flow annulus:

1. The inlet station $q-o$ is assumed to be radial; therefore, angle $\gamma = 0$ and ∂q may be replaced by ∂r in the system of equations (See Fig. 2.4).
2. The flow is inviscid and isentropic prior to reaching the inlet. As a result of this assumption, Eqns. (2.28), (2.29), (2.30), (2.42), and (2.45) reduce to zero, identically.
3. Streamlines at the inlet station are assumed to be flat, i.e., $\phi = 0$, $r_c = \infty$, and $V_m = V_z$ (See Fig. 2.4).
4. There are no blades at the inlet station.
5. Stagnation temperature and pressure are assumed to be constant across the inlet station; therefore, H and s are also constant across the inlet.

These assumptions result in a reduced set of equations for the inlet.

Using the assumptions listed above, the radial equilibrium equation, Eqn. (2.24), reduces to the simple radial equilibrium equation

$$V_z \frac{\partial V_z}{\partial r} = - \frac{V_\theta}{r} \frac{\partial(r V_\theta)}{\partial r} \quad (2.52)$$

Equations (2.47) - (2.49) and (2.51), used to compute entropy, density, temperature, viscosity, and the heat transfer coefficient, remain unchanged.

If the inlet velocity profile contains no V_θ component (zero whirl at the inlet), the right hand side of Eqn. (2.52) reduces to zero and the axial velocity, V_z , must be constant across the inlet.

For the case of the inlet velocity profile containing an arbitrary V_θ distribution with radius, Eqn. (2.52) must be integrated across the inlet to give the profile of the axial velocity, V_z .

The continuity equation is integrated across the inlet from streamline to streamline to obtain the mass flow rate which is used to satisfy continuity for the remaining q -os. The streamline to streamline integration also allows the computation of the fraction of the mass flow which passes between the hub and each streamline outward towards the outer casing. The radial location of each streamline is adjusted as necessary to maintain the same mass flow fraction as occurred at the inlet station. Details of the mass flow integration are covered in a following section.

2.8 Solution Procedure for the Governing Equations in Bladed Flow Regions

The flow annulus of a turbomachine may be divided into unbladed and bladed regions with the bladed regions further divided into rotor and stator blade rows. The solution procedure for spanwise mixing based on streamline curvature through-flow analysis for bladed regions is outlined as follows:

1. The velocities, velocity gradients, and streamline pattern from a previous iteration or initial solution estimate are used to compute the viscous body forces, E_n , E_θ , and E_m , from Eqns. (2.28) - (2.30), and the distributed blade body forces, f_n , f_θ ,

and f_m , from Eqns. (A4) and (A5) in Appendix A.

2. The temperature data obtained from the temperature field in a previous iteration or initial solution estimate are combined with the empirical entropy gradient, the velocity gradients and the streamline pattern to solve the energy equation, Eqn. (2.42), for the meridional entropy gradient along each streamline.
3. The angular momentum equation, Eqn. (2.43), is solved to give the meridional gradient of angular momentum along each streamline.
4. Using the gradients determined from the energy and angular momentum equations in steps 2 and 3, and the meridional body force, E_m , determined in step 1, the meridional gradient of stagnation enthalpy is computed for each streamline using Eqn. (2.41).
5. The entropy, angular momentum, and stagnation enthalpy values are then updated for each streamline and $q-o$ intersection using the gradients determined above.
6. The temperatures and densities are updated for each intersection using Eqns. (2.48) and (2.49). The entropy, stagnation enthalpy, and velocity gradients in the $q-o$ direction are then computed using the data determined in step 5.
7. The radial equilibrium equation is integrated to give the meridional velocity distribution, V_m , along each $q-o$.
8. The new V_m velocity profile computed in step 7 is used in the integration of the continuity equation at each $q-o$. The mass flow rate thus determined is used to adjust the constant of integration in step 7 as needed to achieve the same annulus

mass flow as was determined at the inlet station. The solution procedure iterates on steps 7 and 8 until the difference between the mass flow rates at each $q-o$ and the inlet lies within an arbitrarily chosen tolerance.

9. The streamline radial locations are adjusted by interpolating the mass flow distributions to maintain same fraction of mass flow between the hub and each streamline as at the inlet station.
10. Overall convergence of the solution is determined by comparing the streamline shifts between successive iterations for the flow field with an arbitrarily chosen tolerance. If the streamline shifting is within the tolerance, the flow field solution has converged. If not, the streamline pattern is updated with the results of step 9 and the solution repeated again starting at step 1.

2.9 Solution Procedure for the Governing Equations in Unbladed Flow Regions

The solution procedure for bladed regions differs from the unbladed regions only by the inclusion of the blade body forces and the empirical entropy gradient.

3. SOLUTION METHOD, COMPUTER PROGRAM DEVELOPMENT AND OPERATION

The governing equations developed in Chapter 2 are written in finite difference form and programmed for iterative solution. The computer program was developed for a blade-free annulus with inviscid and adiabatic endwalls. The annulus is essentially axial with the q -os directed radially from the hub to the outer casing. The governing equations are in the form derived by Gallimore (1985) which are a special case of the equations given in Chapter 2.

The following describes the main program involved and the associated subroutines which have been developed. Figure 3.1 is a mapping of the main program and the subroutines called. As stated previously, the solution is iterative with the sequence of execution essentially from left to right across the figure. Execution is repeated numerous times until convergence is obtained and Subroutine OUTPUT is called to print the solution results.

The complete FORTRAN code for the program is contained in Appendix C. Comment lines in the main program and subroutines include a glossary of the FORTRAN variable names used.

3.1 Main Program and Control of the Calculations

The primary purpose of the main program is to control the order in which the subroutines are called to carry out a solution. The main program also prints statements which list the subroutine currently being executed and the number of iterations completed.

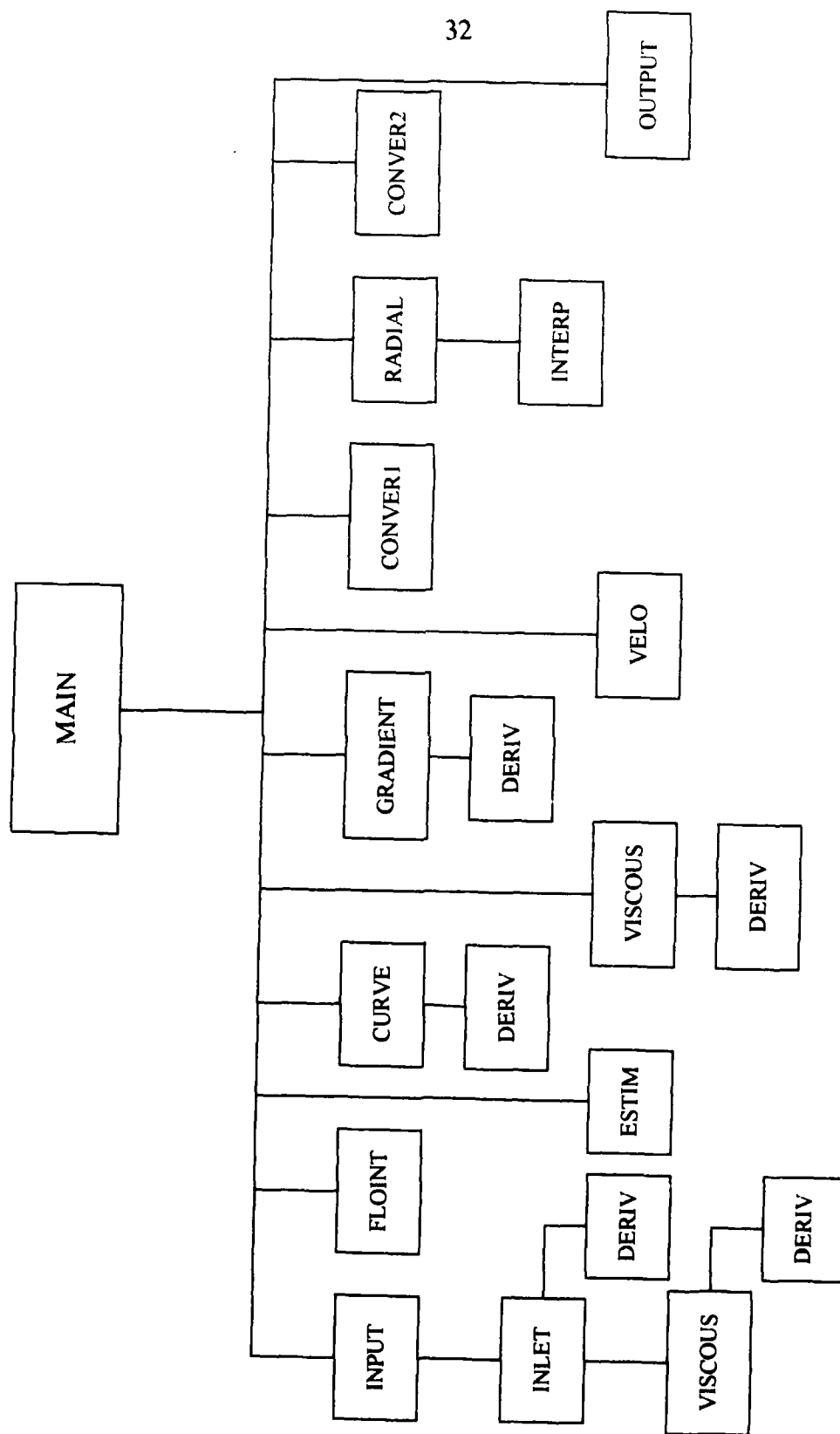


Figure 3.1 Main program and subroutine map

Subroutine INPUT reads the input data for the inlet station $q-o$ and then calls Subroutine INLET as shown in Fig. 3.1. Subroutine INLET computes values for the flow field variables such as static enthalpy and entropy. Subroutine INLET in turn calls DERIV to compute required first and second derivative values for flow field variables. Subroutine VISCOUS is called by INPUT to compute the viscous body forces, E_θ and E_z , and the dissipation function, Φ .

Subroutine FLOINT calculates the mass flow rate and fraction of mass flow rate between adjacent streamlines based on the inlet $q-o$ input data.

Subroutine ESTIM initializes the downstream flow field variables and streamline pattern based on the inlet $q-o$ input data and the hub and casing geometry. Note that INPUT, FLOINT, and ESTIM are called only once at the start of program execution.

Subroutines CURVE through RADIAL comprise the iterative portion of the program for the $q-os$ downstream of the inlet station and implement the solution procedure outlined in Sections 2.8 and 2.9. A global iteration consists of one pass through the annulus executing Subroutines VISCOUS through CONVER1 for all the downstream $q-os$ with the streamline radial locations held constant and is completed once the change in meridional velocity from one pass to the next is less than a user chosen tolerance as determined by Subroutine CONVER1. Subroutines RADIAL and INTERP then compute the new streamline locations for the converged velocity field.

A universal iteration consists of executing Subroutines CURVE through CONVER2 for all the downstream $q-os$. After a global iteration, CONVER2 is called to

check for streamline convergence which is determined by comparing the streamline shifts between successive universal iterations with an arbitrarily chosen tolerance. If convergence has not been attained, the main program starts a new universal iteration at Subroutine CURVE. Once the flow field has converged, Subroutine OUTPUT is called to print the converged flow field values to an output data file.

3.2 Subroutine Descriptions

Each subroutine listing in Appendix C contains a glossary of the FORTRAN variable names used as well as descriptions of the input required and the variables calculated as output. The following descriptions contain the finite difference formulations used.

Subroutine INPUT opens an existing unformatted data file called *input1.dat* and reads the data required by the program. Figure 3.2 shows the meridional plane with arbitrary hub and casing shapes and the FORTRAN variables used in *input1.dat*.

Stagnation temperature and pressure, represented by FORTRAN variables *TEMPIN* and *PRESIN*, are assumed to be constant across the inlet station. *MIXCO*, the mixing coefficient described in Chapter 2, is a function of blade and annulus geometry and the turbomachine operating conditions. *NSTRM* is the number of streamlines in the flow field and *NSTNS* is the number of *q-os*. The index variable, *J*, identifies the streamlines, and *I* identifies the quasi-orthogonals, as shown in Fig. 3.2.

The user supplies the radial location of each streamline at the inlet station, *RADIUS(J,1)*. The tangential velocity for each streamline, *VTHETA(J,1)*, and the axial

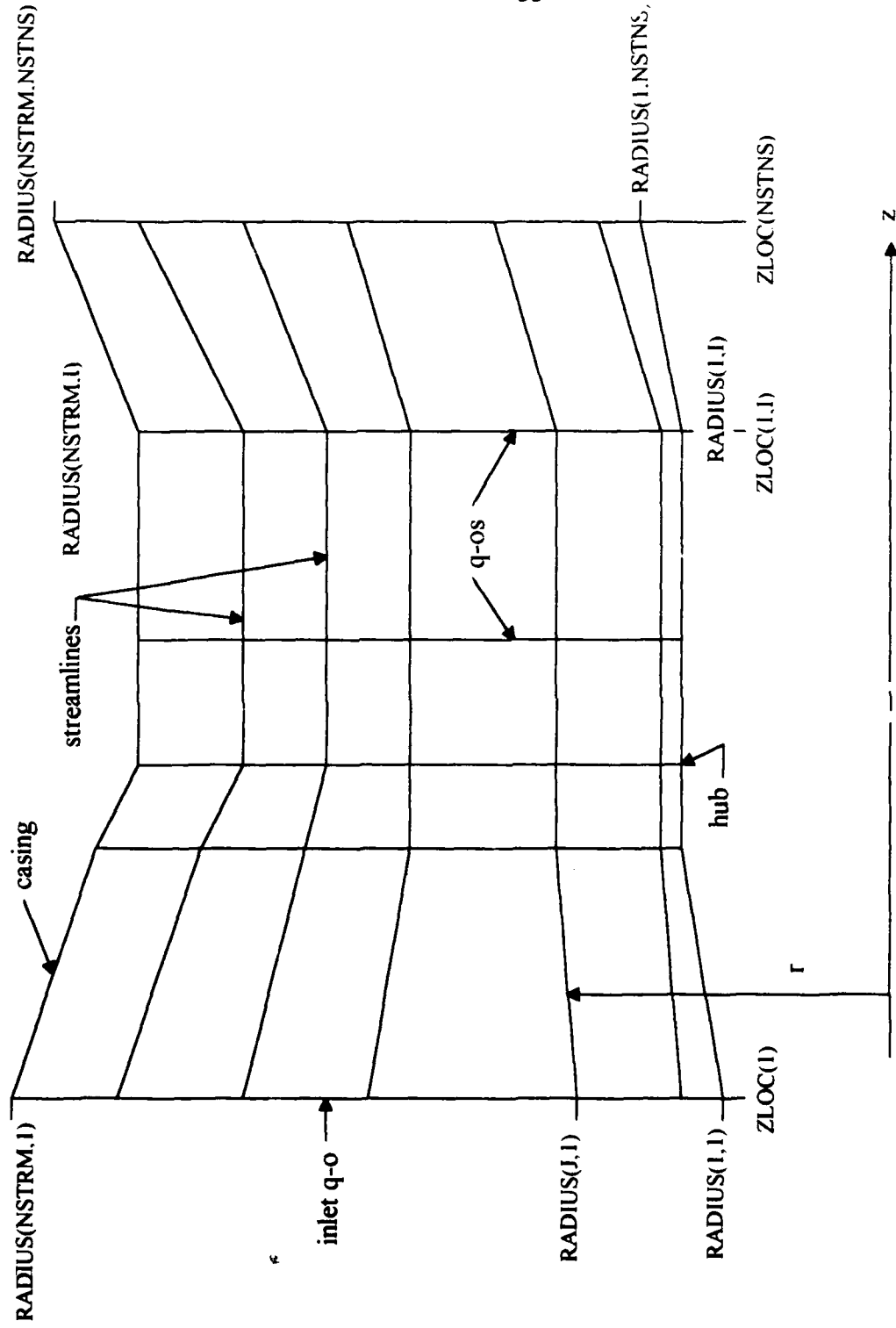


Figure 3.2. Meridional plane, and required hub, casing, streamline, and q-o geometry. (Input flow variables at the inlet q-o are TEMPIN, PRESIN, MIXCO and whirl distribution VTHETA(1,1) ... VTHETA(NSTRM,1))

velocity, $VMID$, at a middle streamline which is determined by

$$JMID = \text{INT}\left(\frac{NSTRM}{2}\right) + 1 \quad (3.1)$$

The streamline spacing prescribed at the inlet from hub to casing is arbitrary.

The q -os in the flow field are limited to the radial direction, although the z direction spacing is arbitrary. The axial location of each q -o is specified by $ZLOC(I)$ and the radial location of the hub and casing on the q -o by $RADIUS(I,I)$, and $RADIUS(NSTRM,I)$. The arrangement of the data in file *input1.dat* is explained in comments included with Subroutine INPUT in Appendix C.

Subroutine INLET computes the flow properties for the inlet q -o using Eqns. (2.47) to (2.49). The flow is assumed to be inviscid and adiabatic prior to reaching the inlet, and the streamlines at the inlet are take to be straight and parallel to the z axis. For a blade-free annulus with radial q -os, the simple radial equilibrium equation give in Eqn. (2.52) applies.

Integration of Eqn. (2.52) along the inlet q -o results in the following expressions for the axial velocity, V_z :

$$V_{z_j}^2 = V_{z_{j-1}}^2 - \frac{(r_j - r_{j-1})}{2} \left[\frac{1}{r_{j-1}} \frac{\partial(rV_\theta)^2}{\partial r} \Big|_{j-1} + \frac{1}{r_j} \frac{\partial(rV_\theta)^2}{\partial r} \Big|_j \right] \quad (j = j_{mid+1} \dots NSTRM) \quad (3.2)$$

$$V_{z_j}^2 = V_{z_{j+1}}^2 - \frac{(r_j - r_{j+1})}{2} \left[\frac{1}{r_{j+1}} \frac{\partial(rV_\theta)^2}{\partial r} \Big|_{j+1} + \frac{1}{r_j} \frac{\partial(rV_\theta)^2}{\partial r} \Big|_j \right] \quad (j = j_{mid-1} \dots 1) \quad (3.3)$$

The integration results in Eqn. (3.3) proceed from the middle streamline, where $V_z = VMID$, to the casing, and in Eqn. (3.4) from the middle streamline to the hub, stepping streamline by streamline. The gradient terms in Eqns. (3.3) and (3.4) are determined in Subroutine DERIV as explained below.

The static temperature, entropy, and density for the inlet $q-o$ are determined using Eqns. (2.47) to (2.49) in the forms:

$$\begin{aligned} T_j &= T^\circ - \frac{V_{z_j}^2 + V_{\theta_j}^2}{2 c_p} \\ s_j &= c_p \ln(T^\circ) - R \ln(p^\circ) \quad (j = 1 \dots NSTRM) \\ \rho_j &= \frac{T_j^{\frac{c_p - R}{R}} e^{\frac{-s_j}{R}}}{R} \end{aligned} \quad (3.4)$$

Subroutine DERIV contains the finite difference schemes needed to compute the first and second derivatives of the distributions of flow field variables in the axial or radial direction. Since the streamline spacing is irregular, the finite difference scheme takes the variable spacing into account based on linear differencing of the variables between streamlines. For an arbitrary variable, $U(r)$ along a $q-o$, the radial gradient at radius r_j is determined with appropriate weighting for variable streamline spacing as

$$\frac{\partial U}{\partial r_j} = \frac{r_{j+1} - r_j}{r_{j+1} - r_{j-1}} \left[\frac{U_j - U_{j-1}}{r_j - r_{j-1}} \right] + \frac{r_j - r_{j-1}}{r_{j+1} - r_{j-1}} \left[\frac{U_{j+1} - U_j}{r_{j+1} - r_j} \right] \quad (j = 2 \dots NSTRM - 1) \quad (3.5)$$

A one-sided finite difference scheme is needed to compute the radial gradients at the hub and outer casing. Applying a second degree polynomial at streamlines $j=1, 2$, and 3 along a $q-o$ (Anderson, et. al., 1988) results in the gradient at the hub radius as

$$\frac{\partial U}{\partial r_{j=1}} = \frac{U_2 - U_1}{r_2 - r_1} - \left[\frac{r_2 - r_1}{r_3 - r_2} \right] \left[\frac{U_3 - U_1}{r_3 - r_1} \right] + \frac{U_2 - U_1}{r_3 - r_1} \quad (3.6)$$

The corresponding gradient at the casing is

$$\begin{aligned} \frac{\partial U}{\partial r_{j=NSTRM}} &= \frac{U_{NSTRM-1} - U_{NSTRM}}{r_{NSTRM-1} - r_{NSTRM}} - \left[\frac{r_{NSTRM-1} - r_{NSTRM}}{r_{NSTRM-2} - r_{NSTRM}} \right] \left[\frac{U_{NSTRM-2} - U_{NSTRM}}{r_{NSTRM-2} - r_{NSTRM}} \right] \\ &+ \frac{U_{NSTRM-1} - U_{NSTRM}}{r_{NSTRM-2} - r_{NSTRM-1}} \end{aligned} \quad (3.7)$$

The second derivative of $U(r)$ at radius r_j is computed as

$$\frac{\partial^2 U}{\partial r^2_j} = \frac{2}{r_{j+1} - r_{j-1}} \left[\frac{U_{j+1} - U_j}{r_{j+1} - r_j} - \frac{U_j - U_{j-1}}{r_j - r_{j-1}} \right] \quad (j = 2 \dots NSTRM - 1) \quad (3.8)$$

Again, a one-sided finite difference scheme is required at the hub and casing, with the results being

$$\frac{\partial^2 U}{\partial r^2}_{j=1} = \frac{2}{r_3 - r_2} \left[\frac{U_3 - U_1}{r_3 - r_1} \right] - \frac{2}{r_2 - r_1} \left[\frac{U_2 - U_1}{r_3 - r_2} \right] \quad (3.9)$$

and

$$\begin{aligned} \frac{\partial^2 U}{\partial r^2}_{j=NSTRM} &= \frac{2}{r_{NSTRM-2} - r_{NSTRM-1}} \left[\frac{U_{NSTRM-2} - U_{NSTRM}}{r_{NSTRM-2} - r_{NSTRM}} \right] \\ &- \frac{2}{r_{NSTRM-1} - r_{NSTRM}} \left[\frac{U_{NSTRM-1} - U_{NSTRM}}{r_{NSTRM-2} - r_{NSTRM-1}} \right] \end{aligned} \quad (3.10)$$

Axial gradients along a streamline where needed in the solution are computed using Eqns. (3.5) to (3.10), but with z replacing r , index i replacing index j , and $NSTNS$ replacing $NSTRM$.

Subroutine FLOINT computes the mass flow rate based on the input data at the inlet $q-o$. The integration technique used is the trapezoidal rule, with integration proceeding from streamline to streamline, starting at the hub and stepping radially outward.

The mass flow rate between adjacent streamlines, j and $j-1$, using data at the inlet $q-o$, is computed by

$$\dot{m}_j = \left(\frac{\rho_j + \rho_{j-1}}{2} \right) \pi (r_j^2 - r_{j-1}^2) \left(\frac{V_{zj} - V_{zj-1}}{2} \right) \quad (j = 2 \dots NSTRM) \quad (3.11)$$

The total mass flow rate, $\dot{m}$, for the annulus is the sum of the mass flow rates

between the streamlines.

The fraction of the total mass flow rate between the hub and each streamline is then computed using

$$fraction_j = \frac{\sum_{k=1}^j \dot{m}_k}{\sum_{k=1}^{NSTRM} \dot{m}_k} \quad (j = 2 \dots NSTRM) \quad (3.12)$$

The individual fractions of the mass flow rate, therefore, identify the streamlines through the annulus.

Since the flow at the annulus walls is inviscid and adiabatic, and the annulus is blade-free, no global energy transfer into or out of the annulus is possible, although energy may be redistributed within the annulus. As a check on energy conservation, the mass weighted stagnation enthalpy for the annulus is computed using

$$\bar{H} = \frac{1}{\dot{m}} \sum_{j=2}^{NSTRM} \dot{m}_j \left(\frac{H_j + H_{j-1}}{2} \right) \quad (3.13)$$

The mass weighted stagnation enthalpy can then be computed for each downstream $q-o$ and the resulting value compared with the inlet value as a check on the flow field calculations.

Subroutine ESTIM estimates, for the initial calculations of the flow field, the static temperature, entropy, density, and streamline radial locations for the $q-os$ downstream

from the inlet for $i = 2$ to $i = NSTNS$, where $NSTNS$ is the number of q -os , or computing stations. ESTIM is executed once only at the start of the calculations.

Since the annulus cross section may change from q -o to q -o as shown in Fig. 3.2, the streamline radial locations for the current, or i th, q -o must be estimated using information from the inlet q -o. The streamline radial locations are compute based on fractions of the span at the inlet according to

$$r_{i,j} = \frac{r_{i,j} - r_{i,1}}{r_{1,NSTRM} - r_{1,1}} (r_{i,NSTRM} - r_{i,1}) + r_{i,1} \quad (j=2...NSTRM-1, i=2...NSTNS) \quad (3.14)$$

The axial velocity for the i th q -o is estimated using the mass flow rate determined at the inlet. The average density at the inlet q -o and throughout the annulus is given by

$$\bar{\rho} = \frac{1}{NSTRM} \sum_{j=1}^{NSTRM} \rho_j \quad (3.15)$$

Using the annulus area for the i th q -o, $\bar{\rho}$, and the total mass flow rate, $\dot{m}$, the average axial velocity for the i th q -o is estimated as

$$\bar{V}_{z_i} = \frac{\dot{m}}{\bar{\rho} \pi (r_{i,NSTRM}^2 - r_{i,1}^2)} \quad (i = 2...NSTNS) \quad (3.16)$$

The stagnation properties are approximated by simply carrying the values downstream from the inlet station. The tangential velocities are similarly carried downstream. The static temperature, entropy, and density for the i th q -o are determined

using Eqns. (3.4)

Subroutine CURVE calculates the streamline curvature, meridional flow angle, and the axial and radial velocities, V_z and V_r , for a $q-o$. CURVE is the first subroutine executed within a universal iteration for the flow field.

The streamline curvature at the determined streamline intersections with the $q-o$ is computed as

$$\frac{1}{r_{c_j}} = \frac{\left(\frac{\partial^2 r}{\partial z^2} \right)_j}{\left[1 + \left(\frac{\partial r}{\partial z} \right)_j^2 \right]^{\frac{3}{2}}} \quad (j = 1..NSTRM) \quad (3.17)$$

where the derivatives of $r(z)$ are determined in DERIV using values of the function at neighboring $q-os$.

The meridional flow angle of a streamline, j , at a $q-o$ is determined from the finite difference form of Eqn. (2.50), or

$$\phi = \frac{1}{2} \left[\arctan \left(\frac{r_{i+1} - r_i}{z_{i+1} - z_i} \right) + \arctan \left(\frac{r_i - r_{i-1}}{z_i - z_{i-1}} \right) \right] \quad (i = 2..NSTNS-1) \quad (3.18)$$

At the inlet $q-o$, $\phi = 0$ and for the last $q-o$ in the flow field, ϕ is determined using the NSTNS and NSTNS - 1 quasi-orthogonals.

The axial and radial velocities at a $q-o$ are then computed as

$$\begin{aligned}
 V_j &= V_{m_j} \cos \phi_j & (j = 1 \dots NSTNS) \\
 V_r &= V_{m_j} \cos \phi_j
 \end{aligned}
 \tag{3.19}$$

where V_m is the meridional velocity computed in subroutine VELO.

Subroutine VISCOUS computes the distributions of the viscous body forces, E_θ and E_z , and the dissipation function, Φ , along a q - o . Using the constraints or assumptions concerning the annulus and q - o geometry listed at the beginning of this chapter, the dissipation function given in Eqn. (2.40) becomes

$$\Phi = \mu \left[\left(\frac{\partial V_z}{\partial r} \right)^2 + \left(\frac{\partial V_\theta}{\partial r} - \frac{V_\theta}{r} \right)^2 \right]
 \tag{3.20}$$

and the viscous body forces in Eqns. (2.29) and (2.30) are written as

$$\begin{aligned}
 E_\theta &= \frac{1}{r} \frac{\partial}{\partial r} \left[r \mu \left(\frac{\partial V_\theta}{\partial r} - \frac{V_\theta}{r} \right) \right] \\
 E_z &= \frac{1}{r} \frac{\partial}{\partial r} \left[r \mu \frac{\partial V_z}{\partial r} \right]
 \end{aligned}
 \tag{3.21}$$

The dissipation function along a given q - o is then computed as

$$\Phi_j = \mu_j \left[\left(\frac{\partial V_z}{\partial r} \right)_j^2 + \left(\left(\frac{\partial V_\theta}{\partial r} \right)_j - \frac{V_{\theta_j}}{r_j} \right)^2 \right] \quad (j = 1 \dots NSTRM)
 \tag{3.22}$$

where the gradient terms are computed by Subroutine DERIV and the viscosity is computed using Eqn. (2.51). The viscosity is computed in Subroutine ESTIM for the first global iteration and in Subroutine GRADIENT for subsequent global iterations.

The viscous body forces are computed using

$$E_{\theta_j} = \frac{1}{r_j} \frac{\partial}{\partial r} \left[r_j \mu_j \left(\left(\frac{\partial V_{\theta}}{\partial r} \right)_j - \frac{V_{\theta_j}}{r_j} \right) \right]_j \quad (j = 1 \dots NSTRM)$$

$$E_{z_j} = - \frac{\partial}{\partial r} \left[r_j \mu_j \left(\frac{\partial V_z}{\partial r} \right)_{jj} \right]$$
(3.23)

Subroutine GRADIENT calls Subroutine DERIV to determine the meridional gradients of entropy, angular momentum, and stagnation enthalpy at a given $q-o$, and then updates these flow field variables at that $q-o$. DERIV is called again to compute the radial gradients needed for integration of the radial equilibrium equation for this $q-o$ in subroutine VELO.

Under the constraints or assumptions listed at the beginning of this chapter, the equations for entropy, angular momentum, and stagnation enthalpy, Eqns. (2.42), (2.46), and (2.45), written for streamline j at the given $q-o$ are:

$$\begin{aligned}
 \left(\frac{\partial s}{\partial m} \right)_j &= \frac{1}{(r \rho T V_m)_j} \frac{\partial}{\partial r} \left[r k \frac{\partial T}{\partial r} \right]_j + \frac{\Phi_j}{(\rho T V_m)_j} \\
 \left(\frac{\partial (r V_\theta)^2}{\partial m} \right)_j &= 2 r_j^2 \left(\frac{V_\theta E_\theta}{V_m \rho} \right)_j \quad (j = 1 \dots NSTRM) \quad (3.24) \\
 \left(\frac{\partial H}{\partial m} \right)_j &= T_j \left(\frac{\partial s}{\partial m} \right)_j + \frac{1}{2 r_j^2} \left(\frac{\partial (r V_\theta)^2}{\partial m} \right)_j + \left(\frac{E_z \cos \phi}{\rho} \right)_j
 \end{aligned}$$

where the radial gradient terms in Eqns. (3.24) are evaluated using DERIV. The heat transfer coefficient, k , is computed according to Eqn. (2.51).

The meridional gradients of entropy, angular momentum, and stagnation enthalpy, computed by Eqns. (3.24), are used to update the flow field variables at the current q -o. The meridional length of each streamline, j , between given adjacent q -os, $i-1$ and i , is computed by

$$\Delta m_j = \sqrt{(r_{i,j} - r_{i-1,j})^2 + (z_{i,j} - z_{i-1,j})^2} \quad (3.25)$$

The entropy, angular momentum, and stagnation enthalpy are updated along the given streamline for the q -o according to

$$\begin{aligned}
 s_{i,j} &= s_{i-1,j} + \Delta m_j \left(\frac{\partial s}{\partial m} \right)_{i,j} \\
 (r V_\theta)^2_{i,j} &= (r V_\theta)^2_{i-1,j} + \Delta m_j \left(\frac{\partial (r V_\theta)^2}{\partial m} \right)_{i,j} \\
 H_{i,j} &= H_{i-1,j} + \Delta m_j \left(\frac{\partial H}{\partial m} \right)_{i,j}
 \end{aligned} \tag{3.26}$$

The change in angular momentum from the previous iteration is relaxed using *RF*, an arbitrary relaxation factor chosen by the user as

$$(r V_\theta)^2 (NEW) = (r V_\theta)^2 (OLD) + RF [(r V_\theta)^2 - (r V_\theta)^2 (OLD)] \tag{3.27}$$

The tangential velocity, V_θ , is computed from the updated angular momentum above, and, in turn, the static temperature, pressure, and density are updated using Eqn. (3.18).

The radial gradients of the updated entropy, angular momentum, and stagnation enthalpy along the $q-o$, required in integration of the radial equilibrium equation, are then computed along the $q-o$ by Subroutine DERIV. The number of the current $q-o$ is sent to GRADIENT in the call list and the main program controls the stepping from $q-o$ to $q-o$, from $i = 2$ to $i = NSTNS$.

Subroutine VELO integrates the radial equilibrium equation to obtain the meridional velocity profile along a given $q-o$. The resulting profile is used to determine the mass flow rate and the constant of integration involved is then adjusted as necessary to

achieve mass continuity. Once mass continuity is satisfied, the main program steps the computations to the next q - o .

Using the assumptions of a blade - free annulus and radially directed quasi-orthogonals, the radial equilibrium equation, Eqn. (2.24), may be rearranged to give

$$\frac{\partial V_m^2}{\partial r} = B - A V_m^2 \quad (3.28)$$

where

$$B = 2 \left[\frac{\partial H}{\partial r} - T \frac{\partial s}{\partial r} - \frac{1}{2r^2} \frac{\partial (r V_\theta)^2}{\partial r} - E_m \sin \phi \right] \quad (3.29)$$

and

$$-A = 2 \left[\frac{\cos \phi}{r_c} + \frac{\sin \phi}{V_m} \frac{\partial V_m}{\partial m} \right] \quad (3.30)$$

where A and B are functions of radius, r , along the q - o .

Equation (3.28) is a linear, first order, ordinary differential equation and may be integrated by using a suitable integrating factor.

Multiplying Eqn. (3.28) by an integrating factor, s , and rearranging yields

$$s \frac{\partial V_m^2}{\partial r} + s A V_m^2 = s B \quad (3.31)$$

For the left - hand side of Eqn. (3.31) to be an exact differential, we require

$$\frac{\partial(s V_m^2)}{\partial r} = s \frac{\partial V_m^2}{\partial r} + V_m^2 \frac{\partial s}{\partial r} \quad (3.32)$$

Comparing Eqns. (3.31) and (3.32), it can be seen that

$$\frac{\partial s}{\partial r} = s A \quad (3.33)$$

from which

$$s = e^{\int A dr} \quad (3.34)$$

where a constant of integration has been omitted for simplicity.

Substituting Eqn. (3.34) into Eqn. (3.31) then yields

$$\frac{\partial}{\partial r} \left[e^{\int A dr} V_m^2 \right] = B e^{\int A dr} \quad (3.35)$$

Equation (3.35) may now be integrated along a q - o from a point r_i where $V_m = V_{m_i}$ to obtain

$$V_m^2 = e^{\int -A dr} \int_{r_1}^r [B e^{\int A dr}] dr + V_{m_1}^2 e^{\int -A dr} \quad (3.36)$$

For a short distance r_1 to r , A and B can be approximated as average values $\bar{A}$ and $\bar{B}$. Therefore, Eqn. (3.36) becomes

$$V_m^2 = e^{-\bar{A}(r-r_1)} \bar{B} \int_{r_1}^r [e^{\bar{A}(r-r_1)}] dr + V_{m_1}^2 e^{-\bar{A}(r-r_1)} \quad (3.37)$$

Evaluating the integral in Eqn. (3.37) and simplifying yields

$$V_m^2 = \frac{\bar{B}}{\bar{A}} [1.0 - e^{-\bar{A}(r-r_1)}] + V_{m_1}^2 e^{-\bar{A}(r-r_1)} \quad (3.38)$$

If $\bar{A} = 0$, Eqn. (3.38) is indeterminate. Applying L'Hopitals rule to evaluate in the limit as $\bar{A}$ goes to zero then gives

$$V_m^2 = \bar{B}(r - r_1) + V_{m_1}^2 \quad (3.39)$$

The forms of Eqns. (3.38) and (3.39) used to integrate the radial equilibrium equation from the middle streamline to the tip are

$$V_{m_j}^2 = \frac{\bar{B}_j}{\bar{A}_j} [1.0 - e^{-\bar{A}_j(r_j - r_{j-1})}] + V_{m_{j-1}}^2 e^{-\bar{A}_j(r_j - r_{j-1})} \quad (3.40)$$

$$V_{m_j}^2 = \bar{B}_j[r_j - r_{j-1}] + V_{m_{j-1}}^2 \quad (j = j_{mid+1} \dots NSTRM)$$

and for integration from the middle streamline to the hub

$$V_{m_j}^2 = \frac{\bar{B}_j}{\bar{A}_j} [1.0 - e^{-\bar{A}_j(r_j - r_{j+1})}] + V_{m_{j+1}}^2 e^{-\bar{A}_j(r_j - r_{j+1})} \quad (3.41)$$

$$V_{m_j}^2 = \bar{B}_j[r_j - r_{j+1}] + V_{m_{j+1}}^2 \quad (j = j_{mid-1} \dots 1)$$

where V_m^2 at j_{mid} is the meridional velocity (VMID) from the previous iteration or the initial solution estimate.

The change in the velocity profile from a previous local iteration is relaxed using the relaxation factor RF as

$$V_m^2(NEW) = V_m^2(OLD) + RF[V_m^2 - V_m^2(OLD)] \quad (3.42)$$

The integration of the radial equilibrium equation may return negative values for the square of the meridional velocity. If this occurs, the square of the meridional velocity is set to 1.0

The mass flow rate and mass weighted stagnation enthalpy are then calculated using Eqns. (3.11) to (3.13) as described for subroutine FLOINT. Mass continuity is checked by comparing the difference between $\dot{m}$ (local) and $\dot{m}$ (inlet) with a user chosen

tolerance. If continuity is not satisfied, a secant method routine (Chapra and Canale, 1988) is used to find a new value of V_m^2 at j_{mid} and the radial equilibrium equation is re-integrated using the new constant of integration. A local iteration is designated as one integration of the radial equilibrium equation and check of mass continuity.

Subroutine RADIAL adjusts the streamline radial locations to match the required fractions of the mass flow rate at the current $q-o$. The meridional velocities which correspond to the new streamline radii are then interpolated from the velocity profile using Subroutine INTERP.

The change in radii between iterations is damped using a relaxation factor (Wilkinson, 1969-70). The optimum relaxation factor as determined by Wilkinson is

$$RF = \frac{1}{1 + \frac{5}{24} (1 - M_m^2) \left(\frac{SPAN}{\Delta z} \right)} \quad (3.43)$$

where M_m is the mean Mach number on a $q-o$, span is the distance between the outer casing and the hub, and Δz is the quasi-orthogonal spacing. Thus, the change in streamline radius at a given $q-o$ is damped according to

$$r(NEW) = r(OLD) + RF[r - r(OLD)] \quad (3.44)$$

where r is the radius interpolated from the velocity profile.

Subroutine INTERP is an interpolation routine using second degree Lagrange interpolating polynomials (Chapra and Canale, 1988). XDATA and YDATA are arrays

of known values of NDATA points each, i.e. $YDATA = f(XDATA)$. Subroutine INTERP computes the ordinate values, YOUT, corresponding to the abscissa points, XIN.

INTERP checks first to ensure that all XIN data points lie within the upper and lower bounds of XDATA and then compares each XIN data point sequentially with the XDATA values to find bracketing values. The interpolation result is then

$$\begin{aligned}
 YOUT = & \frac{[XIN - XDATA(k+1)][XIN - XDATA(k+2)]}{[XDATA(k) - XDATA(k+1)][XDATA(k) - XDATA(k+2)]} YDATA(k) \\
 & + \frac{[XIN - XDATA(k)][XIN - XDATA(k+2)]}{[XDATA(k+1) - XDATA(k)][XDATA(k+1) - XDATA(k+2)]} YDATA(k+1) \\
 & + \frac{[XIN - XDATA(k)][XIN - XDATA(k+1)]}{[XDATA(k+2) - XDATA(k)][XDATA(k+2) - XDATA(k+1)]} YDATA(k+2)
 \end{aligned} \tag{3.45}$$

Subroutines CONVER1 and CONVER2 check the convergence of the meridional velocities and streamline radii through the passage. At each streamline and $q-o$ intersection, the change in velocity and radial location is compared with user chosen tolerances as

$$\begin{aligned}
 \left| \frac{V_m(OLD) - V_m}{V_m} \right| & \leq \text{tolerance (velocity)} \\
 \left| \frac{r(OLD) - r}{r} \right| & \leq \text{tolerance (radial)}
 \end{aligned} \tag{3.46}$$

If convergence is not attained, the old meridional velocity and radial location take the

values of the new velocity and radial location.

The main program calls Subroutine OUTPUT if both the meridional velocities and the streamline radial locations have converged or if the maximum number of iterations desired by the user has been exceeded. If the flow field has not converged, a new iteration is started.

4. EXAMPLE SOLUTIONS

Example Problem 1 is for axial flow through an annular passage. If no tangential velocity profile is included in the input, integration of the radial equilibrium equation, Eqns. (3.2) and (3.3), at the inlet $q-o$ results in a constant axial velocity profile. Because of this velocity condition at the inlet, the viscous body forces, Eqns. (3.23) and the dissipation function, Eqn. (3.20), are all zero and the static temperature profile, Eqn. (3.4), is constant across the inlet. Thus, the meridional gradients of entropy, angular momentum, and stagnation enthalpy in Eqns. (3.24) are all zero.

For constant radius hub and casing shape, initialization of the flow field by Subroutine ESTIM will result in carrying the inlet axial velocity profile and streamline locations to the downstream $q-os$. Integration of the radial equilibrium equation using Eqns. (3.40) and (3.41) simply returns the constant axial velocity since the streamline curvature and meridional flow angle are zero. Because no changes occur, the program will converge for any value of viscosity or convergence tolerance input by the user.

An annular passage with varying hub and casing shape forces the estimation of the flow field for $q-os$ downstream from the inlet to change. For a converging passage, the downstream axial velocity profiles are initialized as constant values from hub to tip; however, the velocity values and the streamline radial locations vary from $q-o$ to $q-o$. The viscous body forces, dissipation function, and meridional gradients of entropy, angular momentum, and stagnation enthalpy remain zero. Integration of the radial equilibrium equation for this case results in a new meridional velocity profile. Thus, the effect of the

varying endwall geometry propagates through the system of governing equations.

The program was run for the case of a converging nozzle with no tangential velocity input. The hub and casing profiles formed a converging passage with smoothly varying first and second derivatives. The hub and casing were symmetric about a centerline of the passage. Convergence was attained after 3 global and 2 universal iterations.

The streamline and quasi-orthogonal locations and the axial, radial, and meridional velocity profiles from the converged solution are shown in Fig. 4.1. As can be seen in Fig. 4.1 the flow velocities are not symmetric about the centerline, although the passage is symmetric. This asymmetry occurs because of the differing areas between streamlines. The full program output is contained in Appendix D.

Example Problem 2 is for axial flow with swirl through a straight annulus. This second test case was run using a constant radius hub and casing shape. The tangential velocity profile at the inlet formed a free vortex profile. The program converged for this case after 2 global and 2 universal iterations. Figure 4.2 shows the streamline and quasi-orthogonal locations and the axial, radial, meridional, and tangential velocity profiles for this test case. The calculations were carried out for 21 streamlines, however, results are shown for every fifth streamline only. The program output is contained in Appendix D.

Tangential shear stresses on the radially inward and outward faces of the streamtubes due to the θ direction velocities should cause the free vortex profile to shift towards a forced vortex profile. The tangential velocity field shown in Fig. 4.2 shows only

a small change in velocities without the expected shift towards a forced vortex profile (Gallimore, 1985). The axial velocity profiles remain essentially constant throughout the flow field with no streamline radial location shifts.

Cuvature calculations are unstable even in inviscid flows and adding viscosity decreases stability even more. It is thought at this time that calculations are not being carried far enough downstream for the flow to shift to the forced vortex profile. Work is continuing in this direction at this time (Grabowska, 1994).

PROGRAM CALCULATION SUMMARY

GLOBAL ITERATIONS TO REACH CONVERGENCE: 3
 UNIVERSAL ITERATIONS TO CONVERGENCE: 2

THE MASS FLOW RATE FOR THE ANNULUS: 0.490 slug/sec

THE RADIAL LOCATIONS (inches) ARE:									
11	12.000	11.990	11.927	11.777	11.543	11.250	10.960	10.722	10.510
10	11.500	11.493	11.446	11.333	11.154	10.922	10.687	10.494	10.320
9	11.000	10.995	10.962	10.884	10.756	10.586	10.409	10.261	10.128
8	10.500	10.497	10.478	10.430	10.352	10.242	10.125	9.961	9.810
7	10.000	9.999	9.992	9.974	9.942	9.892	9.835	9.784	9.738
6	9.500	9.501	9.505	9.515	9.528	9.537	9.541	9.540	9.539
5	9.000	9.002	9.018	9.054	9.112	9.178	9.244	9.325	9.340
4	8.500	8.504	8.531	8.594	8.695	8.819	8.944	9.044	9.133
3	8.000	8.006	8.044	8.134	8.280	8.460	8.643	8.792	8.924
2	7.500	7.508	7.558	7.677	7.867	8.103	8.341	8.536	8.718
1	7.000	7.010	7.073	7.223	7.460	7.750	8.040	8.277	8.490

THE G-Z DIRECTION LOCATIONS (inches) ARE:

THE AXIAL VELOCITIES (ft/sec) ARE:									
11	100.00	100.12	100.76	105.06	113.11	130.07	159.05	197.80	233.74
10	100.00	100.24	101.72	107.97	118.22	136.49	165.27	202.11	235.38
9	100.00	100.31	102.37	110.15	122.46	142.23	170.95	205.96	236.77
8	100.00	100.36	102.77	111.57	125.56	146.80	175.59	209.04	237.85
7	100.00	100.38	102.97	112.31	127.33	149.68	178.64	211.07	238.54
6	100.00	100.37	102.99	112.41	127.73	150.52	179.66	211.79	238.79
5	100.00	100.37	102.86	111.96	126.79	149.25	178.48	211.10	238.58
4	100.00	100.34	102.59	110.97	124.66	146.11	175.31	209.08	237.91
3	100.00	100.29	102.17	109.48	121.52	141.55	170.64	205.96	236.83
2	100.00	100.22	101.57	107.47	117.60	136.10	165.06	202.09	235.41
1	100.00	100.12	100.77	104.98	113.12	130.27	159.14	197.84	233.74

THE RADIAL VELOCITIES (ft/sec) ARE:

11	0.00	-7.27	-21.33	-40.08	-59.44	-75.89	-83.75	-75.99	-49.34	-18.33	-2.47
10	0.00	-5.43	-16.19	-31.40	-48.49	-63.62	-70.66	-63.60	-40.75	-15.01	-2.01
9	0.00	-3.76	-11.35	-22.64	-36.46	-49.41	-55.43	-49.65	-31.45	-11.50	-1.54
8	0.00	-2.23	-6.83	-13.98	-23.66	-33.41	-38.13	-34.18	-21.52	-7.84	-1.05
7	0.00	-0.81	-2.59	-5.56	-10.44	-16.06	-19.15	-17.53	-11.10	-4.07	-0.54
6	0.00	0.52	1.43	2.58	4.81	7.19	9.76	12.50	15.41	18.47	21.67
5	0.00	1.81	5.32	10.48	15.74	19.65	20.64	17.25	10.41	3.64	0.48
4	0.00	3.10	9.17	18.17	28.06	36.35	39.48	34.10	21.04	7.50	0.99
3	0.00	4.41	13.08	25.70	39.54	51.41	56.50	49.77	31.26	11.28	1.50
2	0.00	5.79	17.12	33.10	50.04	64.52	71.27	63.83	40.82	14.92	2.00
1	0.00	7.27	21.34	40.34	59.48	75.63	83.68	76.02	49.50	18.34	2.47

THE MERIDIONAL VELOCITIES (ft/sec) ARE:

11	100.00	100.38	103.00	112.45	127.79	150.59	179.75	211.89	238.80	252.93	255.29
10	100.00	100.38	103.00	112.45	127.78	150.59	179.75	211.88	238.88	252.92	255.27
9	100.00	100.38	103.00	112.45	127.78	150.57	179.72	211.85	238.85	252.89	255.24
8	100.00	100.38	103.00	112.44	127.77	150.55	179.69	211.82	238.82	252.86	255.21
7	100.00	100.38	103.00	112.44	127.76	150.54	179.66	211.80	238.80	252.85	255.19
6	100.00	100.38	103.00	112.44	127.76	150.53	179.66	211.79	238.79	252.83	255.19
5	100.00	100.38	103.00	112.45	127.77	150.54	179.67	211.81	238.81	252.85	255.20
4	100.00	100.38	103.00	112.45	127.78	150.57	179.71	211.84	238.84	252.88	255.23
3	100.00	100.38	103.00	112.45	127.79	150.59	179.75	211.89	238.88	252.92	255.27
2	100.00	100.38	103.00	112.46	127.80	150.62	179.79	211.93	238.92	252.95	255.31
1	100.00	100.38	103.00	112.46	127.81	150.63	179.78	211.94	238.93	252.95	255.31

PROGRAM CALCULATION SUMMARY

GLOBAL ITERATIONS TO REACH CONVERGENCE: 2
 UNIVERSAL ITERATIONS TO CONVERGENCE: 2
 ALL DIMENSIONS IN ft, sec, lbf, R UNLESS NOTED

THE MASS FLOW RATE FOR THE ANNULUS: 0.3301 slug/sec

THE RADIAL LOCATIONS (inches) ARE:

21	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000	10.000
16	9.000	9.000	9.000	9.000	9.000	9.000	9.000	9.000	9.000
11	8.000	8.000	8.000	8.000	8.000	8.000	8.000	8.000	8.000
6	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000
1	6.000	6.000	6.000	6.000	6.000	6.000	6.000	6.000	6.000

THE AXIAL VELOCITIES (ft/sec) ARE:

21	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
16	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
11	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
6	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
1	99.99	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

THE RADIAL VELOCITIES (ft/sec) ARE:

21	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
16	0.00	-0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
11	0.00	-0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
6	0.00	-0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

THE MERIDIONAL VELOCITIES (ft/sec) ARE:

21	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
16	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
11	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
6	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
1	99.99	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

THE THETA VELOCITIES (ft/sec) ARE:

21	7.20	7.20	7.20	7.20	7.20	7.20	7.20	7.20	7.20
16	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00
11	9.00	8.99	8.99	8.98	8.98	8.97	8.97	8.97	8.97
6	10.28	10.26	10.24	10.24	10.25	10.26	10.27	10.28	10.28
1	12.00	12.00	12.01	12.02	12.02	12.03	12.03	12.03	12.03

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APPENDIX A. DISTRIBUTED BLADE BODY FORCES

The component momentum equations, Eqns. (2.10) - (2.12), contain the distributed blade body force terms f_n , f_θ , and f_m . These terms are functions of the blade geometry as described below. Wennerstrom (1974) defined the distributed blade body force terms as functions of both geometry and the entropy generated by flow over the blades.

Figure A2 shows a point on an arbitrary blade with lean in the intrinsic coordinate system, (n, θ, m) , at a distance, r , from the axis of the turbomachine. The quasi-orthogonal, $\bar{q}$, lies in the meridional $(m-n)$ plane at an angle $(\phi + \gamma)$ from the n direction (see also Fig. 2.4). The blade, which is not shown, is described by the mean camber surface. The unit vector, $\hat{B}$, is tangent to the blade surface at an angle ϵ from $\bar{q}$, and lies in the $(q-\theta)$ plane. $\hat{s}$ is a unit vector tangent to the blade surface at an angle β from e_m and lies in the $(\theta-m)$ plane. The blade body force due to pressure differences across the pressure and suction sides of the blade, f_p , is normal to the blade surface.

From Fig. A2, the following unit vectors in the blade surface tangent plane may be written:

$$\begin{aligned}\hat{s} &= \cos\beta \hat{e}_m + \sin\beta \hat{e}_\theta \\ \hat{B} &= \cos\epsilon \sin(\phi + \gamma) \hat{e}_m - \sin\epsilon \hat{e}_\theta + \cos\epsilon \cos(\phi + \gamma) \hat{e}_n\end{aligned}\tag{A1}$$

The unit vector in the direction of f_p is found from the cross product, $\hat{B} \times \hat{s}$ giving

A2

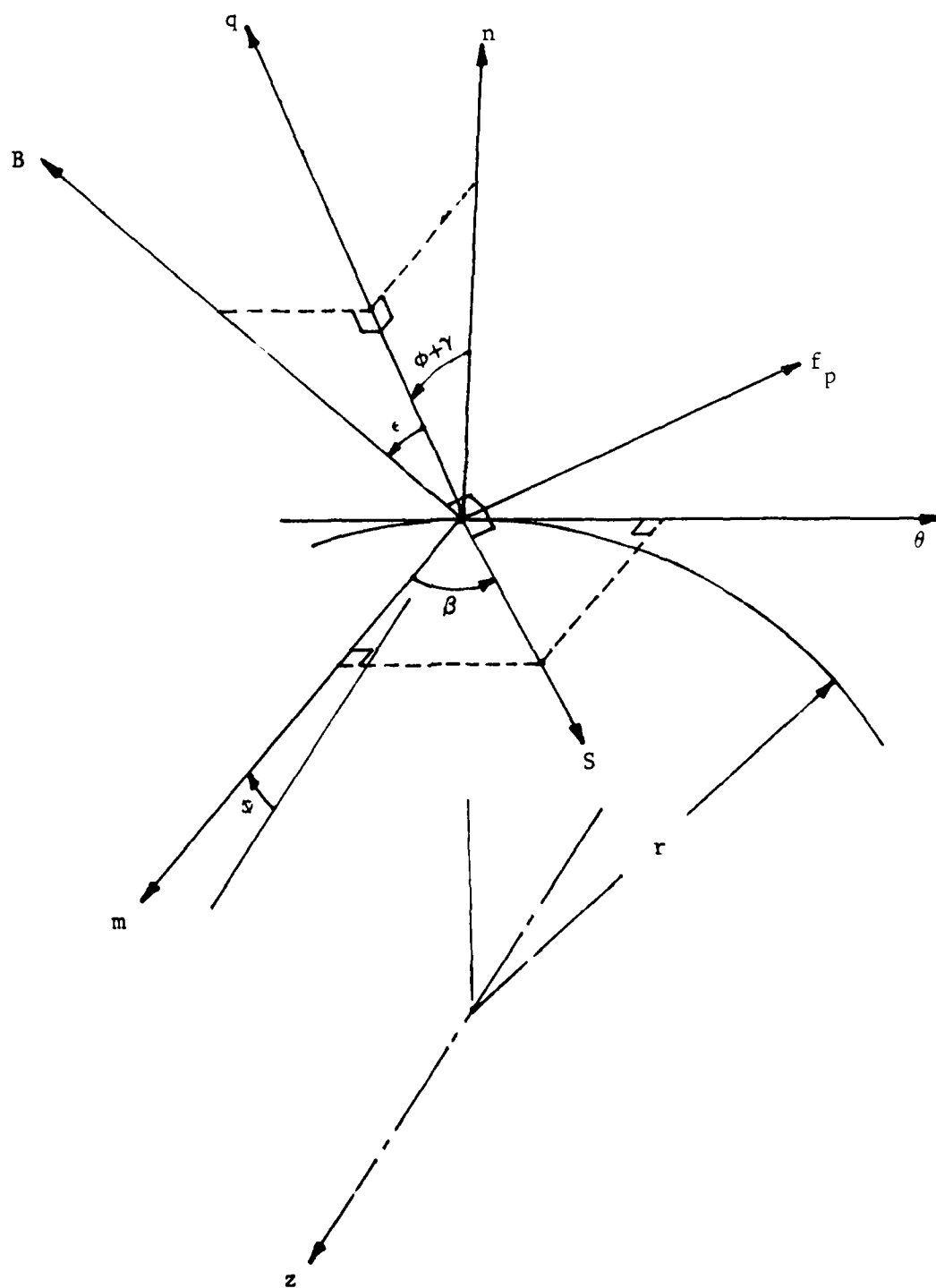


Figure A1. Blade force geometry

$$\frac{\vec{f}_p}{|\vec{f}_p|} = [-\sin\beta \cos\epsilon \cos(\phi + \gamma)] \hat{e}_m + [\cos\beta \cos\epsilon \cos(\phi + \gamma)] \hat{e}_\theta + [\sin\beta \cos\epsilon \sin(\phi + \gamma) + \cos\beta \sin\epsilon] \hat{e}_n \quad (\text{A2})$$

This differs from Wennerstrom's (1974) form of the equation due to an error in the cross product.

The distributed blade body forces for the (n, θ, m) coordinate directions are determined by separating the components of f_p to give

$$\begin{aligned} f_n &= f_p [\sin\beta \cos\epsilon \sin(\phi + \gamma) + \cos\beta \sin\epsilon] \\ f_\theta &= f_p [\cos\beta \cos\epsilon \cos(\phi + \gamma)] \\ f_m &= f_p [\sin\beta \cos\epsilon \cos(\phi + \gamma)] \end{aligned} \quad (\text{A3})$$

The tangential blade body force may be determined by rearranging the tangential momentum equation, Eqn. (2.11), to give

$$f_\theta = \frac{V_m}{r} \frac{\partial}{\partial m} (r V_m \tan\beta) \quad (\text{A4})$$

The magnitude of f_p can then be found from the tangential component equation in Eqn. (A3) and the results substituted into the normal and meridional component equations, yielding

APPENDIX B. GOVERNING EQUATIONS IN CYLINDRICAL COORDINATES

The governing equations for streamline curvature throughflow analysis (see Section 2.6) may be written for cylindrical streamsurfaces with streamline coordinate, m , corresponding to the axial coordinate, z , the normal coordinate, n , to the radius, r , and with the angle $\phi = 0$ and $r_c = \infty$. Thus, in cylindrical coordinates, we obtain,

Viscous Body Forces, (from Eqn. (2.13))

$$\begin{aligned} E_r &= 0 \\ E_\theta &= \frac{1}{r} \frac{\partial}{\partial r} \left[r \mu \left(\frac{\partial V_\theta}{\partial r} - \frac{V_\theta}{r} \right) \right] \\ E_z &= \frac{1}{r} \frac{\partial}{\partial r} \left[r \mu \frac{\partial V_z}{\partial r} \right] \end{aligned} \quad (B1)$$

Entropy/Energy Equation, (from Eqn. (2.41))

$$\frac{\partial s}{\partial z} = \frac{1}{\rho T V_z} \frac{\partial}{\partial r} \left(r k \frac{\partial T}{\partial r} \right) + \frac{\Phi}{\rho T V_z} + \frac{\partial S_e}{\partial z} \quad (B2)$$

where the dissipation function, Φ , from Eqn. (2.40) is written as

$$\Phi = \mu \left\{ \left[\frac{\partial V_\theta}{\partial r} - \frac{V_\theta}{r} \right]^2 + \left[\frac{\partial V_z}{\partial r} \right]^2 \right\} \quad (B3)$$

Meridional Momentum/Enthalpy Equation, (from Eqn. (2.45))

B2

$$\frac{\partial H}{\partial z} = T \frac{\partial s}{\partial z} + \frac{1}{2r^2} \frac{\partial (rV_\theta)^2}{\partial z} + E_z + f_z \quad (\text{B4})$$

Angular Momentum, (from Eqn. (2.46))

$$\frac{1}{2r^2} \frac{\partial (rV_\theta)^2}{\partial z} = \frac{V_\theta}{V_z} (E_\theta + f_\theta) \quad (\text{B5})$$

Radial Equilibrium, (from Eqn. (2.24))

$$V_z \frac{\partial V_z}{\partial r} = \frac{\partial H}{\partial r} - T \frac{\partial s}{\partial r} - \frac{1}{2r^2} \frac{\partial (rV_\theta)^2}{\partial r} - f_r \quad (\text{B6})$$

APPENDIX C. COMPUTER PROGRAM LISTING

The main program and associated subroutines contained in Appendix C are roughly in the order in which they are executed (See Fig. 3.1). Each routine contains comment lines which describe the input and output variables required for operation as well as describing the purpose of each section of code.


```

73 NSTNS = NUMBER OF STATIONS IN FLOW FIELD (q-o)
74 NSTEP - THE LOCATION OF THE STEPS THE DO LOOPS TAKE IN PRINTING OUTPUT
75 ZLOC(1) - THE LOCATION OF THE INLET q-o (USUALLY 0.00)
76 *****0*****
77 VUID IS THE VELOCITY UNITS USED AS CONSTANT OF
78 INTEGRATION TO FIND VELOCITY PROFILE WHICH SATISFIES SIMPLE
79 RADIUS EQUILIBRIUM FOR THE INLET-VTHETA PROFILE (ft/sec)
80 RADIUS,VTHETA THE STREAMLINE LOCATIONS AND THE ACCOMPANYING
81 THETA VELOCITY PROFILE (RADIUS IN inches, VTHETA IN ft/sec)
82 RADIUS,RADIUS,ZLOC - HUB AND OUTER CASING GEOMETRY FOR THE ANNULUS
83 AND THE Z LOCATION FOR EACH. q-os ARE RADIAL ONLY (Z LOCATIONS
84 ARE ALSO Z LOCATION) (RADIUS AND ZLOC ARE INPUT IN inches)
85 *****0*****0*****0*****0*****0*****0*****0*****0*****
86
87 PRINT*, , OPENING INPUT FILE AND READING DATA'
88 OPEN(10, FILE='input1.dat', STATUS='OLD')
89
90 READ(10,*) LP, LLIMIT, MLIMIT, RELAXV, RELAXW, TOLM, TOLV, TOLR
91 READ(10,*) PRESIN, TEMPIN, MIXCO
92 PRINT*, PRESIN, TEMPIN, MIXCO
93
94 READ(10,*) NSTRM, NSTNS, NSTEP
95 PRINT*, NSTRM, NSTNS, NSTEP
96 *****0*****0*****0*****0*****0*****0*****0*****0*****
97
98 READ(10,*) ZLOC(1)
99 PRINT*, ZLOC(1)
100
101 READ(10,*) VMID
102 PRINT*, VMID
103
104 DO 10 J=1,NSTRM
105   READ(10,*) RADIUS(J,1), VTHETA(J,1)
106   PRINT*, J, RADIUS(J,1), VTHETA(J,1)
107 CONTINUE
108
109 PRINT*, , ,
110
111 DO 20 I=2,NSTNS
112   READ(10,*) RADIUS(INSTRM,I), RADIUS(1,1), ZLOC(1)
113   PRINT*, RADIUS(INSTRM,I), RADIUS(1,1), ZLOC(1)
114 CONTINUE
115
116 CLOSE(10)
117
118 *****0*****0*****0*****0*****0*****0*****0*****0*****
119 CALL SUBROUTINE INLET, WHICH COMPUTES ALL THE VARIABLES AND RADIAL
120 GRADIENTS FOR THE INLET q-o.
121 *****0*****0*****0*****0*****0*****0*****0*****0*****
122 CALL INLETCURV, DISSIP, DMHDM, DMHDQ, ENTHAL, ENTROP,
123 ETHETA, EZ, HSTAG, HTCOEF, MEDELTA, MID, MUT, NSTNS, NSTRM,
124 PHI, PRESS, PRESEN, PSTAG, RADIUS, REDLTA, RHO, RVEL, TEMP,
125 TENPIN, TSTAG, VELM, VELTOT, VMID, VTHETA, WHIRL, ZDELTA,
126 ZLOC, ZVEL)
127 *****0*****0*****0*****0*****0*****0*****0*****0*****
128
129 *****0*****0*****0*****0*****0*****0*****0*****0*****
130 ECHO THE INVT VARIABLES TO THE OUTPUT DATA FILE AS REQUIRED BY LP
131 LA VALUES ARE LISTED AT THE LEFT OF THE OUTPUT FILE NAMES.
132 USE LP = 2 TO
133 *****0*****0*****0*****0*****0*****0*****0*****0*****
134 THESE OUTPUT FILES WILL BE USED TO STORE THE DATA FOR REDUCING:
135 1 OUTPUT1 DAT - INLET AND FLOW INTEGRATION DATA - WRITTEN ONCE
136 2 OUTPUT2 DAT - SUBROUTINE ESTIM
137 3 OUTPUT3 DAT - SUBROUTINE CURVE
138 4 OUTPUT4 DAT - SUBROUTINE VISCOUS
139 5 OUTPUT5 DAT - SUBROUTINE GRADEANT
140 6 OUTPUT6 DAT - SUBROUTINE VELO
141 7 OUTPUT7 DAT - SUBROUTINES RADIAL AND INTERP
142 8 OUTPUT8 DAT - SUBROUTINES CONVERGI & 2
143 *****0*****0*****0*****0*****0*****0*****0*****0*****
144 WRITE THESE ONCE
145 FOR EACH GLOBAL
146 ITERATION

```



```

145 PRESS(J,1) = EXP( CP/GASCON * DLOG(TEMP(J,1)) -
146 ENTROP(J,1)/GASCON )
147 IF (PRESS(J,1) .EQ. PSTAG(J,1))
148 PRINT*, 'PRESS2 ERROR IN INLET', J
149
150 ENTHAL HAS UNITS OF ft^2/sec^2
151 ENTHAL(J,1) = HSTAG(J,1) - VELTOT(J,1)**2/2.000
152
153 DENSITY HAS UNITS OF slug/ft^3
154 RHO(J,1) = (PRESS(J,1) * GSUBC)/GASCON * TEMP(J,1)
155 IF (RHO(J,1) .EQ. 0.00)PRINT*, 'RHO ERROR IN INLET', J
156
157 VISCOSITY HAS UNITS OF lbf sec/ft^2 OR slug/ft sec
158 MU(J,1) = RHO(J,1) * MUXCO
159 MU(J,1) = MU(J,1) * CP
160
161 SET AXIAL VELOCITIES WITH UNITS OF ft/sec
162 ZVEL(J,1) = VELM(J,1)
163
164 CONTINUE
165
166 *****0*****0*****0*****0*****0*****0*****0*****
167 CALL VISCOS TO COMPUTE DISSIPATION, ETHETA, AND EZ FOR THE INLET Q=0
168 *****0*****0*****0*****0*****0*****0*****0*****
169 CALL VISCOS(DISSIP, ETHETA, EZ, 1, MUT, NSTNS, NSTRM,
170 RADIUS, ROELTA, VELM, VTHETA, ZVEL)
171
172 *****0*****0*****0*****0*****0*****0*****0*****
173 DHQM AND DSDM ARE CALCULATED FOR INFORMATION ONLY - WILL NOT CHANGE
174 *****0*****0*****0*****0*****0*****0*****0*****
175 DSDM HAS UNITS OF ft/sec^2 R
176 DHQM HAS UNITS OF ft^3/sec^2
177 DHQM HAS UNITS OF ft^3/sec^2
178 *****0*****0*****0*****0*****0*****0*****0*****
179 EOM FOR DSDM THAT DOES NOT ACCOUNT FOR HEAT TRANSFER AT INLET Q=0
180 DO 160 J=2, NSTRM-1
181 DSDM(J,1) = DISSIP(J,1)/(RHO(J,1) * TEMP(J,1) * VELM(J,1))
182 *****0*****0*****0*****0*****0*****0*****0*****
183 DHQM(J,1) = 2.000 * RADIUS(J,1)**2 * VTHETA(J,1)/VELM(J,1)
184 *****0*****0*****0*****0*****0*****0*****0*****
185 DHQM(J,1) = TEMP(J,1) * DSDM(J,1)
186 *****0*****0*****0*****0*****0*****0*****0*****
187 ( EZ(J,1) * COS(PI(J,1)/RHO(J,1)) ) +
188 DHQM(J,1)/(2.000 * RADIUS(J,1)**2)
189
190 CONTINUE
191
192 *****0*****0*****0*****0*****0*****0*****0*****
193 ENDWALL STREAMLINES
194 *****0*****0*****0*****0*****0*****0*****0*****
195 DSDM FOR THE ENDWALL STREAMLINES - NO HEAT TRANSFER AT ENDWALLS
196 DSDM(1,1) = DISSIP(1,1)/(RHO(1,1) * TEMP(1,1) * VELM(1,1))
197 *****0*****0*****0*****0*****0*****0*****0*****
198 DSDM(1,1) = DISSIP(NSTRM,1)/
199 (RHO(NSTRM,1) * TEMP(NSTRM,1) * VELM(NSTRM,1))
200
201 *****0*****0*****0*****0*****0*****0*****0*****
202 COMPUTE DHQM FOR THE ENDWALL STREAMLINES
203 *****0*****0*****0*****0*****0*****0*****0*****
204 DHQM(1,1) = 2.000 * RADIUS(1,1)**2 * VTHETA(1,1)/VELM(1,1)
205 *****0*****0*****0*****0*****0*****0*****0*****
206 DHQM(1,1) = TEMP(1,1) * DSDM(1,1)
207 *****0*****0*****0*****0*****0*****0*****0*****
208 VTHETA(NSTRM,1) = 2.000 * RADIUS(NSTRM,1)**2 *
209 VTHETA(NSTRM,1)/VELM(NSTRM,1)
210 *****0*****0*****0*****0*****0*****0*****0*****
211 (ETHETA(NSTRM,1)/RHO(NSTRM,1))
212 *****0*****0*****0*****0*****0*****0*****0*****
213 DHQM(NSTRM,1) = TEMP(NSTRM,1) * DSDM(NSTRM,1)
214 *****0*****0*****0*****0*****0*****0*****0*****
215 DHQM(NSTRM,1) = 2.000 * RADIUS(NSTRM,1) * DSDM(NSTRM,1)
216 *****0*****0*****0*****0*****0*****0*****0*****
217 DHQM(NSTRM,1) = 2.000 * RADIUS(NSTRM,1)**2 *
218 DHQM(NSTRM,1)/(2.000 * RADIUS(NSTRM,1))
219 *****0*****0*****0*****0*****0*****0*****0*****
220 COMPUTE DHQM FOR THE ENDWALL STREAMLINES
221 *****0*****0*****0*****0*****0*****0*****0*****
222 DHQM(1,1) = TEMP(1,1) * DSDM(1,1)
223 *****0*****0*****0*****0*****0*****0*****0*****
224 DHQM(1,1) = 2.000 * RADIUS(1,1) * DSDM(1,1)
225 *****0*****0*****0*****0*****0*****0*****0*****
226 DHQM(1,1) = 2.000 * RADIUS(1,1)**2 *
227 DHQM(1,1)/(2.000 * RADIUS(1,1))
228 *****0*****0*****0*****0*****0*****0*****0*****
229 DHQM(1,1) = 2.000 * RADIUS(1,1)**2 *
230 DHQM(1,1)/(2.000 * RADIUS(1,1))
231 *****0*****0*****0*****0*****0*****0*****0*****
232 DHQM(1,1) = 2.000 * RADIUS(1,1)**2 *
233 DHQM(1,1)/(2.000 * RADIUS(1,1))
234 *****0*****0*****0*****0*****0*****0*****0*****
235 DHQM(1,1) = 2.000 * RADIUS(1,1)**2 *
236 DHQM(1,1)/(2.000 * RADIUS(1,1))
237 *****0*****0*****0*****0*****0*****0*****0*****
238 DHQM(1,1) = 2.000 * RADIUS(1,1)**2 *
239 DHQM(1,1)/(2.000 * RADIUS(1,1))
240 *****0*****0*****0*****0*****0*****0*****0*****
241 DHQM(1,1) = 2.000 * RADIUS(1,1)**2 *
242 DHQM(1,1)/(2.000 * RADIUS(1,1))
243 *****0*****0*****0*****0*****0*****0*****0*****
244 DHQM(1,1) = 2.000 * RADIUS(1,1)**2 *
245 DHQM(1,1)/(2.000 * RADIUS(1,1))
246 *****0*****0*****0*****0*****0*****0*****0*****
247 DHQM(1,1) = 2.000 * RADIUS(1,1)**2 *
248 DHQM(1,1)/(2.000 * RADIUS(1,1))
249 *****0*****0*****0*****0*****0*****0*****0*****
250 DHQM(1,1) = 2.000 * RADIUS(1,1)**2 *
251 DHQM(1,1)/(2.000 * RADIUS(1,1))
252 *****0*****0*****0*****0*****0*****0*****0*****
253 DHQM(1,1) = 2.000 * RADIUS(1,1)**2 *
254 DHQM(1,1)/(2.000 * RADIUS(1,1))
255 *****0*****0*****0*****0*****0*****0*****0*****
256 DHQM(1,1) = 2.000 * RADIUS(1,1)**2 *
257 DHQM(1,1)/(2.000 * RADIUS(1,1))
258 *****0*****0*****0*****0*****0*****0*****0*****
259 DHQM(1,1) = 2.000 * RADIUS(1,1)**2 *
260 DHQM(1,1)/(2.000 * RADIUS(1,1))
261 *****0*****0*****0*****0*****0*****0*****0*****
262 DHQM(1,1) = 2.000 * RADIUS(1,1)**2 *
263 DHQM(1,1)/(2.000 * RADIUS(1,1))
264 *****0*****0*****0*****0*****0*****0*****0*****
265 DHQM(1,1) = 2.000 * RADIUS(1,1)**2 *
266 DHQM(1,1)/(2.000 * RADIUS(1,1))
267 *****0*****0*****0*****0*****0*****0*****0*****
268 DHQM(1,1) = 2.000 * RADIUS(1,1)**2 *
269 DHQM(1,1)/(2.000 * RADIUS(1,1))
270 *****0*****0*****0*****0*****0*****0*****0*****
271 DHQM(1,1) = 2.000 * RADIUS(1,1)**2 *
272 DHQM(1,1)/(2.000 * RADIUS(1,1))
273 *****0*****0*****0*****0*****0*****0*****0*****
274 DHQM(1,1) = 2.000 * RADIUS(1,1)**2 *
275 DHQM(1,1)/(2.000 * RADIUS(1,1))
276 *****0*****0*****0*****0*****0*****0*****0*****
277 DHQM(1,1) = 2.000 * RADIUS(1,1)**2 *
278 DHQM(1,1)/(2.000 * RADIUS(1,1))
279 *****0*****0*****0*****0*****0*****0*****0*****
280 DHQM(1,1) = 2.000 * RADIUS(1,1)**2 *
281 DHQM(1,1)/(2.000 * RADIUS(1,1))
282 *****0*****0*****0*****0*****0*****0*****0*****
283 DHQM(1,1) = 2.000 * RADIUS(1,1)**2 *
284 DHQM(1,1)/(2.000 * RADIUS(1,1))
285 *****0*****0*****0*****0*****0*****0*****0*****
286 DHQM(1,1) = 2.000 * RADIUS(1,1)**2 *
287 DHQM(1,1)/(2.000 * RADIUS(1,1))
288 *****0*****0*****0*****0*****0*****0*****0*****
289 DHQM(1,1) = 2.000 * RADIUS(1,1)**2 *
290 DHQM(1,1)/(2.000 * RADIUS(1,1))
291 *****0*****0*****0*****0*****0*****0*****0*****
292 DHQM(1,1) = 2.000 * RADIUS(1,1)**2 *
293 DHQM(1,1)/(2.000 * RADIUS(1,1))
294 *****0*****0*****0*****0*****0*****0*****0*****
295 DHQM(1,1) = 2.000 * RADIUS(1,1)**2 *
296 DHQM(1,1)/(2.000 * RADIUS(1,1))
297 *****0*****0*****0*****0*****0*****0*****0*****
298 DHQM(1,1) = 2.000 * RADIUS(1,1)**2 *
299 DHQM(1,1)/(2.000 * RADIUS(1,1))
300 *****0*****0*****0*****0*****0*****0*****0*****
301 DHQM(1,1) = 2.000 * RADIUS(1,1)**2 *
302 DHQM(1,1)/(2.000 * RADIUS(1,1))
303 *****0*****0*****0*****0*****0*****0*****0*****
304 DHQM(1,1) = 2.000 * RADIUS(1,1)**2 *
305 DHQM(1,1)/(2.000 * RADIUS(1,1))
306 *****0*****0*****0*****0*****0*****0*****0*****
307 DHQM(1,1) = 2.000 * RADIUS(1,1)**2 *
308 DHQM(1,1)/(2.000 * RADIUS(1,1))
309 *****0*****0*****0*****0*****0*****0*****0*****
310 DHQM(1,1) = 2.000 * RADIUS(1,1)**2 *
311 DHQM(1,1)/(2.000 * RADIUS(1,1))
312 *****0*****0*****0*****0*****0*****0*****0*****
313 DHQM(1,1) = 2.000 * RADIUS(1,1)**2 *
314 DHQM(1,1)/(2.000 * RADIUS(1,1))
315 *****0*****0*****0*****0*****0*****0*****0*****
316 DHQM(1,1) = 2.000 * RADIUS(1,1)**2 *
317 DHQM(1,1)/(2.000 * RADIUS(1,1))
318 *****0*****0*****0*****0*****0*****0*****0*****
319 DHQM(1,1) = 2.000 * RADIUS(1,1)**2 *
320 DHQM(1,1)/(2.000 * RADIUS(1,1))
321 *****0*****0*****0*****0*****0*****0*****0*****
322 DHQM(1,1) = 2.000 * RADIUS(1,1)**2 *
323 DHQM(1,1)/(2.000 * RADIUS(1,1))
324 *****0*****0*****0*****0*****0*****0*****0*****
325 DHQM(1,1) = 2.000 * RADIUS(1,1)**2 *
326 DHQM(1,1)/(2.000 * RADIUS(1,1))
327 *****0*****0*****0*****0*****0*****0*****0*****
328 DHQM(1,1) = 2.000 * RADIUS(1,1)**2 *
329 DHQM(1,1)/(2.000 * RADIUS(1,1))
330 *****0*****0*****0*****0*****0*****
```

```
289 .....0.....0.....0.....0.....0.....0.....0.....0.....0.....0..
290 .....0.....0.....0.....0.....0.....0.....0.....0.....0.....0..
291 RETURN
292 END
293
294
295
296
297
298
299
300
```



```

145 * WRITE THE DATA FROM SUBROUTINE ESTIM TO THE DATA FILE output2.dat
146 .....0.....0.....0.....0.....0.....0.....0.....0.....0...
147 IF (LP EQ 3 AND I EQ NSTNS OR
148 4 LP EQ 20 AND I EQ NSTNS) THEN
149 WRITE(12,*)
150 WRITE(12,*)
151 WRITE(12,*)
152 WRITE(12,*)
153 WRITE(12,*)
154 WRITE(12,*)
155 WRITE(12,*)
156 DO 210 J=NSTRM,1,-1
157 WRITE(12,1003) J, DTUS(J,K), K=1,1)
158 FORMAT(1X,12,2X,(P8.4,2X))
159 CONTINUE
160
161 WRITE(12,*)
162 WRITE(12,*)
163 WRITE(12,*)
164 WRITE(12,*)
165 DO 220 J=NSTRM,1,-1
166 WRITE(12,1005) J, (DELTA(J,K), K=1,1)
167 FORMAT(1X,12,2X,20(E10.4,2X))
168 CONTINUE
169
170 WRITE(12,*)
171 WRITE(12,*)
172 WRITE(12,*)
173 WRITE(12,*)
174 DO 230 J=NSTRM,1,-1
175 WRITE(12,1003) J, (VELM(J,K), K=1,1)
176 FORMAT(1X,12,2X,20(E10.4,2X))
177 CONTINUE
178
179 WRITE(12,*)
180 WRITE(12,*)
181 WRITE(12,*)
182 WRITE(12,*)
183 DO 240 J=NSTRM,1,-1
184 WRITE(12,1003) J, (ZVEL(J,K), K=1,1)
185 FORMAT(1X,12,2X,20(E10.4,2X))
186 CONTINUE
187
188 WRITE(12,*)
189 WRITE(12,*)
190 WRITE(12,*)
191 WRITE(12,*)
192 DO 250 J=NSTRM,1,-1
193 WRITE(12,1003) J, (VTHETA(J,K), K=1,1)
194 FORMAT(1X,12,2X,20(E14.7,2X))
195 CONTINUE
196
197 WRITE(12,*)
198 WRITE(12,*)
199 WRITE(12,*)
200 WRITE(12,*)
201 DO 260 J=NSTRM,1,-1
202 WRITE(12,1003) J, (STATIC ENTHALPY (ft2/sec2))
203 FORMAT(1X,12,2X,20(E14.7,2X))
204 CONTINUE
205
206 WRITE(12,*)
207 WRITE(12,*)
208 WRITE(12,*)
209 WRITE(12,*)
210 DO 270 J=NSTRM,1,-1
211 WRITE(12,1003) J, (STATIC PRESSURE (lbf/ft2))
212 FORMAT(1X,12,2X,20(E14.7,2X))
213 CONTINUE
214
215 WRITE(12,*)
216 WRITE(12,*)
217 WRITE(12,*)
218 WRITE(12,*)
219 DO 280 J=NSTRM,1,-1
220 WRITE(12,1003) J, (STATIC TEMPERATURE (R))
221 FORMAT(1X,12,2X,20(E14.7,2X))
222 CONTINUE
223
224 WRITE(12,*)
225 WRITE(12,*)
226 WRITE(12,*)
227 WRITE(12,*)
228 DO 290 J=NSTRM,1,-1
229 WRITE(12,1003) J, (HEAT TRANSFER COEFFICIENT (lbf/sec R))
230 FORMAT(1X,12,2X,20(E14.7,2X))
231 CONTINUE
232
233 WRITE(12,*)
234 WRITE(12,*)
235 WRITE(12,*)
236 WRITE(12,*)
237 DO 300 J=NSTRM,1,-1
238 WRITE(12,1003) J, (HTCOEF(J,K), K=1,1)
239 FORMAT(1X,12,2X,20(E14.7,2X))
240 CONTINUE
241
242 WRITE(12,*)
243 WRITE(12,*)
244 WRITE(12,*)
245 WRITE(12,*)
246 DO 310 J=NSTRM,1,-1
247 WRITE(12,1011) J, (MUT(J,K), K=1,1)
248 FORMAT(1X,12,2X,20(E14.7,2X))
249 CONTINUE
250
251 WRITE(12,*)
252 WRITE(12,*)
253 WRITE(12,*)
254 WRITE(12,*)
255 DO 320 J=NSTRM,1,-1
256 WRITE(12,1011) J, (MUT(J,K), K=1,1)
257 FORMAT(1X,12,2X,20(E14.7,2X))
258 CONTINUE
259
260 WRITE(12,*)
261 WRITE(12,*)
262 WRITE(12,*)
263 WRITE(12,*)
264 DO 330 J=NSTRM,1,-1
265 WRITE(12,1011) J, (MUT(J,K), K=1,1)
266 FORMAT(1X,12,2X,20(E14.7,2X))
267 CONTINUE
268
269 WRITE(12,*)
270 WRITE(12,*)
271 WRITE(12,*)
272 WRITE(12,*)
273 DO 340 J=NSTRM,1,-1
274 WRITE(12,1011) J, (MUT(J,K), K=1,1)
275 FORMAT(1X,12,2X,20(E14.7,2X))
276 CONTINUE
277
278 WRITE(12,*)
279 WRITE(12,*)
280 WRITE(12,*)
281 WRITE(12,*)
282 DO 350 J=NSTRM,1,-1
283 WRITE(12,1011) J, (MUT(J,K), K=1,1)
284 FORMAT(1X,12,2X,20(E14.7,2X))
285 CONTINUE
286
287 WRITE(12,*)
288 WRITE(12,*)
289 WRITE(12,*)
290 WRITE(12,*)
291 DO 360 J=NSTRM,1,-1
292 WRITE(12,1011) J, (MUT(J,K), K=1,1)
293 FORMAT(1X,12,2X,20(E14.7,2X))
294 CONTINUE
295
296 WRITE(12,*)
297 WRITE(12,*)
298 WRITE(12,*)
299 WRITE(12,*)
300 DO 370 J=NSTRM,1,-1
301 WRITE(12,1011) J, (MUT(J,K), K=1,1)
302 FORMAT(1X,12,2X,20(E14.7,2X))
303 CONTINUE
304
305 WRITE(12,*)
306 WRITE(12,*)
307 WRITE(12,*)
308 WRITE(12,*)
309 DO 380 J=NSTRM,1,-1
310 WRITE(12,1011) J, (MUT(J,K), K=1,1)
311 FORMAT(1X,12,2X,20(E14.7,2X))
312 CONTINUE
313
314 WRITE(12,*)
315 WRITE(12,*)
316 WRITE(12,*)
317 WRITE(12,*)
318 DO 390 J=NSTRM,1,-1
319 WRITE(12,1011) J, (MUT(J,K), K=1,1)
320 FORMAT(1X,12,2X,20(E14.7,2X))
321 CONTINUE
322
323 WRITE(12,*)
324 WRITE(12,*)
325 WRITE(12,*)
326 WRITE(12,*)
327 DO 400 J=NSTRM,1,-1
328 WRITE(12,1011) J, (MUT(J,K), K=1,1)
329 FORMAT(1X,12,2X,20(E14.7,2X))
330 CONTINUE
331
332 WRITE(12,*)
333 WRITE(12,*)
334 WRITE(12,*)
335 WRITE(12,*)
336 DO 410 J=NSTRM,1,-1
337 WRITE(12,1011) J, (MUT(J,K), K=1,1)
338 FORMAT(1X,12,2X,20(E14.7,2X))
339 CONTINUE
340
341 WRITE(12,*)
342 WRITE(12,*)
343 WRITE(12,*)
344 WRITE(12,*)
345 DO 420 J=NSTRM,1,-1
346 WRITE(12,1011) J, (MUT(J,K), K=1,1)
347 FORMAT(1X,12,2X,20(E14.7,2X))
348 CONTINUE
349
350 WRITE(12,*)
351 WRITE(12,*)
352 WRITE(12,*)
353 WRITE(12,*)
354 DO 430 J=NSTRM,1,-1
355 WRITE(12,1011) J, (MUT(J,K), K=1,1)
356 FORMAT(1X,12,2X,20(E14.7,2X))
357 CONTINUE
358
359 WRITE(12,*)
360 WRITE(12,*)
361 WRITE(12,*)
362 WRITE(12,*)
363 DO 440 J=NSTRM,1,-1
364 WRITE(12,1011) J, (MUT(J,K), K=1,1)
365 FORMAT(1X,12,2X,20(E14.7,2X))
366 CONTINUE
367
368 WRITE(12,*)
369 WRITE(12,*)
370 WRITE(12,*)
371 WRITE(12,*)
372 DO 450 J=NSTRM,1,-1
373 WRITE(12,1011) J, (MUT(J,K), K=1,1)
374 FORMAT(1X,12,2X,20(E14.7,2X))
375 CONTINUE
376
377 WRITE(12,*)
378 WRITE(12,*)
379 WRITE(12,*)
380 WRITE(12,*)
381 DO 460 J=NSTRM,1,-1
382 WRITE(12,1011) J, (MUT(J,K), K=1,1)
383 FORMAT(1X,12,2X,20(E14.7,2X))
384 CONTINUE
385
386 WRITE(12,*)
387 WRITE(12,*)
388 WRITE(12,*)
389 WRITE(12,*)
390 DO 470 J=NSTRM,1,-1
391 WRITE(12,1011) J, (MUT(J,K), K=1,1)
392 FORMAT(1X,12,2X,20(E14.7,2X))
393 CONTINUE
394
395 WRITE(12,*)
396 WRITE(12,*)
397 WRITE(12,*)
398 WRITE(12,*)
399 DO 480 J=NSTRM,1,-1
400 WRITE(12,1011) J, (MUT(J,K), K=1,1)
401 FORMAT(1X,12,2X,20(E14.7,2X))
402 CONTINUE
403
404 WRITE(12,*)
405 WRITE(12,*)
406 WRITE(12,*)
407 WRITE(12,*)
408 DO 490 J=NSTRM,1,-1
409 WRITE(12,1011) J, (MUT(J,K), K=1,1)
410 FORMAT(1X,12,2X,20(E14.7,2X))
411 CONTINUE
412
413 WRITE(12,*)
414 WRITE(12,*)
415 WRITE(12,*)
416 WRITE(12,*)
417 DO 500 J=NSTRM,1,-1
418 WRITE(12,1011) J, (MUT(J,K), K=1,1)
419 FORMAT(1X,12,2X,20(E14.7,2X))
420 CONTINUE
421
422 WRITE(12,*)
423 WRITE(12,*)
424 WRITE(12,*)
425 WRITE(12,*)
426 DO 510 J=NSTRM,1,-1
427 WRITE(12,1011) J, (MUT(J,K), K=1,1)
428 FORMAT(1X,12,2X,20(E14.7,2X))
429 CONTINUE
430
431 WRITE(12,*)
432 WRITE(12,*)
433 WRITE(12,*)
434 WRITE(12,*)
435 DO 520 J=NSTRM,1,-1
436 WRITE(12,1011) J, (MUT(J,K), K=1,1)
437 FORMAT(1X,12,2X,20(E14.7,2X))
438 CONTINUE
439
440 WRITE(12,*)
441 WRITE(12,*)
442 WRITE(12,*)
443 WRITE(12,*)
444 DO 530 J=NSTRM,1,-1
445 WRITE(12,1011) J, (MUT(J,K), K=1,1)
446 FORMAT(1X,12,2X,20
```



```

1  SUBROUTINE CURVE(CURV, I, NSTNS, NSTRM, PHI, RADIUS, RVEL, VELM,
2  ZDELTA, ZVEL)
3
4  *
5  * CURVE I
6  *
7  * PROGRAM VERSION: WHIRL, VARIABLE WALL GEOMETRY
8  *
9  * THIS SUBROUTINE CALCULATES THE CURVATURE, CURV, THE FLOW ANGLE OF
10 * THE FLOW FIELD, PHI, THE Z AND R DIRECTION VELOCITIES, ZVEL AND RVEL,
11 *
12 * INPUT VARIABLES: I, NSTNS, NSTRM, RADIUS, VELM
13 *
14 * OUTPUT VARIABLES: CURV, PHI, RVEL, ZVEL
15 *
16 * VARIABLES INTERNAL TO THE SUBROUTINE ARE: DRDZ AND DRDZ2..
17 *
18 * CURV HAS UNITS OF 1/ft
19 * PHI HAS UNITS OF RADIANS
20 * RVEL AND ZVEL HAVE UNITS OF ft/sec
21 *
22 * DOUBLE PRECISION CP, CURV(51,200), DRDZ(51,200), DRDZ2(51,200),
23 * GAMMA, GASCON, GSURC
24 * MICO, PHI(51,200), PI, RADIUS(51,200), RVEL(51,200),
25 * VELM(51,200), ZDELTA(200), ZVEL(51,200)
26 *
27 INTEGER COUNT, I, LLIMIT, LP, LLIMIT, NSTEP, NSTNS, NSTRM
28
29 COMMON /CONST/ CP, GAMMA, GASCON, GSURC, MICO, PI
30 COMMON /CTRL1/ COUNT, LLIMIT, LP, LLIMIT, NSTEP
31
32 * COMPUTE THE STREAMLINE CURVATURE NOTE THAT THESE ARE Z DIRECTION
33 * DERIVATIVES
34 *
35 * FOR LAST q-o
36 IF(I EQ NSTNS) THEN
37
38 IF(NSTNS EQ 2) THEN
39
40 * SIMPLE BACKWARD DIFFERENCE FOR LAST q-o WITH NSTNS = 2
41 DO 10 J=1, NSTRM
42 DRDZ(J, NSTNS) = (RADIUS(J, NSTNS) - RADIUS(J, NSTNS-1)) /
43 ZDELTA(NSTNS)
44
45 * ASSUME LINEAR 2ND DERIVATIVE IF ONLY 2 q-os EXIST.
46 DRDZ2(J, NSTNS) = 0.0000
47
48 * COMPUTE CURVATURE FOR LAST q-o. CURV HAS UNITS OF 1/ft
49 CURV(J, NSTNS) = DRDZ2(J, NSTNS)
50 ((1.000 + DRDZ(J, NSTNS)**2)**1.500)
51
52 10 CONTINUE
53
54 ELSE
55 * USE POLYNOMIAL DIFFERENCE FOR LAST q-o WITH NSTNS > 2
56 DO 20 J=1, NSTRM
57 DRDZ(J, NSTNS) = (RADIUS(J, NSTNS) - RADIUS(J, NSTNS-1)) /
58 ZDELTA(NSTNS)
59 ((ZDELTA(NSTNS) / ZDELTA(NSTNS-1)) *
60 ((RADIUS(J, NSTNS) - RADIUS(J, NSTNS-2)) /
61 (ZDELTA(NSTNS) + ZDELTA(NSTNS-1)) *
62 (RADIUS(J, NSTNS) - RADIUS(J, NSTNS-1)) /
63 ZDELTA(NSTNS-1))
64 DRDZ2(J, NSTNS) =
65 2.000 * (RADIUS(J, NSTNS-2) - RADIUS(J, NSTNS)) /
66 ((ZDELTA(NSTNS) + ZDELTA(NSTNS-1)) *
67 (ZDELTA(NSTNS) - RADIUS(J, NSTNS-1) - RADIUS(J, NSTNS)) /
68 2.000 * (ZDELTA(NSTNS-1) + ZDELTA(NSTNS))
69 ((ZDELTA(NSTNS-1) + ZDELTA(NSTNS))
70 COMPUTE CURVE FOR LAST q-o. CURV HAS UNITS OF 1/ft
71 CURV(J, NSTNS) = DRDZ2(J, NSTNS) /
72 ((1.000 + DRDZ(J, NSTNS)**2)**1.5)
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20 CONTINUE
21 ENDIF
22 ELSE
23
24 * NOT AT END OF FLOW FIELD, NSTNS > 2, SO FOR MIDDLE
25 * q-o REGION CALCULATE THE DERIVATIVES USING SUBROUTINE DERIV.
26 DO 30 J=1, NSTRM
27 U(J) = RADIUS(J, I)
28 X(J) = ZLOC(J, I)
29 CONTINUE
30
31 CALL DERIV(U, X, NSTRM, DUDX, DZUDX2)
32
33 DO 40 J=1, NSTRM
34 DZUDR(J, I) = DUDX(J)
35 DRDZ2(J, I) = DZUDX2(J)
36
37 CURV(J, I) = DRDZ2(J, I) /
38 ((1.000 + DRDZ(J, I)**2)**1.500)
39
40 CONTINUE
41 ENDIF
42
43 *
44 * THE MERIDIONAL FLOW ANGLE (PHI) HAS UNITS OF RADIANS.
45 * PHI FOR THE LAST COMPUTING STATION
46 IF(I EQ NSTNS) THEN
47 DO 50 J=1, NSTRM
48 PHI(J, NSTNS) = ATAN2((RADIUS(J, NSTNS) - RADIUS(J, NSTNS-1)),
49 ZDELTA(NSTNS))
50 CONTINUE
51
52 ELSE
53 * PHI FOR THE MIDDLE BLOCK OF THE FLOW FIELD
54 DO 60 J=1, NSTRM
55 PHI(J, I) = 0.500 *
56 (ATAN2((RADIUS(J, I+1) - RADIUS(J, I)), ZDELTA(I+1)) +
57 ATAN2((RADIUS(J, I) - RADIUS(J, I-1)), ZDELTA(I)))
58 CONTINUE
59 ENDIF
60
61 *
62 * FIND THE Z AND R COMPONENTS OF THE VELOCITY. ZVEL AND RVEL HAVE
63 * UNITS OF ft/sec
64 * IF THIS IS THE 1ST ITERATION, ZVEL IS KNOWN FROM ESTIM AND INLET.
65 * VELM AND RVEL ARE UNKNOWN.
66 * IF NOT THE FIRST ITERATION, VELM IS KNOWN FROM VELO. ZVEL AND RVEL
67 * ARE KNOWN FROM THE LATEST VELM VALUES.
68 *
69 DO 90 J=1, NSTRM
70 IF(COUNT EQ 1) THEN
71 VELM(J, I) = ZVEL(J, I) / COS(PHI(J, I))
72 RVEL(J, I) = ZVEL(J, I) * TAN(PHI(J, I))
73 ENDIF
74 ZVEL(J, I) = VELM(J, I) * COS(PHI(J, I))
75 RVEL(J, I) = VELM(J, I) * SIN(PHI(J, I))
76 CONTINUE
77
78 *
79 * DATA FROM SUBROUTINE CURVE
80 IF(LP EQ 3 AND I EQ NSTNS OR
81 LP EQ 20 AND I EQ NSTNS) THEN
82 WRITE(13, *) 'q-o dir version I = ', I
83 WRITE(13, *)
84
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145 * AND TEMPERATURE, THE NEW PRESSURE AND DENSITY MAY BE FOUND.
146 * MDLTA HAS UNITS OF ft
147 * HSTAG AND ENTHAL HAVE UNITS OF Btu/lbm
148 * TSTAG AND TEMP HAVE UNITS OF R
149 * ENTROP HAS UNITS OF Btu/lbm R
150 * PRESS AND PSTAG HAVE UNITS OF lbf/ft^2
151 * DENSITY HAS UNITS OF lbm/ft^3
152 * WUT HAS UNITS OF lbf sec/ft^2
153 *****
154 DO 130 J=1,NSTRM
155 *****
156 * VARIABLE WHIRL AND VTHETA UPDATED
157 WHIRL(J,1) = WHIRL(J,1) + MDLTA(J,1) * DHODM(J,1)
158 WHIRL(J,1) = OLDWH(J) + RELAXM * (WHIRL(J,1) - OLDWH(J))
159
160 IF (WHIRL(J,1) .LT. 0.001) THEN
161 PRINT*, 'WHIRL AT Q=0', I, 'STREAMLINE', J, ' IS: ',
162 4 WHIRL(J,1)
163 WHIRL(J,1) = WHIRL(J,1)/2.000
164 PRINT*, ' '
165 ENDIF
166
167 * COMPUTE NEW VTHETA
168 VTHETA(J,1) = SORT(WHIRL(J,1)/RADIUS(J,1))
169
170
171
172 * ENTROPY UPDATED
173 ENTROP(J,1) = ENTROP(J,1-1) + MDLTA(J,1) * DSDM(J,1)
174 IF (ENTROP(J,1) .LE. 0.001) THEN
175 PRINT*, 'ENTROPY AT Q=0', I, 'STREAMLINE', J, ' IS: ',
176 4 ENTROP(J,1)
177 ENTROP(J,1) = ENTROP(J,1)/2.000
178 PRINT*, 'NEG ENTROP BEING SET TO VALUE OF', ENTROP(J,1)
179 PRINT*, ' '
180
181
182 * STAGNATION ENTHALPY UPDATED
183 HSTAG(J,1) = HSTAG(J,1-1) + MDLTA(J,1) * DHODM(J,1)
184 IF (HSTAG(J,1) .LE. 0.001) THEN
185 PRINT*, 'HSTAG AT Q=0', I, 'STREAMLINE', J, ' IS: ',
186 4 HSTAG(J,1)
187 HSTAG(J,1) = HSTAG(J,1)/2.000
188 PRINT*, 'STAG ENTHAL BEING SET TO VALUE OF', HSTAG(J,1)
189 PRINT*, ' '
190
191 * THE REST OF THE FLOW FIELD VARIABLES ARE NOW UPDATED
192 TSTAG(J,1) = HSTAG(J,1)/CP
193
194
195
196
197
198
199
200
201 * (VELM(J,1)**2 + VTHETA(J,1)**2)/2.000
202 IF (ENTHAL(J,1) .LE. 0.001) THEN
203 ENTHAL(J,1) = ENTHAL(J,1)/2.000
204 PRINT*, 'ENTHAL BEING SET TO VALUE OF', ENTHAL(J,1-1)
205 PRINT*, ' '
206
207
208
209
210
211
212 * NOTE
213 ERRORS RESULT IF G IS RAISED TO A LARGE NEGATIVE NUMBER
214 PRESS(J,1) = TEMP(J,1)**(CP/GASCON) * EXP(-ENTROP(J,1)/GASCON)
215 PSTAG(J,1) = PRESS(J,1) * (TSTAG(J,1)/TEMP(J,1))**
216 4 (GAMMA/(GAMMA - 1.000))

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VELTOT(J,1) = SORT(VELM(J,1)**2 + VTHETA(J,1)**2)
RHO(J,1) = (PRESS(J,1) * GSUBC)/(GASCON * TEMP(J,1))
IF (RHO(J,1) .LE. 0.0001) THEN
PRINT*, 'RHO UPDATE ERROR IN GRADIENT'
RHO(J,1) = RHO(J,1)
ENDIF
WUT(J,1) = RHO(J,1) * MIXCO
WTCOEF(J,1) = WUT(J,1) * CP
130 CONTINUE
*****
* ONCE THE NEW HSTAG AND ENTROP HAVE BEEN CALCULATED, THE RADIAL
* GRADIENTS OF STAGNATION ENTHALPY, ENTROPY, AND WHIRL CAN BE FOUND.
*****
DO 150 J=1,NSTRM
U(J) = HSTAG(J,1)
X(J) = RADIUS(J,1)
150 CONTINUE
CALL DERIV(U, X, NSTRM, DUDX, D2UDX2)
DO 160 J=1,NSTRM
DHODQ(J,1) = DUDX(J)
U(J) = ENTROP(J,1)
X(J) = RADIUS(J,1)
160 CONTINUE
CALL DERIV(U, X, NSTRM, DUDX, D2UDX2)
DO 170 J=1,NSTRM
DSDQ(J,1) = DUDX(J)
U(J) = WHIRL(J,1)
X(J) = RADIUS(J,1)
170 CONTINUE
DO 180 J=1,NSTRM
DHODQ(J,1) = DUDX(J)
*****
* DEBUG DATA FROM SUBROUTINE GRADIENT
* PUT THE RESULTS OF THE GRADIENT SUBROUTINE CALCS INTO A FILE SO I
* CAN WONDER WHY THE DAMN THING ISN'T WORKING.
*****
IF (LP .EQ. 5 .AND. I .EQ. NSTNS .OR.
4 LP .EQ. 20 .AND. I .EQ. NSTNS) THEN
WRITE(15,*) ' '
WRITE(15,*) 'gsl-dir version I =', I
WRITE(15,*) ' '
WRITE(15,*) ' '
WRITE(15,*) 'DATA FROM THE GRADIENT SUBROUTINE: outputs.dat'
WRITE(15,*) 'ITERATION COUNT IS: ', COUNT
WRITE(15,*) ' '
WRITE(15,*) ' '
WRITE(15,*) 'J MDLTA'
DO 250 J=NSTRM,1,-NSTEP
WRITE(15,1015) J, (MDLTA(J,K), K=1,NSTNS)
FORMAT(1X, 12, 2X, 20(F16.4, 2X))
250 CONTINUE
WRITE(15,*) ' '
WRITE(15,*) ' '
WRITE(15,*) 'J DWDM'
DO 260 J=NSTRM,1,-NSTEP

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289      WRITE(15,1015) J, (DHQDM(J,K), K=1,NSTNS)
290      CONTINUE
291
292      WRITE(15,*) ' '
293      WRITE(15,*) ' '
294      WRITE(15,*) ' '
295      WRITE(15,*) ' '
296      DO 270 J=NSTRM,1,-NSTEP
297        WRITE(15,1015) J, (TEMP(J,K), K=1,NSTNS)
298      CONTINUE
299
300      WRITE(15,*) ' '
301      WRITE(15,*) ' '
302      WRITE(15,*) ' '
303      WRITE(15,*) ' '
304      DO 280 J=NSTRM,1,-NSTEP
305        WRITE(15,1015) J, (DTQ(J,K), K=1,NSTNS)
306      CONTINUE
307
308      WRITE(15,*) ' '
309      WRITE(15,*) ' '
310      WRITE(15,*) ' '
311      WRITE(15,*) ' '
312      DO 300 J=NSTRM,1,-NSTEP
313        WRITE(15,1015) J, (DSQM(J,K), K=1,NSTNS)
314      CONTINUE
315
316      WRITE(15,*) ' '
317      WRITE(15,*) ' '
318      WRITE(15,*) ' '
319      WRITE(15,*) ' '
320      DO 310 J=NSTRM,1,-NSTEP
321        WRITE(15,1017) J, (ENTROP(J,K), K=1,NSTNS)
322      CONTINUE
323
324      WRITE(15,*) ' '
325      WRITE(15,*) ' '
326      WRITE(15,*) ' '
327      WRITE(15,*) ' '
328      DO 320 J=NSTRM,1,-NSTEP
329        WRITE(15,1017) J, (DSQ(J,K), K=1,NSTNS)
330      CONTINUE
331
332      WRITE(15,*) ' '
333      WRITE(15,*) ' '
334      WRITE(15,*) ' '
335      WRITE(15,*) ' '
336      DO 330 J=NSTRM,1,-NSTEP
337        WRITE(15,1017) J, (DHQDM(J,K), K=1,NSTNS)
338      CONTINUE
339
340      WRITE(15,*) ' '
341      WRITE(15,*) ' '
342      WRITE(15,*) ' '
343      WRITE(15,*) ' '
344      DO 340 J=NSTRM,1,-NSTEP
345        WRITE(15,1019) J, (ENTHAL(J,K), K=1,NSTNS)
346      CONTINUE
347
348      WRITE(15,*) ' '
349      WRITE(15,*) ' '
350      WRITE(15,*) ' '
351      WRITE(15,*) ' '
352      DO 350 J=NSTRM,1,-NSTEP
353        WRITE(15,1019) J, (DHQDQ(J,K), K=1,NSTNS)
354      CONTINUE
355
356      WRITE(15,*) ' '
357      WRITE(15,*) ' '
358      WRITE(15,*) ' '
359      WRITE(15,*) ' '
360
361      WRITE(15,*) ' '
362      DO 400 J=NSTRM,1,-NSTEP
363        WRITE(15,1021) J, (DHQDM(J,K), K=1,NSTNS)
364      CONTINUE
365
366      WRITE(15,*) ' '
367      WRITE(15,*) ' '
368      WRITE(15,*) ' '
369      WRITE(15,*) ' '
370      DO 410 J=NSTRM,1,-NSTEP
371        WRITE(15,1021) J, (WHIRL(J,K), K=1,NSTNS)
372      CONTINUE
373
374      WRITE(15,*) ' '
375      WRITE(15,*) ' '
376      WRITE(15,*) ' '
377      WRITE(15,*) ' '
378      DO 420 J=NSTRM,1,-NSTEP
379        WRITE(15,1021) J, (DHQDQ(J,K), K=1,NSTNS)
380      CONTINUE
381
382      WRITE(15,*) ' '
383      WRITE(15,*) ' '
384      WRITE(15,*) ' '
385      WRITE(15,*) ' '
386      DO 430 J=NSTRM,1,-NSTEP
387        WRITE(15,1021) J, (VTHETA(J,K), K=1,NSTNS)
388      CONTINUE
389
390      ENDIF
391
392      RETURN
393      END

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1  SUBROUTINE VELO(CURV, DHQDQ, DSDQ, DVDM, DMHQ, HOSUM, NSTAG, I,
2  MASTOT, MFRAC, MID, NSTRM, PHI, RADIUS, RHO, TEMP, VELM)
3
4  *****
5  qsl.dlr/velo.f
6
7  *****
8  PROGRAM VERSION: WHIRL, VARIABLE WALL GEOMETRY, AND GALLINORE'S
9  METHOD OF q-o ITERATION.
10
11 *****
12 THIS SUBROUTINE INTEGRATES THE RADIAL EQUILIBRIUM EQUATION TO GET
13 THE MERIDIONAL VELOCITY PROFILE THEN CHECKS AND ADJUSTS VMID UNTIL
14 CONTINUITY IS SATISFIED FOR THE VELOCITY PROFILE.
15
16 *****
17 INPUT VARIABLES: CURV, DHQDQ, DSDQ, DVDM, DMHQ, HOSUM, NSTAG, I,
18 MASTOT, MFRAC, MID, NSTRM, PHI, RADIUS, RHO, TEMP,
19 VELM
20
21 *****
22 OUTPUT VARIABLES: VELM, HOSUM, MFRAC
23
24 *****
25 VARIABLES INTERNAL TO THE SUBROUTINE ARE: AFUN, BFUN, MASFLO, OLDVM2,
26 TUBFLO, VM2NEW, XI, VMID
27
28 *****
29 DOUBLE PRECISION AFUN(51), BFUN(51), CP, CURV(51,200), DHBAR(51),
30 DHQDQ(51,200), DSDQ(51,200), DVDM(51,200), DMHQ(51,200),
31 GAMMA, GASCON, GSUBC, HOSUM(200), NSTAG(51,200), INTRCP,
32 MASFLO, MASTOT, MIDOT1, MIDOT2, MFRAC(51,200), MIXCO,
33 OLDVM2(51), PHI(51,200), PI, RADIUS(51,200), RELAXV, RELAXM,
34 RHO(51,200), SLOPE, TBAR(51), TEMP(51,200), TOLM, TOLR,
35 TUBFLO(51), WBAR(51), VELM(51,200), VMID1, VMID2, VMID3,
36 VMID, VM2NEW(51), XI
37
38 *****
39 INTEGER COUNT, I, ITER, LLIMIT, LP, LLIMIT, NSTEP, NSTRM, MID,
40 TP:P
41
42 *****
43 COMMON /CONST/ CP, GAMMA, GASCON, GSUBC, MIXCO, PI
44 COMMON /CTRL1/ COUNT, LLIMIT, LP, LLIMIT, NSTEP
45 COMMON /CTRL2/ RELAXV, RELAXM, TOLM, TOLV, TOLR
46
47 *****
48 I IS THE STATION NUMBER SENT DOWN FROM THE MAIN PROGRAM
49 ITER IS A LOCAL ITERATION COUNTER. IT RECORDS THE NUMBER OF
50 ITERATIONS REQUIRED TO ACHIEVE MASS FLOW CONVERGENCE AT THIS q-o.
51
52 *****
53 IF(LP .EQ. 6 .AND. I .EQ. NSTNS) THEN
54   LP = LP + 1
55   IF(LP .EQ. 20 .AND. I .EQ. NSTNS) THEN
56     WRITE(16,*) 'qsl.dlr VERSION 1.1'
57     WRITE(16,*) 'DATA OUTPUT FROM SUBROUTINE VELO: output 6 dat'
58     WRITE(16,*) 'GLOBAL ITERATION COUNT (FLOW FIELD) IS: COUNT'
59     ENDF
60
61 *****
62 PREPARE THE DATA FOR USE IN THIS SUBROUTINE
63 VMID - MIDDLE STREAMLINE VALUE FROM INITIAL ESTIMATE OR PREVIOUS
64 GLOBAL ITERATION.
65 OLDVM2 STARTS AS VM^2 FROM THE INITIAL ESTIMATE OR PREVIOUS GLOBAL
66 ITERATION. OLDVM2 IS UPDATED WITH THE NEXT LOCAL ITERATION VALUE
67 COMPUTED BELOW FOR UNDER-RELAXATION OF THE VELOCITY CHANGE
68 (CHANGE FROM OLD LOCAL ITERATION)
69
70 *****
71 VMID1 = VELM(MID, I)
72 VMID = VELM(MID, I)
73 DO 10 J=1, NSTRM
74   OLDVM2(J) = VELM(J, I)**2
75
76 *****
77 10 CONTINUE
78
79 *****
80 SUBROUTINE RETURNS HERE WHEN THE q-o MASS FLOW RATE DOES NOT
81 CONVERGE
82 VM2NEW(MID) = VMID**2
83
84 *****
85 PRINT*, 'ITERATION: STATION NUMBER: ', I
86 *****
87 INTEGRATE THE MOMENTUM EQUATION
88 *****
89 INTEGRATE THE MOMENTUM EQUATION FOR THE CURRENT q-o.
90 *****
91 THE FIRST LOOP INTEGRATES FROM THE MIDLINE TO THE TIP AND GIVES
92 THE SQUARED VELOCITY.
93 DO 20 J = MID+1, NSTRM
94   AFUN(J) = -1 * COS(PHI(J-1, I)) * CURV(J-1, I) +
95     COS(PHI(J, I)) * CURV(J, I) +
96     SIN(PHI(J-1, I)) * DVDM(J-1, I)/VELM(J-1, I) +
97     SIN(PHI(J, I)) * DVDM(J, I)/VELM(J, I)
98
99   DHBAR(J) = DHQDQ(J-1, I) * DMHQDQ(J, I)
100   TBAR(J) = TEMP(J-1, I) * DSDQ(J, I) + TEMP(J, I) * DSDQ(J, I)
101   WBAR(J) = DMHQDQ(J-1, I)/(2.000 * RADIUS(J-1, I)**2) +
102     DMHQDQ(J, I)/(2.000 * RADIUS(J, I)**2)
103
104   BFUN(J) = DHBAR(J) - TBAR(J) - WBAR(J)
105
106   XI = EXP(-AFUN(J) * (RADIUS(J, I) - RADIUS(J-1, I)))
107
108   IF(ABS(AFUN(J)) .LE. 1.0E-8) THEN
109     VM2NEW(J) = VM2NEW(J-1) * BFUN(J) *
110       (RADIUS(J, I) - RADIUS(J-1, I))
111   ELSE
112     VM2NEW(J) = VM2NEW(J-1) * XI *
113       (BFUN(J)/AFUN(J)) * (1.000 - XI)
114   ENDIF
115
116 20 CONTINUE
117
118 *****
119 THE SECOND LOOP INTEGRATES FROM THE MIDLINE TO THE HUB AND
120 GIVES THE SQUARED VELOCITY.
121 DO 30 J = MID-1, 1, -1
122   AFUN(J) = -1 * COS(PHI(J+1, I)) * CURV(J+1, I) +
123     COS(PHI(J, I)) * CURV(J, I) +
124     SIN(PHI(J+1, I)) * DVDM(J+1, I)/VELM(J+1, I) +
125     SIN(PHI(J, I)) * DVDM(J, I)/VELM(J, I)
126
127   DHBAR(J) = DHQDQ(J+1, I) * DMHQDQ(J, I)
128   TBAR(J) = TEMP(J+1, I) * DSDQ(J, I) + TEMP(J, I) * DSDQ(J, I)
129   WBAR(J) = DMHQDQ(J+1, I)/(2.000 * RADIUS(J+1, I)**2) +
130     DMHQDQ(J, I)/(2.000 * RADIUS(J, I)**2)
131
132   BFUN(J) = DHBAR(J) - TBAR(J) - WBAR(J)
133
134   XI = EXP(-AFUN(J) * (RADIUS(J, I) - RADIUS(J+1, I)))
135
136   IF(ABS(AFUN(J)) .LE. 1.0E-10) THEN
137     VM2NEW(J) = VM2NEW(J+1) * BFUN(J) *
138       (RADIUS(J, I) - RADIUS(J+1, I))
139   ELSE
140     VM2NEW(J) = VM2NEW(J+1) * XI *
141       (BFUN(J)/AFUN(J)) * (1.000 - XI)
142   ENDIF
143
144 30 CONTINUE
145
146 *****
147 RELAX THE MERIDIONAL VELOCITY CHANGE: USING LATEST LOCAL ITERATION
148 VELM VALUES TO UPDATE OLDVM2 FOR NEXT LOCAL ITERATION
149
150 *****

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[illegible]


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1  SUBROUTINE RADIAL(I, MFRAC, NSTNS, NSTRM, RADIUS, RADNEW,
2  * DELTA, TEMP, VELM, VELNEW, ZDELTA)
3
4  * *****0*****0*****0*****0*****0*****0*****0*****
5  *          gal.dir/radial.f
6  *
7  * PROGRAM VERSION: WHIRL, VARIABLE WALL GEOMETRY, AND GALLIMORE'S
8  * METHOD OF q-o ITERATION.
9  *
10 * THIS SUBROUTINE ADJUSTS THE RADIAL LOCATION AND FINDS THE RADII WHICH
11 * CORRESPOND TO THE FIXED STREAMTUBE MASS FRACTIONS
12 *
13 * INPUT VARIABLES: I, MDELTA, MFRAC, NSTNS, NSTRM, RADIUS, RDELTA,
14 * TEMP, VELM
15 *
16 * OUTPUT VARIABLES: RADNEW, RADNEW, VELM, VELNEW
17 *
18 * VARIABLES INTERNAL TO THIS SUBROUTINE ARE: AMACH2, MACH2, NUM
19 *
20 * *****0*****0*****0*****0*****0*****0*****0*****
21 * DOUBLE PRECISION AMACH2, CP, GAMMA, GASCON, GSUBC, MACH2,
22 * MFRAC(51,200), MIXCO, PI, RADIUS(51,200), RADNEW(51,200),
23 * DELTA(51,200), RELAX, SUM, TEMP(51,200), VELM(51,200),
24 * VELNEW(51,200), ZDELTA(200)
25 *
26 * INTEGER COUNT, I, LLIMIT, LP, MID, MLIMIT, NSTEP, NSTRM,
27 * NSTNS, NUM
28 *
29 * COMMON /CONST/ CP, GAMMA, GASCON, GSUBC, MIXCO, PI
30 * COMMON /CTRL1/ COUNT, LLIMIT, LP, MLIMIT, NSTEP
31 *
32 * *****0*****0*****0*****0*****0*****0*****0*****
33 * MFRAC(I,1) COMES FROM THE VELOCITY SUBROUTINE VELO. MFRAC CONTAINS
34 * THE FRACTION OF THE FLOW FOR EACH STREAMTUBE IN EACH MFRAC AND
35 * RADIUS CREATE A CURVE (RADIUS AS A FUNCTION OF MFRAC) THAT IS USED
36 * TO FIND THE NEW RADIAL LOCATIONS THAT MATCH THE INLET STREAM TUBE
37 * MASS FRACTIONS IN MFRAC(J,1)
38 *
39 * IF ANY MFRAC VALUES ARE OUTSIDE THE BOUNDS OF MFRAC(J,1) OR THE
40 * OFFENDING VALUE, MFRAC(I,1), IS SET TO BE WITHIN MFRAC(K,1)
41 * NUM IS A MARKER TO TELL WHICH CALL STATEMENT CAUSES ERRORS IN
42 * SUBROUTINE INTERP.
43 * CALL STATEMENT FOR INTERP HAS THE FOLLOWING PATTERN:
44 * X, Y, # OF DATA POINTS - CREATE A CURVE
45 * XIN, YOUT, # OF DATA POINTS
46 * INTERP FINDS YOUT
47 * *****0*****0*****0*****0*****0*****0*****0*****
48 *
49 * INTERPOLATING FOR THE NEW RADII
50 * MFRAC(NSTRM,1) = 1.0000
51 *
52 * NUM IS 1 WHEN MFRAC FOR THE CURRENT q-o IS COMPARED WITH MFRAC FOR
53 * THE INLET q-o. AN ERROR IN SUBROUTINE INTERP WILL PRINT OUT AN
54 * ERROR MESSAGE SAYING THAT THE ERROR IS DUE TO MFRAC VALUES.
55 * NUM = 1
56 *
57 * CALL INTERP(MFRAC(I,1), RADIUS(I,1), NSTRM, MFRAC(I,1),
58 * RADNEW(I,1), NSTRM, NUM)
59 *
60 * *****0*****0*****0*****0*****0*****0*****0*****
61 *
62 * CALCULATE THE RELAXATION FACTOR FOR THE CHANGE IN THE RADIAL
63 * LOCATIONS. RADNEW IS UNDER RELAXED, SO THE CHANGE IS SMALLER THAN
64 * WHAT WAS CALCULATED BY INTERP.
65 * *****0*****0*****0*****0*****0*****0*****0*****
66 * MACH2 IS THE SQUARED M-DIRECTION MACH NO. FOR EACH STREAMLINE ALONG
67 * THE CURRENT q-o. AMACH2 IS THE AVERAGE SQUARED MACH NUMBER FOR THE
68 * ENTIRE q-o.
69 * SUM = 0.000
70 * DO 10 J=1,NSTRM
71 * MACH2 = VELM(J,1)**2/
72
73 * *****0*****0*****0*****0*****0*****0*****0*****
74 *
75 * SUM = SUN + MACH2
76 * AMACH2 = SUN/REAL(NSTRM)
77 *
78 * *****0*****0*****0*****0*****0*****0*****0*****
79 *
80 * THE RELAXATION FACTOR ALREADY HAS A k(min) OF -4.0 INCLUDED (PARABOLIC
81 * COMBUSTION CORRECTION SEE WILKINSON). THE RELAXATION FACTOR DOES
82 * NOT CHANGE WITH RADIUS, IT IS A FUNCTION OF q-o ONLY.
83 * RELAX = 1.000/
84 * ( 1.000 + (5.000/24.000) * (1.000 - AMACH2) *
85 * ((RADIUS(NSTRM,1) - RADIUS(I,1))/ZDELTA(I,1)) )
86 *
87 * THE LOOP DOES NOT INCLUDE THE ENDMALL STREAMLINES SINCE THE ENDMALL
88 * RADIUS NEVER CHANGE LOCATION.
89 * DO 20 J=2,NSTRM-1
90 * RADNEW(J,1) = RADIUS(J,1) + RELAX * (RADNEW(J,1) -
91 * RADIUS(J,1))
92 *
93 * *****0*****0*****0*****0*****0*****0*****0*****
94 *
95 * THE INTERPOLATION ROUTINE AND RELAXATION MAY SET THE NEW ENDMALL
96 * RADIUS OUTSIDE THE BOUNDS OF THE INPUT ANNULUS GEOMETRY. TO ELIMINATE
97 * ANY ERRORS, THIS SECTION RESETS THE NEW WALL RADIAL LOCATIONS TO BE
98 * EQUAL TO THE OLD.
99 * RADNEW(NSTRM,1) = RADIUS(NSTRM,1)
100 * RADNEW(1,1) = RADIUS(1,1)
101 *
102 * *****0*****0*****0*****0*****0*****0*****0*****
103 *
104 * THE M-DIRECTION VELOCITY, VELM, FROM SUBROUTINE VELO AND RADIUS
105 * CREATE A CURVE THAT IS USED TO FIND THE VELOCITIES THAT CORRESPOND TO
106 * THE NEW RADIAL LOCATIONS CALCULATED ABOVE. VELNEW CONTAINS THE NEW
107 * VELOCITIES.
108 * IF THE NEW RADIUS IS OUTSIDE THE BOUNDS OF THE HUB AND TIP, THE
109 * OFFENDING RADIAL LOCATION, RADNEW(K), IS SET TO RADIUS(K)
110 * *****0*****0*****0*****0*****0*****0*****0*****
111 *
112 * INTERPOLATING FOR THE NEW VELOCITY
113 *
114 * NUM IS 2 WHEN RADNEW FOR THE CURRENT q-o IS COMPARED WITH RADOLD FOR
115 * THE CURRENT q-o. AN ERROR IN SUBROUTINE INTERP WILL PRINT OUT AN
116 * ERROR MESSAGE SAYING THAT THE ERROR IS DUE TO RADNEW VALUES.
117 * NUM = 2
118 *
119 * CALL INTERP(RADIUS(1,1), VELM(1,1), NSTRM, RADNEW(1,1),
120 * VELNEW(1,1), NSTRM, NUM)
121 *
122 * *****0*****0*****0*****0*****0*****0*****0*****
123 *
124 * DO 50 J=1,NSTRM
125 * IF(VELNEW(J,1) .LE. 0.0000) THEN
126 * PRINT*, 'VELNEW ERROR IN RADIAL', J
127 * PRINT*, 'VELNEW IS ZERO'
128 * ENDF
129 *
130 * *****0*****0*****0*****0*****0*****0*****0*****
131 *
132 * IF(LP.EQ.7 AND I.EQ.NSTNS OR
133 * LP.EQ.20 AND I.EQ.NSTNS) THEN
134 * WRITE(17,*) 'RESULTS OF THE INTERPOLATION ROUTINE'
135 * WRITE(17,*) 'COUNT =', COUNT, 'I =', I
136 * WRITE(17,*) 'THE AVERAGE MACH'2 IS', MMACH2
137 * WRITE(17,*) 'THE RELAXATION FACTOR IS', RELAX
138 *
139 * *****0*****0*****0*****0*****0*****0*****0*****
140 *
141 * WRITE(17,*) ' '
142 * WRITE(17,*) ' '
143 * DO 110 J=NSTRM,1, -NSTEP
144 * RADIUS
145 * RADNEW'

```

[illegible]

[illegible]

```

145      ((XIN(K) - XDATA(N-2)) * (XIN(K) - XDATA(N))) *
146      XDATA(N-1)/((XDATA(N-1) - XDATA(N-2)) *
147      (XDATA(N-1) - XDATA(N))) *
148      ((XIN(K) - XDATA(N-2)) * (XIN(K) - XDATA(N-1))) *
149      XDATA(N)/((XDATA(N) - XDATA(N-2)) *
150      (XDATA(N) - XDATA(N-1)))
151
152 20 CONTINUE
153
154
155 21 RETURN
156 22 END
157
158
159
160
161
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168
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171

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145 WRITE(21,*) ' '
146 WRITE(21,*) ' ' THE STREAMLINE CURVATURES (1/ft) ARE: '
147 DO 100 J=NSTRM,1,-NSTP
148 WRITE(21,1007) J, (CURV(J,I), I=1,NSTNS,ISTEP)
149
150 CONTINUE
151
152 1007 FORMAT(1X, I2, 1X, 10(F6.3, 1X))
153
154
155 WRITE(21,*) ' '
156 WRITE(21,*) ' ' THE 2-DIRECTION VISCOUS BODY FORCE (1bf/ft^3) IS: '
157 DO 110 J=NSTRM,1,-NSTP
158 WRITE(21,1009) J, (EZ(J,I), I=1,NSTNS,ISTEP)
159
160 CONTINUE
161
162 WRITE(21,*) ' '
163 WRITE(21,*) ' ' THE THETA DIRECTION BODY FORCE (1bf/ft^3) IS: '
164 DO 120 J=NSTRM,1,-NSTP
165 WRITE(21,1009) J, (ETHETA(J,I), I=1,NSTNS,ISTEP)
166
167 CONTINUE
168
169 WRITE(21,*) ' '
170 WRITE(21,*) ' ' THE DISSIPATION FUNCTION (1bf/ft^2) IS: '
171 DO 130 J=NSTRM,1,-NSTP
172 WRITE(21,1009) J, (DISSIP(J,I), I=1,NSTNS,ISTEP)
173
174 CONTINUE
175
176 1009 FORMAT(1X, I2, 1X, 10(E10.4, 2X))
177
178
179 WRITE(21,*) ' '
180 WRITE(21,*) ' ' THE DENSITY FIELD (slug/ft^3) IS: '
181 DO 150 J=NSTRM,1,-NSTP
182 WRITE(21,1011) J, (RHO(J,I), I=1,NSTNS,ISTEP)
183
184 CONTINUE
185
186 1011 FORMAT(1X, I2, 1X, 10(E10.4, 2X))
187
188
189 WRITE(21,*) ' '
190 WRITE(21,*) ' ' THE STAGNATION PRESSURE FIELD (psi) IS: '
191 DO 160 J=NSTRM,1,-NSTP
192 WRITE(21,1013) J, (PSTAG(J,I)/144.0, I=1,NSTNS,ISTEP)
193
194 CONTINUE
195
196 1013 FORMAT(1X, I2, 1X, 10(F6.3, 2X))
197
198
199 WRITE(21,*) ' '
200 WRITE(21,*) ' ' THE STATIC PRESSURE FIELD (psi) IS: '
201 DO 170 J=NSTRM,1,-NSTP
202 WRITE(21,1013) J, (PRESS(J,I)/144.0, I=1,NSTNS,ISTEP)
203
204 CONTINUE
205
206 170 CONTINUE
207
208 DO 210 I=1,NSTNS
209 DO 200 J=1,NSTRM
210 WRITE(21,*) ' '
211 CONTINUE
212
213 WRITE(21,*) ' '
214 WRITE(21,*) ' ' CHANGE IN PRESSURE FROM THE INLET (psi) '
215 WRITE(21,*) ' ' INCREASE IS POSITIVE, DECREASE IS NEGATIVE '
216 DO 220 J=NSTRM,1,-NSTP

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217 WRITE(21,1013) J, (PDELTA(J,I)/144.000, I=1,NSTNS,ISTEP)
218
219 CONTINUE
220
221 1013 FORMAT(1X, I2, 1X, 10(F7.3, 2X))
222
223
224 WRITE(21,*) ' '
225 WRITE(21,*) ' ' THE STATIC TEMPERATURE FIELD (R) IS: '
226 DO 230 J=NSTRM,1,-NSTP
227 WRITE(21,1013) J, (TEMP(J,I), I=1,NSTNS,ISTEP)
228
229 CONTINUE
230
231 WRITE(21,*) ' '
232 WRITE(21,*) ' ' THE STAGNATION TEMPERATURE FIELD (R) IS: '
233 DO 240 J=NSTRM,1,-NSTP
234 WRITE(21,1013) J, (TSTAG(J,I), I=1,NSTNS,ISTEP)
235
236 CONTINUE
237
238 240 CONTINUE
239
240
241 WRITE(21,*) ' '
242 WRITE(21,*) ' ' THE TURBULENT VISCOSITY IS: '
243 DO 250 J=NSTRM,1,-NSTP
244 WRITE(21,1015) J, (MUT(J,I), I=1,NSTNS,ISTEP)
245
246 CONTINUE
247
248 250 CONTINUE
249
250
251 WRITE(21,*) ' '
252 WRITE(21,*) ' ' THE HEAT TRANSFER COEFFICIENT IS: '
253 DO 260 J=NSTRM,1,-NSTP
254 WRITE(21,1015) J, (HTCOEF(J,I), I=1,NSTNS,ISTEP)
255
256 CONTINUE
257
258 260 CONTINUE
259
260
261 1015 FORMAT(1X, I2, 1X, 10(E10.4, 2X))
262
263
264 * CHECK CONSERVATION OF ANGULAR MOMENTUM output written to output.9.dat
265 DO 66 I=1,NSTNS
266 SPAN(I) = RADIUS(NSTRM,I) - RADIUS(1,I)
267 IF (SPAN(I) .LT. 0.00) PRINT*, 'SPAN ERROR IN OUTPUT 9 q-o-', I
268 DO 65 J=1,NSTRM
269 X(J,I) = VTHETA(J,I)/ZVEL(J,I)
270 RADRAT(J,I) = (RADIUS(J,I) - RADIUS(1,I))/SPAN(I)
271 IF (RADRAT(J,I) .LT. 0.00) PRINT*,
272 'RADRAT ERROR IN OUTPUT 9 q-o-', I
273 REYN(I) = ZVEL(MID,1) * ZLOC(I) * RHO(MID,1)/MUT(J,I)
274
275 CONTINUE
276
277 66 CONTINUE
278
279 DO 67 J=NSTRM,1,-1
280 WRITE(19,1002) X(J,I), RADRAT(J,I),
281 X(J,NSTNS), RADRAT(J,NSTNS)
282
283 CONTINUE
284 1002 FORMAT(1X, F6.4, 2X, F5.3, 6X, F6.4, 2X, F5.3)
285
286 WRITE(19,*) ' '
287 WRITE(19,*) ' REYN(1), REYN(NSTNS)
288
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C32

CLOSE (21)
RETURN
END

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292
293
294

APPENDIX D. RESULTS FOR EXAMPLE PROBLEMS

Appendix D contains the full program output for the two example problems described in Chapter 4.

DATA FROM THE INPUT SUBROUTINE
ALL DIMENSIONS ARE IN ft. sec. 1bf. R UNLESS NOTED.

```
CONSTANT STAGNATION PRESS AT INLET:    2116.8000000000000000 psf
CONSTANT STAGNATION TEMP AT INLET:     519.6700000000000000 R
THE Z LOCATION OF INLET:                 0.000000000000000E+00 in
THE Z LOCATION FOR LAST STATION:        5.500000000000000 in
```

```

THE NUMBER OF STREAMLINES IS: 11
THE NUMBER OF COMPUTING STATIONS (q-o) IS: 12
THE MASS FLOW TOLERANCE USED IS: 1.0000000000000000E-02
THE RADIAL CHANGE TOLERANCE IS: 5.0000000000000000E-02
VELOCITY RELAXATION FACTOR USED: 1.0000000000000000

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THE CONSTANTS USED ARE:
GASCON: 1715.8400000000000
GSUBC: 1.000000000000000
CP: 6009.0700000000000
GAMMA: 1.400000000000000
PI: 3.141592653589793
      ft^2/sec^2 R      ft^2/sec^2 R
      slug ft/lbf sec^2      slug ft/lbf sec^2
      ft^2/sec^2 R      ft^2/sec^2 R

```

J	RADIUS ft	TOTAL		V_E_L_O_C_I_T_I_E_S		INPUT	
		ft/sec	axial	ft/sec	THETA ft/sec		
1	1.0000	100.0000	100.0000	100.0000	0.0000	0.0000	0.0000
10	0.9583	100.0000	100.0000	100.0000	0.0000	0.0000	0.0000
9	0.9167	100.0000	100.0000	100.0000	0.0000	0.0000	0.0000
8	0.8750	100.0000	100.0000	100.0000	0.0000	0.0000	0.0000
7	0.8333	100.0000	100.0000	100.0000	0.0000	0.0000	0.0000
6	0.7917	100.0000	100.0000	100.0000	0.0000	0.0000	0.0000
5	0.7500	100.0000	100.0000	100.0000	0.0000	0.0000	0.0000
4	0.7083	100.0000	100.0000	100.0000	0.0000	0.0000	0.0000
3	0.6667	100.0000	100.0000	100.0000	0.0000	0.0000	0.0000
2	0.6250	100.0000	100.0000	100.0000	0.0000	0.0000	0.0000
1	0.5833	100.0000	100.0000	100.0000	0.0000	0.0000	0.0000

PROGRAM CALCULATION SUMMARY

GLOBAL ITERATIONS TO REACH CONVERGENCE: 3
UNIVERSAL ITERATIONS TO CONVERGENCE: 2
ALL DIMENSIONS IN ft. sec. lbf. R UNLESS NOTED

THE MASS FLOW RATE FOR THE ANNULUS: 0.4900552816037199 slug/sec

[illegible]

THE q-o Z DIRECTION LOCATIONS (inches) ARE:

THE THETA DIRECTION BODY FORCE (lbf/ft ³) IS:									
11	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
10	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
9	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
8	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
7	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
6	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
5	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
4	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
3	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
2	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
1	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00

THE DISSIPATION FUNCTION (lbf/ft ²) IS:									
11	0.0000E+00	0.1086E-02	0.2154E+00	0.2250E+01	0.8372E+01	0.1752E+02	0.2345E+02	0.1615E+02	0.3066E+01
10	0.0000E+00	0.1546E-02	0.1135E+00	0.1326E+01	0.5773E+01	0.1366E+02	0.1895E+02	0.1263E+02	0.2239E+01
9	0.0000E+00	0.6199E-03	0.4849E-01	0.6508E+00	0.3430E+01	0.9397E+01	0.1368E+02	0.8834E+01	0.1457E+01
8	0.0000E+00	0.1911E-03	0.1532E-01	0.2318E+00	0.1472E+01	0.4720E+01	0.7318E+01	0.4668E+01	0.7223E+00
7	0.0000E+00	0.7031E-04	0.4688E-02	0.1849E+00	0.2872E+01	0.1132E+01	0.1386E+01	0.2052E+00	0.2473E-02
6	0.0000E+00	0.8919E-04	0.6843E-02	0.1003E+00	0.5535E+00	0.1130E+01	0.2029E+01	0.1130E+01	0.1849E+00
5	0.0000E+00	0.2684E-03	0.2084E-01	0.2971E+00	0.1618E+00	0.4881E+01	0.2128E+01	0.4201E+01	0.6220E+00
4	0.0000E+00	0.5879E-03	0.4507E-01	0.5946E+00	0.2970E+01	0.7595E+01	0.6892E+01	0.7596E+01	0.1243E+01
3	0.0000E+00	0.1145E-02	0.8482E-01	0.5970E+00	0.1280E+01	0.1027E+02	0.1474E+02	0.1003E+02	0.1810E+01
2	0.0000E+00	0.1980E-02	0.1418E+00	0.1502E+01	0.5584E+01	0.1194E+02	0.1653E+02	0.1174E+02	0.2335E+01
1	0.0000E+00	0.1980E-02	0.1418E+00	0.1502E+01	0.5584E+01	0.1194E+02	0.1653E+02	0.1174E+02	0.2335E+01

THE DENSITY FIELD (slug/ft ³) IS:									
11	0.2364E-02	0.2364E-02	0.2364E-02	0.2364E-02	0.2364E-02	0.2352E-02	0.2343E-02	0.2331E-02	0.2319E-02
10	0.2364E-02	0.2364E-02	0.2364E-02	0.2364E-02	0.2364E-02	0.2352E-02	0.2343E-02	0.2331E-02	0.2319E-02
9	0.2364E-02	0.2364E-02	0.2364E-02	0.2364E-02	0.2364E-02	0.2352E-02	0.2343E-02	0.2331E-02	0.2319E-02
8	0.2364E-02	0.2364E-02	0.2364E-02	0.2364E-02	0.2364E-02	0.2352E-02	0.2343E-02	0.2331E-02	0.2319E-02
7	0.2364E-02	0.2364E-02	0.2364E-02	0.2364E-02	0.2364E-02	0.2352E-02	0.2343E-02	0.2331E-02	0.2319E-02
6	0.2364E-02	0.2364E-02	0.2364E-02	0.2364E-02	0.2364E-02	0.2352E-02	0.2343E-02	0.2331E-02	0.2319E-02
5	0.2364E-02	0.2364E-02	0.2364E-02	0.2364E-02	0.2364E-02	0.2352E-02	0.2343E-02	0.2331E-02	0.2319E-02
4	0.2364E-02	0.2364E-02	0.2364E-02	0.2364E-02	0.2364E-02	0.2352E-02	0.2343E-02	0.2331E-02	0.2319E-02
3	0.2364E-02	0.2364E-02	0.2364E-02	0.2364E-02	0.2364E-02	0.2352E-02	0.2343E-02	0.2331E-02	0.2319E-02
2	0.2364E-02	0.2364E-02	0.2364E-02	0.2364E-02	0.2364E-02	0.2352E-02	0.2343E-02	0.2331E-02	0.2319E-02
1	0.2364E-02	0.2364E-02	0.2364E-02	0.2364E-02	0.2364E-02	0.2352E-02	0.2343E-02	0.2331E-02	0.2319E-02

THE STAGNATION PRESSURE FIELD (psi) IS:									
11	14.700	14.700	14.700	14.700	14.700	14.699	14.699	14.699	14.699
10	14.700	14.700	14.700	14.700	14.700	14.699	14.699	14.699	14.699
9	14.700	14.700	14.700	14.700	14.700	14.699	14.699	14.699	14.699
8	14.700	14.700	14.700	14.700	14.700	14.699	14.699	14.699	14.699
7	14.700	14.700	14.700	14.700	14.700	14.699	14.699	14.699	14.699
6	14.700	14.700	14.700	14.700	14.700	14.699	14.699	14.699	14.699
5	14.700	14.700	14.700	14.700	14.700	14.699	14.699	14.699	14.699
4	14.700	14.700	14.700	14.700	14.700	14.699	14.699	14.699	14.699
3	14.700	14.700	14.700	14.700	14.700	14.699	14.699	14.699	14.699
2	14.700	14.700	14.700	14.700	14.700	14.700	14.700	14.700	14.700
1	14.700	14.700	14.700	14.700	14.700	14.700	14.700	14.700	14.700

THE STATIC PRESSURE FIELD (psi) IS:									
11	14.618	14.617	14.613	14.596	14.566	14.514	14.435	14.333	14.234
10	14.618	14.617	14.613	14.596	14.566	14.514	14.435	14.333	14.234
9	14.618	14.617	14.613	14.596	14.566	14.514	14.435	14.333	14.234
8	14.618	14.617	14.613	14.596	14.566	14.514	14.435	14.333	14.234
7	14.618	14.617	14.613	14.596	14.566	14.514	14.435	14.333	14.234
6	14.618	14.617	14.613	14.596	14.566	14.514	14.435	14.333	14.234
5	14.618	14.617	14.613	14.596	14.566	14.514	14.435	14.333	14.234
4	14.618	14.617	14.613	14.596	14.566	14.514	14.435	14.333	14.234
3	14.618	14.617	14.613	14.596	14.566	14.514	14.435	14.333	14.234
2	14.618	14.617	14.613	14.596	14.566	14.514	14.435	14.333	14.234
1	14.618	14.617	14.613	14.596	14.566	14.514	14.435	14.333	14.234

THE STATIC PRESSURE FIELD (psi) IS:									
11	14.618	14.617	14.613	14.596	14.566	14.514	14.435	14.333	14.234
10	14.618	14.617	14.613	14.596	14.566	14.514	14.435	14.333	14.234
9	14.618	14.617	14.613	14.596	14.566	14.514	14.435	14.333	14.234
8	14.618	14.617	14.613	14.596	14.566	14.514	14.435	14.333	14.234
7	14.618	14.617	14.613	14.596	14.566	14.514	14.435	14.333	14.234
6	14.618	14.617	14.613	14.596	14.566	14.514	14.435	14.333	14.234
5	14.618	14.617	14.613	14.596	14.566	14.514	14.435	14.333	14.234
4	14.618	14.617	14.613	14.596	14.566	14.514	14.435	14.333	14.234
3	14.618	14.617	14.613	14.596	14.566	14.514	14.435	14.333	14.234
2	14.618	14.617	14.613	14.596	14.566	14.514	14.435	14.333	14.234
1	14.618	14.617	14.613	14.596	14.566	14.514	14.435	14.333	14.234

[illegible]

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Joan Boggs
STINFO Program Manager