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GREEN'S FUNCTIONS FOR AN ANISOTROPIC MEDIUM: PART II. TWO-LAYER CASE

ARCON Corporation

Saba Mudaliar

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APPROVED: Zachary O. White

ZACHARY O. WHITE
Project Engineer

FOR THE COMMANDER John K. Schindler

JOHN K. SCHINDLER
Director
Electromagnetics & Reliability Directorate

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13. ABSTRACT (Maximum 200 words)

The Dyadic Green's Functions (DGF) of a two-layer biaxially anisotropic medium are derived. The principal coordinate system of the anisotropic medium is allowed to have arbitrary orientation with respect to the layer geometry. The formulation is based on the unbounded Dyadic Green's Function derived in Part I of the sequel. Using the matrix method the coefficients of the two-layer DGF are expressed in terms of half-space Fresnel reflection and transmission coefficients. To complete this procedure the various relevant half-space Fresnel coefficients are derived. The form in which the results are presented has a physically meaningful and compact structure. A numerical example is provided where we have computed the reflectivities.

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1. INTRODUCTION

In Part I [Mudaliar, 1993][1] we obtained the dyadic Green's function (DGF) of a biaxially anisotropic medium for the unbounded case. It may be noted that the results obtained there are in a form very convenient for deriving the DGF's for anisotropic layered structures. Indeed that was one of the underlying motives of Part I. We consider in this report the two-layer case.

The plan of the report is as follows. In Section 2 we briefly describe the geometry of the problem. We formulate the solutions of the problem in Section 3. In order to facilitate the evaluation of the coefficients of the DGF's we introduce amplitude vectors and adopt a matrix method in Section 4. The procedure is completed in Section 5 by deriving various half-space Fresnel coefficients. Finally the report concludes in Secion 6.

2. GEOMMETRY OF THE PROBLEM

The geometry of the problem is shown in Figure 1. It consists of three regions: Region 0 ($z > 0$) is an isotropic medium with permittivity ε_0, Region 1 ($0 > z > -d$) is the biaxially anisotropic medium with permittivity $\bar{\bar{\varepsilon}}$ and Region 2 ($z < -d$) is an isotropic medium of permittivity ε_2. All the three regions have the same permeability μ. In this coordinate system the permittivity $\bar{\bar{\varepsilon}}$ is represented by the following symmetric matrix.

$$\bar{\bar{\epsilon}} = \begin{bmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{12} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{13} & \epsilon_{23} & \epsilon_{33} \end{bmatrix} \tag{1}$$

where the elements ϵ_{ij} are explained in (3) of Part I.

3. FORMULATION

We have a unit impulse electric current source in Region 0 away from the boundary. Our interest is to find the DGF's: $\bar{\bar{G}}_{00}(\bar{r},\bar{r}')$, $\bar{\bar{G}}_{10}(\bar{r},\bar{r}')$ and $\bar{\bar{G}}_{20}(\bar{r},\bar{r}')$. Here as usual the second subscript denotes the region where the source is located while the first subscript denotes the region containing the observation point. In order to obtain the above DGF's we need to solve the system of equations given in Appendix A. Based on the knowledge about the structure of the DGF for the unbounded case obtained in Part I and that of the DGF's for the layered isotropic media[2] we can now construct solutions for our two-layer problem as follows.

$$\bar{\bar{G}}_{00}(\bar{r},\bar{r}') = \frac{i}{8\pi^2} \int_{-\infty}^{\infty} d^2\bar{k}_\rho \frac{1}{k_{0z}} \Big\{ \Big[\hat{h}_0^- e^{i\bar{\kappa}_0\cdot\bar{r}} + R_{hh} \hat{h}_0^+ e^{i\bar{k}_0\cdot\bar{r}}$$

$$+ R_{hv} \hat{v}_0^+ e^{i\bar{k}_0\cdot\bar{r}} \Big] \hat{h}_0^-$$

$$+ \Big[\hat{v}_0^- e^{i\bar{\kappa}_0\cdot\bar{r}} + R_{vv} \hat{v}_0^+ e^{i\bar{k}_0\cdot\bar{r}}$$

$$+ R_{vh} \hat{h}_0^+ e^{i\bar{k}_0\cdot\bar{r}} \Big] \hat{v}_0^- \Big\} e^{-i\bar{\kappa}_0\cdot\bar{r}'} \tag{2}$$

$$\bar{\bar{G}}_{10}(\bar{r},\bar{r}') = \frac{i}{8\pi^2} \int_{-\infty}^{\infty} d^2\bar{k}_\rho \frac{1}{k_{0z}} \Big\{ \Big[A_{ha} \hat{a}^- e^{i\bar{\kappa}^a\cdot\bar{r}} + B_{ha} \hat{a}^+ e^{i\bar{k}^a\cdot\bar{r}}$$

$$+ A_{hb} \hat{b}^- e^{i\bar{\kappa}^b\cdot\bar{r}} + B_{hb} \hat{b}^+ e^{i\bar{k}^b\cdot\bar{r}} \Big] \hat{h}_0^-$$

$$+ \Big[A_{va} \hat{a}^- e^{i\bar{\kappa}^a\cdot\bar{r}} + B_{va} \hat{a}^+ e^{i\bar{k}^a\cdot\bar{r}}$$

$$+ A_{vb} \hat{b}^- e^{i\bar{\kappa}^b\cdot\bar{r}} + B_{vb} \hat{b}^+ e^{i\bar{k}^b\cdot\bar{r}} \Big] \hat{v}_0^- \Big\} e^{-i\bar{\kappa}_0\cdot\bar{r}'} \tag{3}$$

$$\bar{\bar{G}}_{20}(\bar{r},\bar{r}') = \frac{i}{8\pi^2} \int_{-\infty}^{\infty} d^2\bar{k}_\rho \, \frac{1}{k_{0z}} \left\{ \left[X_{hh}\, \hat{h}_2^{-} + X_{hv}\, \hat{v}_2^{-} \right] \hat{h}_0^{-} \right.$$

$$\left. + \left[X_{vv}\, \hat{v}_2^{-} + X_{vh}\, \hat{h}_2^{-} \right] \hat{v}_0^{-} \right\} e^{i\bar{\kappa}_2 \cdot \bar{r}} \; e^{-i\bar{\kappa}_0 \cdot \bar{r}'} \tag{4}$$

where for $n = 0,2$

$$\hat{h}_n^{\pm} = \frac{1}{k_\rho} \bar{k}_n \times \hat{z} \tag{5a}$$

$$\hat{v}_n^{+} = \frac{1}{k_n} \bar{k}_n \times \hat{h}_n^{+} \tag{5b}$$

$$\hat{v}_n^{-} = \frac{1}{k_n} \bar{\kappa}_n \times \hat{h}_n^{-} \tag{5c}$$

$$\bar{k}_n = \bar{k}_\rho + \hat{z}\, k_{nz} \tag{6a}$$

$$\bar{\kappa}_n = \bar{k}_\rho - \hat{z}\, k_{nz} \tag{6b}$$

$$k_{nz} = \left(k_n^2 - k_\rho^2 \right)^{1/2} \tag{7a}$$

$$k_n^2 = \omega^2 \mu \epsilon_n \tag{7b}$$

and $\hat{a}^{\pm}$,$\hat{b}^{\pm}$, $\bar{k}^a$, $\bar{\kappa}^a$, $\bar{k}^b$, $\bar{\kappa}^b$ are defined in Appendix B. Here $\hat{h}$ and $\hat{v}$ denote the horizontal and vertical polarizations, which are two characteristic linear polarizations in an isotropic medium. $\hat{a}$ and $\hat{b}$ denote the polarizations of the a-wave and the b-wave, which are two characteristic (extraordinary) waves in a biaxially anisotropic medium. Note that the superscripts + and − indicate the upward and downward propagating waves, respectively. The task here is to determine the sixteen reflection and transmission coefficients: R's, A's, B's and X's.

4. MATRIX METHOD

The unknown coefficients appearing in (2), (3) and (4) can be evaluated by using (A4) and (A5). But this is at best a tedius procedure; moreover, the results thus obtained are in a very complicated form. We therefore follow an alternative method[3] in which we first express the unknowns in terms of half-space Fresnel coefficients. Later in the next section we evaluate the various half-space Fresnel coefficients.

In this procedure we denote the amplitude vectors of various waves as $\bar{p}$, $\bar{q}$, $\bar{P}$, $\bar{Q}$, $\bar{s}$ (see Figure 2). These are two-element vectors whose elements are the characteristic components of the corresponding wave. For example

$$\bar{p} = \begin{bmatrix} p_h \\ p_v \end{bmatrix}$$

where p_h and p_v are the amplitudes of horizontally and vertically polarized components of the p-wave in an isotropic medium. Similarly

$$\bar{P} = \begin{bmatrix} P_a \\ P_b \end{bmatrix}$$

where P_a and P_b are the a- and b- components of the P-wave in a biaxially anisotropic medium and so on. Accordingly these amplitude vectors satisfy the following equations

$$\begin{bmatrix} \bar{q} \\ \bar{P} \end{bmatrix} = \begin{bmatrix} \bar{\bar{R}}_{01} & \bar{\bar{X}}_{10} \\ \bar{\bar{X}}_{01} & \bar{\bar{R}}_{10} \end{bmatrix} \begin{bmatrix} \bar{p} \\ \bar{Q} \end{bmatrix} \tag{8}$$

$$\begin{bmatrix} \bar{Q} \\ \bar{s} \end{bmatrix} = \begin{bmatrix} \bar{\bar{R}}_{12} \\ \bar{\bar{X}}_{12} \end{bmatrix} \bar{P} \tag{9}$$

where $\bar{\bar{R}}_{01}$, $\bar{\bar{X}}_{01}$, $\bar{\bar{R}}_{10}$, $\bar{\bar{X}}_{10}$, $\bar{\bar{R}}_{12}$ and $\bar{\bar{X}}_{12}$ are the various half-space reflection and transmission matrices defined as follows

$$\bar{\bar{R}}_{01} = \begin{bmatrix} R^{01}_{hh} & R^{01}_{vh} \\ R^{01}_{hv} & R^{01}_{vv} \end{bmatrix} \tag{10a}$$

$$\bar{\bar{X}}_{01} = \begin{bmatrix} X_{ha} & X_{va} \\ X_{hb} & X_{vb} \end{bmatrix} \tag{10b}$$

$$\bar{\bar{R}}_{10} = \begin{bmatrix} R_{aa} & R_{ba} \\ R_{ab} & R_{bb} \end{bmatrix} \tag{11a}$$

$$\bar{\bar{X}}_{10} = \begin{bmatrix} X_{ah} & X_{bh} \\ X_{av} & X_{bv} \end{bmatrix} \tag{11b}$$

$$\bar{\bar{R}}_{12} = \begin{bmatrix} R_{aa}^{12} \exp\left[i\left(-k_z^{ad}+k_z^{au}\right)d \right] & R_{ba}^{12} \exp\left[i\left(-k_z^{bd}+k_z^{au}\right)d \right] \\ R_{ab}^{12} \exp\left[i\left(-k_z^{ad}+k_z^{bu}\right)d \right] & R_{bb}^{12} \exp\left[i\left(-k_z^{bd}+k_z^{bu}\right)d \right] \end{bmatrix} \tag{12a}$$

$$\bar{\bar{X}}_{12} = \begin{bmatrix} X_{ah}^{12} \exp\left[i\left(-k_z^{ad}-k_{2z}\right)d \right] & X_{bh}^{12} \exp\left[i\left(-k_z^{bd}-k_{2z}\right)d \right] \\ X_{av}^{12} \exp\left[i\left(-k_z^{ad}-k_{2z}\right)d \right] & X_{bv}^{12} \exp\left[i\left(-k_z^{bd}-k_{2z}\right)d \right] \end{bmatrix} \tag{12b}$$

Note that the exponential terms are included in (12) to take into account the phase shift at the boundary, $z = -d$. For a given incident wave corresponding to $\bar{p}$, Equations (8) and (9) represent four equations for four unknowns $\bar{q}$, $\bar{P}$, $\bar{Q}$ and $\bar{s}$. The solution is readily obtained and is given as follows.

$$\bar{q} = \bar{\bar{R}}\,\bar{p} \tag{13a}$$

$$\bar{P} = \bar{\bar{A}}\,\bar{p} \tag{13b}$$

$$\bar{Q} = \bar{\bar{B}}\,\bar{p} \tag{13c}$$

$$\bar{s} = \bar{\bar{X}}\,\bar{p} \tag{13d}$$

where

$$\bar{\bar{R}} = \begin{bmatrix} R_{hh} & R_{vh} \\ R_{hv} & R_{vv} \end{bmatrix} = \bar{\bar{R}}_{01} + \bar{\bar{X}}_{10}\,\bar{\bar{R}}_{12}\,(\bar{\bar{I}} - \bar{\bar{R}}_{10}\,\bar{\bar{R}}_{12})^{-1}\,\bar{\bar{X}}_{01} \tag{14a}$$

$$\bar{\bar{A}} = \begin{bmatrix} A_{ha} & A_{va} \\ A_{hb} & A_{vb} \end{bmatrix} = (\bar{\bar{I}} - \bar{\bar{R}}_{10}\,\bar{\bar{R}}_{12})^{-1}\,\bar{\bar{X}}_{01} \tag{14b}$$

$$\bar{\bar{B}} = \begin{bmatrix} B_{ha} & B_{va} \\ B_{hb} & B_{vb} \end{bmatrix} = \bar{\bar{R}}_{12}\,(\bar{\bar{I}} - \bar{\bar{R}}_{10}\,\bar{\bar{R}}_{12})^{-1}\,\bar{\bar{X}}_{01} \tag{14c}$$

$$\bar{\bar{X}} = \begin{bmatrix} X_{hh} & X_{vh} \\ X_{hv} & X_{vv} \end{bmatrix} = \bar{\bar{X}}_{12}\,(\bar{\bar{I}} - \bar{\bar{R}}_{10}\,\bar{\bar{R}}_{12})^{-1}\,\bar{\bar{X}}_{01} \tag{14d}$$

We note that through (14) we have represented all the unknown coefficients in the two-layer DGF's in terms of half-space Fresnel coefficients. In the uniaxial limit, i.e. when $\epsilon_x = \epsilon_y$ and $\psi_2 = 0$, our results agree with those of Lee and Kong[3]. We shall take up the task of deriving explicit expressions for these half-space Fresnel coefficients in the next section.

5. HALF-SPACE FRESNEL COEFFICIENTS

We notice that for our two-layer problem there are three situations to consider.

SITUATION 1

Here we have free space in the region above the interface z = 0 and the biaxially anisotropic medium in the region below (Figure 2). A plane wave from above is incident on the interface. Depending on the polarization of the incident wave there are two cases to consider.

(i) h-wave incidence

For this case the electric fields in the two regions may be formulated as follows.

$$\bar{E}_0(\bar{r}) = \hat{h}_0^- e^{i\bar{\kappa}_0\cdot\bar{r}} + R_{hh}^{01}\hat{h}_0^+ e^{i\bar{k}_0\cdot\bar{r}} + R_{hv}^{01}\hat{v}_0^+ e^{i\bar{k}_0\cdot\bar{r}} , \quad z > 0 \tag{15a}$$

$$\bar{E}_1(\bar{r}) = X_{ha}\hat{a}^- e^{i\bar{\kappa}^a\cdot\bar{r}} + X_{hb}\hat{b}^- e^{i\bar{\kappa}^b\cdot\bar{r}} , \quad z < 0 \tag{15b}$$

The associated boundary conditions are given as follows.

$$\hat{z} \times \bar{E}_0(\bar{r}) = \hat{z} \times \bar{E}_1(\bar{r}) \qquad \text{at} \quad z = 0 \tag{16a}$$

$$\hat{z} \times \nabla \times \bar{E}_0(\bar{r}) = \hat{z} \times \nabla \times \bar{E}_1(\bar{r}) \qquad \text{at} \quad z = 0 \tag{16b}$$

On substituting (15) in (16) we obtain the following solutions for the Fresnel coefficients.

$$R_{hh}^{01} = -1 + X_{ha}\, v_{a-}^{+} + X_{hb}\, v_{b-}^{+} \tag{17a}$$

$$R_{hv}^{01} = X_{ha}\, h_{a-}^{+} + X_{hb}\, h_{b-}^{+} \tag{17b}$$

$$X_{ha} = -2k_{0z} \left(h_x^- \, y_{b-}^{+} - h_y^- \, x_{b-}^{+} \right) / \Delta^{+} \tag{17c}$$

$$X_{hb} = -2k_{0z} \left(h_y^- \, x_{a-}^{+} - h_x^- \, y_{a-}^{+} \right) / \Delta^{+} \tag{17d}$$

where all the unknown quantities are defined in Appendix C.

<u>(ii) v-wave incidence</u>

Here the electric fields in the two regions are formulated as follows.

$$\bar{E}_0(\bar{r}) = \hat{v}_0^{-} e^{i\bar{\kappa}_0\cdot\bar{r}} + R_{vv}^{01}\, \hat{v}_0^{+} e^{i\bar{k}_0\cdot\bar{r}} + R_{vh}^{01}\, \hat{h}_0^{+} e^{i\bar{k}_0\cdot\bar{r}} \quad , \; z > 0 \tag{18a}$$

$$\bar{E}_1(\bar{r}) = X_{va}\,\hat{a}\,e^{i\bar{\kappa}^a\cdot\bar{r}} + X_{vb}\,\hat{b}\,e^{i\bar{\kappa}^b\cdot\bar{r}}\quad, z < 0 \tag{18b}$$

By substituting (18) in (16), we obtain the following solutions.

$$R^{01}_{vv} = -1 + X_{va}\,h^+_{a-} + X_{vb}\,h^+_{b-} \tag{19a}$$

$$R^{01}_{vh} = X_{va}\,v^+_{a-} + X_{vb}\,v^+_{b-} \tag{19b}$$

$$X_{va} = \left[\left(T_{v^+x} - T_{v^-x} \right) y^+_{b-} - \left(T_{v^+y} - T_{v^-x} \right) x^+_{b-} \right] / \Delta^+ \tag{19c}$$

$$X_{vb} = -\left[\left(T_{v^+x} - T_{v^-x} \right) y^+_{a-} - \left(T_{v^+y} - T_{v^-x} \right) x^+_{a-} \right] / \Delta^- \tag{19d}$$

<u>SITUATION 2</u>

Here we have the biaxially anisotropic medium above the boundary $z = 0$ and the isotropic medium 2 below it. A plane wave from above impinges on the interface. There are two cases to consider depending upon the type of the incident wave .

<u>(i) a-wave incidence</u>

The electric fields $\bar{E}_1$ and $\bar{E}_2$ in the two regions are formulated as follows.

$$\bar{E}_1(\bar{r}) = \hat{a}^{-}\, e^{i\bar{\kappa}^{a}\cdot\bar{r}} + R^{12}_{aa}\, \hat{a}^{+}\, e^{i\bar{k}^{a}\cdot\bar{r}} + R^{12}_{ab}\, \hat{b}^{+}\, e^{i\bar{k}^{b}\cdot\bar{r}} \quad , \; z > 0 \tag{20a}$$

$$\bar{E}_2(\bar{r}) = X^{12}_{ah}\, \hat{h}^{-}_2\, e^{i\bar{\kappa}_2\cdot\bar{r}} + X^{12}_{av}\, \hat{v}^{-}_2\, e^{i\bar{\kappa}_2\cdot\bar{r}} \quad , \; z < 0 \tag{20b}$$

Substituting (20) in (16) we obtain the following solutions.

$$R^{12}_{aa} = -\left(x^{-}_{a^-}\, y^{-}_{b^+} - x^{-}_{b^+}\, y^{-}_{a^-} \right) / \Delta^{+} \tag{21a}$$

$$R^{12}_{ab} = -\left(x^{-}_{a^+}\, y^{-}_{a^-} - x^{-}_{a^-}\, y^{-}_{a^+} \right) / \Delta^{+} \tag{21b}$$

$$X^{12}_{ah} = R^{12}_{aa} v^{-}_{a^+} + R^{12}_{ab} v^{-}_{b^+} + v^{-}_{a^-} \tag{21c}$$

$$X^{av}_{12} = R^{12}_{aa} h^{-}_{a^+} + R^{12}_{ab} h^{-}_{b^+} + h^{-}_{a^-} \tag{21d}$$

where

$$\xi^{j}_{\zeta^i} = k^{j}_{\zeta^i}\, T_{v^j \xi} + v^{j}_{\zeta^i}\, k_{2z}\, h^{j}_{\xi} - T_{\zeta^i \xi} \tag{22}$$

for $\xi = \{x, y\}$, $\zeta = \{a, b\}$, $\{i, j\} = \{+, -\}$

All other quantities are given in Appendix C; but we have to replace

k_{0z} by k_{2z}.

(i) b-wave incidence

The electric fields $\bar{E}_1$ and $\bar{E}_2$ in the two regions are formulated as follows.

$$\bar{E}_1(\bar{r}) = \hat{b}^- e^{i\bar{\kappa}^b \cdot \bar{r}} + R_{bb}^{12} \hat{b}^+ e^{i\bar{k}^b \cdot \bar{r}} + R_{ba}^{12} \hat{a}^+ e^{i\bar{k}^a \cdot \bar{r}} \quad , z > 0 \tag{23a}$$

$$\bar{E}_2(\bar{r}) = X_{bh}^{12} \hat{h}_2^- e^{i\bar{\kappa}_2 \cdot \bar{r}} + X_{bv}^{12} \hat{v}_2^- e^{i\bar{\kappa}_2 \cdot \bar{r}} \quad , z < 0 \tag{23b}$$

From (23) and (16) we obtain the following solutions.

$$R_{bb}^{12} = R_{aa}^{12}$$

$$R_{ba}^{12} = R_{ab}^{12} \qquad \left|\begin{array}{c} \text{Replacements} \\ a \to b \\ b \to a \end{array}\right| \tag{24}$$

$$X_{bh}^{12} = X_{ah}^{12}$$

$$X_{bv}^{12} = X_{av}^{12}$$

SITUATION 3

In this situation the geometry is the same as in Situation 1; the only difference is that the wave is incident from below. Once again there are two cases to consider.

(i) a-wave incidence

The electric fields in the two regions are formulated as

$$\bar{E}_0(\bar{r}) = X_{ah}\,\hat{h}_0^{+}\,e^{i\bar{k}_0\cdot\bar{r}} + X_{av}\,\hat{v}_0^{+}\,e^{i\bar{k}_0\cdot\bar{r}} \quad , \; z > 0 \tag{25a}$$

$$\bar{E}_1(\bar{r}) = \hat{a}^{+}\,e^{i\bar{k}^{a}\cdot\bar{r}} + R_{aa}\,\hat{a}^{-}\,e^{i\bar{\kappa}^{a}\cdot\bar{r}} + R_{ab}\,\hat{b}^{-}\,e^{i\bar{\kappa}^{b}\cdot\bar{r}} \quad , \; z < 0 \tag{25b}$$

Applying the boundary conditions (16), we have the following solutions.

$$\begin{array}{lcl|c|}
R_{aa} & = & R_{aa}^{12} & \\
R_{ab} & = & R_{ab}^{12} & \text{Replacement of} \\
 & & & \text{Superscripts} \\
X_{ah} & = & X_{ah}^{12} & + \rightarrow - \\
 & & & - \rightarrow + \\
X_{av} & = & X_{av}^{12} &
\end{array} \tag{26}$$

Besides the above replacements, also note the following changes: $\xi^{j}_{\zeta i}$ is here defined by (A2) and in other equations k_{2z} is replaced by k_{0z}.

(ii) b-wave incidence

The electric fields in the two regions are formulated as

$$\bar{E}_0(\bar{r}) = X_{bh}\,\hat{h}_0^{+}\, e^{i\bar{k}_0\cdot\bar{r}} + X_{bv}\,\hat{v}_0^{+}\, e^{i\bar{k}_0\cdot\bar{r}} \quad , \; z > 0 \tag{27a}$$

$$\bar{E}_1(\bar{r}) = \hat{b}^{+}\, e^{i\bar{k}^{b}\cdot\bar{r}} + R_{bb}\,\hat{b}^{-}\, e^{i\bar{\kappa}^{b}\cdot\bar{r}} + R_{ba}\,\hat{a}^{-}\, e^{i\bar{\kappa}^{a}\cdot\bar{r}} \quad , \; z < 0 \tag{27b}$$

From (27) and (16) we have the following solutions.

$$\begin{array}{lc|c|}
R_{bb} = R_{aa} & & \\
R_{ba} = R_{ab} & & \text{Replacements} \\
 & & a \rightarrow b \\
X_{bh} = X_{ah} & & b \rightarrow a \\
X_{bv} = X_{av} & &
\end{array} \tag{28}$$

It is to be noted that in the uniaxial limit, our results (17), (19), (21), (24), (26) and (28) agree with corresponding results of Lee and Kong[3].

6. NUMERICAL EXAMPLE

One quantity of interest in our results is the reflection coefficient of the two layer medium. To illustrate its characteristics we have taken the following example. Region 0 is free space with permittivity ϵ_0 while

Region 2 has permittivity $(6.0+i.006)\epsilon_0$. The anisotropic medium is characterised by the following permittivities: $\epsilon_x = (2.8+i.001)\epsilon_0$, $\epsilon_y = (3.0+i.002)\epsilon_0$, $\epsilon_z = (3.2+i.003)\epsilon_0$. The tilt angles are $\psi_1 = 20°$ and $\psi_2 = 40°$. The azimuthal angle of incident wave is 45°. The thickness of the layer is 1.0 m and the frequency is chosen as 10 GHz. In Figure 3a we have plotted the reflectivities, $r_h = |R_{hh}|^2 + |R_{hv}|^2$ and $r_v = |R_{vv}|^2 + |R_{vh}|^2$. The behaviour here appears to be a familiar one. For small angles on incidence we notice that $r_h = r_v$ and there is a null (Brewster angle!) at around 60° for r_v. The anisotropic nature of the medium is not very evident by looking at this reflectivity plot. This is perhaps the axis of symmetry is very close to the incident plane. Next we consider the case when $\psi_2 = 80°$. Since the axis of symmetry is well away from the incident plane the anisotropic behaviour is very apparent in Figure 3b. Just to illustrate the contributions of like-polarised and cross-polarised reflectivities to r_v we have plotted $|R_{vv}|^2$ and $|R_{vh}|^2$in Figure 3c. Perhaps in this example the cross-polarised reflectivity is rather small compared to the like polarised reflectilvity. But the fact that there exists cross-polarization is significant.

7. CONCLUSION

We have derived in this report the dyadic Green's function of a two layer biaxially anisotropic medium. The formulation is based on the solution for the unbounded case obtained in Part I. But the evaluation of the various coefficients required the use of a matrix method which expresses these two-layer coefficients in terms of half-space reflection and transmission coefficients. This procedure is completed by deriving explicit expressions for the various half-space Fresnel reflection and transmission coefficients. To illustrate the computational simplicity we have provided a numerical example where we have plotted the reflectivities versus the incident angle.

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APPENDIX A

The DGF's satisfy the following equations

$$\nabla \times \nabla \times \bar{\bar{G}}_{00}(\bar{r},\bar{r}') - \omega^2\mu\,\epsilon_0\,\bar{\bar{G}}_{00}(\bar{r},\bar{r}') = \bar{\bar{I}}\,\delta(\bar{r} - \bar{r}') \tag{A1}$$

$$\nabla \times \nabla \times \bar{\bar{G}}_{10}(\bar{r},\bar{r}') - \omega^2\mu\,\bar{\bar{\epsilon}}\,\bar{\bar{G}}_{10}(\bar{r},\bar{r}') = 0 \tag{A2}$$

$$\nabla \times \nabla \times \bar{\bar{G}}_{20}(\bar{r},\bar{r}') - \omega^2\mu\,\epsilon_2\,\bar{\bar{G}}_{20}(\bar{r},\bar{r}') = 0 \tag{A3}$$

where ω is the angular frequency. The boundary conditions associated with these DGF's are given as follows:

$$\hat{z} \times \bar{\bar{G}}_{00}(\bar{r},\bar{r}') = \hat{z} \times \bar{\bar{G}}_{10}(\bar{r},\bar{r}') \quad \text{at } z = 0 \tag{A4a}$$

$$\hat{z} \times \nabla \times \bar{\bar{G}}_{00}(\bar{r},\bar{r}') = \hat{z} \times \nabla \times \bar{\bar{G}}_{10}(\bar{r},\bar{r}') \quad \text{at } z = 0 \tag{A4b}$$

$$\hat{z} \times \bar{\bar{G}}_{10}(\bar{r},\bar{r}') = \hat{z} \times \bar{\bar{G}}_{20}(\bar{r},\bar{r}') \quad \text{at } z = -d \tag{A5a}$$

$$\hat{z} \times \nabla \times \bar{\bar{G}}_{10}(\bar{r},\bar{r}') = \hat{z} \times \nabla \times \bar{\bar{G}}_{20}(\bar{r},\bar{r}') \quad \text{at } z = -d \tag{A5b}$$

APPENDIX B

$$\hat{\mathrm{a}}^{+} = \left(\nu^{au} \right)^{-1/2} \bar{\bar{\chi}} \cdot \hat{a}^{+} \tag{B1a}$$

$$\hat{\mathrm{a}}^{-} = \left(\nu^{ad} \right)^{-1/2} \bar{\bar{\chi}} \cdot \hat{a}^{-} \tag{B1b}$$

$$\hat{\mathrm{b}}^{+} = \left(\nu^{bu} \right)^{-1/2} \bar{\bar{\chi}} \cdot \hat{b}^{+} \tag{B2a}$$

$$\hat{\mathrm{b}}^{-} = \left(\nu^{bd} \right)^{-1/2} \bar{\bar{\chi}} \cdot \hat{b}^{-} \tag{B2b}$$

$$\nu^{\ell u} = \hat{\ell}^{+} \cdot \bar{\bar{\chi}}^{2} \cdot \hat{\ell}^{+} , \qquad \ell = a \text{ or } b \tag{B3a}$$

$$\nu^{\ell d} = \hat{\ell}^{-} \cdot \bar{\bar{\chi}}^{2} \cdot \hat{\ell}^{-} , \qquad \ell = a \text{ or } b \tag{B3b}$$

where

$$\hat{a}^{+} = \frac{1}{h^{au}} \left[\frac{\hat{k}^{a} \times \hat{o}_{1}}{|\hat{k}^{a} \times \hat{o}_{1}|} + \frac{\hat{k}^{a} \times \hat{o}_{2}}{|\hat{k}^{a} \times \hat{o}_{2}|} \right] \tag{B4a}$$

$$\hat{b}^{+} = \frac{1}{h^{bu}} \left[\frac{\hat{k}^{b} \times (\hat{k}^{b} \times \hat{o}_{1})}{|\hat{k}^{b} \times \hat{o}_{1}|} + \frac{\hat{k}^{b} \times (\hat{k}^{b} \times \hat{o}_{2})}{|\hat{k}^{b} \times \hat{o}_{2}|} \right] \tag{B4b}$$

$$\bar{k}^{\zeta} = \hat{\rho}\, k_{\rho} + \hat{z}\, k_{z}^{\zeta u} , \quad \zeta = a \text{ or } b \tag{B5}$$

$$h^{\zeta u} = \sqrt{2} \left\{ 1 + \frac{ (\hat{k}^{\zeta} \times \hat{o}_{1}) \cdot (\hat{k}^{\zeta} \times \hat{o}_{2}) }{ |\hat{k}^{\zeta} \times \hat{o}_{1}| \; |\hat{k}^{\zeta} \times \hat{o}_{2}| } \right\}^{1/2} \tag{B6}$$

$$\hat{k}^{\zeta} = \bar{k}^{\zeta} / k^{\zeta} \tag{B7}$$

$$\left\{ \begin{array}{c} \hat{o}_1 \\ \hat{o}_2 \end{array} \right\} = \hat{x} \left(\pm g_1 \cos \phi_2 + g_2 \sin \phi_1 \sin \phi_2 \right)$$

$$+ \hat{y} \left(\mp g_1 \sin \phi_2 + g_2 \sin \phi_1 \cos \phi_2 \right) + \hat{z} \cos \phi_1 \tag{B8}$$

$$g_1 = \left[\frac{\varepsilon_z(\varepsilon_y - \varepsilon_x)}{\varepsilon_y(\varepsilon_z - \varepsilon_x)} \right]^{1/2} \tag{B9a}$$

$$g_2 = \left[\frac{\varepsilon_x(\varepsilon_z - \varepsilon_y)}{\varepsilon_y(\varepsilon_z - \varepsilon_x)} \right]^{1/2} \tag{B9b}$$

For definiteness we have assumed in the above equations that $\varepsilon_x < \varepsilon_y < \varepsilon_z$.

$$\hat{a}^{-} = \frac{1}{h^{ad}} \left[\frac{\hat{\kappa}^a \times \hat{o}_1}{|\hat{\kappa}^a \times \hat{o}_1|} + \frac{\hat{\kappa}^a \times \hat{o}_2}{|\hat{\kappa}^a \times \hat{o}_2|} \right] \tag{B10a}$$

$$\hat{b}^{-} = \frac{1}{h^{bd}} \left[\frac{\hat{\kappa}^b \times (\hat{\kappa}^b \times \hat{o}_1)}{|\hat{\kappa}^b \times \hat{o}_1|} + \frac{\hat{\kappa}^b \times (\hat{\kappa}^b \times \hat{o}_2)}{|\hat{\kappa}^b \times \hat{o}_2|} \right] \tag{B10b}$$

$$\bar{\kappa}^{\zeta} = \hat{\rho}\, k_{\rho} + \hat{z}\, k_z^{\zeta d} \quad , \quad \zeta = a \text{ or } b \tag{B11}$$

$$h^{\zeta d} = h^{\zeta u} \left\{ u \rightarrow d \right\} \tag{B12}$$

APPENDIX C

$$\eta^{j}_{\zeta i} = \pm \left(\eta^{j}_{x}\, \zeta^{i}_{y} - \eta^{j}_{y}\, \zeta^{i}_{x} \right) / \delta^{j} \tag{C1}$$

where

$$\left\{ i,\ j \right\} = \left\{ +,\ - \right\}$$

$$\eta = \left\{ h,\ v \right\}$$

$$\zeta = \left\{ a,\ b \right\}$$

$$\eta = \begin{cases} h & \rightarrow\ + \\ v & \rightarrow\ - \end{cases}$$

$$\xi^{j}_{\zeta i} = h^{j}_{\zeta i}\, T_{vi\xi} + v^{j}_{\zeta i} \left(- k_{0z} \right) h^{i}_{\xi} - T_{\zeta i \xi} \tag{C2}$$

where $\xi = \left\{ x,\ y \right\}$

$$\delta^{i} = h^{i}_{x}\, v^{i}_{y} - h^{i}_{y}\, v^{i}_{x} \tag{C3}$$

$$T_{\zeta i \xi} = k_{\xi}\, \zeta^{i}_{z} - k^{\zeta i}_{z}\, \zeta^{i}_{\xi} \tag{C4}$$

$$ {}^{T}v^{j}_{\xi} = k_{\xi}\, v^{j}_{z} - j\,(-\,k_{0z})\, v^{j}_{\xi} \tag{C5} $$

$$ \Delta^{+} = x^{+}_{a-}\, y^{+}_{b-} - x^{+}_{b-}\, y^{+}_{a-} \tag{C6} $$

$$ \Delta^{-} = \Delta^{+} \left\{ \begin{array}{ccc} + & \rightarrow & - \\ - & \rightarrow & + \end{array} \right\} \tag{C7} $$

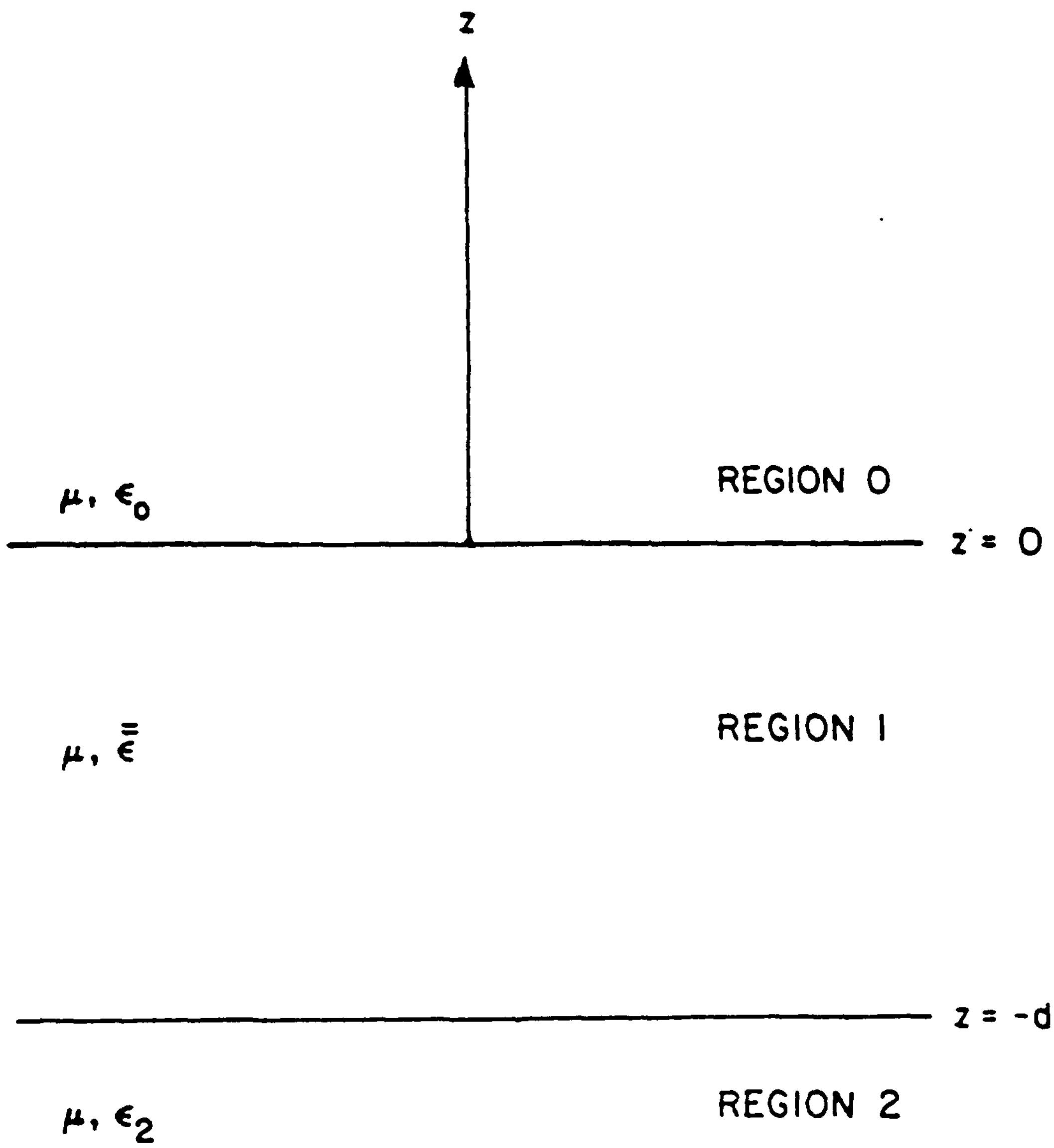


Figure 1. Geometry of the Problem

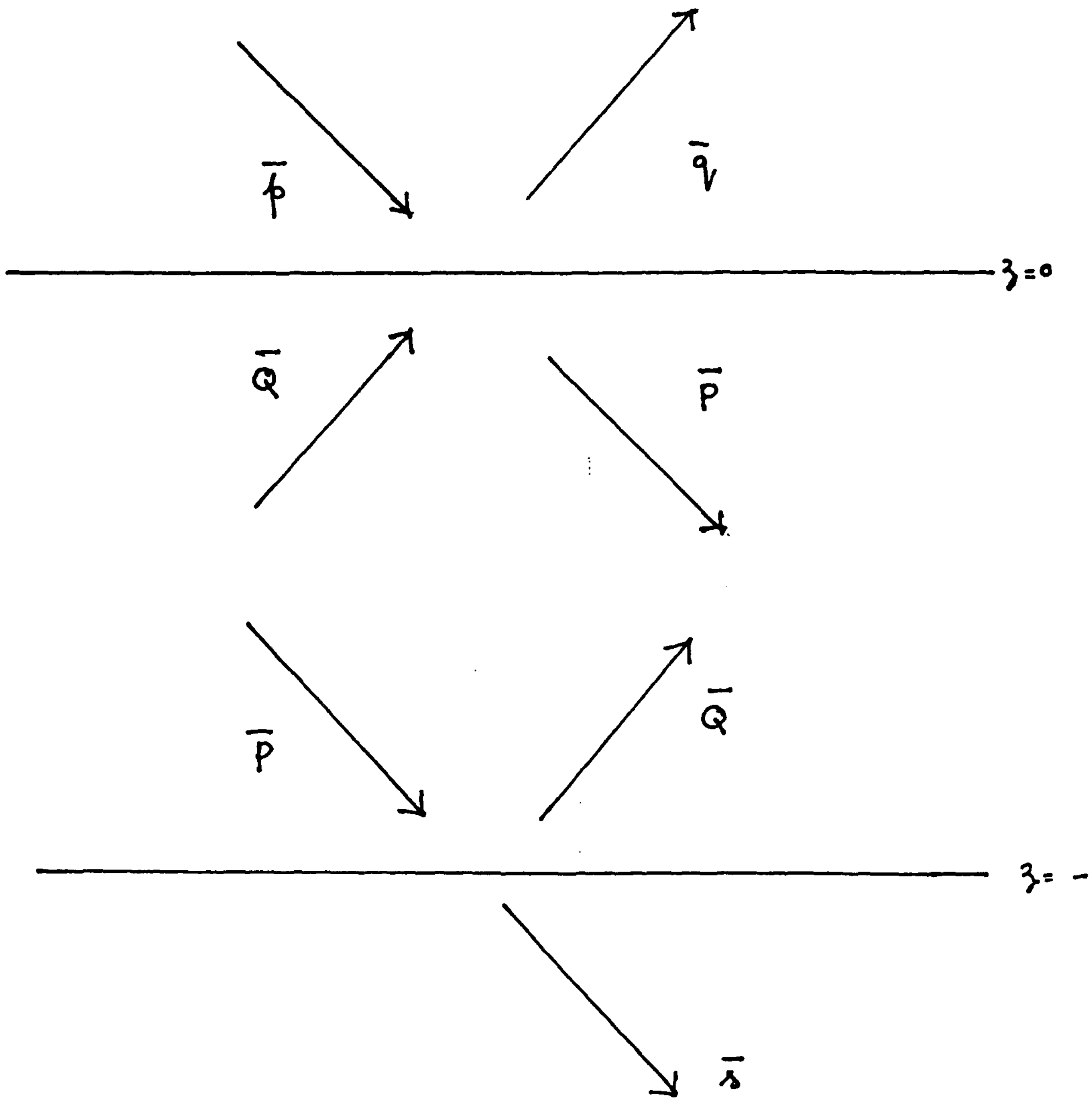
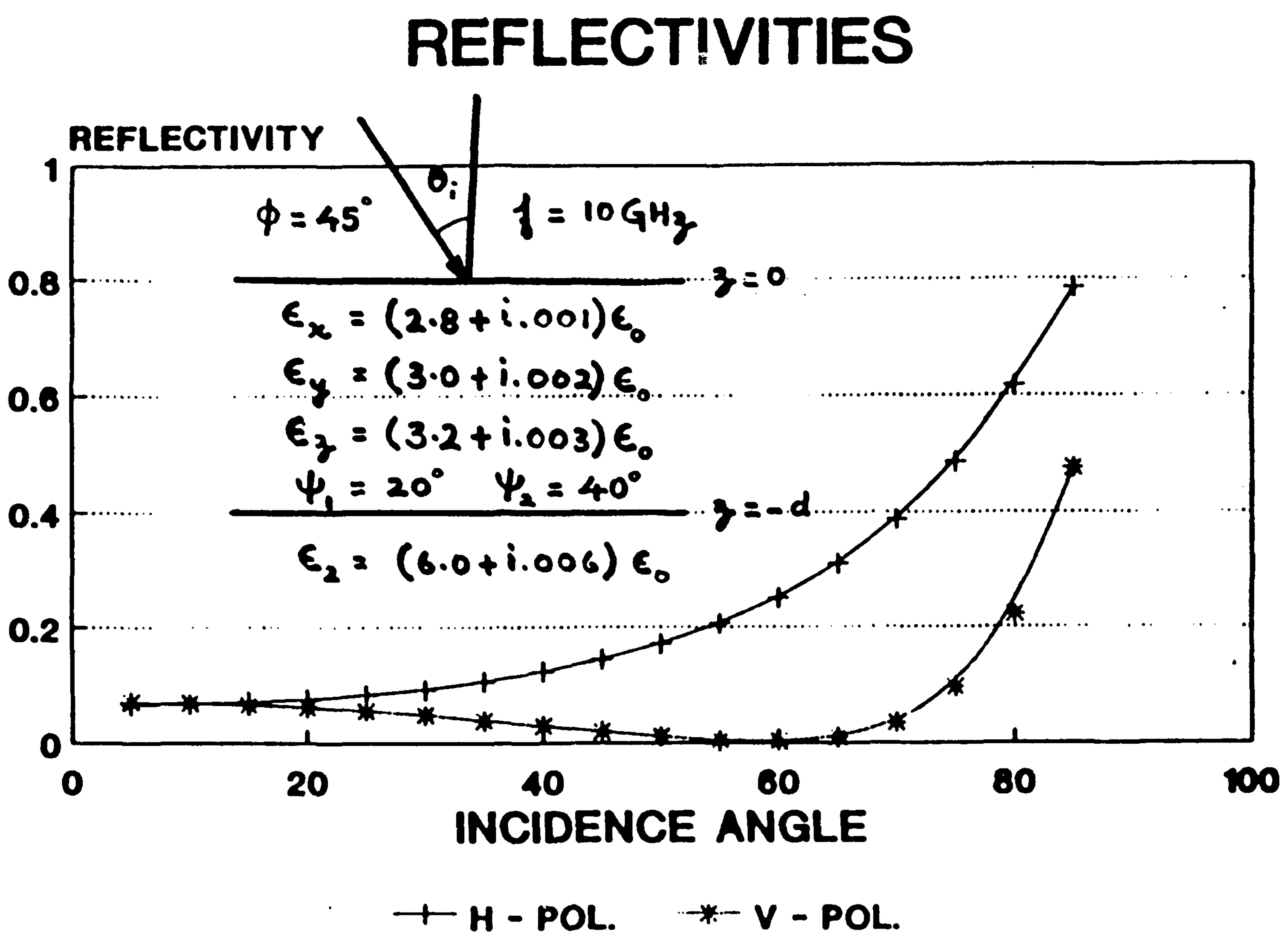


Figure 2. Amplitude Vectors of Waves in Two-Layer Medium

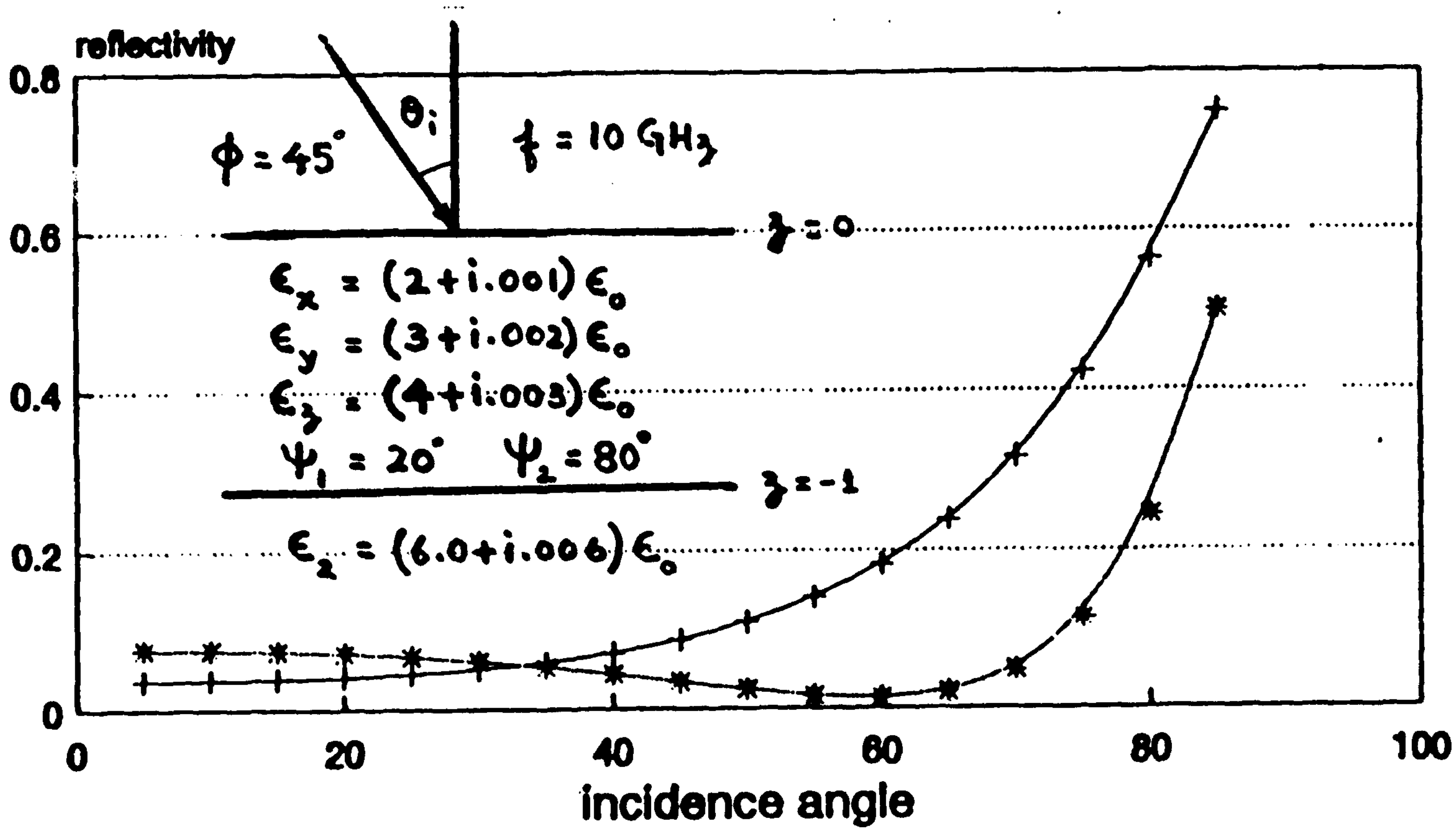


$r_h \equiv |R_{hh}|^2 + |R_{hv}|^2 \qquad r_v \equiv |R_{vv}|^2 + |R_{vh}|^2$

Figure 3a

Figure 3. Reflectivities Versus Incident Angle

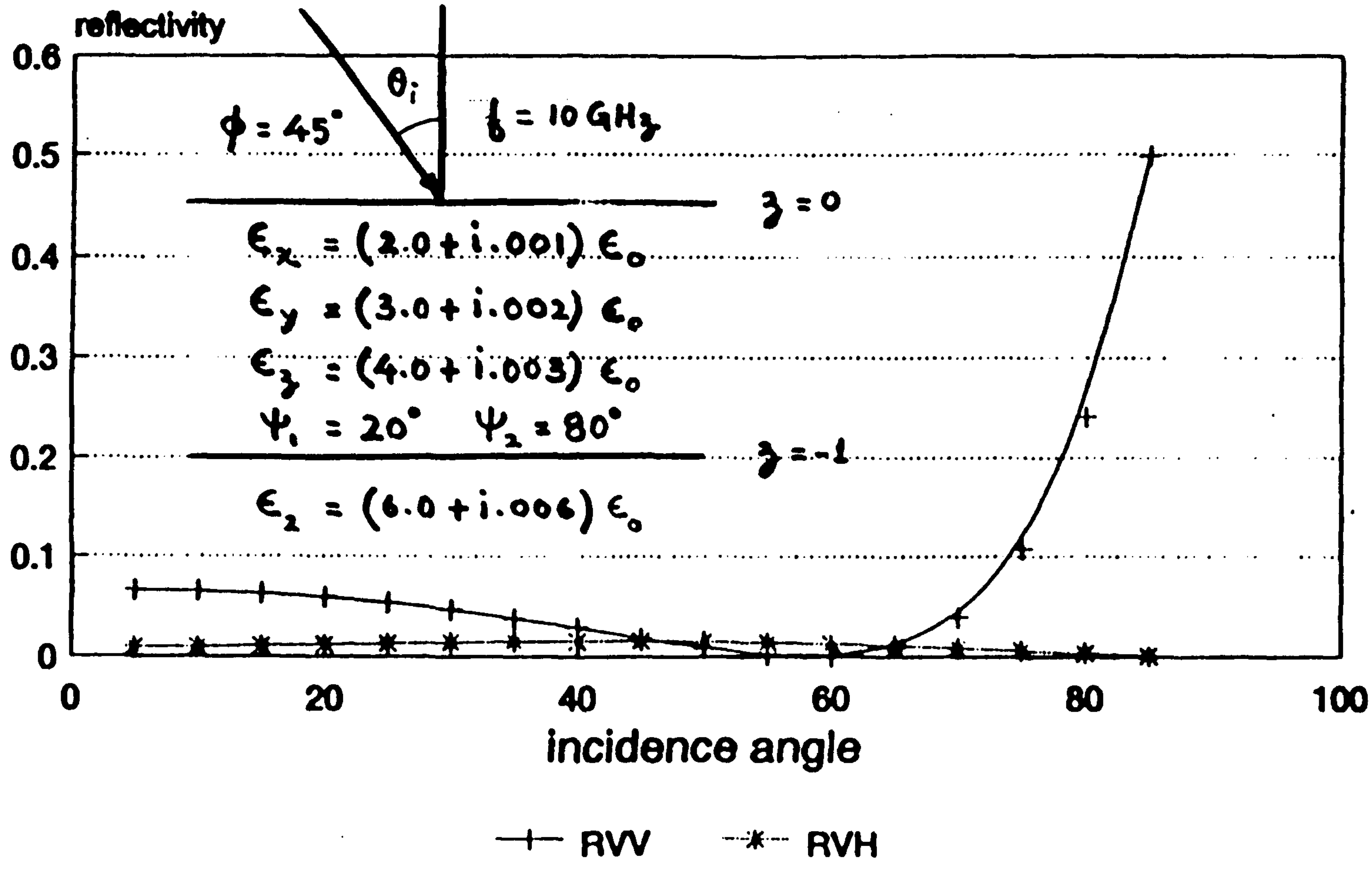
reflectivities 3



$r_h \equiv |R_{hh}|^2 + |R_{hv}|^2 \qquad r_v \equiv |R_{vv}|^2 + |R_{vh}|^2$

Figure 3b

reflectivities 2



$\text{RVV} \equiv |R_{vv}|^2 \qquad \text{RVH} \equiv |R_{vh}|^2$

Figure 3c

MISSION

OF

ROME LABORATORY

Mission. The mission of Rome Laboratory is to advance the science and technologies of command, control, communications and intelligence and to transition them into systems to meet customer needs. To achieve this, Rome Lab:

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