(2)

The Earth Satellite Program (ESP) Version 2.0

M 93B0000070
February 1994

R. Callwood
L. M. Gaffney

MITRE
Bedford, Massachusetts

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	February 1994	

4. TITLE AND SUBTITLE

The Earth Satellite Program (ESP) Version 2.0

5. FUNDING NUMBERS

F19628-94-C-0001

6. AUTHOR(S)

R. Callwood, L. M. Gaffney

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

The MITRE Corporation
202 Burlington Road
Bedford, MA 01730-1420

8. PERFORMING ORGANIZATION REPORT NUMBER

M 93B0000070

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

Electronic Systems Center
Air Force Materiel Command
Hanscom Air Force Base
Bedford, MA 01731

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release; distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

The Earth Satellite Program (ESP) version 2.0 is a user-friendly, highly graphical, Macintosh-based program that was developed for the Air Force Milstar program to support satellite constellation and ground site terminal design studies. The computer program generates satellite ground tracks; computes elevation angles, ranges, and range rates; determines outage zones for a constellation of satellites; and generates spot beam projections. Any elliptical Earth-orbit may be accommodated. A fourth-order Runge-Kutta integrator is used to propagate satellite orbits. Perturbations due to the Earth's oblateness (J_2 effects), atmospheric drag, and solar radiation pressure are included. This paper contains a user's guide to the program, presents the underlying mathematical models, and includes several sample outputs and the software on diskette.

14. SUBJECT TERMS

satellite, constellation, satellite orbits, spot beam projections

15. NUMBER OF PAGES

102

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	Unlimited

The Earth Satellite Program (ESP) Version 2.0

M 93B0000070
February 1994

R. Callwood
L. M. Gaffney

Contract Sponsor ESC
Contract No. F19628-94-C-0001
Project No. 634M
Dept. D063

MITRE
Bedford, Massachusetts

ABSTRACT

The Earth Satellite Program (ESP) version 2.0 is a user-friendly, highly graphical, Macintosh-based program that was developed for the Air Force Milstar program to support satellite constellation and ground site terminal design studies. The computer program generates satellite ground tracks; computes elevation angles, ranges, and range rates; determines outage zones for a constellation of satellites; and generates spot beam projections. Any elliptical Earth-orbit may be accommodated. A fourth-order Runge-Kutta integrator is used to propagate satellite orbits. Perturbations due to the Earth's oblateness (J_2 effects), atmospheric drag, and solar radiation pressure are included. This paper contains a user's guide to the program, presents the underlying mathematical models, and includes several sample outputs and the software on diskette.

ACKNOWLEDGMENTS

The authors express their gratitude to the many individuals who contributed to the success of this effort. The earlier computer programs of M. J. Foodman served as a point of departure for the ESP model. Special appreciation goes to Dr. J. H. Kwok of the Jet Propulsion Laboratory (JPL). His Artificial Satellite Analysis Program (ASAP) was an important source for the mathematical models for accelerations due to atmospheric drag and solar radiation pressure. The atmospheric density data file was adapted directly. ASAP was also an invaluable source of clarification as well as a means of validation for ESP. We thank MITRE's Cartography Services and G. S. Marzot for their assistance with the world map database and M. L. Robinson and M. A. Thomas for their clarification of the beam pattern function. We also acknowledge the careful attention of the reviewers of earlier drafts of this document: M. J. Foodman, Dr. N. D. Hulkower, D. V. Marsicano, Dr. R. A. Moynihan, M. S. Pavloff, C. M. Plummer, and B. E. Wolfinger. In addition, we are grateful to J. B. MacGillivray for her careful preparation of the manuscript. This effort was sponsored by the United States Air Force Electronic Systems Center.

TABLE OF CONTENTS

SECTION		PAGE
1	Introduction	1
1.1	Overview	1
1.2	Purpose	1
1.3	Report Organization	2
2	User's Guide	3
2.1	Overview	3
2.2	Program Requirements	3
2.3	Sample Usage	3
2.4	Using the Program	4
2.4.1	The Apple Menu	6
2.4.2	The File Menu	6
2.4.3	Window Manipulation	8
2.4.4	The Edit Menu	9
2.4.5	The Options Menu	9
2.4.6	The Windows Menu	22
2.5	Printing and Opening Documents from the Finder	22
2.6	Orbital Parameters Conversion Utility	22
3	Analytical Foundations	25
3.1	Overview	25
3.2	Fundamentals	25
3.2.1	Orbital Elements	25
3.2.2	Osculating Orbit	27
3.2.3	Additional Parameters	28
3.3	Equations of Motion	30
3.3.1	Perturbations Due to J_2 Effects	32
3.3.2	Perturbations Due to Atmospheric Drag	33
3.3.3	Perturbations Due to Solar Radiation Pressure	37
3.4	Integration Scheme	39
3.5	Initial Conditions	41
3.6	Determination of Satellite Ground Track	43
3.7	Range and Range Rate Calculations	44
3.8	Visibility Test	46
3.9	Spot Beam Projections	47
3.10	Loss Contours	53
3.11	Horizon Generation	54

SECTION		PAGE

4	Programming Considerations	57
4.1	Introduction	57
4.2	Floating Point Precision	57
4.3	Convergence and Numerical Tolerance	57
4.4	The Runge-Kutta Procedure	58
4.5	The Main Loops	59
4.6	Numeric Error Avoidance	60
4.7	Function and Procedure Testing	60
	List of References	63
	Appendix	65
	Distribution List	103

LIST OF FIGURES

FIGURE		PAGE
2-1	ESP Icons	5
2-2	New Document Dialog Box	5
2-3	Print Options Dialog Box	8
2-4	First Ground Track Dialog Box	10
2-5	Second Ground Track Dialog Box	10
2-6	First Elevation Angles Dialog Box	11
2-7	Second Elevation Angles Dialog Box	11
2-8	First Outage Zones Dialog Box	12
2-9	Second Outage Zones Dialog Box	12
2-10	First Spot Beam Projections Dialog Box	13
2-11	Second Spot Beam Projections Dialog Box	13
2-12	Geodetic (L) Versus Geocentric (L') Latitude	15
2-13	Second Spot Beam Projections Dialog Box with Contour Input Fields	18
2-14	Map Zooming	20
2-15	Outage Zone Recalculation Dialog Box	21
2-16	Sample Convert Parameters Dialog Box	23
2-17	Sample Convert Parameters Output	23
3-1	Semimajor Axis, Eccentricity, and Time of Pericenter Passage	26
3-2	Euler Angles	27
3-3	Osculating Orbit	28
3-4	Additional Parameters	29

FIGURE		PAGE
3-5	Two-body Geometry	31
3-6	Earth Cross-section	35
3-7	Test for Shadowing	38
3-8	Rotated Coordinate System	41
3-9	Subsatellite Right Ascension and Declination	43
3-10	Range Geometry	45
3-11	Elevation Angle Geometry	46
3-12	Spot Beam Projection Geometry	48
3-13	Spot Beam Angle Relationships	48
3-14	Azimuth Angle	50
3-15	Maximum θ Value	52
A-1	Geosynchronous Orbit Ground Track - Graphical Output	71
A-2	Geosynchronous Orbit Outage Zones - Graphical Output	79
A-3	Geosynchronous Orbit Spot Beam - Graphical Output	83
A-4	Geosynchronous Orbit Loss Contours - Graphical Output	87
A-5	Low-Earth Orbit Ground Track - Graphical Output	91
A-6	Low-Earth Orbit Outage Zones - Graphical Output	93
A-7	Low-Earth Orbit Spot Beams - Graphical Output	95
A-8	Molniya Orbit Ground Track - Graphical Output	99
A-9	Molniya Orbit Outage Zones - Graphical Output	101

LIST OF TABLES

TABLE		PAGE
A-1	Geosynchronous Orbit Ground Track - Echoed Inputs	68
A-2	Geosynchronous Orbit Ground Track - Sample Tabular Output	69
A-3	Geosynchronous Orbit Elevation Angles - Echoed Inputs	73
A-4	Geosynchronous Orbit Elevation Angles - Sample Tabular Output	74
A-5	Geosynchronous Orbit Outage Zones - Echoed Input	75
A-6	Geosynchronous Orbit Outage Zones - Sample Tabular Output	77
A-7	Geosynchronous Orbit Spot Beam - Echoed Inputs	81
A-8	Geosynchronous Orbit Spot Beam - Sample Tabular Output	82
A-9	Geosynchronous Orbit Loss Contours - Echoed Inputs	85
A-10	Geosynchronous Orbit Loss Contours - Sample Tabular Output	86

SECTION 1

INTRODUCTION

1.1 OVERVIEW

The Earth Satellite Program (ESP) is an orbital analysis software tool for Earth-orbiting satellites. The main features of the program, in a broad sense, are coverage, tracking, and mapping. More specifically, the program has the capability to

(a) generate the ground track of a satellite, that is, the projection of the satellite on the surface of the Earth as the satellite moves in its orbit over time.

(b) determine outage zones over a region of the Earth for a constellation of up to 200 satellites and a user-specified minimum elevation angle. An outage zone is a set of ground site locations that are unable to see a user-specified minimum number of satellites in the constellation over some time interval.

(c) project single or multiple antenna spot beams of a user-specified beam width in any direction onto the Earth's surface and track the location of these beams over time. The option to generate contours of equal loss for a single beam is available.

(d) calculate the elevation angle and range from ground site locations to a satellite, each as a function of time. Range rate is also included.

For each of these features, output is provided in tabular form. In addition, items (a) through (c) may be plotted on all or a portion of a world map. Satellite orbits are propagated using a fourth-order Runge-Kutta integration scheme. The program runs on a Macintosh computer and is written in THINK Pascal [4, 5]. The software on diskette is also attached.

1.2 PURPOSE

This effort evolved from some computer programs with similar features that were developed in the 1980s by MITRE D090. Previous programs were limited to circular, two-body orbits, assumed a radially symmetric, point mass Earth, and were prompt-driven. In contrast, ESP accommodates any elliptical orbit and includes the perturbations due to a non-spherical Earth, atmospheric drag, and solar radiation pressure. It is also simple to use; being Macintosh-based, it adheres to the standard Macintosh look-and-feel by implementing dialog boxes, pull-down menus, menu commands, icons, scroll bars, etc. Furthermore, unlike its predecessors, its mapping capability is an integral part of the program.

ESP was developed for the Milstar program to support system design trade-off analyses for both ground and space segments. In particular, it may be used to determine those satellite constellations or ground site terminal antenna designs, or some combination thereof, that provide adequate or even optimal coverage of the Earth. Because it is easy to learn and use and typically generates results in a matter of minutes, the program is a valuable tool for providing quick turn-around responses to questions that may arise from the System Program Office (SPO). However, nothing in the program restricts it to Milstar applications. ESP is quite general in nature and has successfully been used to support a number of programs at the MITRE Corporation.

ESP 2.0 is an upgrade from version 1.0 which was released in June 1991. Major enhancements include spot beam projections, the capability to enter larger constellations and to require multiple satellites to be in view of any ground site in the outage feature, and a more sophisticated mapping capability that includes map zooming.

1.3 REPORT ORGANIZATION

This paper documents the capabilities, usage, and mathematical models of ESP version 2.0, including recent enhancements and all features in version 1.0. It is organized into five major sections. An overview of the content of each of the sections that follows is presented here.

Section 2 serves as a user's guide. It begins with a discussion of some examples of specific kinds of problems that may be analyzed using the program. A detailed description of how to operate the program then follows. This section assumes a general familiarity with Macintosh-based applications that rely on a mouse.

The analytical foundations are provided in section 3. For the user who is unfamiliar with the six classical orbital elements, which are required inputs for all aspects of the program, the initial subsections are prerequisite reading. A detailed description of the equations of motion, perturbations, and integration algorithm is included. This section also explains how the satellite position vector is used to generate the ground track and perform the elevation angle, range, and range rate calculations, and how the spot beam projections, horizon, and loss contours are derived.

Section 4 contains a discussion of computer programming considerations. Since computers can represent numbers with only finite precision, errors in floating point calculations are inevitable. This section details how ESP handles the problems of floating point calculations. It also describes how the program was tested and validated.

Finally, some example runs illustrating the program's main features are provided in the appendix. This section includes a brief description of the various types of orbits analyzed. All three types of program outputs — echoed input data and tabular and graphical results — are shown. However, since the non-graphical output is generally quite lengthy, only a sample of this type of output is given for each feature.

SECTION 2

USER'S GUIDE

2.1 OVERVIEW

This section describes how to use ESP 2.0. The program presents the standard Macintosh user interface. For a user familiar with operating Macintosh programs in general, exploring the program should not be difficult. Specifically, the user should know how to click and drag items, invoke pull-down menu commands, size windows, and use other features common to Macintosh programs.

2.2 PROGRAM REQUIREMENTS

ESP 2.0 requires a Macintosh II with four megabytes of random access memory (RAM), System Software version 6.0.5 or later, and a high density floppy disk drive. The specific features required by the program are an MC68881/882 mathematics coprocessor, Color QuickDraw[1], and a large monitor (at least 12 inches if monochrome, 13 inches if color), all of which are standard on most Macintosh IIs. If any of these features are not present, ESP will inform the user of the problem and then halt. A color monitor is recommended but not required. The files required to run ESP 2.0 are ESP itself, ESPmap, and ESPdensity. These files are on the diskette provided with this document.

Certain Macintosh initialization files, or INITs, that operate in the background may interfere with ESP. If the user experiences problems running ESP, the INITs should be removed until ESP runs normally. This is done by systematically taking them out of the system folder and restarting the Macintosh. In System 7.0, INITs are referred to as system extensions. They can be found in the Extensions folder within the System folder.

2.3 SAMPLE USAGE

Before exploring ESP's various menus and commands, some examples of the program's utility and application are presented. The intent of this section is to provide the reader with a flavor of the various problems that may be analyzed.

The satellite ground track indicates the location of the satellite with respect to the geography of the rotating Earth. Knowledge of this location is important when addressing various coverage issues. By inspection, one can obtain a general sense as to what regions of the Earth are likely to have coverage. The ground track may be used as a tool in the design of satellite constellations that provide continuous world-wide coverage. By examining the individual ground traces of several different orbits, one may gain insight into the types of orbit combinations that optimize coverage. (This is especially useful in conjunction with the outage

---

[1]Color QuickDraw is the part of the Macintosh system software that supports color graphics. It is not the same as a color monitor. Color QuickDraw is not present on some older Macintosh models.

zone feature.) A typical run propagates a satellite trajectory over a 24-hour period. Longer runs are permitted by the program; however, no studies have been made as to how the accuracy of the model degrades over time. For such longer runs there are more appropriate tools available, such as ASAP listed in the reference section.

The elevation angle calculation may be used to determine how low a ground site terminal antenna must be pointed in order to maintain coverage. In general, the elevation angle from a given ground site to a satellite will vary as the satellite moves in its orbit. The elevation calculation will indicate the elevation angle at user-specified times over the course of the run. The ground site antennae should then be designed so that the ground site is capable of contacting the satellite over the range of angles calculated. Alternatively, the elevation angle calculation may be used to assess sensitivities of orbital parameters to a minimum elevation angle constraint. For example, by calculating the elevation angles from ground sites to a satellite in geosynchronous[2] orbit with various inclinations, the program can be used to determine the maximum tolerable inclination drift as a function of minimum elevation angle.

The range, which is the distance from a given ground site to the satellite, is useful for determining the delay time between the transmitting and receiving of a signal. The range can be divided by the speed of light to obtain the time required for the signal to travel. The time derivative of range, or range rate, is useful in determining the extent of the Doppler shift on signals. With this information available, the ground site terminals can then be designed to take the changing frequencies into account.

The outage zones calculation is an important tool for determining what satellite configurations and antenna designs and placements will result in optimal coverage. Several satellite orbits may be entered at once, whereupon ESP will determine those ground sites that are not covered, and an approximation of the outage time at each site. The orbit of each satellite can be adjusted individually so that gaps in coverage may be filled. The number of satellites, minimum antenna elevation angle, and minimum number of satellites in view can each be adjusted individually in order to assess the sensitivity of coverage to the constellation's configuration. These kinds of analyses can assist in determining the trade-off between more satellites and more powerful or complex antennae.

The spot beam feature, in addition to simply determining beam coverage, can be used to assess the sensitivity of beam coverage to satellite placement and/or beam width. Since a given beam width corresponds (approximately) to some elevation angle, this feature can also be used to generate elevation angle contours. In addition, contours of equal decibel loss show how signal strength decreases as a function of distance from the beam center.

2.4 USING THE PROGRAM

The user is presumed to have a basic knowledge of the six elements of an orbit and of elementary perturbed two-body theory. An adequate background can be obtained by reading section 3 of this document.

---

[2]"Geosynchronous" is defined in appendix A.

As with any Macintosh program, ESP is started by double-clicking its icon (see figure 2-1) or by selecting it and choosing Open from the Finder's File menu. In order for the program to run properly, the files ESPmap and ESPdensity must be in the same folder as ESP. If the support files are not present, ESP will still run. If ESPmap cannot be found or loaded, ground tracks, outage zones, and spot beams will still be plotted, but on a blank rectangle. The only visible geographical frame of reference will be the longitude and latitude labels along the edges of the plot. If ESPdensity cannot be found or loaded, ESP will compute the orbits as if there were no atmosphere. After ESP loads the support files, a dialog box, shown in figure 2-2 and described in section 2.4.2 under New..., is displayed asking the user to choose a calculation type.

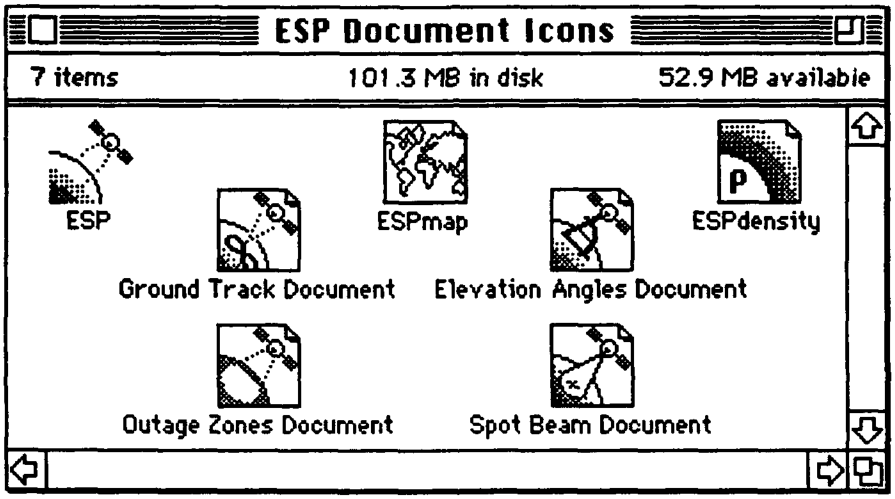


Figure 2-1. ESP Icons

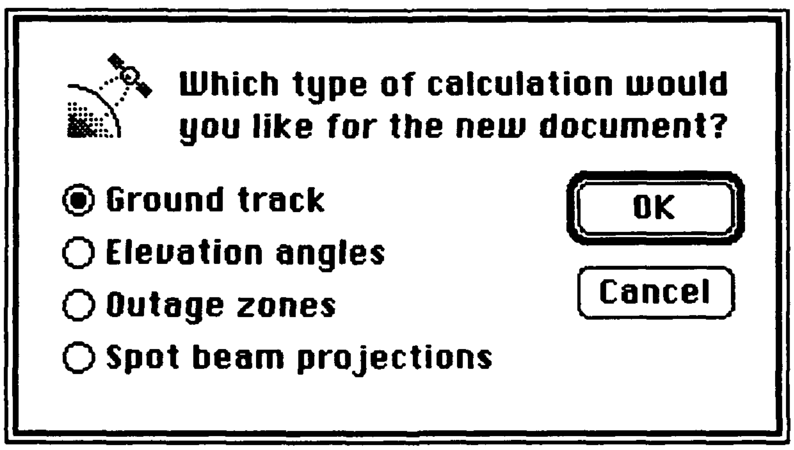


Figure 2-2. New Document Dialog Box

The user may also invoke ESP from any previously saved ESP documents (saving is described in section 2.4.2, under Save and Save As...). This is accomplished by selecting one or more of these documents and then either double-clicking on one of them or choosing Open from the Finder's File menu. The method is the same as when opening any other Macintosh document. In this case the dialog box in figure 2-2 will not be displayed, nor will the user be prompted to enter new inputs.

Within ESP, five menus are displayed across the top of the screen. The first menu is the usual apple menu. The second menu is the File menu, which contains commands for opening, closing, and printing documents. The third menu, the Edit menu, is not used in the current version, but is fully available for use by desk accessories. The next menu is the Options menu, which is used to invoke any of the program's four main features; it contains all commands for calculations and drawing maps. Finally, the Windows menu enables the user to reactivate document windows that have been temporarily closed or hidden by other windows.

Occasionally the user may request an operation, such as New or Open, that would result in more windows than ESP can handle. (The limit is currently 100 windows at a time.) If this happens, a message will appear stating that too many windows are open. In this situation, the user must permanently close some other window, as described in section 2.4.3.

2.4.1 The Apple Menu

<u>About ESP...</u>

This command displays the version, authors, and copyright notice.

<u>Desk Accessories</u>

Any desk accessory installed on the system may be invoked as usual. When a desk accessory is opened, the Edit menu, normally dormant in ESP, is made available for copying, cutting, and pasting. Some of the commands in the File menu, which are inappropriate for a desk accessory, are disabled. If the Macintosh is running MultiFinder or System 7.0, the desk accessory will temporarily hide ESP's menus and substitute its own.

2.4.2 The File Menu

<u>New...</u>

The New... command creates a new untitled window. A dialog box is presented for selecting the desired type of calculation: ground track, outage zones, elevation angles, or spot beam projections (see figure 2-2). The elevation angles option includes range and range rate calculations. After a calculation type is selected, two dialog boxes are displayed in succession requesting inputs for the orbital calculations. These dialog boxes are described in section 2.4.5.

Open...

The Open... command presents the standard dialog box for opening documents. Only documents created by ESP will be listed. A particular document is opened by either selecting its name in the list and clicking the "Open" button or by double-clicking its name. If a document was created by an earlier version of ESP, the program will notify the user that if the document is saved again, it will be saved in version 2.0 format. The version number of a document can be determined from the Finder in the usual manner with the Get Info command under the File menu.

Close

This command closes the active window. If the window is the only window open fo particular document and the document has not been saved since its last calculation, the user will be given an opportunity to save the document.

Save

This command saves the active window and its inputs. If the document represented by the window has both a map and tabular output, both representations of the data are automatically saved as one file in addition to the input dialog boxes. If the document does not yet have a title, this command is equivalent to the Save As... command.

Save As...

This command saves the active window under a new title. A dialog box will appear for entering the desired name. The new title will be reflected in the document's window or windows. If a document already exists on the disk with the same name, the user will be given the choice of either replacing the existing document or choosing a different title.

Revert to Saved...

The Revert command has not been implemented in this release of ESP. One can, however, retrieve the saved version of a document by doing the following. First, all windows associated with the document must be closed. A message will then appear asking the user whether or not to save the document. The user should click the "No" button, then choose Open from the File menu, and select the document.

Page Setup...

This command presents a dialog box that allows the user to change the printer settings. The dialog box will be different for different printers. Example settings that can be changed are reduction/enlargement, page orientation, and printer effects.

Print...

This command enables the user to print the active document's calculation results, and the map, if the document contains ground track, outage zone, or spot beam calculations. If a map is available, a dialog box will be presented asking which portions of the output are to be printed (see figure 2-3). The output is divided into three categories: the echoed input, the tabular output, and the map. This feature was designed so that the user may print the inputs and the textual output on a laser or impact printer and the map on a color printer. It is also useful if the user desires only part of the output. If the computer is on a network with more than one printer, the Page Setup command should be executed after switching to a different printer. Once the output types to be printed are selected (or immediately after selecting Print... in the case of an elevation angles document), the standard print dialog box is presented and asks for page numbers and other information specific to the job. Print spooling is supported. For legibility, the printout uses a different character font from that on the screen.

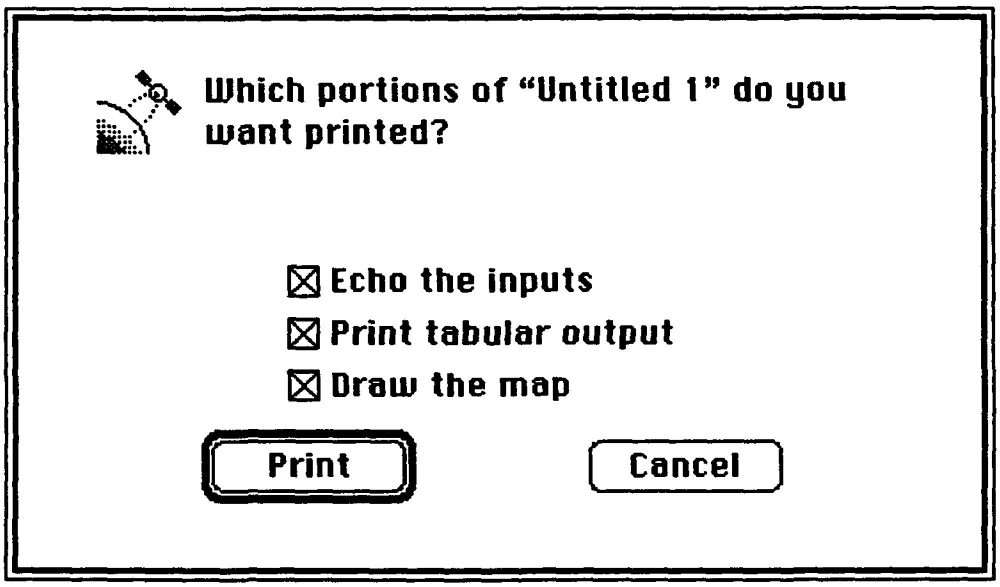


Figure 2-3. Print Options Dialog Box

Quit ESP

This command terminates execution of ESP. If any documents have not been saved, the user is given the opportunity to save them.

2.4.3 Window Manipulation

This subsection describes properties of ESP windows that are not present in all Macintosh programs.

Although the height of a text window may be changed by dragging the size icon in the lower right corner of the window, the width cannot be changed. (Both the width and the height of a map window can be adjusted, as in most Macintosh applications.)

Outage zone and spot beam documents have a horizontal scroll bar at the bottom of their text windows. This scroll bar does not scroll between columns but is used instead to switch between outage zones or individual beam projections.

Holding down the command key (⌘) or the option key while clicking the close box in the upper left corner of a window hides the window temporarily. The window is still in memory, however, and may be recalled using the Windows menu. Clicking the close box without holding down either of these keys closes the window permanently. Note that this behavior is different from that of ESP 1.0, where clicking the close box without the command or option key hid the window temporarily.

If a message appears asking the user to close some windows, the user must close a window permanently, i.e., without using the command or option keys.

2.4.4 The Edit Menu

At this time, ESP does not use the editing commands (Undo, Cut, Copy, Paste, and Clear). However, these commands are made available when a desk accessory is opened.

2.4.5 The Options Menu

The first four commands in this menu are Calculate Ground Track..., Calculate Elevation Angles..., Calculate Outage Zones..., and Calculate Spot Beam Projections.... After choosing one of these commands, the user goes through the same process for each. Two dialog boxes appear in succession, in which the user fills in information about the satellite or constellation and the particular run. Not all the input fields are needed for the ground track, elevation angle, or spot beam calculations, so certain fields are grayed out in these cases. Any information entered into these fields will be ignored. The dialog boxes are shown in figures 2-4 through 2-11 with their default entries.

The first dialog box solicits information about the satellite or constellation. The scroll bar and the field for the number of satellites are used in defining constellations of satellites. Since constellations are used only for the outage zone calculations, the discussion of these fields will be deferred to the section on outage zones. The next seven fields are for inputting the six classical orbital elements of the satellite. These elements are explained in section 3.2.1. As indicated in the dialog box, all angles are in degrees, and distance is in kilometers. The inclination must be between 0° and 180° inclusive. The longitude of the node and the argument of perigee must be greater than or equal to 0° and less than 360°. Dates in the dialog boxes must be in the form YYYYMMDD (e.g. 19930704 for 4 July 1993) and times must be in the form HHMMSS using twenty-four hour time (e.g., 130000 for 1:00 PM).

Note that the longitude of the node, referred to above, is measured relative to the vernal equinox and is not the same as the more familiar longitude system, which is based on the Greenwich, England prime meridian. The vernal equinox is based on a coordinate system that is fixed with respect to the stars, whereas the prime meridian rotates with the Earth. Hence, the subsatellite longitude, which is relative to the prime meridian, is in general often not equal to the longitude of the node. For example, for the default date and time inputs the difference

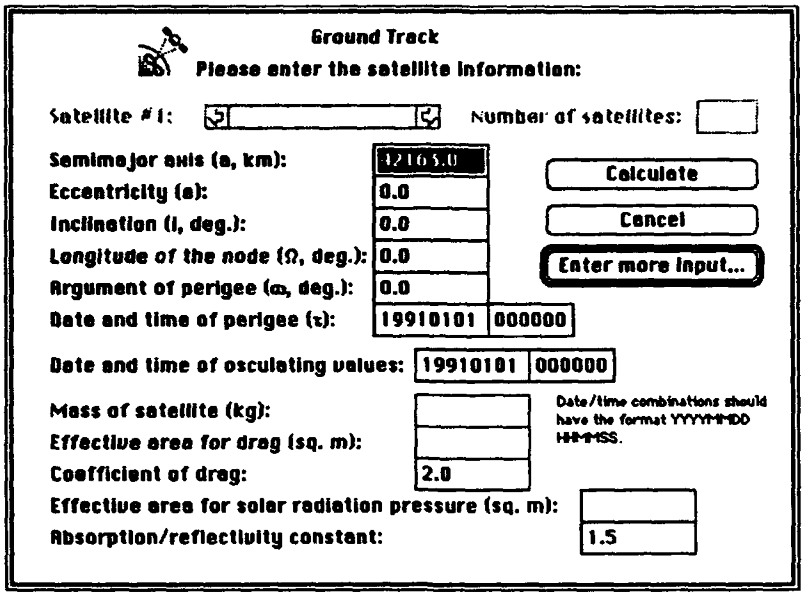


Figure 2-4. First Ground Track Dialog Box

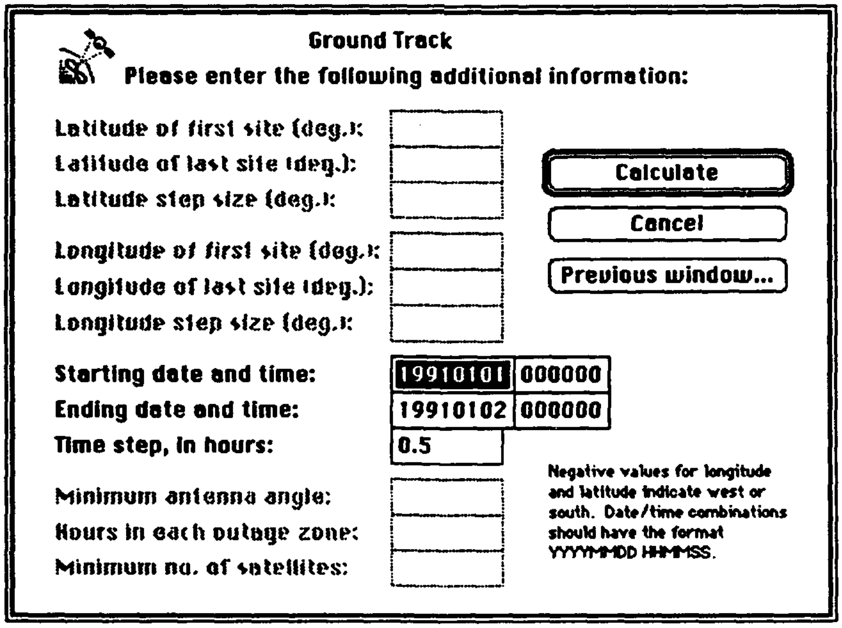


Figure 2-5. Second Ground Track Dialog Box

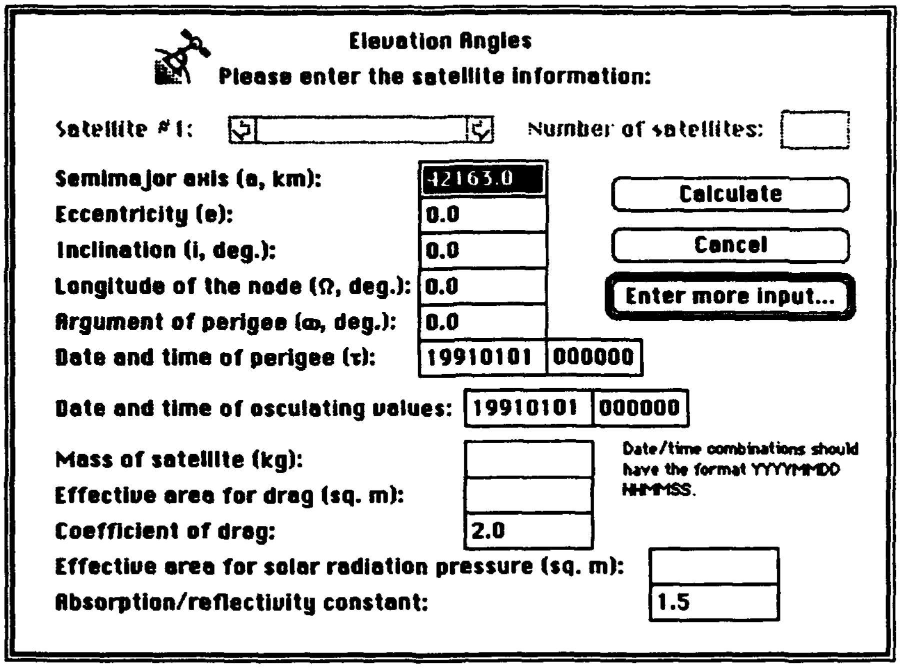


Figure 2-6. First Elevation Angles Dialog Box

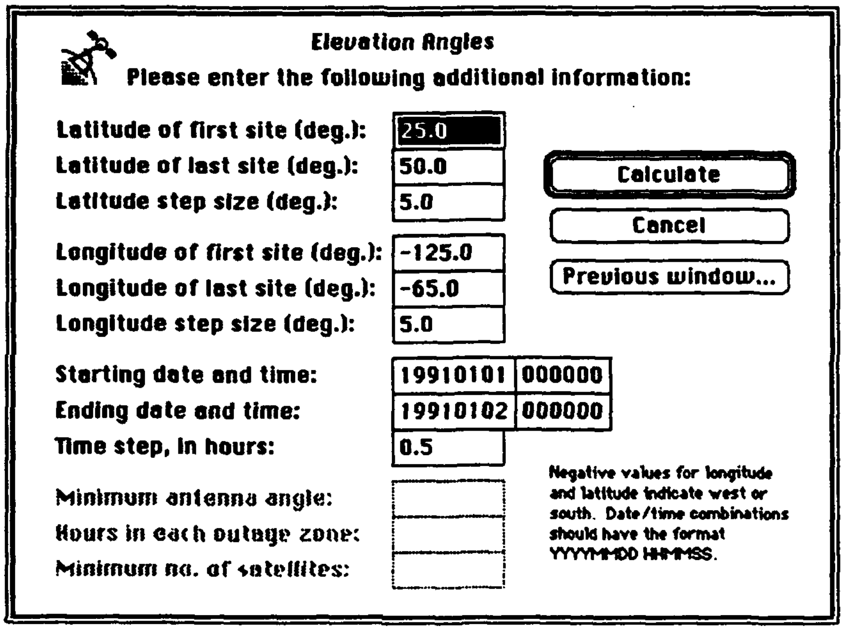


Figure 2-7. Second Elevation Angles Dialog Box

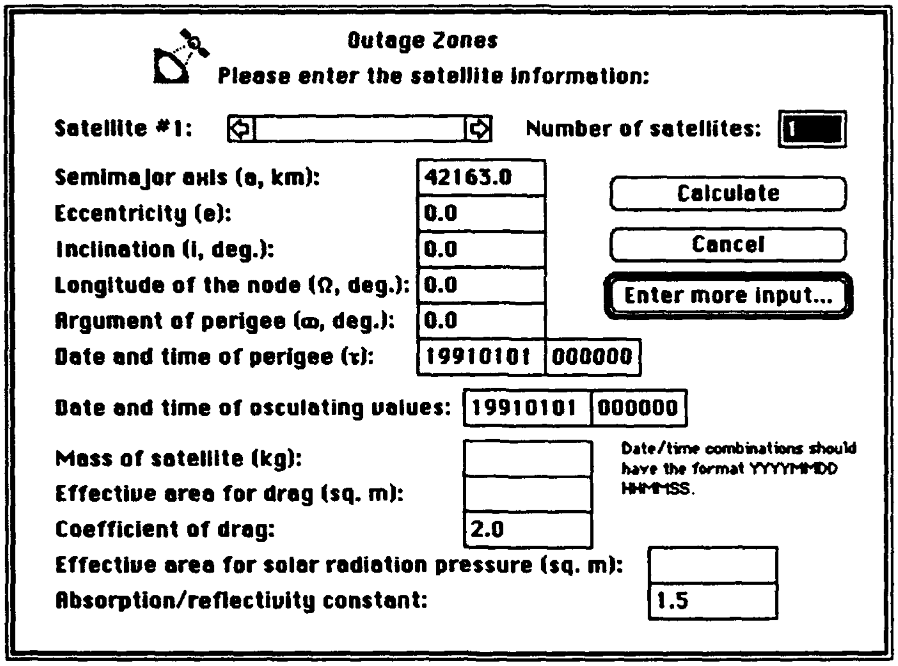


Figure 2-8. First Outage Zones Dialog Box

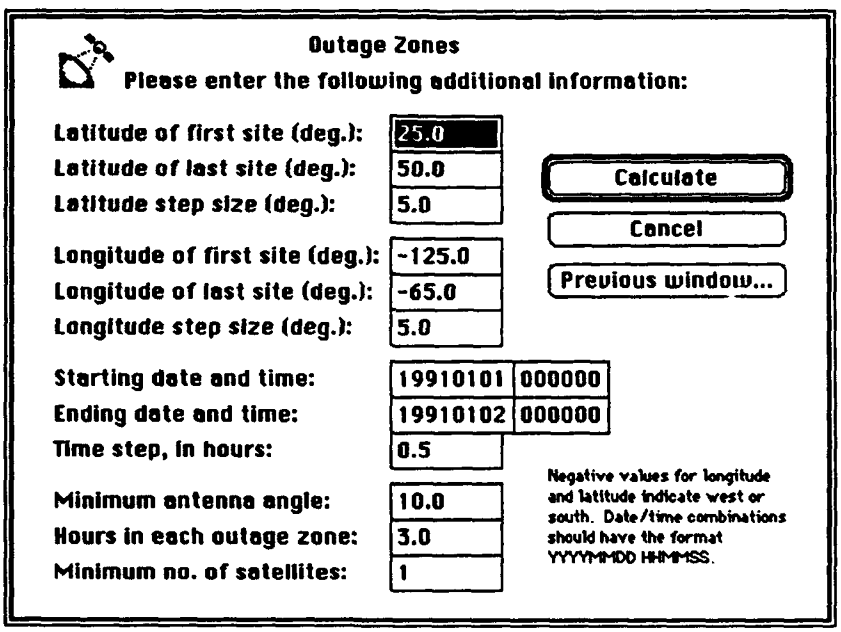


Figure 2-9. Second Outage Zones Dialog Box

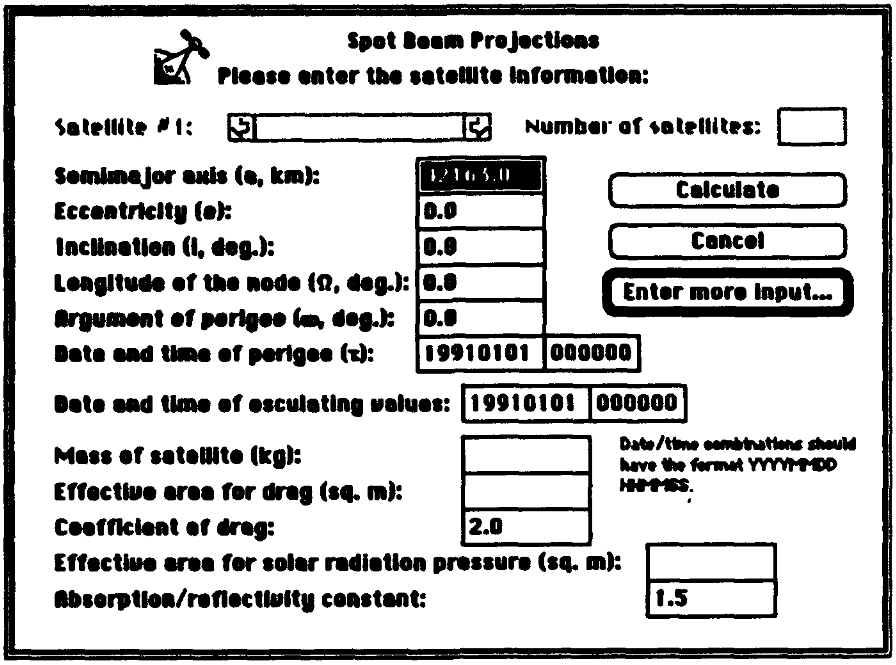


Figure 2-10. First Spot Beam Projections Dialog Box

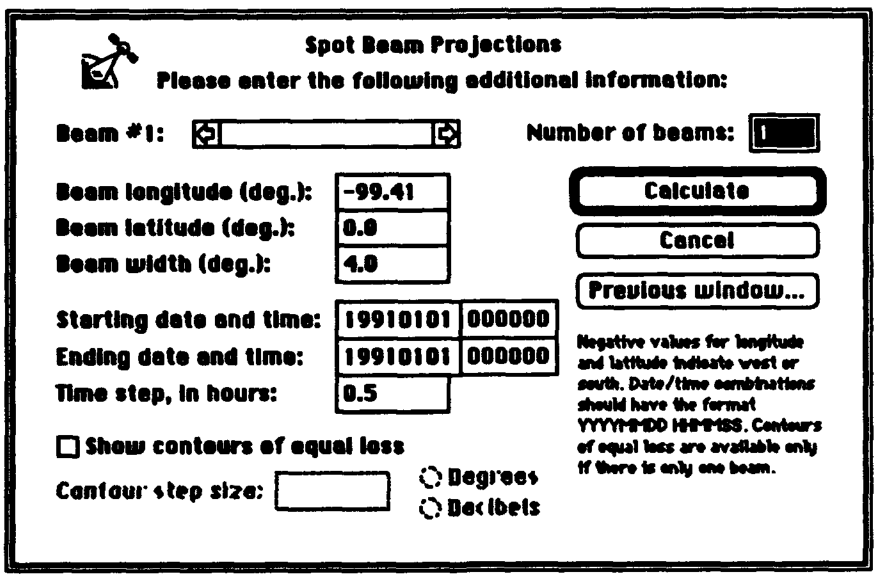


Figure 2-11. Second Spot Beam Projections Dialog Box

between the two values is 99°.41. A program that calculates the longitude of the ascending node for a geosynchronous satellite centered on a particular subsatellite longitude is supplied on the diskette and is described in section 2.6.

ESP uses ephemeris time (ET) rather than universal time (UT). Ephemeris time is a uniform measure defined by the orbital motions of the planets. Universal time (sometimes referred to as Zulu or Greenwich mean time), on the other hand, is based on the diurnal rotation of the Earth and the motion of the Earth in its orbit about the sun. Differences between the two measures at a given instant are slight, generally no more than a few minutes in the twentieth century. Reference 16 provides the following approximation for the difference (in minutes) for dates beyond 1987:

$$\Delta T = \mathrm{ET} - \mathrm{UT} = 0.41 + 1.2053T + 0.4992T^2$$

where T represents the time elapsed in centuries from noon on 31 December 1899 (Julian date 2415020.0) to present. Exact values for past years and a projected value for 1993 can be obtained from *The Astronomical Almanac for the Year 1993* [17]. This almanac is updated annually.

The default semi-major axis is 42,163 km, which is the radius of a geosynchronous orbit. (Geosynchronous orbits are defined in section A.1.) The eccentricity, inclination, longitude of the node, and argument of perigee are all initially set to zero. The default date and time of perigee is January 1, 1991 at midnight.

The date and time of the osculating values, or epoch time, is the time at which the six orbital elements are known. It is, in effect, the time of the initial conditions. Due to perturbations on the orbit of the satellite, the satellite's six orbital elements are functions of time, so this information is important. The date and time of the osculating values must be before or at the date and time of perigee. The default is January 1, 1991 at midnight.

The remaining fields are for inputting information that is required for atmospheric drag and solar radiation pressure calculations. To include these effects, positive values for the mass of the satellite (in kilograms) and the effective areas (in square meters) must be entered. To make a run without one or both of these effects, the appropriate field(s) should be filled with a zero or left blank. Both of these effects may also be ignored by leaving the mass field blank. If an area field is omitted or set to zero, "N/A" will appear on the printout for the respective field. "N/A" will also be printed for both areas and the mass if the mass is omitted. The coefficient of drag has a default value of 2.0. The absorption/reflectivity constant ranges from 0.0 to 2.0. It assumes a value less than 1.0 for varying degrees of transparency, 1.0 for a totally absorbent surface (i.e., a black surface), and 2.0 for a totally reflective surface. An example is aluminum, which has a value of 1.96. The program uses 1.5 as a default.

After entering the satellite information, the user should press the button marked "Enter more input..." The second dialog box will appear. This dialog box concerns itself with aspects of the particular run. Since it is similar for the ground track, outage zone, and elevation angle calculations it is described here. The second dialog box for the spot beam calculation is very different; it is described in the section on spot beam projections.

For the elevation and outage zones calculations the first six fields of this second dialog box determine a quadrille grid of ground sites to be included in the calculation. The longitudes of the first and last sites must be between -180° and 180° inclusive, where the positive direction is east of Greenwich and the negative direction is west of Greenwich. The latitude entries must be between -90° and 90° inclusive, where the positive direction is north of the equator and the negative direction is south of the equator. The step size in both cases must be positive. The ending longitude and latitude are included in the grid even if the entire range in either case is not an integral multiple of the step size. The default entries form a grid over the forty-eight contiguous United States (CONUS) with sites at five-degree intervals. A run for a single site can be generated by setting the ending latitude and longitude equal to the starting latitude and longitude, respectively.

ESP uses geocentric latitude, which is the angle between the equatorial plane and the line passing through the ground site and the center of the Earth. Geodetic latitude, which is often the basis for maps [2], is the angle between the equatorial plane and the line perpendicular to the surface of the Earth at the ground site (see figure 2-12). Due to the Earth's oblateness, the two latitude values of a given ground site are generally different. The maximum difference between the two values is approximately 0.19 degrees, which occurs at -45° and 45° geodetic latitude and corresponds to a difference of about 13 miles. As the latitude approaches the equator or the poles, this difference approaches zero. Geodetic latitude (L) can be easily converted to geocentric latitude (L') using the following formula:

$$L' = \arctan(0.9933056 \tan L)$$

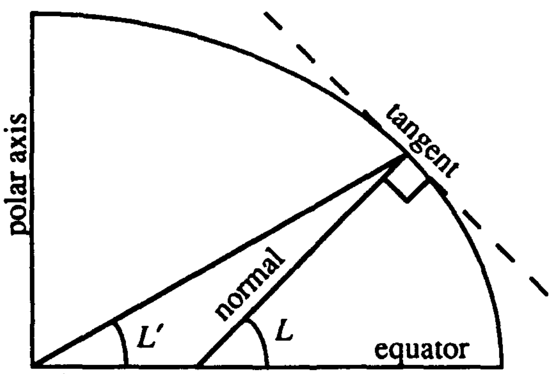


Figure 2-12. Geodetic (L) Versus Geocentric (L') Latitude

The next three fields are the starting date and time, the ending date and time, and the time step. The starting and ending dates and times are given in the format described above. The time step is given in hours. The ending time is included in the calculation even if the entire time of the run is not a multiple of the time step. Example time step values are given in the appendix. It should be noted that the time step field is used for the output, not for the orbital calculations. The calculations have their own internal time step of approximately one minute.

The last three fields, which are used only for outage zone calculations, are described in the section on outage zones.

The user may return to the first dialog box, if desired, by pressing the button marked "Previous window..." The user may switch back and forth between the two dialog boxes as often as needed. Once satisfied with the inputs, the user should press the "Calculate" button. After a few moments during which the satellite's orbit is calculated (or orbits, if there are more than one satellite), the results will be displayed in the text window. The outputs for each type of calculation are described in their respective sections. If a map window is already displayed, it will be updated.

Calculate Ground Track...

This option calculates an individual satellite's ground track and altitude as a function of time. It is only available when the active window is a ground track window. The ground track dialog boxes are shown in figures 2-4 and 2-5. Since only one satellite is accommodated, the scroll bar and the field for the number of satellites are grayed out in the first dialog box. In the second dialog box, only the starting date and time, the ending date and time, and the time step are required.

The ground track output consists of four columns. These columns contain the date and time, the subsatellite longitude, the subsatellite latitude, and the satellite's altitude, respectively. If at some time the satellite crashes into the Earth, the user will be alerted with a dialog box. After the user dismisses the dialog box, the output line at that point will be displayed in red (if a color monitor is used) and the calculation will halt.

Calculate Elevation Angles...

For a given set of ground site locations, this option calculates the elevation angles and ranges to a satellite over time. Range rate as a function of time is also included. This option is available if the active window is an elevation window. The elevation dialog boxes are shown in figures 2-6 and 2-7. As is the case with the ground track dialog, the scroll bar and the field for the number of satellites are grayed out in the first dialog box. The information to be entered is the same as for the ground track. In the second dialog box, all fields are used except the last three.

The elevation output consists of six columns. Each line of the output will show a date and time, the site longitude, the site latitude, the elevation angle, the range, and the range rate. If an elevation angle is negative and a color monitor is used, the angle will be displayed in blue.

Calculate Outage Zones...

This option determines the coverage provided by a constellation of satellites over time. It is available when the current window is an outage zone window. The coverage may apply to the entire Earth's surface or to a portion that can be defined by a grid of latitude and longitude coordinates. The first dialog box is shown in figure 2-8. In addition to the fields described above, the first two fields are made available. The number of satellites field is for setting the size of the constellation. Up to 200 satellites may be entered. Since each satellite has its own orbital elements and characteristics, this information must be entered separately for each. However, the information entered for the first satellite is inherited by all subsequent satellites in the constellation. The scroll bar allows the user to switch to the remaining satellites and edit their characteristics. If the number of satellites is increased in a document that was read from the disk, the new satellites will inherit the characteristics of the last satellite in the original constellation.

The second dialog box, shown in figure 2-9, uses all the input fields. The minimum antenna angle determines how low a satellite can be with respect to the horizon while still maintaining coverage. When the elevation angle from a given ground site to a satellite is below this minimum, the satellite cannot communicate with the ground site and will be considered blacked out. Only a single value can be entered; this will be used for all satellites and ground site locations.

An outage zone is a set of ground sites that lack coverage from a minimum number of satellites in the constellation for certain amounts of time. The hours in each outage zone field determines the size of the interval for each time length category. For example, if there were three hours in each outage zone, then zone one would contain all sites that are out for up to three hours, zone two would contain all sites that are out between three and six hours, etc. The program can accommodate a maximum of six zones; therefore the last zone often contains sites that are out over a longer time interval. Assuming the above example is over a twenty-four hour time period, zone six would contain those sites that are out between fifteen and twenty-four hours. Each range is inclusive of the upper bound, but exclusive of the lower bound. For example, a site with no coverage for exactly three hours would be in zone one, not zone two. A site that is covered for the entire simulation run is not considered to be in any zone.

The coverage at each site is checked at the user-defined time intervals. In other words, if the user specified a step size of a half hour, the coverage would be checked every half hour. Thus, the outage time calculated should be considered an approximation. If a site is covered at both ends of a time interval, it is assumed to be covered for the entire interval. If a site cannot contact at least the minimum number of satellites at both ends of a time interval, it is assumed to be blacked out for the entire interval. If a site is covered at only one end of a time interval, it is assumed to be covered for half the interval. The total outage time for a site is thus an integral multiple of one-half the time step.

The last input is the minimum number of satellites required to be in view (above the minimum elevation angle) in order for a site to be considered covered.

When the calculation is completed, the text window will contain three columns listing the site longitude and latitude and an approximate total outage time for each site that was at least partially out. The output itself is organized by outage zone. The user can switch between zones by manipulating the horizontal scroll bar at the bottom of the window.

Calculate Spot Beam Projection

This option generates projections of satellite spot beams onto the Earth and optionally generates contours of equal decibel (dB) loss. The first dialog box is the same as for the ground track and elevation angle documents. Only one satellite may be entered per run. The second dialog box is completely different from that of the other calculation types (see figure 2-11). The user may specify more than one beam from the satellite, using the input field labeled "Number of Beams." The scroll bar is used to switch between beam inputs. The beam longitude and latitude specify the point on the Earth at which the center of each beam is aimed. The beam width is the angle that subtends the beam cone. By default, there is one beam aimed approximately directly below the satellite, with a width of four degrees.

The starting date and time, the ending date and time, and the time step specify the times at which spot beam calculations are made. These fields are the same as those so labeled in the dialog boxes for the other document types.

When the box labeled "Show contours of equal loss" is checked, the program will generate loss contours corresponding to a single projection at a single instant in time. This feature is available when the beam width is 30° or less. A circular aperture uniform distribution beam pattern function is used. The mathematical description is provided in section 3.10. When this option is selected, certain fields are grayed out (see figure 2-13). These are

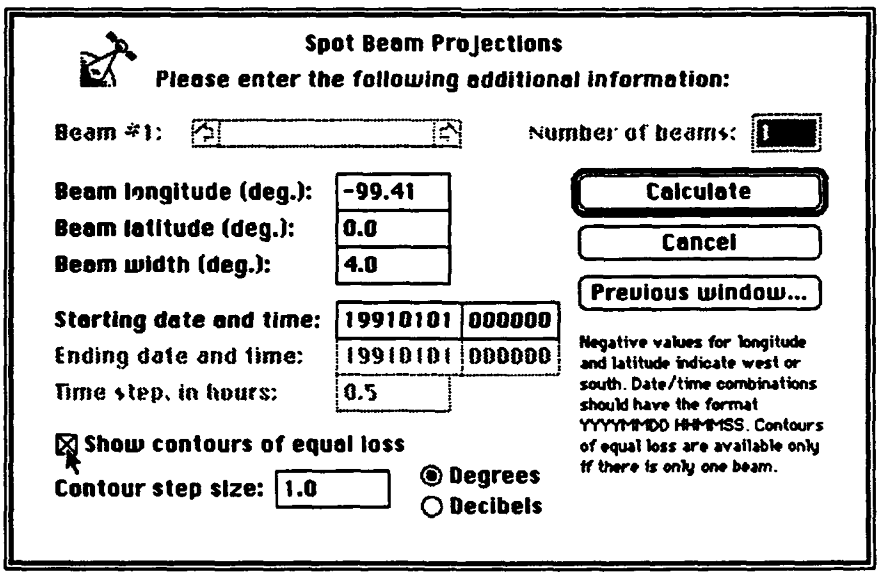


Figure 2-13. Second Spot Beam Projections Dialog Box with Contour Input Fields

the number of beams field, the scroll bar, the ending date and time fields, and the time step field. The contour step size field is used to specify the intervals for generating the loss contours. The buttons marked "Degrees" and "Decibels" define its unit. By default, the "Degrees" button is selected, and the contour step size is set to one. The specified beam width is assumed to correspond to -3 dB. Additional higher loss contours are generated according to the step size, up to a loss value of -17.6 dB, as explained in section 3.10.

To show contours for one beam from a document that contains calculations for several beams, the user should scroll to the desired beam before checking on the contour loss box. The other beams will then be discarded. For this reason it is recommended that the user compute the contours on a copy of the original multibeam document. (A copy can be easily made with the Save As... command from the File menu or the Duplicate command from the Finder.)

If any beam's pointing location is not visible from the satellite (i.e., falls below the horizon), the user will be notified. If none of the pointing locations are visible from the satellite, the dialog boxes will be presented to the user for editing.

The spot beam feature displays the map by default instead of the text window. The spot beam projections and/or contours are displayed as solid curves on the map. If the calculation was done over time and the user has a color monitor, the colors of the curves are varied to reflect projections at different times. All projections at a given time are displayed in the same color, though the color associated with each time is not necessarily unique. When plotting contours of equal loss each contour is labeled with the decibel loss.

If any beam is projected partially beyond the horizon and the calculation is done for only one instant in time, the satellite's horizon contour is also displayed. The horizon is represented as a dashed blue line.

The text window is available as usual via the Show Text Output command in the Options menu (described later in this section). The text window displays points along each spot beam projection, and points along the horizon or loss contours if applicable. Other information, such as beam width and pointing location, is displayed above the column headers.

Draw a Map

This command plots the data from a text window onto a map of the Earth. The command is available if the active window is a ground track, outage zone, or spot beam text window and a calculation has already been performed for that window. No map is drawn for elevation angle calculations. If the map has already been plotted, this command simply brings the map window to the front of the screen.

The map data consists of approximately 200,000 points taken from the World Data Bank II from National Technical Information Service. The data includes U. S. state and international boundaries, current as of the breakup of the Soviet Union (January 1992). Germany, Yemen, the former Czechoslovakia, and the former Yugoslavia are each represented as a single country. The map data has a resolution of one minute of arc.

The map projection used is equidistant cylindrical, a projection commonly used in scientific plotting. In this projection both the meridians of longitude and the parallels of latitude are equally spaced. The poles are represented as the horizontal lines at the top and bottom edges of the map. The longitude and latitude scales are labeled along the edges of the map. When the mouse cursor is located on the map, the longitude and latitude of the tip of the arrow are displayed at the bottom of the window.

The ground track map shows the ground trace against a map of the world. The outage map covers an area encompassing the ground site grid entered by the user in the second dialog box and plots every ground site that was not covered for one hundred percent of the time. The symbol used to plot each site indicates how much time the site was blacked out. A different symbol is used for each outage zone, as indicated on the map legend. The spot beam map displays spot beams and possibly loss contours and the horizon as described above on a map of the world. Examples of all three types of maps are shown in the appendices.

The user can expand a rectangular portion of the map ("zoom in") by moving the mouse cursor to one corner of the desired rectangle, pressing the mouse button, and dragging a rectangle to cover the desired area. A sample rectangle is shown in figure 2-14. Once the zoom operation has been performed, the displayed area can be reenlarged by dragging an interior rectangle or shifted slightly by dragging the rectangle outside of the map area.

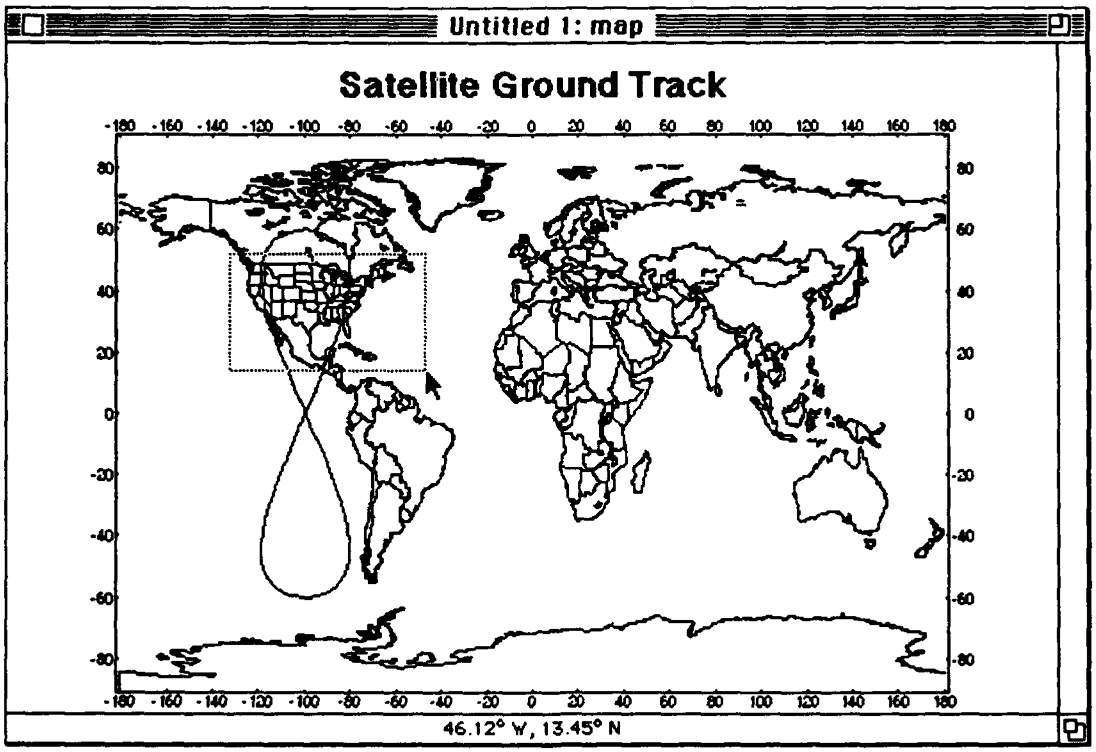


Figure 2-14. Map Zooming

When expanding an outage zone map, the user is given the opportunity to recalculate the outage zones within the new rectangle. The dialog box shown in figure 2-15 appears on the screen. The user can enter new step sizes for the calculation. The default step sizes are taken from the original document. Pressing the "Yes" button starts the recalculation. Pressing "No" zooms in without recalculating. Pressing "Cancel" cancels the zoom operation altogether. If the user presses "Yes," the recalculated area becomes the new "zoomed out" state. In other words, the user may not zoom out to the state prior to the recalculation. (Zooming out is described below.)

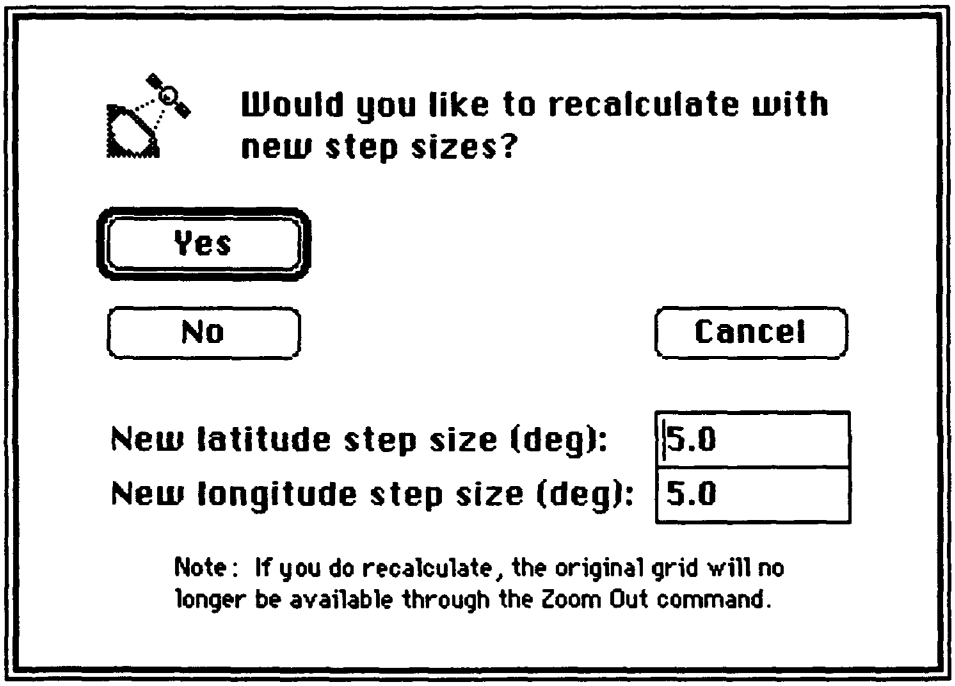


Figure 2-15. Outage Zone Recalculation Dialog Box

<u>Show Text Output</u>

This command is available if the active window is a map window. It simply brings to the front of the screen the corresponding text window, redisplaying it if it were closed or hidden.

<u>Zoom Out</u>

This command resets the display area of the map. For ground track and spot beam projection documents, this is the entire world. For outage zone documents, this is the area encompassing the ground site grid.

Show Minute Features

This command determines how much detail will be displayed on the map. Selecting this command toggles it on or off; the on state is indicated by the presence of a check mark. When the feature is activated, all map features are plotted, no matter how small. When this feature is turned off, the smallest islands, lakes, and countries are not displayed. The trade-off is in execution time.

2.4.6 The Windows Menu

This menu contains a list of all the windows currently in memory, including those temporarily hidden. Selecting a window from this menu brings it to the front on the screen, redisplaying it if it were hidden.

2.5 PRINTING AND OPENING DOCUMENTS FROM THE FINDER

The printing of documents from the finder is supported by ESP. To do this the user first selects the icons of the desired files and then chooses the Print command from the File menu. The Page Setup dialog box is presented first, and the options set here will apply to all the documents. Separate dialog boxes will then be presented for each document for entering the final job settings, e.g.. the pages to be printed and the number of copies.

ESP can also be launched by opening any of its documents from the finder. The icons of the different types of documents are shown in figure 2-1.

2.6 ORBITAL PARAMETERS CONVERSION UTILITY

In order to accommodate different orbital parameter sets, a conversion utility called Convert Parameters is provided on the diskette. The program's operation is simple. When started, the program displays the dialog box shown in figure 2-16. The user first selects one of the five conversions shown in the dialog box. The input fields in the dialog box will change according to the selected conversion. For those conversions that require date and time inputs, the defaults are the same as those of ESP. After entering the inputs the user should press the "Convert" button. A window will be displayed with the converted parameter(s). Shown in figure 2-17 is the output window that shows the results corresponding to the inputs in figure 2-16. The user can make as many conversions as desired by selecting Convert... from the Edit menu.

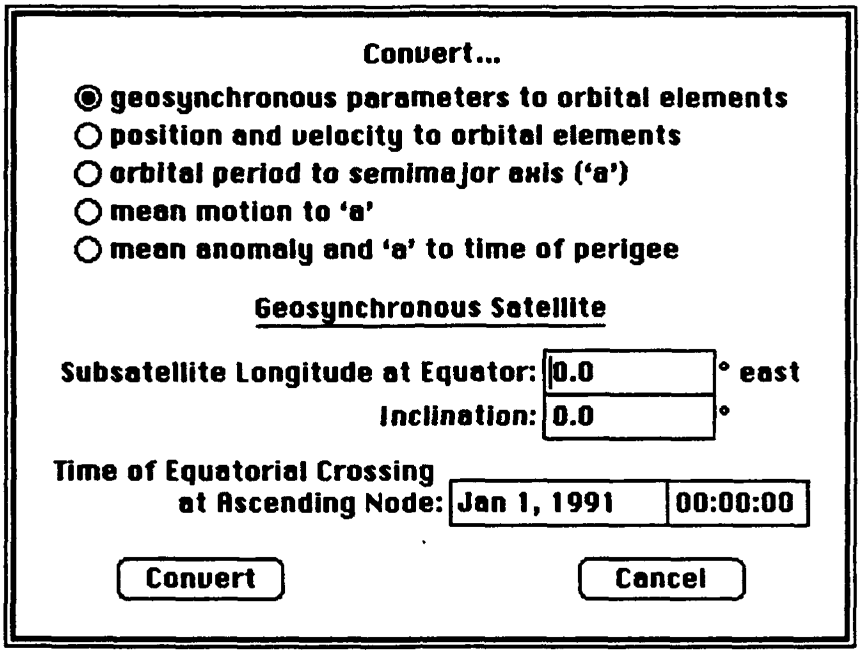


Figure 2-16. Sample Convert Parameters Dialog Box

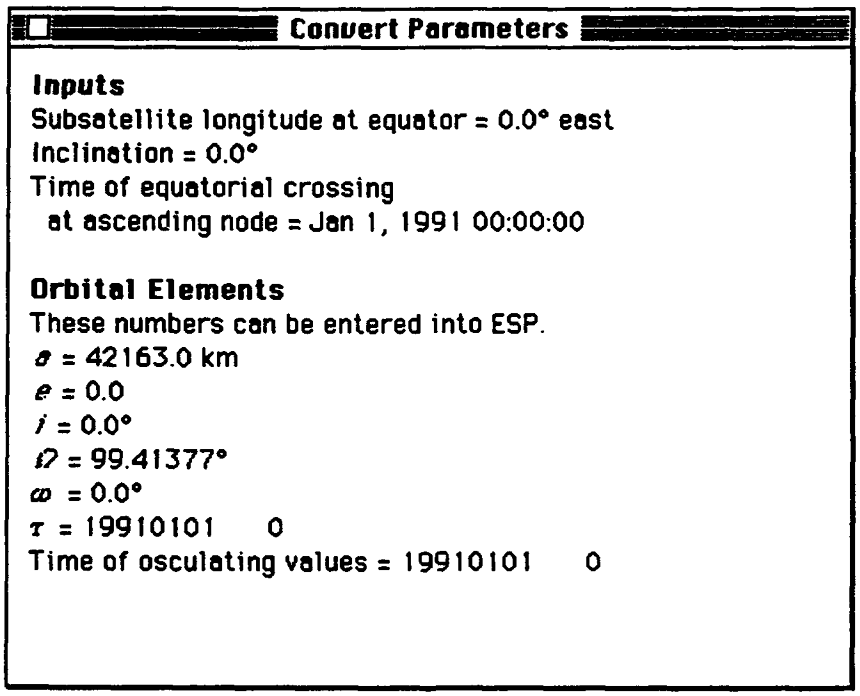


Figure 2-17. Sample Convert Parameters Output

SECTION 3

ANALYTICAL FOUNDATIONS

3.1 OVERVIEW

ESP 2.0 includes perturbations to a satellite's orbit that are due to the Earth's oblateness (J_2 effects), atmospheric drag, and solar radiation pressure. Satellite orbits are propagated using a classical fourth-order Runge-Kutta algorithm with a fixed step size. This provides a good approximation to a satellite's position and velocity vectors, each as a function of time. Initial conditions are determined from the values of the satellite parameters that are input by the user. Major outputs of the program — the satellite ground track; elevation angle, range and range rate calculations; and spot beam projections — are derived from the satellite position vector as a function of time. This section provides the mathematical details for the propagation scheme as well as all secondary calculations.

3.2 FUNDAMENTALS

To begin, a review of some astrodynamics fundamentals is essential. Consider the two-body problem, that is, the simplified scenario in which the sole force governing two masses is that of their mutual gravitational attraction. In this circumstance, the relative motion is confined to a plane and described by a conic section. Furthermore, there are six integrals or constants of motion that completely define the behavior of the particles. These six constants are known as the *orbital elements*; they may assume several forms and define the motion for any type of conic. The classical representation of these elements in the context of closed orbits (circles and ellipses) is the basis for ESP and subsequent discussion.

3.2.1 Orbital Elements

The first three elements are independent of the frame of reference and are shown in figure 3-1. The size of the orbit is defined by the *semimajor axis*, a, which is one-half the length of the chord passing through the foci of the ellipse. The shape is determined by the *eccentricity*, e, which is defined as the ratio of one-half the distance between the foci (denoted by c in the figure) over the semimajor axis (i.e., c/a). For closed orbits, the eccentricity ranges from zero, for circles, up to but not including one, for ellipses. The third element pinpoints satellite position on the orbit as a function of time; it is the time of closest approach to the central attracting body, known as the *time of pericenter passage* (or in the case of Earth-orbiting satellites, the *time of perigee passage*), τ.

The remaining three elements do in fact depend on the reference frame. Typically, when dealing with Earth-orbiting satellites, one uses the geocentric-equatorial coordinate frame, otherwise known as the Earth-Centered Inertial (ECI) coordinate frame. In this system the origin of coordinates is taken as the center of the Earth, with the x-y plane coincident with the

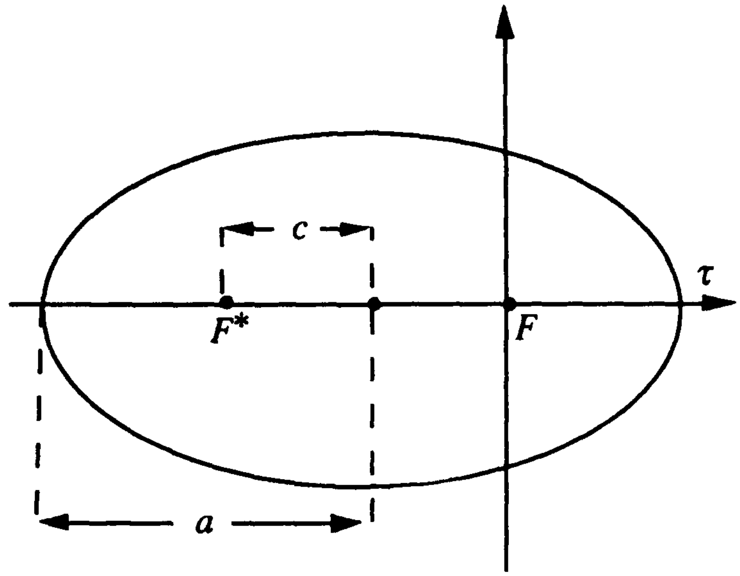


Figure 3-1. Semimajor Axis, Eccentricity, and Time of Pericenter Passage

equatorial plane. The positive *x*-axis is in the direction of vernal equinox[1], the positive *z*-axis is along the axis of the Earth's rotation and points due north, and the *y*-axis is chosen so that a right-handed coordinate system is formed. It is important to note that this frame does not rotate as the Earth rotates diurnally on its axis. However, it is not an inertial frame in the true sense due to precession[2]. A fixed frame of reference may be established by defining the coordinate system at some epoch. ESP uses the mean[3] equinox and Earth equator of the standard epoch of 1950.0[4]. An illustration of the ECI coordinate frame as it pertains to the discussion that follows is provided in figure 3-2.

---

[1]The vernal equinox is defined as that point along the line of intersection of the Earth's equatorial plane and the ecliptic plane (the plane of the Sun-Earth motion) at which the Sun crosses from south to north in its apparent motion about the Earth.

[2]*Luni-solar precession* refers to the motion of the Earth's polar axis about the surface of a cone with period 26,000 years. It is caused by the gravitational attraction of the Sun and Moon on the Earth's equatorial bulge and results in a westward shift of the vernal equinox at a rate of about 50 arc-seconds per year. There is also an oscillation (or nodding) of the Earth's polar axis as it sweeps out this cone. This effect is imposed by the gravitational attraction of the Moon, has period 18.6 years, and is referred to as *nutation*. In addition, *planetary precession*, caused by the gravitational attraction of the planets, results in a slow shift in the orientation of the ecliptic plane.

[3]*Mean* equinox and equator neglects the relatively short-period variations caused by nutation.

[4]The standard epoch of 1950.0 is defined as the beginning of the Besselian year, or the Julian ephemeris date 2433282.423357. The reference planes are fixed at this instant. The time reference epoch that appears in the calculations of later sections (sections 3.3.3 and 3.6) is the nearest even date, which is zero hours on 1 January 1950, or the Julian ephemeris date 2433282.5.

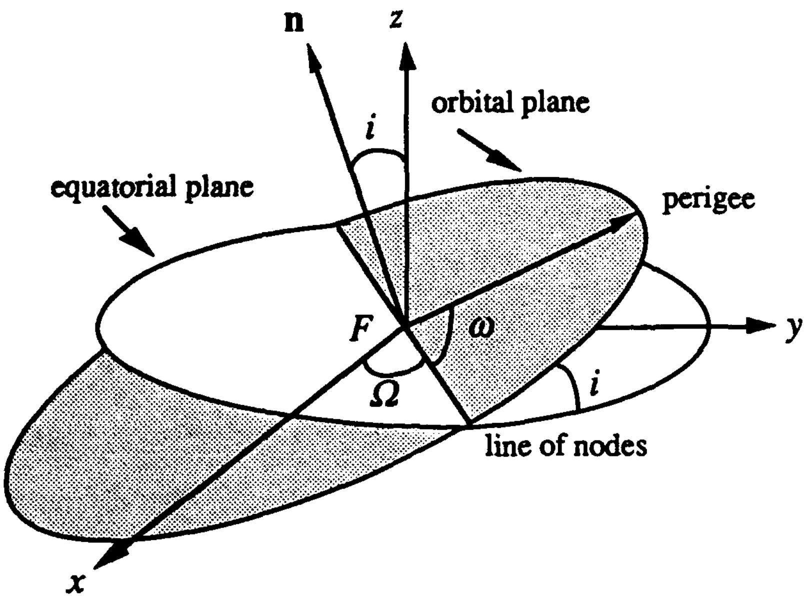


Figure 3-2. Euler Angles

Now that the frame of reference has been established, the remaining three elements may be defined. First note that the equatorial plane and the orbital plane (the plane of the orbiting body's motion) intersect in a line (when the orbital plane is different from the equatorial plane), which is referred to as the *line of nodes*. The ascending (descending) node is that point along the line of nodes where the satellite crosses from south (north) to north (south) in its orbit. The angle in the equatorial plane between the positive x-axis and the direction of the ascending node is the *longitude of the ascending node*, Ω. (For the case in which the orbital plane and the equatorial plane are the same, Ω is arbitrary.) The angle in the orbital plane measured from the direction of the ascending node to the direction of perigee is the *argument of perigee*, ω. Finally, the angle between the equatorial plane and the orbital plane is the *inclination*[5], i. These three angles collectively are known as the Euler angles.

3.2.2 Osculating Orbit

If the basic premise of two-body motion held in real life, then it would be a simple task to determine the satellite position and velocity vectors as functions of time. (This would be accomplished exactly according to the method provided in section 3.5.) Unfortunately, there is a complex interplay of forces that affects the motion of any body in space. In actuality, the six elements described above are not constants but functions of time, and the true motion of a body orbiting the Earth is not a perfect ellipse but some irregularly shaped path. At any instant the true orbit is tangent to a perfectly elliptical orbit, known as the *osculating orbit*, with corresponding orbital elements identical at that instant. The instant is thus referred to as the *time of the*

---

[5]The inclination is often equivalently defined as the angle between the positive z-axis and the orbit normal vector (the vector **n** in figure 3-2).

osculating values (or frequently, the *epoch time*). The tangential orbit may be regarded as that orbit which would result if all perturbations were turned off at that instant. Figure 3-3 depicts the relationship between the actual and osculating orbits.

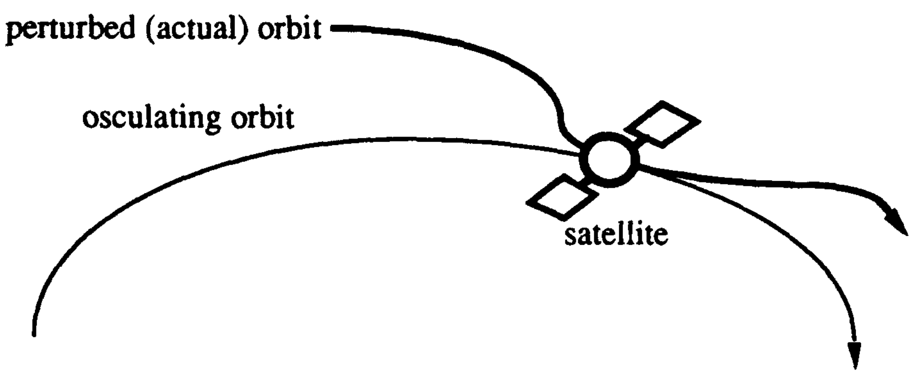


Figure 3-3. Osculating Orbit

Given the six orbital elements of the osculating orbit at some particular instant, the position and velocity are relatively easily determined at that instant. Subsequent positions and velocities can then be estimated using these initial conditions and the equations of motion in conjunction with an integration scheme.

3.2.3 Additional Parameters

In addition to the orbital elements discussed above, there are six other parameters of direct interest. The first three are illustrated in figure 3-4. The *true anomaly*, f, is the angle in the orbital plane measured from the direction of perigee (often referred to as the *line of apsides* or *apsidal line*) to the satellite position vector, $\mathbf{r}$. The *eccentric anomaly*, E, may be obtained from the true anomaly geometrically as follows. First, an auxiliary circle with radius equal to the semimajor axis a is circumscribed about the ellipse. Next, a perpendicular is dropped from the satellite position vector to the major axis and extended to the point P on the circle. The eccentric anomaly is the resultant angle formed between the line of apsides and the line segment OP. There are also algebraic relationships between f and E, however these are not of concern in the present analysis. The third parameter shown in figure 3-4 is the *semiminor axis*, b; it is one-half the length of the chord passing through the center of the ellipse and perpendicular to the major axis. The semiminor axis is related algebraically to the semimajor axis and eccentricity as follows:

$$b = a\sqrt{1-e^2} \qquad (3\text{-}1)$$

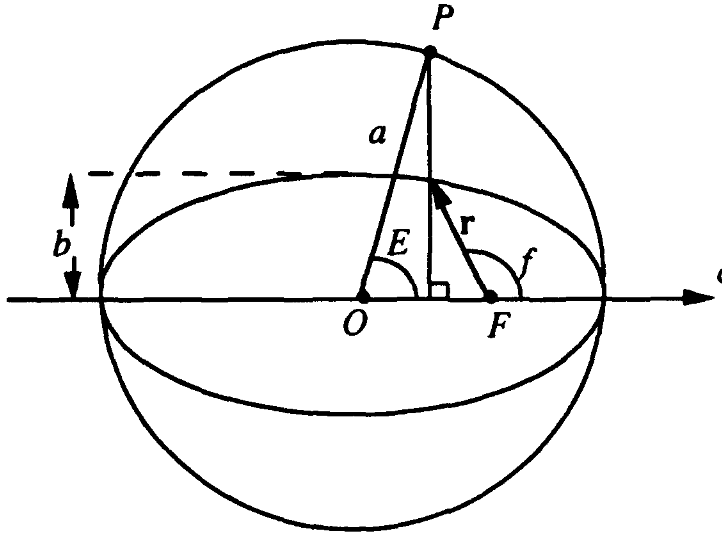


Figure 3-4. Additional Parameters

The remaining three parameters needed are defined algebraically. The time required for a satellite to make one complete pass through its orbit, or the *orbit period, P*, is a function of the semimajor axis a and the gravitational parameter μ:

$$P = 2\pi\sqrt{\frac{a^3}{\mu}} \tag{3-2}$$

For an Earth-orbiting satellite,

$$\mu = G(m_e + m_s)$$

where G is the universal gravitational constant, and m_e and m_s are the masses of the Earth and satellite, respectively. Since the mass of the satellite is negligible in comparison with the mass of the Earth,

$$\mu \approx Gm_e = 398600.45 \quad \text{km}^3 / \text{sec}^2$$

Note that equation 3-2 can be rearranged to determine the semimajor axis associated with a given orbit period

$$a = \left(\frac{\mu P^2}{4\pi^2}\right)^{\frac{1}{3}} \tag{3-3}$$

The average angular motion (radians per unit time) or *mean motion, n,* is defined in terms of the orbit period by

$$n = \frac{2\pi}{P} = \sqrt{\frac{\mu}{a^3}}$$

The final parameter is the *mean anomaly, M*. Regardless of the type of motion, M is always a function of time. It is defined in terms of the mean motion n and the time of pericenter passage τ. Explicitly,

$$M(t) = n(t - \tau)$$

$M(t)$ may be interpreted geometrically as the angle measured from the line of apsides to the position vector obtained at time t if the satellite were moving with constant angular velocity n. Often the time of the osculating values, t_{osc} and the time of pericenter passage, τ, are provided implicitly by the single parameter $M(t_{osc})$.

3.3 EQUATIONS OF MOTION

In the case of two-body motion, the sole force exerted on the two mass particles, m_1 and m_2, is that of their mutual gravitational attraction. In accordance with Newton's Law of Universal Gravitation, the magnitude of this force is directly proportional to the product of the masses and inversely proportional to the square of the distance between them. The direction of the force exerted by m_2 on m_1 is along the line of sight from m_1 to m_2 (and oppositely oriented is the force exerted by m_1 on m_2). Based on Newton's Second Law[6], the motion of each of the masses with respect to a common origin O (refer to figure 3-5) is expressed algebraically as follows:

$$m_i \ddot{\mathbf{r}}_i = \frac{G m_1 m_2}{r^2} (-1)^{i+1} \frac{\mathbf{r}}{r} \qquad i = 1,2$$

where $\mathbf{r}_i$ represents the position of mass m_i relative to origin O, $\mathbf{r}$ is the position of m_2 relative to m_1 (i.e., $\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$), and r is the magnitude of the vector $\mathbf{r}$.

Subtraction of the case $i = 2$ from the case $i = 1$ yields the following equation that completely describes the relative motion of the particles:

$$\ddot{\mathbf{r}} = -\frac{\mu}{r^3}\mathbf{r} \qquad (3\text{-}4)$$

[6]*Newton's Second Law* states that the rate of change of momentum is proportional to the force impressed and is in the same direction as that force [1]. For a body with constant mass m, it is expressed mathematically as $\mathbf{F} = m\mathbf{a}$, where $\mathbf{F}$ is the net force exerted on the body and $\mathbf{a}$ is the acceleration of the body.

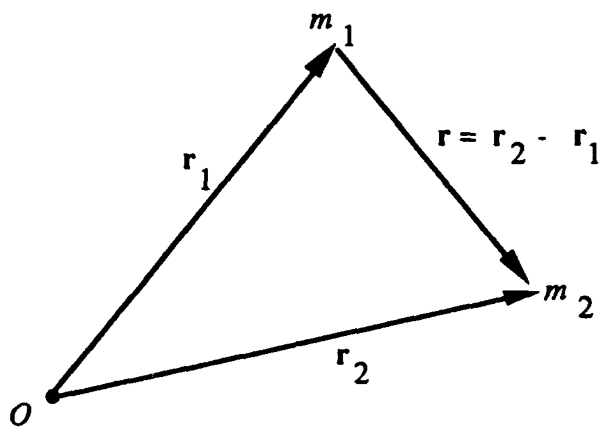


Figure 3-5. Two-body Geometry

This second-order equation is often expressed equivalently as a system of two first-order equations:

$$\frac{d\mathbf{r}}{dt} = \mathbf{v} \qquad \frac{d\mathbf{v}}{dt} = -\frac{\mu}{r^3}\mathbf{r}$$

where **v** is the velocity of m_2 relative to m_1. In the presence of additional forces, equation 3-4 becomes:

$$\ddot{\mathbf{r}} = -\frac{\mu}{r^3}\mathbf{r} + \mathbf{F} \tag{3-5}$$

where **F** represents the net disturbing force acting on the system. **F** is simply the sum of all the individual disturbing forces, i.e.,

$$\mathbf{F} = \mathbf{f}_1 + \mathbf{f}_2 + \mathbf{f}_3 + \ldots$$

where each $\mathbf{f}_i$ is an individual disturbing force. Since these individual disturbing forces cause the motion to deviate from a perfect conic, they are commonly referred to as *perturbations*.

ESP 2.0 takes into account the effects of three perturbations, namely, the oblateness of the Earth (J_2 effects), atmospheric drag, and solar radiation pressure. The mathematical model for each is presented in detail in the subsections that follow. As a notational convenience, they will be denoted by $\mathbf{f}_1$, $\mathbf{f}_2$, and $\mathbf{f}_3$, respectively. Also, the satellite position and velocity vectors with respect to the ECI coordinate frame will be denoted by

$$\mathbf{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \mathbf{v} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$$

respectively.

3.3.1 Perturbations Due to J_2 Effects

The major force influencing the motion of an Earth-orbiting satellite is the Earth's gravitational field. If the Earth were a perfectly smooth sphere with a uniform mass distribution, then equation 3-4 would completely describe the motion resulting from this force. In actuality, the Earth is pear-shaped, meaning it is more massive below the equatorial belt than above. Under the assumption that the Earth is axially symmetric, the additional force exerted on a satellite due to the Earth's uneven mass distribution is determined from the Earth's gravitational potential function

$$V(r,z) = \frac{\mu}{r}\left[1 - \sum_{k=2}^{\infty} J_k \left(\frac{R_{eq}}{r}\right)^k P_k\left(\frac{z}{r}\right)\right]$$

where J_k = empirically determined constants
R_{eq} = equatorial radius of the Earth = 6378.14 km
P_k = Legendre polynomials, $k = 0, 1, 2, \ldots$

In the above expression, the first two terms are dominant, a result of the fact that

(a) the coefficients J_k are all very close to zero, and become significantly more so as k increases (as an indication, J_3 is smaller than J_2 by three orders of magnitude)

(b) $\left(\frac{R_{eq}}{r}\right) << 1$

and (c) $\left|P_k\left(\frac{z}{r}\right)\right| \leq 1$

Neglecting all subsequent terms results in

$$V(r,z) = \frac{\mu}{r}\left[1 - J_2\left(\frac{R_{eq}}{r}\right)^2 P_2\left(\frac{z}{r}\right)\right]$$

where J_2 = zonal harmonic of degree two = 0.00108263 [3]

and
$$P_2(\upsilon) = \frac{1}{2}(3\upsilon^2 - 1)$$

The force per unit mass acting upon the satellite due to J_2 is found by taking the gradient of

$$V^* = V - \frac{\mu}{r}$$

with respect to the position vector **r**, i.e.,

$$\mathbf{f}_1 = \left[\frac{\partial V^*}{\partial \mathbf{r}}\right]^{\mathrm{T}} = \left[\frac{\partial V^*}{\partial x} \quad \frac{\partial V^*}{\partial y} \quad \frac{\partial V^*}{\partial z}\right]^{\mathrm{T}}$$

where
$$\frac{\partial V^*}{\partial x} = -\frac{3}{2}\,\frac{J_2 R_{eq}^2}{r^5}\mu x\left[1 - \frac{5z^2}{r^2}\right]$$

$$\frac{\partial V^*}{\partial y} = -\frac{3}{2}\,\frac{J_2 R_{eq}^2}{r^5}\mu y\left[1 - \frac{5z^2}{r^2}\right]$$

$$\frac{\partial V^*}{\partial z} = -\frac{3}{2}\,\frac{J_2 R_{eq}^2}{r^5}\mu z\left[3 - \frac{5z^2}{r^2}\right]$$

Note that the contribution to the net force that is a result of the first term of V, μ/r, is included as the first term in equation 3-5.

3.3.2 Perturbations Due to Atmospheric Drag

Atmospheric drag is an important force influencing the motion of satellites in low-Earth orbit. The general effect of this force is that it causes energy to dissipate. Drag is directly proportional to atmospheric density which in turn decreases with altitude above the Earth's surface. It is also a function of a satellite's cross-sectional area-to-mass ratio. For any given system, the lower the orbit the greater the impact of drag. In general, orbits between 100 km and 200 km above the Earth will decay in a matter of a few revolutions, whereas orbits higher than 500 km will have lifetimes measured in years. The impact of drag is essentially negligible for orbits that are higher than 1000 km above the Earth.

Mathematically, the force exerted on a satellite due to drag is expressed as follows:

$$\mathbf{f}_2 = -\frac{1}{2}\frac{C_D A_D}{M}\rho v_{rel}\mathbf{v}_{rel}$$

where C_D = coefficient of drag, dependent on vehicle shape and attitude; several sources recommend a value of 2.0
A_D = effective satellite area for drag (cross-sectional area perpendicular to the direction of motion)
M = satellite mass
ρ = atmospheric density corresponding to satellite altitude
$\mathbf{v}_{rel}$ = velocity of the satellite relative to the rotating atmosphere
v_{rel} = magnitude of $\mathbf{v}_{rel}$

ESP uses a static atmospheric density model based on the 1976 U. S. Standard Atmosphere and adapted from the JPL tool ASAP[7]. This tabular model provides atmospheric density as a function of altitude, for altitudes ranging from 86 km to 1000 km. Altitudes lower than 86 km, though highly unlikely, are assumed to have density equivalent to that at 86 km. Altitudes higher than 1000 km are assumed to have zero density. The densities of altitudes between values in the table are determined by linear interpolation.

The altitude, h, of the satellite at any instant is given by

$$h = r - R_e$$

where r is the magnitude of the position vector, and R_e is the radius of the Earth. ESP models the Earth as an oblate spheroid in the determination of R_e. In general, the radius, ζ, of an oblate spheroid is given by

$$\zeta = \sqrt{\frac{a_e^2(1-e_e^2)}{1-e_e^2\cos^2\delta}} \qquad (3\text{-}6)$$

where a_e and e_e are the semimajor axis and eccentricity, respectively, of an elliptical cross-section through the poles, and δ is the geocentric latitude. An expression for the eccentricity of

[7]The Artificial Satellite Analysis Program (ASAP) [15] is a sophisticated trajectory propagator useful for analyzing orbit behavior over a period of several months. It includes perturbations on the spacecraft orbit due to non-sphericity (up to a 40 × 40 field) of the central body, luni-solar effects, drag and solar radiation pressure, and uses an eighth-order Runge-Kutta integration scheme with variable step size control. The software runs on a PC and takes as input a lengthy column-formatted file. Major outputs are time histories of the orbital elements/satellite state in a variety of representations.

this cross-section in terms of the semimajor and semiminor axes is found by rearranging equation 3-1:

$$e_e = \frac{\sqrt{a_e^2 - b_e^2}}{a_e}$$

For the Earth, a_e and b_e are the equatorial and polar radii, respectively, as illustrated in figure 3-6. Their values are given by

$$a_e \approx 6378 \text{ km}$$
$$b_e \approx 6357 \text{ km}$$

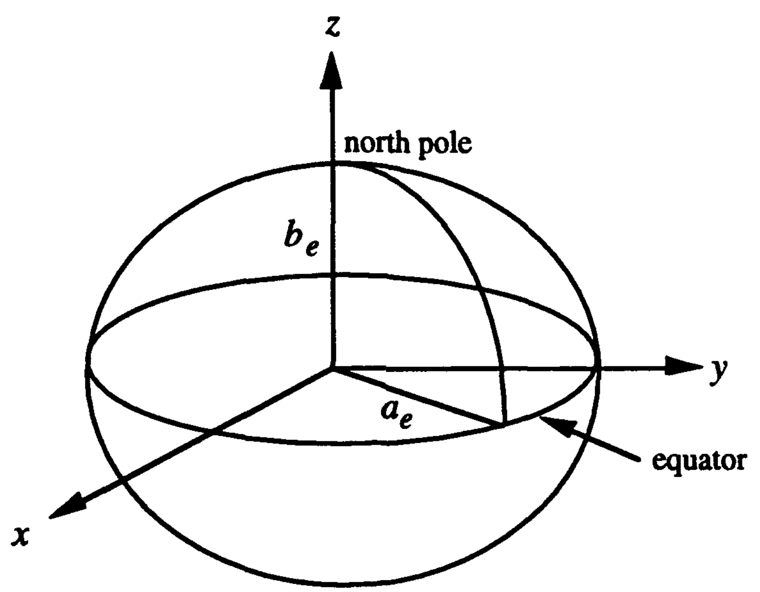


Figure 3-6. Earth Cross-section

which leads to the following widely referenced ([2], [8], [9], [10], [11], [15]) value for e_e:

$$e_e = 0.08182$$

(Note that this result is obtained using more precise values for a_e and b_e than the approximations indicated above.) Also, at any particular time the latitude δ corresponding to the satellite position vector **r** is given by

$$\delta = \arcsin\left(\frac{z}{r}\right)$$

as shown in section 3.6.

Finally, the velocity of the satellite relative to the rotating atmosphere is needed. This is obtained by direct application of the Coriolis Theorem. This theorem provides a relationship between a vector in a fixed coordinate system F and the same vector in another coordinate system R, rotating with respect to F with angular velocity vector $\boldsymbol{\omega}$. For any vector $\mathbf{a}$ in F, this relationship is expressed as follows:

$$\left.\frac{d\mathbf{a}}{dt}\right|_F = \left.\frac{d\mathbf{a}}{dt}\right|_R + \boldsymbol{\omega} \times \mathbf{a}$$

where in general,

$$\left.\frac{d\mathbf{a}}{dt}\right|_A$$

denotes the time derivative of $\mathbf{a}$ with respect to the coordinate frame A.

Now the Earth is rotating in the ECI frame with mean angular velocity

$$\boldsymbol{\omega}_e = \begin{bmatrix} 0 \\ 0 \\ \omega_e \end{bmatrix}$$

where

$$\omega_e = 15^\circ.041067178 \text{ per hour}$$

Application of the above theorem to the position vector $\mathbf{r}$ in the fixed or ECI coordinate frame yields the velocity of the satellite relative to the rotating atmosphere, as follows:

$$\mathbf{v}_{rel} = \left.\frac{d\mathbf{r}}{dt}\right|_R = \left.\frac{d\mathbf{r}}{dt}\right|_F - \boldsymbol{\omega}_e \times \mathbf{r}$$

$$= \mathbf{v} - \boldsymbol{\omega}_e \times \mathbf{r}$$

$$= \begin{bmatrix} v_x + y\omega_e \\ v_y - x\omega_e \\ v_z \end{bmatrix}$$

3.3.3 Perturbations Due to Solar Radiation Pressure

Solar radiation pressure is caused by the light energy, or photons, of the Sun. Depending on where the satellite is in its orbit, it may add/subtract energy to/from the orbit. As one might expect, the force exerted on a satellite due to solar radiation pressure is a function of the position of the Sun relative to the Earth. Like drag, it is also a function of a satellite's cross-sectional area-to-mass ratio (although the effective satellite area for solar radiation may very well differ from that for atmospheric drag). In extreme cases where the area-to-mass ratio is very large (e.g., balloon satellites) the effect of solar radiation is to "blow" the satellite off course, a phenomenon referred to as "solar sailing" and one that may be desirable for spacecraft navigation or attitude control. This force is modelled mathematically as follows:

$$\mathbf{f}_3 = -\frac{\gamma P_S A_S}{M}\hat{\mathbf{r}}_S$$

where

γ	=	reflectivity/absorption constant, $0 \le \gamma \le 2$
	< 1	for translucent material
	$= 1$	for perfectly absorbent material
	$= 2$	for perfectly reflective material
P_S	=	radiation pressure (force per unit area) on a perfectly absorbing surface at 1 astronomical unit[8]
	=	4.4×10^{-3} kg/km-sec^2
A_S	=	effective satellite area for solar radiation pressure (cross-sectional area exposed to the Sun)
M	=	satellite mass
$\hat{\mathbf{r}}_S$	=	unit vector to the Sun in the ECI frame, i.e., direction of the Sun with respect to the Earth

The direction of solar radiation pressure is actually assumed to be along the line-of-sight from the Sun to the satellite, or $\mathbf{r} - \mathbf{r}_S$. Since the Sun is located at a large distance from both the Earth and a satellite orbiting the Earth, $\mathbf{r} - \mathbf{r}_S \approx -\mathbf{r}_S$. The position vector to the Sun in the ECI frame, $\mathbf{r}_S$, is obtained by considering the Sun as a body orbiting the Earth. Given the orbital elements of the Sun at time t, $\mathbf{r}_S(t)$ is determined exactly according to the method outlined in section 3.5. Reference 21 provides the following relationships for the mean orbital elements of the Sun with respect to the mean equinox and Earth equator of 1950.0 (distance in kilometers, and angles in degrees):

$$a_S = 1.49597927 \times 10^8$$

$$e_S = 1.67301085 \times 10^{-2} - 4.1926 \times 10^{-5} T - 1.26 \times 10^{-7} T^2$$

---

[8]An *astronomical unit* (A. U.) is defined as the mean distance from the Earth to the Sun, or approximately 150 million kilometers.

$$i_s = 23.4457888616 - 1.30141669 \times 10^{-2} T - 9.445 \times 10^{-7} T^2 + 5.000 \times 10^{-7} T^3$$

$$\omega_s = 282.08053 + 3.2328 \times 10^{-1} T + 1.5 \times 10^{-4} T^2$$

$$M_s = 358.000682 + 9.856002623 \times 10^{-1} d - 1.550000 \times 10^{-4} T^2 - 3.3333 \times 10^{-6} T^3$$

where d = number of ephemeris days from reference epoch 1 January 1950 to time t

and T = number of Julian centuries from reference epoch 1 January 1950 to time t
$= d/36525$

Note that the longitude of the ascending node, Ω_s, is zero since the positive x-axis is defined as the direction of the vernal equinox. Also note that the first four elements listed above are essentially fixed for the duration of the run. Thus, they are calculated once at starting time. The mean anomaly, however, determines the location of the Sun in its orbit relative to the Earth as a function of time. Therefore, it must be recalculated with each evaluation of the function f_3.

In the case in which the Earth blocks or "shadows" the Sun from the satellite, there is no solar radiation pressure effect. Shadowing may occur if the angle θ subtending the vectors $\mathbf{r}$ and $\mathbf{r}_s$ is greater than 90° (refer to figure 3-7). θ is calculated from the dot product as follows:

$$\theta = \arccos\left(\frac{\mathbf{r} \cdot \mathbf{r}_S}{r\ r_S}\right)$$

If θ does in fact exceed 90°, the Earth will block the line-of-sight vector from the Sun to the satellite when

$$r \sin\theta < R_{atm}$$

where R_{atm} denotes the radius of the Earth plus the obscuring atmosphere and is approximated by R_e + 90 km [15].

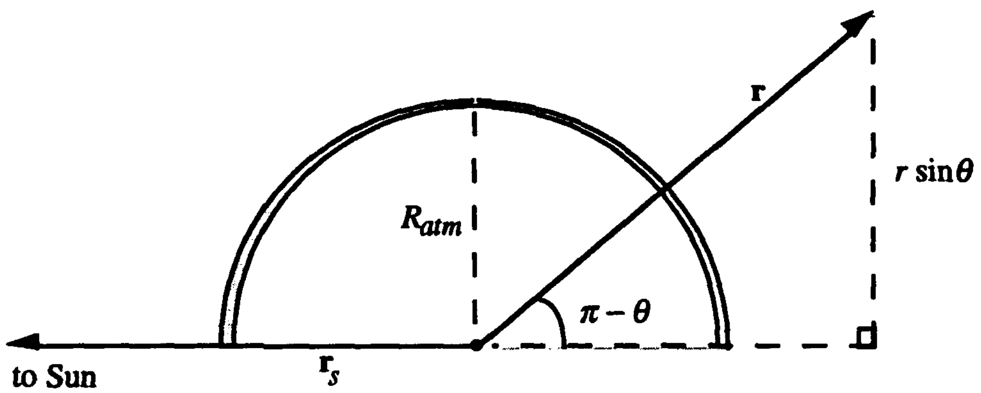


Figure 3-7. Test for Shadowing

3.4 INTEGRATION SCHEME

The equations of motion in section 3.3 have no closed-form solution. However, approximations to a satellite's position and velocity vectors as functions of time can be obtained using numerical methods. The scheme used by ESP is a classical fourth-order Runge-Kutta algorithm.

To develop a general understanding of the theory behind this algorithm, consider the equations for two-body motion

$$\frac{d\mathbf{r}}{dt} = \mathbf{v} \qquad \frac{d\mathbf{v}}{dt} = \mathbf{g}(\mathbf{r}) = -\frac{\mu}{r^3}\mathbf{r} \tag{3-7}$$

with initial conditions at time t_o, $\mathbf{r}(t_o)$ and $\mathbf{v}(t_o)$. The position and velocity at some later time $t_o + \Delta t$ can be found by expanding $\mathbf{r}(t)$ and $\mathbf{v}(t)$ each as a Taylor series about the value t_o. As an example, first-order approximations are given by

$$\mathbf{r}(t_o + \Delta t) = \mathbf{r}(t_o) + \left.\frac{d\mathbf{r}}{dt}\right|_{t_o} \Delta t + O(\Delta t)^2$$

$$\mathbf{v}(t_o + \Delta t) = \mathbf{v}(t_o) + \left.\frac{d\mathbf{v}}{dt}\right|_{t_o} \Delta t + O(\Delta t)^2$$

where terms of the order $(\Delta t)^2$ and higher are neglected. (Note that this example serves for illustration purposes only. As section 3.2 indicates, the six orbital elements explicitly define two-body motion; as such, an integration scheme is neither needed nor desirable.) In general, an n^{th} order algorithm neglects terms of order $(\Delta t)^{n+1}$ and higher. Accuracy is improved by using a higher order algorithm or a smaller time increment, or both. However, when moving to higher orders, the derivatives on the right-hand side of the above equations can become quite complex. Even with a second-order algorithm, the third term in the expansion for the velocity vector includes

$$\left.\frac{d^2\mathbf{v}}{dt^2}\right|_{t_o} = \left.\frac{\partial \mathbf{g}}{\partial \mathbf{r}}\frac{d\mathbf{r}}{dt}\right|_{t_o}$$

which involves calculating a 3×3 matrix. Moving to a third-order algorithm would involve a 3-dimensional array, and the computations become increasingly more complicated with orders beyond. These cumbersome calculations can be avoided by considering Taylor series expansions of appropriate vector sums and making substitutions. The coefficients in these sums can be chosen in such a way that the higher order algorithms all follow the same general format. An underlying goal in making these substitutions is to minimize the number of function evaluations (i.e., evaluations of the vector $\mathbf{g}$), otherwise known as the *number of stages*. Inspection of the equations of motion with just three perturbations included illustrates the desirability of attaining this goal. The details for these conversions are themselves long and tedious and are deferred to reference 3.

The general form of equations 3-7 does not hold when including atmospheric drag and solar radiation pressure effects. In particular, the vector **g** is no longer strictly a function of **r** but now also a function of **v** and t. This problem is handled by defining the following 6×1 state vector:

$$\mathbf{s} = \begin{bmatrix} \mathbf{r} \\ \mathbf{v} \end{bmatrix}$$

so that

$$\frac{d\mathbf{s}}{dt} = \mathbf{f}(t,\mathbf{s})$$

Given the initial value $\mathbf{s}_o$ defined at time t_o, the classical four-stage, fourth-order algorithm for approximating the state at time $t_o + \Delta t$ is as follows:

$$\mathbf{s} = \mathbf{s}_o + \frac{\Delta t}{6}(\mathbf{k}_0 + 2\mathbf{k}_1 + 2\mathbf{k}_2 + \mathbf{k}_3) + O(\Delta t)^5$$

where

$$\mathbf{k}_0 = \mathbf{f}(t_o,\mathbf{s}_o)$$

$$\mathbf{k}_1 = \mathbf{f}\left(t_o + \frac{\Delta t}{2},\mathbf{s}_o + \frac{\Delta t}{2}\mathbf{k}_0\right)$$

$$\mathbf{k}_2 = \mathbf{f}\left(t_o + \frac{\Delta t}{2},\mathbf{s}_o + \frac{\Delta t}{2}\mathbf{k}_1\right)$$

$$\mathbf{k}_3 = \mathbf{f}(t_o + \Delta t,\mathbf{s}_o + \Delta t\mathbf{k}_2)$$

The new value for **s** corresponding to time $t_o + \Delta t$ then serves as the initial condition for obtaining the state at time $t_o + 2\Delta t$, and the process repeats.

ESP 2.0 uses a fixed step size (Δt) of approximately one minute. The error introduced due to a fixed step size and truncation of the series expansions after five terms was found to be minimal, as addressed in section 4.4.

3.5 INITIAL CONDITIONS

This section provides the details for determining satellite position and velocity vectors at some instant given the six orbital elements at that instant (a, e, i, Ω, ω, τ, and t_{osc}, or equivalently, a, e, i, Ω, ω, and $M(t_{osc})$). The following method is used to obtain initial values for the integration scheme as well as for updating the position vector to the Sun as a function of time.

First consider the (x', y', z') coordinate system obtained by rotating the ECI coordinate frame through the three Euler angles, Ω, ω, and i (see figure 3-8). In the rotated system, the x'-y' plane corresponds to the orbital plane with the positive x' direction toward perigee. The position vector in the rotated frame may be expressed in terms of the eccentric anomaly, semimajor, and semiminor axes, as follows:

$$\mathbf{r} = \begin{bmatrix} a\cos E - ae \\ b\sin E \\ 0 \end{bmatrix}$$

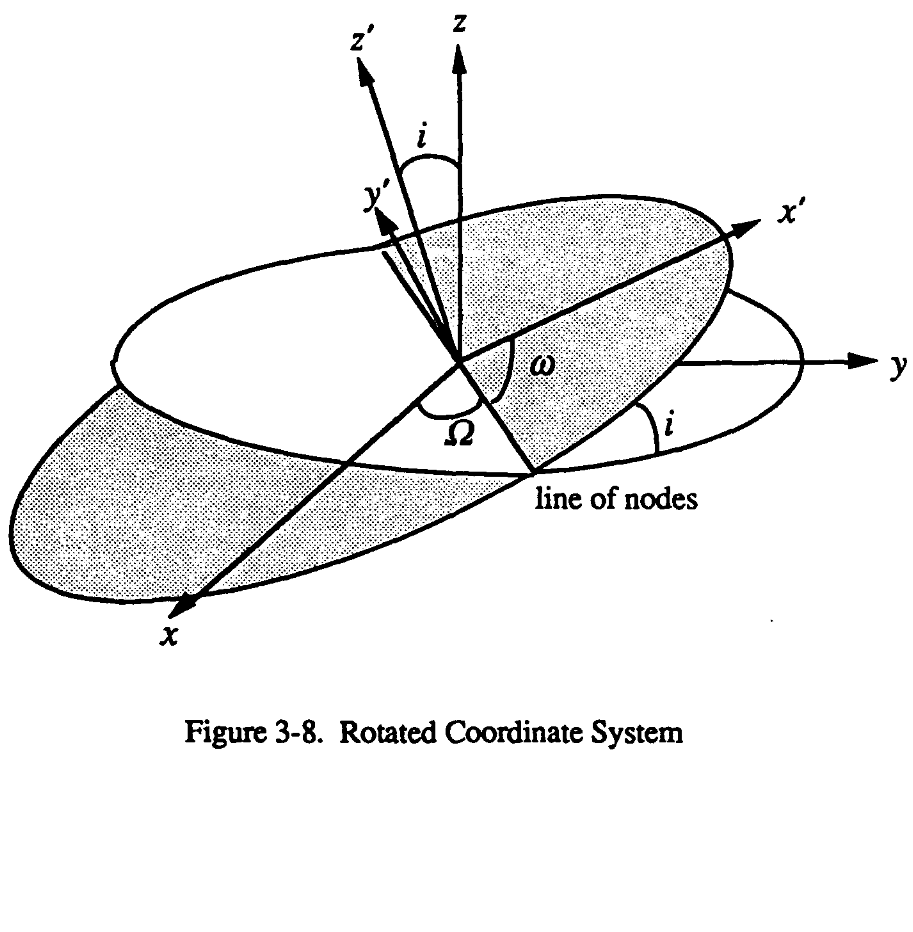


Figure 3-8. Rotated Coordinate System

The velocity vector is calculated by differentiating **r** with respect to time, which results in

$$\mathbf{v} = \frac{na}{r}\begin{bmatrix} -a\sin E \\ b\cos E \\ 0 \end{bmatrix}$$

since

$$\frac{dE}{dt} = \frac{na}{r}$$

The z' component of these vectors is naturally zero since both lie in the plane of motion.

The eccentric anomaly is related to the mean anomaly and eccentricity by Kepler's equation

$$E = M + e\sin E$$

which is transcendental in E. It is easily shown that this equation has a unique solution, and a variety of numerical solution techniques exist. Reference 3 provides a simple proof as well as several methods that may be used to solve for E. ESP uses Newton's method, which is an iterative technique. An initial guess is taken as $M + e \sin M$; subsequent values are determined from the relation

$$E_{k+1} = E_k - \frac{f(E_k)}{f'(E_k)} \tag{3-8}$$

where

$$f(E) = E - e\sin E - M$$

until successive values agree to some desired level of accuracy. (Accuracy is further addressed in section 4.) This technique actually works quite well, often converging within four iterations. An exception occurs when e is very close to one and E is close to zero or 2π. In this case, the denominator in equation 3-8 approaches zero and convergence difficulties may be encountered. However, with an initial guess taken as π, this problem should be avoided [2].

The position and velocity in the ECI frame are then obtained by left multiplying the position and velocity in the rotated frame by the rotation matrix

$$\mathbf{R} = \begin{bmatrix} \cos\Omega\cos\omega - \sin\Omega\sin\omega\cos i & -\cos\Omega\sin\omega - \sin\Omega\cos\omega\cos i & \sin\Omega\sin i \\ \sin\Omega\cos\omega + \cos\Omega\sin\omega\cos i & -\sin\Omega\sin\omega + \cos\Omega\cos\omega\cos i & -\cos\Omega\sin i \\ \sin\omega\sin i & \cos\omega\sin i & \cos i \end{bmatrix}$$

whose entries are the direction cosines of the nine angles formed between the axes of the two coordinate systems.

3.6 DETERMINATION OF SATELLITE GROUND TRACK

At any particular instant the satellite position vector intersects the surface of the Earth at a point referred to as the subsatellite point. The satellite ground track is generated by connecting a discrete set of subsatellite points that result as the satellite moves in its orbit over time. The location of a particular subsatellite point in the ECI frame is obtained by projecting the satellite position onto the Earth and can be described by the two angles, α_{sat} and δ_{sat}, shown in figure 3-9. These angles are commonly referred to as the *right ascension* and *declination*, respectively. They are related to the satellite's Cartesian coordinates as follows:

$$\alpha_{sat} = \arctan\left(\frac{y}{x}\right)$$

$$\delta_{sat} = \arcsin\left(\frac{z}{r}\right)$$

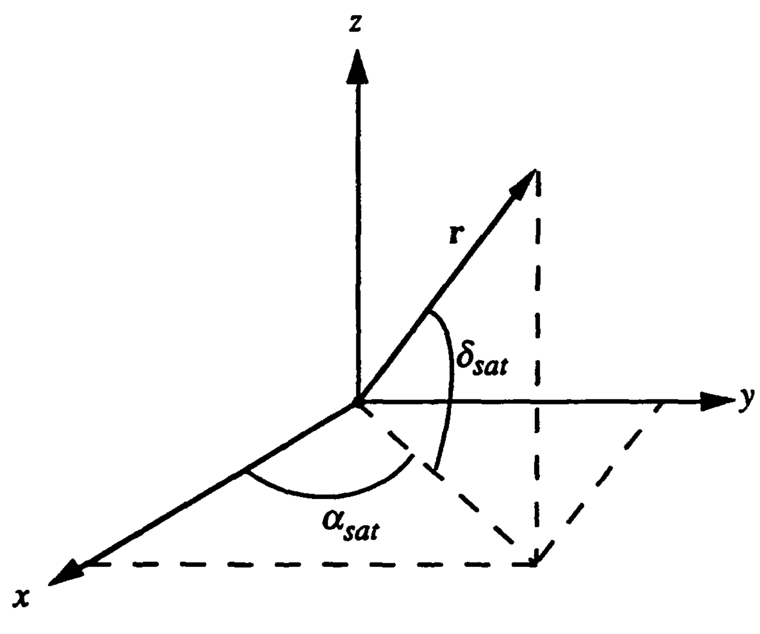


Figure 3-9. Subsatellite Right Ascension and Declination

Note that the geocentric latitude is synonymous with the declination δ_{sat}. However, the longitude is measured from the prime meridian (the 0° meridian passing through Greenwich, England). The longitude, γ_{sat}, of the subsatellite point can be obtained from the subsatellite and prime meridian right ascensions, α_{sat} and PM, respectively, by the following relation

$$\gamma_{sat} = \alpha_{sat} - \text{PM}$$

Reference 9 provides the following approximation for the ephemeris position of PM at time t in the standard coordinate system of 1950.0:

$$\text{PM} = 99$$
$$^{\circ}.87 + 24(\omega_e)(d)$$

where as before,

ω_e = mean rotation rate of the Earth = 15°.041067178 per hour
and d = number of ephemeris days from reference epoch 1 January 1950 to time t

Also note that the typical problems that occur with standard arctangent functions (e.g. when the argument is equal to 90° or 270° or falls in quadrants II or III) are avoided by using a "smart" arctangent function, as described in section 4.6.

3.7 RANGE AND RANGE RATE CALCULATIONS

Range is defined as the distance from a given ground site location to the satellite. At any particular instant, this distance is determined as a function of the position vectors in the ECI frame to the site and the satellite. The position vector to the site is defined in terms of its latitude and longitude coordinates. As described above, the declination, δ_{site}, is equivalent to the latitude, and the right ascension, α_{site}, is a function of the longitude and the prime meridian location:

$$\delta_{site} = \text{site latitude}$$

$$\alpha_{site} = \text{PM} + \text{site longitude}$$

Also, the magnitude of this vector, r_{site}, is identical to the radius of the Earth at the given location; it is determined as a function of the site latitude using equation 3-6. The vector to the site in the ECI frame is then obtained using spherical coordinates.

$$\mathbf{r}_{site} = \begin{bmatrix} x_{site} \\ y_{site} \\ z_{site} \end{bmatrix} = r_{site} \begin{bmatrix} \cos(\delta_{site})\cos(\alpha_{site}) \\ \cos(\delta_{site})\sin(\alpha_{site}) \\ \sin(\delta_{site}) \end{bmatrix}$$

As shown in figure 3-10, the line-of-sight vector from the ground site to the satellite is calculated as the vector subtraction,

$$\mathbf{d} = \mathbf{r} - \mathbf{r}_{site}$$

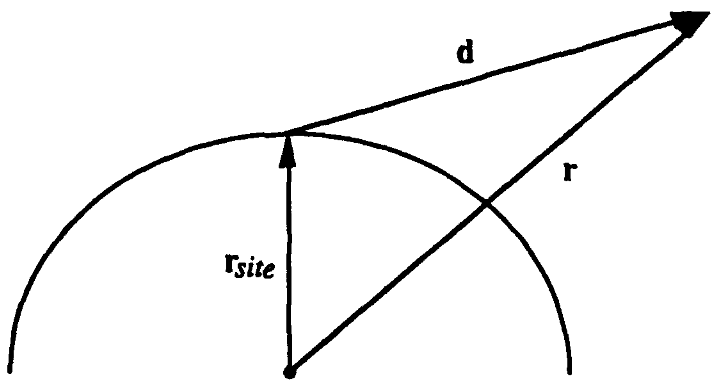


Figure 3-10. Range Geometry

The range is then simply the magnitude of the vector **d**, i.e.,

$$|\mathbf{d}| = \left[(x - x_{site})^2 + (y - y_{site})^2 + (z - z_{site})^2\right]^{\frac{1}{2}}$$

Range rate is calculated as the time derivative of the range, or

$$\frac{d\left[|\mathbf{d}|\right]}{dt} = \frac{\mathbf{d} \cdot \dot{\mathbf{d}}}{|\mathbf{d}|}$$

with the time derivative of the vector **d** given by

$$\dot{\mathbf{d}} = \begin{bmatrix} \dot{x} - \dot{x}_{site} \\ \dot{y} - \dot{y}_{site} \\ \dot{z} - \dot{z}_{site} \end{bmatrix} = \begin{bmatrix} v_x + r_{site}\cos(\delta_{site})\sin(\alpha_{site})\omega_e \\ v_y - r_{site}\cos(\delta_{site})\cos(\alpha_{site})\omega_e \\ v_z - 0 \end{bmatrix}$$

since

$$\Delta\alpha_{site} = \omega_e \Delta t \quad => \quad \frac{d\left[\alpha_{site}\right]}{dt} = \omega_e$$

$$\Delta\delta_{site} = 0 \quad => \quad \frac{d\left[\delta_{site}\right]}{dt} = 0$$

3.8 VISIBILITY TEST

In general, visibility between a particular ground site location and a single satellite is possible when the line-of-sight connecting them is above the horizon. In other words, the elevation angle measured from the horizon to the vector **d** must be greater than or equal to zero. Figure 3-11 illustrates the geometry. The vector **a** is normal to the horizon and intersects the *x-y* plane in the geodetic latitude, *L*. *L* is related to the site declination (otherwise known as the geocentric latitude), δ_{site}, by the following expression:

$$L = \arctan\left(\frac{1}{\left(1-e_e^2\right)}\tan(\delta_{site})\right)$$

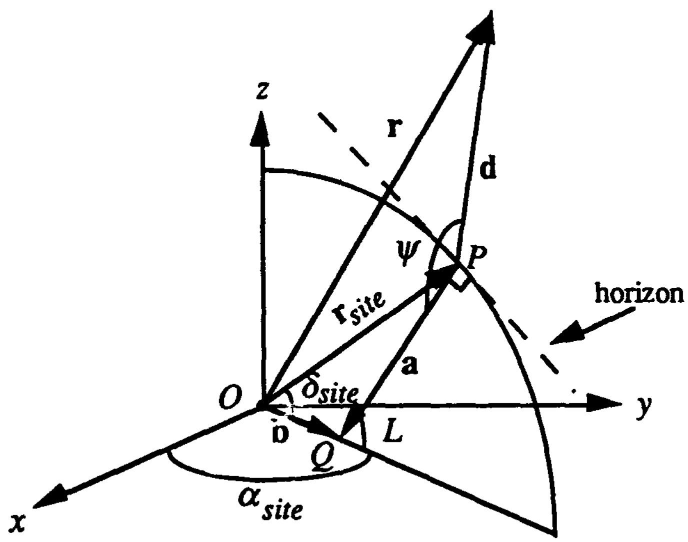


Figure 3-11. Elevation Angle Geometry

where e_e is the eccentricity of an elliptical cross-section through the Earth (equal to 0.08182) as defined in section 3.3.2. (For a derivation, see reference 10.) Thus, the planar triangle *OPQ* has known angles $|\delta_{site}|$, $|L|-|\delta_{site}|$, and $180°-|L|$. The vector **b** is therefore determined as follows:

$$\mathbf{b} = b\begin{bmatrix}\cos(\alpha_{site})\\ \sin(\alpha_{site})\\ 0\end{bmatrix}$$

where α_{site} is the site right ascension and the magnitude, b, is calculated from the planar triangle law of sines as

$$b = \frac{r_{site}\sin(|L| - |\delta_{site}|)}{\sin(180^\circ - |L|)} \tag{3-9}$$

Note that when $|L| = 0$ then $\mathbf{b} = \mathbf{0}$; thus, the singularity in equation 3-9 is easily avoided. The vector $\mathbf{a}$ is then given by the vector subtraction

$$\mathbf{a} = \mathbf{b} - \mathbf{r}_{site}$$

Finally, the angle ψ between the two vectors $\mathbf{a}$ and $\mathbf{d}$ is calculated from the dot product as

$$\psi = \arccos\left(\frac{\mathbf{a}\cdot\mathbf{d}}{a\,d}\right) \tag{3-10}$$

The elevation angle is simply $\psi - 90^\circ$. A negative value indicates that the satellite is below the horizon.

In the outage portion of the program, the user inputs a minimum elevation angle below which satellites and ground sites cannot see each other. Coverage is obtained at a given ground site location only when the elevation angle measured from this site is above this minimum elevation angle for the user-specified minimum number of satellites in the constellation.

3.9 SPOT BEAM PROJECTIONS

The term spot beam refers to a satellite beam that is narrow in comparison to an Earth coverage beam. A spot beam projection is defined as the contour of intersection of such a beam and the surface of the Earth. ESP assumes that the beam is a circular cone. The user specifies a pointing location for the beam centroid, i.e., longitude and latitude coordinates on the surface of the Earth, and a beam width, defined as the angle that subtends the full cone. The technique used for generating these projections is based on the one described in reference 1 with modifications which are described below. The basic geometry is illustrated in figure 3-12.

The idea behind the technique is to determine a sufficient number of discrete points along the intersection of the cone and the Earth's surface such that, when these points are connected, the resultant curve will appear smooth. In the terminology of reference 1, these intersection points are referred to as *generator points*. The coordinates of an individual generator point are derived from the angle θ_e on the surface of the Earth (which may alternatively be thought of as a central Earth angle) that connects the generator point with the subsatellite point, specifically, from the magnitude of θ_e and its orientation relative to the subsatellite point. A free parameter

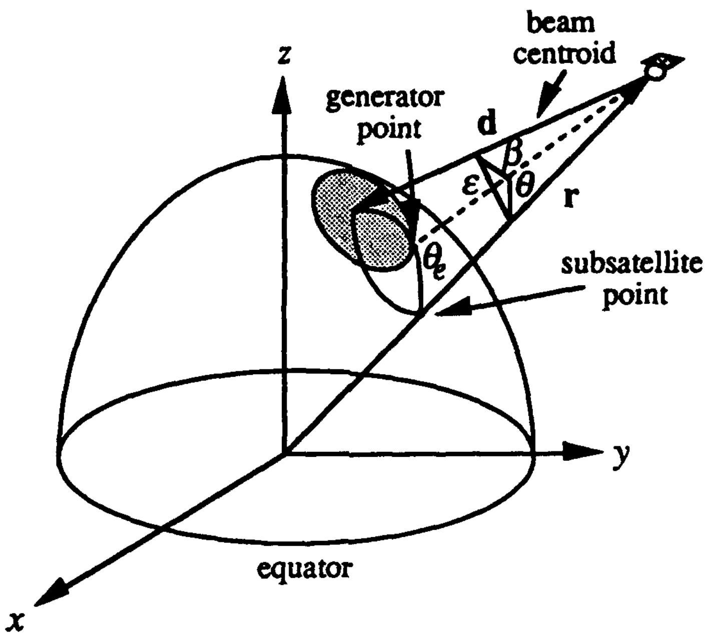


Figure 3-12. Spot Beam Projection Geometry

γ is used to determine a complete set of generator points. Figure 3-13 illustrates how a specific generator point corresponds to some angle γ and how a full set of generator points may be obtained by varying γ from 0° to 360°.

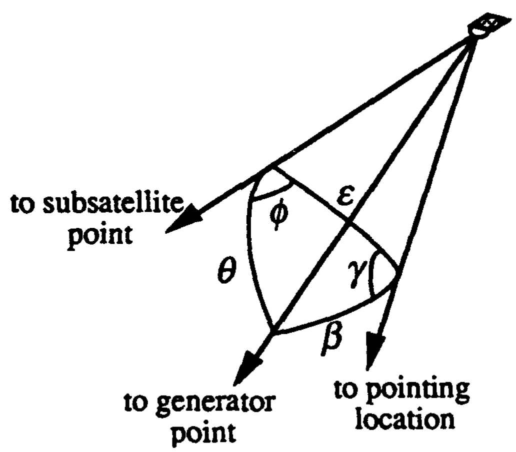


Figure 3-13. Spot Beam Angle Relationships

Application of the technique first requires calculation of two auxiliary angles that are derived from the beam centroid and satellite positions. These are the elevation and azimuth angles, denoted ε and az respectively, of the beam centroid with respect to the satellite. The angle ε is depicted in figure 3-12 and evaluated from the dot product as

$$\varepsilon = \arccos\left(\frac{-\mathbf{r}\cdot\mathbf{d}}{rd}\right)$$

The angle az, illustrated by the angle χ in figure 3-14, is defined in the usual manner as the positive angle measured clockwise from north to the pointing location. In general, az is calculated as a function of the smallest angle χ measured from north to the pointing location as follows:

$$az = \begin{cases} \chi, & \text{if } \gamma_{sat} \text{ lies to the left of } \gamma_{cen} \\ 360^\circ - \chi, & \text{if } \gamma_{sat} \text{ lies to the right of } \gamma_{cen} \end{cases}$$

where γ_{sat} and γ_{cen} denote the longitudes of the subsatellite and beam centroid points, respectively. The adjustment that occurs when γ_{sat} lies to the right of γ_{cen} is necessary since in such cases, az, by definition, assumes values larger than 180°. The angle χ, in turn, is a function of the angle b_o (also shown in figure 3-14). Both χ and b_o are calculated from the spherical triangle law of cosines for sides as follows:

$$\chi = \arccos\left(\frac{\cos(90^\circ-\delta_{cen}) - \cos(90^\circ-\delta_{sat})\cos(b_o)}{\sin(90^\circ-\delta_{sat})\sin(b_o)}\right) \tag{3-11}$$

$$b_o = \arccos\left(\cos(90^\circ-\delta_{sat})\cos(90^\circ-\delta_{cen}) + \sin(90^\circ-\delta_{\mathrm{sat}})\sin(90^\circ-\delta_{cen})\cos(\Delta long_{cen})\right)$$

with δ_{sat} and δ_{cen} denoting the latitudes of the subsatellite and centroid locations, and $\Delta long_{cen}$ taken as the magnitude of the difference between γ_{sat} and γ_{cen}.

The individual steps of the technique are outlined below. The calculations rely heavily on trigonometric relationships for spherical triangles. The details are deferred to reference 1:

1) Calculate β = (beam width)/2

2) Set the free parameter γ, $0^\circ \le \gamma < 360^\circ$

3) Calculate the side θ and then the interior angle ϕ of the spherical triangle shown in figure 3-13:

$$\theta = \arccos(\cos\varepsilon\cos\beta + \sin\varepsilon\sin\beta\cos\gamma)$$

$$\phi = \arcsin(\sin\beta\sin\gamma / \sin\theta) \tag{3-12}$$

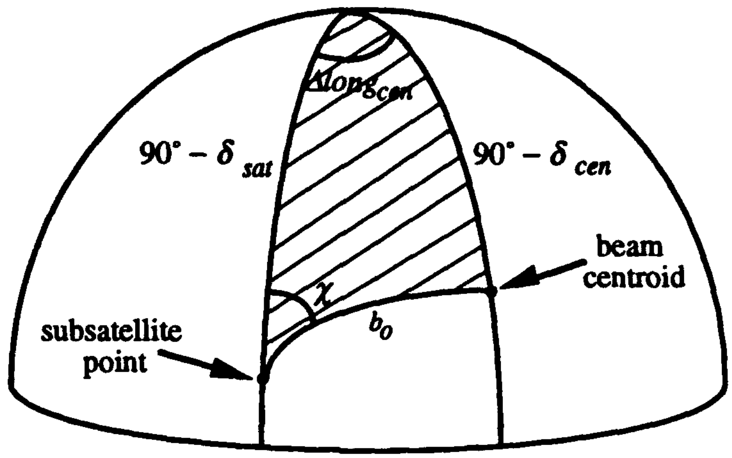


Figure 3-14. Azimuth Angle

4) Calculate the angle on the surface of the Earth, θ_e, which corresponds to θ; assume a constant Earth radius, R_e:

$$\theta_e = \arcsin\left(\frac{r\sin\theta}{R_e}\right) - \theta \tag{3-13}$$

5) Calculate the generator point latitude, δ_{gen}, and the difference, $\Delta long_{gen}$, in the generator point longitude relative to the subsatellite longitude:

$$\delta_{gen} = \arcsin\left(\sin\delta_{sat}\cos\theta_e + \cos\delta_{sat}\sin\theta_e\cos(az+\phi)\right) \tag{3-14}$$

$$\Delta long_{gen} = \arcsin\left(\frac{\sin(az+\phi)\sin\theta_e}{\cos(\delta_{gen})}\right) \tag{3-15}$$

6) Increment γ and repeat steps (3) through (5).

7) Repeat step (6) until γ sweeps out 360°.

ESP uses a fixed step size for γ of 2°.8125, so chosen to yield 128 generator points for every projection. This value was found to produce projections that are smooth in appearance regardless of the size of the projection contour.

In the above formulation, there are problems under certain circumstances with inverse trigonometric functions returning the incorrect quadrant. One example is evaluation of the angle ϕ by equation 3-12 when $\varepsilon < \beta$, i.e., when the projection contains the subsatellite point. The problem arises because in this circumstance, ϕ assumes values larger in magnitude than 90°, however, the arcsine function by itself is incapable of distinguishing angles in quadrants II and III from quadrants I and IV (by default it returns values in quadrant I when positive and quadrant IV when negative). A similar problem occurs with equation 3-15 when the satellite is near the poles. ESP handles these circumstances by supplementing the sine evaluations with cosine evaluations, and using a "smart" arctangent function, which is described in section 4.6. (A similar problem is generally not encountered with the arccosine function since in many cases the angle involved is known to fall within the range of arccosine values, i.e., within the interval [0°, 180°].)

The above technique has also been modified so that it is consistent with the Earth model described by equation 3-6, in which the radius of the Earth is a function of geocentric latitude. An iteration technique is used on steps (4) and (5) as follows. An initial value θ_{e_0} is calculated from equation 3-13 with R_e set equal to the radius of the Earth at the centroid location. Equations 3-14 and 3-15 are evaluated with θ_e set equal to θ_{e_0} to arrive at initial generator point coordinates. The radius of the Earth corresponding to the calculated generator point latitude is then used in equation 3-13 to calculate a new value for θ_e, which in turn is used in equations 3-14 and 3-15 to calculate new generator point coordinates. The process is repeated until subsequent values of the generator point coordinates are sufficiently close.

There are also a number of potential singularities (e.g. zero denominators) in the above formulation that require special attention. Some are easily eliminated by considering their geometric interpretation. For example, equation 3-11 need not be evaluated when b_0 is equal to zero since this implies *az* is equal to zero. Similarly, equation 3-12 (and all subsequent equations as well) is not necessary when θ is equal to zero, since in this case, the generator point coordinates are identical to the subsatellite point coordinates. Another example is evaluation of $\cos\phi$ (used to supplement equation 3-12) when $\varepsilon = 0$. In this case, the beam pointing location is the subsatellite point and the spherical triangle used to define ϕ (shown in figure 3-13) does not exist. Geometrically, ϕ becomes a free parameter and is taken equal to γ, thereby eliminating the use of the cosine function. In a few other instances, namely equation 3-11 when δ_{sat} is equal to ±90° and equation 3-15 when δ_{gen} is equal to ±90°, the singularities are avoided by substituting a latitude value which is very close, as described in section 4.6. This is done to preserve the geometry upon which the algorithm is based and thus ensure a continuous projection curve; it is necessary since the longitude of a point at the poles is arbitrary.

Throughout the algorithm, ESP checks to see if the beam shines over the horizon. This occurs when there exist values of θ, calculated by step (3), that exceed the maximum possible θ, denoted θ_{max} and shown in figure 3-15. The check is important since in general θ greater than

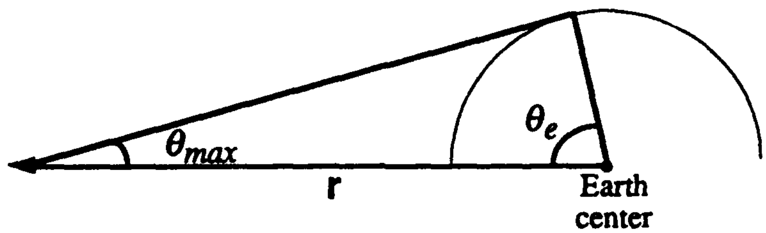


Figure 3-15. Maximum θ Value

θ_{max} means the planar triangle which is the basis for calculating θ_e does not exist. The result is that evaluation of equation 3-13 requires evaluation of the arcsine function with an argument greater than one. Since θ_{max} is a function of the radius of the Earth, which is variable, an initial guess is conservatively chosen as

$$\theta_{max} \approx \arcsin\left(\frac{6356.75}{r}\right)$$

This approximation is considered conservative since it represents the minimum possible θ_{max}, corresponding to the Earth's radius at the poles. Thus, when θ exceeds θ_{max}, there may or may not be a problem. In such cases, two potential generator points are calculated. The *first* is calculated in the usual manner as described above with θ_e set equal to an estimate of the maximum possible value, if necessary, to avoid the arcsine problem. The second point is calculated to fall on the horizon by iterating on θ_e until the elevation angle at the resultant generator point is sufficiently close to zero. Subsequent values for θ_e are determined from previous values as follows

$$\theta_{e_{i+1}} = \theta_{e_i} + \Delta$$

where the parameter Δ is chosen relative to the elevation angle as

$$\Delta = (\text{elevation angle})/2$$

This choice is based on an analysis of the sensitivity of elevation angle to θ_e, which showed that as the elevation angle varied from +0.5 to -0.5 degrees, θ_e increased by an approximate equal amount of one degree, regardless of orbital altitude. In all of the test cases, this technique worked well, with the number of iterations eight or fewer and most often four. Given the horizon point, the estimate for θ_{max} is updated based on the corresponding radius of the Earth. A choice is then made between the two potential points according to whether or not θ exceeds the updated value. ESP plots and connects all of the projection points up to and including any end points that fall directly on the horizon. Closure of a projection that shines over the horizon is achieved by overlaying a plot of the horizon contour, whose generation is described in section 3.11.

3.10 LOSS CONTOURS

In the case in which a single beam projection is of interest and the beam width corresponding to 3 dB loss (specified by the user) is less than 30°, ESP offers the option to plot contours of equal loss. A circular aperture uniform illumination beam pattern function (one of the more common pattern functions for circular conical beams) is used. The following expression is used to approximate voltage, $P(\eta)$, as a function of the angle η off boresight:

$$P(\eta) = \left(\frac{2J_1\left(K_1 \frac{\sin\eta}{\eta_o}\right)}{\left(K_1 \frac{\sin\eta}{\eta_o}\right)} \right) \tag{3-16}$$

where η_o represents the beam width corresponding to 3 dB loss, $J_1(x)$ is the first order Bessel function of the first kind, and K_1 is a constant equal to 3.2328. This approximation, which removes the explicit dependency of voltage on the radius of aperture and wavelength, is reasonably accurate for values of η_o less than or equal to 30° (which correspond to a radius of aperture less than or approximately equal to the wavelength). For such values, the 3 dB beam width is accurate to within 0.3 degrees, and the corresponding loss value is accurate to within 0.06 dB.

ESP takes η_o, by default, as the value entered by the user in the beam width field of the dialog box. The user has the option of plotting contours with either a uniform degree or dB step. In the case of the former, a loop is performed on η with loss calculated as a function of voltage as given below:

$$\text{Loss} = 20\log\left(P(\eta)\right)$$

In the case of the latter, the problem is solved backward with a loop over loss values. The result is that for a given loss value, equation 3-16 must be solved for η. Rearranged, the problem is to solve

$$f(\eta) = J_1(C_2 \sin\eta) - C_1 C_2 \sin\eta = 0 \tag{3-17}$$

where

$$C_1 = \frac{10^{\left(\frac{\text{loss}}{20}\right)}}{2}$$

$$C_2 = \frac{K_1}{\eta_o}$$

Since equation 3-17 is transcendental in η (i.e., it cannot be solved explicitly for η), it is solved using a Newton iteration scheme similar to the one described in section 3.5, with the function $f(\eta)$ given as above and $f'(\eta)$ given by

$$f'(\eta) = \left(J_0(C_2 \sin\eta) - \frac{J_1(C_2 \sin\eta)}{(C_2 \sin\eta)} \right) C_2 \cos\eta - C_1 C_2 \cos\eta$$

where $J_0(x)$ is the zero order Bessel function of the first kind. For a given loss value, an initial guess for η is taken as one-half the beam width of the previous contour. This technique was found to work quite well, with convergence occurring in most cases within four or five iterations.

Regardless of the step units, contours are plotted for loss values up to but not including -17.6 dB. This value corresponds to the power at the first side lobe of the beam pattern function, where the notion of a contour breaks down. The Bessel functions are evaluated according to polynomial approximations provided in reference 22.

3.11 HORIZON GENERATION

A horizon point is defined as a ground location from which the elevation angle to the satellite is zero. A horizon contour is a curve representing the set of points that lie on the horizon for a particular satellite at a particular instant. This contour is often of interest since it defines a boundary beyond which ground sites are incapable of seeing the satellite at some point in time (although points on and along the inner edge of this contour are often unable to see the satellite due to terrain irregularities and tall buildings). The technique for generating the horizon contour is similar to the technique for generating satellite ground tracks and spot beam contours in that a number of discrete points along the contour are found and connected. In the notation of section 3.8, the problem is to find all right ascension-declination pairs, (α_o, δ_o), such that $\mathbf{d} \cdot \mathbf{a} = 0$ (refer to figure 3-11). Substitution of the vectors $\mathbf{d}$ and $\mathbf{a}$, with $\mathbf{r}_{site}$ denoting the vector to the horizon point with coordinates $(\alpha_o\ \delta_o)$ and otherwise the same notation as used in section 3.8, results in the following equation:

$$\cos\alpha_o + \tilde{\beta}\sin\alpha_o + \tilde{\gamma} = 0 \qquad (3\text{-}18)$$

where

$$\tilde{\beta} = \frac{y}{x}$$

$$\tilde{\gamma} = \frac{r_{site}(r_{site} - b\cos\delta_o - z\sin\delta_o)}{x(b - r_{site}\cos\delta_o)}$$

For a specific value for δ_o, the above coefficients, $\tilde{\beta}$ and $\tilde{\gamma}$, are constant. Elimination of $\sin\alpha_o$ in equation 3-18 reduces the problem to solving a quadratic in $\cos\alpha_o$:

$$\tilde{a}\cos^2(\alpha_o) + \tilde{b}\cos\alpha_o + \tilde{c} = 0 \tag{3-19}$$

where

$$\tilde{a} = 1 + \tilde{\beta}^2$$

$$\tilde{b} = 2\tilde{\gamma}$$

$$\tilde{c} = \tilde{\gamma}^2 - \tilde{\beta}^2$$

The idea is then to set δ_o and use the quadratic formula to solve equation 3-19 for $\cos\alpha_o$. There may be at most two solutions to the quadratic which translates into four possible right ascension values, since a solution λ for $\cos\alpha_o$ implies that both $\arccos\lambda$ and $360° - \arccos\lambda$ are possibilities. The elevation angle for each possible coordinate pair is then calculated as described in section 3.8 to determine which possibilities correspond to points along the horizon. A complete set of horizon points is obtained by iterating this procedure over latitude values δ_o ranging from -90° to 90°. Right ascension coordinates are converted to longitude values for plotting purposes as described in section 3.6.

SECTION 4

PROGRAMMING CONSIDERATIONS

4.1 INTRODUCTION

This section addresses testing and questions of numerical accuracy within ESP. As with any computation-intensive program, measures had to be taken to deal with the unavoidable errors resulting from the finite precision with which computers represent floating-point numbers.

4.2 FLOATING POINT PRECISION

ESP uses double precision floating point numbers for time, date, prime meridian, and certain horizon calculations, and single precision for all other floating point operations. ESP uses Julian dates, where the basic unit of time is a day. Julian date values are currently so large (for example, 1 January 1990 is 2447892.5) that single precision floating point numbers give a resolution of about one day. Double precision is used to represent hours, minutes, and seconds. This is particularly important when calculating the prime meridian location and the orbital elements of the Sun as functions of time. In both examples, double precision is used to ensure sufficient precision in determining the time elapsed from the 1950 reference epoch to the present.

4.3 CONVERGENCE AND NUMERICAL TOLERANCE

ESP performs several operations that require the difference between two values to be "close." An example is the use of Newton's method, which derives successive estimates of the root of an equation from previous estimates. ESP uses Newton's method to solve Kepler's equation for eccentric anomaly and to convert a spot beam decibal loss value into beam width. For termination, this algorithm requires a maximum tolerance value for the difference between successive estimates.

In general, it is possible for the difference between estimates never to fall below a given tolerance. If this happens in an iterative algorithm, the program will fall into an infinite loop. At the same time, it is important that the tolerance be small enough to insure sufficient precision in the result. It is therefore important to allow a tolerance slightly larger than the minimum non-zero difference between any two estimates. (Unlike in real arithmetic, in floating point arithmetic there *can* be a minimum non-zero difference between values.) This minimum will vary with the size of the estimates. The program therefore chooses tolerances relative to the estimates, using the formula

$$(|E_1| + |E_2|)\,(k)$$

where E_1 and E_2 are the two values being compared, and k is 10^{-7} for single precision and 2×10^{-16} for double precision. This formula results in tolerances very near the limits of the Macintosh's precision.

In stand-alone tests of the Kepler function, estimates invariably converged to a difference of zero within five iterations. However, since there has not been a rigorous proof that the difference between successive estimates will always converge to zero, the tolerance is an important safeguard to prevent an infinite loop. In similar tests of the spot beam decibel loss-to-angle conversion, the tolerance was found to be necessary, since the difference between estimates did not always converge to exactly zero (although the difference invariably converged to within the tolerance within seven iterations, and usually within five).

In the unlikely event that convergence does not take place after a reasonable number of iterations, an error message is presented to the user, and calculation proceeds normally after the user dismisses the error message. This error message has never been generated in actual use within D063. "Reasonable" is defined as ten iterations in this case.

A tolerance is also used to calculate the horizon. This calculation uses the quadratic formula to locate points along the horizon. In particular, the discriminant of the formula determines whether or not such points exist at a given latitude. If the absolute value of the discriminant falls below the tolerance calculated from the formula above (with E_1 equal to zero and E_2 equal to the discriminant), the discriminant is assumed to be zero. This allows certain points to be located which would otherwise be skipped due to a very small negative discriminant. Since the program searches for points at every degree of latitude, skipping such a point would create an error in the graphical output.

4.4 THE RUNGE-KUTTA PROCEDURE

ESP uses two levels of time steps in its calculations. The higher level time step is the one entered by the user in the dialog box. In order to ensure accurate results from the numerical integration, the calculations are done at intervals of about one minute or less. These one-minute intervals will be referred to as the internal time interval; the one-minute step size will be referred to as the internal time step. It is at the internal time intervals that ESP calculates the satellite position and velocity. (The internal time step is derived by dividing the user-specified time step into equal intervals. Therefore, the internal time step will not be exactly one minute if the user-specified step size is not an exact integer number of minutes.)

Each call to the Runge-Kutta procedure calculates the satellite position and velocity over one user-specified (higher level) time interval. The procedure itself calculates the satellite position and velocity many times, using the internal time step. If each new time were derived by adding the internal time step to the previous time, there would be the potential for significant round-off errors. The time at the end of the procedure's calculations might not match the time that would be expected based on the user inputs. More seriously, the output would show satellite positions and velocities that do not match the printed times.

Although the Runge-Kutta procedure can take measures to minimize any mismatch between the calculated time at the end of its run and the actual user-specified time, it cannot guarantee that the two numbers will be equal each time. Therefore, each time the procedure is called (except for the first), the calculated ending time from the previous call is used as the starting time for the new higher level interval. This is done in order to insure consistency between the time values and the satellite positions and velocities from one call of the procedure to the next. The user-specified interval ending time is passed to the procedure, giving it a target time and insuring that the times used in the calculations do not stray from the user-specified times.

Within the Runge-Kutta procedure itself, measures were taken to minimize round-off errors in the internal time interval calculation. The procedure determines the internal time values in a two step process, relying entirely on integers for loop control. First, the number of iterations required over the higher level interval is determined. Then, on every pass through the loop, the precise values of the beginning and the end of each internal time interval are recalculated from the loop control values and the initial time of the higher level interval passed into the procedure. The variable that contains the initial time of the higher level interval is never changed during this process. In this way, the cumulative errors of repeated floating point additions are avoided. Upon exit from the procedure, the ending time of the higher level interval is calculated in the same manner based on its initial time. Only one floating point addition is used for the ending time value.

The Runge-Kutta algorithm inherently introduces error, since the equations are derived from truncated Taylor series expansions where terms of the order $(\Delta t)^5$ and higher are disregarded. As a means of assessing the magnitude of the error due to truncation and a fixed time step Δt, results for a variety of orbit types (low-Earth, Molniya, and geosynchronous) were compared with results from ASAP, which is significantly more sophisticated in its implementation of an eighth-order algorithm with variable step size control. The outcome was quite favorable; over a 24-hour time interval, position vector components agreed to within one kilometer at worst and often within tens of meters. This consistency of results is a source of confidence as well as validation, since ASAP is in turn a well-tested tool with results in agreement with other orbit propagators and more importantly, actual tracking data.

4.5 THE MAIN LOOPS

For the main loops over time, longitude and latitude, all of the loop control parameters are floating point. Due to the inexact nature of floating point control parameters, it is not unusual for a loop to iterate one time too few when using such parameters. This can be demonstrated with an example from a hypothetical machine with a precision of three decimal digits. The number two-thirds would be represented as 0.667. A loop from 0.0 to 10.0 by two-thirds would execute only fifteen times if 10.0 / 0.667 is truncated, and would end at 9.33 instead of 10.0.

To solve this problem, the correct number of iterations is determined before any loop is executed. A tolerance factor relative to the size of the difference between the stopping and starting parameters is added to that difference. The adjusted difference is then divided by the step size. When the loop is executed, an integer variable is used for counting, using the derived number of iterations. The proper floating point value for each pass is derived in turn.

The tolerance used is large enough to account for round-off error, but small enough to avoid overstepping the upper limit, so that, for example, a loop from 0 to 10 with a step of 1.5 would not end at 10.5.

If the range of the loop is not a multiple of the step size (within the tolerance), the computed number of iterations is increased by one in order to include the upper bound entered by the user.

4.6 NUMERIC ERROR AVOIDANCE

ESP uses a "smart" arctangent function, which has a range of $(-\pi, \pi]$ and handles 90° and 270° properly. The function takes two arguments and is defined as

$$\arctan2\,(y, x) = \arctan\,(y\,/\,x)$$

with special cases to handle $x = 0$ and testing of the signs of x and y to determine the proper quadrant of the result. For the reader familiar with the FORTRAN programming language, this function is equivalent to FORTRAN's function ATAN2.

The program's arcsine and arccosine implementations use arctan2 rather than the standard Pascal arctan, so arcsin (1) and arccos (0) do not result in runtime errors. Occasionally, as a result of round-off errors, a number slightly greater than 1 or less than -1 may be passed to one of these two routines during the course of calculation. In these rare cases a message is presented to the user, a 1 (or -1) is substituted for the incorrect argument, and calculation proceeds normally.

As noted in section 3.9, a singularity occurs during spot beam calculations when the subsatellite point or a spot beam generator point is at a pole. In addition, the geometry breaks down when either point is very near a pole. In these instances, 89.999° is substituted for 90°. The substitution does not affect the output because the output is given to two decimal places.

4.7 FUNCTION AND PROCEDURE TESTING

Where possible, the functions and procedures in ESP were individually tested and validated. Some specific examples are cited below:

- The Julian date function was verified by comparison with the JPL software tool QUICK [20]. This interactive calculator-like program has been extensively used and tested at JPL since its initial version in 1971.

- As described above, the Newton iteration schemes used to solve Kepler's equation and to calculate beam width as a function of decibel loss were tested for convergence. In addition, the solutions were verified for a variety of examples by substituting them back into the original equations.

- The Runge-Kutta algorithm was validated via trigonometric examples. Values for the sine and cosine functions were estimated and found to agree with the actual values to a minimum of five (and often six or seven) decimal places.

- The inverse trigonometric functions were tested with angles ranging from 0° to 360° to verify that the results returned were in the proper quadrant.

- Various altitudes were passed into the atmospheric density function to insure it produced the same values as in the density input tables.

- Sanity checks were made on the elevation and outage outputs to insure that they reflected the calculated satellite positions.

- The coded polynomial approximations for $J_0(x)$ and $J_1(x)$ were verified by comparison with tabulated values provided in reference 22.

- The spot beam projection and horizon algorithms were tested by examining numerous possible scenarios (e.g., for the relative locations of the pointing location and subsatellite point and a variety of orbit types). Doing so highlighted a number of singularities (particularly in the case of polar orbits) that have since been eliminated.

LIST OF REFERENCES

1. Adamy, D. L., December 1974, "ESV Antenna Footprints," *Microwave Journal*, pp. 57-59.

2. Bate, R. R., et al., 1971, *Fundamentals of Astrodynamics*, Dover Publications Inc., New York.

3. Battin, R. H., 1987, *An Introduction to the Mathematics and Methods of Astrodynamics*, American Institute of Aeronautics and Astronautics, Inc., New York.

4. Borenstein, P., et al., 1989-1991, *THINK Pascal Object-Oriented Programming Manual*, Symantec Corporation, Cupertino, CA.

5. Borenstein, P., et al., 1988-1991, *THINK Pascal User Manual*, Symantec Corporation, Cupertino, CA.

6. Bousquet, M., and G. Maral, 1990, "Orbital Aspects and Useful Relations from Earth Satellite Geometry in the Frame of Future Mobile Satellite Systems," AIAA-90-0874-CP.

7. Callwood, R., L. M. Gaffney, and R. A. Pomponi, 1993, "The Earth Satellite Program (ESP): A User-Friendly Constellation Design Tool," AIAA-93-1128.

8. Chobotov, V. A., 1991, *Orbital Mechanics*, American Institute of Aeronautics and Astronautics, Inc., Washington, DC.

9. Davies, M. E., et al., 1983, "Report of the IAU Working Group on Cartographic Coordinates and Rotational Elements of the Planets and Satellites: 1982," *Celestial Mechanics* 29, pp. 309-321.

10. Escobal P. R., 1965, *Methods of Orbit Determination*, John Wiley and Sons, Inc., New York.

11. Herrick, S., 1971, *Astrodynamics Volume I*, Van Nostrand Reinhold Company, London.

12. H. M. Nautical Almanac Office, 1961, *Explanatory Supplement to the Astronomical Ephemeris and the American Ephemeris and Nautical Almanac*, Her Majesty's Stationery Office, London.

13. *Inside Macintosh*, Vol. 1-5, 1985-1988, Apple Computer, Inc., Addison-Wesley Publishing Co., Inc.

14. Jensen, J., et al., 1962, *Design Guide to Orbital Flight*, Martin 'arietta Corporation, Maryland.

15. Kwok, J. H., April 20, 1987, *The Artificial Satellite Analysis Program (ASAP) Version 2.0*, EM 312/87-153, Jet Propulsion Laboratory.

16. Meeus, J., 1988, *Astronomical Formulae for Calculators*, Willmann-Bell, Inc., Virginia.

17. Nautical Almanac Office United States Naval Observatory, 1992, *The Astronomical Almanac For The Year 1993*, U.S. Government Printing Office, Washington D.C.

18. Roy, A. E., 1988, *Orbital Motion*, Adam Hilger, Bristol and Philadelphia.

19. Ruddy, J. M., et al., March 11-15, 1990, "Concept for a Cost/Technology-Driven Mobile Satellite Communications (MOBILSATCOM) System," AIAA-90-0864 presented at the 13th International Communication Satellite Systems Conference.

20. Skinner, D. L., 1989, "QUICK: An Interactive Software Environment for Engineering Design," American Institute of Aeronautics and Astronautics, Inc.

21. Sturms, F. M., January 15, 1971, *Polynomial Expressions for Planetary Equators and Orbit Elements with Respect to the Mean 1950.0 Coordinate System*, NASA Technical Report 32-1508.

22. United States Department of Commerce, 1970, *Handbook of Mathematical Functions With Formulas, Graphs, and Mathematical Tables*, National Bureau of Standards Applied Mathematics Series 55, Ninth printing.

APPENDIX

SAMPLE RUNS

A.1 SUMMARY

This appendix contains a sample of the program's outputs. It is organized into three subsections according to orbit type. A summary of the content of the individual subsections including a brief description of the orbits presented is provided below. Unless otherwise stated, satellite orbits were propagated over 24 hours, and outage feature runs have the minimum elevation angle parameter set to 10° and the minimum number of satellites in view parameter set to one. Additional examples which show how the program has been used since its initial release in 1991 to support a variety of MITRE projects can be found in reference 7.

Section A.2 provides all types of program outputs for geosynchronous orbits; it consists of the echoed input data as well as both tabular and graphical results. It should be noted that the tabular output shown is just a one page sample in all cases, since the complete output is often quite lengthy and would require numerous pages to duplicate. By definition, a geosynchronous orbit is a circular orbit with period equal to the period of the Earth's rotation about its axis (approximately 23.93 hours). The semimajor axis is calculated to be 42,163 km from equation 3-3. A satellite in geosynchronous orbit is often considered ideal for communications purposes since it resides over the same general region of the Earth. This is obvious from the ground track and becomes remarkably more pronounced as the inclination angle approaches zero. In all of the examples in this section, satellite orbits have a 65° inclination. Figure A-1 shows the ground track for a satellite centered on 99° W longitude. Table A-4 presents hourly elevation angle, range, and range rate calculations for the same satellite as it travels through its orbit and a set of four ground sites located in western Europe. Figure A-2 is an outage plot for a constellation of three satellites with parameters Ω, ω, and τ chosen so that each follows approximately the same ground track depicted in figure A-1. Such a constellation provides near perfect coverage for one side of the globe (as can be seen by the areas that have no markings in the plot) and no coverage for the other side (as can be seen by the large region that falls in zone 6). The output time step for both the ground track and outage zones is chosen relative to the orbit period and is one-half hour; this ensures both a smooth ground track and reasonably accurate coverage results. The last two examples show the program's spot beam and loss contours features for the same satellite as in figure A-1, although the projections are calculated at different points along its orbit. In figure A-3, a 5° beam is pointing in the Boston area, whereas in figure A-4, a 4° beam is pointing directly below the satellite. Both plots have been enlarged via the zoom feature described in section 2.4.5 under Draw a Map.

The remaining sections contain only the graphical outputs. Section A.3 is based on typical MOBILSATCOM orbits [19], which are highly inclined, circular low-Earth orbits. The satellite altitude is approximately 1000 km (which corresponds to a semimajor axis of 7378 km) and the orbital inclination is 80°. Figure A-5 shows the ground track. As can be seen from the plot, the satellite travels around the Earth several times over the course of a day (13.7 to be exact), since the orbit period is small (105 minutes) relative to the period of the Earth's daily rotation. Figure A-6 shows the coverage plot for a 96-satellite constellation that consists of eight different rings equally spaced in Ω with 12 equally spaced satellites per ring. Satellites in adjacent planes are half way out of phase, and a minimum of two satellites must be in view for ground sites to be considered covered. In both examples a time step of three minutes was used in order to obtain sufficient resolution for the ground track and outage zones. Examination of the program's text output indicates that the maximum cumulative outage over the course of a day is 4.3 hours. An example of multiple beam projections is shown in figure A-7, in which there are three 35° spot beams located on the same satellite as in figure A-5. Since the beams shine partially over the horizon, the horizon contour is plotted as well.

Section A.4 presents results for Molniya orbits. These are defined as highly eccentric 12-hour orbits (semimajor axis equal to 26,610 km) with inclination given by

$$\arccos\left(\frac{1}{\sqrt{5}}\right) \approx 63.4^\circ$$

This value is referred to as the *critical inclination*. At this inclination, the line of apsides will on the average remain constant over time. In contrast, the line of apsides will regress (advance) when the orbit inclination is greater (less) than this value due to J_2 effects. Figure A-8 shows the ground track for a satellite in a Molniya orbit with an eccentricity of 0.72 and with perigee in the southern hemisphere. In order to close the ground track, it was necessary to propagate the orbit over 24 hours, 8 minutes, and 57 seconds, since it takes half that time to generate the portion of the track from the start of the loop centered on 100° W longitude to the start of the loop centered on 78° E longitude. A major advantage of such orbits is that a large portion of the Earth's surface is visible at high elevation angles for long intervals of time. This is a result of the fact that the satellite spends a large percentage of its time near apogee (approximately 10 hours to form each of the ground track loops in figure A-8) and thus a relatively small period of time near perigee. Figure A-9 shows the coverage plot for a constellation of three satellites and a minimum elevation angle of 30°. All satellites have perigee in the southern hemisphere and Ω and τ are chosen so that all three follow approximately the same ground track shown in figure A-8. In both examples, a small output time step was used (3.6 minutes) since the satellite is moving so rapidly through perigee (approximately 9.6 km/s versus 1.6 km/s at apogee).

SECTION A.2

GEOSYNCHRONOUS ORBITS

Table A-1. Geosynchronous Orbit Ground Track - Echoed Inputs

Satellite Information

Semimajor axis	a = 42163.0 km
Eccentricity	e = 0.0
Inclination	i = 65.0° north
Longitude of the node	Ω = 90.0° east
Argument of perigee	ω = 0.0° east
Time of perigee	τ = Jan 1, 1991 0:00:00

The time of the osculating values is Jan 1, 1991 0:00:00.

Satellite mass	1500.0 kg
Effective area for drag	8.0 square m
Coefficient of drag	2.0
Effective area for solar radiation	10.0 square m
Absorption/reflectivity constant	1.5

Inputs For This Run

Starting time of the run	Jan 1, 1991 0:00:00
Ending time of the run	Jan 2, 1991 0:00:00
Time step	0.5 hour

Table A-2. Geosynchronous Orbit Ground Track - Sample Tabular Output

Date and time		Subsatellite longitude (degrees)	Subsatellite latitude (degrees)	Satellite altitude (km)
Jan 1, 1991	0:00:00	-9.41	0.00	35784.86
Jan 1, 1991	0:30:00	-13.74	6.81	35785.15
Jan 1, 1991	1:00:00	-17.98	13.60	35785.99
Jan 1, 1991	1:30:00	-22.02	20.35	35787.33
Jan 1, 1991	2:00:00	-25.74	27.02	35789.09
Jan 1, 1991	2:30:00	-28.99	33.58	35791.12
Jan 1, 1991	3:00:00	-31.54	39.96	35793.29
Jan 1, 1991	3:30:00	-33.08	46.09	35795.44
Jan 1, 1991	4:00:00	-33.19	51.83	35797.43
Jan 1, 1991	4:30:00	-31.26	56.98	35799.10
Jan 1, 1991	5:00:00	-26.62	61.20	35800.36
Jan 1, 1991	5:30:00	-18.94	64.03	35801.10
Jan 1, 1991	6:00:00	-9.07	65.00	35801.28
Jan 1, 1991	6:30:00	0.72	63.89	35800.88
Jan 1, 1991	7:00:00	8.20	60.95	35799.91
Jan 1, 1991	7:30:00	12.64	56.66	35798.43
Jan 1, 1991	8:00:00	14.42	51.46	35796.54
Jan 1, 1991	8:30:00	14.20	45.69	35794.38
Jan 1, 1991	9:00:00	12.57	39.54	35792.07
Jan 1, 1991	9:30:00	9.97	33.14	35789.78
Jan 1, 1991	10:00:00	6.68	26.57	35787.65
Jan 1, 1991	10:30:00	2.94	19.90	35785.84
Jan 1, 1991	11:00:00	-1.12	13.15	35784.45
Jan 1, 1991	11:30:00	-5.37	6.35	35783.59
Jan 1, 1991	12:00:00	-9.70	-0.46	35783.31
Jan 1, 1991	12:30:00	-14.02	-7.27	35783.64
Jan 1, 1991	13:00:00	-18.24	-14.06	35784.54
Jan 1, 1991	13:30:00	-22.27	-20.80	35785.96
Jan 1, 1991	14:00:00	-25.96	-27.47	35787.80
Jan 1, 1991	14:30:00	-29.17	-34.01	35789.93
Jan 1, 1991	15:00:00	-31.66	-40.38	35792.22
Jan 1, 1991	15:30:00	-33.13	-46.49	35794.49
Jan 1, 1991	16:00:00	-33.12	-52.20	35796.61
Jan 1, 1991	16:30:00	-31.02	-57.30	35798.42
Jan 1, 1991	17:00:00	-26.17	-61.44	35799.82
Jan 1, 1991	17:30:00	-18.30	-64.16	35800.71
Jan 1, 1991	18:00:00	-8.35	-64.99	35801.03
Jan 1, 1991	18:30:00	1.33	-63.75	35800.78
Jan 1, 1991	19:00:00	8.61	-60.70	35799.95
Jan 1, 1991	19:30:00	12.86	-56.33	35798.65
Jan 1, 1991	20:00:00	14.48	-51.09	35796.93
Jan 1, 1991	20:30:00	14.14	-45.28	35794.91
Jan 1, 1991	21:00:00	12.44	-39.11	35792.76
Jan 1, 1991	21:30:00	9.78	-32.70	35790.63
Jan 1, 1991	22:00:00	6.46	-26.13	35788.64
Jan 1, 1991	22:30:00	2.68	-19.44	35786.95
Jan 1, 1991	23:00:00	-1.39	-12.69	35785.69
Jan 1, 1991	23:30:00	-5.64	-5.89	35784.93

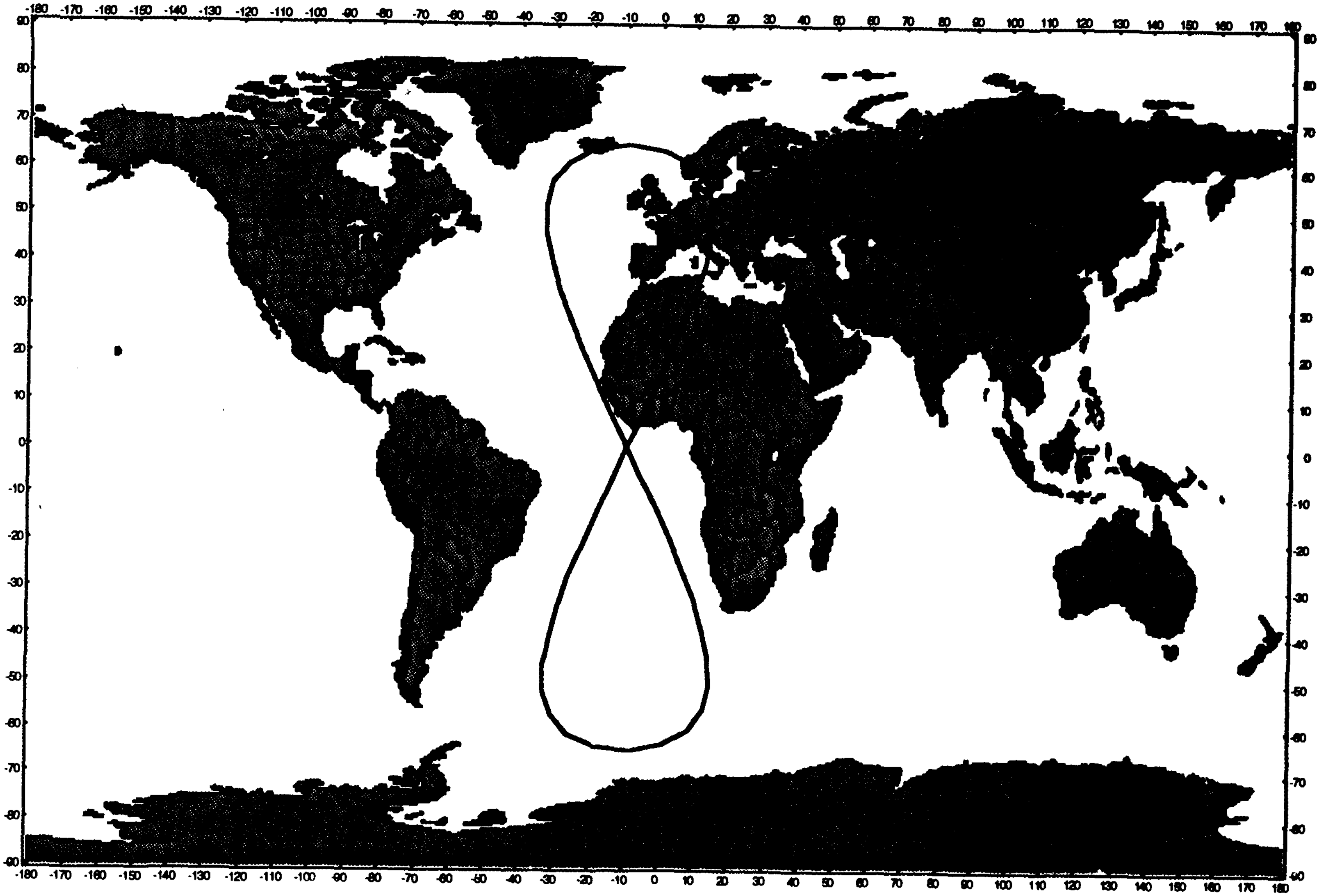


Figure A-1. Geosynchronous Orbit Ground Track - Graphical Output

Table A-3. Geosynchronous Orbit Elevation Angles - Echoed Inputs

<u>Satellite Information</u>

Semimajor axis	a = 42163.0 km
Eccentricity	e = 0.0
Inclination	i = 65.0° north
Longitude of the node	Ω = 90.0° east
Argument of perigee	ω = 0.0° east
Time of perigee	τ = Jan 1, 1991 0:00:00

The time of the osculating values is Jan 1, 1991 0:00:00.

Satellite mass	1500.0 kg
Effective area for drag	8.0 square m
Coefficient of drag	2.0
Effective area for solar radiation	10.0 square m
Absorption/reflectivity constant	1.5

<u>Inputs For This Run</u>

Latitude of first site	45.0° north
Latitude of last site	50.0° north
Latitude step size	5.0° north
Longitude of first site	0.0° east
Longitude of last site	10.0° east
Longitude step size	10.0° east
Starting time of the run	Jan 1, 1991 0:00:00
Ending time of the run	Jan 2, 1991 0:00:00
Time step	1.0 hour

Table A-4. Geosynchronous Orbit Elevation Angles - Sample Tabular Output

Date and time		Site longitude (degrees)	Site latitude (degrees)	Elevation angle (degrees)	Range (km)	Range rate (km/hr)
Jan 1, 1991	0:00:00	0.00	45.00	37.15	37996.08	-1064.89
Jan 1, 1991	0:00:00	0.00	50.00	31.81	38442.80	-1161.25
Jan 1, 1991	0:00:00	10.00	45.00	34.52	38212.26	-931.75
Jan 1, 1991	0:00:00	10.00	50.00	29.62	38637.02	-1041.15
Jan 1, 1991	1:00:00	0.00	45.00	49.29	37120.71	-682.18
Jan 1, 1991	1:00:00	0.00	50.00	44.41	37450.37	-817.47
Jan 1, 1991	1:00:00	10.00	45.00	44.33	37457.39	-580.54
Jan 1, 1991	1:00:00	10.00	50.00	40.31	37753.78	-724.79
Jan 1, 1991	2:00:00	0.00	45.00	58.03	36620.92	-331.09
Jan 1, 1991	2:00:00	0.00	50.00	54.63	36804.07	-484.46
Jan 1, 1991	2:00:00	10.00	45.00	50.83	37030.08	-291.42
Jan 1, 1991	2:00:00	10.00	50.00	48.56	37174.26	-447.04
Jan 1, 1991	3:00:00	0.00	45.00	62.34	36419.11	-96.42
Jan 1, 1991	3:00:00	0.00	50.00	61.62	36451.44	-239.92
Jan 1, 1991	3:00:00	10.00	45.00	54.25	36831.39	-129.83
Jan 1, 1991	3:00:00	10.00	50.00	54.35	36825.97	-268.82
Jan 1, 1991	4:00:00	0.00	45.00	63.36	36378.05	-9.77
Jan 1, 1991	4:00:00	0.00	50.00	65.66	36283.13	-116.84
Jan 1, 1991	4:00:00	10.00	45.00	56.26	36723.95	-105.83
Jan 1, 1991	4:00:00	10.00	50.00	58.67	36598.43	-203.52
Jan 1, 1991	5:00:00	0.00	45.00	63.87	36358.62	-44.51
Jan 1, 1991	5:00:00	0.00	50.00	68.40	36183.15	-96.20
Jan 1, 1991	5:00:00	10.00	45.00	58.92	36587.77	-176.54
Jan 1, 1991	5:00:00	10.00	50.00	63.10	36392.46	-216.53
Jan 1, 1991	6:00:00	0.00	45.00	66.03	36271.47	-131.30
Jan 1, 1991	6:00:00	0.00	50.00	71.71	36076.04	-119.03
Jan 1, 1991	6:00:00	10.00	45.00	63.74	36365.07	-264.25
Jan 1, 1991	6:00:00	10.00	50.00	69.05	36161.57	-240.56
Jan 1, 1991	7:00:00	0.00	45.00	70.62	36108.01	-182.38
Jan 1, 1991	7:00:00	0.00	50.00	76.22	35956.23	-108.72
Jan 1, 1991	7:00:00	10.00	45.00	71.42	36083.16	-280.81
Jan 1, 1991	7:00:00	10.00	50.00	77.25	35933.55	-198.49
Jan 1, 1991	8:00:00	0.00	45.00	76.54	35945.58	-118.02
Jan 1, 1991	8:00:00	0.00	50.00	79.19	35892.90	3.34
Jan 1, 1991	8:00:00	10.00	45.00	81.82	35851.60	-154.36
Jan 1, 1991	8:00:00	10.00	50.00	86.37	35807.38	-29.44
Jan 1, 1991	9:00:00	0.00	45.00	77.25	35926.00	106.47
Jan 1, 1991	9:00:00	0.00	50.00	73.75	36009.08	252.12
Jan 1, 1991	9:00:00	10.00	45.00	83.01	35832.25	143.35
Jan 1, 1991	9:00:00	10.00	50.00	77.33	35924.09	285.89
Jan 1, 1991	10:00:00	0.00	45.00	67.31	36208.38	478.33
Jan 1, 1991	10:00:00	0.00	50.00	61.76	36437.41	619.82
Jan 1, 1991	10:00:00	10.00	45.00	67.97	36184.36	577.07
Jan 1, 1991	10:00:00	10.00	50.00	62.24	36415.72	709.07
Jan 1, 1991	11:00:00	0.00	45.00	52.72	36907.14	922.26
Jan 1, 1991	11:00:00	0.00	50.00	47.07	37264.76	1034.54
Jan 1, 1991	11:00:00	10.00	45.00	51.18	37000.12	1051.06
Jan 1, 1991	11:00:00	10.00	50.00	45.84	37348.47	1150.27

Table A-5. Geosynchronous Orbit Outage Zones - Echoed Inputs

<u>Satellite Number 1</u>

Semimajor axis	a = 42163.0 km
Eccentricity	e = 0.0
Inclination	i = 65.0° north
Longitude of the node	Ω = 90.0° east
Argument of perigee	ω = 0.0° east
Time of perigee	τ = Jan 1, 1991 0:00:00

The time of the osculating values is Jan 1, 1991 0:00:00.

Satellite mass	1500.0 kg
Effective area for drag	8.0 square m
Coefficient of drag	2.0
Effective area for solar radiation	10.0 square m
Absorption/reflectivity constant	1.5

<u>Satellite Number 2</u>

Semi-major axis	a = 42163.0 km
Eccentricity	e = 0.0
Inclination	i = 65.0° north
Longitude of the node	Ω = 210.0° east
Argument of perigee	ω = 240.0° east
Time of perigee	τ = Jan 1, 1991 0:00:00

The time of the osculating values is Jan 1, 1991 0:00:00.

Satellite mass	1500.0 kg
Effective area for drag	8.0 square m
Coefficient of drag	2.0
Effective area for solar radiation	10.0 square m
Absorption/reflectivity constant	1.5

<u>Satellite Number 3</u>

Semi-major axis	a = 42163.0 km
Eccentricity	e = 0.0
Inclination	i = 65.0° north
Longitude of the node	Ω = 330.0° east
Argument of perigee	ω = 120.0° east
Time of perigee	τ = Jan 1, 1991 0:00:00

The time of the osculating values is Jan 1, 1991 0:00:00.

Satellite mass	1500.0 kg
Effective area for drag	8.0 square m
Coefficient of drag	2.0
Effective area for solar radiation	10.0 square m
Absorption/reflectivity constant	1.5

Table A-5. (Concluded)

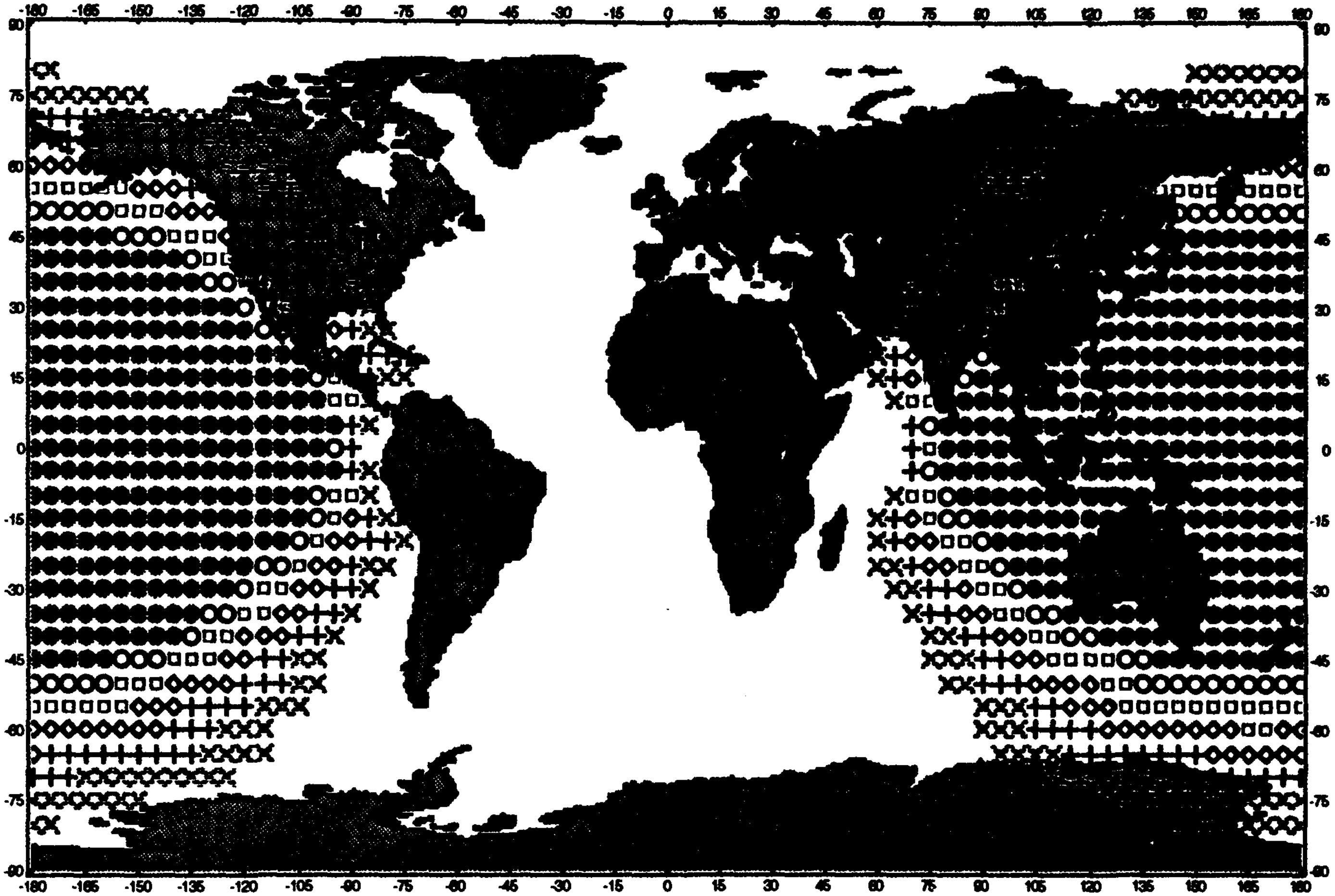
Inputs For This Run

Latitude of first site	-90.0° north
Latitude of last site	90.0° north
Latitude step size	5.0° north
Longitude of first site	-180.0° east
Longitude of last site	180.0° east
Longitude step size	5.0° east
Starting time of the run	Jan 1, 1991 0:00:00
Ending time of the run	Jan 2, 1991 0:00:00
Time step	0.5 hour
Minimum elevation angle	10.0°
Size of each outage zone	3.0 hours
Minimum number of satellites required for coverage	1 satellite

Table A-6. Geosynchronous Orbit Outage Zones - Sample Tabular Output

Zone 1 (0.00 to 3.00 hours)

Site longitude (degrees)	Site latitude (degrees)	Total outage time (hours)
-180.00	-80.00	1.00
-180.00	-75.00	2.50
-180.00	75.00	2.50
-180.00	80.00	1.00
-175.00	-80.00	0.50
-175.00	-75.00	3.00
-175.00	75.00	2.50
-175.00	80.00	0.50
-170.00	-75.00	3.00
-170.00	75.00	3.00
-165.00	-75.00	3.00
-165.00	-70.00	3.00
-165.00	75.00	2.50
-160.00	-75.00	2.50
-160.00	-70.00	3.00
-160.00	70.00	3.00
-160.00	75.00	2.50
-155.00	-75.00	2.00
-155.00	-70.00	3.00
-155.00	70.00	3.00
-155.00	75.00	2.00
-150.00	-75.00	1.00
-150.00	-70.00	3.00
-150.00	70.00	3.00
-150.00	75.00	1.00
-145.00	-70.00	3.00
-145.00	70.00	3.00
-140.00	-70.00	3.00
-140.00	70.00	3.00
-135.00	-70.00	1.50
-135.00	70.00	1.50
-130.00	-70.00	1.50
-130.00	-65.00	2.00
-130.00	65.00	2.00
-130.00	70.00	1.50
-125.00	-70.00	1.50
-125.00	-65.00	1.50
-125.00	-60.00	3.00
-125.00	60.00	3.00
-125.00	65.00	1.50
-125.00	70.00	1.50
-120.00	-65.00	1.50
-120.00	-60.00	3.00
-120.00	60.00	3.00
-120.00	65.00	1.50
-115.00	-65.00	1.50



✕ zone 1: 0.0 to 3.0 hrs.
✚ zone 2: 3.0 to 6.0 hrs.
◇ zone 3: 6.0 to 9.0 hrs.
□ zone 4: 9.0 to 12.0 hrs.
○ zone 5: 12.0 to 15.0 hrs.
● zone 6: more than 15.0 hrs.

Figure A-2. Geosynchronous Orbit Outage Zones - Graphical Output

Table A-7. Geosynchronous Orbit Spot Beam - Echoed Inputs

Satellite Information

Semimajor axis	a = 42163.0 km
Eccentricity	e = 0.0
Inclination	i = 65.0° north
Longitude of the node	Ω = 90.0° east
Argument of perigee	ω = 0.0° east
Time of perigee	τ = Jan 1, 1991 0:00:00

The time of the osculating values is Jan 1, 1991 0:00:00.

Satellite mass	1500.0 kg
Effective area for drag	8.0 square m
Coefficient of drag	2.0
Effective area for solar radiation	10.0 square m
Absorption/reflectivity constant	1.5

Inputs For This Run

Starting time of the run	Jan 1, 1991 3:30:00
Ending time of the run	Jan 1, 1991 3:30:00
Time step	0.5 hour

Contours of equal loss were not calculated.

Number of beams 1

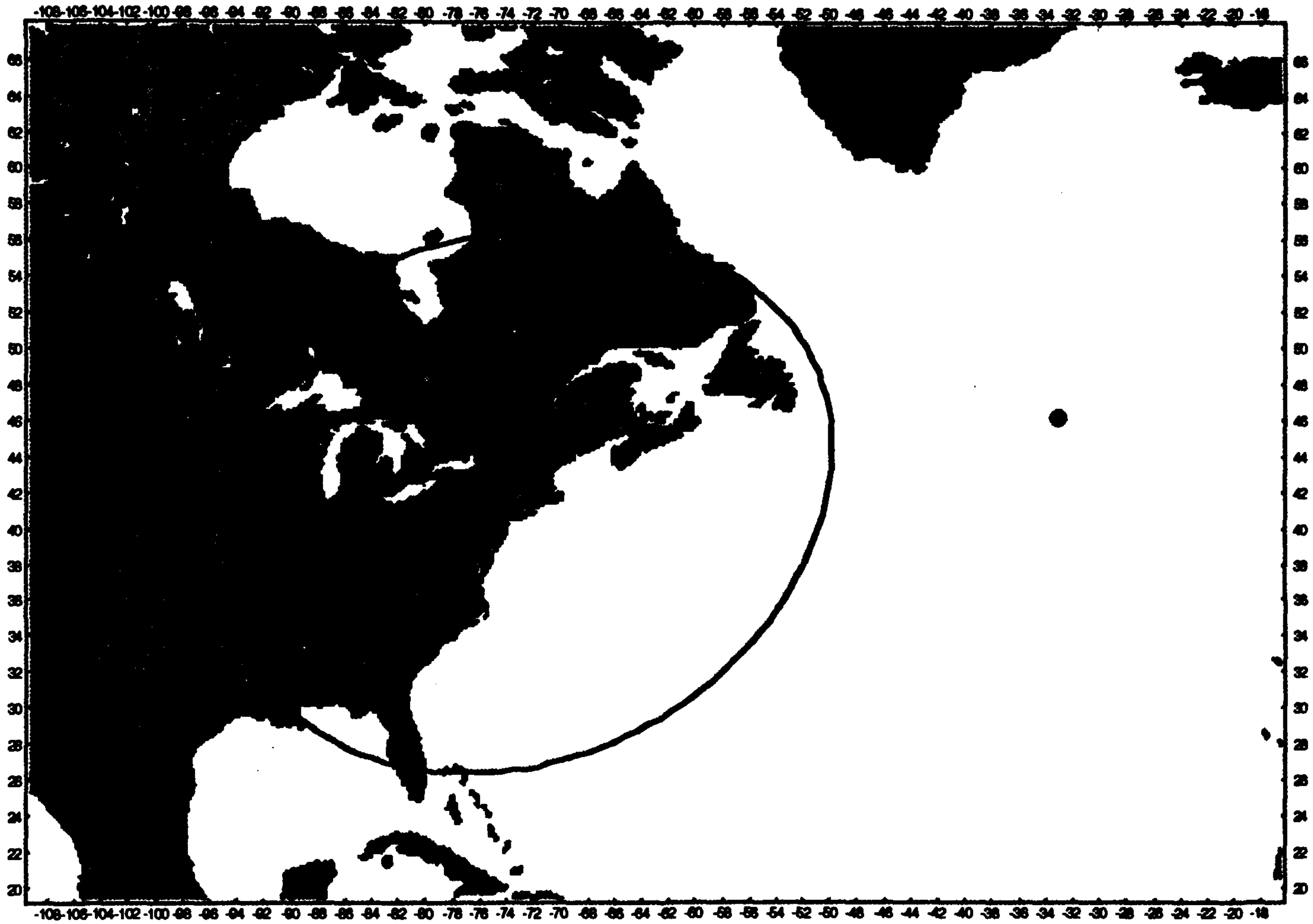
Beam Number	Centroid Longitude	Centroid Latitude	Beam Width
1	-70.90°	42.20°	5.00°

Table A-8. Geosynchronous Orbit Spot Beam - Sample Tabular Output

Beam #1 directed at (-70.90, 42.20)
Subsatellite Point: (-33.08, 46.09)

Beam width: 5.00°
Jan 1, 1991 3:30:00

Projection Points Longitude (degrees)	Projection Points Latitude (degrees)	Projection Points Longitude (degrees)	Projection Points Latitude (degrees)
-82.05	54.96	-66.13	28.05
-83.34	54.54	-65.24	28.36
-84.57	54.07	-64.35	28.68
-85.74	53.56	-63.48	29.03
-86.86	53.02	-62.63	29.40
-87.91	52.44	-61.80	29.79
-88.90	51.82	-60.98	30.20
-89.83	51.18	-60.18	30.63
-90.69	50.51	-59.40	31.09
-91.48	49.81	-58.64	31.56
-92.21	49.09	-57.91	32.05
-92.87	48.35	-57.20	32.55
-93.47	47.59	-56.51	33.08
-94.00	46.82	-55.85	33.62
-94.47	46.03	-55.21	34.18
-94.87	45.24	-54.61	34.75
-95.21	44.44	-54.03	35.34
-95.49	43.63	-53.49	35.94
-95.72	42.82	-52.97	36.56
-95.88	42.02	-52.49	37.18
-95.99	41.21	-52.04	37.82
-96.04	40.41	-51.63	38.47
-96.04	39.62	-51.26	39.13
-95.98	38.84	-50.93	39.79
-95.87	38.07	-50.63	40.47
-95.71	37.32	-50.38	41.15
-95.50	36.58	-50.17	41.83
-95.24	35.85	-50.01	42.52
-94.94	35.15	-49.89	43.21
-94.58	34.47	-49.82	43.91
-94.19	33.80	-49.80	44.60
-93.75	33.17	-49.84	45.29
-93.27	32.55	-49.93	45.98
-92.74	31.97	-50.07	46.67
-92.18	31.41	-50.27	47.35
-91.58	30.88	-50.53	48.02
-90.95	30.37	-50.85	48.68
-90.28	29.90	-51.24	49.33
-89.58	29.45	-51.69	49.97
-88.85	29.04	-52.20	50.59
-88.10	28.65	-52.79	51.20
-87.31	28.30	-53.44	51.78
-86.50	27.97	-54.16	52.35
-85.67	27.67	-54.95	52.89
-84.82	27.41	-55.80	53.40



+ Beam Centroid Location
● Subsatellite Location
— The Beam
---- The Horizon

Figure A-3. Geosynchronous Orbit Spot Beam - Graphical Output

Table A-9. Geosynchronous Orbit Loss Contours - Echoed Inputs

Satellite Information

Semimajor axis	a = 42163.0 km
Eccentricity	e = 0.0
Inclination	i = 65.0° north
Longitude of the node	Ω = 90.0° east
Argument of perigee	ω = 0.0° east
Time of perigee	τ = Jan 1, 1991 0:00:00

The time of the osculating values is Jan 1, 1991 0:00:00.

Satellite mass	1500.0 kg
Effective area for drag	8.0 square m
Coefficient of drag	2.0
Effective area for solar radiation	10.0 square m
Absorption/reflectivity constant	1.5

Inputs For This Run

Starting time of the run	Jan 1, 1991 0:00:00
Ending time of the run	N/A
Time step	N/A

Contours of equal loss are calculated every 2.0 dB.

Number of beams 1

Beam Number	Centroid Longitude	Centroid Latitude	Beam Width
1	-9.41°	0.00°	4.00°

Table A-10. Geosynchronous Orbit Loss Contours - Sample Tabular Output

Contour beam width: 4.00°
Subsatellite Point: (-9.41, 0.00)

Loss: -3.00 dB
Jan 1, 1991 12:00:00 AM

Projection Points		Projection Points	
Longitude (degrees)	Latitude (degrees)	Longitude (degrees)	Latitude (degrees)
1.92	-0.00	-20.75	0.00
1.91	-0.55	-20.74	0.55
1.87	-1.10	-20.70	1.10
1.81	-1.65	-20.63	1.65
1.71	-2.20	-20.54	2.20
1.59	-2.74	-20.42	2.74
1.45	-3.27	-20.28	3.27
1.28	-3.80	-20.11	3.80
1.08	-4.31	-19.91	4.31
0.86	-4.82	-19.69	4.82
0.62	-5.32	-19.44	5.32
0.35	-5.80	-19.17	5.80
0.05	-6.27	-18.88	6.27
-0.26	-6.73	-18.56	6.73
-0.60	-7.17	-18.23	7.17
-0.96	-7.59	-17.87	7.59
-1.34	-7.99	-17.48	7.99
-1.74	-8.38	-17.08	8.38
-2.16	-8.74	-16.66	8.74
-2.60	-9.09	-16.23	9.09
-3.06	-9.41	-15.77	9.41
-3.53	-9.71	-15.30	9.71
-4.01	-9.99	-14.81	9.99
-4.51	-10.24	-14.31	10.24
-5.03	-10.47	-13.80	10.47
-5.55	-10.67	-13.28	10.67
-6.08	-10.85	-12.75	10.85
-6.62	-11.00	-12.20	11.00
-7.17	-11.12	-11.65	11.12
-7.73	-11.22	-11.10	11.22
-8.29	-11.28	-10.54	11.28
-8.85	-11.33	-9.98	11.33
-9.41	-11.34	-9.41	11.34
-9.98	-11.33	-8.85	11.33
-10.54	-11.28	-8.29	11.28
-11.10	-11.22	-7.73	11.22
-11.65	-11.12	-7.17	11.12
-12.20	-11.00	-6.62	11.00
-12.75	-10.85	-6.08	10.85
-13.28	-10.67	-5.55	10.67
-13.80	-10.47	-5.03	10.47
-14.31	-10.24	-4.51	10.24
-14.81	-9.99	-4.01	9.99
-15.30	-9.71	-3.53	9.71
-15.77	-9.41	-3.06	9.41

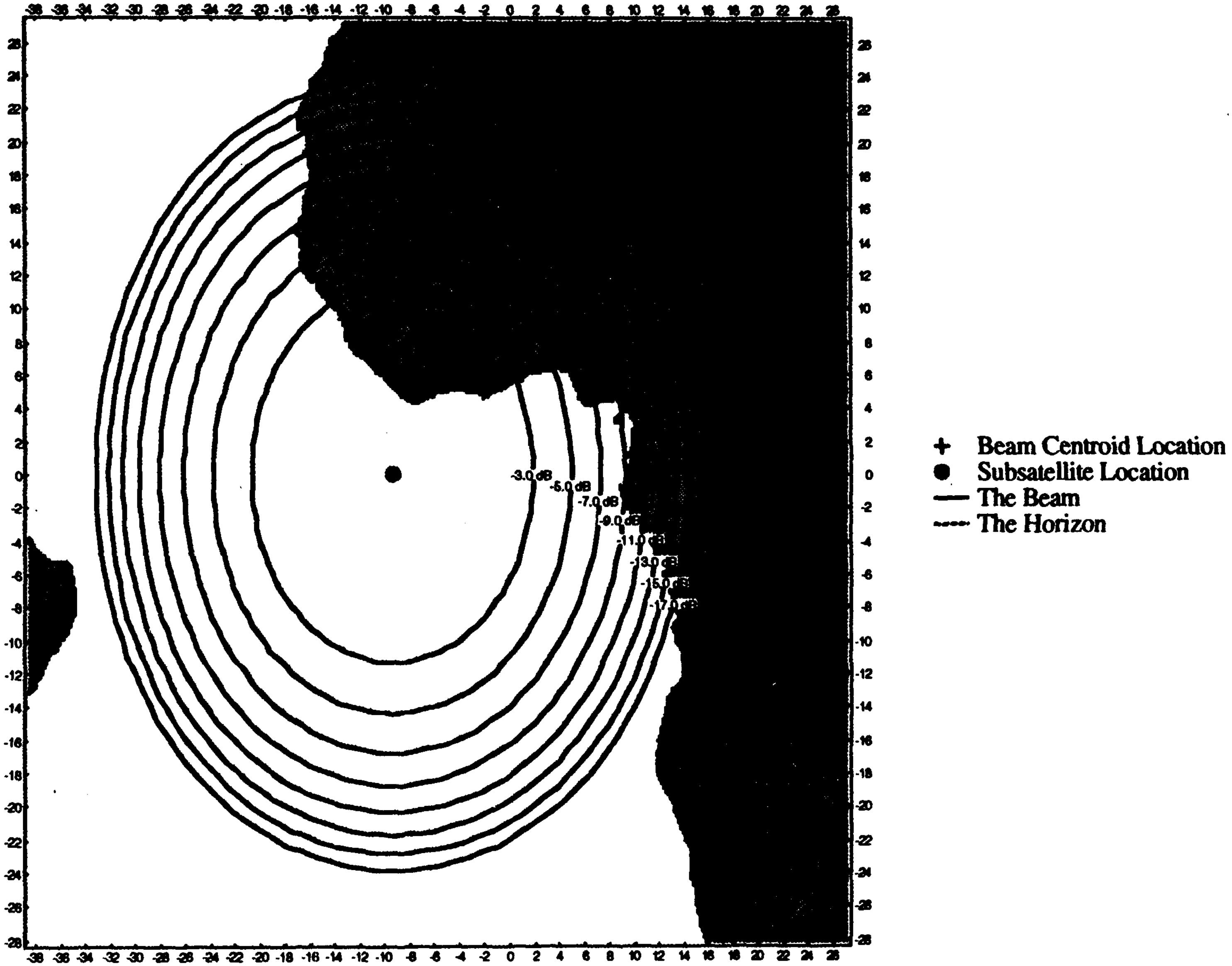


Figure A-4. Geosynchronous Orbit Loss Contours - Graphical Output

SECTION A.3

LOW-EARTH ORBITS

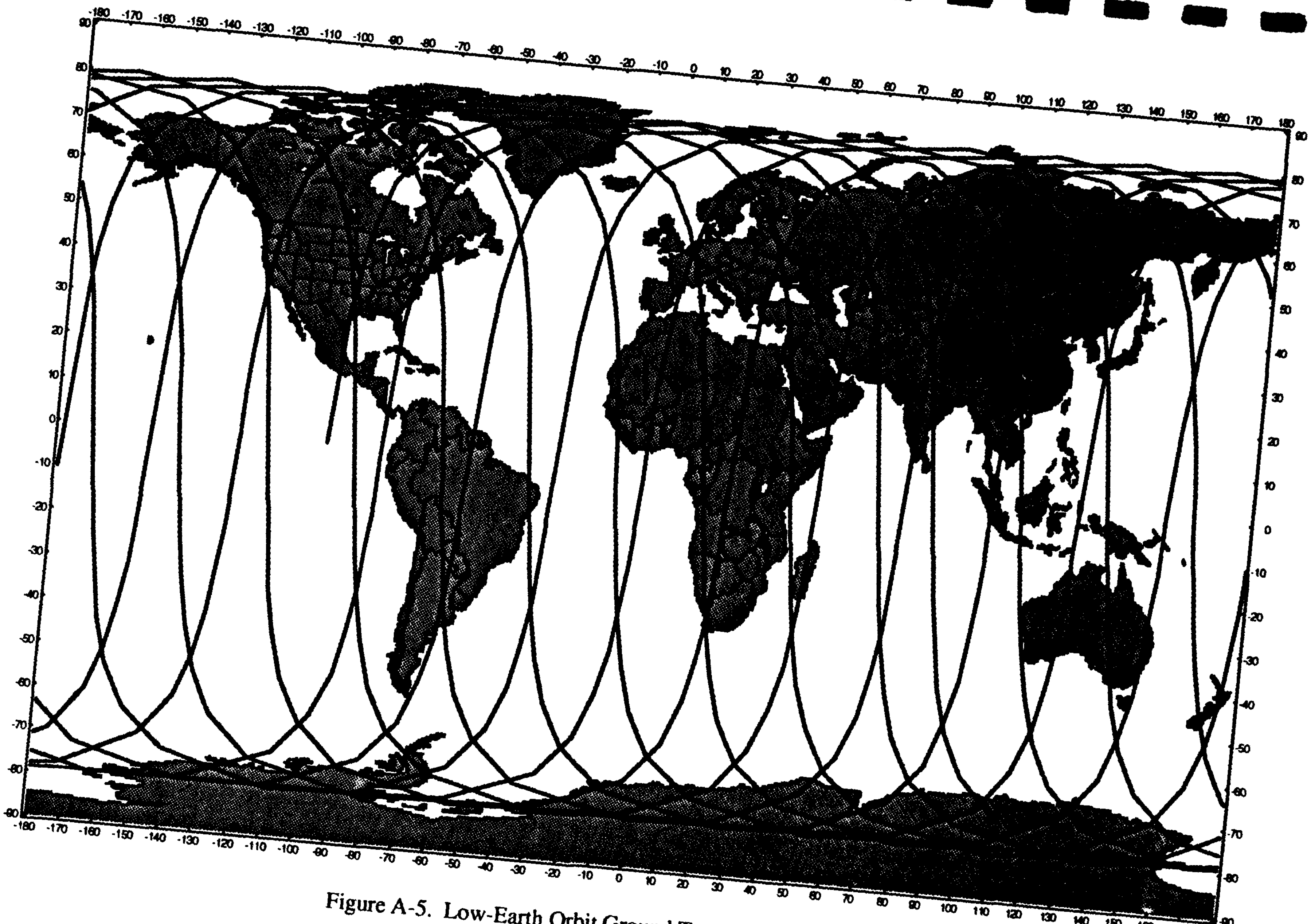
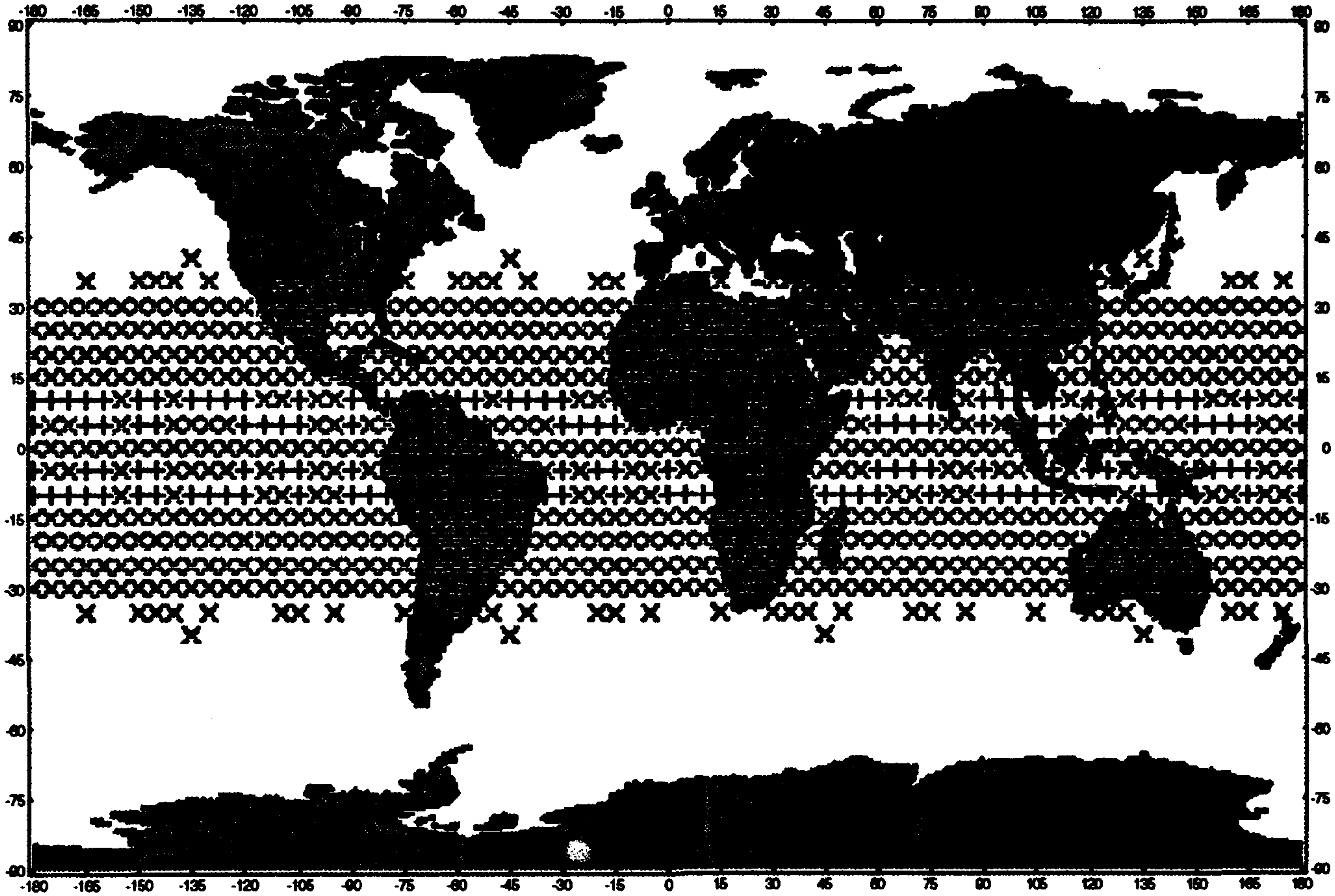


Figure A-5. Low-Earth Orbit Ground Track - Graphical Output



- ✖ zone 1: 0.0 to 4.0 hrs.
- ✚ zone 2: 4.0 to 8.0 hrs.
- ◇ zone 3: 8.0 to 12.0 hrs.
- □ zone 4: 12.0 to 16.0 hrs.
- ○ zone 5: 16.0 to 20.0 hrs.
- ● zone 6: more than 20.0 hrs.

Figure A-6. Low-Earth Orbit Outage Zones - Graphical Output

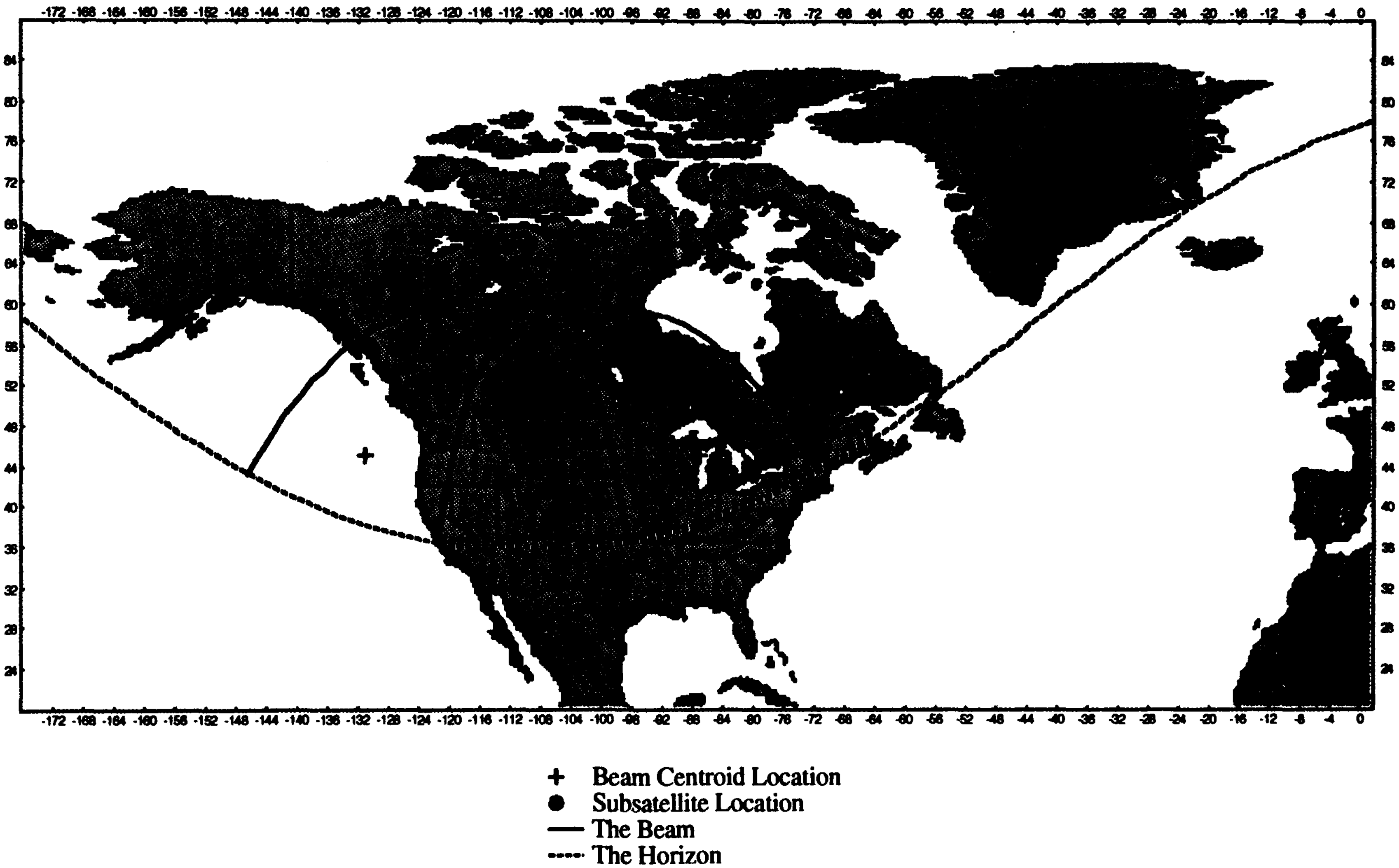


Figure A-7. Low-Earth Orbit Spot Beams - Graphical Output

SECTION A.4

MOLNIYA ORBITS

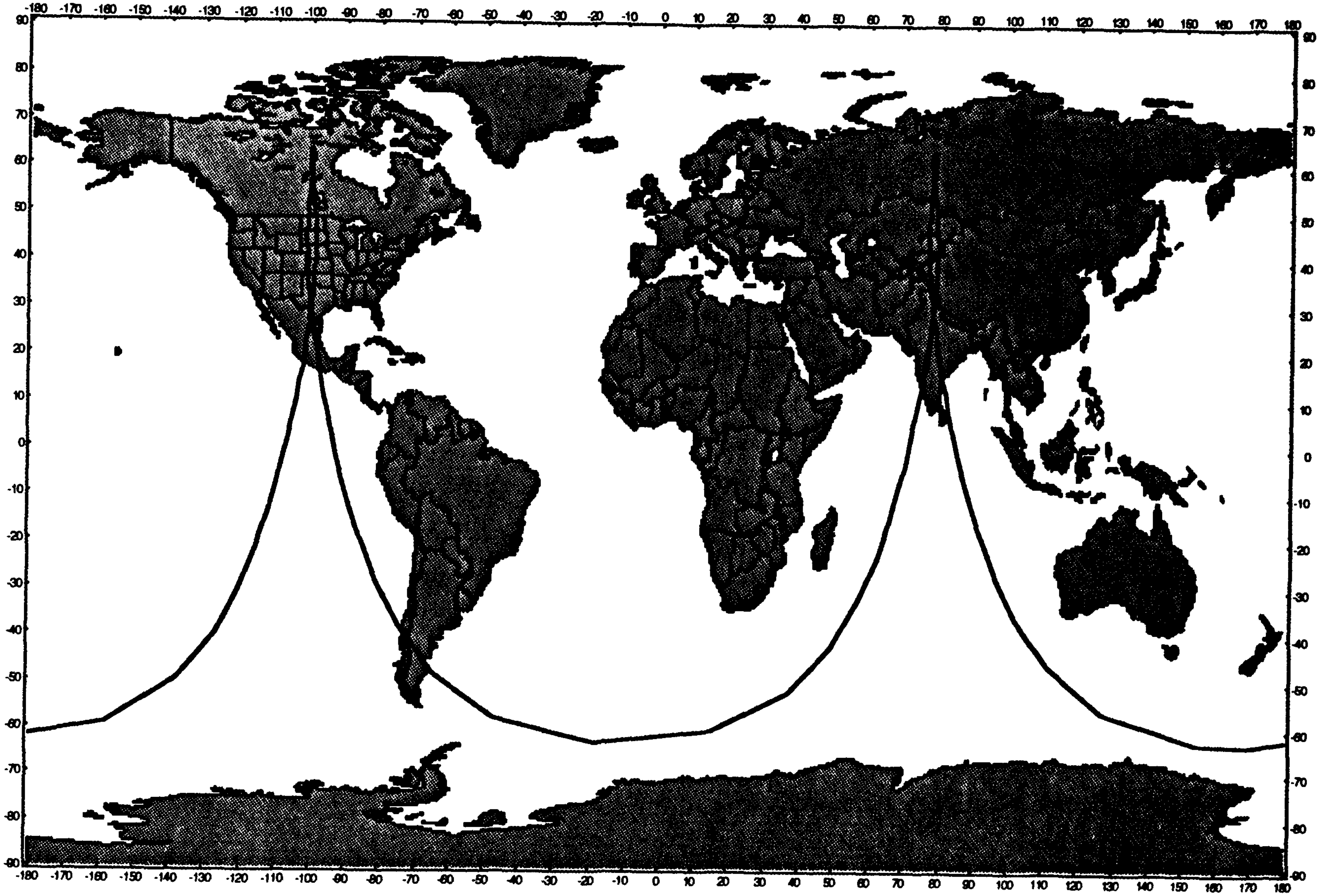
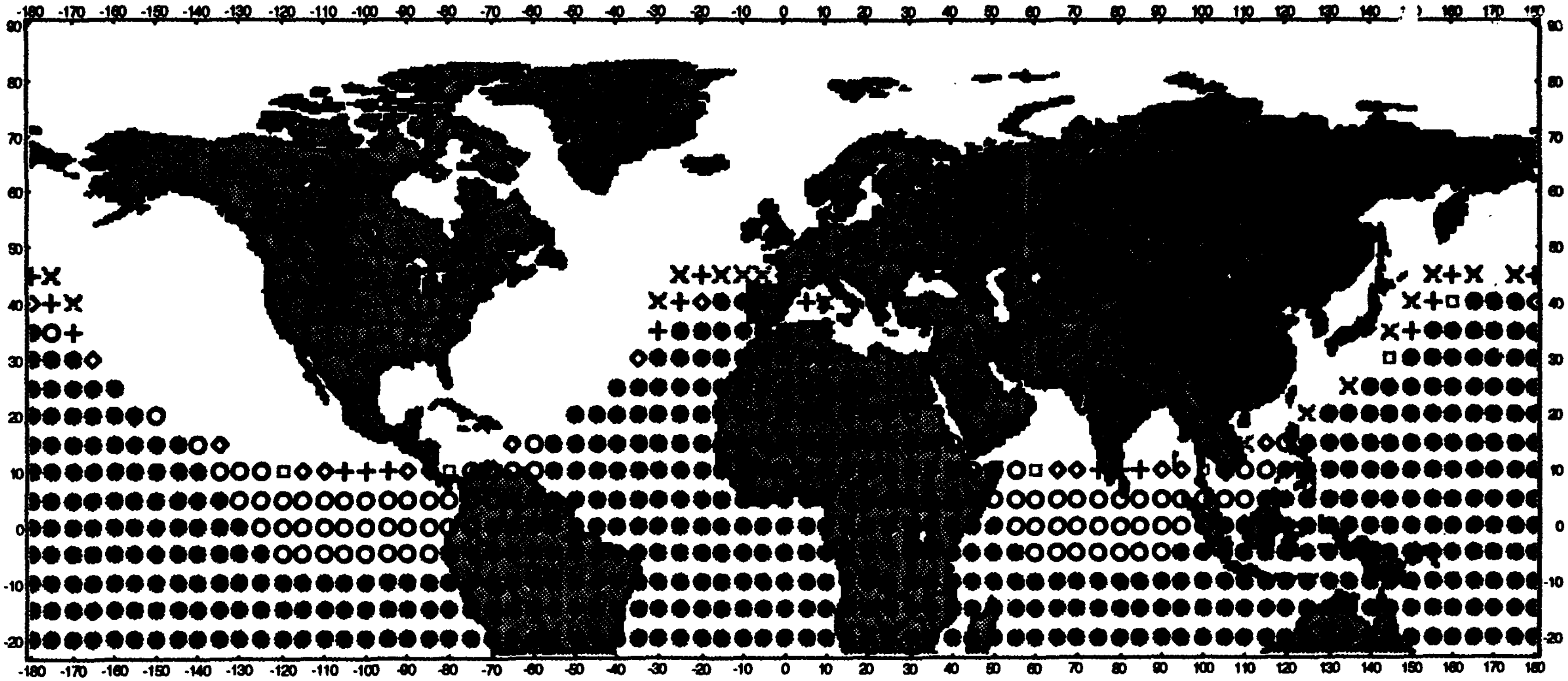


Figure A-8. Molniya Orbit Ground Track - Graphical Output



× zone 1: 0.0 to 3.0 hrs.
+ zone 2: 3.0 to 6.0 hrs.
◇ zone 3: 6.0 to 9.0 hrs.
□ zone 4: 9.0 to 12.0 hrs.
○ zone 5: 12.0 to 15.0 hrs.
● zone 6: more than 15.0 hrs.

Figure A-9. Molniya Orbit Outage Zones - Graphical Output