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Collisionless Dynamics of the Magnetosphere

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SUMMARY

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Theory : Substorms in the magnetosphere cause the generation of major electromagnetic disturbances and energetic particles. We examine the role of the collisionless tearing instability as a possible mechanism for substorms. Global asymptotic magnetotail equilbria which are slowly varying in the Earth-Sun direction are constructed, including all three components of the magnetic field. Some of these equilibria are analyzed for stability with respect to collisionless electron tearing modes. It is found that the ion tearing instability, which has been widely invoked as a possible trigger for subtorms, does not exist. The B_y field is demonstrated to have a destabilizing effect on electron tearing modes. Regimes in which collisionless tearing modes can grow are delineated.

THE COLLISIONLESS TERRELLA EXPERIMENT

The "Collisionless Terrella Experiment", or CTX, is a relatively new laboratory experiment built at Columbia University in order to directly observe the unique and fundamental properties of collisionless radial transport of plasma trapped within planetary magnetospheres. Collisionless radial transport in a dipole-confined plasma occurs only when wave-particle interactions are sufficiently intense and broad-banded and when the particle motion satisfies well-defined conditions for chaos. The primary goals of our experiments are (1) to directly observe the conditions required for the onset of radial transport, (2) to study the evolution of the plasma profiles undergoing radial transport, and (3) to develop and test a generalized model for radial transport applicable to trapped plasma in the earth's magnetosphere. This laboratory program is particularly exciting since (1) CTX is the first collisionless dipole laboratory experiment capable of observing collisionless radial transport, (2) plasma waves can be produced in the laboratory which break only a particle's third adiabatic invariant making radial transport relatively simple to characterize, and (3) the impact of collisionless transport on a plasma's global profile is predicted to have an unique signature that should be readily identified.

Our first experiments with CTX have focused on the dynamics of an "artificial radiation belt" consisting of trapped electrons with energies between 10 and 30 keV. The electrons are created by adjusting our microwave plasma source for direct cyclotron heating of magnetically-trapped electrons. The electrons generated have a collisional mean-free-path longer than 1,000 drift orbits about the equator, and they provide the ideal laboratory medium with which to study colllisionless radial transport in dipole geometry.

Figure 1 shows the time history of a typical CTX discharge and the production of an energetic electron belt. The discharge duration was arbitrarily programmed to last approximately 0.5 sec, and the several signals represent the time history of the ion-saturation current to a Langmuir probe, the hydrogen gas pressure, the forward and reflected microwave power, and the x-ray spectra as recorded by a krypton proportional counter. The experiment is fully computer controlled, and the gas pressure, pulse length, and heating power can be programmed independently for each discharge. For discharges similar to that shown in Figure 1, intense fluctuations are observed both during the microwave heating and during the "afterglow" when the heating power has been switched-off. These fluctuations only occur in the presence of the energetic electron belt, and we have characterized them as a complex and nonlinear development of drift-resonant hot electron interchange instabilities (HEI). In some respects, the HEI is the electrostatic "analog" of symmetric electromagnetic kinetic Alfvén instabilities occurring in planetary magnetospheres.

Figure 2 shows the probe fluctuations on a faster, 10 msec, time-scale. During the microwave heating, quasi-periodic "bursts" are observed; whereas, during the afterglow, the probe signals are seen to evolve more gradually. The quasi-periodic bursts correspond to rapid radial transport of electrons from the belt region. This is observed directly with a gridded electron detector and indirectly by large and negative decrease in the probe's floating potential.

By using very high-speed data recorders, we have identified the drift-resonant instabilities inducing this rapid electron transport. Figure 3 shows a frequency spectrogram of the instabilities occurring both during the microwave heating and during the afterglow. During the heating, the quasi-periodic pulsations consist of relatively wide-band signals ranging from 0.1 MHz $\leq f < 2$ MHz. In contrast, the instabilities observed during the afterglow consist of a multi-mode collection of relatively coherent rising tones. Closer examination of the quasi-periodic bursts during heating also show a collection of rising tones. The difference in the frequency spectra can be linked to the energy of the electron belt as measured with the x-ray proportional counter. When the average belt energy is relatively low, $\langle E \rangle \sim 10$ keV, (such as found during the heating), the wave frequency is also relatively low. During the afterglow, when the cooler electrons scattering into the terrella's polar regions, $\langle E \rangle$ increases, and the frequency of drift-resonant instabilities also increase. The rate of rise of the frequency is also linked to the average energy of the trapped electron belt.

Multiple probes are used to determine the azimuthal and radial structure of the drift-resonant waves. When the probes are located on the same flux surface, the two probes indicate that the waves propagate in the electron drift direction, and the wave spectrum consists of multiple azimuthal mode numbers, m, as well as multiple frequencies. When the probes are separated radially, we observe the radial mode structure to be relatively broad with complex phase-fronts characteristics.

Although the CTX experiment has been operating for less than one year, our early results already have important implications for understanding the basic process of collisionless radial transport in a dipole magnetic field:

- First, transport in a dipole magnetic field seems to require multiple modes. This is a unique property of the dipole magnetic geometry resulting from the strong radial dependence of a particle's azimuthal drift frequency, $\omega_d \propto L^{-2}$, where L is the equatorial radius of a flux surface. Global transport can only occur with multiple modes since a particle does not remain correlated with a single wave as it diffuses radially.
- Secondly, the radial transport rate is fast. In CTX, large electron bursts lasting only 10's of drift periods are observed to cause significant transport.

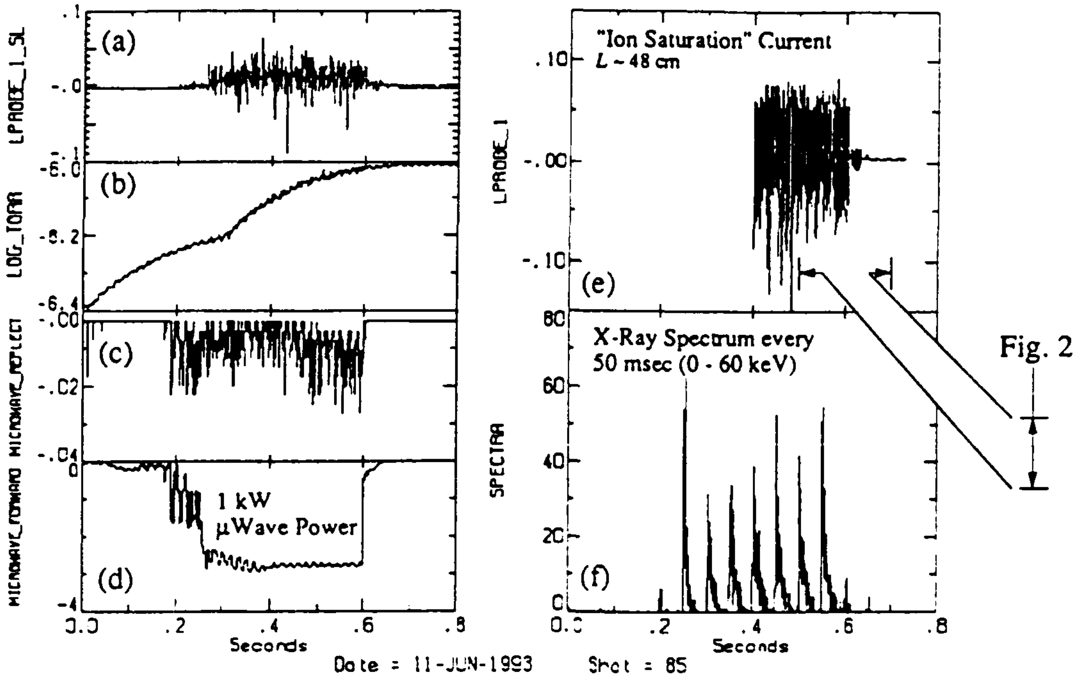

Figure 1. Data from a typical pulsed discharge of the CTX experiment during the production of an energetic electron belt using microwave heating. (a) Langmuir probe ion saturation current, (b) hydrogen gas pressure, (c) reflected microwave power, (d) forward power, (e) probe fluctuations, (f) x-ray spectra measured every 50 msec.

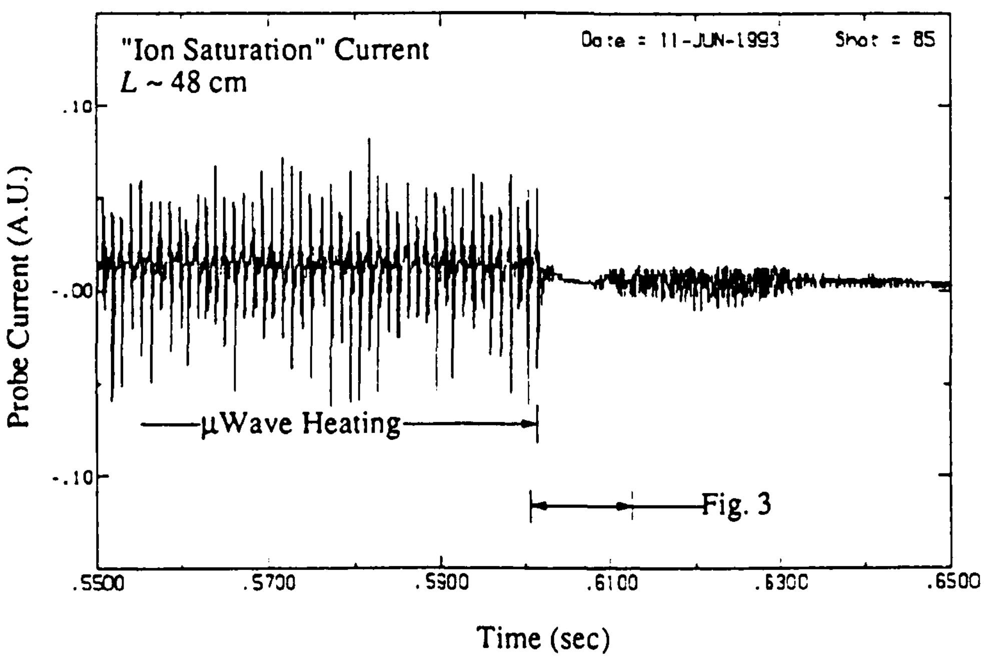

Figure 2. A fast-time scale detail of the fluctuations of the ion saturation current during the heating and during the "afterglow".

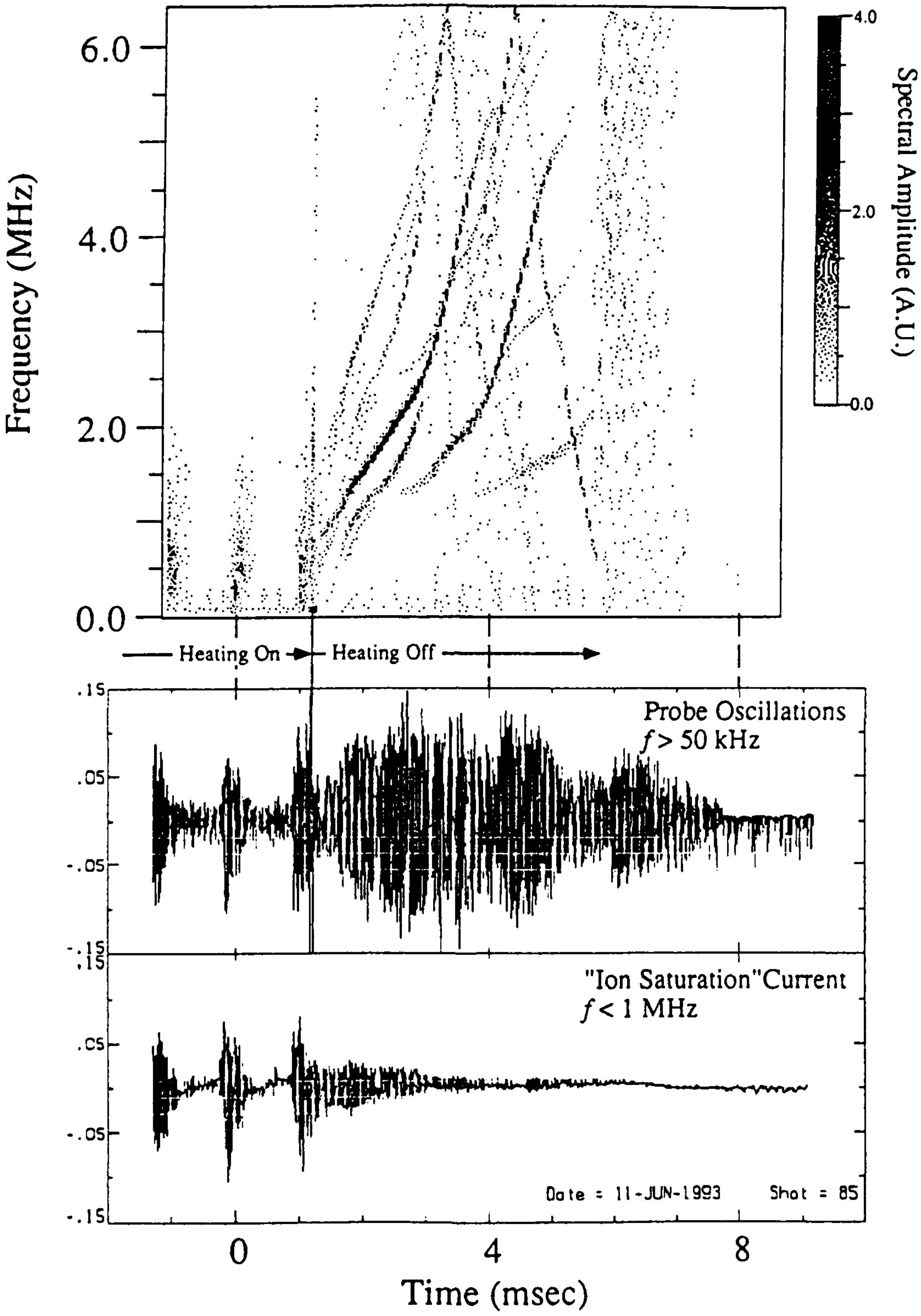

Figure 3. Frequency spectrograms of rising-tone, multiple-mode drift-resonant instabilities. The drift-frequency of the energetic electron belt is approximately 1 MHz, and multiple-probe measurements show the waves propagate in the electron drift direction.

- Finally, drift-resonant instabilities lead to transport causing self-stabilization and continuous, quasi-periodic bursting. This is observation is important. It implies that the transport-inducing waves also decrease the radial pressure gradient of the electron belt—but do not destroy electron confinement altogether. For a dipole magnetic field, the marginally stable pressure profile scales like $p \propto L^{-20/3}$.

THEORY RESEARCH ON SUBSTORMS

During substorms, the magnetic configuration of the magnetospheric plasma is rearranged drastically. The large electromagnetic disturbances and fluxes of energetic particles can interfere seriously with the performance of spacecrafts. As discussed in our original proposal (Section 2.1), the ion tearing instability has been considered by many as a trigger for substorms over the last two decades. We have treated this problem analytically, extending and questioning earlier results significantly.

We have revisited the problem of collisionless tearing in the earth's magnetotail, and gone beyond extant theories in two significant ways. Firstly, we have included the B_y field allowed for a spatially dependent B_n in our equilibrium model. Secondly, we have demonstrated that the ion tearing instability, which has been suggested by many as a possible trigger for substorms, does not exist. We find, instead, that if there is any form of collisionless tearing, it must be the electron tearing instability. We have delineated regimes in which the electron tearing mode can be excited. The conditions that favor the growth of electron tearing are those in which the B_y field is large and the B_n field is very small. However, if the B_n field is significant, the electron tearing instability is weak, can account for the slow growth phase, but not for the rapid current disruption and dipolarization phase.

The results have significant implications for observations. They demonstrate that the B_y field must be included in a global model of magnetotail equilibria, and call for systematic studies correlating B_y and B_n fields with the occurrence of substorm onset. Our results also question the prevalent wisdom among many that the collisionless tearing instability can provide a universal trigger for substorms.

For further details, we refer to the attached paper which is to be published in the Journal of Geophysical Research.

Global Asymptotic Equilibria and Collisionless Tearing Stability of Magnetotail Plasmas

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Asymptotic tail equilibria which are slowly varying in the Earth-Sun direction are constructed, including all three components of the magnetic field. These equilibria allow for spatial dependencies in B_n and B_y. Some of these equilibria are analyzed for stability with respect to collisionless electron tearing modes using a fluid model which predicts, to within a numerical factor of $\pi^{1/2}$, the growth rates derived from kinetic theory. No ion tearing instability is found. The B_y field is demonstrated to have a destabilizing effect on electron tearing modes. In the asymptotic equilibria considered here, electron tearing modes can grow in the presence of B_y in those regions where the stabilizing effect of electron bounce is small. Implications for numerical simulations and observations are discussed.

1. INTRODUCTION

Ever since Ness [1965] reported observational evidence for a neutral sheet in the Earth's magnetotail, the collisionless tearing instability has claimed much attention as a possible mechanism for magnetic reconnection in the tail. *Coppi et al.* [1966] considered a simple neutral sheet in which oppositely directed magnetic fields $B_x = B_x(z)$ face each other across the $z = 0$ line (in the $x - z$ plane) and demonstrated that such a sheet is unstable to the collisionless tearing instability [*Furth*, 1962; *Laval et al.*, 1966]. (We use here the standard solar magnetospheric coordinates (x,y,z), with the x axis in the Earth-Sun direction, the z axis in the south-north direction, and the y axis, which defines an ignorable direction, is chosen to make the coordinate system right-handed.) In the simple magnetic geometry considered by Coppi et al., the neutral line $z = 0$ is the source of the separatrix. Far away from the separatrix, the plasma obeys the ideal magnetohydrodynamic (MHD) equations. The departure from ideal MHD behavior occurs in a narrow region near the separatrix. By considering the energetics of the instability, Coppi et al. demonstrated that the dominant contribution to the inverse Landau damping effect comes from electrons, not ions. Thus this mode came to be known as the "electron tearing" instability.

We consider now the effect of a large, constant B_y field superimposed on the model of Coppi et al. The presence of B_y introduces magnetic shear in the model. The separatrix in the $x - z$ plane grows out of the $z = 0$ line. *Drake and Lee* [1977] showed that collisionless tearing modes are unstable in this geometry. (Strictly speaking, the results of *Drake and Lee* [1977] hold for a low-beta plasma without temperature gradients, as shown by *Cowley et al.* [1986]). Since the electrons carry the perturbed parallel current near the separatrix and provide the mechanism for reconnection through their small but finite inertia, this instability too can be classified as electron tearing.

Neither of the two cases discussed above are representative of the Earth's magnetotail. Much emphasis has been placed in the literature on the two-dimensional model

$$B = B_0 \tanh\frac{z}{\lambda}\hat{x} + B_n\hat{z}, \qquad (1)$$

with constant B_0, λ, and B_n [*Schindler*, 1974; *Galeev and Zelenyi*, 1976; *Lembege and Pellat*, 1982; *Büchner and Zelenyi*, 1987; *Büchner et al.*, 1991; *Pellat et al.*, 1991; *Kuznetsova and Zelenyi*, 1991]. For nonzero values of B_n, this configuration has no magnetic separatrix. (For useful discussions of the role of separatrices in magnetic reconnection, we refer the reader to *Greene* [1988] and *Lau and Finn* [1990].) The absence of a separatrix implies that reconnection or tearing (in the sense of affecting a topological change) cannot really happen for significant values of B_n. It is widely believed that the "ion tearing" instability can occur in these circumstances, but the subject remains a matter of lively debate [*Schindler*, 1974; *Galeev and Zelenyi*, 1976; *Coroniti*, 1980; *Lembege and Pellat*, 1982; *Büchner and Zelenyi*, 1987; *Pellat et al.*, 1991; *Kuznetsova and Zelenyi*, 1991]. The analysis given in this paper turns out to support the point of view recently expressed by *Pellat et al.* [1991], who have questioned the existence of the ion tearing mode. This point of view has significant implications for electromagnetic particle simulations of collisionless tearing [*Terasawa*, 1981; *Hamilton and Eastwood*, 1982; *Swift*, 1983; *Ambrosiano et al.*, 1986; *Swift and Allen*, 1987; *Pritchett et al.*, 1989; *Zwingmann et al.*, 1990; *Pritchett et al.*, 1991] which can shed valuable light on this controversial issue.

Recently, we showed that the inclusion of a constant B_y field in the model (1) can qualitatively change its stability properties [*Wang et al.*, 1990; hereafter WBL]. Our model, referred to here as the three-component model, breaks with the tradition of using two-component models in theoretical analyses of collisionless instabilities in the magnetotail. However, just as in the two-component model, the formation of a magnetic separatrix is inhibited by the presence of a significant B_n field. It is therefore not surprising that we found that the electron tearing instability has a stabler parameter space and is much harder to excite in the three-component magnetotail than in a configuration with $B_n = 0$. Furthermore, the growth rate of the mode is slow, consistent with the growth phase, but not the expansion phase of a substorm.

We now develop an equilibrium model which is more realistic than considered heretofore in analytical studies of collisionless tearing. As in WBL, we include the B_y field. Fairfield and others have noted that the B_y field is a persistent feature of the magnetotail [*Fairfield*, 1979; *Cattell and Mozer*, 1982; *Lui*, 1983; *McComas et al.*, 1986; *Tsurutani et al.*, 1984; *Sibeck et al.*, 1985]. Voigt and coworkers [*Voigt and Hilmer*, 1987; *Hau and Voigt*, 1992] have shown, based on some analytical and numerical examples, that the requirements of global magnetostatic equilibrium of the magnetotail should include B_y. The possible role of the B_y field in observations of substorm dynamics was pointed out as early as 1978 by Akasofu and coworkers [*Akasofu et al.*, 1978]. From a detailed examination of IMP data, Akasofu et al. stressed the need for a three-component analysis of magnetic fields in the magnetotail during substorms. Though there are a number of other events reported in the literature in which an enhancement in B_y and a reduction in B_n is observed prior to the onset of a substorm [*Nishida et al.*, 1983; *Bieber et al.*, 1984; *Lepping*, 1987; *Takahashi et al.*, 1987; *Lui et al.*, 1988; *Lopez et al.*, 1989], no systematic studies of substorm events with correlated variations in B_y and B_n are available yet.

One of the main improvements of the present paper over WBL is the development of asymptotic equilibria with spatial dependencies in B_n and B_y, that is, $B_n = B_n(x,z)$ and $B_y = B_y(x,z)$. These equilibria, and the single-particle motions in them, are described in section 2. We show that these equilibria change qualitatively our current understanding of collisionless instabilities in the magnetotail by introducing new global features not captured adequately by the model,

$$\mathbf{B} = B_0 \tanh\frac{z}{\lambda}\,\hat{x} + B_y\,\hat{y} + B_n\,\hat{z}\,, \tag{2}$$

with B_y and B_n constant. In this geometry, the global bounce period τ_{be} for electrons is much shorter than the growth time of the instability when $B_y \sim B_z$. WBL has been criticized for neglecting the stabilizing effect of this bounce [*Büchner et al.*, 1991; *Pritchett et al.*, 1991]. This criticism would be justified except for the fact that the model (2) is itself globally rather crude and underestimates severely the bounce period in a stretched magnetotail. If we must include the effect of electron bounce, then it is preferable to do so in an equilibrium model which captures the global features of the magnetotail with greater realism than equation (2). And that is precisely what we achieve by allowing for spatially varying B_n in the new equilibrium model. We then show that there are regions where $\gamma\tau_{be} \geq 1$, and the electron tearing mode grows, with a growth rate γ, as predicted by WBL. Furthermore, when we consider the special case $B_n = \text{const}$ in which $\gamma\tau_{be} << 1$, we find no ion tearing, contrary to the findings of *Büchner et al.* [1991]. The persistence of slow electron tearing, and the absence of ion tearing, are recurrent themes that are explored in detail for both two-component ($B_y = 0$) and three-component ($B_y \neq 0$) equilibria in sections 4 and 5.

Though the electron tearing instability can account for the growth phase of a substorm, it is not sufficiently rapid to account for the current disruption and diversion that occurs at the onset of the expansion phase [*Takahashi et al.*, 1987; *Lui et al.*, 1988]. Elsewhere [*Wang et al.*, 1991], we have discussed that nonlinear mode-coupling effects may lead to a significant enhancement of the linear growth rate. It was implicitly assumed in that discussion that a linearly unstable mode will grow to sufficiently large amplitude that it can couple to other unstable modes. However, that possibility was explored in the context of the equilibrium represented by (2) which, as discussed here, has certain limitations. We suggest here that the circumstances in which collisionless tearing can grow as a robust instability, not only in the linear regime but also in the nonlinear regime, must involve reduction of B_n to extremely small values. We therefore point to the possibilities inherent in the three-component asymptotic equilibria calculated in this paper. An interesting property of these equilibria is that B_n may be reduced to zero at near-Earth distances when B_y is space-dependent. This possibility was first noted by *Hau and Voigt* [1992] whose profiles for B_y were different from ours, and who concluded that for their class of profiles, the B_y value required to reduce B_n to zero is much larger than the average value of B_y in the plasma sheet. For our class of profiles, we find that the reduction of B_n to zero occurs for values of B_y more in accord with observed values. When B_n vanishes, a separatrix can be formed, and tearing instabilities can grow. Whether the nonlinear evolution of these instabilities can actually account for the rapid current disruption and diversion observed in near-Earth regions remains an open question.

The layout of this paper is as follows. In section 2 we obtain some asymptotic tail equilibria, both with and without B_y. In section 3 we develop a fluid model, including a generalized Ohm's law, which allows the treatment of collisionless tearing modes, and benchmark the predictions of this model with known results from kinetic theory. In section 4 we use the energy integral derived from our fluid model to analyze the stability of the two-dimensional magnetotail without B_y and constant B_n. We find that the ion tearing mode does not occur, and the only possible instability, under certain conditions, is an electron tearing mode. In section 5 it is shown that the inclusion of B_y and a spatially varying B_n can further destabilize the electron tearing instability. We conclude in section 6 with a summary of our results and their implications for observations.

2. Some Asymptotic Equilibria

2.1. *Two-Component Equilibria*

We begin by considering some asymptotic two-component equilibria, i.e., equilibria for which $B_y = 0$. Assuming that the y coordinate is ignorable, the magnetic field B may be written as

$$\mathbf{B} = \nabla\psi \times \hat{y} \,, \tag{3}$$

where ψ is a flux function. In equilibrium, for a charged particle of type α, the energy H_α and the momentum $P_{\alpha y}$ are conserved. Assuming that there is no equilibrium electrostatic field, these constants are

$$H_\alpha = \frac{1}{2} m_\alpha v_\alpha^2 \,, \tag{4}$$

$$P_{\alpha y} = m_\alpha v_{\alpha y} + q_\alpha \psi / c \,, \tag{5}$$

where m_α is the mass of the charge particle of type α, q_α, its charge, and c is the speed of light. An equilibrium distribution function can be written as

$$\begin{aligned} f_\alpha &= f_\alpha (H_\alpha, P_{\alpha y}) \,, \\ &= n_0 \left(\frac{m_\alpha}{2\pi T_\alpha} \right)^{3/2} \exp\left(\frac{P_{\alpha y} V_{\alpha y} - H_\alpha}{T_\alpha} \right), \end{aligned} \tag{6}$$

where V_α is the drift velocity, T_α is the temperature (in energy units) for particles of type α ($= e, i$ for a hydrogen plasma), and n_0 is the average number density of electrons and protons. The temperature T_α is taken to be a constant. From the relation

$$n_\alpha = \int d\mathbf{v}_\alpha f_\alpha \,, \tag{7}$$

and the requirement of quasi-neutrality, $n_e = n_i$, we obtain the condition $V_{iy}/T_i = -V_{ey}/T_e$. The y component of Ampere's law gives

$$\nabla^2\psi = -\frac{4\pi}{c}\sum_\alpha q_\alpha \int d\mathbf{v}_\alpha \, v_y f_\alpha \, .$$

$$= -\frac{B_0}{\lambda}\exp\left\{\frac{2\psi}{B_0\lambda}\right\} , \tag{8}$$

where

$$\lambda \equiv \frac{2c\,(T_i + T_e)}{eB_0\,(V_{iy} - V_{ey})} , \tag{9}$$

defines a characteristic equilibrium length scale, and the constant B_0 is determined by the relation

$$B_0^2/8\pi = n_0 \; (T_e + T_i) \; . \tag{10}$$

B_0 is a measure of the lobe magnetic field. We scale the variables $\mathbf{x}/\lambda \rightarrow \mathbf{x}$, $\psi/B_0\lambda \rightarrow \psi$, $p/(B_0{}^2/4\pi) \rightarrow p$, $\mathbf{B}/B_0 \rightarrow \mathbf{B}$, and introduce a small, positive parameter $\varepsilon \sim \partial/\partial x << \partial/\partial z \sim 1$. In this approximation, a large class of equilibrium solutions of (8) is given by [*Birn et al.*, 1975; *Birn*, 1979; *Zwingmann and Schindler*, 1980; *Lembege and Pellat*, 1982; *Zwingmann*, 1983]

$$\psi \simeq -\log\,\{\cosh\,[z\,f(\varepsilon x)]\,/\,f(\varepsilon x)\,\} , \tag{11}$$

where $f(\varepsilon x)$ is a slowly varying function of x. It follows that

$$B_x = -\frac{\partial\psi}{\partial z} = f(\varepsilon x)\tanh\,[z f(\varepsilon x)] \; , \tag{12}$$

$$B_z \equiv B_n = \frac{\partial\psi}{\partial x} = \varepsilon\left\{\frac{f'(\varepsilon x)}{f(\varepsilon x)} - z f'(\varepsilon x)\tanh[z f(\varepsilon x)]\right\}, \tag{13}$$

where prime denotes differentiation with respect to the argument. Following *Lembege and Pellat* [1982], we take $f(\varepsilon x) = \exp(\varepsilon x)$ $(x < 0)$. As a first approximation, almost all analytical studies replace (12) and (13) with

$$B_x = \tanh z , \tag{14}$$

$$B_n = \varepsilon . \tag{15}$$

However, this is valid only in the region $|x| \leq 1$, $|z| \rightarrow 0$. It cannot be assumed that this approximation holds for large x close to the $z = 0$ line, for the equilibrium pressure balance condition implies that $\partial p/\partial x = -\varepsilon$ which, in turn, gives $p = p_0 - \varepsilon x$, with p_0 constant. This means that the pressure increases with x, which is unrealistic for the distant tail. A better approximation for B_n than (15) is

$$B_n = \varepsilon\,(1 - z\tanh z) . \tag{16}$$

However, (16) does not represent realistically the x dependence of $B_n(x,z)$ which should tend to zero as $|x| \rightarrow \infty$. It is possible to improve on (16) by taking

$$f(\varepsilon x) = \exp\,[\varepsilon x/(1 - \varepsilon x)] \quad (x < 0) . \tag{17}$$

We recall that the distance x is measured in units of $\lambda \sim 1\ R_E$. For specificity, we take $\varepsilon = 0.1$, and define $L_0 \equiv \varepsilon^{-1} = 10$. The scale L_0 ($\sim 10 R_E$) enables us to define three separate regions:

1. Near-Earth region, $|x| << L_0$

In this region, using (17) in (12) and (13), we get

$$B_x \simeq \tanh z , \tag{18}$$

$$B_n \simeq \varepsilon\,(1 - z \tanh z) . \tag{19}$$

2. Middle region, $|x| \sim L_0$

In this region, since $\varepsilon x \sim 1$, we get

$$B_x \simeq e^{1/2} \tanh\,(e^{1/2} z) , \tag{20}$$

$$B_n \simeq \frac{\varepsilon}{4}\ [1 - e^{1/2} z \tanh\ (e^{1/2} z)] . \tag{21}$$

If we define $B' \equiv e^{-1/2} B$, $z' \equiv e^{1/2} z$ and $\varepsilon' \equiv e^{-1/2}\,\varepsilon/4$, then (20) and (21) become

$$B_x' \simeq \tanh z', \tag{22}$$

$$B_n' \simeq \varepsilon'\,(1 - z' \tanh z'), \tag{23}$$

which is the same as (18) and (19). In other words, the middle region has the same structure as the near-Earth region, except that the magnetic field in it is somewhat weaker and the current sheet is somewhat wider.

3. Distant-tail region, $|x| >> L_0$

In the distant-tail region, taking the limit $x/L_0 \rightarrow -\infty$, we get

$$B_x \simeq e^{-1} \tanh\,(e^{-1} z) , \tag{24}$$

$$B_n \simeq 0 . \tag{25}$$

Equations (24) and (25) describe essentially the configuration first considered by *Coppi et al.* [1966], who found electron tearing modes to be unstable. We caution that though the qualitative features of the magnetic field as described by (24) and (25) are reasonable, the far distant-tail region is outside the strict domain of validity of the asymptotic solution (13). Here we do not pursue this matter further, for the main focus of this paper is on the investigation of collisionless tearing modes in the near-Earth and middle regions. We also note that the near-Earth configuration described above is not totally realistic for $|x| \leq 5 R_E$ because at these distances, the Earth's dipole field, not included in the model, plays a dominant role. Matching to the dipole field can be carried out, in principle, but is not germane to our considerations here. Figure 1 shows a plot of our asymptotic two-dimensional model with the origin set (arbitrarily) at $x = -5\ R_E$, $z = 0$. Fig. 1

We now discuss some features of the single-particle orbits. For a particle of mass m_α and charge q_α gyrating in magnetic field B, the Larmor frequency is $\omega_{c\alpha} = q_\alpha B/m_\alpha c$, and the typical Larmor radius is $\rho_{c\alpha} = v_{t\alpha}/\omega_{c\alpha}$, where $v_{t\alpha} \equiv (2T_\alpha/m_\alpha)^{1/2}$ is the thermal velocity. Using typical tail parameters (see, for instance, *Lui* [1987]), we get $\rho_{ci} \sim (0.5 -$

$1)R_E$. In both regions 1 and 2, $\rho_{ci} \geq 1$, $\rho_{ce} << 1$ (scaled by λ). Hence electrons may be treated in the guiding-center approximation, but the ions are essentially unmagnetized. Because of the z dependence of $B_n(x,z)$, the field lines are more stretched as z increases than in the case of constant B_n (see Figure 1.). In the Appendix, we show that this has the consequence that the average bounce period of electrons in both regions 2 and 3 can be substantially larger than the bounce period with constant B_n.

It is interesting to note that the large separation in the magnitude of the Larmor frequency ω_{ce} and the average bounce frequency $\omega_{be} \equiv 2\pi\tau_{be}^{-1}$ in our asymptotic equilibria (in those regions where B_n is weak) diminishes the possibility of low-order resonances between the Larmor and bounce frequencies for most particles, and the chaos that can result from such resonances [*Chen and Palmadesso*, 1986; *Büchner and Zelenyi*, 1987]. Hence, for equilibria with spatially varying B_n, we will not concern ourselves here with intrinsic stochastic diffusion as a possible mechanism for the restoration of ion tearing. As to whether stochastic diffusion can destabilize the ion tearing if B_n is constant has been the subject of debate recently [*Pellat et al.*, 1991; *Kuznetsova and Zelenyi*, 1991] and is an issue we shall address in section 4.

2.2. *Three-Component Equilibria*

We now consider equilibria which are symmetric in y but with all three components of the magnetic field nonzero. The magnetic field B is represented as

$$\mathbf{B} = B_y \hat{y} + \nabla\psi \times \hat{y} . \tag{26}$$

The condition of magnetostatic equilibrium gives the Grad-Shafranov equation [*Voigt and Hilmer*, 1987; *Paranicas and Bhattacharjee*, 1989; *Hau and Voigt*, 1992]

$$\nabla^2\psi + \frac{d}{d\psi}\left(p + \frac{B_y^2}{2}\right) = 0 , \tag{27}$$

where $p = p(\psi)$ and $B_y = B_y(\psi)$ are two free functions. If we set $B_y = 0$ and take $p = \exp(2\psi)$, we recover (8) (in dimensionless variables). We note that (8) also holds for $B_y =$ const, in which case a class of asymptotic solutions can be again constructed using (11). The B_x and B_z components for this class of solutions has already been given in section 2.1.

We now consider the effect of a nonzero $dB_y^2/d\psi$ on the solution (11). Since ψ is small and negative near $z = 0$, we make the expansion

$$B_y^2 = b_0^2 - 2b_1\psi + ... , \tag{28}$$

when b_0 and b_1 are positive constants. If we order $b_0 \sim b_1 \sim |\psi| \sim \varepsilon$, then near $z = 0$ (11) can be modified as

$$\psi = -\log\{\cosh[z/f(\varepsilon x)]/f(\varepsilon x)\} + b_1 x^2/2 , \tag{29}$$

with $f(\varepsilon x)$ specified by (17). This yields $B_n = \varepsilon + b_1 x$ which implies that B_n vanishes on the $z = 0$ plane at $x = -b_1/\varepsilon$. Hence an X-point appears on the $z = 0$ line (in the $x-z$ plane) at near-Earth distances ($5\ R_E \leq |x| \leq 10\ R_E$). That the spatial dependence of B_y in a magnetostatic equilibrium can lead to the reduction of B_x to zero on the $z = 0$ plane has been recognized by *Hau and Voigt* [1992]. However, for their class of equilibrium profiles, they find that the B_y required to reduce B_x to zero is much larger than the observed average B_y in the plasma sheet. For our class of pressure and B_y profiles, this limitation is overcome because the average values of both B_x and B_y are of the same order.

The particle orbits and drift motions in the presence of B_y has been considered by WBL and will not be repeated here. Simple considerations of particle orbits near $z = 0$, where (2) holds, shows that the main stabilizing effect of B_x is to remove electrons from the $z = 0$ plane where the separatrix tends to form in the absence of B_x. The addition of the B_y field provides the electrons with another guided channel for motion near the $z = 0$ plane. Then, as noted by WBL, the growth of the electron tearing instability depends on the competing effects of B_y and B_z. As far as the effect of electron bounce is concerned, we show in the Appendix that the bounce period in the stretched magnetotail increases due to the x dependence of B_x; hence the condition $\gamma\tau_{be} \geq 1$ is satisfied in the middle region. This, in turn, implies that the stabilizing effect of the electron bounce is weakened, and that the electron tearing mode can grow in the linear regime not only for $B_y >> B_z$ but also for $B_y \sim B_z$ near the $z = 0$ plane. However, we repeat for emphasis that in the latter case, the growth rate is small, and the mode is likely to saturate nonlinearly at a low amplitude.

In this section we have made use of dimensionless variables in order to keep the notation simple. In the remainder of the paper, we shall return to using the primitive physical variables.

3. Energy Integral

In WBL, the stability of the plasma sheet was analyzed by asymptotic matching of the normal mode equations in the inner region, where finite particle inertia provides the reconnection mechanism, with the equations in the outer region, governed by ideal MHD. The technical details of such an approach are somewhat different from those involved in the Lyapunov functional method (developed by *Laval et al.* [1966]) which relies on the existence of an energy integral. We review, at first, the stability criteria that follow from the energy integral. From the linearized Maxwell's equations, it follows that

$$\int dx \left[\frac{\partial}{\partial t} \left\{ \frac{1}{8\pi} \left(\mathbf{B}_1^2 + \mathbf{E}_1^2 \right) \right\} + \mathbf{J}_1 \cdot \mathbf{E}_1 \right] = -\oint d\mathbf{a} \cdot \frac{c}{4\pi} \mathbf{E}_1 \times \mathbf{B}_1 , \tag{30}$$

where all perturbed quantities are designated by the subscript 1. We assume that the boundary conditions on the surface bounding the plasma volume ensure that the surface term vanishes. Then, it can be shown [*Laval et al.*, 1966] that

$$\int dx\, \mathbf{J}_1 \cdot \mathbf{E}_1 = -\frac{\partial}{\partial t} \int dx\, \frac{1}{2} \sum_\alpha \left[\frac{q_\alpha}{c} \int dv\, v_y\, \frac{\partial f_\alpha}{\partial \psi}\, \psi_1^2 + \int dv\, \tilde{f}_{1\alpha}^{\,2} \Big/ \frac{\partial f_\alpha}{\partial H_\alpha} \right] . \tag{31}$$

where

$$\tilde{f}_{1\alpha} \equiv f_{1\alpha} - \frac{\partial f_\alpha}{\partial \psi}\, \psi_1 \,. \tag{32}$$

From (30) and (31), it follows that

$$\frac{\partial}{\partial t}\, \delta^2 \varepsilon = 0\,, \tag{33}$$

where

$$\delta^2 \varepsilon \equiv \int dx \left[\frac{1}{8\pi} \left\{ \mathbf{B}_1^2 + \mathbf{E}_1^2 \right\} \right.$$

$$\left. - \frac{1}{2} \sum_\alpha \left\{ \frac{q_\alpha}{c} \int d\mathbf{v} v_y \frac{\partial f_\alpha}{\partial \psi} \psi_1^2 + \int d\mathbf{v}\, \tilde{f}_{1\alpha}{}^2 \Big/ \frac{\partial f_\alpha}{\partial H_\alpha} \right\} \right]. \tag{34}$$

The energy integral $\delta^2\varepsilon$ is a quadratic form. If $\delta^2\varepsilon$ is positive-definite for all nontrivial permissible perturbations, then the system is stable [*Kruskal and Oberman*, 1958; *Laval et al.*, 1966]. In other words, a sufficient condition for stability is

$$\delta^2 \varepsilon \geq 0\,. \tag{35}$$

Furthermore, for a Maxwellian distribution, since, $\partial f_\alpha / \partial H_\alpha = -f_\alpha/T_\alpha$, we get

$$-\frac{1}{2} \sum_\alpha \int d\mathbf{v}\, \tilde{f}_{1\alpha}^{\,2} \Big/ \frac{\partial f_\alpha}{\partial H_\alpha} = \frac{1}{2} \sum_\alpha T_\alpha \int d\mathbf{v}\, \frac{\tilde{f}_{1\alpha}^{\,2}}{f_\alpha} \geq 0\,. \tag{36}$$

Equation (36) implies that for a Maxwellian distribution a sufficient condition for stability is

$$\delta^2 W \equiv \int dx \left[\frac{1}{8\pi} \left\{ \mathbf{B}_1^2 + \mathbf{E}_1^2 \right\} \right.$$

$$\left. - \frac{1}{2} \sum_\alpha \frac{q_\alpha}{c} \int d\mathbf{v}\, v_y \frac{\partial f_\alpha}{\partial \psi} \psi_1^2 \right] \geq 0. \tag{37}$$

Since collisionless tearing modes have low frequency, the electrical energy $E_1{}^2/8\pi$ is much smaller than the magnetic energy $B_1{}^2/8\pi$, and can be neglected. The sufficient condition (37) can be rewritten as

$$\delta^2 \mathrm{W} \equiv \int d\mathrm{x} \left[\frac{\mathbf{B}_1^2}{8\pi} - \frac{1}{2} \sum_\alpha \frac{q_\alpha}{c} \int d\mathrm{v}\, \mathrm{v}_y \frac{\partial f_\alpha}{\partial \psi} \psi_1^2 \right] \geq 0\,. \tag{38}$$

One of the difficulties presented by the energy integral $\delta^2\varepsilon$ *is* that the physical interpretation of some terms is not transparent. We take for instance, the last term on the right-hand-side of (34). *Lembege and Pellat* (1982) showed, by using a Schwartz inequality, that the term has a lower bound which can be attributed to the compressibility of the electron fluid. This interpretation has been invoked repeatedly in the literature, but it is worth noting that it was meant to hold for the lower bound, and not the term itself. In fact, it is clear that there is more to the original term than electron compressibility. The function $\delta^2\varepsilon$ represents the second variation of the total energy ε which is a sum of the electromagnetic field energy and the kinetic energy of the fluid. Since the perturbed kinetic energy of the fluid is always positive definite, it must be associated with a manifestly positive definite term in $\delta^2\varepsilon$. Thus it would seem that the last term in (34) should involve the perturbed kinetic energy of the fluid plasma, but this is not obvious. (The physical interpretation of the kinetic terms is much more transparent in the energy principle of *Kruskal and Oberman* [1958] for a guiding-center plasma, but this is not the model underlying the functional (34) which has been derived from the full Vlasov equation.)

In view of the interpretational difficulties of a fully kinetic treatment, we propose a different approach for the study of collisionless tearing modes in the magnetotail. This approach uses fluid equations, and the mechanism for reconnection is provided by finite particle inertia in the generalized Ohm's law. That this is a reasonable model for collisionless tearing modes was suggested by *Furth* [1962, 1964]. The point of view we adopt here is that the energetics of collisionless tearing modes is describable within a fluid model by using a generalized Ohm's law. The wave-particle resonance condition (which is a kinetic effect) is included in such a model by simply equating the growth rate γ to kv_{te}, where k is the wave number and v_{te} the electron thermal speed. (For sheared configurations, k is replaced by $k_\parallel$, the component of k parallel to the magnetic field.) This ad hoc resonance condition misses the detailed structure of the particle distribution functions. However, we shall demonstrate that the growth rate calculated from the model equations agrees, except for an overall multiplicative factor of $\pi^{1/2}$, with the results of a fully kinetic treatment. One of the advantages of the fluid model is that the energetics of the instability is much easier to interpret physically. This will enable us to formulate a stability condition which is both necessary and sufficient.

The linearized fluid equations are

$$\rho\frac{\partial \mathbf{u}}{\partial t} = \frac{\mathbf{J}\times\mathbf{B}_1}{c} + \frac{\mathbf{J}_1\times\mathbf{B}}{c} - \nabla p_1 , \qquad (39)$$

$$\frac{\partial \rho_1}{\partial t} + \nabla\cdot(\rho\mathbf{u}) = 0 , \qquad (40)$$

$$\mathbf{E}_1 + \frac{\mathbf{u}\times\mathbf{B}}{c} = \frac{m_e}{ne^2}\frac{\partial \mathbf{J}_1}{\partial t} , \qquad (41)$$

$$\nabla\times\mathbf{E}_1 = -\frac{1}{c}\frac{\partial \mathbf{B}_1}{\partial t} , \qquad (42)$$

$$\nabla\times\mathbf{B}_1 = \frac{4\pi}{c}\mathbf{J}_1 , \qquad (43)$$

$$p_1 = n_1(T_e + T_i) , \qquad (44)$$

where ρ_1 is the perturbed mass density, $\mathbf{u}$ is the perturbed fluid velocity, p_1 is the perturbed pressure, n_1 is the perturbed number density ($n_1 = n_{e1} = n_{i1}$ by quasi-neutrality), and the unsubscripted variables represent equilibrium quantities. We note that the generalized Ohm's law (41) has a term proportional to the electron mass, but none proportional to the ion mass. This can be readily seen by considering the more general form (see, for instance, *Krall and Trivelpiece* [1986])

$$\frac{\partial \mathbf{J}_1}{\partial t} = ne^2 \left(\frac{1}{m_e} + \frac{1}{m_i}\right) \left(\mathbf{E}_1 + \frac{\mathbf{u} \times \mathbf{B}}{c}\right),$$

which reduces in the limit $m_e/m_i << 1$ to the form (41). Apart from terms involving finite particle inertia, the generalized Ohm's law contains other terms such as the Hall term, the electron pressure gradient term, as well as terms involving the anisotropic stress tensor. It can be shown that the first two do not qualitatively change our results for the class of equilibria considered here. However, anisotropies in the stress tensor, which are an additional source of free energy, may alter some of our conclusions. We do not consider pressure anisotropies here because it is questionable whether an instability that is primarily driven by such a source of free energy should be classified as a tearing mode.

We now use (39) – (44) to calculate the different terms in (30) (neglecting, of course, the term $E_1{}^2/8\pi$ which is much smaller than the term $B_1{}^2/8\pi$). We get

$$\mathbf{J}_1 \cdot \mathbf{E}_1 = \frac{\mathbf{J}_1 \times \mathbf{B} \cdot \mathbf{u}}{c} + \frac{\partial}{\partial t}\left(\frac{m_e}{2ne^2} J_1{}^2\right). \tag{45}$$

The first term on the right-hand-side of (45) can be calculated from the momentum equation (39) which gives

$$\frac{\mathbf{J}_1 \times \mathbf{B} \cdot \mathbf{u}}{c} = \frac{\partial}{\partial t}\left(\frac{1}{2}\rho u^2\right)$$

$$+ \nabla \cdot (p_1 \mathbf{u}) - p_1 \nabla \cdot \mathbf{u} + \mathbf{u} \cdot \frac{\mathbf{J} \times \mathbf{B}_1}{c}. \tag{46}$$

Writing

$$\mathbf{B}_1 = \nabla \psi_1 \times \hat{y} + B_{y1} \hat{y}, \tag{47}$$

we get

$$\mathbf{u} \cdot \frac{\mathbf{J} \times \mathbf{B}_1}{c} = \nabla \cdot \left(\mathbf{u} \frac{J \psi_1}{c}\right)$$

$$- \frac{J \psi_1}{c} \nabla \cdot \mathbf{u} + \frac{\psi_1}{c} \mathbf{u} \cdot \nabla J. \tag{48}$$

Since $\partial \psi_1/\partial t + \mathbf{u} \cdot \nabla \psi = 0$, and $J = J(\psi)$, we obtain

$$\mathbf{u} \cdot \nabla J = -\frac{\partial \psi_1}{\partial t} J'(\psi) = \frac{c}{4\pi} \frac{\partial \psi_1}{\partial t} \frac{\psi'''(z,\varepsilon x)}{\psi'(z,\varepsilon x)}, \tag{49}$$

where the prime in $\psi'(z,\varepsilon x)$ indicates differentiation with respect to z. Defining $F \equiv \psi'(z,\varepsilon x)$ and substituting (46) – (49) in (45), we get

$$\mathbf{J}_1 \cdot \mathbf{E}_1 = \frac{\partial}{\partial t}\left[\frac{1}{2}\rho u^2 + 2\pi \frac{J_1^2}{\omega_p^2} + \frac{F''}{8\pi F} \psi_1^2\right]$$

$$+\nabla\cdot(\tilde{p}_1\mathbf{u})-\tilde{p}_1\nabla\cdot\mathbf{u}\,, \tag{50}$$

where $\omega_p{}^2 = 4\pi ne^2/m_e$ is the plasma frequency, and

$$\tilde{p}_1 = p_1 - p'(\psi)\,\psi_1 = p_1 - J\,\psi_1/c\,. \tag{51}$$

Equation (30) can then be cast in the form,

$$\frac{\partial}{\partial t}\,\delta^2\varepsilon + \oint d\mathbf{a}\cdot\left[\frac{c}{4\pi}\,\mathbf{E}_1\times\mathbf{B}_1 + \tilde{p}_1\,\mathbf{u}\right] = 0\,, \tag{52}$$

where

$$\delta^2\varepsilon = \delta^2W_f + \delta^2W_c + \delta^2K + \delta^2Q \equiv \delta^2U + \delta^2K + \delta^2Q\,. \tag{53}$$

Here

$$\delta^2W_f = \frac{1}{8\pi}\int dx\,\left(B_1^2 + \frac{F''}{F}\,\psi_1^2\right), \tag{54}$$

is the free energy of the magnetic field;

$$\frac{\partial}{\partial t}\,\delta^2W_c \equiv -\int dx\,\tilde{p}_1\,\nabla\cdot\mathbf{u}\,, \tag{55}$$

can be attributed to plasma compression, and

$$\delta^2Q \equiv \int dx\left(2\pi/\omega_p^2\right)J_1^2\,. \tag{56}$$

is the "dissipation" due to finite electron inertia. Note that the *fluid kinetic energy* $\delta^2K \equiv \int dx\,(1/2)\rho u^2$, and the dissipation δ^2Q are positive definite quantities. If the boundary in (52) is chosen such that the surface term vanishes, then a sufficient condition for stability is

$$\delta^2U \equiv \delta^2W_f + \delta^2W_c \geq 0\,. \tag{57}$$

In order to develop confidence in the fluid model, we now benchmark it with standard results from kinetic theory. For simplicity, we consider an incompressible fluid for which $\delta^2W_c = 0$. Then, there are three terms in the energy integral (53) among which δ^2K is the kinetic energy and always positive definite. It is clear that a tearing instability can occur if and only if there is magnetic free energy available, i.e., $\delta^2W_f < 0$, and there exists simultaneously, a mechanism for dissipation causing δ^2Q (which is always positive definite).

Let us first consider the equilibrium configuration (2) with $B_n = 0$. For this case the growth rate of the collisionless tearing instability is known analytically. The mathematical problem divides itself neatly into two regions: the outer region, away from $z = 0$, where the plasma is in quasistatic equilibrium, and the inner region near $z = 0$ where inertial and dissipative effects are important. We rewrite (54) in the form

$$\delta^2W_f = \frac{1}{16\pi}\int dx\,\left[\tilde{\psi}'^2 + k^2\tilde{\psi}^2 + \frac{F''}{F}\,\tilde{\psi}^2\right], \tag{58}$$

where $\psi_1 = \tilde{\psi}(z,t)\cos kx$, $F \equiv \tanh z/\lambda$ and prime denotes derivative with respect to z. In the outer region, neglecting inertia and dissipation, we get

$$\tilde{\psi}'' - k^2\tilde{\psi} - (F''/F)\,\tilde{\psi} = 0\,. \tag{59}$$

Furth [1963] observed that if the first term in the integrand of (58) is integrated by parts and (59) is used, $\delta^2 W_f$ reduces to

$$\delta^2 W_f = -\frac{a_{xy}}{16\pi} \tilde{\psi} \tilde{\psi}' \Big|_{z=0-}^{z=0+} .$$

$$= -\frac{a_{xy}}{16\pi} \Delta_0' \tilde{\psi}^2(0) , \qquad (60)$$

where $\int dx = a_{xy} \int dz$, $a_{xy} = \int dx dy$ and the parameter Δ_0' is defined by the relation,

$$\Delta_0' \equiv \frac{\tilde{\psi}'(0+) - \tilde{\psi}'(0-)}{\tilde{\psi}(0)} . \qquad (61)$$

(In obtaining (60) , we have used the boundary conditions $\tilde{\psi}(-\infty) = \tilde{\psi}(+\infty) = 0$.) For the equilibrium (2) with $B_n = 0$, we get (see, for instance, WBL)

$$\Delta_0' = \frac{2}{k\lambda^2}(1 - k^2\lambda^2) . \qquad (62)$$

On the interface between the inner and outer regions, the Poynting flux is

$$\oint d\mathbf{a} \cdot \frac{c}{4\pi} \mathbf{E}_1 \times \mathbf{B}_1$$

$$= \frac{c}{4\pi} \left[\left\{ \int_{z=0-} dx dy - \int_{z=0+} dx dy \right\} \hat{z} \cdot \mathbf{E}_1 \times \mathbf{B}_1 \right]$$

$$= \frac{a_{xy}}{16\pi} \Delta_0' \frac{\partial}{\partial t} \tilde{\psi}^2(0) = -\frac{\partial}{\partial t} \delta^2 \varepsilon_{\text{outer}} . \qquad (63)$$

Thus the magnetic free energy in the outer region is spent as dissipation and kinetic energy in the inner region.

To determine the dissipation in the inner region, we use the generalized Ohm's law,

$$-\frac{1}{c} \frac{\partial \tilde{\psi}}{\partial t} = \frac{m_e}{ne^2} \frac{\partial \tilde{J}_y}{\partial t} , \qquad (64)$$

where $J_{1y} = \tilde{J}_y \cos kx$. Equation (64) gives

$$\tilde{J}_y = -\frac{c}{4\pi} k_0^2 \tilde{\psi} , \qquad (65)$$

where $k_0 \equiv \omega_p/c$. The dissipation caused by electron inertia in the inner region is

$$\delta^2 Q = a_{xy} \frac{k_0^2}{16\pi} \int_{z=-d_e}^{z=+d_e} dz \, \tilde{\psi}^2$$

$$= a_{xy} \frac{k_0^2}{8\pi} d_e \tilde{\psi}(0)^2 . \tag{66}$$

where d_e is the width of the tearing layer.

Requiring that the system energy be equal to its equilibrium value [*Kruskal and Oberman*, 1958], we get

$$\delta^2 \varepsilon = \delta^2 W_f + \delta^2 K + \delta^2 Q = 0 . \tag{67}$$

For the electron tearing instability, the fluid kinetic energy $\delta^2 K$, which is dominantly due to ions, is much smaller than the electron dissipation $\delta^2 Q$. Hence, from the relation

$$\delta^2 W_f + \delta^2 Q = 0 , \tag{68}$$

we obtain the tearing layer width [*Drake and Lee*, 1977]

$$d_e = d_e(k) = \frac{\Delta_0'}{2k_0^2} . \tag{69}$$

The growth rate of the instability can be determined by using the wave-particle resonance condition. In the case $B_y = 0$ this condition is

$$\gamma = k\, v_{te} , \tag{70}$$

where k is determined from (62) by the requirement $\Delta_0' > 0$ ($\delta^2 W_f < 0$). From (62) and (70) we obtain

$$\gamma = \frac{v_{te}}{k_0^2 \lambda^2 d_e} (1 - k^2 \lambda^2) , \tag{71}$$

$$= \left(\frac{v_{te}}{\lambda}\right) \left(\frac{\rho_e}{\lambda}\right)^{3/2} \left(1 + \frac{T_i}{T_e}\right) (1 - k^2 \lambda^2) . \tag{72}$$

In writing (72) we have made use of the equilibrium relation (10). The growth rate (72) is larger by the factor $\pi^{1/2}$ than the result obtained from kinetic theory by *Laval et al.* [1966].

In the case $B_y \neq 0$ the resonance condition (70) is modified to

$$\gamma = k_{\parallel} v_{te} . \tag{73}$$

where $k_{\parallel} = k\, d_e / \lambda$. Equations (62) and (73) then yield

$$\gamma = \frac{k v_{te} \Delta_0'}{2 k_0^2 \lambda} , \tag{74}$$

which is also larger by the factor $\pi^{1/2}$ than the result of *Drake and Lee* [1977].

Equations (71) and (74) demonstrate that the fluid model is reliable as a predictor of the parametric dependencies and the order of magnitude of the growth rate of collisionless tearing modes when $B_z = 0$, both with and without B_y. The numerical factor missed by the model has to do with the precise details of the electron distribution function.

In the last paragraph, we have used the term "collisionless tearing" instead of "electron tearing," though the latter name is commonly used. It has been shown elsewhere (for the case $B_y \neq$

0) that the full ion dynamics gives small logarithmic corrections to (74) (WBL, Appendix A]. In other words, the result (74) already includes the ion response, and there is no ion tearing branch of the dispersion equation. That being so, it is redundant to call this tearing instability electron tearing because in this case there is no other form of tearing.

4. Effect of Constant B_n

So far, we have benchmarked the fluid model by reproducing known results from kinetic theory for equilibria with $B_n = 0$. We presume that a fluid model which has been demonstrated to be reliable for $B_n = 0$ will work for equilibria with $B_n \neq 0$, representative of the Earth's magnetotail. As discussed in WBL, what will change in the presence of a nonzero B_n are the particle orbits. We consider, at first, the case $B_n =$ constant (equation (15)) which has engendered considerable controversy in the literature.

In the presence of B_n the perturbed current has a component J_{1z} in the z direction which generates a perturbed magnetic field component B_{1y}. In WBL, this component has been calculated by writing $B_1 = \nabla \times A_1$, using the Coulomb gauge $\nabla \cdot A_1 = 0$ and the approximation $\partial/\partial x << \partial/\partial z$ for perturbed quantities. Here we denote $\psi_1 \equiv A_{1y}$, $\chi_1 \equiv A_{1z}$, and write

$$B_1^2 \equiv \psi_1'^2 + \left(\frac{\partial \psi_1}{\partial x}\right)^2 + B_{1y}^2 \qquad (75)$$

Substituting (75) in (54), we have

$$\delta^2 W_f = \frac{1}{8\pi} \int dx \left[\psi_1'^2 + \left(\frac{\partial \psi_1}{\partial x}\right)^2 + \frac{F''}{F} \psi_1^2 + B_{1y}^2 \right]. \quad (76)$$

We take $\chi_1 = \tilde{\chi}_1(z,t) \cos kx$. In the outer region, we have [WBL, Appendix B]

$$\tilde{\chi}'' - k^2 \tilde{\chi} \simeq 0, \qquad (77)$$

which has a solution of the form $e^{-k|z|}$. This solution has a jump continuity in its logarithmic derivative, specified by

$$\Delta_2' \equiv \frac{\tilde{\chi}'(0+) - \tilde{\chi}'(0-)}{\tilde{\chi}(0)} \simeq -2k. \qquad (78)$$

In the Coulomb gauge, we can now write $B_{1y}^2 = k^{-2}(\tilde{\chi}'' - k^2 \tilde{\chi})^2$. Then, the contribution of the last term in the integrand of (76) is $-(a_{xy}/16\pi)\, \Delta_2' \, \tilde{\chi}^2(0)$. This has a clear physical interpretation: B_n reduces the magnetic free energy available to the tearing instability and hence has a stabilizing effect.

The computation of the first three terms in the integrand of (76) is somewhat more involved. One of the complications introduced by B_n is that it introduces a phase shift that, in effect, couples the cosine solution in x, i.e., $\tilde{\psi}_c(z,t) \cos kx$ with the sine solutions in x, i.e., $\tilde{\psi}_s(z,t) \sin kx$. In the outer region equation,

$$\psi' \frac{\partial}{\partial x} \nabla^2 \psi_1 = \psi''' \frac{\partial \psi_1}{\partial x} + B_n \frac{\partial}{\partial z} \nabla^2 \psi_1, \qquad (79)$$

substituting

$$\psi_1 = \tilde{\psi}_c(z,t)\cos kx + \tilde{\psi}_s(z,t)\sin kx\ , \qquad (80)$$

we obtain

$$h''' \tilde{\psi}_c = h'(\tilde{\psi}_c'' - k^2\psi_c) - \frac{\varepsilon}{k\lambda}(\tilde{\psi}_s''' - k^2\tilde{\psi}_s')\ , \qquad (81)$$

$$h''' \tilde{\psi}_s = h'(\tilde{\psi}_s'' - k^2\psi_s) + \frac{\varepsilon}{k\lambda}(\tilde{\psi}_c''' - k^2\tilde{\psi}_c')\ , \qquad (82)$$

where $h' \equiv \lambda^{-1} \tanh z/\lambda$. Following the method outlined in Appendix B of WBL, we get

$$\tilde{\psi}_c = \tilde{\psi}_c^{(0)} \pm \varepsilon\lambda b h' \tilde{\psi}_s^{(0)} + O(\varepsilon^2)\ , \qquad (83)$$

$$\tilde{\psi}_s = \tilde{\psi}_s^{(0)} \mp \varepsilon\lambda b h' \tilde{\psi}_c^{(0)} + O(\varepsilon^2)\ , \qquad (84)$$

where the upper sign in (83) and (84) corresponds to $z > 0$, the lower sign to $z < 0$, and $b \equiv 4/(k\Delta_0'\lambda^2)$. The leading order solutions $\tilde{\psi}_c^{(0)}$ and $\tilde{\psi}_s^{(0)}$ obey

$$h''' \ \tilde{\psi}_{c,s}^{(0)} = h'\left(\tilde{\psi}_{c,s}^{(0)''} - k^2\lambda^2\,\tilde{\psi}_{c,s}^{(0)}\right)\ . \qquad (85)$$

We now use (81) – (85) to calculate the first three terms in $\delta^2 W_f$. These are

$$\delta^2 W_\psi \equiv \frac{1}{8\pi}\int dx\left[\psi_1'^2 + \left(\frac{\partial\psi_1}{\partial x}\right)^2 + (F''/F)\,\psi_1^2\right]\ ,$$

$$= \frac{a_{xy}}{16\pi}\int dx\left[\tilde{\psi}_c'^2 + k^2\tilde{\psi}_c^2 + \frac{h'''}{h'}\tilde{\psi}_c^2\right.$$

$$\left. + \tilde{\psi}_s'^2 + k^2\tilde{\psi}_s^2 + \frac{h'''}{h'}\tilde{\psi}_s^2\right]\ . \qquad (86)$$

Integrating the right-hand side of (86) by parts and using (81) and (82), we get

$$\delta^2 W_\psi = -\frac{a_{xy}}{16\pi}\ \Delta_0'\,\tilde{\psi}^2(0) + \frac{\varepsilon}{k\lambda}\left[\int dz\,\frac{\tilde{\psi}_c^{(0)}}{h'}\left(\frac{h'''}{h'}\tilde{\psi}_s^{(0)}\right)'\right.$$

$$\left. - \int dz\,\frac{\tilde{\psi}_s^{(0)}}{h'}\left(\frac{h'''}{h'}\tilde{\psi}_c^{(0)}\right)'\right]\ . \qquad (87)$$

where $\tilde{\psi}^2(0) \equiv \tilde{\psi}_c^2(0) + \tilde{\psi}_s^2(0)$. Since $\tilde{\psi}_c^{(0)}$ and $\tilde{\psi}_s^{(0)}$ are both solutions of (85) and obey the same boundary conditions, the last two terms of (87) cancel each other exactly. Defining

$$\Delta_r' \equiv \frac{\Delta_0'\,\tilde{\psi}^2(0) + \Delta_\chi'\,\tilde{\chi}^2(0)}{\tilde{\psi}^2(0) + \tilde{\chi}^2(0)}\ , \qquad (88)$$

and using (87), we get

$$\delta^2 W_f = -\frac{a_{xy}}{16\pi}\ \Delta_r'\left(\tilde{\psi}^2(0) + \tilde{\chi}^2(0)\right)\ . \qquad (89)$$

We now evaluate $\delta^2 W_c$, which is due to plasma compressibility. If B_n is large and constant, then the inequality $\gamma\tau_{be} << 1$ is satisfied. Under these conditions the electrons have a stabilizing compressional contribution due to the bouncing motion between mirror points along a field line. However, the effect of ion compressibility, which comes into play because of the quasi-neutrality constraint $n_{e1} = n_{i1}$, is larger than the effect of electron compressibility. The magnetic free energy $\delta^2 W_f$ is generally not large enough to provide energy for compressing the ions, and the mode is stabilized unless the wavelength is very large. In order to demonstrate this, we calculate $\delta^2 W_c$ from the fluid equations. From the linearized continuity equation, we get

$$\nabla \cdot \mathbf{u} = -\frac{1}{n}\frac{\partial n_1}{\partial t} - \mathbf{u}\cdot\nabla\psi \frac{d}{d\psi}\ln n . \qquad (90)$$

Using the equilibrium relation (7), and defining

$$\tilde{n} = n_1 - \frac{dn}{d\psi}\psi_1 , \qquad (91)$$

we rewrite (90) in the form

$$\nabla \cdot \mathbf{u} = -\frac{1}{n}\frac{\partial \tilde{n}}{\partial t} - \frac{2}{B_0\lambda}\left(\frac{\partial \psi_1}{\partial t} + \mathbf{u}\cdot\nabla\psi\right). \qquad (92)$$

Averaging (92) over an electron bounce period, we have

$$<\nabla \cdot \mathbf{u}> = -\frac{\gamma}{n}<\tilde{n}> - \frac{2}{B_0\lambda}\left(\gamma <\psi_1> + B_n <u_{1x}>\right) , \qquad (93)$$

where $< >$ indicates an average over the rapid bounce motion of the electrons. Since the bounce motion involves dominantly the outer region, we neglect the electron inertia term in averaging Ohm's law (41). We then obtain

$$< E_{1y} > = -\frac{1}{c}\left< \frac{\partial \psi_1}{\partial t} \right> = \frac{<v_{1x}> B_n}{c} . \qquad (94)$$

Using (44), (51), (93), and (94), we get

$$<\tilde{p}_1 \nabla\cdot\mathbf{u}> = -\frac{k^2}{2B_n^{\ 2}} n_0 (T_e + T_i) \left< \frac{\partial}{\partial t}\psi_1^2 \right>$$

$$= \frac{k^2 B_0^{\ 2}}{16\pi B_n^{\ 2}} \left< \frac{\partial}{\partial t}\psi_1^2 \right> . \qquad (95)$$

Hereafter, to simplify notation, we shall drop the averaging sign. Equation (95) then gives

$$\delta^2 W_c = \int \frac{dx}{16\pi} \frac{k^2 B_0^{\ 2}}{2B_n^2} \tilde{\psi}^2 . \qquad (96)$$

Equation (96) can be rewritten as

$$\delta^2 W_c = \frac{a_{xy}}{16\pi} \frac{k^2 B_0^{\ 2}}{2B_n^{\ 2}} z_0 \tilde{\psi}^2(0) . \qquad (97)$$

where

$$z_0 = 2\int_0^{\infty} dz \exp(-2kz)\left[1+\tanh z/\lambda\right]^2. \qquad (98)$$

Equation (97) agrees with (91) of *Galeev* [1984] (except for the area term a_{xy} which has been taken to be unity by Galeev without loss of generality). The constant z_0 is estimated by Galeev from physical arguments; here, we evaluate (98) asymptotically to obtain

$$z_0 \simeq 1/k\ . \qquad (99)$$

Note that an upper bound for $\delta^2 W_f$ is obtained by setting $\tilde{\chi}$ (0) equal to zero. Hence, from (57), a sufficient condition for stability is

$$-\Delta_0' + k/\varepsilon^2 \geq 0\,, \qquad (100)$$

which for $\varepsilon << 1$, reduces to

$$k\lambda/\varepsilon \geq 2\,. \qquad (101)$$

Equation (101) is close to the sufficient condition for stability $k\lambda/\varepsilon \geq 4/\pi$, derived by *Lembege and Pellat* [1982]. The inequality (101) implies that all wavelengths smaller than $\pi\lambda/\varepsilon$ are stable. For example, if we take $\lambda \sim 1\ R_E$, $\varepsilon \sim 0.1$, we find that wavelengths smaller than 30 R_E are stable. Of course, this does not necessarily mean that wavelengths larger than $\pi\lambda/\varepsilon$ are unstable because violation of (101) does not imply instability. If one proceeds with the hypothesis that instability is possible for $k\lambda/\varepsilon < 2$, it can be shown, following *Lembege and Pellat* [1982], that a long-wavelength ion tearing mode is impossible. We refer the reader to the work of *Lembege and Pellat* [1982] for further details.

Attempts have been made to restore the ion tearing instability by invoking pitch-angle diffusion [*Coroniti*, 1980; *Galeev*, 1984 and references therein] or intrinsic chaotic diffusion [*Büchner et al.*, 1987]. We now demonstrate that even in the presence of these effects, the most that we can get is some form of weak electron tearing and that there is no ion tearing. At first we note that occasionally, a source of some confusion in the literature has been the misleading premise that it is electron compressibility that stabilizes tearing in the presence of a constant B_n. From this premise follows the argument that if the electrons are removed by pitch angle scattering or intrinsic stochastic diffusion, then it is possible to neglect the electrons while the ions tear field-lines. Our fluid model clearly indicates that electron compressibility is less of a factor than ion compressibility for conditions typical of the magnetotail. Inspection of (95) shows that both ions and electrons contribute to $\delta^2 W_c$, but the electron contribution to $\delta^2 W_c$ may be apportioned as $[T_e/(T_i+T_e)]\ \delta^2 W_c$, whereas the ion contribution is $[T_i/(T_i+T_e)]\ \delta^2 W_c$. Since $T_i \approx 5T_e$ is typical in the magnetotail, this apportionment indicates that the dominant contribution to fluid compressibility comes from ions.

In order to pinpoint the differences between our results and others in the literature, we refer the reader to the review by *Galeev* [1984]. Galeev's equation (91) gives the energy spent for plasma compression, in agreement with our $\delta^2 W_c$. Subsequently, in the presence of pitch angle scattering, Galeev attributes a compressional term similar to $\delta^2 W_c$ entirely to electrons (see his equation (96)), and yet another contribution due to ions (his equation (97)). Our fluid model yields the compressional energy,

$$\delta^2 W_c = \frac{a_{xy}}{16\pi} \frac{k^2 B_0^{\ 2}}{2B_n^2} \frac{\gamma}{\gamma + \nu_{eff}} z_0 \tilde{\psi}^2(0) , \qquad (102)$$

where ν_{eff} is the bounce-averaged effective collision frequency. Note that when $\nu_{eff} = 0$, (102) reduces to (97), as it should. (If $\nu_{eff} >> \gamma$, then the factor $\gamma/(\gamma + \nu_{eff})$ can be approximated by γ/ν_{eff}). Thus $\delta^2 W_c$ (our equation (102)) includes the compressional effect due to both electrons and ions, and that there is no separate ion contribution as Galeev's equation (97) suggests. For large ν_{eff} (or equivalently, large stochastic diffusion), the stabilizing effect of $\delta^2 W_c$ can be strongly reduced. Under these conditions, it is possible, in principle, to recover an electron tearing instability, but there is no ion tearing. This conclusion supports the recent results of *Pellat et al.* [1991] who question the very existence of ion tearing, but contradicts the findings of *Kuznetsova and Zelenyi* [1991].

In view of the controversy in analytical theories, much can be learned from particle simulations. Unfortunately, electromagnetic particle simulations of collisionless tearing inevitably involve making compromising choices on such parameters as m_e/m_i, the system size (which determines the range of unstable wavelengths), and the spatial grid size. We have cited several such simulations earlier, and it is fair to say that in all of them, an instability with the theoretically predicted growth rate and characteristics of ion tearing has been very difficult to find. Since we believe that both electron and ion dynamics (which are tied by the constraint of quasi-neutrality) should be retained in simulations of collisionless tearing, we first comment on reported results from two-species simulations that include a B_n field. *Swift and Allen* [1987, P.10,015] report that their previous unpublished work showed "no evidence of the development of any type of instability." They also attribute correctly the observed stability to ion compressibility. *Zwingmann et al.* [1990] report results mostly for the mass ratio $m_e/m_i = 1$, with some discussion of a case with $m_e/m_i = 1/10$. As they note, the case $m_e/m_i = 1$ cannot distinguish between electron and ion tearing. (If an ion tearing mode exists, its growth rate should be much larger than the electron tearing growth rate when the mass ratio is realistic.) Their results show significant discrepancies with theoretical predictions [*Schindler*, 1974]. In particular, the growth rate observed in the simulation is up to an order of magnitude less than predicted by theory. We attribute the growth of the instability in these simulations for small values of B_n to electron tearing, not ion tearing. This hypothesis can be tested, of course, by a study which computes the growth rate as a function of m_e/m_i.

Apart from two-species simulations, there are one-species simulations of the ion tearing mode in which the electrons are involved only as a static charge-neutralizing background [*Terasawa*, 1981; *Hamilton and Eastwood*, 1982; *Swift*, 1983; *Ambrosiano and Lee*, 1983; *Pritchett et al.*, 1991]. It is clear from our previous discussion that these simulations cannot realistically simulate electron tearing modes. Furthermore, any inference on the viability of the ion tearing mode from these simulations is questionable because electron dynamics has been eliminated arbitrarily for reasons of computational convenience.

We conclude this section with the remark that unless the B_n field is very small, field lines cannot reconnect to form islands in the linear regime. The striking contrast between configurations with $B_n = 0$ which tear easily to form magnetic islands and configurations in which significant values of B_n inhibit tearing is illustrated well by Figures 6.2.4 and 6.2.9 in Galeev's review paper. In Figure 6.2.4, islands develop at the separatrix where collisionless reconnection provides accessibility to a state of lower energy. In Figure 6.1.9, there is no well-defined separatrix, and the system sustains global compressional oscillations.

5. Effects of B_y and Spatially Varying B_n

WBL considered the effect of a constant B_y field superimposed on the two-dimensional configuration of section 4. Their analysis of the electron tearing mode dealt with the inner region dynamics using kinetic theory, but global aspects of the dynamics such as the bounce motion was neglected. The aim of the present effort is to explore the consequences of these global effects in the context of the improved asymptotic equilibria developed in section 2.

At first, we consider equilibria in which B_y is constant, and the spatial dependencies of B_x and B_n are described in region 1 (near-Earth) by (18) and (19), in region 2 (middle) by (20) and (21), and in region 3 (distant-tail) by (22) and (23), respectively. As noted in section 2, the spatial structure of B_n in the near-Earth and middle regions are similar, except that the magnetic field is weaker and the current sheet is wider in the middle region. It is shown in Appendix A that the bounce period τ_{be} in region 2 is larger by an order of magnitude than τ_{be} in region 1. In region 3, since B_n is vanishingly small, τ_{be} is extremely large.

Certain conditions must be fulfilled for the electron tearing instability to occur. First, there must be magnetic free energy available to drive the instability; i.e., we must have $\delta^2 W_f < 0$, where $\delta^2 W_f$ is given by (89). This means that the stability parameter Δ_t' must be positive. In standard analyses of collisionless stability of the tail (see, for instance, *Galeev* [1984, and references therein]), Δ_t' is replaced by Δ_0'. Note that this overestimates the range of unstable wavelengths because Δ_2' (equation (78)) is negative.

Second, the stabilizing compressional energy $\delta^2 W_c$ due to the bounce motion of electrons should not exceed the destabilizing term $\delta^2 W_f$. We show, a posteriori, that in region 1, $\gamma\tau_{be} << 1$, but in regions 2 and 3, we have $\gamma\tau_{be} \geq 1$ and $\gamma\tau_{be} >> 1$, respectively. The compressional stabilization is thus significant in region 1 but not so in regions 2 and 3.

In regions 2 and 3, the effect of the electron bounce can be neglected. We are then back in the framework of WBL who obtained the dispersion equation for electron tearing modes neglecting electron bounce. From a kinetic analysis, carried out in Appendix B, we obtain the complex frequency $\omega = \omega_R + i\gamma$, where

$$\omega_R = \omega_* - \frac{k v_{te} s_0}{2\sqrt{\pi}\, k_0^2 L_s} \Delta_i' , \tag{103}$$

$$\gamma = \frac{1}{\sqrt{\pi}} \frac{k v_{te} s_0}{2 k_0^2 L_S} \Delta_r' . \tag{104}$$

in region 2. Here $\omega_* = k V_{ez}$. s_0 is a constant which can be shown to have a numerical value of approximately 10 under typical conditions. $L_s = \lambda(B_y^2 + B_n^2)^{1/2}/B_0$ and $\Delta_i' = 8\varepsilon/(k\Delta_0' \lambda^3) << \Delta_r'$. For comparison, we recall that if $B_n \simeq 0$, the real and imaginary parts of the complex growth rate are, respectively [*Drake and Lee*, 1977], given by

$$\omega_R \simeq \omega_* . \tag{105}$$

$$\gamma = \frac{1}{\sqrt{\pi}} \frac{k v_{te}}{2 k_0^2 L_s} \Delta_0' . \tag{106}$$

In (103) and (104), all equilibrium quantities are evaluated at $z = 0$. In particular, since $\tilde{\psi}(0) / \tilde{\chi}(0) \simeq B_y/B_n(z = 0)$, we can write

$$\Delta_r' \simeq \frac{\alpha^2 \Delta_0' + \varepsilon^2 \Delta_2'}{\alpha^2 + \varepsilon^2}, \tag{107}$$

where $\alpha \equiv B_y/B_0$. If $\alpha \sim \varepsilon$, a necessary condition for instability is

$$\Delta_r' = \Delta_0' + \Delta_2' > 0 , \tag{108}$$

which gives

$$k\lambda < 1 / \sqrt{2} . \tag{109}$$

Equation (109) implies that wavelengths larger than $2\sqrt{2}\,\pi\lambda$ ($\simeq 9 R_E$ for $\lambda \simeq 1 R_E$) may be unstable. In order for the instability to grow, however, it must also satisfy the condition $\gamma\tau_{be} \geq 1$. A viable class of instabilities is obtained for $k^{-1} \geq 3\lambda$; these do obey the condition $\gamma\tau_{be} \geq 1$ for $s_0 \simeq 10$, $v_{te} \simeq 2\lambda/s$.

If $B_y << B_z$, i.e., $\varepsilon >> \alpha$, the condition $\Delta_r' > 0$ for instability reduces to $k\lambda < \alpha/\varepsilon$. As shown by WBL, this condition predicts unstable wavelengths which are much too large to account for reconnection events in the near-Earth and middle regions.

We note that (104) and (106) have been obtained from a kinetic analysis, and except for a factor of $\pi^{1/2}$, can also be obtained from the fluid model. The fluid analog of (106) is (74), derived in section 3. The fluid analog of (104) has been derived in Appendix B, and is given by

$$\gamma \simeq \frac{k v_{te} s_0}{2 k_0^2 L_s} \Delta_r' . \tag{110}$$

As before, the results from the fluid and kinetic calculations differ by a multiplicative factor of $\pi^{1/2}$.

In regions where $\gamma\tau_{be} \geq 1$, the destabilizing effect of B_y may be understood as follows. If $B_y = 0$, *Galeev* [1984] points out that the energy spent for plasma compression is the work done by the perturbed plasma current generated by the perturbed pressure gradient,

$$\frac{\partial \tilde{p}_1}{\partial x} = \frac{\tilde{J}_{1y} B_n}{c} . \tag{111}$$

If $B_y \neq 0$, (111) changes to

$$\frac{\partial \tilde{p}_1}{\partial x} = \frac{1}{c} (\tilde{J}_{1y} B_n - \tilde{J}_{1z} B_y) . \tag{112}$$

Now, if $\gamma\tau_{be} << 1$, then the bounce average of (112) should be taken, and the second term on the right-hand side of (112) averages to zero. On the other hand, if $\gamma\tau_{be} \geq 1$, or $\gamma\tau_{be} >> 1$, then there is no bounce average to be taken, and the second term tends to reduce the first term because $\tilde{J}_{1y} / \tilde{J}_{1z} \sim B_y / B_z$.

There is some evidence in the numerical simulations of *Swift and Allen* [1987] that the presence of B_y enhances the tearing activity near $z = 0$ compared with the case $B_y = 0$. (See their section 4.3.) Clearly, there is a need for two-species simulations including B_y using either asymptotic equilibria of the kind developed in this paper, or numerical solutions of the equilibrium Grad-Shafranov equation [cf. *Voigt and Hilmer*, 1987].

Finally, we comment on equilibria with spatially varying B_y, discussed in section 2.2. There we show that, for a class of pressure profiles, the spatial variation of B_y can cause the formation of an X point on the $z = 0$ line. We show, furthermore, that this can occur at near-Earth distances for average values of B_y and B_n characteristic of the plasma sheet. The configuration thus formed is likely to be highly magnetically stressed. Under such conditions, rapid reconnection may occur at the separatrix in both the linear and nonlinear regimes. We conjecture that the collisionless reconnection rate in this geometry is likely to be much larger than the rates derived in this paper. Such a geometry calls for a separate treatment, and the exploration of that possibility is left to future work.

6. Conclusions

This paper makes two main contributions to the problem of collisionless tearing modes in the Earth's magnetotail. The first involves the development of asymptotic magnetotail equilibria including all three components of the magnetic field, with realism in the modeling of the normal component of the magnetic field, $B_n(x,z)$. The second involves the development of a fluid model that is physically transparent and accurate in reproducing the parametric dependencies of the growth rates of collisionless tearing modes calculated from kinetic theory.

One of the significant conclusions of this paper is that the ion tearing mode, which has been the subject of considerable research and controversy over the last two decades, does not occur. This is true for both two- and three-component models of the magnetotail. We are not the first to suggest this, because *Lembege and Pellat* [1982] and *Pellat et al.* [1991] have preceded us, albeit in the context of the simple two-component equilibrium (1). We find that for the two and three-component equilibria given in this paper that if there is a collisionless tearing instability in the magnetotail, it is the electron tearing mode.

There are certain conditions that must be fulfilled for the electron tearing mode to be seen. A significant value of B_n in the two-component magnetotail, represented by (1), tends to suppress the instability. The reason for this strong stabilization can be understood in dynamical terms. In our view, the dynamics are a symptom of a deeper cause which has to do with geometry. The main cause of the stabilization of the tearing mode is that B_n destroys the separatrix at $z = 0$. By contrast, if we set $B_n = 0$ but include a B_y field, the separatrix at $z = 0$ is undisturbed, and electron tearing modes can easily occur.

The main difficulty posed by the magnetotail is that all three components of the magnetic field can be significant. Then, the circumstances that favor electron tearing are those that minimize global features of the dynamics such as electron bounce, and keep the electron confined near $z = 0$. It is intuitively clear that the B_y field tends to confine electrons near $z = 0$, and hence helps the electron tearing instability grow. The asymptotic equilibria presented in this paper have regions where the stabilizing effect of electron bounce can be neglected and where the presence of $B_y \sim B_n$ can cause the excitation of electron tearing. A tearing instability in which field lines actually undergo genuine topological change does not occur unless B_n is very small. Unless topological change occurs, the instability is likely to saturate nonlinearly at a relatively low amplitude. We do not believe that such a weak instability can account for the dramatic signatures associated with current disruption and diversion during substorms.

The instability is more interesting when B_n is zero. We have demonstrated that if we include B_y and allow it to vary spatially in a three-dimensional magnetotail equilibrium, then B_n can vanish at near-Earth distances. The linear as well as the nonlinear growth of electron tearing modes in such a configuration is likely to lead to interesting results and will be investigated in the near future.

An important challenge for a theory of substorms is that it should account not only for the violent activity that is associated with substorms, but also identify conditions under which the magnetotail is stable. A universal instability that occurs always and spontaneously is likely not to be a correct explanation because that would suggest the magnetotail is always unstable, which is not observed to be the case. In this work, we have identified conditions under which electron tearing modes may be unstable and delineated regimes when they are not. It is our hope that this paper, as well as its forerunner (WBL), will stimulate a reexamination of old as well as new data in substorms with a renewed emphasis on the B_y field. Observationalists, many of whom we have cited here, have been aware over the last 15 years of the ubiquitous presence of the B_y field, varying spatially as well as in time before and during different phases of a substorm. What is required is a more systematic study correlating B_y and B_z with the occurrence of substorm onset. More two-species electromagnetic particle simulations, including all three components of the magnetic field, are also required, both to check analytic theory and to model realistically global features of magnetotail equilibria.

Appendix A: Bounce Motion Of Electrons

An electron gyrating along a field line in the magnetotail may, under certain conditions, bounce between two turning points. Since $B_n(x, z) = B_n(x, -z)$, the z coordinates of the turning points may be written as $z = \pm z_t$, where the constant z_t is determined by the parallel energy of the electron,

$$\frac{1}{2} m v_{\|}^2 = \frac{1}{2} m v^2 - \mu B \equiv K - \mu B , \qquad \text{(A1)}$$

where μ is the magnetic moment. The bounce period is defined as

$$\tau_b = \oint \frac{dl}{v_{\|}} , \qquad \text{(A2)}$$

where l is the coordinate along a field line. Using the relation $dl / B = dl / B_z$, we have

$$\tau_b = 4 \int_0^{z_t} \left(\frac{m}{2K} \right)^{1/2} \frac{B}{B_z} \frac{dz}{(1 - \mu B / K)^{1/2}} . \qquad \text{(A3)}$$

We recall that $B_x \simeq \tanh z$ and $B_n \simeq \varepsilon (1 - z \tanh z)$. For $|z| \leq 1$, we use the approximations $B_x \simeq z$ and $B_n \simeq \varepsilon (1 - z^2)$. Since $B_n (z = \pm 1) \simeq 0$, we need consider only the domain $0 \leq |z| < 1$. Equation (A3) may be then approximated as

$$\tau_b \simeq \frac{4}{\varepsilon v_{te}} \int_0^{z_t} dz \frac{[z^2 + \varepsilon^2 (1 - z^2)^2]^{1/2}}{(1 - z^2) [1 - B(z) / B(z_t)]^{1/2}} . \qquad \text{(A4)}$$

Defining $u^2 = 1 - z/z_t$, we get

$$\tau_b \simeq \frac{8 z_t^2}{\varepsilon v_{te}} \int_0^1 du \frac{1 - u^2}{1 - z_t^2 (1 - u^2)^2} . \qquad \text{(A5)}$$

If we take $B_n = \varepsilon$ everywhere, as in *Lembege and Pellat* [1982], we get

$$\tau_b \simeq \frac{8 z_t^2}{\varepsilon v_{te}} \int_0^1 (1 - u^2) du = \frac{16}{3 \varepsilon v_{te}} z_t^2 . \qquad \text{(A6)}$$

For $z_t \sim 1$, equation (A6) predicts a bounce period of $\tau_b \simeq 25 s$ (for $\varepsilon \sim 0.1$ and $v_{te} \sim 2/s$). We show below that this is much smaller than the bounce period in our asymptotic equilibria.

The integral (A5) can be evaluated exactly to give

$$\tau_b \simeq \frac{4}{\varepsilon v_{te}} \left[(1 - z_t^2)^{-1/2} \tan^{-1} \left\{ z_t / (1 - z_t^2)^{-1/2} \right\} - \frac{1}{2} (1 + z_t^2)^{-1/2} \log \frac{(1 + z_t^2)^{1/2} + z_t}{(1 + z_t^2)^{1/2} - z_t} \right] . \qquad \text{(A7)}$$

Clearly, $\tau_b \rightarrow \infty$, as $|z_t| \rightarrow 1$; this means that some electrons are lost and some have very large bounce periods. An average bounce period for the confined electrons can be obtained by averaging over the distribution function of electrons. For fixed z we have

$$< \tau_b > = \int_{-v_L}^{v_L} dv_{\parallel} \left(\frac{m_e}{2\pi T_e} \right)^{1/2} \exp\left(\frac{-m\, v_{\parallel}^2}{2T_e} \right) \tau_b \, . \qquad \text{(A8)}$$

where v_L is the maximum parallel velocity above which the electron is lost. Note that (A8) underestimates the bounce period because it should be normalized by the fraction of confined electrons which is smaller than 1. Since all bouncing electrons pass through $z = 0$, we take the distribution at $z = 0$ to be Maxwellian. We estimate $v_L = v_{te} \, [B(z = 1)/\varepsilon - 1]^{1/2}$. Asymptotic evaluation of the integral (A8) gives $< \tau_b > = 3e\pi/\varepsilon v_{te}$. For $\varepsilon \simeq 0.1$, $v_{te} \simeq 2/s$, we get $< \tau_b > = 10^2 s$ in region 1. In region 2, since B_n is space-dependent, ε is replaced by $\varepsilon' = e^{-1/2} \varepsilon/4$, and the length scale is amplified by $e^{1/2}$ (see section (2.1)). Then $< \tau_b >$ is amplified by a numerical factor of approximately $4e^{1/2}$, which gives $<\tau_b> \geq 10^3 s$. For $\gamma \simeq 10^{-3} s$, we thus have $\gamma < \tau_b > \ll 1$ in region 1, but $\gamma < \tau_b > \gtrsim 1$ in region 2.

Appendix B: Dispersion Relation For Electron Tearing Modem

In order to keep this paper self-contained, we review here the derivation of the dispersion relation for electron tearing modes using kinetic theory. The main effort lies in calculating the perturbed current J_1 from the perturbed distribution function $f_{1\alpha}$ by means of the relation

$$\mathrm{J}_1 = \sum_\alpha q_\alpha \int d\mathrm{v} \, \mathrm{v} f_{1\alpha} \, . \qquad \text{(B1)}$$

It is convenient to use (32) to write

$$f_{1\alpha} = \frac{\partial f_\alpha}{\partial \psi} \psi_1 + \tilde{f}_{1\alpha} \, . \qquad \text{(B2)}$$

We recall that [*Laval et al.*, 1966; *Galeev*, 1984]

$$\sum_\alpha q_\alpha \int d\mathrm{v} \, \mathrm{v} \frac{\partial f_\alpha}{\partial \psi} \psi_1 = \frac{c}{2\pi} \frac{1}{\lambda^2 \cosh^2(z/\lambda)} \psi_1 \hat{y} \, . \qquad \text{(B3)}$$

Hence, from Ampere's law,

$$\mathrm{J}_1 = -\frac{c}{4\pi} \nabla^2 \mathrm{A}_1 = \sum_\alpha q_\alpha \int d\mathrm{v} \, \mathrm{v} f_{1\alpha} \, . \qquad \text{(B4)}$$

we get

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) \mathrm{A}_1 + \frac{2}{\lambda^2 \cosh^2(z/\lambda)} \psi_1 \hat{y}$$

$$= -\frac{4\pi}{c} \sum_\alpha q_\alpha \int d\mathrm{v} \, \mathrm{v} \, \tilde{f}_{1\alpha} \, . \qquad \text{(B5)}$$

From the linearized Vlasov equation it follows that

$$\frac{d}{dt} \tilde{f}_{1\alpha} = -\frac{q_\alpha}{T_\alpha} \mathrm{E}_1 \cdot \mathrm{v} f_{0\alpha} \, . \qquad \text{(B6)}$$

where $d/dt \equiv \partial/\partial t + \mathbf{v}\cdot\nabla$ and $f_{0\alpha}$ is the Maxwellian distribution. Integrating (B6) along characteristics and substituting the result in (B5), we get

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\mathbf{A}_1 + \frac{2}{\lambda^2\cosh^2(z/\lambda)}\ \psi_1\hat{y}$$

$$= -\frac{4\pi}{c}\sum_\alpha \frac{q_\alpha^2}{T_\alpha}\int d\mathbf{v}\ \mathbf{v}(t)\int^t \mathbf{v}(t')\cdot\mathbf{E}_1(t')f_{0\alpha}(t')\,dt'$$

$$\equiv -\frac{4\pi}{c}\ \overleftrightarrow{\sigma}\cdot\mathbf{E}_1\,, \qquad \text{(B7)}$$

where $\overleftrightarrow{\sigma}$ is the collisionless conductivity tensor which can also be written as (see, for instance, *Horton and Tajma* [1990]),

$$\overleftrightarrow{\sigma} \equiv -\sum_\alpha \frac{q_\alpha^2}{T_\alpha}\int d\mathbf{v}\int_0^\infty d\tau\ \mathbf{v}(t)\ \mathbf{v}(t-\tau)$$

$$\times f_{0\alpha}(t-\tau)\exp\left[i\omega\tau - ik\int_{t-\tau}^{t} v_x(t')\,dt'\right]. \qquad \text{(B8)}$$

Near $z=0$, in the inner region $|\partial/\partial z| >> |\partial/\partial x| \sim 1/\lambda$. Then $\overleftrightarrow{\sigma}$ can be diagonalized in the form

$$\overleftrightarrow{\sigma} = \begin{pmatrix} \sigma_x & 0 & 0 \\ 0 & \sigma_\tau & 0 \\ 0 & 0 & \sigma_\parallel \end{pmatrix}, \qquad \text{(B9)}$$

where $\hat{x}=\hat{x}$, $\hat{\tau}=\alpha_z\hat{y}-\alpha_y\hat{z}$, $\hat{b}\equiv\mathbf{B}/B=\alpha_y\hat{y}+\alpha_z\hat{z}$ with $\alpha_y = B_y/B$ and $\alpha_z = B_z/B$. We write $E_{1\tau}=\alpha_z E_{1y}-\alpha_y E_{1z}$, $E_{1\parallel}=\alpha_y E_{1y}+\alpha_z E_{1z}$, since $\alpha_y^2+\alpha_z^2=1$.

Defining $\psi_1 \equiv A_{1y}$, $\chi_1 \equiv A_{1z}$, and writing, as in WBL, any perturbed quantity such as ψ in the form $\tilde{\psi}(z)\exp(ikx-i\omega t)$, we get

$$-\frac{c}{4\pi}\frac{d^2\tilde{\psi}}{dz^2} = \hat{y}\cdot\overleftrightarrow{\sigma}\cdot\tilde{\mathbf{E}},$$

$$= \alpha_y\sigma_\parallel\tilde{E}_\parallel + \alpha_z\sigma_\perp\tilde{E}_\tau,$$

$$= (\alpha_y^2\sigma_\parallel + \alpha_z^2\sigma_\perp)\,\tilde{E}_y + \alpha_y\alpha_z(\sigma_\parallel-\sigma_\perp)\,\tilde{E}_z\,, \qquad \text{(B10)}$$

where $\sigma_\perp \equiv \sigma_x = \sigma_\tau$. Similarly,

$$-\frac{c}{4\pi}\frac{d^2\tilde{\chi}}{dz^2} = \left(\alpha_y^2\sigma_\perp + \alpha_z^2\sigma_\parallel\right)\tilde{E}_z$$

$$+\ \alpha_y\alpha_z(\sigma_\parallel-\sigma_\perp)\,\tilde{E}_y\ . \qquad \text{(B11)}$$

The parallel conductivity $\sigma_\parallel$ can be calculated by using the drift-kinetic approximation for electrons. It can be shown that [*Drake and Lee*, 1977; WBL, 1990]

$$\tilde{J}_\parallel = i\frac{\omega_p^2}{4\pi\,\omega^2}(\omega-\omega^*)\,s^2 Z'(s)\,\tilde{E}_\parallel, \qquad \text{(B12)}$$

where

$$\sigma_\parallel = i\frac{\omega_p^2}{4\pi\,\omega^2}(\omega-\omega^*)\,s^2 Z'(s)\,, \qquad \text{(B13)}$$

where $\omega^* = kV_{dz}$, $s = \omega/(k_\parallel v_{te})$, and $Z(s)$ is the plasma dispersion function.

The perpendicular conductivity $\sigma_\perp$ is mostly due to the ion polarization drift, since the $E \times B$ drift carries no current. The full ion response [*Cowley et al.* 1986; WBL] essentially reduces to

$$\tilde{\mathbf{J}}_\perp = \frac{n m_i c^2}{B^2}(-i\omega\,\tilde{\mathbf{E}}_\perp) \simeq -i\omega\frac{c^2}{4\pi V_A^2}\,\tilde{\mathbf{E}}_\perp\,. \qquad \text{(B14)}$$

where $V_A = B/(4\pi n\, m_i)^{1/2}$ is the Alfvén velocity. Note that

$$\frac{\sigma_\perp}{\sigma_\parallel} \sim \frac{c^2}{V_A^2}\,\frac{\omega^2}{\omega_p^2 s^2}$$

$$= \frac{k_\parallel v_{te}^2}{k_0^2 V_A^2} << \frac{k^2}{k_0^2}\,\frac{m_i}{m_e} \sim 10^{-2} << 1\,, \qquad \text{(B15)}$$

for magnetotail plasmas. We then recover the result derived by WBL, i.e.,

$$\frac{d^2}{dz^2}\begin{pmatrix}\tilde{\psi}\\ \tilde{\chi}\end{pmatrix}$$

$$\simeq -\frac{4\pi i\omega}{c^2}\,\sigma_\parallel\begin{pmatrix}\alpha_y^2 & \alpha_y\alpha_z\\ \alpha_y\alpha_z & \alpha_z^2\end{pmatrix}\begin{pmatrix}\tilde{\psi}\\ \tilde{\chi}\end{pmatrix}. \qquad \text{(B16)}$$

We now introduce two characteristic frequencies. One is $\omega_e \equiv k_\parallel v_{te}$. The other is $\omega_\varepsilon \equiv \tau_\varepsilon^{-1}$, where τ_ε is the time it takes for an electron to travel a characteristic distance k^{-1} along x [WBL]. From the field-line equation

$$\frac{dz}{B_z} = \frac{dx}{B_x}, \qquad \text{(B17)}$$

it follows that if an electron travels k^{-1} along x, it must travel z_0 along z, where z_0 is given by the relation

$$z_0^2 = \lambda^2\,[1-\exp(2\varepsilon/k\lambda)]\,. \qquad \text{(B18)}$$

We then obtain [WBL]

$$\tau_\varepsilon = \int_0^{k^{-1}} \frac{dx}{\bar{v}_{ex}} = \int_0^{z_0} \frac{dx}{\bar{v}_{ez}} ,$$

$$= \frac{\pi}{2\varepsilon\, v_{te}} \exp(\varepsilon/k\lambda) . \qquad \text{(B19)}$$

We introduce the strained inner variable

$$\xi = \frac{z}{\delta_e} \equiv \frac{1}{s} , \qquad \text{(B20)}$$

where $\delta_e \equiv (\omega/k\, v_{te})\ (L_s/s_0)$ and $s_0 = 1$ if $\omega_e > \omega_\varepsilon$ (as in region 3), but $s_0 = \omega_b/\omega_e$ if $\omega_e < \omega_\varepsilon$ (as in region 2). The shear length L_s is defined by $L_s \equiv \lambda\, B(z = 0)\, B_0$. With these definitions, (B16) can be rewritten as

$$\frac{d^2}{dz^2} \begin{pmatrix} \tilde{\psi} \\ \tilde{\chi} \end{pmatrix}$$

$$= f(\xi) \begin{pmatrix} \alpha_0^2 & \alpha_0\varepsilon_0 \\ \alpha_0\varepsilon_0 & \varepsilon_0^2 \end{pmatrix} \begin{pmatrix} \tilde{\psi} \\ \tilde{\chi} \end{pmatrix} , \qquad \text{(B21)}$$

where $\alpha_0 \equiv B_y(0)/B_0$, $\varepsilon_0 \equiv B_z(0)B_0$, and

$$f(\xi) \equiv \frac{(1 - \omega^*/\omega)\, \delta_e^2\, \xi^{-2}\, k_0^2}{\alpha_0^2 + \varepsilon_0^2}\, Z'(\xi^{-1}) , \qquad \text{(B22)}$$

which is the same as (77) of WBL.

Equation (B21) can be diagonalized as

$$\frac{d^2}{d\xi^2} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = f(\xi) \begin{pmatrix} \alpha_0^2 + \varepsilon_0^2 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} , \qquad \text{(B23)}$$

where

$$a_1 = \alpha_0\tilde{\psi} + \varepsilon_0\tilde{\chi} , \qquad \text{(B24)}$$

$$a_2 = -\varepsilon_0\tilde{\psi} + \alpha_0\tilde{\chi} . \qquad \text{(B25)}$$

As shown by WBL, $a_2\, (\chi = 0) = 0$, which yields

$$\tilde{\psi}(0) = (\alpha_0/\varepsilon_0)\, \tilde{\chi}(0) . \qquad \text{(B26)}$$

The solution for a_1 is different in different regions.

In region 1, where $\gamma\tau_b << 1$, the integration over z involves a bounce average which extends over the outer region. We can then see the stabilizing effect of this bounce by noting that it tends to cancel out the perturbed current in the inner region and gives $\tilde{\psi} = \tilde{\chi} = 0$. In regions 2 and 3, however, we have $\gamma\tau_b \geq 1$ and $\gamma\tau_b >> 1$, respectively. Integrating (B23) over the inner region, and using the "constant ψ" approximation, we get the jump condition

$$\frac{da_1}{d\xi}\Big|_{\text{inner}} = (\alpha_0^2+\varepsilon_0^2)\, a_1(0) \int_{\text{inner}} d\xi\, f(\xi). \qquad \text{(B27)}$$

In order to match the inner region to the outer region, we use the relations

$$\Delta_1' \equiv \frac{d}{dz}\log\tilde{\psi}_{\text{outer}}\Big|_{0-}^{0+} = \frac{d}{dz}\log\tilde{\psi}_{\text{outer}}\Big|_{-\infty}^{+\infty}, \qquad \text{(B28)}$$

$$\Delta_2' \equiv \frac{d}{dz}\log\tilde{\chi}_{\text{outer}}\Big|_{0-}^{0+} = \frac{d}{dz}\log\tilde{\chi}_{\text{innerr}}\Big|_{-\infty}^{+\infty} \qquad \text{(B29)}$$

The left-hand side of (B27) can then be written as

$$\frac{da_1}{d\xi}\Big|_{\text{inner}} = \delta_e \frac{da_1}{d\xi}_{\text{inner}}\Big|_{-\infty}^{+\infty} = \delta_e \frac{da_1}{d\xi}_{\text{outer}}\Big|_{-\infty}^{+\infty}$$

$$= \delta_e\left[\alpha_0\,\tilde{\psi}(0)\,\Delta_1' + \varepsilon_0\,\tilde{\chi}(0)\,\Delta_2'\right]. \qquad \text{(B30)}$$

The right-hand side of (B27) gives

$$\int_{\text{inner}} d\xi\, f(\xi) = \int_{-\infty}^{+\infty} d\xi\, f(\xi) = 2\int_{\infty}^{0} ds\,\frac{d\xi}{ds}\Big|_{\xi=1/s} f(s^{-1})$$

$$= -\frac{2\delta_e^2\, k_0^2\,(1-\omega^*/\omega)}{\alpha_0^2+\varepsilon_0^2}\, Z(0)\ . \qquad \text{(B31)}$$

Using (B30) and (B31), we obtain the dispersion equation

$$D(k,\omega) = \frac{k\, v_{te}\, \Delta' S_0}{2k_0^2 L_s\, \omega} + \left(1-\frac{\omega^*}{\omega}\right) Z(0) = 0\,, \qquad \text{(B32)}$$

where

$$\Delta' \equiv \frac{\alpha_0^2\Delta_1' + \varepsilon_0^2\Delta_2'}{\alpha_0^2+\varepsilon_0^2}. \qquad \text{(B33)}$$

Equation (B32) is slightly different from the analogous equation derived by WBL. The difference can be attributed primarily to a slightly different treatment of the asymptotic matching condition; here we have extended the outer solution all the way to $z = 0$ instead of taking it to the outer limits of the inner region. In the rotation of WBL, $\Delta' = \Delta_r' + i\,\Delta_i'$; however, if we recall that $\Delta_i' \sim \varepsilon << \Delta_r'$, it is clear that the results derived by WBL and here do not differ significantly. (In order to avoid possible confusion, we point out that ω^* has been neglected in certain results derived in WBL; this affects the real part of ω, but not the growth rate.) Solving the dispersion equation (B32), we obtain ω_R and γ, given, respectively, by (103) and (104) in region 2 and (105) and (106) in region 3.

The growth rate derived from kinetic theory can also be obtained from the fluid model. The fluid analog of (106) has already been derived in section 3. Here we derive easily the anolog of (104). Recalling the heuristic discussion given by WBL (see section 3, WBL), we have $\gamma \approx \omega_\xi \equiv \omega_\varepsilon k_\parallel v_{te}/\omega_e = \omega_\varepsilon k d_e/(\lambda \omega_e)$. Here $d_e = \Delta'_r/2k_0^2$, which is obtained from (68), with Δ'_r given by (88). If we now use (B25), we get (110) which is larger than the growth rate derived from kinetic theory by the factor $\pi^{1/2}$.

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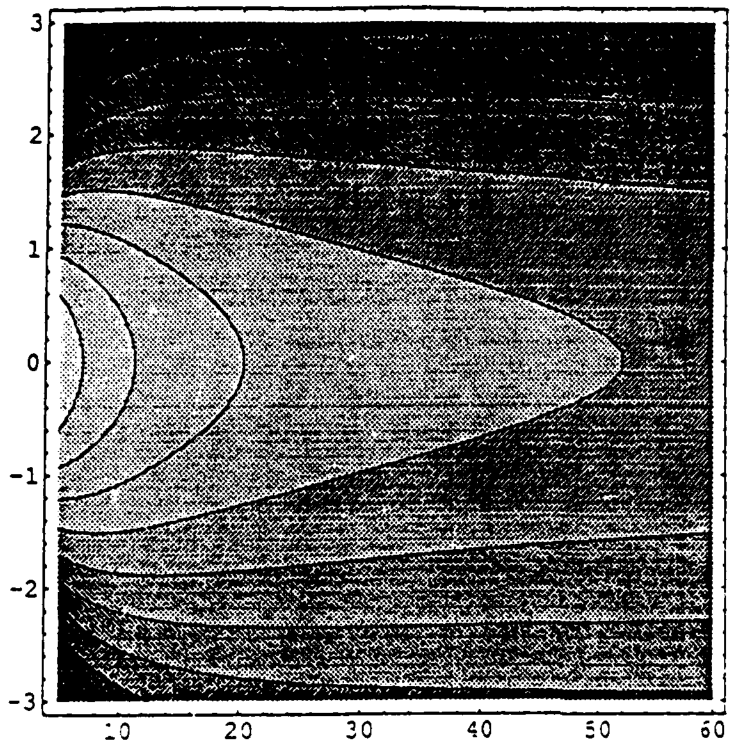

Figure 1.