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Two Russian Professors, Dr. Krasnoschekov and Dr. Savin, met with several faculty members at San Diego State University to discuss combat modeling, including a complete derivation and discussion of the space-time combat model. Also included in the discussions, were a history of the recent development of combat modeling in the former Soviet Union, a discussion of systems of models and models for systems for representing complex, technical processes, a discussion of operations research techniques and applications to combat modeling problems, a description of a methodology for the optimal design of complex systems, including design of combat aircraft, and several other models that they were interested in explaining.

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FINAL TECHNICAL REPORT

RECENT DEVELOPMENTS IN SOVIET COMBAT MODELING

US ARO Grant No. DAAL03-92-G-0403

by

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BRIEF SUMMARY OF RESEARCH FINDINGS

Professors Krasnoschekov and Savin arrived on schedule in San Diego on 21 September 1993. Beginning that week, they and several faculty members at San Diego State University began a series of regularly arranged seminars. The seminars met generally three days per week for about 2-3 hours for each session. In between the planned seminars, there were informal meetings to discuss more about particular details of their presentation as well as several meeting devoted to the topic of how their understanding of the space-time combat model compared to the assumptions we made about it previously.

The topics for the seminars were mutually agreed upon and contained a discussion of combat modeling, including a complete derivation and discussion of a the space-time combat model, a history of the recent development of combat modeling in the former Soviet Union, a discussion of systems of models and models for systems for representing complex, technical processes, a discussion of operations research techniques and applications to combat modeling problems, a description of a methodology for the optimal design of complex systems, including design of combat aircraft, and several other models that they were interested in explaining. The seminars were presented in an open, non-formal atmosphere with much discussion and interaction between the participants. Some problems of semantics were encountered between their terminology and ours, but after much discussion we believe we achieved a complete understanding of what they presented to us.

In general, it was perceived that they presented a complete and honest account of their involvement with combat modeling, which they claim ended about 1979. Since that time they have mainly worked on problems associated with the design of technical systems. At the beginning, we did not make a connection between the combat modeling problems associated with the particular space-time model and the systems of models and hierachical structure of models presented in later seminars. Eventually it became apparent that the more complex systems of models are precisely what is needed to close the model equations in the space-time combat model and determine tactics which are in some sense optimal and feasible. It is now believed that this more extensive and elaborate system of models forms the basis for the staff-level model developed in the former Soviet Union in the 1970's.

This report is extracted from a more complete version of a report of an on-going and more extensive project that was sponsored by USA TRAC-WSMR through the Scientific Services Program and funded through Battelle. In addition to what is described here, that version also contains a listing of a program for solving the model equations and an analysis of numerical results The complete version can be obtained by request through Mr. Peter Shugart at TRAC-WSMR.

1. BACKGROUND AND PURPOSE OF THIS REPORT

In the early 1970's, a space-time combat model was described by Yu. Chuyev[1] for homogeneous, large-scale forces on a two-dimensional battlefield. This model is a natural extension of Lanchester's equations to the case when the forces are not uniformily distributed on the battlefield, but may be concentrated in various regions according to density functions which measure force-equivalents per unit area. Since that time, such space-time combat models have been cited regularly in the Soviet military operations research literature and various attributes of the models have been claimed. Among these are that the models provide a rational basis for selecting optimal troop formations and that the outcome of the models conform well with certain historical data. In Chuyev's works, the model was never completely described or carefully derived, so that the exact assumptions could not be ascertained. Also, a procedure for solving the model equations was not given and it was also not clear which quantities in the models should be considered as unknowns and which are parameters.

In a definitive book on model construction that appeared in 1983, P. S. Krasnoshchekov and A. A. Petrov [2] describe a space-time combat model of Chuyev's type. They provide a complete derivation of the model based on principles from fluid dynamics and continuum mechanics and also give a thorough and scientifically sound discussion of the interpretation of the quantities in the equation. This account provides much better insight into the assumptions behind the model and to its capabilities and limitations. There is no doubt that Chuyev and Krasnoschekov-Petrov are speaking about the same class of combat models. From now on this type of model will be called a *space-time model for combat interactions* . While Krasnoschekov's model is clearly a generalization of Chuyev's, they have the same essential characteristics and both are examples of a class of differential equations studied originally by L. Euler in the beginning of the 18th century.

In the first phase of this project, a numerical procedure (algorithm) was constructed for solving the partial differential equations of Chuyev's model in the case of a one-dimensional battlefield. The goal in this phase of the project is to extend this procedure to the two-dimensional case and provide an algorithm capable of solving the model equations on a PC or similar sized computer with a reasonably fast turn-around time. To do this several problems needed to be addressed. Some of these dealt with further assumptions necessary to complete the model.

While the description of the model in the book of Krasnoshchekov-Petrov is far more substantial than Chuyev's initial account, it does not provide everything needed to run and test the model. These assumptions are related specifically to the functional forms for the velocity and attrition functions. The other major problems concerned developing numerical procedures that will solve the equations with reasonable accuracy and time. This problem is also dependent on how accurate the solutions need to be for certain applications and how quickly the solutions need to be computed. A subsequent problem that needs to be addressed is the stability of the model with respect to changes in the data and sensitivity of the model to non-smooth data.

During the course of work on this project, a unique opportunity arose. This was created by a suggestion from Peter Shugart of TRAC-WSMR that it might be possible to invite P. S. Krasnoschekov himself to the USA to critique our work on the model and learn more from him about the development of combat modeling in the former Soviet Union. This additional project was funded by a grant through the US Army Research Office.. A letter was sent to Krasnoschekov in the summer of 1992, inviting him to come to San Diego for a period of 3 months during the fall of 1993 for the purpose of discussing his work on modeling of combat.

He responded that while he would very much like to come, his knowledge of English was essentially non-existent, he had never before traveled to a Western country, and therefore was reluctant to accept our invitation. He then suggested that a colleague of his at the Academy of Sciences, Gennadiy I. Savin, was also knowledgeable about combat modeling, had a good command of English, and the two would be willing to come for 2 months with no additional funding required; they would split the stipend that was originally intended for Krasnoschekov alone. Since this gave us 4 man-months for the cost of the originally estimated and budgeted 3, it was acceptable to the Army and we agreed to the counter-proposal.

In September of 1993, P.S. Krasnoschekov and G. Savin arrived in San Diego to begin their stay. During the course of it, they presented an intensive schedule of seminars on modeling interactions and also modeling large scale complex systems. It turned out that the latter topics also have significant application to modeling large scale military operations. This, as it turned out, provides not only the structure and logical apparatus in which the combat environment was modeled, but also provided a means for "closing" the original combat model, which was the source of our interest. Thus we not only learned more about the Chuyev model, but where and how this model fits into the decision-making process at the staff and operations level.

In addition to these formal seminars presented by Krasnoschekov and Savin, there were extensive informal discussions of the Chuyev model and Krasnoschekov's role in its development Krasnoschekov gave us a personal history of his involvement with combat modeling from 1964 until 1979 and revealed to us a great deal about how the personalities involved interacted and the work proceeded. He also critiqued our approach, both with regard to assumptions we had made to close the model as well as his view of the difficulties in using numerical algorithms to calculate solutions. We also critiqued some of his views and together came to a better understanding of some of the critical problems for this model.

The purpose of this report is to first discuss the impact of the newly acquired information, to give a current status in the space-time model, specifically where we now stand with regard to the assumptions made to close the model, and where and how the resulting solutions should be used. As a result of the seminars, there are approximately 300 transparencies which were left with us, many in the original Russian, and a manuscript for a book (also in Russian) for further models for long range combat. Also, a log was maintained of the daily activities in the seminar and our impressions of what was presented. This log is included in the complete version of the report supplied to USA-TRAC.

2. HISTORY AND RECENT DEVELOPMENT OF COMBAT MODELING IN THE FORMER SOVIET UNION ACCORDING TO P.S. KRASNOSCHEKOV

Professor Pavel S. Krasnoschekov is currently an Academician in the Computing Center of the Russian Academy of Sciences in Moskow, its Deputy Director, and also Head of Department of Mathematical Modeling of Systems and Decisions. This places him in the uppermost echelon of academics in Russia and in an influential position as the chief of from 800 to 1000 mathematicians, physicists, and computer scientists involved in modeling a wide variety of activities from economic to physical to military. He has been intimately involved in the development of the so-called continuous media, time-space model for combat interactions since its inception. This involvement is attested to in N. N. Moiseyev's book [3] and it would not be unrealistic to point to Krasnoschekov as Moiseyev's hand-picked successor.

In the course of discussions of combat modeling in the former Soviet Union and how it evolved, the following history was compiled. It reflects, of course, some of Krasnoschekov's prejudices and biases, but based on other collateral evidence, it seems to be an accurate summary.

1964: Krasnoschekov received his Ph.D. from Moskow's Steklov Institute, majoring in classical applied mathematics, specifically gas and fluid mechanics. He was then recognized by Moiseyev and invited to join the Computer Center of the Soviet Academy of Sciences (CCAS), where the monetary rewards far exceeded that from other academic jobs.

1965: Krasnoschekov began work at the CCAS. His main jobs were to work with Petrov on economic modeling and with a group lead by Moiseyev on military modeling.

1966: In a seminar at the CCAS, Lebedev presented some empirical results and a conjecture on the relationship between the rate of movement of the front line and the ratio or correlation of forces, which was based on some historical data, some from the Napoleonic wars. In the ensuing discussion, Krasnoschekov indicated that a factor is missing in Lebedev's conjecture and in the presence of Chuyev and Kuzmin suggested that there should be a continuous, space-time model for combat, analogous to the Euler equations of fluid mechanics, which might account for such relationships.

1967: In another seminar at the CCAS, Chuyev and Kuzmin wrote down "their" equations, which turned out to be analogous to the well known Euler differential equations of fluid mechanics. Upon hearing that there was no derivation, justification, or even complete description or understanding of these equations, Krasnoschekov soundly criticized Chuyev and Kuzmin. After being silenced by Moiseyev, Krasnoschekov began work on a systematic derivation of some equations for a continuous media, space-time model for combat using some principles analogous to the development of continuum mechanics from the Boltzmann equations. After deriving these equations, Krasnoschekov was able to derive a functional relationship similar to Lebedev's as a first approximation to the relative rate of motion of the frontline, subject to certain simplifying assumptions.

1968: As a result of this work, which was judged the best of the year from the CCAS, Krasnoschekov was invited to present his results to the Presidium of the Academy of Sciences.

1970: Krasnoschekov was surprised by the appearance of Chuyev's book [1] in print for two reasons. First, because he had thought that the equations were in some sense "classified", and secondly because there was no real justification and the model described was as incomplete and ambiguous as Chuyev's and Kuzmin's original presentation back in 1967.

1971: G. Savin received his M.S. degree under Krasnoshekov's direction which began their joint involvement in military operations research.

1975: Krasnoschekov and Petrov began their book [2] on Mathematical Modeling, based on lectures at the CCAS and Moskcw State University. Krasnoschekov used as justification for his chapter on combat modeling (modeling of conflict interactions) the fact that Chuyev already published the equations in the open literature.

1977: G. Savin completed his Ph.D. under Krasnoschekov's direction. This involved Systems of Models for large-scale, complex processes with applications to combat modeling. It reflected a new direction taken by Krasnoschekov in his research, which involved a greater use of operations research techniques and a lesser role on classical applied mathematics and differential equations.

1979: As a result of a Ph. D. thesis by a military graduate student of Savin, an implementation of the System of Models approach was made for operations at the Theater level. This required coding about one million lines of ALGOL and involved some extensive trials. A copy of this model supposedly still exists in the CCAS and is used from time to time. In one allusion to the model, Krasnoschekov told us about a trial pitting the model against a staff of "experts" (using their traditional approach), who both analyzed and planned an operation. The experts arrived at a different solution, which Krasnoschekov proved was neither optimal nor feasible, whereas using the model provided an "optimal" variant in some sense which the experts later acknowledged. After this, Krasnoschekov and Savin claimed to lose interest in combat modeling due to lack of funding by the military and because of opportunities to use their theory for projects in the design and control of complex technical processes. For this work and its application to the design of the SU-27 aircraft, Krasnoschekov received a prize as Hero of Soviet Workers.

1992: Krasnoschekov receives offer to visit San Diego to consult on combat modeling problems. He first believes this to be practical joke by some of colleagues in the CCAS.

1993: Krasnoschekov and Savin arrive in San Diego and begin again, after a period of 14 years, some joint work and discussions on combat modeling problems.

3. CURRENT STATUS OF THE SPACE-TIME MODEL.

The equations which Chuyev and Kuzmin wrote down and also the ones derived by Krasnoschekov can be classified as Euler-type differential equations. There are several ways to derive such equations for a combat model. One way is to think of the amount of force-density moving into a small region, the amount moving out, and the amount destroyed due to enemy action. Another way is to proceed from the discrete, microscopic level to the continuous, macro level by stochastic averaging, similar to the derivation of the fluid equations in continuum mechanics. A special advantage of the latter approach (followed by Krasnoschekov) is that the attrition-producing terms in the macro-level equations are a direct consequence of the assumptions made at the micro-level and even the parameters which enter into the continuous media equations can, in principle, be traced to quantities and assumptions concerned with the attrition producing terms at the micro-level. Even more importantly according to Krasnoschekov, is that certain "averaging parameters" naturally enter into the final result, which influence the resolution and interpretation of the validity of the solutions of the continuous media equation. For a derivation of the equations, see Chapter 13 of [2].

In addition to the so-called Euler equations for homogeneous combat units, Krasnoschekov has also discussed analogous equations in the case of a mix of heterogenous forces. The equations look rather similar and this writer does not understand any real theoretical distinction between the two cases, since all quantities and parameters in the equations can also depend upon position. As long as the assumption that no two distinct types of forces could simultaneously occupy the same ground, it is possible to rewrite the Euler equations from a heterogeneous situation into a homogeneous one by just renaming the density functions. But this may not necessarily be so practical or convenient, because the heterogeneous model would display a natural and important decomposition of forces and means. Here, we will treat only the case of homogeneous forces, but these methods, techniques, and analysis can be directly extended to the more general (in some sense) case.

The Euler equations for the continuous media, space-time homogenous equations have the general form:

$$\partial\rho/\partial t + \mathrm{div}\,(\rho\, U) = -\, f_e$$

$$\partial\rho_e/\partial t + \mathrm{div}\,(\rho_e\, U) = -\, f \quad .$$

In these equations, ρ and ρ_e represent the density of forces per unit area, with the subscript "e" denoting "enemy" forces. They are assumed to depend upon time t, and the spatial variables x and y. The quantity U is the velocity vector of the field, having two components in general and the symbol "div" denotes the divergence of the vector field. Lastly, the quantities f and f_e on the right hand sides are known as the attrition- or destruction- producing terms. Without them, the equations would represent the flow of two non-viscous, compressible fluids without sources or sinks. These are essentially the same equations written down by Chuyev and Kuzmin, but they expanded the divergence term using the usual definition as a sum of partial derivatives with respect to Cartesian coordinates x and y. There are good practical reasons for not doing this, however, and treating the divergence as a single expression rather than a sum. This will be discussed again in the section on numerical algorithms. After having written down these equations, there are a number of natural problems which arise immediately:

a. What assumptions, beyond those mentioned already above, are implicitly made about the combat model when these equations are used?

b. How can the model or the equations be "closed"?

c. How can the equations be solved?

d. What do the solutions mean?

e. Where and how should this information be used?

In the remainder of this section, these problems will be addressed as well as a discussion of Krasnoschekov's critique of what we had previously done and our response to the issues he raised.

a. The Euler Equations.

There are no extra or hidden assumptions in the equations beyond those mentioned above. They simply represent "conservation laws" for the forces involved. Not even the assumption that the forces are separated is implicit in the equations. The real assumptions involve the next step of "closing" the model, which is where the real work is involved.

b. Closing the Model.

To close the model means to add assumptions or extra conditions so that the equations will have one and only one solution, which represents the averaged densities of the forces as combat develops. There are several such assumptions or specifications to be made. A main one is to decide what the attrition-producing terms depend upon and to determine how to calculate them. A second assumption concerns the velocity vector U of the flow and how to obtain it. A third concerns choosing appropriate boundary and initial conditions on the density functions. Krasnoschekov's derivation produces a function of the form

$$f(t, x, y, \rho, \rho_e) = c \iint P(\text{hit at } (x,y))(\text{rate of fire})(\text{average of firing units})\, dA.$$

This is a symbolic formula, but it does contain some assumptions. One is that the number of casualties is directly proportional to the number of enemy units firing at a particular point. The double integral just means that the units which can fire on position (x,y) are averaged with respect to their target priorities and rates of fire, with the casualties in this homogeneous situation then just being a constant times this amount. The constant is sometimes called the combat effectiveness coefficient and corresponds to the probability of a kill, given that the target was hit. This constant also needs to be determined in closing the model and how this can be done is briefly discussed in the section on Systems of Models.

The next quantity that needs to be determined is the velocity of the flow. According to Krasnoschekov, this must be treated as a control parameter in the model. The reason for this, as opposed to situations in physics, is that while nature usually acts in some optimal way, there is no reason to assume the same about combat, where human decisions are a major factor. A force, even with great numerical superiority, could decide to stay put or even withdraw according to the commander's wishes. So there is no natural way to determine what this will be and the equations themselves do not yield any information about this. To determine what the velocity should be, it is necessary to "synthesize" a variant that meets the assigned objectives of the operation. Moreover, the selection of the variant should be accomplished in a way that meets certain optimality criteria identified by the commander. More about this will be said in the next section on Systems Of Models.

At this point it is germain to mention Krasnoschekov's work on Lebedev's conjecture. What he actually showed is that if forces are being "pumped" toward the front in such a way that no separation or overrunning of the two forces is allowed (i.e., a unique "frontline" is maintained), then (in the first approximation, modulo some other conditions) the front will move at a rate proportional to the rate at which the forces of the attacker are moving toward the front, where the proportionality factor depends upon the correlation of forces. The functional relationship is given by a so-called "Witch of Agnesi" curve. (See [2], Chapter 13, for a proof of this statement.)

The expression given by Chuyev for main component of the velocity vector is essentially of this type, but with an incorrect interpretation. In Krasnoschekov's derivation he obtains an expression for the *relative* velocity of the frontline with respect to the rate at which the forces are being pumped toward it, whereas Chuyev claims that the forces themselves should (by some unknown reason) move with a velocity that is proportional to the maximum that the force would be capable of moving if it were completely unopposed. Krasnoschekov said this statement has no validity and is utter nonsense.

Finally, after the attrition producing terms and velocity vectors have been determined, the zone of combat should be specified (say with fixed lateral boundaries) and initial densities distribution of both forces at the beginning of combat should be specified. There are natural smoothness conditions that should be observed for the initial densities just because the equations contain partial derivatives, which must exist, but also for the solution to be computable it is necessary that the densities not be too wild.

c. Solving the Equations.

There are two possiblities for solutions of the equations, once the model has been closed properly. One possibility is to obtain a closed-form, explicit solution, say as some elementary function. For such equations such explicit solutions are highly unlikely. Nevertheless, Krasnoschekov has found some in the one-dimensional (only one spatial component) case under some very stringent assumptions on the shape of the density functions, the probability of a hit (target distribution function), the rate of fire, and the rate of movement of the forces. These he calls "analytic" solutions, which is not consistent with Western terminology, which would use the same term in another sense. Be that as it may, it is interesting that such explicit solutions exist. Briefly, what is needed is that the densities are decreasing exponentials and the other quantities in the attrition function are such that the solutions will also be decreasing exponentials (both as one moves away from the front as well as in time). Maybe this is somehow expected because in the spatially independent case of Lanchester's equations, the solutions are also decreasing exponentials. Krasnoschekov values these solutions greatly because they enabled him to show Lebedev's relation, but in our opinion he has placed too much emphasis on them. When he was asked what to do in other cases (when the densities are not like this) he first responded that one could make an expansion using a sum of such exponentials. But when we disputed this claim, saying that the required calculations would even be more difficult than a numerical solution, he recanted and was not so certain. Also, it was pointed out to him that while it worked luckily in one spatial dimension, it was much more unlikely to work in two, without some even more stringent and un-motivated assumptions. So it is by no means clear that this approach can be of any help in general.

An alternative is to compute solutions numerically, that is at certain grid points in space and time. This is usually the most that one can expect for solutions of partial differential equations, especially ones of the above type. All numerical procedures basically involve replacing the differential equations by a system of algebraic equations with a finite number of unknowns. Some ways to bring this about is to replace the derivatives by certain differences and the integral by some finite approximation. There are many different ways to do this and most will lead to undesirable situations which are either unstable numerically, ill-conditioned, or both. Moreover, most methods will suffer from the fact that while the exact differential equations are conservation laws, the approximate system of equations, in general, will have solutions that no longer conserve total mass. Depending upon whether the approximate equations immediately determine the updated values of the solutions, or whether one needs to solve the system of equations to obtain the new values, the method is called either "explicit" or "implicit".

An advantage of explicit methods is speed of calculation, whereas implicit methods can take a great deal more time since the solution of the system of equations at each iteration involves an extra step. The amount of work increases significantly with a decrease in the grid-size and the resolution of the model.

The first approach we used in the one spatial dimensional case was based on the Lax-Wendroff approach, an explicit numerical method. The algorithm obtained is especially fast and requires very little storage, however it some serious drawbacks. A major one is that to make the method stable, it is convenient to link the time step size with the spatial step size. This is not a problem with one spatial dimension, but does not extend naturally to two dimensions. Another is that it does not conserve mass very well and it is also quite sensitive to perturbations in the data.

The second approach, which works quite nicely for two spatial dimensions and has relatively good numerical behavior is based on the Crank-Nicholson method. This is an implicit method that requires solving a system of linear equations to obtain the updated densities. In order to make the calculations fewer and quicker, a modified method was developed which uses the full scope of the Crank-Nicholson approximation for derivatives in the direction of the main component of the velocity vector, but uses a simplified approximation for other, lesser, component. This results in a tridiagonal system of linear equations which can be solved using a very fast and efficient algorithm. Another simplification of the procedure is to calculate the attrition term using only values of the densities at the previous time-step, as opposed to the current time step. As long as the time interval is sufficiently small, for example, smaller than the average time between firing and its effect of creating a casualty, then such an assumption seems reasonable and greatly reduces the cost of calculating the up-dated solution.

Some extensive tests were made using this algorithm and it proved to be an acceptable means for calculating the solutions within a certain degree of accuracy and with relatively good conservation of mass properties. This means that the algorithm could be valuable in the so-called "synthesis" mode of the Systems of Models, where a relatively good and fast calculation is required, but is probably not good enough for the "analysis" or reproduction mode, which requires more accuracy with not nearly the emphasis on speed of calculation. This was discussed with Krasnoschekov and he concurred with that assessment.

In the 1980's another method which gives rise to numerical algorithms for calculating solutions of certain types of partial differential equations was developed in the former Soviet Union, particularly at the Computing Center of the Academy of Sciences. it is especially designed to work very well on equations which are composed of expressions such as the divergence operator, and in particular, equations that are of conservation type. A main feature of the method is that it is specifically designed so that the difference scheme also conserves mass. (Most difference schemes such as those discusses above will not satisfy this property.)

The basic idea is that to generate a difference scheme, all the quantities in the differential equations are first rewritten in terms of some "basic" operators, such as the divergence operator, using some standard types of vector-calculus identities Then each one of these basic operators is approximated, not by the usual difference type scheme that is associated with its representation as a derivative, but instead with an approximation arising from its corresponding invariant integral representation. This creates, of course, a significantly more complex coefficient matrix for the system of linear equations to be solved. The are many ways to choose grid points for the integral approximation and this is where the developer can have some positive or negative impact on the result.

Another "variable" in the method is to choose an algorithm for solving the system of linear equations which is compatible and well suited to the particular structure of these equations. Both of these choices can have a substantial impact on the overall accuracy of the method and the speed of calculation. While on a supercomputer, these problems may not be significant for equations of the type we are considering, to make the calculations on a slower and smaller computer requires that an extra effort be made on the numerical analysis before the arithmetic operations are performed. It is exactly this type of analysis which was and still is being emphasized in the former Soviet Union. The development of a numerical algorithm based on this so-called "support operator" approach was beyond the scope of the currently funded project when we learned about it and its relation to the solution of differential equations of the type being considered. Not only would the algorithms themselves have to be redesigned, but this would require completely new subroutines to be coded, tested, and evaluated.

d. Meaning of the solutions.

According to Krasnoschekov's derivation, either the exact, analytic solutions of the equations or their numerical approximations represent "averaged densities" of the forces with respect to certain time and space averaging parameters. The meaning of the averaged densities is that of expected value. The averaging parameters come about and can be calculated based on the explicit assumptions and form of the attrition terms. For example, the sizes of the regions for effective fire, the accuracy parameters for the weapons, and the p obability of a kill given a hit will influence them. These averaging parameters restrict considerably the range of validity in which the solutions can and should be used as faithful representations of the combat situation. Krasnoschekov is aware of the debate over whether a deterministic model can represent the expected value for the solutions of a stochastic model. He claims within the range of validity specified by the averaging parameters, the solutions of the deterministic model are indeed the expected values. On the other hand, he did not show how to prove this or give a probabilistic error estimate for the variance from the expected value.

e. Using the solutions.

The equations for the space-time model should be used, according to Krasnoschekov, in a "narrow band" of the front line, also called the zone of "nearest interaction", where forces fire and manuever continuously and all forces are within effective range of the opposing forces. This does not mean that the equations are invalid for any particular reason farther away from the front line, just that they are unnecessarily complicated, and simpler, nonpartial differential equations may be used instead there to obtain equivalent results. These other zones of combat and the equations will be discussed in the next Section on Systems of Models. The particular parameters associated with this zone of nearest interaction are not fixed, but depend on tactics and accuracy of weapons. Krasnoschekov indicated that in the 1970's, this zone of nearest interaction was considered to be only several (maybe 3-5) kilometers wide for a combat model at the Front (theater) level. This seems to be rather narrow by modern day standards of weapons accuracy and mobility, but nevertneless it indicates that the depth of forces is much less than previously imagined. For combat at lower levels, the depth of this zone of nearest interaction will generally decrease.

SUMMARY OF KNOWLEDGE OF THE SPACE-TIME COMBAT MODEL:

1) The equations should be used for "nearest interaction" combat. For combat in the other zones, less complicated models are used.

2) Some of the assumptions that were used to complete the model are consistent with Krasnoschekov's approach, but his idea is that the velocity and some of the quantities entering the attrition function should be treated as control variables. For simplicity some of these were taken as constants in closing the model in our procedure, but it would be an easy modification to put in any other expressions, say for the firing rates and target distribution functions.

3) The differential equations can be solved explicitly in one spatial dimension and under certain very restrictive assumptions on the density functions. This does not seem to be possible in two dimensions and for other types of density functions an expansion or approximation technique would likely be as complicated or more so than solving the equations numerically.

4) A numerical algorithm for solving the equations based on the Crank-Nicolson approach has been coded and is operational. The results are good, but do not reflect the required conservation of mass that one would need for a high resolution application. A better approach for this purpose is now known, but it has not been applied to solve these problems.

4. OVERVIEW OF MODELS OF SYSTEMS AND SYSTEMS OF MODELS

As mentioned in Section 3, the space-time model for combat is just one of a set of models to depict combat losses at the front (theater) level. Krasnoschekov has proposed decomposing the area of operations into three zones of combat with corresponding models appropriate to those zones. The first is the zone nearest the frontline, called the zone of *"nearest interaction"*, in which all forces are more or less equally vulnerable to other similar forces and forces fire and manuever simultaneously. This would include mainly various types of infantry units and tank units. For this zone either a homogeneous space-time model as described above in Section 3 or a heterogeneous analog of it could be used. The heterogeneous analog was described in other works by Krasnoschekov which were presented to us.

The next closest zone of combat is called the zone of *"medium interaction"* and includes longer range artillery and combat forces which are being brought forward to the zone of nearest interaction, but which do not necessarily have the means of creating casualties at that range, only sustaining them. Principally they move forward according to some scheme of manuever and eventually reach the closest zone after sustaining some losses. In this zone, fire and manuever can be treated independently and for the purpose of modeling this kind of attrition, a system of generalized Lanchester equations is suggested. These are used to portray counter-battery fires, fires of artillery against infantry and tank units, and any other types of longer range interactions. Some degree of interdiction against support units would also be included here. In this zone a major role would also be played by an optimization model to maximize the effect of the artillery, for example, on the available targets given the available resources.

The zone of *"farthest interaction"* corresponds to very long range artillery or rockets using weapons of mass destruction (i.e., nuclear fire) for fire against significant targets in the nearer zones. For this purpose, and because these casualties take place in a short time interval, a relatively simple one-sided attrition model may be used to calculate the effects of this level of combat. Another type of model Krasnoschekov indicated was used here was a decision model based on game theory to decide the optimal times to use such weapons, given probable consequences for initiating the use of them.

In addition to these kinds of models, a large number of supporting sub-models are required, for example, to calculate movement of forces, supply, to account for variable factors such as weather, terrain, and also uncertainties caused by lack of information.

The natural problem arises of how to construct a comprehensive system so that all the models at the various levels can interact appropriately with each other and be used to plan and predict the outcome of a large-scale operations. In particular, such a system would be used to select a variant of a plan that meets certain criteria specified by the commander, determine the missions of subordinate units, and accomplish this in a manner which is in some sense "optimal", based on criteria established by the decision maker. This is a problem in the design and control of large-scale complicated technical systems, which was a much studied subject in the 1970's in the former Soviet Union due to a large variety of potential applications for industrial processes, centralized economic planning, and military operations. A solution involves what Krasnoschekov and Savin call a *System of Models and Models of Systems.* For this they have developed a comprehensive theoretical basis, designed procedures for making the required calculations, and have constructed software for the implementation.

The basic idea is to create a hierachical system of models at various levels which are strongly inter-connected and self-similar at the levels. A military command structure is particularly well suited to this approach. The system is designed to operate in two types of modes, called "sythesis" and "reproduction" (or "analysis") . Basically, the synthesis mode determines a variant of an operational plan that should meet the objectives. The reproduction mode then predicts an expected outcome for the selected objective. It is important to realize that these two modes are highly interrelated and interdependent, not separate and independent events. "Without analysis, there can be no synthesis", according to V. Lenin. This means that they can't decide on a "best" variant without making some calculations that indicate that the variant has a chance of working and to supply the decision-maker with some rough calculations and options to support making a choice.

For the synthesis mode, the calculations are not required to be as exact or accurate as in the reproduction mode. The reason for this is that for synthesis it is only necessary to distinguish between two variants or to determine if a variant is feasible, while in the reproduction phase a more exact or accurate solution is required that will more closely "track" the expected outcome. In particular, for the zone of nearest interaction, the degree of accuracy of a solution for the space-time equations does not need to be as great as for the corresponding calculations required in the synthesis mode.

On the other hand, to be useful they must be fast. Krasnoschekov and Savin mentioned that all calculations in support of the synthesis mode must be done in at least real time. This is because a large number of variants must be checked and the results used by operator/ controllers in a man/machine interactive loop. So in the synthesis mode accuracy is sacrificed for speed and a fast numerical method giving a relatively good approximation is preferred over a more exact, but slower method.

In the hierarchical structure, an operation at the front (theater) level involves systems for modeling at that level, at the Army level, and down to division level, by reason that a three-level span of control is the largest that is practical. For the corresponding planning operation at the lower levels, another system of three analogous models is created. To support these models it is necessary to solve problems of disaggregation and aggregation to relate the results of the calculations between the various levels at various levels and determine the parameters needed for the model equations. This is important in the implementation of the model since it relates the combat effectiveness parameters at one level to those at higher and lower levels. So in particular, if the coeffients at one level are known, they can be determined at the other levels.

This seems to work easily as long as the models are all deterministic differential equations. For the models corresponding to the lowest levels of combat the differential equations are stochastic. Krasnoschekov indicated that he had considered the coefficient aggregation problem from stochastic to deterministic equations and knew how to solve it, but did not explicitly discuss the solution in the seminars. Presumably, this is closely related to the derivation of the deterministic equations and the representation of the attrition-producing function in his book [2] and in the new material he presented.

In addition to this top-down, bottom-up structure of the system of models, there is also a kind of horizontal structure, coming from a set of task- related activities that forces at that level could engage in. The structure of models described in the seminar lectures and notes reflect a systematic, logically well-conceived and developed approach, but the details and interconnections between the submodels and decision rules for implementing the system of models are still incomplete.

As a consequence of synthesis mode, a variant of the operational plan is selected. It includes a plan for allocating resources to the battle and expected rates of advance, etc. In particular, the outcome produces the velocities required to close the space-time model in the sense that it provides an estimate for the forces that can be introduced into combat in the zone of nearest interaction at various times and how they should be allocated and their velocity controlled to achieve the objective.

Another important feature of the system of models described in the seminars related to creating a common time-line for decision making and interface between the models at different levels. This is important in setting up communication between models at different levels and different models at the same level, indicating when certain calculations have to be performed and decisions made based on them.

In addition to the deterministic stucture of the model, a significant effort was made in the System of Models to account for uncertainties on the battlefield and how to effectively deal with them. These included, for example, uncertainties created by lack of intelligence on enemy order of battle and enemy intentions and uncertainties due to friendly communications and control. The basis for treating these uncertainties and having to make decisions in spite of them was laid on a foundation of game theory. Another feature of this part of the analysis involved considering games when two players do not necessarily have opposite interests in mind.

As indicated above, our present knowledge of the System of Models described in the seminars by Krasnoschekov and Savin is not complete in all aspects, but enough was discussed and defended so that the general impression was that such a System was based on a solid theoretical foundation and the missing pieces were likely to be constructable. The following is a summary of some of the salient points of this approach:

1) The problems solved in the System of Models are very strongly tied to the problems of closing the space-time model discussed in Section 3. In particular, the questions of how to choose the velocity functions and how to identify the parameters are handled with this procedure.

2) In using the model for operational planning, the results of the individual submodels, such as finding the solutions of the equations in the space-time model, are used on an interactive basis to supply an operator with the quantitative means for making a decision at some selected times. This is where the "optimality" that Chuyev discussed in his book must come in. Based on a list of criteria which the decision-maker specifies (for example, arrival at an objective, friendly loses, enemy loses, expenditure of supplies), the operator is supplied with alternatives meeting those objectives (if possible) and is then required to choose one. This procedure is iterated until a comprehensive variant is attained which meets criteria as well as possible. A particularly interesting feature of this approach is that the decision-maker gets to redefine his criteria and rank their importance as he sees the process develop and understand what is possible and what is not.

3) Since many such operators are required to run the model at the theater level and the entire process involves making many similar calculations at various levels in the hierarchical structure, a special, dedicated, system of software was designed and developed to streamline the operation of the System. No details about this software system were given, but it was stated that without such software, it would have been difficult or impossible to implement the model. On the other hand, this software represented the best that was possible in the 1970's in the former Soviet Union on very small computers by modern day standards.

5. SEMINAR LISTINGS

SAN DIEGO STATE UNIVERSITY DEPARTMENT OF MATHEMATICAL SCIENCES

SPECIAL SEMINAR ON MATHEMATICAL MODELING AND SYSTEMS

Professors P. Krasnoschekov and G. Savin, Russian Academy of Sciences, Moskow

MWF 10-12, BAM-207

TOPICS

WEEK OF SEPTEMBER 21-23

Overview of operations research activities at the Computation Center of the Russian Academy of Sciences, with a discussion of work they wished to present during the coming weeks of the seminar. Historical remarks on combat modeling.

WEEK OF SEPTEMBER 27- OCTOBER 1:

Modeling of large-scale, continuous media interactions in time and space.

1. Introduction to models of war interaction.
2. Derivation of the equations for a combat model in Euler variables
3. Transformation of equations to Lagrange variables
4. Relationships to modeling of complex systems, hierarchical design problems

WEEK OF OCTOBER 4-8:

Modeling of large-scale, continuous media interactions-continued.

1. Comparison of Lagrange and Euler forms of the equations.
2. Derivation of the Lebedev relation for movement of the front line.
3. Critical observations and remarks on combat modeling

WEEK OF OCTOBER 11-15:

Systems of mathematical modeling for the design of large-scale, complex processes including combat modeling

1. Simulation Systems
2. Synthesis of Systems
3. Realization of Systems

WEEK OF OCTOBER 18-22:

Principles of the mathematical theory related to the design of complex technical systems

1. Design as a problem of multi-criteria optimization
2. Hierarchical systems of design
3. Fundamental systems of criteria

WEEK OF OCTOBER 25-29:

Mathematical models of Operations Research

1. Combining models, information and mixed strategies
2. Efficiency of multi-criteria tasks
3. Applications to large-scale conflict models

WEEK OF NOVEMBER 1-5:

Application of the design-optimization process to the construction of a flying object.

1. Construction of a model
2. Construction of a designing image
3. Finding image characteristics

WEEK OF NOVEMBER 15-17

Review and analysis of continuous media, space-time equations and their role in a staff model.

1. Assumptions governing the continuous media model equations.
2. Synthesis and Reproduction Models for front level operations.
3. Proposal for new book on combat modeling and military opertions research techniques.

6. REFERENCES

[1] Chuyev, Yu. Research of Military Operations, 1970, Moskow. (Translated as JPRS 53366; 15 June 1971).

[2] Krasnoschekov, P. and Petrov, A. A., Principles of Mathematical Modeling, Moskow, 1983.

[3] Moiseyev, N. N., Mathematics conducts an experiment, FTD-ID(RS) T-0776-82.

[4] Lutz, Donald A. and Castillo, Jose E., Soviet Combat Modeling, Report for USA- TRAC WSMR, 25 Feb 1994.