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NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

JAVELIN ANALYSIS USING MATHEMATICAL MODELING

by

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June, 1994

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REPORT DOCUMENTATION PAGE	Form Approved OMB No. 0704

1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE June 1994	3. REPORT TYPE AND DATES COVERED Master's Thesis

4. TITLE AND SUBTITLE: JAVELIN ANALYSIS USING MATHEMATICAL MODELING	5. FUNDING NUMBERS
6. AUTHOR(S) CPT(P) Archie Wilmer III	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Postgraduate School Monterey CA 93943-5000	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.

12a. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution is unlimited.	12b. DISTRIBUTION CODE

13. ABSTRACT *(maximum 200 words)*

The Javelin antitank weapon system will replace the Dragon in Infantry and Combat Engineer battalions on a one for one basis. The tactics and techniques for Javelin employment will closely mirror those used for Dragon and TOW missile systems. This thesis examines the effectiveness of Javelin versus Dragon and examines a different employment method. The thesis goals are:

- To investigate the suitability of Janus(A) to model the employment of various weapon systems.
- To compare Javelin against Dragon, using measures of effectiveness and performance outlined in the Test and Evaluation Plan (TEP).
- To examine how changes in the composition and tactical employment of anti-armor weapon systems influence their lethality and survivability, potentially suggesting a more effective employment method.

The data generated from the simulated force-on-force scenarios is analyzed using graphical, statistical, and mathematical modeling techniques. The results could benefit the Army's Training and Doctrine Command in their combat development and combat effectiveness analysis.

14. SUBJECT TERMS Janus(A), Simulation, Javelin, Modeling.	15. NUMBER OF PAGES 98
	16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT UL

NSN 7540-01-280-5500 Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. 239-18

JAVELIN ANALYSIS
USING
MATHEMATICAL MODELING

by

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Submitted in partial fulfillment
of the requirements for the degree of

MASTER OF SCIENCE IN APPLIED MATHEMATICS

from the

NAVAL POSTGRADUATE SCHOOL
June 1994

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I. INTRODUCTION

A. BACKGROUND

The United States Army is acquiring a new medium range anti-armor missile system called Javelin. It will replace Dragon, the current medium range anti-armor system. Javelin offers a significant increase in capabilities over Dragon. The Army must determine how Javelin can best be employed to profit from these increased capabilities. For now, the tactics and techniques for Javelin employment closely mirror, in many respects, those used for the Dragon and TOW missile systems [Ref. 5:para. 3b].

This thesis presents a method for examining weapon system effectiveness. It examines how changes in the combination and tactical employment of long and medium range anti-armor weapon systems influence their lethality and survivability. Potentially, the results of this study could suggest a more effective employment method for the Javelin anti-armor weapon system.

The approach in this study includes modeling combat engagements using Janus(A), a high-resolution combat simulation. Janus(A) provides a way to design simulated weapon systems, establish combat scenarios, and conduct experiments that are difficult or infeasible to conduct in the real world. (Appendix D presents a step-by-step tutorial on designing weapon systems in Janus(A)). Then statistical analyses and mathematical modeling methods are used to analyze the generated data from the experiment. Analysis of variance (ANOVA) tables provide indication of the significant factors. The mathematical modeling process allows a more focused examination on specific aspects of the study.

B. MODELING PROCESS

Using the modeling process, a modeler constructs and uses models to better understand real-world systems. The modeling process consists roughly of the following steps:

- Given some real-world system or behavior, gather sufficient information to formulate a model.
- Analyze the model and reach mathematical conclusions.
- Interpret the model and make predictions or offer explanations.
- Gather data to test the conclusions of the model against the real-world system.

The types of models used may differ in both appearance and purpose. Mathematical models include graphical, symbolic, simulation, and experimental constructs. A symbolic model is generally a formula, equation, or system of equations. Another model type is miniature replication, like a model aircraft or submarine.

Sometimes it is infeasible to observe the behavior of interest directly or to conduct experiments. In instances where the behavior cannot be formulated analytically, or data collected directly, the modeler may simulate the behavior indirectly using a computer. The simulation serves as a stand-in for the actual system. Experiments performed on the simulation model often provide valuable insights with respect to the actual behavior of interest. For this study, the Janus(A) simulation model approximates the very complex stochastic processes found in combat engagements.

C. EXPERIMENTAL DESIGN

Experiments are often performed to compare two treatments; for example, to compare two different fertilizers, machines, methods, processes, or materials. The objectives are to determine whether there is any real difference between them, to estimate that difference, and to measure the precision of the estimate. Experimenters must avoid the pitfalls often found when designing experiments. By understanding and respecting the important assumptions associated with experimental design, serious errors can be avoided [Ref. 1:p. 19].

When an operation or experiment is repeated under nearly the same conditions, the observed results are rarely identical. The fluctuation that occurs from one repetition to another is called experimental variation or error. The influence of experimental error in data analysis is a paramount consideration in planning the generation of data in the design of an experiment [Ref. 1:p. 24].

Experimenters must adhere to several important principles when assessing the possible difference between two treatments. Experiments need to be comparative. For example, in testing a modification, the modified and unmodified procedures should be run side by side in the same experiment: there should be genuine replication. Usually, treatment runs should be carried out several times. Furthermore, this procedure should be done in a way that variation among replicates gives an accurate measure of errors affecting comparisons made between the runs. Whenever appropriate, blocking (pairing) should be used to reduce the error. Similarity of basic conditions for pairs of runs provide a basis for blocking; for example, those runs made on the same day, from the same blend of raw materials, with animals from the same litter, or on shoes from the same boy, and so forth. Having eliminated "known" sources of discrepancy, either by holding certain parameters constant during the experiment or by blocking, the unknown discrepancies should be forced by randomization to contribute homogeneously to both treatment runs. This methodology generates an estimate of error appropriate to the comparisons made and validates standard tests [Ref. 1:pp. 105-106].

II. ANTI-ARMOR DOCTRINE AND WEAPONS

A. ANTI-ARMOR WARFARE

In Air Defense Artillery (ADA), the employment guideline of *defense in depth* means positioning ADA weapons so that threat aircraft encounter an increasing volume of fire as they approach a specific defended asset [Ref. 2:p. 128]. The same general guideline exists for the employment of anti-armor weapons. As with ADA weapon systems, each type of anti-armor system is most effective in defense of a particular range over the battlefield. Usually, an effective anti-armor defense requires combining two or more weapon systems and overlapping their coverage. This defense enhances the probability of destroying enemy armor vehicles by forcing the enemy to encounter an increasing volume of fire from an increasing number of anti-armor weapons (which complicates his ability to accomplish his mission). Enemy defeat is more certain when each anti-armor weapon system is employed to maximize its design capabilities.

B. DRAGON

The M47 Dragon is currently the Army's medium range anti-armor weapon system. It is composed of three major parts: a day tracker, a night tracker, and a tube-launched missile round. The round contains a high explosive warhead. A Dragon gunner fires the weapon from a kneeling position while supporting the system with the shoulder. Once the gunner fires the missile, he guides it to the target using one of the trackers. As the missile clears the launch tube, a guidance-wire bobbin starts dispensing its teflon-insulated line. This electrical wire links the tracker to the missile. Infrared radiation emitted by a flare located in the aft end of the missile is focussed by sensor optics on to a detector in the tracker. The missile position and the gunner's line of sight to the target are compared by the tracker electronics, and trajectory correction signals are transmitted to the missile over the wire link. In response to the incoming signals, the missile computer selects and fires pairs of motors which adjust the flight of the missile. The minimum arming distance of the missile is 65 meters and has a maximum range of 1000 meters. The missile's time of flight to maximum range is 10 seconds. During this flight time, the gunner remains exposed to enemy fire. Additionally, a smoke trail points the enemy toward the gunner's position.

C. JAVELIN

The Javelin system is also a medium range anti-armor weapon system, but offers a significant increase in capabilities over Dragon. Javelin reduces gunner vulnerability by having a smaller back-blast, increased lethality, and greater range. A Javelin system can hit a target at 2000 meters in 16 seconds. Its missile consists of an imaging infrared seeker, feature-based tracker, tandem shape-charged warheads, dual in-line eject and flight motors, and attack guidance controls for either top attack or direct fire profiles. The tandem warheads and top attack flight profile increase the lethality of the Javelin. The system is "fire-and-forget" capable. This enables the gunner to move immediately to a new location or to seek cover after firing. Javelin fires using a soft launch so the gunner can fire the system from inside buildings, bunkers or enclosed areas. This feature also reduces the system signature during firing.

D. ITV

One of the Army's long range anti-armor weapon systems is the Improved TOW Vehicle (ITV) M901. TOW is the acronym for tube-launched, optically tracked, wire-guided missile. The ITV is an armored two-tube TOW launcher integrated into the standard M113A1 chassis. It provides light armor protection to the crew against suppression by artillery and small arms fire. Its primary mission is to destroy enemy armor. It is also effective against point targets, such as bunkers and crew-served vehicles. The ITV has a minimum range of 65 meters and a maximum range of 3750 meters. It uses a shape-charged warhead that does not rely on speed to penetrate the target. The warhead can penetrate over 19 inches of armor.

E. MEASURES OF EFFECTIVENESS

> The operational effectiveness of an anti-armor weapon system [or combination of weapon systems] is characterized by its ability to defeat armor forces and survive in a realistic battlefield environment with uncertainty [Ref. 6:pp. 2-1, 2-2].

The Javelin Test and Evaluation Plan (TEP) documents the planning for Initial Operational Test and Evaluation (IOTE). The TEP focuses on the operational effectiveness and suitability of the system in accomplishing its designed mission. It also contains the criteria used to measure the weapon's operational effectiveness.

This study examines the effectiveness of two anti-armor weapon systems employed together. The mixture of a medium range with a long range weapon system is considered here as a combination. The measures of effectiveness and performance for the Javelin are applied to the combination as a whole. Thus, the evaluation concept compares Dragon and ITV performance against Javelin and ITV performance using the criteria outlined in the Javelin Test and Evaluation Plan. The performance comparison is based on two factors; namely, composition and tactical employment method. Table I defines the composition of forces.

TABLE I. COMPOSITION OF FORCES

Blue Force

Composition	Dragon	Javelin	ITV	Total
A	9	0	10	19
B	0	9	10	19

Red Force = 13 T-72 Tanks

Composition A contains a Blue force of nine Dragon and ten ITV weapon systems. Composition B contains a Blue force o nine Javelin and ten ITV weapon systems. The Red force is a heavy armored threat of thirteen T-72 tanks.

TABLE II. DEFINITION OF FACTORS

Factor$_1$ - COMPOSITION
A - Composition A against the Red force.
B - Composition B against the Red force.

Factor$_2$ - EMPLOYMENT METHOD
1 - Antiarmor systems are relatively "on-line".
2 - Medium range system is 500 meters forward.
3 - Medium range system is 1500 meters forward.

In addition to comparing two compositions, this study compares three employment methods. Employment method #1 consists of positioning both the medium and long range systems relatively "on-line". That is, all the weapon systems are positioned such that generally no system fires from the rear of another system. Employment method #2 consists of positioning the medium range systems roughly 500 meters forward of the ITV systems. Employment method #3 consists of positioning the medium range system roughly 1500 meters forward of the ITV. Table II shows the three employment methods as the second factor in our study. Appendix A, pages 57-60, contains Janus(A) printouts of the initial positioning of blue forces using the various employment methods.

Each combination will undergo similar scenario events to test the following hypotheses:

- A Javelin and ITV combination equals or improves unit performance.
- A Javelin and ITV combination increases the number of threat armored forces destroyed.
- More Javelin and ITV systems survive engagement while accomplishing their anti-armor mission.

Furthermore, to examine a potentially more effective employment technique, the experiment tests three additional hypotheses:

- A Javelin and ITV combination using employment method #3 equals or improves unit performance.
- A Javelin and ITV combination using employment method #3 increases the number of threat armored forces destroyed.
- More Javelin and ITV systems survive while using employment method #3 in accomplishing their anti-armor mission.

These six hypotheses are the alternate hypotheses of interest. The accompanying null hypotheses claim that there is no difference in the effectiveness of the two combinations or three employment methods.

One performance measure is the amount of overlap of the shot distributions. Chapter IV explains this measure of performance. Loss Exchange Ratio (LER) and Force Exchange Ratio (FER) provide other measures of performance. LER is a simple ratio of the count of attackers killed (A_k) and defenders killed (D_k).

$$LER = \frac{A_k}{D_k}$$

FER normalizes the LER by force size by introducing the number of attackers (M) and the number of defenders (N) so the proportion of each killed is presented:

$$FER = \frac{\frac{A_k}{M}}{\frac{D_k}{N}} .$$

The final measure of performance is the number of survivals for the Blue force. The number of survivals equals the number of defenders (N) minus the defenders killed (D_k):

$$SURVIVALS = N - D_k .$$

III. STATISTICAL ANALYSIS

A. DESIGN OF THE EXPERIMENT

The evaluation concept compares Dragon and ITV performance with Javelin and ITV performance based on two factors, composition and tactical employment method to test the following alternate hypotheses:

- A Javelin and ITV combination equals or improves unit performance.
- A Javelin and ITV combination increases the number of threat armored forces destroyed.
- More Javelin and ITV systems survive engagement while accomplishing their anti-armor mission.
- A Javelin and ITV combination using employment method #3 equals or improves unit performance.
- A Javelin and ITV combination using employment method #3 increases the number of threat armored forces destroyed.
- More Javelin and ITV systems survive while using employment method #3 in accomplishing their anti-armor mission.

The null hypotheses claim that there is no difference in the effectiveness of the two combinations or three employment methods. In the first three alternate hypotheses, the comparison is made against a Dragon and ITV combination. In the last three alternate hypotheses, the comparison is made against a Javelin and ITV combination using employment methods

#1 and #2. The Janus(A) scenario runs generate LER, FER, and survival data to measure the performance composition and employment combination. In short, does the data present sufficient evidence to indicate a difference in the effectiveness of the compositions and employment methods?

Under the general classification of "two factors with replication", the statistical model is fixed (both factors are fixed). The arrangement for the Javelin and Dragon experiment is a 3 X 2 fixed two-factor design, replicated ten times. There is no blocking, and both factors, (factor_1 = compositions) and (factor_2 = employments), are of equal interest, as is the possibility that these factors interact. If they interact, they will not behave in an additive manner. Instead, the mean difference in observations between the compositions is different for different employments. The arrangement for the employment experiment is a one-factor analysis of variance design, replicated ten times. Appendix B, starting on page 61, contains the general designs of the two experiments.

The analysis, in general, supposes that there are n levels of some factor E (n=3 employments), k levels of some factor C (k=2 compositions or treatments), and r replications (r=10 runs per scenario). The corresponding sums of squares, S_E for factor E, S_C for factor C, S_I for the interaction between E and C, S_e for error, and S for total, are given by the formulas and table in Appendix B, page 63.

B. ANALYSIS OF RESULTS

Based on the data generated from the Janus(A) runs (See Appendix C, pages 64-66), the following tables show the ANOVA results. The first three tables show the results of the Javelin and ITV combination compared with the Dragon and ITV combination.

TABLE III. TWO-FACTOR ANOVA TABLE (LER)

source of variation	sum of squares	degrees of freedom	mean square	ratio of mean square
employment	1.3702	2	0.6851	3.4391
composition	26.1360	1	26.1360	131.1987
interaction	3.2971	2	1.6486	8.2757
error	10.7573	54	0.1992	
total	41.5606	59		

Critical Values:

$F_{2,54}(.01) = 5.0212$ and $F_{1,54}(.01) = 7.1288$

Reject the null hypothesis if the ratio of mean square value is greater than the appropriate critical value.

If the null hypotheses were true, the value of the ratio of mean squares would follow an F distribution. This value is the significance level for the source of variation. The critical values are $F_{2,\ 54}(.01) = 5.0212$ and $F_{1,\ 54}(.01) = 7.1288$. Thus, the data presents significant evidence to indicate a difference in the effect of compositions.

TABLE IV. TWO-FACTOR ANOVA TABLE (FER)

source of variation	sum of squares	degrees of freedom	mean square	ratio of mean square
employment	2.8920	2	1.4460	3.4117
composition	55.8542	1	55.8542	131.7808
interaction	7.1002	2	3.5501	8.3760
error	22.8874	54	0.4238	
total	88.7339	59		

Critical values:

$F_{2,54}(.01) = 5.0212$ and $F_{1,54}(.01) = 7.1288$

Reject the null hypothesis if the ratio of mean square value is greater than the appropriate critical value.

TABLE V. TWO-FACTOR ANOVA TABLE (SURVIVALS)

source of variation	sum of squares	degrees of freedom	mean square	ratio of mean square
employment	34.5333	2	17.2667	3.0204
composition	1430.8167	1	1430.8167	250.2886
interaction	84.1333	2	42.0667	7.3586
error	308.7000	54	5.7167	
total	1858.1833	59		

Critical Values:

$F_{2,54}(.01) = 5.0212$ and $F_{1,54}(.01) = 7.1288$

Reject the null hypothesis if the ratio of mean square value is greater than the appropriate critical value.

In the one factor case, if the null hypotheses were true; that is, if none of the employment methods were significantly different, then the ratio of mean squares would follow an F distribution. The critical value equals $F_{2,27}(.01) = 5.4881$. Thus, there is significant evidence to indicate a difference in the effect of employment methods using each of the measures of performance.

TABLE VI. ONE-FACTOR ANOVA TABLE (LER)

source of variation	sum of squares	degrees of freedom	mean square	ratio of mean square
employment	4.3526	2	2.1763	7.0051
error	8.3883	27	0.3107	
total	12.7409	29		

Critical Value:

$F_{2,27}(.01) = 5.4881$

Reject the null hypothesis if the ratio of mean square value is greater than the appropriate critical value.

TABLE VII. ONE-FACTOR ANOVA TABLE (FER)

source of variation	sum of squares	degrees of freedom	mean square	ratio of mean square
employment	9.2619	2	4.6309	6.9924
error	17.8815	27	0.6623	
total	27.1434	29		

Critical Value:

$F_{2,27}(.01) = 5.4881$

Reject the null hypothesis if the ratio of mean square value is greater than the appropriate critical value.

TABLE VIII. ONE-FACTOR ANOVA TABLE (SURVIVALS)

source of variation	sum of squares	degrees of freedom	mean square	ratio of mean square
employment	109.0667	2	54.5333	8.2211
error	179.1000	27	6.6333	
total	288.1667	29		

Critical Value:

$F_{2,27}(.01) = 5.4881$

Reject the null hypothesis if the ratio of mean square value is greater than the appropriate critical value.

C. SUMMARY

Statistical analysis reveals that we can reject the null hypotheses. We can reject the claim that there is no difference in the effectiveness of the two combinations or the three employment methods. The results show that the combination and employment method of the anti-armor weapon systems in this study indeed influence their lethality and survivability. The data from the simulated battles indicate that the Javelin and ITV combinations are more effective than the Dragon and ITV combinations. Further, the Javelin and ITV combination using employment method #3 is more effective than the same combination using employment methods #1 and #2. Chapter V includes more discussion on the statistical analysis.

IV. MATHEMATICAL MODELING

A. ANTI-ARMOR WARFARE REVISITED

Each type of anti-armor system is most effective in defense of a particular range over the battlefield. Consider the Javelin and ITV employed together as a combined system. Figure 1 depicts the coverage area of the combined system.

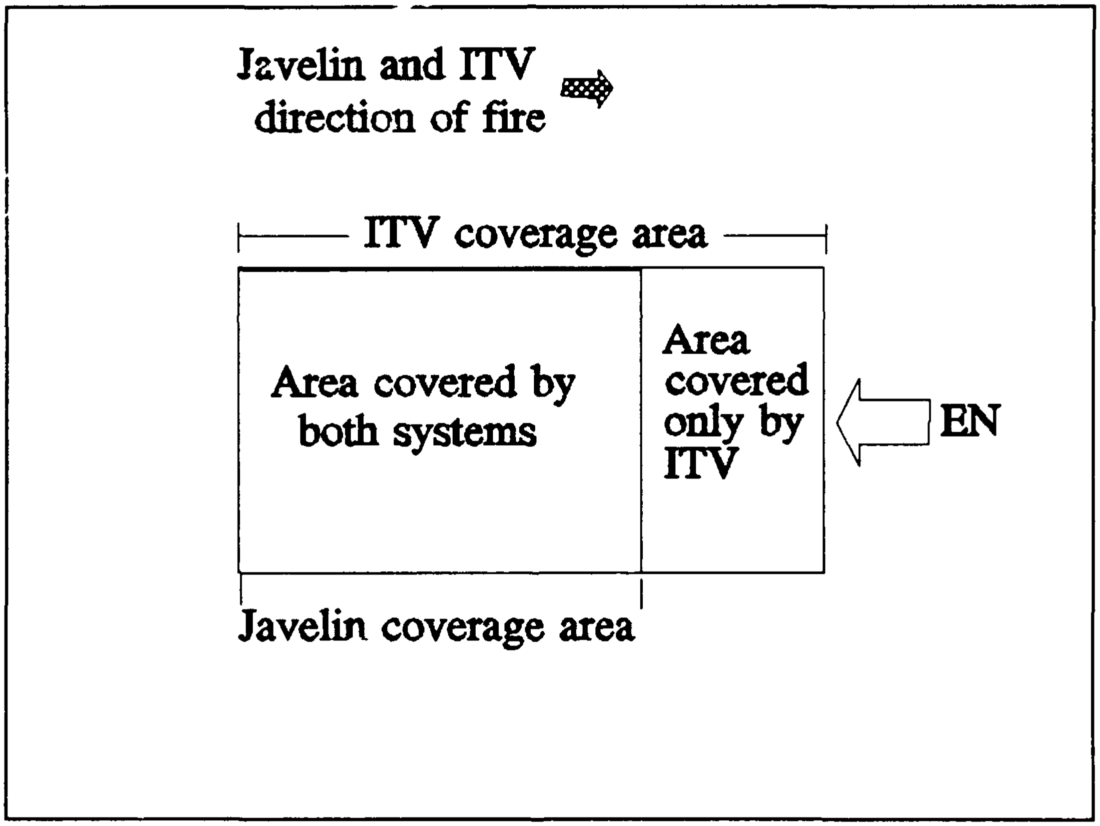

Figure 1. Coverage (Systems On-line)

If the weapons are employed relatively on-line and all weapons fire in the same general direction, then the engagement area (i.e., any threat within this area could be engaged if detected) consists of the area inside the maximum range of either the Javelin or ITV systems. Since ITV has the greater maximum range, any area covered by Javelin is also covered by ITV. However, the area beyond the maximum range of Javelin, the ITV would have to cover alone out to its maximum range. This sparsely defended portion of the battle zone could be quite large, up to the difference of the maximum effective ranges of the two systems. During an engagement with the opposing force, the ITV must defend this area alone with no possible support from the other system. As an alternative, the ITV could hold its fire until the opposing force moved into the area covered by both systems. This scenario would allow both the Javelin and the ITV to engage the enemy simultaneously and gain a numerical advantage. However, this negates the long range capability of the ITV. If the ITV begins to engage while the enemy is in the sparsely defended portion, the scenario is a one-on-one fight. Given the composition of forces in our study, engagements occurring in this area of the battle zone place ITV at a numerical disadvantage. The ITV systems would be destroyed leaving the Javelin systems to defend alone (another one-on-one battle) and Javelin would lose. Essentially, this case happened in the simulated battles where the initial employment of weapons were on-line. Tables XVI and XVII on pages 67 and 68, respectively, display the shot data from the battles. The data is shown graphically in Figure 2. The horizontal axis represents the distance (one unit is one hundred meters) from the position of the ITVs to the target of the shot. The vertical axis represents the total number of shots fired at the given target range.

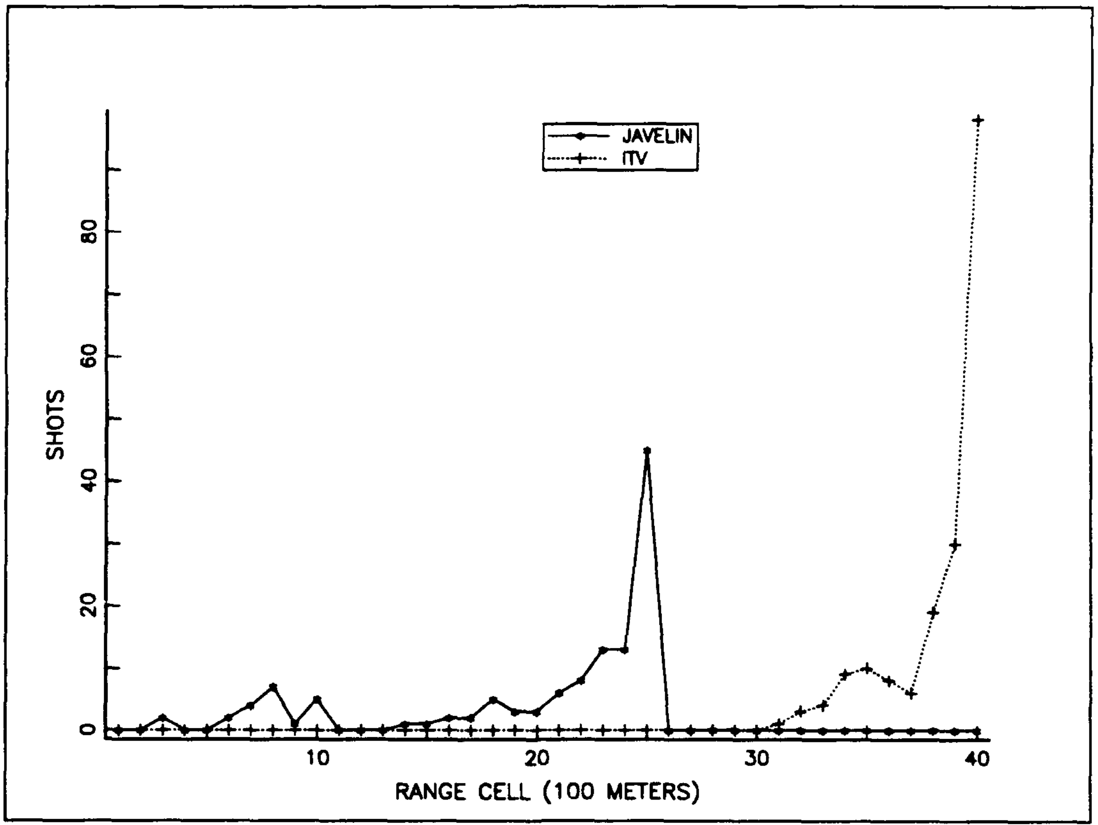

Figure 2. Distribution of Shots (Systems On-line)

Notice the large gap that occurs between the prominent peaks. The distance of the gap corresponds to the difference between the maximum effective ranges of the weapon systems based on their positioning. The peak to the right corresponds to the shots from the ITV as it engages at maximum effective range. The gap from range cell 25 (2500 meters) to 30 (3000 meters) occurs because all the ITV systems are destroyed in the one-on-one engagement. Javelin cannot support the fight at these range distances. No shots are fired in this range by the friendly force as the remaining opposing force continues to move forward. The opposing force enters the effective range for Javelin, more shots are fired represented by the peak at range cell 25.

Next, placing the Javelin weapon systems initially 500 meters forward of the ITV systems, the coverage area changes as depicted in Figure 3. This effect narrows the sparsely covered portion and shifts the maximum range of the Javelin closer to the forward edge of the battle zone.

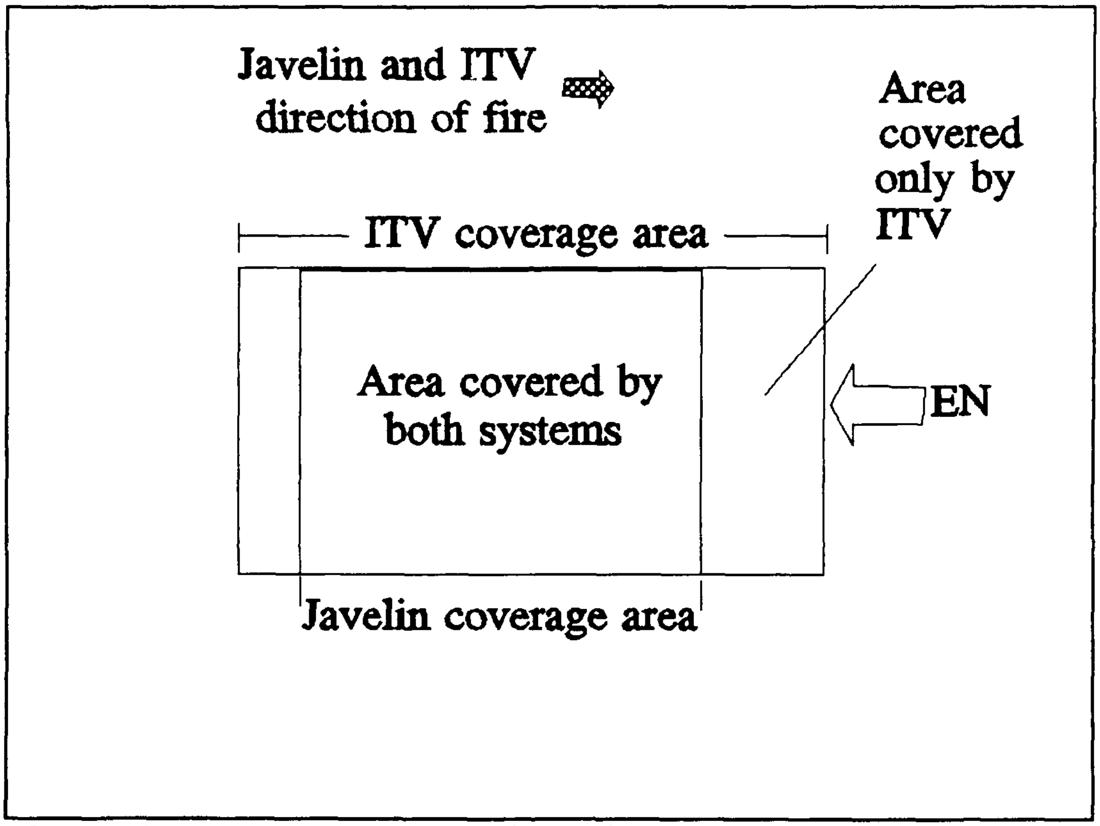

Figure 3. Coverage (Javelin shifted 500m forward)

After conducting scenario runs, the shot distributions appear as shown in Figure 4. The distribution for Javelin now appears shifted 500 meters and the gap between the peaks appears much smaller.

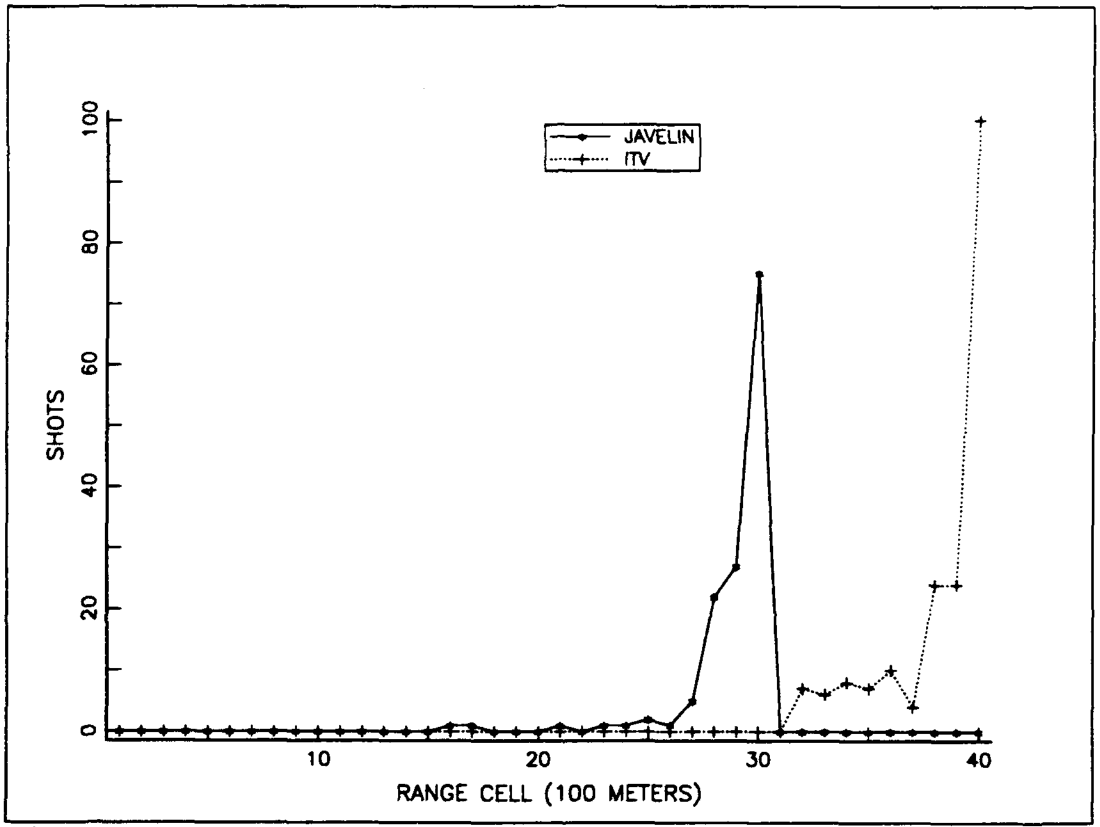

Figure 4. Distribution of Shots (Javelin shifted 500m forward)

Third, after positioning the Javelin systems initially 1500 meters forward of the ITV systems, the coverage area changes again as depicted in Figure 5. Now the shot distribution for Javelin appears shifted 1500 meters as in Figure 6, the peaks almost coincide, and the gap is virtually eliminated. Effectively, this third employment method allows both Javelin and ITV systems to engage the opposing force at the same time, thus supporting each other in the fight.

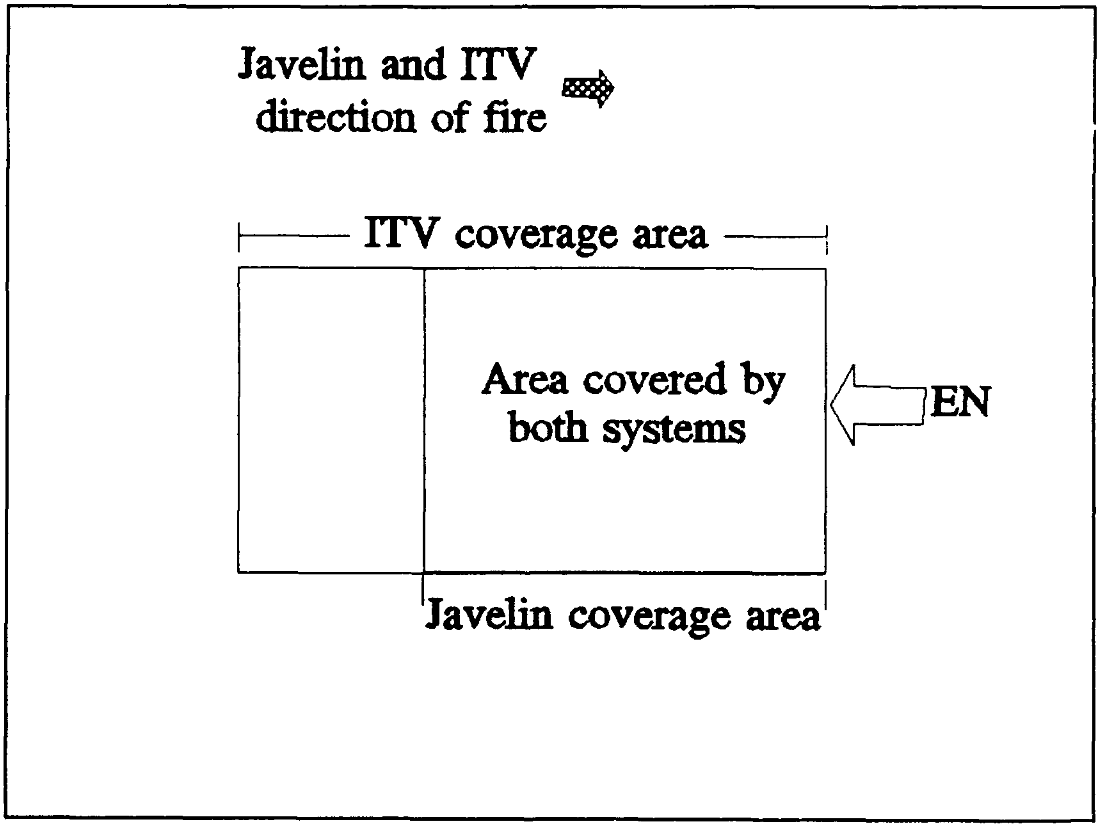

Figure 5. Coverage (Javelin shifted 1500m forward)

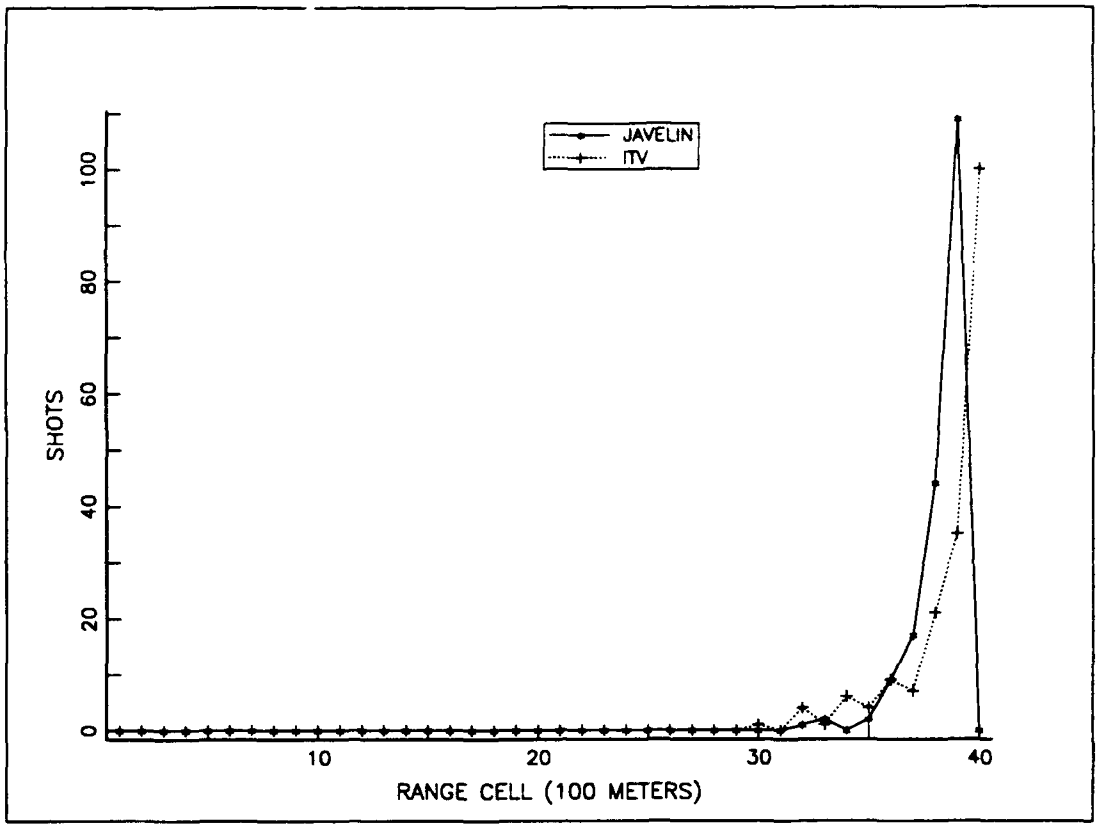

Figure 6. Distribution of Shots (Javelin shifted 1500m forward)

If we fit continuous function curves through the points of the shot distributions and graph the curves, we establish a "shot signature". This signature can answer questions such as: (1) How many shots occur at a given range? (2) At what target ranges do both systems fire shots? (3) Are there gaps where no shots are fired by either weapon system? (4) How large are the gaps? (5) How much do the areas under the curves overlap? Examining the shot signature and analyzing the fitted function curves can answer these questions.

Graphically we see that in the scenario using employmen method #3 there are range cells containing shots for both Javelin and ITV. The curves of the signature overlap throughout a contiguous subset of range cells. The key to more effective and lethal weapons employment would be to ensure that this overlapping occurs over the same target ranges as the planned engagement area. Over this range, threat forces encounter an increased volume of fire from an increased number of weapon systems.

B. MODEL SELECTION

The approach used examines various families of curve models. Within each family, we select the curve that "best fits" the trend of the data by applying the least-squares criterion. A "best fitting" least-squares curve is the curve of a function f(x) whose parameters minimize the sum:

$$\sum_{i=1}^{m} [y_i - f(x_i)]^2$$

The Chebyshev criterion (which minimizes the largest absolute deviation) could also be used. Later in this chapter we will discuss why we select the least-squares over the Chebyshev criterion. Also later we will establish a criterion to compare the different curve models and use it to select the model most appropriate for our analysis.

Using the problem solving process, the problem statement is as follows:

- Model the distribution of shots as a function of range for the Javelin/ITV battles.

The following are the assumptions:

- Janus(A) adequately simulates real-world combat engagements.
- There are two forces with elements similar to those defined earlier.
- Blue (friendly) and Red (opposing) initial forces are fixed (no replacements).
- Blue force weapon systems are stationary and fire from partial defilade defensive positions.
- Red force attacks in formation similar to Soviet doctrine.
- Terrain is flat, desert, open plains.
- Weather conditions are similar to summer.
- The least-squares criterion is applied.

1. Graphical Analysis

A graphical analysis of the data provides important information. The shots increase as the range increases. Near the maximum effective range of the weapon, shots increase dramatically and reach a maximum value at the maximum effective range. At several range cells, no shots are fired. Thus, the shot value is zero for that range cell. This information is critical as we examine the alternative models. Ideally, the best model must behave in a way similar to these characteristics.

Transforming the data is a technique that will help start our search for the best fitting function curve. Using this technique transforms the data so that the points lie approximately in a straight line. Listed below is the ladder of transformation functions [Ref. 3:p. 171].

$$\begin{array}{c} \vdots \\ z^3 \\ z^2 \\ z \text{ (no change)} \\ \left.\begin{array}{c} \sqrt{z} \\ \log z \\ \frac{-1}{\sqrt{z}} \\ \frac{-1}{z} \end{array}\right\} * \\ \frac{-1}{z^2} \\ \vdots \end{array}$$

* *denotes most often used transformations.*

The inverse of the transformation function provides an initial form of the function we seek. The transformation function used is a matter of trial and error, and experience. Our graphical analysis provides information we can use to narrow our search. Zeros for many data values restrict us from using the transformation functions below $log(z)$, inclusively. Using a transformation of the form z^p, where $p > 1$, transforms the data farther away from a straight line. The remaining transformation is of the form z^p, where $0 < p < 1$. Letting $p = 1/n$, where $n > 1$, we can rewrite the form as $z^{1/n}$. So applying the transformation yields:

$$y = f(x)$$
$$(y)^{\frac{1}{n}} = g(y) = (f(x))^{\frac{1}{n}}$$

Because we want the data to lie approximately in a straight line passing through the origin, this implies:

$$(f(x))^{\frac{1}{n}} \approx ax$$

where a is constant and the value of n is fixed. Thus,

$$(y)^{\frac{1}{n}} \approx ax$$
$$y = f(x) \approx (ax)^n$$

for some fixed $n > 1$. Thus, we start our search with the curve family of the form $y = Ax^n$, where A is a constant and n is fixed.

2. Model Alternatives

a. *One-Term Models*

Application of the least-squares criterion to fit a curve of the form $y=Ax^n$ ($f(x)=ax^n$), where *n* is fixed, to the given collection of data points in Tables XVI and XVII on pages 67 and 68, respectively, requires the minimization of *S* (the sum of squared deviations),

$$S = \sum_{i=1}^{m} [y_i - f(x_i)]^2 = \sum_{i=1}^{m} [y_i - ax_i^n]^2$$

A necessary condition for optimality is that the derivative *dS/da* equals zero. Taking the derivative, setting it equal to zero and solving for "a" yields

$$a = \frac{\sum x_i^n y_i}{\sum x_i^{2n}}$$

One observation is that the many $y_i=0$ data values tend to make the value of "a" close to zero.

The following discussion [Ref 3:pp. 100-101] validates our least-squares assumption. Application of the Chebyshev criterion normally requires optimization of a linear program to determine the solution, whereas the least-squares criterion requires only the calculus of several variables. Suppose the Chebyshev criterion is applied and the resulting optimization problem solved to yield the function $f_1(x)$. The absolute deviations resulting from the fit are defined as:

$$|y_i - f_1(x_i)| = c_i, \quad i = 1, 2, \ldots, m$$

Now, define $c_{\max}$ as the largest of the absolute deviations c_i. Thus $c_{\max}$ is the minimal largest absolute deviation obtainable and $c_i \le c_{\max}$ for every i.

On the other hand, suppose the least-squares criterion is applied and the resulting optimization problem solved to yield the function $f_2(x)$. The absolute deviations resulting from this fit are defined as:

$$|y_i - f_2(x_i)| = d_i, \quad i = 1, 2, \ldots, m$$

Define $d_{\max}$ as the largest of the absolute deviations d_i for every i. Because of the Chebyshev criterion $c_{\max} \le d_{\max}$ and because of the least-squares criterion, it must be true that

$$\sum_{i=1}^{m} d_i^2 \le \sum_{i=1}^{m} c_i^2$$

Since $c_i \le c_{\max}$ for every i, this implies

$$\sum_{i=1}^{m} d_i^2 \le \sum_{i=1}^{m} c_i^2$$

$$\sum_{i=1}^{m} d_i^2 \le m c_{\max}^2$$

$$\sqrt{\frac{\sum_{i=1}^{m} d_i^2}{m}} \le c_{\max}$$

For ease of discussion define

$$D = \sqrt{\frac{\sum_{i=1}^{m} d_i^2}{m}}$$

Thus, $D \le c_{\max} \le d_{\max}$ which is very revealing.

Suppose it is more convenient to apply the least-squares criterion, but there is concern about the largest absolute deviation $c_{\max}$ that may result. If we compute D, a lower bound on $c_{\max}$ is obtained, and $d_{\max}$ gives an upper bound. So, if there is a considerable difference between D and $d_{\max}$, the modeler should consider applying the Chebyshev criterion.

Applying the least-squares criterion to the Javelin (on-line) shot data for n=17 yields the parameter a=7.2613E-23 with S=248.76. Computing D and $d_{\max}$ yields $D = (S/m)^{1/2} = 3.1544$, while $d_{\max} = 8.1158$. Since the difference between D and $d_{\max}$ is not considerable, the assumption of applying the least-squares criterion is sufficiently valid and application of the Chebyshev criterion would provide little gain.

The problem of zeros for many of the data values is acute for the two models $y=Ax^N$ (N is variable) and $y=Ae^{Bx}$. Transformed least-squares fits of these models are obtained by applying the logarithm to both sides of the equations

$$y_i = ax_i^n \quad and \quad y_i = ae^{bx_i}$$

which yields

$$\ln(y_i) = \ln(a) + n\ln(x_i) \quad \text{and} \quad \ln(y_i) = \ln(a) + bx_i$$

Notice that for $y_i=0$, $\ln(y_i)$ does not exist. In addition, the transformation would not yield a least-squares fit to the original data, but to the transformed data. Thus, examination of these models is unproductive.

The one-term models were solved using a computer spreadsheet package. Because of the automatic calculation capability of the spreadsheet program, I could enter a value for n and immediately obtain the least-squares value for the constant "a" and the sum of squares value S_n. After testing other values such as $n-1$ and $n+1$, then comparing S_n to S_{n-1} and S_{n+1}, I could find a better fitting function. The exponent of the "best fit" function was the n that met the criteria

$$S_n \le S_{n-1} \text{ and } S_n \le S_{n+1}.$$

The following equations "best fit" the data for the one-term models:

Employ Method 1: *Javelin*, $y = (7.2613\times10^{-23})x^{17}$, *for* $0\le x\le 25$
ITV, $y = (1.2757\times10^{-59})x^{38}$, *for* $0\le x\le 40$

Employ Method 2: *Javelin*, $y = (2.6154\times10^{-34})x^{24}$, *for* $0\le x\le 30$
ITV, $y = (2.0236\times10^{-64})x^{41}$, *for* $0\le x\le 40$

Employ Method 3: *Javelin*, $y = (2.2417\times10^{-54})x^{35}$, *for* $0\le x\le 39$
ITV, $y = (3.3257\times10^{-53})x^{34}$, *for* $0\le x\le 40$

The following figures show the graphs of the one-term model fits to the data.

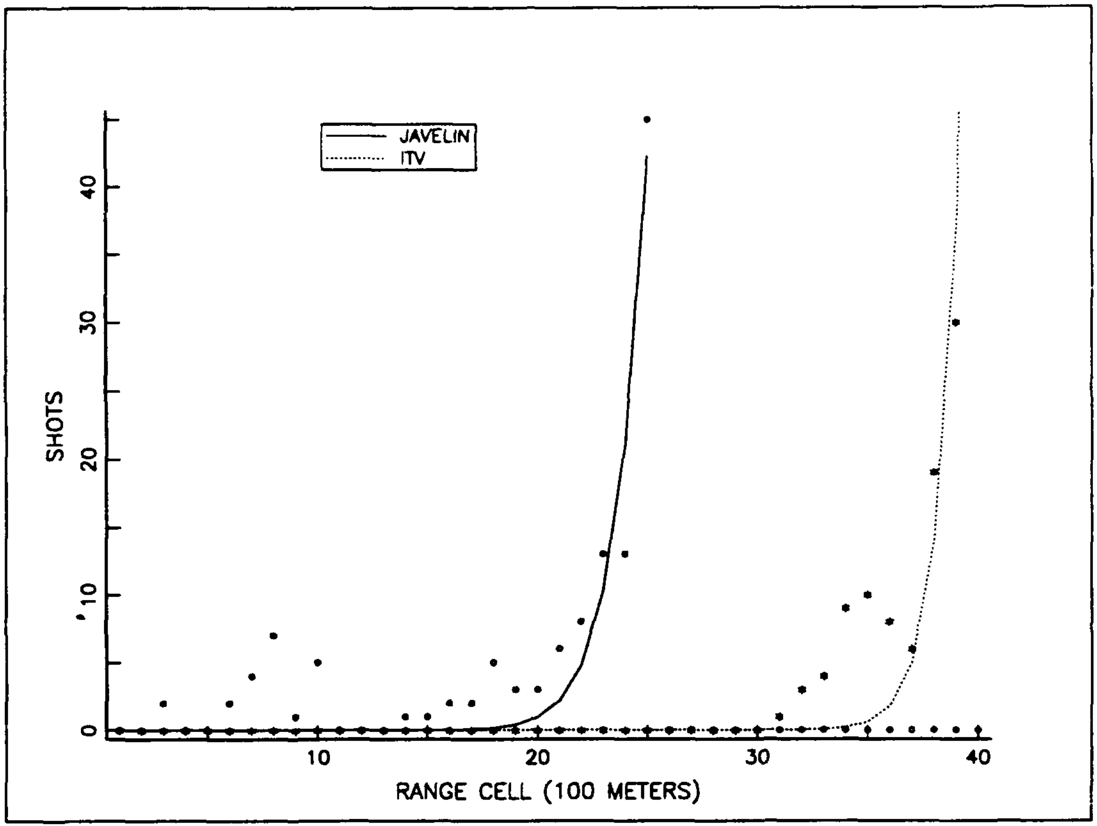

Figure 7. One-Term Model Fits (Employment Method #1)

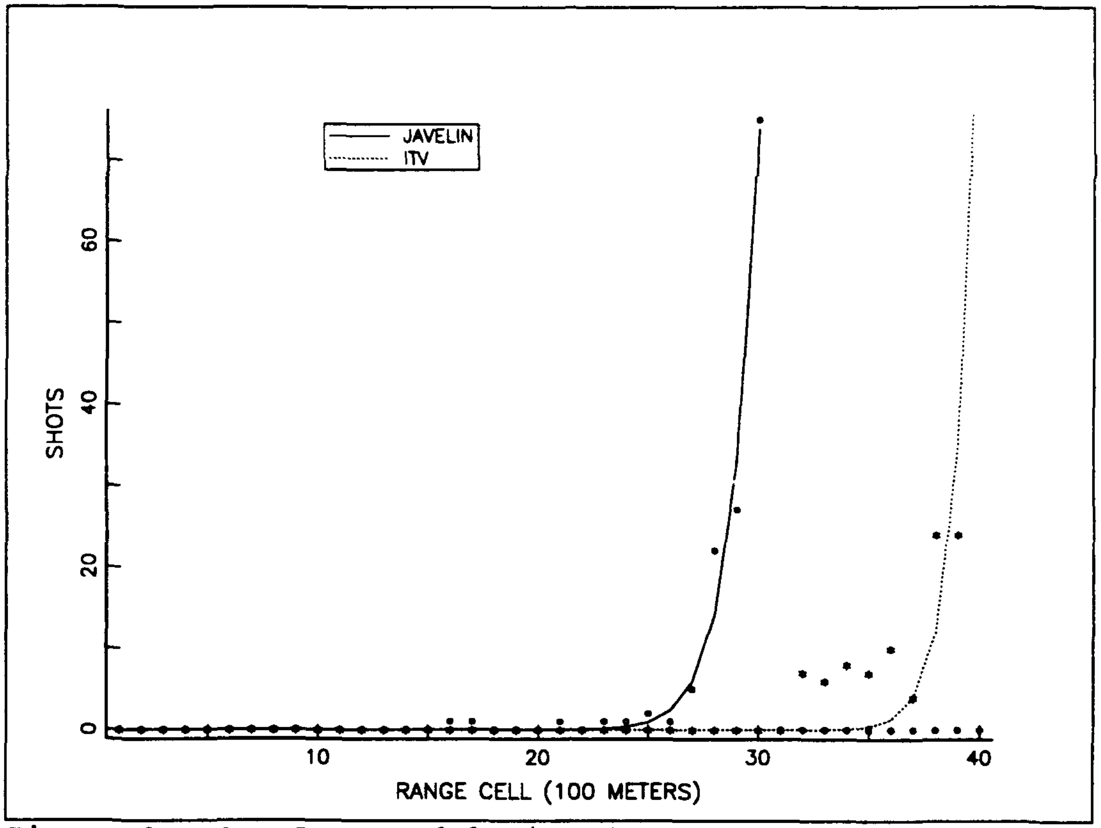

Figure 8. One-Term Model Fits (Employment Method #2)

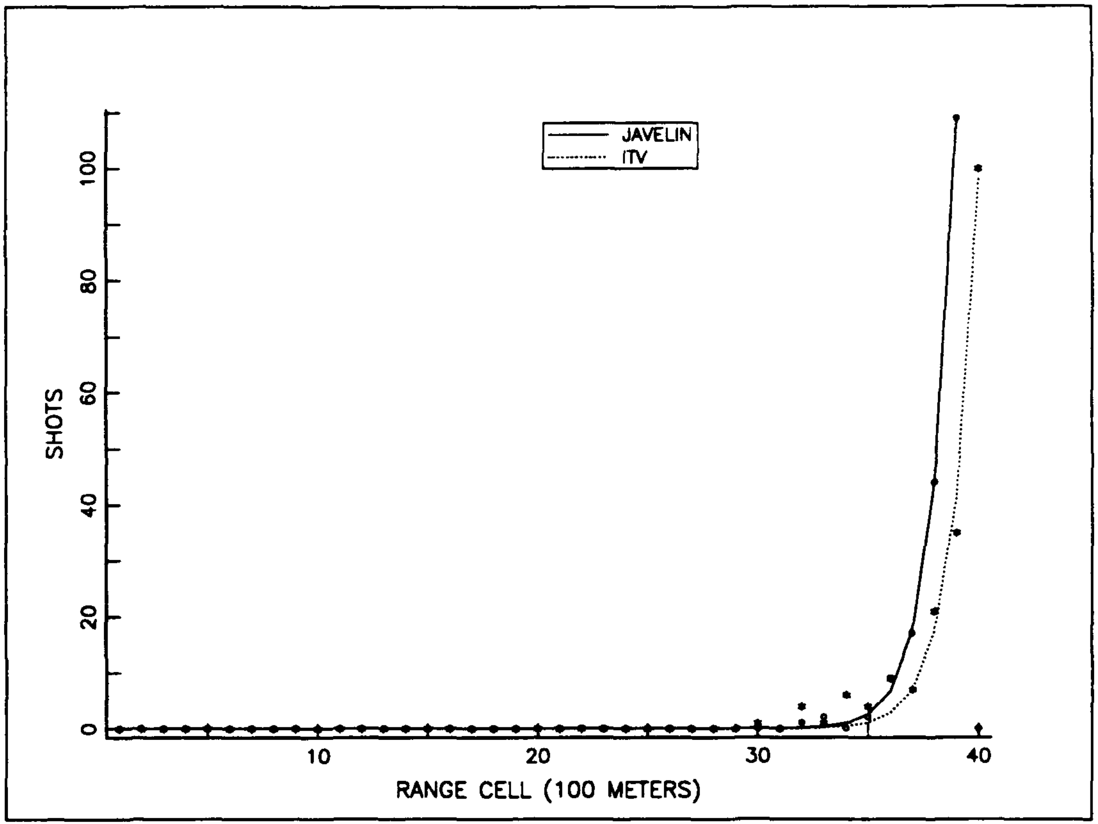

Figure 9. One-Term Model Fits (Employment Method #3)

b. Polynomial Models Using Divided Differences

We can construct a divided difference table for the data (See Appendix C, page 69). A divided difference table would suggest the order of the polynomial that best passes through a chosen subset of the data. Like the transformation technique, this method would also provide a starting point for determining a polynomial curve that might fit the data. The difference table could suggest which order polynomials would be poor models. Examination of the divided difference table reveals that the magnitude of values do not get small until after the fifth divided difference, suggesting that very low-order polynomials (less than fifth order) are not appropriate models.

For the polynomial models, generally the higher the degree of the function, the better the fit. I limited the degree to less than or equal to nine. Higher degree functions tend to oscillate more near the ends of the interval and tend to be more sensitive to small changes or errors in the input data. The models were solved on a computer with statistical and curve fitting software. The following figures show fitted polynomial curves of degree nine.

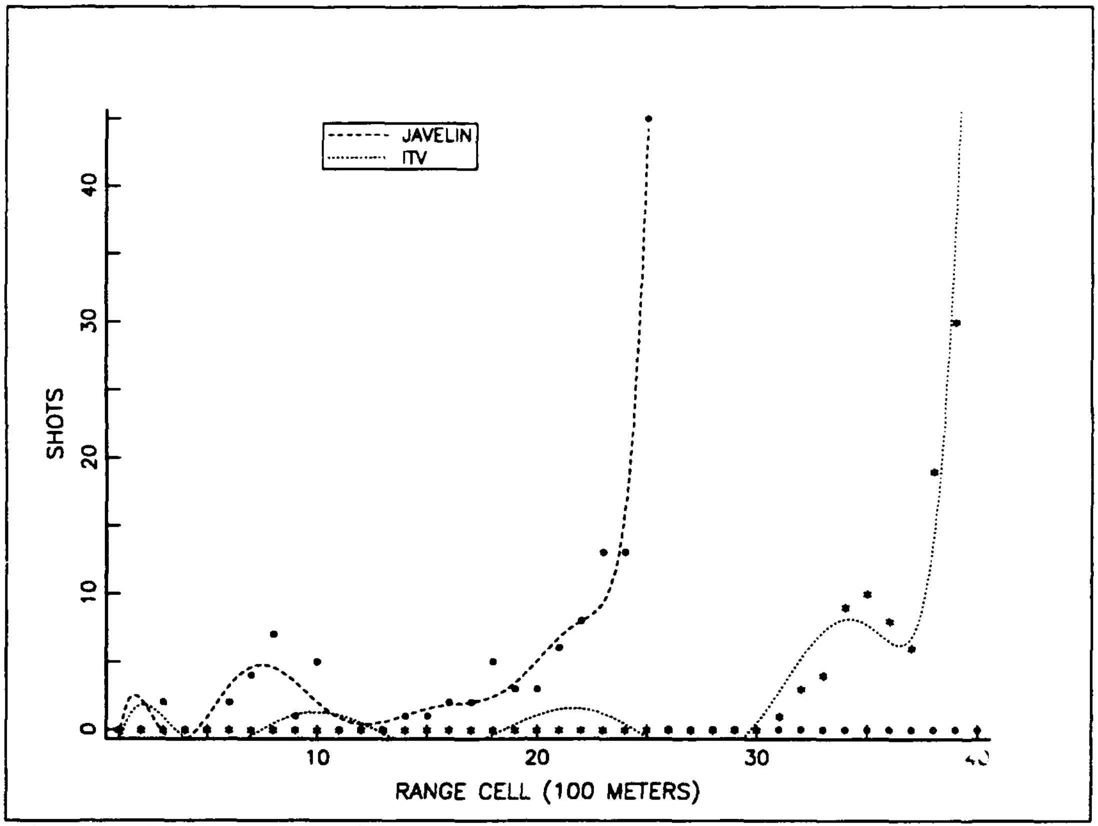

Figure 10. Polynomial Fits (Employment Method #1)

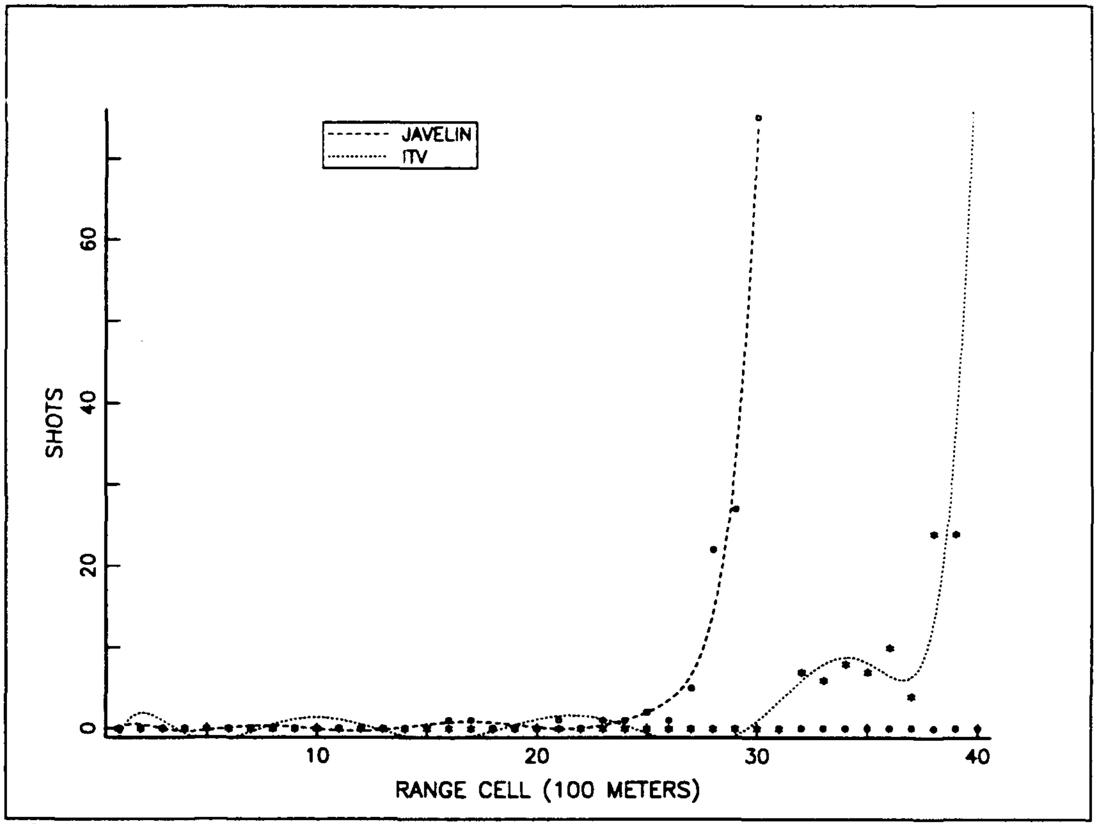

Figure 11. Polynomial Fits (Employment Method #2)

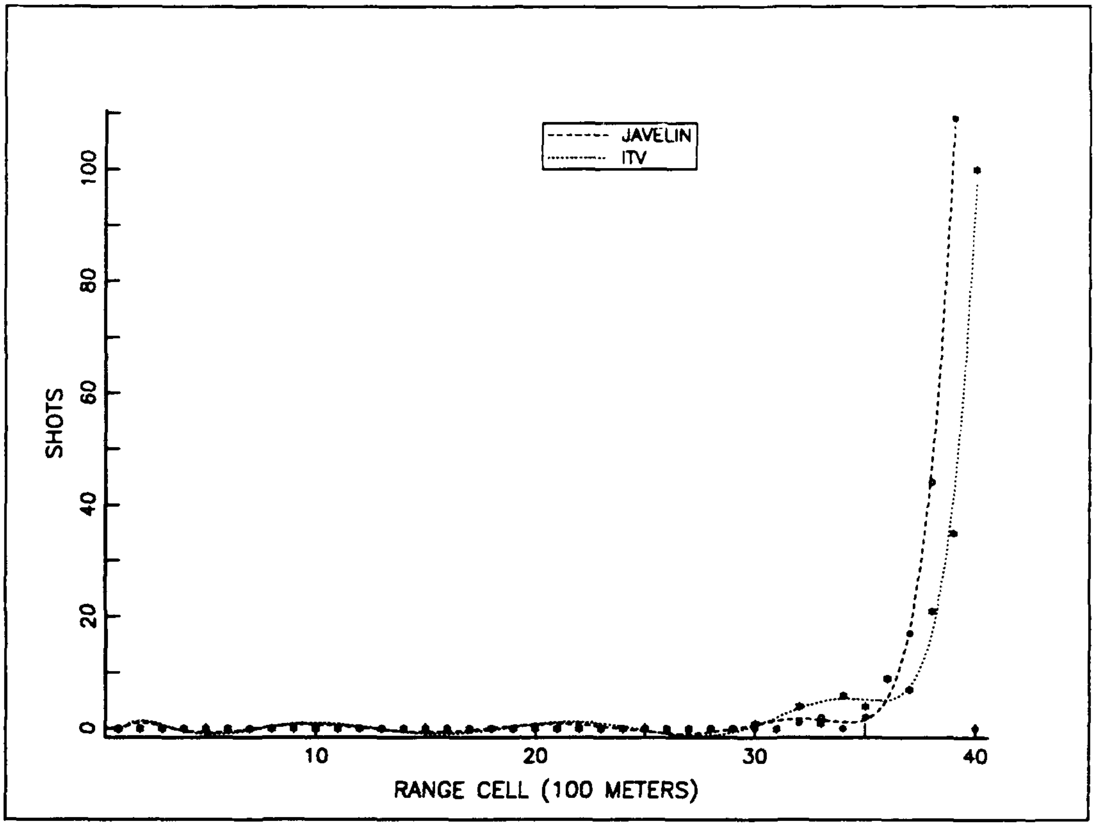

Figure 12. Polynomial Fits (Employment Method #3)

c. *Spline Models*

Spline interpolation techniques provide excellent models for making interpolating predictions based on the data. Allowing the variation in *y* to occur linearly between each range cell establishes a linear spline interpolation. When *x* is in the *i*th interval, where $x_i \le x < x_{i+1}$, the model used takes the form

$$S_i(x) = a_i + b_i x \qquad \text{for } x \in [x_i, x_{i+1})$$

The spline $S_i(x)$ must pass through the points (x_i, y_i) and (x_{i+1}, y_{i+1}). The values of a_i and b_i are found by solving the system of linear equations

$$\begin{gathered} a_i + b_i x_i = y_i \\ a_i + b_i x_{i+1} = y_{i+1} \end{gathered}$$

Figure 2, Figure 4, and Figure 6 display linear spline graphs.

A cubic spline model is a continuous function with continuous first and second derivatives consisting of cubic polynomial segments [Ref. 3:p. 204]. By using different cubic polynomials between successive pairs of data points, we can capture the trend of the data regardless of the nature of the underlying relationship [Ref. 3:p. 201]. The cubic splines are determined according to the following criteria:

- Each spline must pass through the two adjacent data points.
- At interior nodes, the first and second derivatives of adjacent splines are equal.

- If the two exterior splines have first derivatives at the end nodes specified to be constant known values, the spline type is clamped.
- If the exterior splines have second derivatives at the end nodes equal to zero: the first derivative assumes a constant "natural" value, the spline type is natural.

The values of the unknown coefficients are found by solving a system of linear equations. For large data sets, like the ones in this study, the solution of the cubic spline equations is best performed on a computer. The next three figures show the graphs of the cubic spline fits to the data.

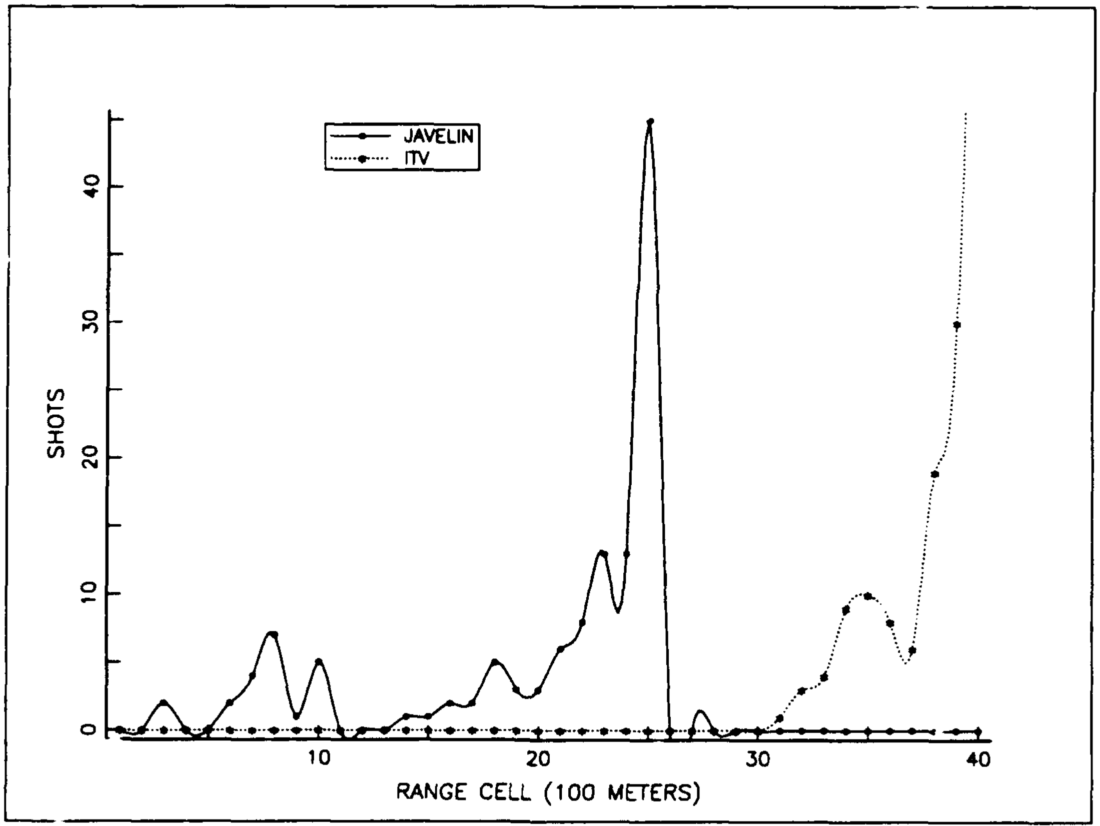

Figure 13. Cubic Spline Fits (Employment Method #1)

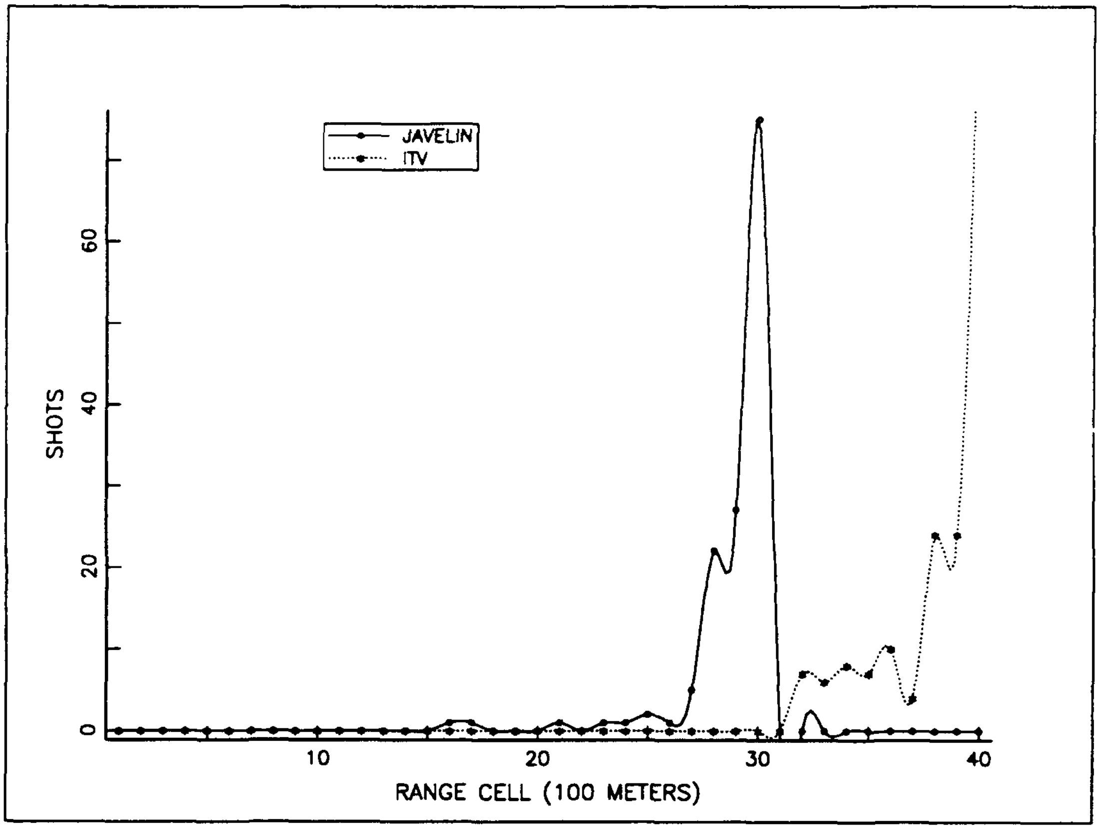

Figure 14. Cubic Spline Fits (Employment Method #2)

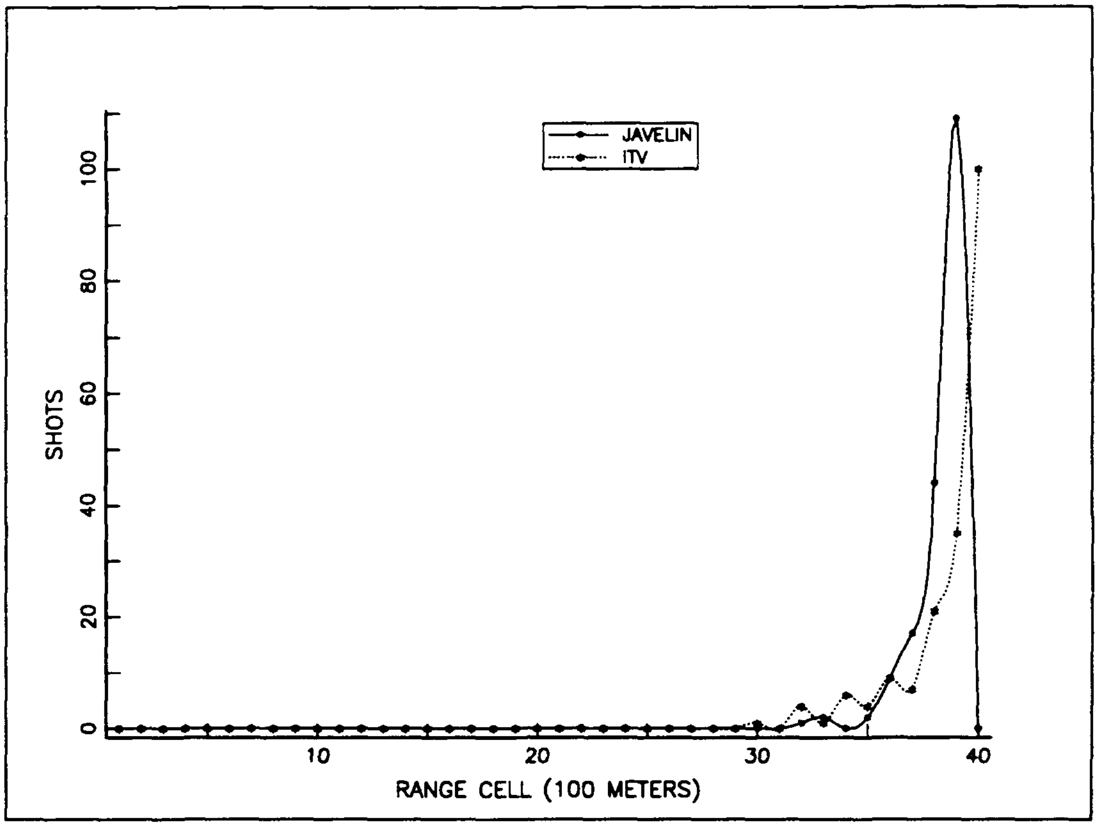

Figure 15. Cubic Spline Fits (Employment Method #3)

3. Model comparison and selection

Our measure of effectiveness is the amount of overlap in the shot signature. Let's clarify and refine the definition of this measure. Suppose we have a typical shot signature S with two curves A and B as shown in Figure 16.

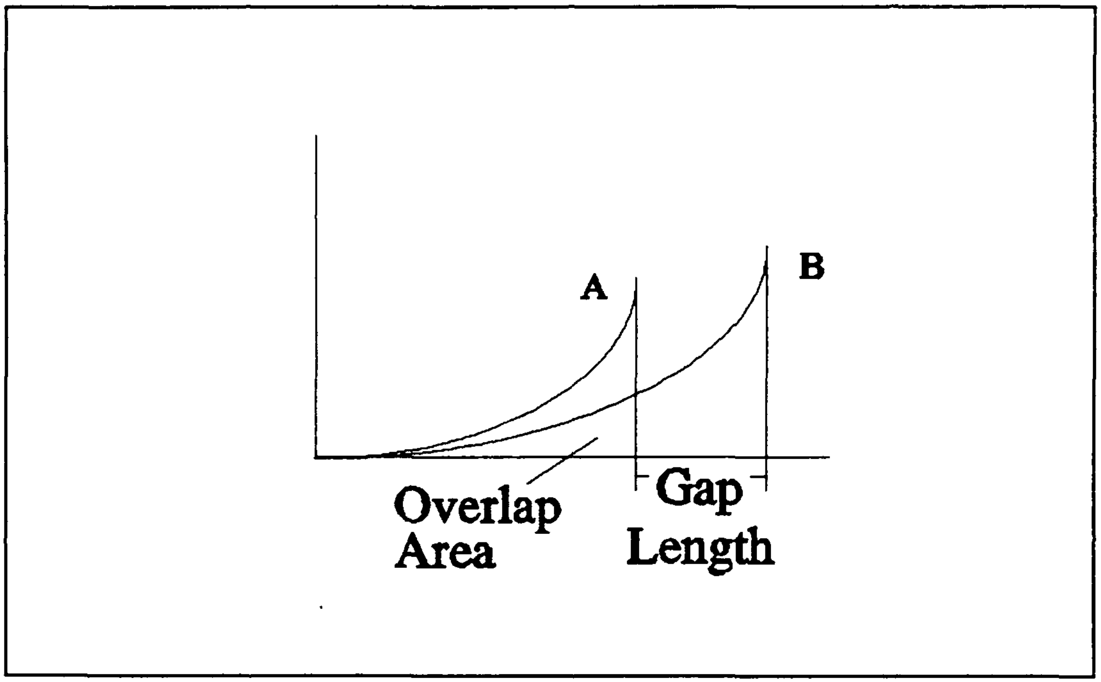

Figure 16. Typical Shot Signature

Two quantifiable attributes of the signature are as follows:

- GAP LENGTH - the horizontal distance (measured in meters) from the highest point (peak) on one curve to the highest point (peak) on the other curve. This distance can be determined either by visual inspection or by solving for the maximum value of each curve function.
- OVERLAP AREA - the amount of intersection of the areas under the curves. This amount can be determined by integrating over the area of intersection.

If we relate these two attributes to weapon system effectiveness, we can measure and compare signatures.

Gap length is a measure of the sparsely covered portion of the battle zone discussed earlier in this chapter. Small gap values are better than large values. The smaller the value, the narrower the sparsely covered portion. This narrowing means that the maximum range of the Javelin shifts closer to the forward edge of the battle zone. This effect increases the number of anti-armor weapons that can engage the enemy near the forward edge (an important aspect of anti-armor defense). More importantly, this effect allows Javelin to better support the ITV in the engagement. A large gap value means that the Javelin and ITV systems, though employed as a combined system, are not able to engage the opposing force as a combined system (i.e., simultaneously). Thus, the smaller the gap value, the more effective the tactical employment. When comparing two shot signatures, the signature with the smaller gap length value has the more effective employment. If two shot signatures have nearly equal gap length values, we then compare overlap area.

Overlap area is a measure of the increasing volume of fire discussed in the anti-armor warfare section in Chapter II. Using similar reasoning as with gap length, we conclude that large overlap area values are better than small values. The larger the overlap area value, the more effective the employment. If two shot signatures have equal gap length values and equal overlap area values, then we conclude the effectiveness of their employments are equal.

It is now simple to establish our criterion for model selection. We select the model that best (easily) allows for determining the measure of a shot signature. For example, suppose we have only the model equations (as on page 35 for the one-term model) and we do not have graphs of any of the signatures. Of the four model alternatives (one-term, polynomial, linear spline, and cubic spline) the easiest model to solve for its maximum value is the one-term model. For the polynomial and cubic spline models, we must take derivatives, set them equal to zero and then solve for the root of that equation. In the case of cubic splines, we must perform this same procedure piece-wise for each spline. In either case the procedure is time consuming. For the linear spline, we must compare all the spline segment end-points (that is, find the largest value of the independent variable) and find the dependent value to which it corresponds. For the one-term model (a strictly monotone increasing continuous function over its interval), the maximum value always occurs at the right boundary of the dependent variable interval (proof omitted).

Of the four alternative models, the one-term model is, by its nature, the easiest to integrate. In addition, we can answer other performance questions using the one-term model.

For example, to determine the target ranges where shots will occur, we simply solve the equations for y greater than or equal to one.

$$1 \leq y = ax^{n}$$

$$1 \leq ax^{n}$$

$$(a^{-1})^{\frac{1}{n}} \leq x.$$

Thus, for $1 \leq y$ the lower bound on x yields:

Employ Method 1: *Javelin*, $20.06 \leq x \leq 25$
ITV, $35.47 \leq x \leq 40$

Employ Method 2: *Javelin*, $25.08 \leq x \leq 30$
ITV, $35.77 \leq x \leq 40$

Employ Method 3: *Javelin*, $34.11 \leq x \leq 39$
ITV, $34.95 \leq x \leq 40$

The following advantages and limitations pertain specifically to measuring our defined attributes and may not apply or be true in general. Keep in mind that each shot signature contains two function curves that may or may not intersect and if so, then at one or more points.

ONE-TERM MODEL

Advantages:

- The model captures the trend of the data in a simple form.
- Finding the maximum value is simple because the functions are strictly monotone increasing over the defined intervals.
- Finding the overlap area is relatively easy. We simply integrate the functions over the appropriate interval. This can be done without using a computer.

Limitation:

- A loss of fidelity occurs because the curves do not pass through every data point.

POLYNOMIAL MODEL

Advantages:

- In general, polynomial functions are relatively easy to integrate and differentiate. However, finding the maximum value without a graph of the signature includes solving for the roots of the first derivatives. Finding the overlap area by integration is easy if the curves do not intersect.

Neutral:

- The curves may or may not pass through every data point.

Limitations:

- Finding the overlap area includes searching for points of intersection (if the curves intersect) and integrating multiterm functions piece-wise over the interval.

LINEAR SPLINE MODEL

Advantage:

- The curves pass through every data point.

Limitations:

- The functions are systems of equations. Their curves are not smooth and differentiable at every point of the interval. Finding the maximum value over the interval includes comparing all the spline segment end-points.

- Finding the overlap area includes integrating piece-wise over each spline segment.

CUBIC SPLINE MODEL

Advantage:

- The curve passes through every data point.

Limitations:

- The function is a system of equations. Finding the maximum value without a graph may include differentiating piece-wise over each spline segment.
- Finding the overlap area includes integrating piece-wise over each spline segment.

After considering the advantages and limitations of each model, we conclude that the one-term model best allows for determining the measure of a shot signature. So, we select the one-term model for the analysis.

$$\textit{Employ Method 1:}\quad \textit{Gap} = 4000 - 2500 = 1500\ \textit{meters}$$

$$\textit{Overlap} = \int_0^{25} (1.2757E\text{-}59)\, x^{38}\, dx$$

$$= (1.2757E\text{-}59)\, \frac{x^{39}}{39}\Big|_0^{25}$$

$$= (1.2757E\text{-}59)\, \frac{25^{39}}{39} \approx 0$$

$$\textit{Employ Method 2:}\quad \textit{Gap} = 4000 - 3000 = 1000\ \textit{meters}$$

$$\textit{Overlap} = \int_0^{30} (2.0236E\text{-}64)\, x^{41}\, dx$$

$$= (2.0236E\text{-}64)\, \frac{30^{42}}{42} = 0.0005$$

$$\textit{Employ Method 3:}\quad \textit{Gap} = 4000 - 3900 = 100\ \textit{meters}$$

$$\textit{Overlap} = \int_0^{39} (3.3257E\text{-}53)\, x^{34}\, dx$$

$$= (3.3257E\text{-}53)\, \frac{39^{35}}{35} = 46.25$$

C. SUMMARY

Based on mathematical modeling analysis, we can compare the shot distributions in our study. We find that the Javelin and ITV combination using employment method #3 is more effective than the same combination using employment methods #1 and #2, the same results reached using a statistical analysis.

V. CONCLUSIONS

> Leaders must understand the techniques of controlling and integrating all available fires. They must understand the capabilities of their weapons and supporting weapons. They must be experts at positioning and employing these systems [Ref. 7:para 1-5.b.].

The result of this study was to present a method to help the Army determine the best employment method for the Javelin weapon system. In doing so, this study presented an examination of weapon system effectiveness. It examined how changes in the combination and tactical employment of long and medium range anti-armor weapon systems influence their lethality and survivability. Specifically, this study compared the effectiveness of the Javelin, with the Dragon, using measures of performance outlined in the Test and Evaluation Plan. Additionally, this study presented a more focused examination of Javelin's tactical employment.

Combat engagements were simulated using Janus(A), a high resolution combat simulation. Data results, generated from the scenario runs, were analyzed using statistical, graphical, and mathematical methods. A major assumption of this study was that Janus(A) simulation adequately approximates the very complex processes found in real combat engagements. Confidence in the generated data depended on the validity of this assumption. Proof of this validity is beyond the scope of this study. Nonetheless, interpretation of the statistical and mathematical analysis is limited to the data in this study. The conclusions pertain only to the models used. More research could validate the conclusions by testing them against new observations and data.

Using statistical analysis techniques, the data from the experiment presented sufficient evidence to indicate a difference in the effectiveness of the compositions and employment methods. The composition factor was quite significant. Results revealed that the Javelin/ITV combination is much more effective in anti-armor defense than the Dragon/ITV combination and that a significant interaction exists between the factors of composition and employment method. Interaction suggests that the effectiveness of a combined system is influenced by the composition of forces and the employment method.

Further statistical analysis revealed that a Javelin/ITV combination employment with the Javelin systems positioned 1500 meters forward of the ITV systems was more effective than an employment with the Javelin systems on-line with the ITV or only 500 meters forward of the ITV. Analysis of a shot signature using mathematical modeling techniques provided deeper insight. The performance measures were gap length and overlap area. The existence of a large gap meant that the medium and long range weapon systems, though employed as a combined system, were not able to engage the opposing force as a combined system. Existence of an overlap meant that over several contiguous target ranges, both systems fired shots at the enemy and he encountered an increased volume of fire. This effect increased the combined system's ability to defeat armor forces and survive, thus increasing its effectiveness.

APPENDIX A: INITIAL EMPLOYMENT SETTINGS

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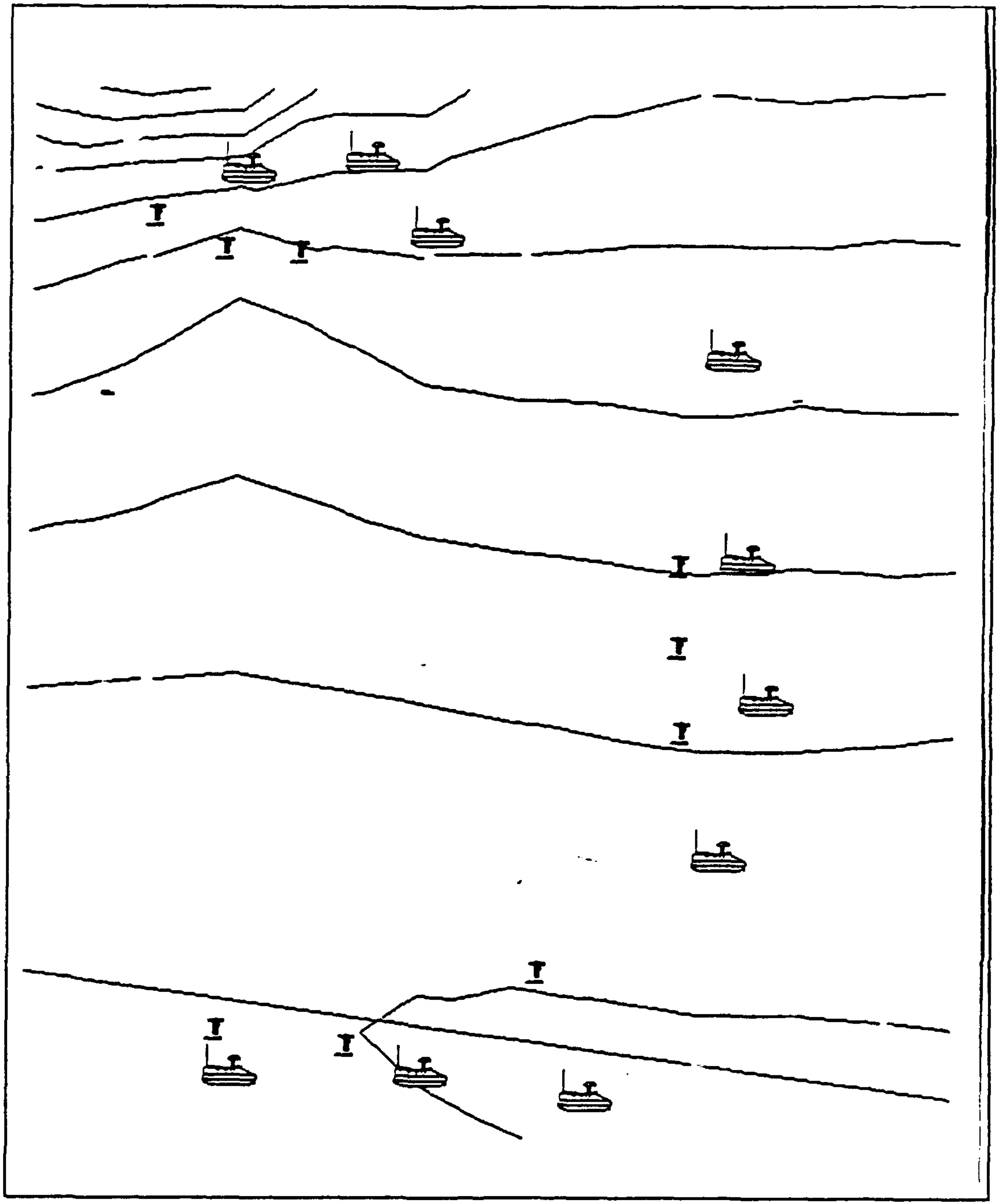

Figure 17. Weapon Systems (Employment Method #1)

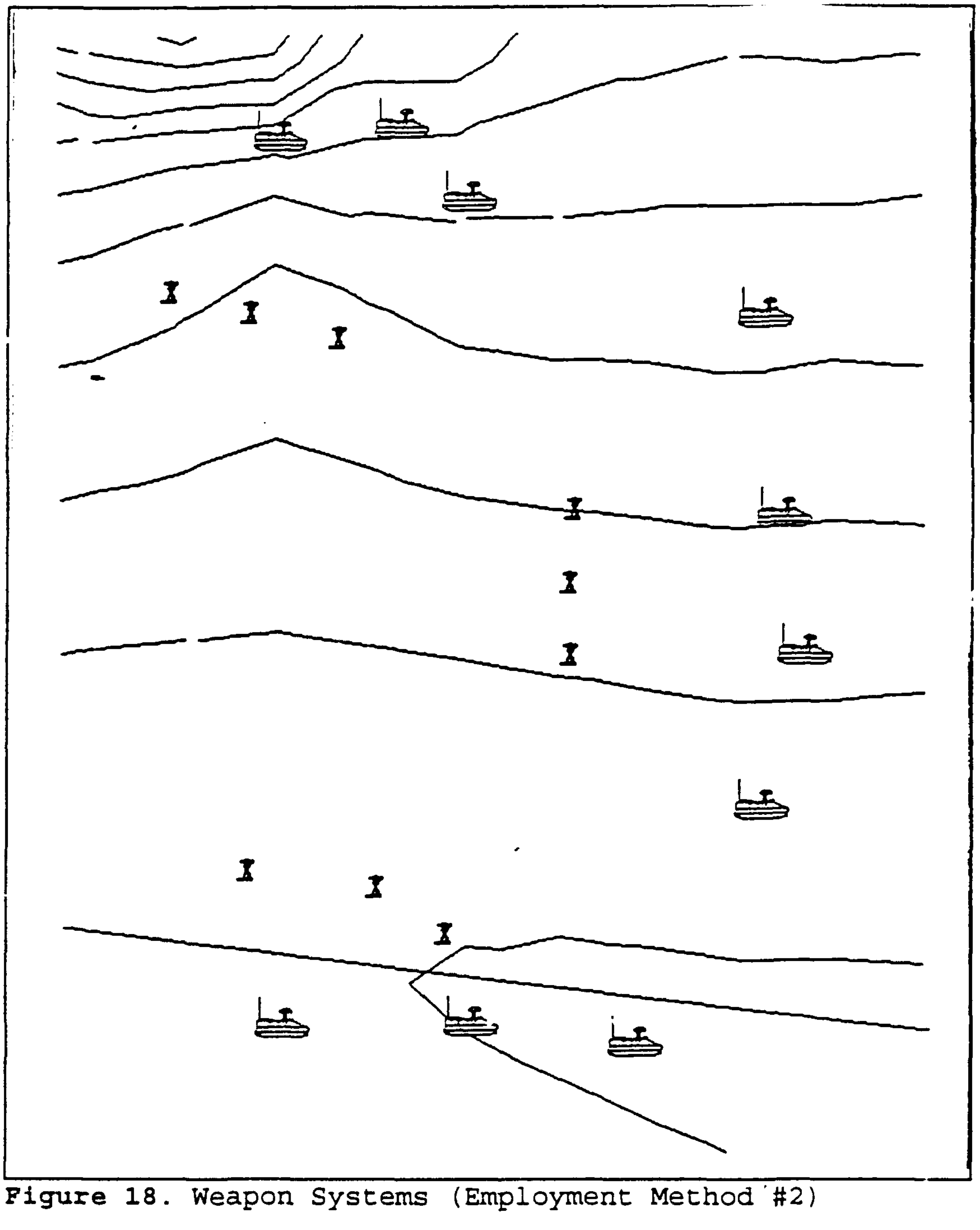

Figure 18. Weapon Systems (Employment Method #2)

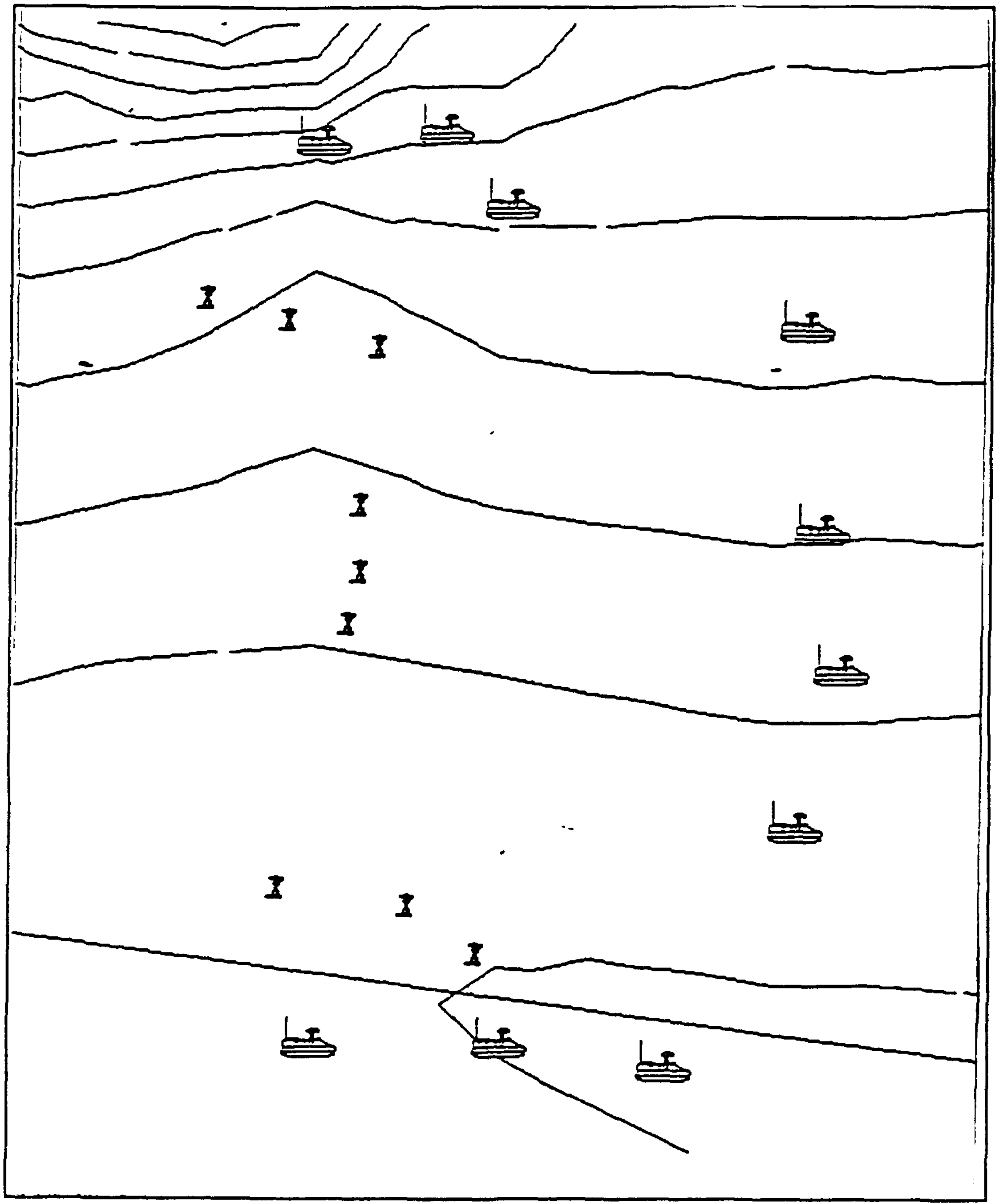

Figure 19. Weapon Systems (Employment Method #3)

APPENDIX B: DESIGN OF EXPERIMENTS

TABLE IX. 3 X 2 FACTOR DESIGN, WITH REPLICATIONS

observation = $y(\text{factor}_1, \text{factor}_2, \text{replication})$

	factor$_1$		
	A	B	
factor$_2$ 1	y_{A11} y_{A12} . . . y_{A1r} . . . $y_{A,1,10}$	y_{B11} y_{B12} . . . y_{B1r} . . . $y_{B,1,10}$	r=replications=10
factor$_2$ 2	y_{A21} y_{A22} . . . $y_{A,2,10}$	y_{B21} y_{B22} . . . $y_{B,2,10}$	
factor$_2$ 3	y_{A31} y_{A32} . . . $y_{A,3,10}$	y_{B32} y_{B32} . . . $y_{B,3,10}$	

TABLE X. 3 X 1 FACTOR DESIGN, WITH REPLICATIONS

observation = $y(\text{treatment}_i, \text{replication})$

treatment_1	y_{11}	
	y_{12}	
	.	
	.	
	.	
	y_{1r}	r=replications=10
	.	
	.	
	.	
	$y_{1,10}$	
treatment_2	y_{21}	
	y_{22}	
	.	
	.	
	.	
	$y_{2,10}$	
treatment_3	y_{31}	
	y_{32}	
	.	
	.	
	.	
	$y_{3,10}$	

SUM OF SQUARES FORMULAS AND ANOVA TABLES

$$S_E = rk\sum_{i} (\overline{y_i} - \overline{y})^2$$

$$S_C = rn\sum_{t} (\overline{y_t} - \overline{y})^2$$

$$S_I = r\sum_{t}\sum_{i} (\overline{y_{ti}} - \overline{y_t} - \overline{y_i} + \overline{y})^2$$

$$S_e = \sum_{t}\sum_{i}\sum_{j} (y_{tij} - \overline{y_{ti}})^2$$

$$S = \sum_{t}\sum_{i}\sum_{j} (y_{tij} - \overline{y})^2$$

TABLE XI. ANOVA TABLE (3 X 2 DESIGN)

source of variation	sum of squares	degrees of freedom	mean square	ratio of mean squares
employment s^2_E/s^2_e	S_E	n-1	$s^2_E=S_E/(n-1)$	
composition s^2_C/s^2_e	S_C	k-1	$s^2_C=S_C/(k-1)$	
interaction s^2_I/s^2_e	S_I	(n-1)(k-1)	$s^2_I=S_I/(n-1)(k-1)$	
error	S_e	nk(r-1)	$s^2_e=S_e/nk(r-1)$	
total	S	nkr-1		

TABLE XII. ANOVA TABLE (3 X 1 DESIGN)

source of variation	sum of squares	degrees of freedom	mean square	ratio of mean squares
employment s^2_E/s^2_e	S_E	n-1	$s^2_E=S_E/(n-1)$	
error	S_e	n(r-1)	$s^2_e=S_e/n(r-1)$	
total	S	nr-1		

APPENDIX C: GENERATED DATA

TABLE XIII. LOSS EXCHANGE RATIO (LER) DATA

	Compositions (with ITV)	
	A (Dragon)	B (Javelin)
Employment 1 (on-line)	1.86	1.86
	0.37	0.87
	0.37	1.44
	0.42	1.08
	0.26	1.44
	0.42	1.44
	0.21	0.93
	0.26	1.00
	0.47	0.80
	0.37	2.60
Employment 2 (500m shift)	0.11	1.63
	0.26	1.18
	0.21	1.44
	0.21	2.17
	0.47	1.18
	0.32	1.30
	0.53	2.17
	0.37	1.30
	0.47	1.18
	0.26	1.18
Employment 3 (1500m shift)	0.26	1.44
	0.11	2.17
	0.37	3.25
	0.21	2.60
	0.16	1.86
	0.21	2.60
	0.21	1.44
	0.21	3.25
	0.37	1.63
	0.37	1.86

TABLE XIV. FORCE EXCHANGE RATIO (FER) DATA

	Compositions (with ITV)	
	A (Dragon)	B (Javelin)
Employment 1 (on-line)	2.71	2.71
	0.54	1.27
	0.54	2.11
	0.62	1.58
	0.38	2.11
	0.62	2.11
	0.31	1.36
	0.38	1.46
	0.69	1.17
	0.54	3.80
Employment 2 (500m shift)	0.15	2.38
	0.38	1.73
	0.31	2.11
	0.31	3.17
	0.69	1.73
	0.46	1.90
	0.77	3.17
	0.54	1.90
	0.69	1.73
	0.38	1.73
Employment 3 (1500m shift)	0.38	2.11
	0.15	3.17
	0.54	4.75
	0.31	3.80
	0.23	2.71
	0.31	3.80
	0.31	2.11
	0.31	4.75
	0.54	2.38
	0.54	2.71

TABLE XV. SURVIVAL DATA

	Compositions (with ITV)	
	A (Dragon)	B (Javelin)
Employment 1 (on-line)	12	12
	0	4
	0	10
	0	7
	0	10
	0	10
	0	5
	0	5
	0	4
	0	14
Employment 2 (500m shift)	0	11
	0	8
	0	10
	0	13
	0	8
	0	9
	0	13
	0	9
	0	8
	0	8
Employment 3 (1500m shift)	0	10
	0	13
	0	15
	0	14
	0	12
	0	14
	0	11
	0	15
	0	11
	0	12

TABLE XVI. DISTRIBUTION OF SHOTS DATA - PART 1

	Employment Method					
	Number 1		Number 2		Number 3	
Range Cell	Javelin	ITV	Javelin	ITV	Javelin	ITV
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	2	0	0	0	0	0
4	0	0	0	0	0	0
5	0	0	0	0	0	0
6	2	0	0	0	0	0
7	4	0	0	0	0	0
8	7	0	0	0	0	0
9	1	0	0	0	0	0
10	5	0	0	0	0	0
11	0	0	0	0	0	0
12	0	0	0	0	0	0
13	0	0	0	0	0	0
14	1	0	0	0	0	0
15	1	0	0	0	0	0
16	2	0	1	0	0	0
17	2	0	1	0	0	0
18	5	0	0	0	0	0
19	3	0	0	0	0	0
20	3	0	0	0	0	0

TABLE XVII. DISTRIBUTION OF SHOTS DATA - PART 2

	Employment Method					
	Number 1		Number 2		Number 3	
Range Cell	Javelin	ITV	Javelin	ITV	Javelin	ITV
21	6	0	1	0	0	0
22	8	0	0	0	0	0
23	13	0	1	0	0	0
24	13	0	1	0	0	0
25	45	0	2	0	0	0
26	0	0	1	0	0	0
27	0	0	5	0	0	0
28	0	0	22	0	0	0
29	0	0	27	0	0	0
30	0	0	75	0	0	1
31	0	1	0	0	0	0
32	0	3	0	7	1	4
33	0	4	0	6	2	1
34	0	9	0	8	0	6
35	0	10	0	7	2	4
36	0	8	0	10	9	9
37	0	6	0	4	17	7
38	0	19	0	24	44	21
39	0	30	0	24	109	35
40	0	98	0	100	0	100

TABLE XVIII. DIVIDED DIFFERENCE TABLE

Data		Divided Differences				
x_i	y_i	Δ	Δ^2	Δ^3	Δ^4	Δ^5
1	0					
2	0	0.000				
3	2	2.000	1.000			
4	0	-2.000	-2.000	-1.000		
5	0	0.000	1.000	1.000	0.500	
6	2	2.000	1.000	0.000	-0.250	-0.150
7	4	2.000	0.000	-0.333	-0.083	0.033
8	7	3.000	0.500	0.167	0.125	0.042
9	1	-6.000	-4.500	-1.667	-0.458	-0.117
10	5	4.000	5.000	3.167	1.208	0.333
11	0	-5.000	-4.500	-3.167	-1.583	-0.558
12	0	0.000	2.500	2.333	1.375	0.592
13	0	0.000	0.000	-0.833	-0.792	-0.433
14	1	1.000	0.500	0.167	0.250	0.208
15	1	0.000	-0.500	-0.333	-0.125	-0.075
16	2	1.000	0.500	0.333	0.167	0.058
17	2	0.000	-0.500	-0.333	-0.167	-0.067
18	5	3.000	1.500	0.667	0.250	0.083
19	3	-2.000	-2.500	-1.333	-0.500	-0.150
20	3	0.000	1.000	1.167	0.625	0.225
21	6	3.000	1.500	0.167	-0.250	-0.175
22	8	2.000	-0.500	-0.667	-0.208	0.083
23	13	5.000	1.500	0.667	0.333	0.108
24	13	0.000	-2.500	-1.333	-0.500	-0.167
25	45	32.000	16.000	6.167	1.875	0.475
26	0	-45.000	-38.500	-18.167	-6.083	-1.592
27	0	0.000	22.500	20.333	9.625	3.142
28	0	0.000	0.000	-7.500	-6.958	-3.317
29	0	0.000	0.000	0.000	1.875	1.767
30	0	0.000	0.000	0.000	0.000	-0.375
31	0	0.000	0.000	0.000	0.000	0.000
32	0	0.000	0.000	0.000	0.000	0.000
...				...		
40	0	0.000	0.000	0.000	0.000	0.000

APPENDIX D: WEAPON SYSTEM MODELING IN JANUS(A)

A. BACKGROUND

Janus(A) is a high-resolution interactive ground combat computer simulation model. It represents weapons and weapon systems with a relational data structure. In Janus(A), a weapon is a device specifically designed to fire a projectile. A weapon system (or simply system) is a weapon or collection of weapons combined with a human gunner, wheeled or tracked vehicle, or aircraft used as a means of mobility. [For example, the Javelin weapon is designed to fire an anti-armor missile. The Javelin weapon system is the Javelin weapon together with a soldier who carries and fires the weapon. Another example is the ITV weapon system, a M113A1 (an armored, tracked chassis) modified to carry a TOW weapon and an automatic weapon.] By modifying the relationships and data that define an existing weapon or system in Janus, the user can represent new weapons or systems. The details required to change specific relationships depend on the amount, quality and availability of performance data. Limited information restricts the amount of detail that can be incorporated into model data. (For example, modeling new weapons often relies on performance estimates or design specifications and prototypes may not exist to make actual measurements.) The problem is often severe for information on probability of hit-and-kill data because the values depend heavily on how actual soldiers employ the new weapon in a particular battle. However, there is typically no experience to estimate the probabilities, so the data originates from detailed engineering models instead. These are products of the weapon development process so they may not reflect actual weapon performance during a battle. The usual procedure is to get the best data available, model the situation as precisely as possible, and attempt to understand the implications of what is left out or assumed away.

B. GENERAL PROCEDURES

Before you make any changes, ensure you are using the Developmental Database, not the Master Database. At the Janus Data Development and Administration menu, select the Change Data Base (CD) option. Do not make changes to the master database.

The following procedure uses existing weapons and systems as a starting point and makes modifications where needed to define new relationships. There are five primary steps:

- Modify a copy of an existing weapon definition, creating a new weapon.
- Define new probability of hit (Ph) and probability of kill (Pk) relationships for the new weapon.

- Link the new weapon definition to the new Ph/Pk relationships.
- Modify an existing system definition to create a new system.
- Link the new system definition with the appropriate new weapon definition.

1. Modify an existing weapon definition

Step 1. From the Janus Data Development and Administration menu, select the Combat Systems Data Base (DD) option. The Combat Systems Data Editor provides access to a series of programs used to view and modify weapon performance characteristics and system definitions. You can assign weapon definition numbers in similar fashion to assigning systems. A procedure for creating a new weapon definition is to find an equivalent definition that already exists. This can be done by selecting the appropriate weapon from the old, or base-case system.

a. Review weapons' association

Step 2. Select the SYSTEMS (SY) option.

Step 3. Select the Weapons and Ordnance (WW) option.

Step 4. Select the side and system number. Viewing the data for the system selected, you will see the following six categories of entries·

- Wpn/Ord Number - Relative: There can be up to 15 weapons associated with each system.
- Wpn/Ord Number - Absolute: There may be up to 250 weapon definitions.
- Wpn/Ord Name
- Basic Load
- Upload Time
- Rel Wpn/Ord to use if Ammo Expended: This means that if one weapon runs out of ammunition, then an alternative can be selected.

Step 5. Review the weapons for the system you entered. Find a weapon that is roughly equivalent to the new weapon you want to create.

Step 6. Note and remember the weapon's Absolute Weapon Number; you will need this number later. Next, find an Absolute Weapon Number that does not currently contain a weapon definition. This step is important because if you copy a new definition to an existing weapon definition, you overwrite and lose the old definition.

Step 7. Exit to the Combat Systems Data Editor menu.

Step 8. Select the WEAPONS (WP) option.

<u>Step 9.</u> Select the Characteristics (CC) option. Notice that there is no weapon definition corresponding to some Absolute Weapon Numbers. This means that this number is available for a new weapon definition and its use will not conflict with any other weapon definitions.

<u>Step 10.</u> Note and remember an available number.

<u>Step 11.</u> Exit to the Combat Systems Data Editor menu.

b. Copy a weapon definition

<u>Step 12.</u> Select the UTILITIES (UU) option.

<u>Step 13.</u> Select the Copy Weapon (CW) option.

<u>Step 14.</u> Enter the side (B or R).

<u>Step 15.</u> Enter the FROM: (Old Absolute Weapon number).

<u>Step 16.</u> Enter the TO: (New, unused Absolute Weapon number).

<u>Step 17.</u> Exit to the UTILITIES menu.

c. Check the new weapon definition

<u>Step 18.</u> Select the Differences of Weapons (DW) option.

<u>Step 19.</u> Enter the side (B or R).

<u>Step 20.</u> Enter the First System: (Old Absolute Weapon number).

<u>Step 21.</u> Enter the Second System: (New Absolute Weapon number). Press ENTER to run the comparison function.

Results will appear on the monitor and show that there are no differences between the two weapons.

Step 22. Exit to the Combat Systems Data Editor menu.

2. Define new Ph/Pk relationships

The definition of a weapon has two parts. First, there is information that defines the effect the weapon will have on a potential target. This information includes probability of hit-and-kill data. Second, there is information that defines the weapon's attributes, whatever the target (speed of round, basic load, etc.).

The Janus database allows a total of 4000 Ph/Pk data sets to define the relationship between weapons and target systems. These probabilities are expressed as a function of the range between the firing weapon and the target. By convention, you should define new Ph and Pk tables with identical table numbers. Thus, Ph table 1285 relates to Pk table 1285. This makes it easier to check for errors. All current database probability tables use numbers less than 2000. Thus, you can employ data set numbers from 2000-4000 without changing existing data structures.

a. Enter Ph data

Step 1. Select the WEAPONS (WP) option from the Combat Systems Data Editor menu.

Step 2. Select the PH Data Sets (PH) option.

Step 3. Enter 2000. Observe that this data set contains no entries. If it does, exit the data set and select 2001. Continue until you arrive at a data set with no entries.

Janus uses a piecewise continuous function composed of four line segments to describe the probability of hit for a given weapon as a function of range, measured in meters from the firing weapon to the target. Suppose a new weapon has a Ph value of 0.95 at 500 meters, 0.80 at 1000 meters, 0.75 at 1500 meters, 0.60 at 2000 meters, and 0.50 at 2500 meters. Figure 20 shows a graph of the four line segments that approximate the probability of hit for the new weapon.

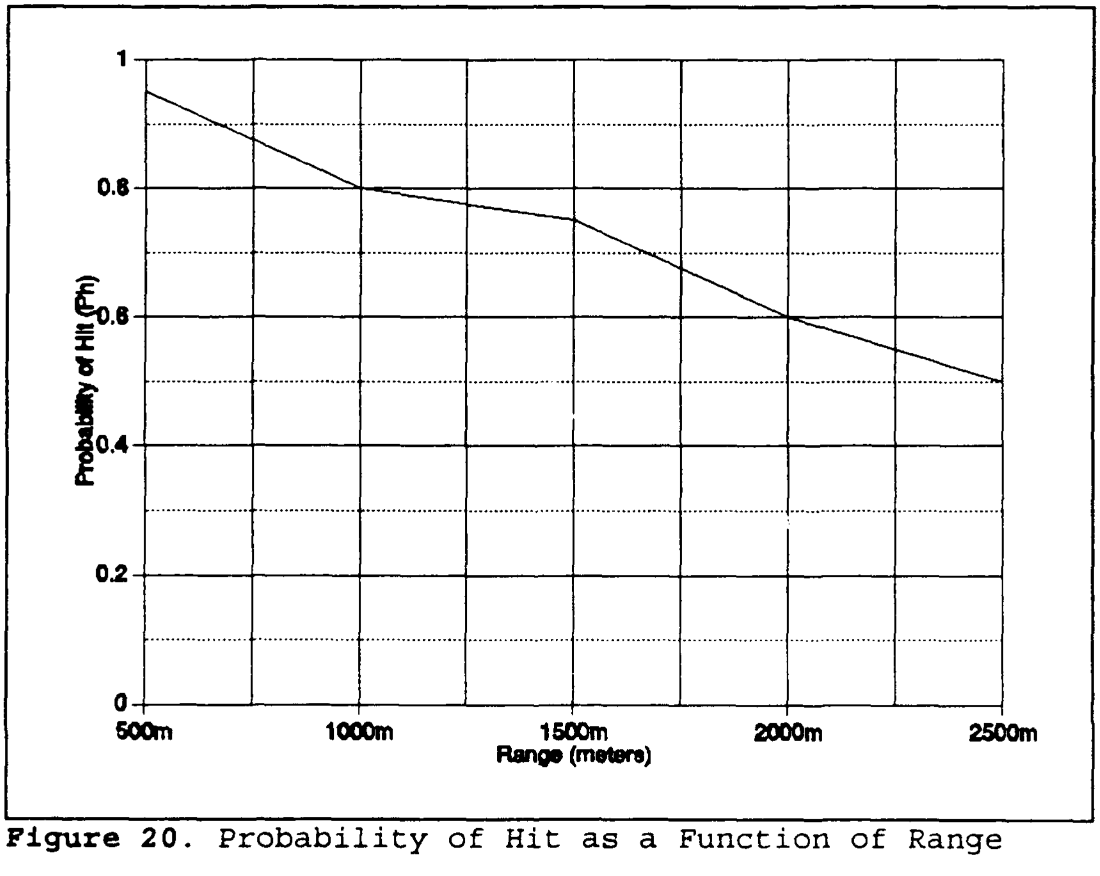

Figure 20. Probability of Hit as a Function of Range

In this example you would enter the range values 500, 1000, 1500, 2000, and 2500 in the five spaces across the top of the screen, from smallest to largest. The first entry should reflect the minimum weapon range.

The Ph values depend on the relationships between the firing weapon and the target, stationary (S) or moving (M); the target posture, defilade (D) or exposed (E); and the target aspect, flank (F) or head-on (H). The first letter in the column entries refers to the firing weapon's posture. The next three letters denote the target posture. If the new weapon system cannot be fired while moving, enter data only for the stationary case. Furthermore, if available data does not refer to target posture or aspect, enter the same data for all possible target postures.

There must be a different Ph/Pk data set established for each enemy target the new weapon can engage. Thus, if there are four different enemy systems that the new system can engage, you must make four Ph/Pk data sets, one for each enemy system.

Step 4. After entering all data for a set, exit by pressing ENTER.

Step 5. After entering all data sets applicable, exit to the Weapons Data Menu.

b. Enter Pk data

Step 6. Select the PK Data Sets (PK) option. There are normally four categories of "kill" in most simulations:

- Mobility - Mobility loss, but repairable.
- Firepower - Weapon function loss, but repairable.
- Mobility/Firepower - Both mobility and firepower function are lost, but repairable.
- Catastrophic Kill - No possibility of repair.

In Janus, only the Mobility/Firepower category exists. The omission of the first two categories is not serious since most Janus scenarios do not address battlefield logistics in terms of a repair or repair rate measure of effectiveness. Omission of the catastrophic kill category makes little difference since the Janus detection algorithm does not distinguish between mobility/firepower and catastrophic kills. The use of a single kill category means the design of Janus battle calculus addresses attrition relationships.

Step 7. Enter 2000 in the Probability of Kill Data Sets menu. The monitor will display an entry field with five data columns similar to that for the Ph data. Pk data is likewise approximated with four piecewise continuous line segments.

Step 8. Enter the range information with the minimum range necessarily the first entry. The range breaks need not be identical to the Ph data. The "best" values depend on the shape of the Pk function curve.

Step 9. Enter the Pk data corresponding to the enemy systems, as appropriate. If there is no information on the mobility and firepower rows, then leave them blank (or zero).

Step 10. After entering all the data, exit to the Combat Systems Data Editor menu.

3. Link the new weapon definition with the new Ph/Pk relationships

Step 1. Select the WEAPONS (WP) option from the Combat Systems Data Editor menu.

Step 2. Select the Hit and Kill Data Set Assignments by WEAPON (WW) option.

Step 3. On the Hit and Kill Data Set Pointers menu, enter the side (B or R) and the new Absolute Weapon Number.

Step 4. Using the editing keys, assign the data sets created to their appropriate enemy target systems.

Step 5. When finished, return to the Combat Systems Data Editor menu.

a. *Set weapon characteristics*

Step 6. Select the WEAPONS (WP) option.

Step 7. Select Characteristics (CC) option.

Step 8. Locate and rename the new weapon.

Step 9. Edit the entries corresponding to the new Absolute Weapon number using the weapon performance data.

Step 10. When finished, return to the Weapons Data Menu.

b. *Set round guidance option*

Step 11. Select the Round Guidance (RR) option.

Step 12. For the new Absolute Weapon number, set the weapon/round guidance mode, as appropriate.

Step 13. Select the appropriate "fire on the move" value from the table at the top of the monitor display.

Step 14. When finished, return to the Weapons Data menu.

c. *Set MOPP effects*

Step 15. Select the MOPP Effects (MM) option. MOPP effects on friendly weapon performance are expressed in terms of a time factor and probability of hit degradation. A time factor greater than 1.0 will increase weapon lay, aim, and reload times proportionally. Ph factors less than 1.0 will have a similar effect on the hit probability. The standard entries for these values are 1.4 for time degradation and 1.0 for Ph. No changes are needed on this screen.

Step 16. Exit to the Combat Systems Data Editor menu.

4. Modify an existing system definition

The procedure for copying a system definition is essentially equivalent to that for copying weapon definitions. The first step is to find an appropriate system number to place the new definition, insuring that no old definition is overwritten.

Step 1. From the Combat Systems Data Editor menu, select SYSTEMS (SY) option.

Step 2. Select the Characteristics (CC) option.

Step 3. Select the General (GG) option.

Step 4. View the data associated with an old system.

Step 5. Note and remember a system number that is empty and available.

Step 6. Exit to the Combat Systems Data Editor menu.

a. *Copy a system definition*

Step 7. Select the UTILITIES (UU) option.

Step 8. Enter Copy System (CS) option.

Step 9. Select side (B or R).

Step 10. Select FROM: (Old Absolute System number).

Step 11. Select TO: (New, unused Absolute System number).

Step 12. When finished, exit to the UTILITIES Menu.

b. *Check the new system definition*

Step 13. Run the Differences of Systems (DS) option to check the number of differences. There should be no differences.

Step 14. Select side (B or R).

Step 15. Select First System: (Old Absolute System number).

Step 16. Select Second System: (New Absolute System number). Press ENTER to run the comparison function. Results appear on the monitor and show that there are no differences between the two systems.

Step 17. Return to the Combat Systems Data Editor Menu.

c. *Modify a system definition*

Step 18. From the Combat Systems Data Editor menu, select the SYSTEMS (SY) option.

Step 19. Select the Characteristics (CC) option.

Step 20. Select the General (GG) option.

Step 21. Locate and rename the new system.

Step 22. Enter the data elements appropriate for your new system.

Step 23. When finished, exit to the Systems Characteristics menu.

Step 24. Select the Functionality (FF) option.

Step 25. Make changes as necessary.

Step 26. When finished, exit to the System Characteristics menu.

Step 27. Select the Detection Data (DD) option. The data on this screen is used to calibrate the Janus detection algorithm for the particular system and this is where sensor types are assigned to each system. Notice that each system can have two sensors, a primary and alternate.

Step 28. Modify the detection dimension data to correspond to the new system.

Step 29. When finished viewing this screen, exit to the Systems Data menu.

5. Link the new system definition with the new weapon definitions

Step 1. Select the Weapons and Ordinance (WW) option.

Step 2. Select the new system number.

Step 3. In the row corresponding to relative weapon 1, enter the new Absolute Weapon Number. This will associate the new weapon with the new system. Also, notice that this part of the database contains the elements to change a weapon's basic load. In the defense mode the basic load may have to be increased. Enter data as appropriate.

Step 4. Exit to the Systems Data menu.

Step 5. Select the Weapon Selection by Firing System (FF) option.

Step 6. Select the new system.

Step 7. Check to insure the new system will engage the appropriate enemy systems.

Step 8. When finished, exit to the Systems Data menu.

Step 9. Select Weapon Selection by Target System (TT) option.

Step 10. Check to insure the correct weapon and Ph/Pk data and firing weapon is associated with the correct target.

Step 11. Exit to the Combat Systems Data Editor menu.

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