

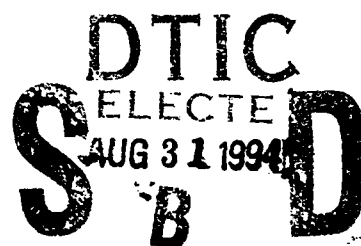
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Mathematical Analysis of Actuator Forces in a Scissor Lift

H. Spackman



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Mathematical Analysis of Actuator Forces in a Scissor Lift

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1.0 ABSTRACT

In 1985, NCCOSC began development of a tele-operated vehicle as part of the U.S. Marine Corps Land-Air Tele-Robotics Systems Program. One of the required vehicle components is a rigid, light-weight, and compact lift mechanism capable of deploying a surveillance package 10 feet above the vehicle bed. The lift mechanism that was eventually built and implemented was a 3-level scissor lift. In order to analyze the forces throughout the lift structure, a set of mathematical equations was derived. From these equations it was discovered that prudent placement of a lift's actuator can significantly reduce the forces required of the actuator and the stress levels in the adjacent scissor members. The purpose of this paper is to present the equations that were derived for analyzing the actuator forces. Using these equations, a designer can quickly determine the optimal locations for mounting an actuator and the resulting forces.

2.0 INTRODUCTION

In reference (1), equations are derived for determining the reaction forces throughout a scissor lift. To facilitate analysis, reference (1) divides the problem into two parts. In the first part, equations for a basic scissor structure - a scissor structure with no actuators and with all four bottom joints pinned to "ground" - are derived. In the second part, equations for calculating the actuator forces are derived. A later section discusses how these equations can be used to calculate the reaction forces throughout the scissor structure.

The actuator force equations in reference (1) are derived assuming that one or both of the actuator ends lie along the longitudinal axis of the scissor members. In practice, this is seldom the case because increased mechanical advantage and improved fitment can be obtained by offsetting the actuator ends. An attempt was made to derive equations that are valid for any actuator placement, but the solutions were not found until after publication of reference (1). The purpose of this paper is to present the equations for the more general case.

3.0 DERIVATION OF EQUATIONS

As in reference (1), the actuator force equations will be derived assuming conservation of energy and quasistatic equilibrium. The first assumption implies that frictional forces are negligible, and the second that acceleration is negligible. In the material that follows, an equation referred to as the fundamental equation is first derived. This equation expresses actuator force as a function of the derivative of lift height to actuator length, i.e.,

$$F = f\left(\frac{dh}{dl}\right).$$

Following this, equations for dh/dl are derived for the two possible methods of mounting actuators. Substituting the equations for dh/dl into the fundamental equation gives the final result.

3.1 DERIVATION OF THE FUNDAMENTAL EQUATION

An n-level lift is shown in figure (1) along with the possible loads that can be applied to the top of the lift and some dimensional nomenclature. Of the six possible applied loads (see figure (1)) only H_x and H_y result in work as the lift elevates, and H_x results in work only if the platform translates in the x-direction as the lift elevates.

Because the lift itself is a significant part of the load, its weight must be included in the derivation. Let B equal the total weight of the lift, B_x and B_z the components of B in the x, and z directions respectively, and B_y the component of B in the negative y direction. As with the applied loads, only B_x and B_y result in work as the lift elevates. If the lift's mass is evenly distributed then the weight of the lift can be modeled by placing $B_y/2$ at the top of the lift and $B_x/2$ at the translating side of the lift. The validity of this statement is illustrated as follows.

Consider the mass shown in figure (2). The total potential energy of the mass can be approximated by

$$E = \lim_{\Delta m_i \rightarrow 0} \sum g \rho u w \Delta y_i y_i$$

where g = acceleration due to gravity
 ρ = density.

Passing to the limit gives

$$E = \int_0^h g \rho u w y dy.$$

Now, if the mass of the object is evenly distributed over its height then

$$\rho g u w = \frac{W}{h}$$

where W = total weight
 h = height.

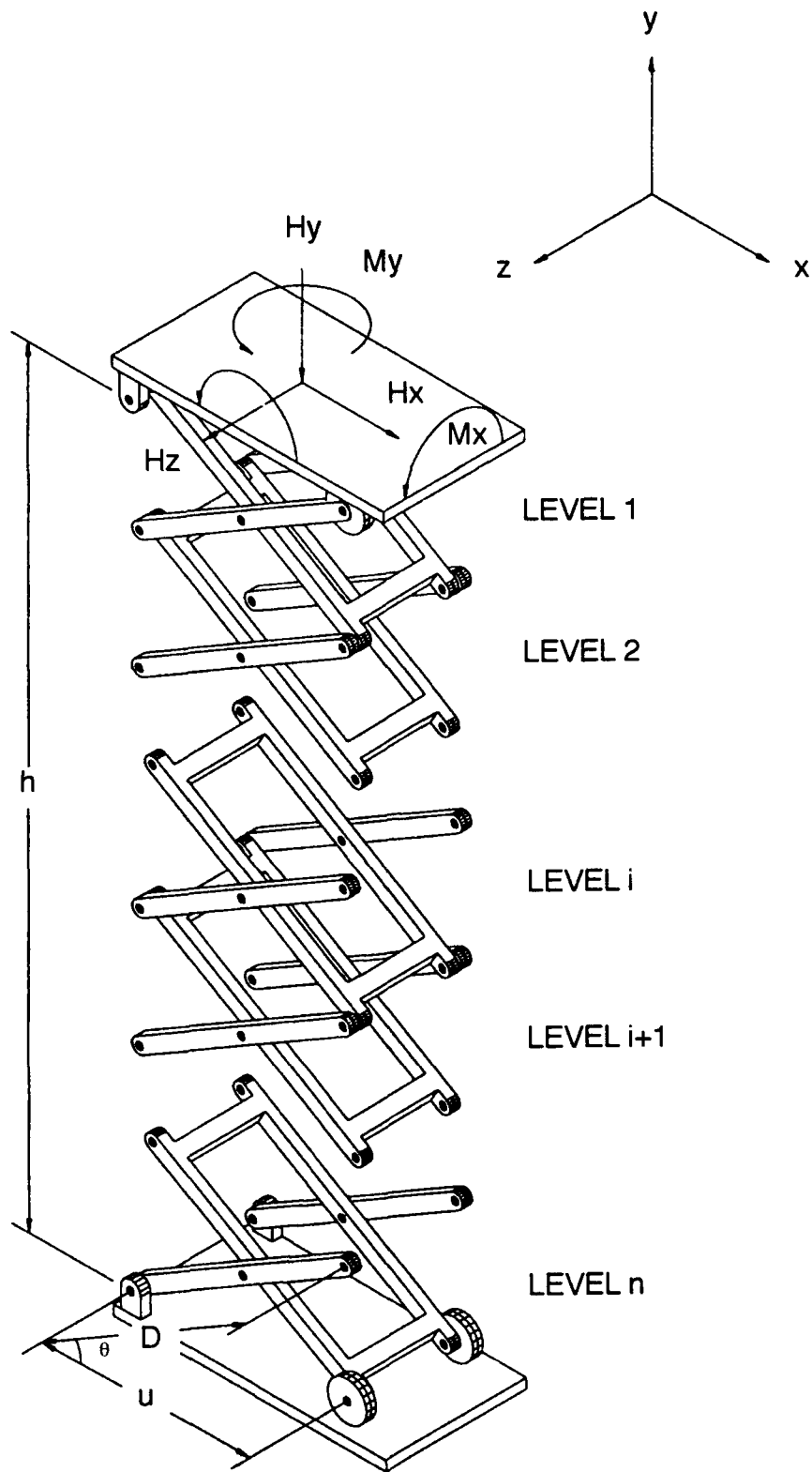


Figure 1. Nomenclature.

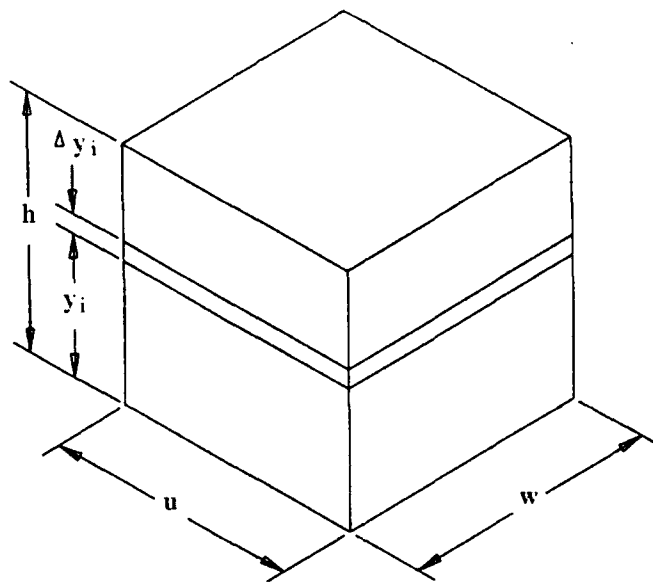


Figure 2. Arbitrary Mass.

Making the appropriate substitution results in

$$\begin{aligned}
 E &= \int_0^h \frac{W}{h} y dy \\
 &= \frac{W}{2h} y^2 \Big|_0^h \\
 &= \frac{Wh}{2} .
 \end{aligned}$$

Now, if the block height increases but the total weight stays the same then

$$\begin{aligned}
 \Delta E &= E_2 - E_1 \\
 &= \frac{W}{2} (h_2 - h_1) .
 \end{aligned}$$

This is the desired result.

Figure (3) shows a four level lift with the loads that result in work as the lift elevates. Remember that H_x contributes only if the platform is attached to the joints that translate in the x-direction as the lift elevates. In most designs this is not the case, and H_x would be disregarded.

Now, the assumption of conservation of energy requires that work in equals work out. In other words, if F = actuator force and l = actuator length then

$$\int_{l_1}^{l_2} F dl = \int_{h_1}^{h_2} \left(H_y + \frac{B_y}{2} \right) dh - \int_{u_1}^{u_2} \left(H_x + \frac{B_x}{2} \right) du .$$

Taking the derivative of both sides with respect to l results in

$$F = \left(H_y + \frac{B_y}{2} \right) \frac{dh}{dl} - \left(H_x + \frac{B_x}{2} \right) \frac{du}{dl} . \quad (\text{EQ 1})$$

But an expression for du/dl in terms of dh/dl can be found as follows. From figure (1) it is clear that

$$h = n(D^2 - u^2)^{1/2}$$

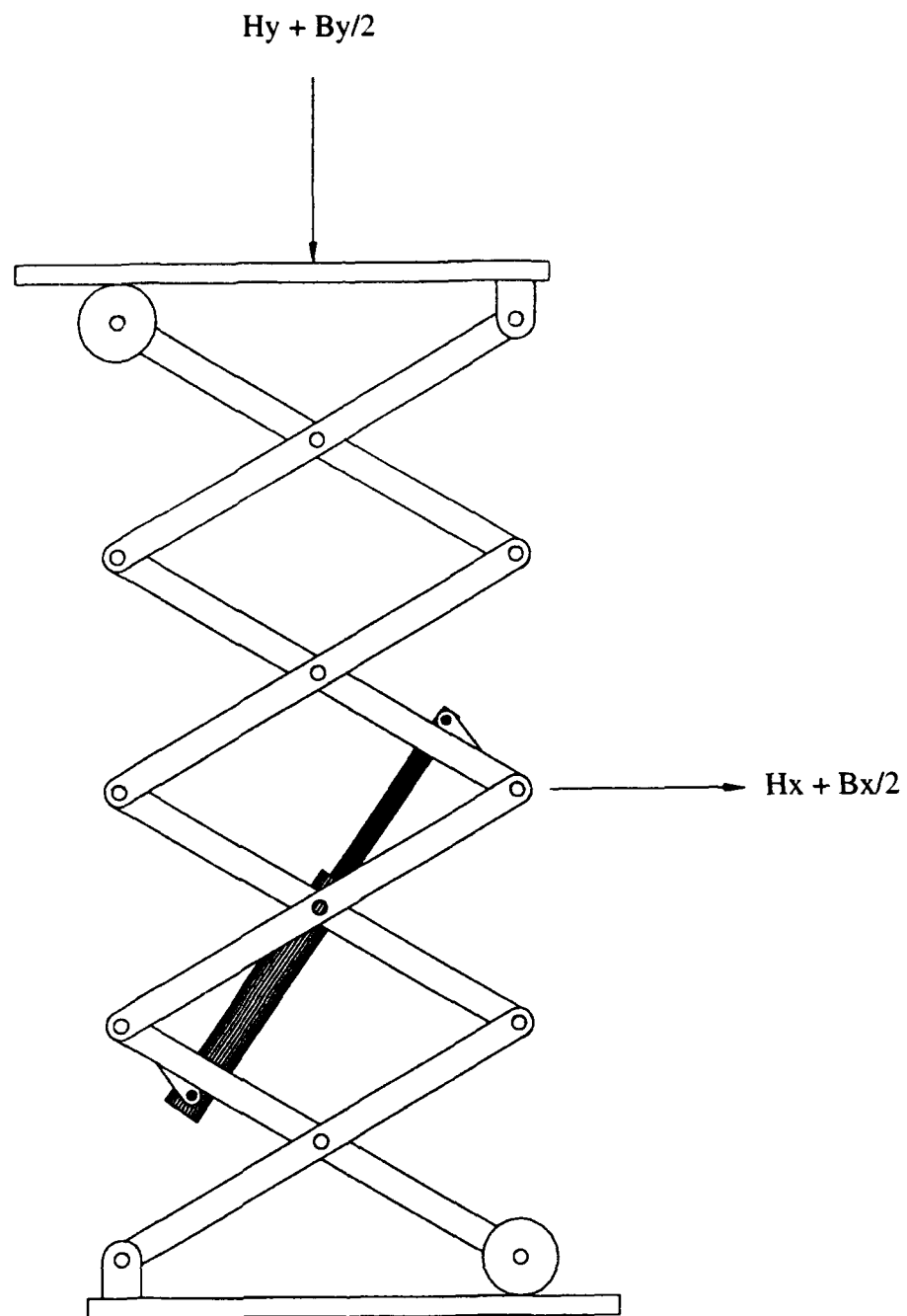


Figure 3. 4-level lift with loads that result in work.

$$\begin{aligned}
\frac{dh}{dl} &= \frac{1}{2} n (D^2 - u^2)^{-1/2} (-2u) \frac{du}{dl} \\
&= \frac{-nu}{(D^2 - u^2)^{1/2}} \frac{du}{dl} \\
&= \frac{-n}{\tan \theta} \frac{du}{dl}
\end{aligned}$$

Solving for du/dl gives

$$\frac{du}{dl} = -\frac{\tan \theta}{n} \frac{dh}{dl}$$

Substituting this last equation into equation 1 results in

$$F = \left[\left(H_y + \frac{B_y}{2} \right) + \left(H_x + \frac{B_x}{2} \right) \frac{\tan \theta}{n} \right] \frac{dh}{dl} \quad (\text{EQ 2})$$

This is the fundamental equation.

Having derived the fundamental equation, the next step is to derive expressions for dh/dl as a function of actuator placement and θ (see figure 1). Two options exist for mounting actuators. The first option is to attach both ends of the actuator to scissor members. In this case both ends of the actuator translate as the lift elevates. The second option is to attach one end of the actuator to a scissor member and the other end to a fixed point. In the next two sections equations for determining dh/dl are derived for each option.

3.2 DERIVATION OF dh/dl FOR OPTION 1

Figure (4) shows an n -level lift at several stages of deployment. The fully retracted and fully extended positions are, of course, impractical but are included to show useful terminology. The x - y coordinates of the offset point on the positively sloping member of level i are given by

$$\begin{aligned}
x &= x_0 \cos \theta + x_{90} \sin \theta \\
y &= y_{90} \sin \theta + y_0 \cos \theta
\end{aligned}$$

(Notice that x_{90} is negative in figure (4)). However, these formulas are also valid for offset

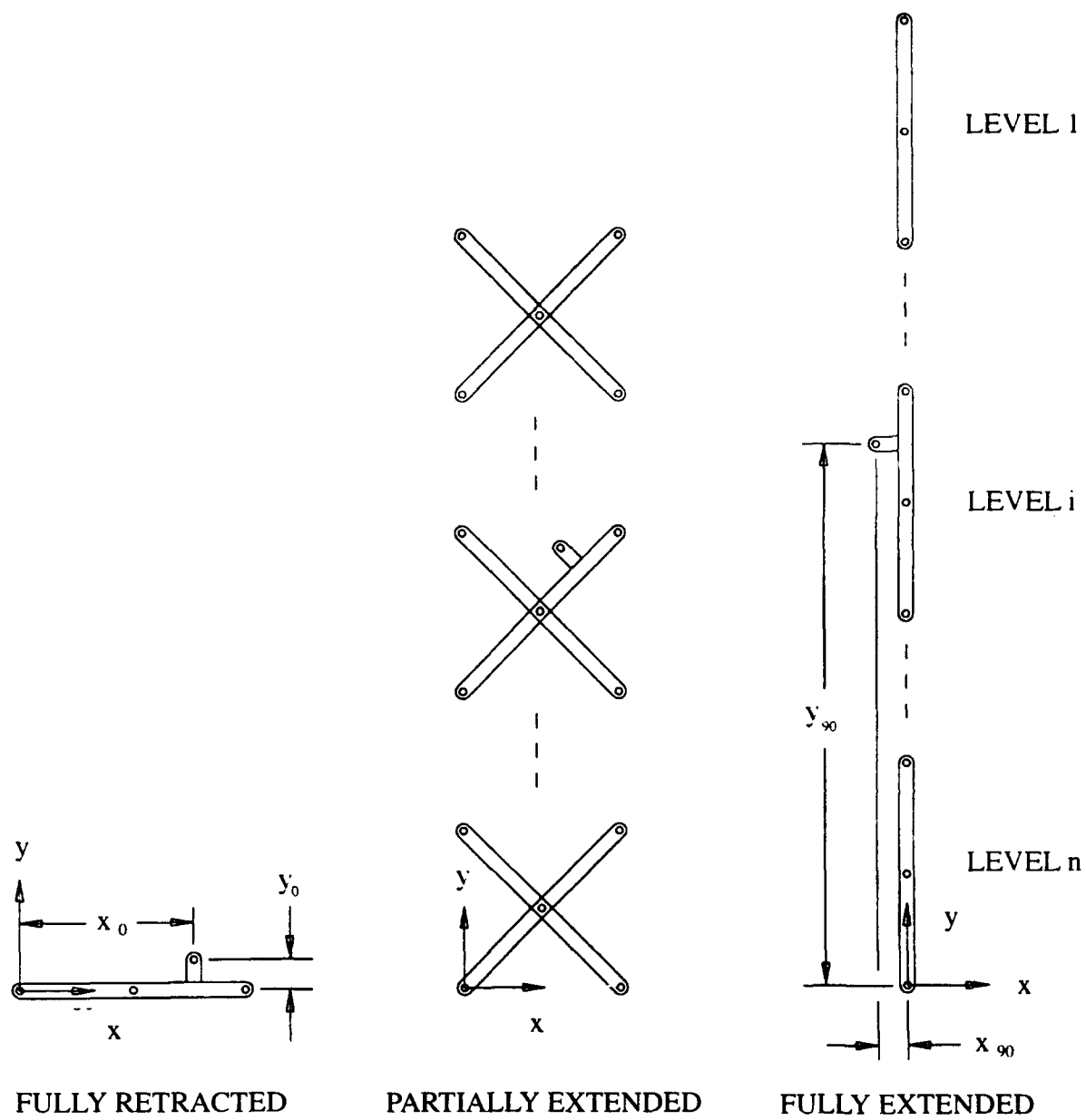


Figure 4. n-level lift at various stages of deployment.

points associated with positively and negatively sloping members of any level. Now given any two points A and B, the distance between the points is given by

$$\begin{aligned}
 l^2 &= (x_B - x_A)^2 + (y_B - y_A)^2 \\
 &= [(x_{B90} - x_{A0}) \cos \theta + (x_{B90} - x_{A90}) \sin \theta]^2 + [(y_{B90} - y_{A90}) \sin \theta + (y_{B0} - y_{A0}) \cos \theta]^2 \\
 &= (a \cos \theta + b \sin \theta)^2 + (c \sin \theta + d \cos \theta)^2 \quad (\text{EQ 3})
 \end{aligned}$$

where

$$\begin{aligned}
 a &= x_{B0} - x_{A0} & c &= y_{B90} - y_{A90} \\
 b &= x_{B90} - x_{A90} & d &= y_{B0} - y_{A0}
 \end{aligned}$$

Taking the derivative of l with respect to h gives

$$2l \frac{dl}{dh} = [2(a \cos \theta + b \sin \theta)(-a \sin \theta + b \cos \theta) + 2(c \sin \theta + d \cos \theta)(c \cos \theta - d \sin \theta)] \frac{d\theta}{dh},$$

and solving for $\frac{dh}{dl}$ results in

$$\frac{dh}{dl} = \frac{2l \frac{d\theta}{dh}}{2(a \cos \theta + b \sin \theta)(-a \sin \theta + b \cos \theta) + 2(c \sin \theta + d \cos \theta)(c \cos \theta - d \sin \theta)}.$$

Solving for l in equation (3) gives

$$l = [(a \cos \theta + b \sin \theta)^2 + (c \sin \theta + d \cos \theta)^2]^{1/2}.$$

From figure (1) it is clear that

$$h = nD \sin \theta \quad (\text{EQ 4})$$

$$\frac{dh}{d\theta} = nD \cos \theta.$$

Making the appropriate substitutions results in

$$\frac{dh}{dl} = \frac{ns \cos \theta [(a \cos \theta + b \sin \theta)^2 + (c \sin \theta + d \cos \theta)^2]^{1/2}}{(a \cos \theta + b \sin \theta)(-a \sin \theta + b \cos \theta) + (c \sin \theta + d \cos \theta)(c \cos \theta - d \sin \theta)} \quad (\text{EQ 5})$$

Equations (2) and (4) together give the final result.

3.3 DERIVATION OF dh/dl FOR OPTION 2

The equation for dh/dl for the 2nd option is derived using a slightly different approach. Figure 5 shows the incremental displacement of an offset point Q (presumably where one end of the actuator is attached). The coordinates of point Q are given by

$$x_Q = x_{Q0} \cos \theta + x_{Q90} \sin \theta \quad (\text{EQ 6})$$

$$y_Q = y_{Q90} \sin \theta + y_{Q0} \cos \theta \quad (\text{EQ 7})$$

Taking the derivative of both equations results in

$$\frac{dx_Q}{d\theta} = -x_{Q0} \sin \theta + x_{Q90} \cos \theta$$

$$\frac{dy_Q}{d\theta} = y_{Q90} \cos \theta - y_{Q0} \sin \theta \quad (\text{EQ 8})$$

Now, $\frac{dy_Q}{dx_Q}$ can be found by dividing $\frac{dy_Q}{d\theta}$ by $\frac{dx_Q}{d\theta}$. Doing so results in

$$\frac{dy_Q}{dx_Q} = \frac{y_{Q90} \cos \theta - y_{Q0} \sin \theta}{-x_{Q0} \sin \theta + x_{Q90} \cos \theta}$$

An equation for ξ (see figure (5)), then, is

$$\xi = \text{atan}\left(\frac{y_{Q90} \cos \theta - y_{Q0} \sin \theta}{x_{Q0} \sin \theta - x_{Q90} \cos \theta}\right)$$

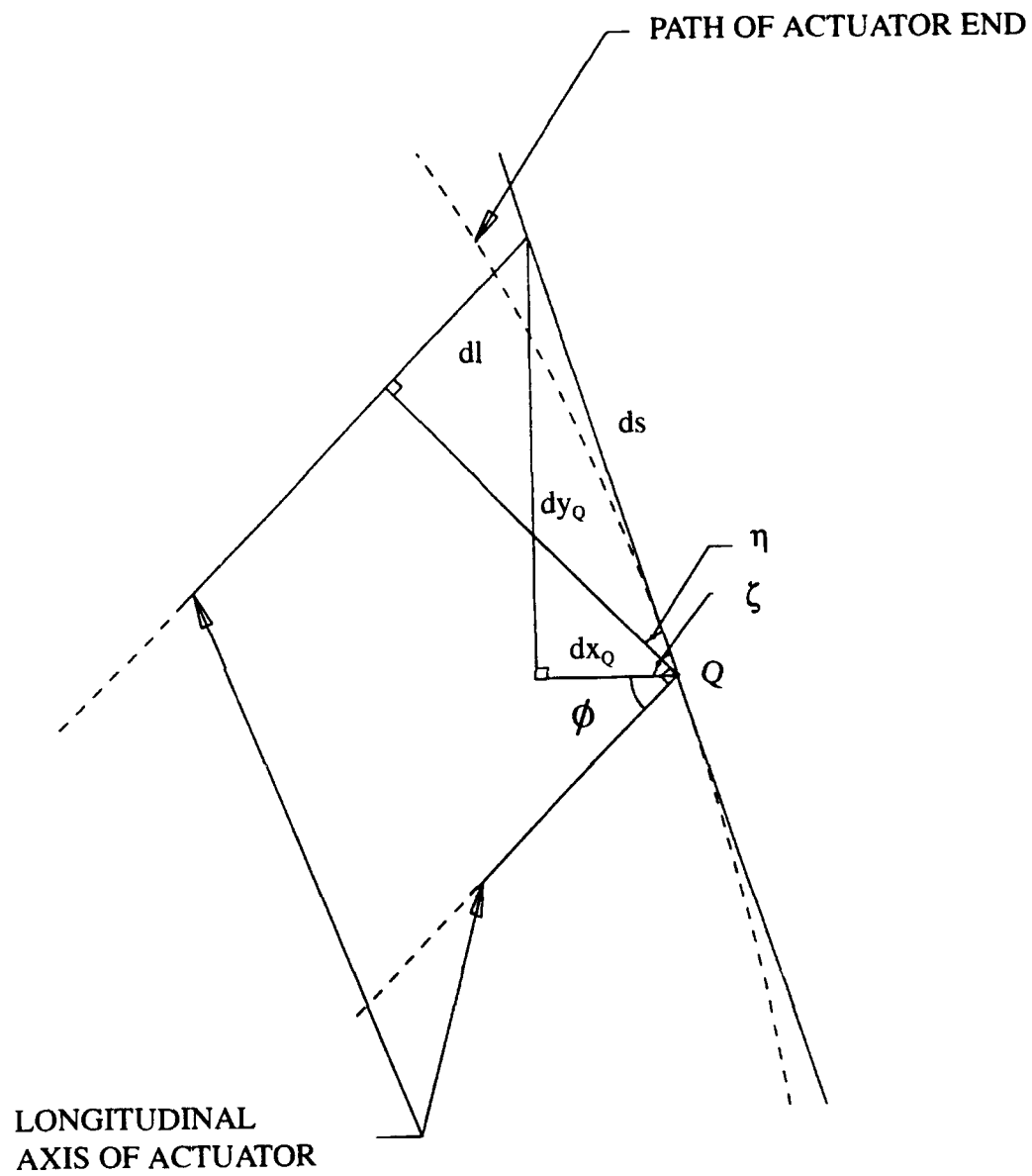


Figure 5. Incremental displacement of actuator end.

From figure 5 it is clear that

$$\frac{dy_Q}{ds} = \sin \zeta, \quad (\text{EQ 9})$$

and that

$$\begin{aligned} \frac{ds}{dl} &= \frac{1}{\sin \eta} \\ &= \frac{-1}{\cos (\eta + 90)} \\ &= \frac{-1}{\cos (\phi + \zeta)}. \end{aligned} \quad (\text{EQ 10})$$

Taking the derivative of EQ (4) with respect to y_Q results in

$$\frac{dh}{dy_Q} = nD \cos \theta \frac{d\theta}{dy_Q}.$$

But, from equation 6 we have

$$\frac{d\theta}{dy_Q} = \frac{1}{y_{Q90} \cos \theta - y_{Q0} \sin \theta},$$

which when substituted into the previous equation results in

$$\frac{dh}{dy_Q} = \frac{nD}{y_{Q90} - y_{Q0} \tan \theta}. \quad (\text{EQ 11})$$

Now, we can combine equations 9, 10, and 11 to obtain

$$\begin{aligned}\frac{dh}{dl} &= \frac{dh}{dy_Q} \frac{dy_Q}{ds} \frac{ds}{dl} \\ &= \frac{-nD \sin \zeta}{(y_{Q90} - y_{Q0} \tan \theta) \cos (\phi + \zeta)}.\end{aligned}$$

If the other end of the actuator is attached to a fixed point, P, then

$$\phi = \text{atan}\left(\frac{y_Q - y_P}{x_Q - x_P}\right).$$

In summary

$$x_Q = x_{Q0} \cos \theta + x_{Q90} \sin \theta$$

$$y_Q = y_{Q90} \sin \theta + y_{Q0} \cos \theta$$

$$\phi = \text{atan}\left(\frac{y_Q - y_P}{x_Q - x_P}\right)$$

$$\zeta = \text{atan}\left(\frac{y_{Q90} \cos \theta - y_{Q0} \sin \theta}{x_{Q0} \sin \theta - x_{Q90} \cos \theta}\right)$$

$$\frac{dh}{dl} = \frac{-nD \sin \zeta}{(y_{Q90} - y_{Q0} \tan \theta) \cos (\zeta + \phi)}.$$

4.0 CONCLUSION

A critical component of scissor lift design is the placement of the lift's actuator(s). Prudent placement can reduce the force required of the actuator(s) and reduce stress levels in the scissor structure. By assuming conservation of energy and quasi-static equilibrium, it is possible to derive an equation that gives actuator force as a function of the derivative of lift height to actuator length, i.e., $F = f(dh/dl)$. This equation is referred to as the fundamental equation in this paper, and the result is equation (2) of section 3.1. Having found this equation, the next step is to derive an equation for dh/dl as a function of angular displacement of the scissor members. However, there are two possible options for mounting an actuator, and the derivation of dh/dl is different for each option. The first option is to attach both ends of the actuator to scissor members, and the second is to attach one end of

the actuator to a scissor member and the other end to a fixed point. The derivations of dh/dl for the two options are given in section 3.2 and 3.3 respectively. By combining the fundamental equation with the appropriate equation for dh/dl , the designer can quickly determine the optimal locations for mounting the actuator.

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