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MICROWAVE HEATING AND JOINING OF CERAMIC CYLINDERS: A MATHEMATICAL MODEL

Michael R. Booty
Gregory A. Kriegsmann

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Microwave Heating and Joining of Ceramic Cylinders: A Mathematical Model

MICHAEL R. BOOTY AND GREGORY A. KRIEGSMANN

Department of Mathematics
Center for Applied Mathematics and Statistics
New Jersey Institute of Technology
University Heights
Newark, NJ 07102

Abstract

A thin cylindrical ceramic sample is placed in a single mode microwave applicator in such a way that the electric field strength is allowed to vary along its axis. The sample can either be a single rod or two rods butted together. We present a simple mathematical model which describes the microwave heating process. It is built on the assumption that the Biot number of the material is small, and that the electric field is known and uniform throughout the cylinder's cross-section. The model takes the form of a nonlinear parabolic equation of reaction-diffusion type, with a spatially varying reaction term that corresponds to the spatial variation of the electromagnetic field strength in the waveguide. The equation is analyzed and a solution is found which develops a hot spot near the center of the cylindrical sample and which then propagates outwards until it stabilizes. The propagation and stabilization phenomenon concentrates the microwave energy in a localized region about the center where elevated temperatures may be desirable.

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1. Introduction.

The use of microwaves to sinter or join ceramics is rapidly gaining acceptance in industry where the efficient production of high quality materials is important. Efficiencies are increased because microwaves penetrate a material and rapidly deposit energy there, in direct contrast to conventional heating schemes where heat diffuses into a material from its surface. The price paid is the need of control systems to prevent thermal runaway and other related instabilities.

The control and dependability of these processes requires a deep understanding of the inherent physics which are described by a formidable nonlinear initial boundary value problem. This is comprised of the time-harmonic version of Maxwell's equations, the heat equation, an equation of state relating the effective electrical conductivity to the temperature, and a thermal boundary conditions on the surface of the ceramic material which balances conduction, convection, and thermal radiation. The nonlinear character arises from the dependence of the electric field upon the effective electrical conductivity, which is a function of the temperature, the dependence of the temperature upon the microwave power deposition, which is proportional to the product of the effective electrical conductivity and the the magnitude of the electric field squared, and the radiative heat loss, which varies as the fourth power of the temperature. In addition, the boundary value character of the problem is also challenging because the electromagnetic fields and the ceramic material are confined in a cavity or waveguide applicator of a complicated geometry.

The systematic analysis of these equations under a variety of physical limits has primarily been restricted to one dimensional geometries (see reference 4 and the bibliography therein), but has recently been extended to three dimensions [4] in the small Biot number limit. However, in all these cases the effect of the waveguide applicator or cavity were neglected, i.e., the ceramic samples were irradiated by plane waves in free space. Nonetheless, the small Biot number theory predicts the phenomenon of thermal runaway and suggests methods for its control.

The problem we model and study in this paper is concerned with sintering and joining of ceramic fibers in a microwave applicator. As such, it is not described by the theories mentioned above, because of the applicator and also because of the small aspect ratio a/d of the fibers. In this paper. we take into account the effects of the applicator by assuming that the electric field is uniform throughout the cylinder's cross-section and known along its length. That is, the ceramic cylinder is thin enough not to perturb the electric field to leading order. Thus, the heating process will be modeled by a nonlinear heat equation and boundary condition.

There are three small parameters that arise from a dimensional analysis of the simplified model problem. The first is the aspect ratio defined above, the second is the Biot number B_1, which is a measure of convective heat loss at the surface, and the third is B_2 is a measure of radiative heat loss there. As described above, we have developed an asymptotic theory to study the microwave heating of ceramic slabs and other compact geometries [3,4] as $B_1 \to 0$. In these studies $B_2 \sim B_1$ so that both physical effects of radiation and convection have been incorporated into the theory. This asymptotic theory can be employed to analyze the present problem with the proviso that the parameter $\epsilon^2 = (a/d)^2/B_1$ is order one. The net result is that the temperature remains spatially uniform across the ceramic's cross-section and satisfies a nonlinear reaction-diffusion equation along its length. In this equation the reaction term accounts for adsorption of microwave energy and loss of thermal energy, which arises from convection and radiation at the sample boundaries, and the diffusion coefficient is ϵ^2. The later will be taken as small (which is the case for fibers) to allow an analysis of this equation.

Two types of problems naturally arise depending upon the orientation of the ceramic cylinder in the applicator. If the sample is placed so that the electric field has no spatial variation along its axis, then the reaction-diffusion equation has constant coefficients. Equations of this type have received considerable study because of their applicability in a wide variety of physical settings [9]. That the present equation supports traveling transition layers comes as no mathematical surprise. However, in the present physical context it does explain the mechanism for the formation and propagation of hot-spots [5,6] which are seen in experiments [8,11].

On the other hand, if the ceramic fiber is placed so that the electric field varies along its axis, then the equation has a spatially varying reaction term. The analysis and understanding of the solutions of these types of equations are less well understood than those described above [1]. We shall analyze the equation below using standard matched asymptotic methods and show how its solution evolves into a hot spot, propagates outward from its inception, and stabilizes to form a region of elevated temperature. This stable region of elevated temperature can be exploited in fiber sintering and has already been used in joining processes [2]. Moreover, our analysis shows how the size of the spot depends upon the temperature dependence of the material's thermal properties. These dependencies are often ignored because the experiments are more sensitive to changes in the effective electrical conductivity with temperature. However, they are essential in understanding the final width of the spot.

2. Formulation.

A thin cylindrical ceramic sample is positioned in a single mode waveguide applicator, so that the electric field along its axis varies as the fundamental mode of the waveguide. The sample is held in place at its ends by two thermally insulated, microwave transparent push-rods. Although the electric field is altered by the presence of the ceramic, for the present analysis we assume that this effect is negligible and that the time-harmonic electric field is given by

$$\mathbf{E} = E_0 \sin(\pi Z/d)\mathbf{j} \tag{1}$$

where d is the height of both the cylinder and the guide, E_0 is the strength of the incident mode, and $\mathbf{j}$ is a unit vector perpendicular to the axis of the cylinder. This assumption effectively decouples the equations for the electromagnetic field from the equation for the energy of the sample, and allows us to focus solely on the sample's thermal field. In addition, we assume that the sample is thin enough to ensure that variations in the electric field are negligible across its circular cross-section.

In light of these assumptions, the temperature, T, satisfies the energy equation

$$\frac{\partial}{\partial t}(\rho C_P T) = \nabla \cdot (K \nabla T) + \frac{\sigma(T)}{2} E_0^2 \sin^2(\pi Z/d), \quad 0 < Z < d, \quad 0 < R < a \tag{2}$$

where $R = \sqrt{X^2 + Y^2}$ is the radial distance from the cylinder's axis, ρ is the density of the ceramic, C_P is its specific heat, K is its thermal conductivity, σ is its effective electrical conductivity, and a is the radius of the sample. Although variations of the thermal parameters K and C_P are small over the temperature range required for sintering or joining when compared to the change in the electrical conductivity, they are included in the following analysis, since they may have a profound effect on the dynamics of the heating process.

We also require that the temperature satisfies the surface heat balance

$$K\frac{\partial}{\partial r}T + h(T - T_A) + se(T^4 - T_A^4) = 0, \quad R = a, \quad 0 < Z < d \tag{3a}$$

where h is a constant corresponding to heat loss from the surface by convection, s is a constant for radiative heat loss, e is the emissivity of the surface, and T_A is the ambient temperature of the surrounding medium. To simplify the analysis that follows, we assume that the ambient temperature remains constant. At the ends of the sample, we prescribe the boundary conditions

$$\frac{\partial}{\partial Z}T = 0, \qquad Z = 0, d; \quad \text{and} \quad 0 < R < a \tag{3b}$$

and we take the initial temperature of the sample to coincide with the ambient temperature, i.e.,

$$T(X, Y, Z, 0) = T_A. \tag{4}$$

Equations (2)-(4) constitute a nonlinear initial boundary value problem for the temperature T within the sample. The nonlinear character of this simplified problem is caused by the dependence of the electrical conductivity σ and thermal parameters K and C_P on the temperature and by the radiative losses at the sample boundary. This is the generalization of the mathematical models for microwave heating as studied by Tian using finite difference simulations [10] and by Kriegsmann using asymptotic methods [5,6].

3. The Simplified Theory.

There are three small parameters that arise from the nondimensionalization of equations (2-4). The first is the Biot number $B_1 = ha/K_A$, where $K_A = K(T_A)$ is the value of the thermal conductivity at the ambient temperature T_A, and the second is $B_2 = seaT_A^3/K_A$. The former is a measure of the relative effects of convection and conduction and the latter is a measure of the relative effects of radiation and conduction. Typical values of B_1 and B_2 for ceramics are of the order of 0.01, see, e.g., [7]. The third small parameter is the fineness or aspect ratio of the cylinder a/d.

In recent studies [3,4] of the microwave heating of ceramic slabs and other compact geometries we have utilized the size of these three parameters to obtain an asymptotic approximation to the temperature. Similar methods can be employed for the nonlinear initial boundary value problem (2-4) but will not be reported in detail here. The result of this analysis is that, as $B_1 \to 0$ with B_2/B_1 held fixed (so that the effects of radiation and convection are of equal importance) and with $(a/d)^2/B_1$ held fixed, the temperature of the sample T is given by

$$T(X, Y, Z, t) = U(Z, t) + O(B_1) \tag{5}$$

where $U(Z,t)$ is the leading order approximation to the temperature, which is independent of the cross-sectional or transverse coordinates X and Y, and $O(B_1)$ represents a contribution which is of the same (small) order as B_1. We note that this $O(B_1)$ error term does depend upon X and Y and thus represents a small nonuniform heating across the cross-section of the sample; the term will not be calculated here.

The leading order term $U(Z,t)$ of (5) satisfies the dimensionless initial boundary value problem

$$\frac{\partial}{\partial \eta}(\Gamma(1+u)) = \epsilon^2 \frac{\partial}{\partial z}(k \frac{\partial}{\partial z} u) + pf(u)\sin^2(\pi z) - 2\{u + \alpha[(u+1)^4 - 1]\}, \qquad 0 < z < 1, \tag{6a}$$

where the boundary conditions (3b) and initial condition (4) become, respectively,

$$\frac{\partial}{\partial z} u = 0, \qquad z = 0, 1 \tag{6b}$$

$$u(z,0) = 0, \qquad 0 < z < 1. \tag{6c}$$

In arriving at equations (6a-c) we have introduced the dimensionless variables

$$z = Z/d, \quad u = \frac{U}{T_A} - 1, \quad \eta = \frac{h}{a(\rho C_P)_A} t \tag{7a}$$

where Z is nondimensionalized with respect to the cylinder length d, u is the relative deviation of T from T_A, and t is nondimensionalized with respect to the ambient convective time $a(\rho C_P)_A/h$. We have also introduced the dimensionless parameters

$$p = \frac{a\sigma_A E_0^2}{2hT_A}, \quad \alpha = \frac{B_2}{B_1}, \quad \epsilon^2 = \frac{(a/d)^2}{B_1} \tag{7b}$$

and the dimensionless functions

$$f(u) = \frac{\sigma((T_A(1+u))}{\sigma_A}, \quad k = K/K_A, \quad \Gamma = \frac{\rho C_P}{(\rho C_P)_A} \tag{7}$$

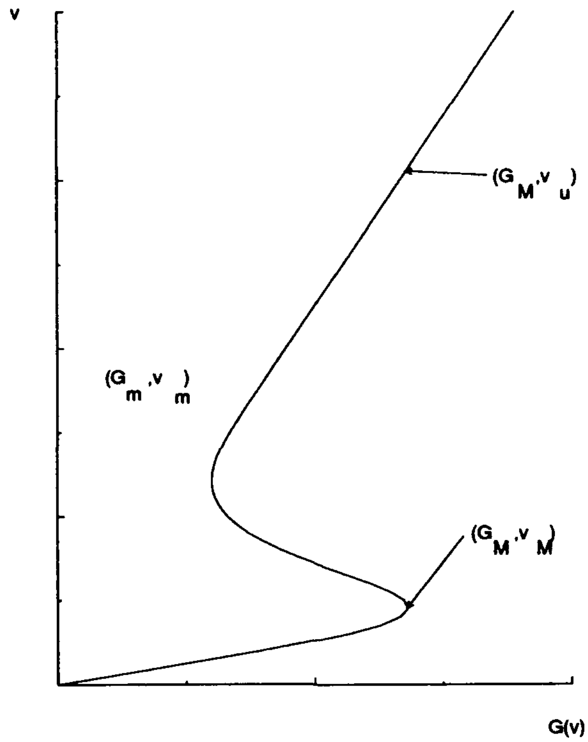
where $\sigma_A = \sigma(T_A)$ is the effective electrical conductivity at the ambient temperature, α is the ratio of the convective and radiative Biot numbers, and p is a dimensionless power.

The nonlinear initial boundary value problem (6) constitutes the mathematical statement of our small Biot number theory for the heating of the ceramic rod.

4. Analysis.

We note here that in some applications where ceramic fibers are sintered in a single mode applicator [11] the parameter ϵ is very small. For other experiments such as joining, ϵ may not be as small. The asymptotic limit $\epsilon \to 0$ is quite relevant in the former case and is expected to give qualitative results in the later. In mathematical terms, the theory which follows is strictly valid for the ordering $B_1 << \epsilon^2 << 1$. That is, $B_1 << a/d << \sqrt{B_1}$, so that the diffusion term in (6c) is larger than the neglected terms of $O(B_1)$.

FIGURE 1



Setting $\epsilon = 0$ in (6a), we obtain the ordinary differential equation

$$\frac{\partial}{\partial \eta}(\Gamma(1+u)) = pf(u)\sin^2(\pi z) - 2\{u + \alpha[(u+1)^4 - 1]\} \tag{8}$$

the solution of which depends upon z parametrically and satisfies the initial conditions (6c).

A reasonable model for the effective electrical conductivity leads to the function f being given by the Arrhenius-like law [12]

$$f(u) = 1 + c_1 e^{-c_2/u} \tag{9}$$

where c_1 and c_2 are constants. If we fix z and define $P = p\sin^2(\pi z)$, then the solution of (8) and (6c) increases monotonically from its initial value $u = 0$ to a terminal value v, which is given implicitly by the solution of

$$P = G(v) \equiv \frac{2\{v + \alpha[(v+1)^4 - 1]\}}{f(v)}. \tag{10}$$

A graph of $G(v)$ is shown in Figure 1, from which we deduce that there can be either one or three solutions of (10) depending upon the value of P. If $P < G_m$ then the terminal solution v lies on the lower branch, whereas if $P > G_M$ then it lies on the upper branch. If $G_m < P < G_M$, then there are three solutions; one on the upper branch, another on the lower branch, and the third solution on the middle branch. A simple analysis of (8) shows that solutions on the upper and lower branches are stable, and that solutions on the middle branch are unstable.

We observe that, because of the spatial variation of the power P along the axis of the waveguide and sample, at different points along its axis the ceramic sample experiences different values of P, and so there is the possibility that a steady temperature distribution may be on the upper branch in one part of the sample while it is on the lower branch in the remainder. This is indeed the case if we take the dimensionless power $p > G_M$. If we define z_1 by

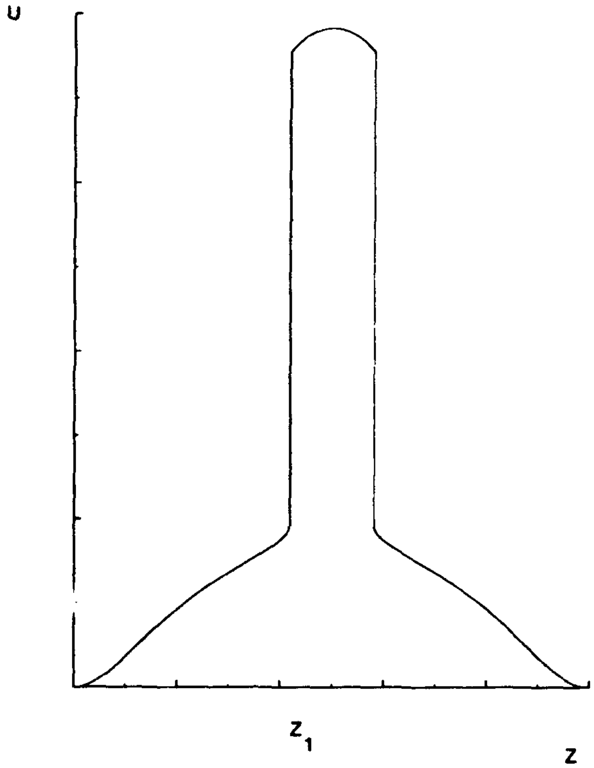
$$z_1 = \frac{1}{\pi} arcsin(\sqrt{G_M/p}) \quad \text{and} \quad z_2 = 1 - z_1, \tag{11}$$

then $P > G_M$ in the interval $z_1 < z < z_2$, and a steady temperature distribution in that part of the sample must lie on the upper branch. We have sketched this in Figure 2.

If we try to resolve the discontinuity in this steady state approximation by introducing boundary layers at z_1 and z_2, within which the diffusion term of (6a) is important, then we are immediately struck with the disconcerting fact that such a solution does not exist. To see this, we introduce the stretched variable $\bar{z} = \dfrac{z - z_1}{\epsilon}$ into (6a), set $\dfrac{\partial}{\partial \eta} = 0$, and obtain

$$\frac{d}{d\bar{z}}(k \frac{d}{d\bar{z}} u) + f(u)[G_M - G(u)] = 0, \qquad |\bar{z}| < \infty \tag{12a}$$

FIGURE 2



where G is defined in (10). The boundary conditions for this equation are deduced by asymptotic matching of its solution as $\bar{z} \to \pm\infty$ with solutions of (10). We find from a straightforward analysis that $u \to v_u + o(1)$ as $\bar{z} \to \infty$ where v_u is the value of u on the upper branch corresponding to G_M (see Figure 1). We deduce from a similar analysis that $u \to v_M + \frac{c}{\bar{z}^2}$ as $\bar{z} \to -\infty$. In either case we have

$$\frac{d}{d\bar{z}}u \to 0 \quad \text{as} \quad \bar{z} \to \pm\infty. \tag{12b}$$

Upon multiplying (12a) by $k\dfrac{d}{d\bar{z}}u$, integrating the result from $\pm\infty$, and using (12b) we obtain

$$\int_{v_M}^{v_u} k(u)f(u)[G_M - G(u)]\,du = 0. \tag{13}$$

However, from the definition of G_M we deduce that since $v_M < u < v_u$ the integrand is positive so that (13) can not be true, and a steady solution of this type does not exist.

We resolve this apparent contradiction by removing the constraint that $\frac{\partial}{\partial\eta} = 0$ in the above analysis. That is, we shall look for a traveling wave solution of (6a) which has the form

$$u = \phi(\bar{z}), \quad \text{where} \quad \bar{z} = \frac{z - z_1 - C(\tau)}{\epsilon} \quad \text{and} \quad \tau = \epsilon\eta. \tag{14}$$

Inserting this ansatz into (6a) we obtain, at leading order,

$$\frac{d}{d\bar{z}}(k\frac{d}{d\bar{z}}\phi) + C'\frac{d}{d\bar{z}}(\Gamma(1+\phi)) + f(\phi)[p\sin^2\pi(z_1 + C) - G(\phi)] = 0 \tag{15a}$$

where the prime on C denotes a derivative with respect to its slow time argument $\tau = \epsilon\eta$, and the equation is to be solved on the interval $-\infty < \bar{z} < \infty$. We deduce similar boundary conditions to the above and find again that, as in (12b),

$$\frac{d}{d\bar{z}}\phi \to 0 \quad \text{as} \quad \bar{z} \to \pm\infty. \tag{15b}$$

Now however, asymptotic matching implies that the solution ϕ has the limits

$$\phi \to \phi_\pm(C) \quad \text{as} \quad \bar{z} \to \pm\infty, \tag{15c}$$

where $\phi_+(C) > \phi_-(C)$ are those roots of $p\sin^2(z_1 + C) = G(\phi)$ which lie on the upper and lower branch of the S-shaped response curve of (10), respectively. Multiplying (15a) by $k\frac{d}{d\bar{z}}\phi$, integrating the resulting expression, and applying (15b) and (15c), we deduce that

$$C' = -\frac{\int_{\phi_-(C)}^{\phi_+(C)} k(\phi)f(\phi)[p\sin^2\pi(z_1 + C) - G(\phi)]\,d\phi}{\int_{-\infty}^{\infty} k(\phi)(\frac{d}{d\bar{z}}\phi)^2[\Gamma + \dot{\Gamma}(1+\phi)]\,d\bar{z}} \tag{16}$$

where the dot above Γ denotes its derivative with respect to ϕ.

Equation (16) is a first order nonlinear ordinary differential equation for the position of the slowly moving traveling wave, or front, which has the initial condition $C(0) = 0$ and connects the solutions on the upper and lower branches of (10). We note that the dependence of the right hand side on C is implicit, and, in particular, the integrand in the denominator depends upon C through the solution of the boundary value problem (15).

We can now deduce the dynamics of the heating process when $p > G_M$, by using (16). Initially, the rod heats according to (8), so that a discontinuous temperature profile begins to form, with discontinuities at $z = z_1$ and $z = z_2$ and a hot spot, where $z_1 < z < z_2$, as shown in Figure 2. At this point in time, considering the dynamics of the left half of the sample $0 < z < 1/2$ alone since the solution is symmetric about the sample's midpoint $z = 1/2$, $C = 0$ and from (16) C' is negative, if we assume that the term $\Gamma + \dot{\Gamma}(1+\phi) > 0$ as will be the case for physically realistic applications. Thus, C decreases, and the front begins to move to the left, from $z = z_1$. This elevates a larger portion of the rod to the higher temperatures of the upper branch, i.e., the hot spot begins to grow.

This description is given under the proviso that

$$\frac{d}{d\phi}\Gamma(\phi)(1+\phi) > 0$$

which, from the definitions of ϕ and Γ, is equivalent to the statement that the internal energy density of the ceramic, $\rho C_P T$, is an increasing function of the temperature. This is true in ceramics and in almost all materials, away from phase transitions.

We now turn to the further development and stabilization of the hot spot. Noting the definitions of $\phi_\pm(C)$, it follows that the derivative with respect to C of the numerator on the right hand side of (16), i.e.,

$$N = \int_{\phi_-(C)}^{\phi_+(C)} k(\phi)f(\phi)[p\sin^2\pi(z_1 + C) - G(\phi)]\,d\phi, \tag{17}$$

is

$$\frac{\partial}{\partial C}N = \pi p \int_{\phi_-(C)}^{\phi_+(C)} k(\phi)f(\phi)\sin 2\pi(z_1 + C)\,du, \tag{18}$$

which is strictly positive, since $0 < z_1 + C < 1/2$, and k and f are positive. Also, since $\phi_+(0) = v_u$, $\phi_-(0) = v_M$, and $G_M = p\sin^2(\pi z_1)$, we deduce from (17) that $N(0) > 0$. Similarly, if we define $\tilde{C}$ so that $G_m = p\sin^2 \pi(z_1 + \tilde{C})$, then $\phi_+(\tilde{C}) = v_m$, $\phi_-(\tilde{C})$ is the corresponding point on the lower branch of the S-shaped curve, and consequently $N(\tilde{C}) < 0$. Hence, there is a unique $C = C_* < 0$ such that $N(C_*) = 0$. Since the denominator, $D(C)$, of (16) is, for the reasons given above, strictly positive, we deduce that the solution of the differential equation $C' = -N(C)/D(C)$ with initial condition $C(0) = 0$ is monotone decreasing and tends to C_* as $\tau \to \infty$, with its final approach being exponential in τ. Thus, the hot spot grows in size and finally stabilizes to occupy the region $z_1 + C_* < z < 1 - z_1 - C_*$.

We can now consider the influence of a temperature-dependent thermal conductivity on the equilibrated value C_*. First we recall that C_* is such that

$$N(C_*) = \int_{\phi_-(C_*)}^{\phi_+(C_*)} k(\phi)f(\phi)[p\sin^2 \pi(z_1 + C_*) - G(\phi)]\, d\phi = 0, \qquad (19)$$

and consider the case when the thermal conductivity is constant, so that $k(\phi) = 1$. Then, since $\phi_-(C)$ and $\phi_+(C)$ are such that the local power at the boundaries of the hot spot $P = p\sin^2(z_1 + C_*) = G(\phi_\pm(C_*))$, the graphs of $f(u)G(u)$ and $Pf(u)$ intersect transversally at $u = \phi_-(C_*)$, $u = \phi_+(C_*)$, and, from (19), also at some value between, i.e., on the interval $\phi_-(C_*) < u < \phi_+(C_*)$. Equation (19) implies that the unique value C_* is such that the areas of the two lobes between the graphs of $f(u)G(u)$ and $Pf(u)$ are equal. When $k(\phi)$ is not constant but, for example, is a monotone increasing function of ϕ with $k(0) = 1$, as is the case for typical ceramics, the influence of the temperature-dependence with C fixed is such as to increase the area of the right hand lobe (at larger ϕ) more than that of the left hand lobe. This increases $N(C)$, so that, since $\frac{\partial}{\partial C}N > 0$, the temperature-dependence of k is such as to decrease the equilibrating value C_* to more negative values, and hence increase the final width of the hot spot in the steady state.

5. Conclusion.

The implications of the analysis for the sintering of ceramic fibers and joining of ceramic cylinders is now evident. In the first case, the hot spot forms, propagates, and then stabilizes. If the temperature in the relatively warm region of the hot spot is sufficient for sintering, then the fiber can be slowly pulled through the guide, thus insuring that the entire sample is processed. This is used as a means of sintering ceramics in practise [11], and the rate at which the fiber is to be drawn is found experimentally. In the second case, the hot spot is to encompass the butt joint at which the two ceramic cylinders are to be joined, and, if the temperature in this region is sufficient for the materials to fuse, then a strong joint can be obtained [2].

We close by briefly describing another type of solution that is possible if the applied electric field has a minimum at the center of the fiber. This may occur by exciting the applicator in one of its higher spatial modes. If the maximum of the electric field is such that $P > G_M$ and the minimum is such that $P < G_M$, then hot spots will form at both ends of the fiber. These spots will grow in size and stabilize according to the mechanism described at the end of the last section.

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Michael R. Booty
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Institute for Computer Applications in Science
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13. ABSTRACT *(Maximum 200 words)*
A thin cylindrical ceramic sample is placed in a single mode microwave applicator in such a way that the electric field strength is allowed to vary along its axis. The sample can either be a single rod or two rods butted together. We present a simple mathematical model which describes the microwave heating process. It is built on the assumption that the Biot number of the material is small, and that the electric field is known and uniform throughout the cylinder's cross-section. The model takes the form of a non-linear parabolic equation of reaction-diffusion type, with a spatially varying reaction term that corresponds to the spatial variation of the electromagnetic field strength in the waveguide. The equation is analyzed and a solution is found which develops a hot spot near the center of the cylindrical sample and which then propagates outwards until it stabilizes. The propagation and stabilization phenomenon concentrates the microwave energy in a localized region about the center where elevated temperatures may be desirable.

14. SUBJECT TERMS
microwave heating, heat transfer, hot spots, reaction-diffusion equations

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