ARMY RESEARCH LABORATORY

Vulnerability Analyses for the Command and Control Vehicle (C2V): A Supporting Penetration Equation for Aluminum-Kevlar Composites

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13. ABSTRACT *(Maximum 200 words)*

The Logistical and Tactical Targets Branch (LTTB) of the Ballistic Vulnerability Lethality Division (BVLD) has a requirement to conduct vulnerability analyses for the Command and Control Vehicle (C2V). Analysts in LTTB recognized the need for a single penetration algorithm for evaluating the aluminum-Kevlar composites serving as C2V armor materials. Existing penetration equations were not developed to address composites. The Systems Analysis Branch (SAB) designed and executed an experimental program to provide support for the development of a penetration algorithm for armor-piercing (AP) projectiles against aluminum-Kevlar composites. The Simulation Technology Division (STD) assisted SAB in the program design and developed an algorithm for predicting residual velocity behind aluminum-Kevlar composites. This report documents the experimental program, results, and modeling efforts.

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aluminum, Kevlar, C2V, composite armor, penetration equations, 14.5-mm B32, 7.62-mm B32, regression modeling

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1. INTRODUCTION

The Logistical and Tactical Targets Branch (LTTB) of the Ballistic Vulnerability Lethality Division (BVLD) has an FY95 requirement to conduct vulnerability analyses for the Command and Control Vehicle (C2V). Analysts in LTTB realized that the penetration equations for aluminum-Kevlar composites, the C2V's armor materials, were not very robust. In fact, current practice is to use the Project THOR equation for aluminum and to apply a locally developed equation for Kevlar. LTTB analysts realized that it would be advantageous to have a single penetration algorithm for the composite instead of using two separate equations that were developed for penetration through each individual material. Thus, LTTB requested that the Systems Analysis Branch (SAB) plan and execute an experimental program designed to gather sufficient data for use in the development of a penetration algorithm for armor-piercing (AP) projectiles vs. aluminum-Kevlar composites. SAB, in turn, enlisted the Simulation Technology Division (STD) to aid in the data analysis and algorithm development portion of this effort.

LTTB stated a desire for an algorithm that would calculate residual masses and velocities for various sizes of small caliber AP projectiles as they perforate aluminum-Kevlar composites of varying thicknesses. After LTTB expressed its needs, the scope of the effort had to be determined. Since all AP projectiles could not be fired at all combinations of aluminum and Kevlar thicknesses, it was decided, for expediency and cost concerns, that only projectiles and materials which SAB had on hand would be used. The final test matrix included 7.62-mm and 14.5-mm armor-piercing incendiary (API) B32 steel core projectiles, 30.5-cm × 30.5-cm composite targets of various thicknesses, and shotline obliquities of 0° and 45°. The experimental strategy section of this report will provide greater detail on the matrix and explain how the various thicknesses were chosen. The remainder of this report will document the experimental design, experimental results, and data analysis.

2. METHODOLOGY

2.1 <u>Experimental Strategy.</u>

2.1.1 Stages. Data were collected in two stages. Experimental results from testing according to a pilot design provided support for decisions as to how to make efficient use of resources to provide a basis for the modeling effort. There were two principal advantages to this approach. First, no complete agreement existed among planners regarding which combinations of test parameters would yield useful data. (Since modeling residual velocity and mass was the goal, it followed that perforation should occur, but that great overmatch should not, if useful data were to be obtained.) Thus, rather than implement an experimental design involving all the resources and running the risk of weak data to support modeling, a pilot design was first run using partial resources. It is important to emphasize an implementation of a fixed initial design. A haphazard firing program (i.e., in which a design change was made after each shot or series of shots) was not conducted.

The second advantage was that pilot data were available for preliminary analysis. This allowed for the validity of theorized models to be tested and provided for a rationale—augment data to strengthen model support—for filling out the data set. New ground was being broken in the theorized model because two plates, one aluminum and one Kevlar, each with varying thicknesses, were combined as the target. Previous modeling involved only one material for the target plate (e.g., Bely, Bodt, and Schumacher 1992; Holloway et al. 1978).

2.1.2 Design Matrix. Careful consideration was given to the experimental design matrix (Table 1) due to the limited resources available. The two projectiles, 7.62-mm B32 and the 14.5-mm B32, were chosen because of their availability in sufficient quantities and their similarity excepting scale. Thus, it was felt that any algorithm developed should be valid through the range of projectile sizes that they bracket.

The availability of Kevlar was the limiting factor for the matrix. Only one thickness, 19 mm, was available, and since more than one thickness was desired, two pieces were used for some targets. The matrix ended up with composites containing 19 mm and 38 mm of Kevlar. The small number of available targets limited the shotline obliquities to 0° and 45°.

The thickness of the aluminum portion of the composites to be used also required some consideration. It was decided that three thicknesses of aluminum would be used. One thickness was to approximate that being considered for the C2V, and the other two were to bracket the first while taking into account the known capability of the projectiles against aluminum armor. The first thickness chosen was 32 mm which was to represent that of the C2V. The thickest dimension chosen was 44.5 mm which at high obliquities represents a challenging target for the 14.5-mm projectile. Sources, such as M113 Live Fire Test Data (Grote et al. 1990), were consulted to verify this assumption about the 14.5-mm projectile. On the other

Table 1. Design Matrix

Factor	Levels		
Projectile Mass (g)	5.40	40.50	
Aluminum Thickness (mm)	12.70	31.75	44.45
Kevlar Thickness (mm)	19.05	38.10	
Target Obliquity (deg)	0	45	

side of the spectrum, 13 mm was chosen since it was determined that 13 mm would probably be the smallest thickness of aluminum for which LTTB would have an interest.

2.1.3 V-ratio. After parameters involving obliquity, thickness, and mass are set, the only remaining freedom of selection is velocity. A very simple but helpful tool in assessing the overall quality of the data set in terms of velocity and, thereby, helpful in selecting specific velocities is the v-ratio. The v-ratio is defined as (Vs − Vr) / Vs, where V denotes velocity and the subscripts "s" and "r" denote the modifiers striking and residual, respectively. The v-ratio is defined on the interval [0,1]. To explain why this is helpful we must first briefly explain data collection for regression modeling, the analytical tool that is discussed more fully in section 2.3.

Consider a simple linear model, that is, one having the form of a straight line with a single response and a single explanatory variable. If the practicable range of the explanatory variable is bounded in the interval [L,U] and if the underlying model is known to be linear, a sensible test design would involve placing an equal number of points at L and U. This design would be optimal in the sense that the variance of the slope estimate is minimized. However, if the exact form of the model is not known, it is advisable to space the values of the explanatory variable over the practicable range allowing for alternative model forms to be investigated (Neter and Wasserman 1974).

In our situation, neither the model form nor the practicable range [L,U] with respect to velocity was known. Indeed, the practicable range would likely change as other experimental conditions were varied. In the spirit of the design strategy described in the previous paragraph, v-ratio values scattered within [0,1] would be most useful. With this fact in mind, the quality of data with respect to velocity in stage 1 could be assessed and the augmentation of data in stage 2 could be planned.

2.2 Experimental Setup. The experimental setup was typical of ballistic work. The 14.5-mm projectile was fired from a Soviet 14.5-mm gun that was fastened to a fixed mount. The 7.62-mm projectile was fired from a Mann barrel that was also on a fixed mount. The 14.5-mm bullet core is shown in Figure 1. The 14.5 mm was initially fired at a range of 100 ft but, as weather became inclement, all firings were conducted indoors at a distance of about 10 ft. Break screens were used to obtain the striking and residual velocities of the projectiles—projectile breakup was not an issue. The target plates were clamped to a rigid test stand, and a series of celotex and plywood panels were used to catch the projectiles. Figure 2 is a schematic of the experimental setup.

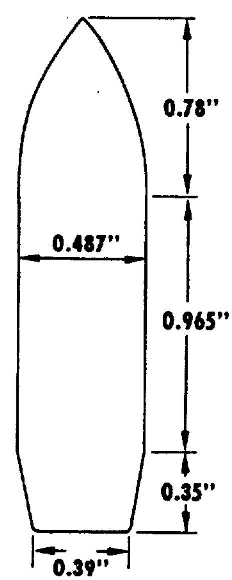

Figure 1. Core dimensions for the 14.5-mm B32 projectile.

The composite armor targets were cut into squares with 30.5-cm sides in order to facilitate handling by range personnel. The aluminum and Kevlar pieces were simply taped and clamped together to form the composite. The aluminum alloy used was 5083 with a modulus of elasticity of 68,950 MPa, a density of 2.66 g/cm^3, and a Poisson's ratio of 0.336. The Kevlar used was Kevlar 29, 3000 Denier, 34 plies, with an area density of 2.2–2.25 g/cm^2.

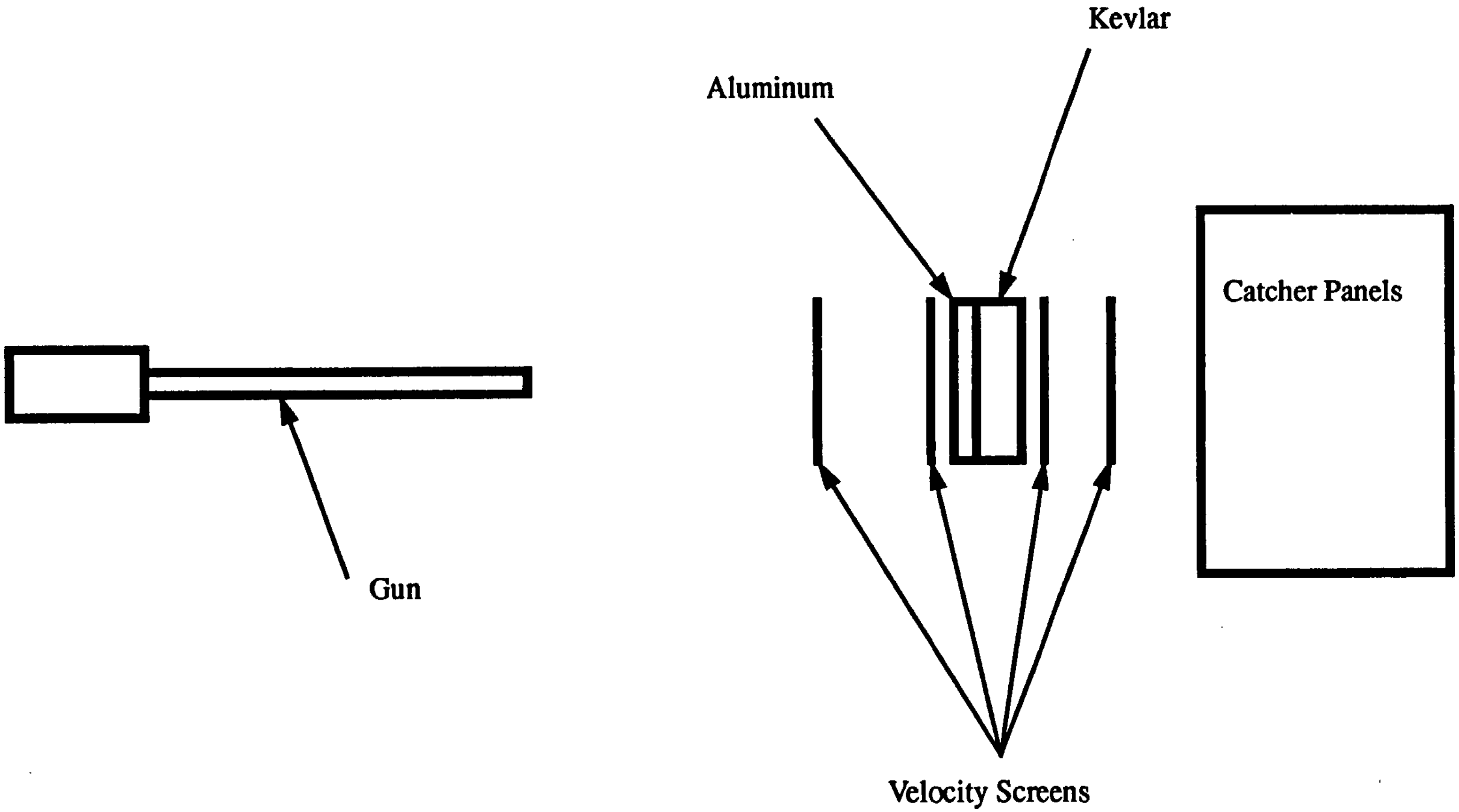

Figure 2. Schematic of experimental setup.

2.3 Modeling Approach. With only minor exceptions with regard to model form and the least-squares approach, the development of the penetration equations proceeded as in Bely, Bodt, and Schumacher (1992). The model form proposed for residual velocity in this study was

$$V_r = V_s - 10^a A^b K^c M_s^d (\sec \theta)^e V_s^f, \tag{1}$$

where

V_r = fragment residual velocity (m/s)
V_s = fragment striking velocity (m/s)
A = aluminum thickness (mm)
K = Kevlar thickness (mm)
M_s = fragment striking mass (g)
θ = obliquity (°)

and a, b, c, d, e, and f are empirically derived regression model coefficients. In Bely, Bodt, and Schumacher (1992), the model included a fragment shape factor not included here. Since the 7.62 mm and 14.5 mm scaled nearly proportionally (aspect ratios of 4.3 and 4.5, respectively), their fragment shape factor was not a variable and was absorbed in the leading constant, 10^a. Another modification was the use of two plate thickness factors, one for aluminum and one for Kevlar. To our knowledge, two thickness factors in such a model have not been tried previously.

In order to apply multiple linear regression to this intrinsically linear model, (1) was transformed to the form

$$\begin{aligned}\log(V_s - V_r) = a + b \log(A) + c \log(K) + d \log(M_s) \\ + e \log(\sec \theta) + f \log(V_s).\end{aligned} \tag{2}$$

Regression was accomplished using the package SYSTAT/W 5.2, with modeling diagnostics carried out in accordance with Belsley, Kuh, and Welsch (1980). Model performance is reported in terms of the root-mean squared (RMS) error, the square root of the average residual squared. Diagnostics and analysis of variance results are reported in terms of the transformed metric (2). RMS results are expressed in original units.

3. RESULTS

3.1 Stage 1. Data appear in Tables 2 and 3, and also in Appendix A. Results from the initial stage of testing yielded the ***shaded*** values reported in Tables 2 and 3. Two entries are made for each cell: the striking velocity of the projectile (upper left), and the v-ratio (lower right). Not all cells have entries because either penetration for those factor level combinations was not likely to occur or it was likely to be an extreme overmatch.

Plausibility of the model form was assessed with preliminary regression modeling using (2). The initial fit was reasonable but with two unusual observations. When those observations were treated as outliers and removed from the analysis, the model was greatly improved in terms of the percentage of variation in the response explained by the predictor variables and in terms of the behavior of residuals, the difference between the observed response value and its predicted value. Further, all predictor coefficients were statistically significant ($\alpha = 0.01$). This was particularly important to the dual thickness approach. With this preliminary analysis, we determined that the proposed model form was plausible, statistically. Detail regarding the analysis at this stage appears as Appendix B.

Stage 1 experimentation showed that a model for residual mass would not be feasible. The steel core of the projectile tended to either pass through the target intact or not pass through at all. This was true in all but a few cases.

Augmenting the data base was accomplished using the v-ratio approach with the remaining resources. The strategy was to target combinations of experimental conditions from stage 1 that were not well represented, or for which inconsistent results (e.g., higher striking velocity yielding a lower residual velocity) were reported. Not all points requested were obtained, but one can see, in Tables 2 and 3, that the stage 2 firing, ***unshaded*** values, resulted in a wider range of v-ratio values, thus strengthening the support for the stage 2 modeling effort.

3.2 Stage 2. The completed firing program yielded 56 data values. All data gathered appear in Appendix A. Case 23 was excluded from the analysis due to a missing value for the residual velocity. Cases 1, 18, 25, 48, and 49 were eliminated during the modeling effort. Rationale for the elimination of these points appears as part of the stage 2 analysis detailed in Appendix C.

Table 2. Data for the 7.62-mm B32 Projectile (M_s = 5.4 g)

Aluminum (mm)	12.70		31.75		44.45	
Kevlar (mm)	19.05	38.10	19.05	38.10	19.05	38.10
0° Obliquity	787[a] .26[b]	786 .42	816 .48	868 .50		
	622 .54	740 .47	865 .49	789 .87		
	536 .74	655 .64	753 .71	789 .88		
	512 1.00	622 .86	753 .79	754 1.00		
			613 1.00			
45° Obliquity	777[c] .27	791 1.00	755[c] 1.00			
	865 .61					
	790 .69					
	802 .78					
	783 .83					

[a] Striking velocity (m/s) is expressed in upper left corner of table cells.
[b] V-ratio is expressed in lower right corner of table cells.
[c] Determined an outlier in the regression analysis.

The final model developed using (2) explained 81% of the variation in residual velocity. The standard error of prediction of the response was reduced slightly relative to stage 1. All model inputs were found to contribute to the prediction, including the separate inputs for Kevlar and aluminum thicknesses. In the final model, all goodness criteria were met.

In terms of equation (1), the model is

$$V_r = V_s - 10^{5.859} A^{.832} K^{.493} M_s^{-.404} (\sec\theta)^{2.512} V_s^{-1.641}. \tag{3}$$

Table 3. Data for the 14.5-mm B32 Projectile (M_s = 40.5 g)

Aluminum (mm)	12.70		31.75		44.45	
Kevlar (mm)	19.05	38.10	19.05	38.10	19.05	38.10
0° Obliquity			843[a] .16[b]	757[c] .21	915 .29	913 .33
			660 .37	715[c] .22	899 .30	903 .39
			572 .46	807 .31	784 .37	781 .86
			800[c] .79	638 .61	793 .38	
			536 .80	619 .76	723 .56	
			472 1.00	618 1.00	680 .66	
45° Obliquity			819 .41	808 .67	918 .49	918 .81
			780 .66		858 .91	791 1.00
			672 1.00		791 1.00	
			708 1.00			

[a] Striking velocity (m/s) is expressed in upper left corner of table cells.
[b] V-ratio is expressed in lower right corner of table cells.
[c] Determined an outlier in the regression analysis.

When using the model, negative residual velocities are set to zero. Therefore, the predicted residual velocity was set to zero for the four values predicted to be less than zero. The RMS was then computed based on the 50 data values remaining after outlier and missing observation removal. The RMS was found to be 78 m/s, loosely conveying the average absolute difference between the predicted and true residual velocity values. Figure 3 shows the model predicted residual velocities (VRHAT) plotted against the observed residual velocities (VR) for all obtainable residual velocities. Error bars representing 78 m/s to either side of the line VRHAT = VR and outlier case numbers are included. The explained variation in original units, with the convention that negative residual velocities are set to zero, is somewhat better than reported for the transformed metric. The proportion of variation explained was taken to be one minus the

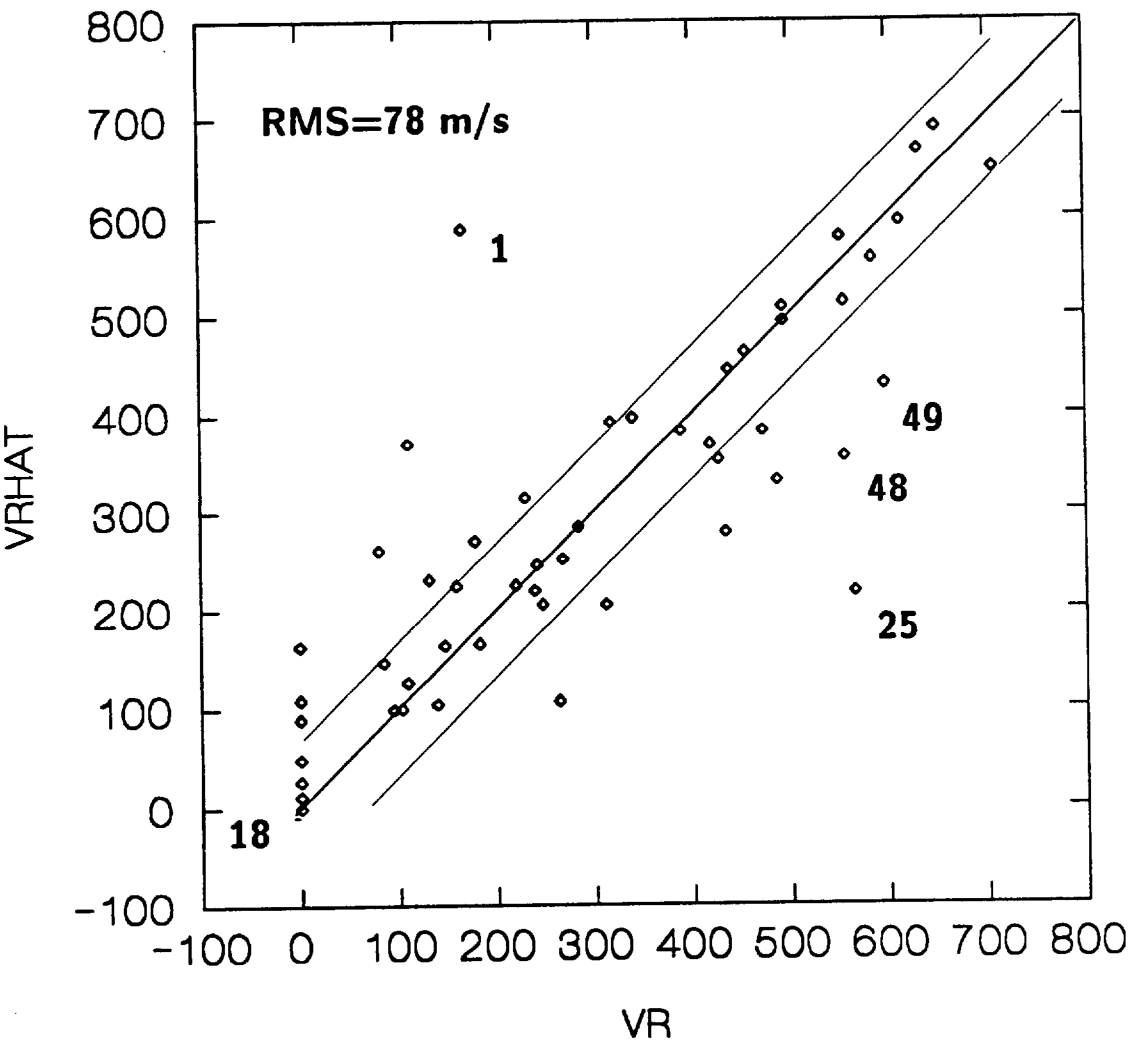

Figure 3. Predicted residual velocities (VRHAT) plotted against observed residual velocities (VR) for the final model expressed as equation (3).

ratio of the sum of squared residuals to the sum of squared deviations for residual velocity. Expressed as a percentage, 86% of the variation in residual velocity is explained by (3).

4. DISCUSSION

4.1 General Comments. In this program we implemented a staged sequence of experimentation to gather data supporting a regression analysis. A benefit of the staging was that it allowed a preliminary evaluation of the model before all resources had been expended. The initial analysis guided us in the augmentation of data to support the final model.

The dual thickness modification on the original THOR equation proved to be effective in handling the situation where Kevlar is affixed to the back of an aluminum plate. This modification has not been explored from foundations of physics. It was merely a convenient modeling convention which worked.

Frequently, firing programs are conducted with the intent of gathering information on residual mass and velocity. The v-ratio concept seems to help guide the range personnel in gathering more informative data for the modeling effort. It was effective because only one experimental parameter, velocity, was being varied. It is not straightforward to answer how to achieve, or to monitor the achievement of, the optimal selection of "regression" data where two or more experimental parameters are jointly varied.

4.2 Model Use. It is our suggestion that the model above only be applied over the ranges for aluminum thickness, Kevlar thickness, obliquity, and velocity included in this program. Additionally, the model is only appropriate for scaled versions of the two projectiles considered, with masses between 5.4 g and 40.5 g. It is further suggested that any negative residual velocity be regarded as zero.

With regard to residual mass, experimentation showed that projectile breakup was not an issue. Thus, if the residual velocity is nonzero, the mass should be taken to be the entire projectile core.

The English units equivalent of this model with feet per second instead of meters per second and inches instead of millimeters requires only a change in the exponent for 10. In the English units model, the exponent should be 2.426.

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5. REFERENCES

Belsley, D., E. Kuh, and R. Welsch. Regression Diagnostics. New York: John Wiley and Sons, 1980.

Bely, D., B. Bodt, and R. Schumacher. "M74 Bomblet Penetration Evaluation." BRL-TR-3400, U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD, 1992.

Grote, R., J. Martin, S. Powell, and W. Thompson. "M113 Family of Vehicles Vulnerability Test Program, Vol. I: Results Summary and Analysis (U)." BRL-TR-3162 (Secret), U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD, 1990.

Holloway, G., M. Danish, and J. Matts. "Penetration Relations for Tungsten Alloy Fragments vs. Selected Target Range Materials." BRL-TR-02087, U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD, 1978.

Neter, J., and W. Wasserman. Applied Linear Statistical Models. Homewood, IL: Richard D. Irwin, Inc., 1974.

The Johns Hopkins University. "The Resistance of Various Metallic Materials to Perforation by Steel Fragments; Empirical Relationships for Fragment Residual Velocity and Residual Weight." Technical Report No. 47, Ballistic Analysis Laboratory, Institute for Cooperative Research, Project THOR, 1961.

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APPENDIX A:

FIRING RESULTS

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Case	Aluminum (mm)	Kevlar (mm)	Angle (deg)	V_s (m/s)	V_r (m/s)	M_s (g)
1	31.75	19.05	0	800	167	40.5
2	31.75	19.05	45	819	487	40.5
3	31.75	38.10	0	807	556	40.5
4	31.75	38.10	45	808	264	40.5
5	44.45	19.05	0	793	494	40.5
6	44.45	19.05	0	784	494	40.5
7	44.45	19.05	0	899	632	40.5
8	44.45	19.05	0	915	650	40.5
9	44.45	19.05	45	791	0	40.5
10	44.45	19.05	45	918	473	40.5
11	44.45	38.10	0	781	111	40.5
12	44.45	38.10	0	913	0	40.5
13	44.45	38.10	45	791	0	40.5
14	44.45	38.10	45	918	183	40.5
15	31.75	19.05	0	865	438	5.4
16	31.75	19.05	0	753	159	5.4
17	31.75	19.05	0	613	0	5.4
18	31.75	19.05	45	755	0	5.4
19	31.75	38.10	0	754	0	5.4
20	31.75	38.10	0	789	95	5.4
21	12.70	19.05	0	512	0	5.4
22	12.70	19.05	0	787	585	5.4
23	12.70	19.05	45	752	***	5.4
24	12.70	19.05	45	790	242	5.4
25	12.70	19.05	45	777	566	5.4
26	12.70	38.10	0	786	455	5.4
27	12.70	38.10	45	791	0	5.4
28	12.70	19.05	0	536	139	5.4
29	12.70	19.05	0	622	284	5.4
30	12.70	19.05	45	783	132	5.4
31	12.70	19.05	45	802	179	5.4
32	12.70	19.05	45	865	340	5.4
33	12.70	38.10	0	740	389	5.4
34	12.70	38.10	0	655	239	5.4
35	12.70	38.10	0	622	85	5.4
36	31.75	19.05	0	816	428	5.4
37	31.75	19.05	0	753	220	5.4
38	31.75	19.05	0	843	708	40.5
39	31.75	19.05	0	572	312	40.5
40	31.75	19.05	0	660	419	40.5
41	31.75	19.05	0	472	0	40.5
42	31.75	19.05	0	536	109	40.5
43	31.75	19.05	45	672	0	40.5
44	31.75	19.05	45	780	268	40.5
45	31.75	19.05	45	708	0	40.5
46	31.75	38.10	0	868	435	5.4
47	31.75	38.10	0	789	103	5.4
48	31.75	38.10	0	715	556	40.5
49	31.75	38.10	0	757	597	40.5
50	31.75	38.10	0	618	0	40.5
51	31.75	38.10	0	619	147	40.5
52	31.75	38.10	0	638	247	40.5
53	44.45	19.05	0	723	318	40.5
54	44.45	19.05	0	680	230	40.5
55	44.45	19.05	45	858	81	40.5
56	44.45	38.10	0	903	553	40.5

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APPENDIX B:

STAGE 1 ANALYSIS

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In stage 1, 27 shots were taken. Figure B-1 shows the output from a regression analysis carried out in the transformed metric suggested by equation (2). Case 23 had a missing value for the residual velocity and was removed from consideration. All coefficients were indicated as significant; their standardized values indicating, for example, that the aluminum thickness had the greatest influence on residual velocity prediction and the Kevlar thickness had the least.

DEP VAR: LOGVDIFF N: 26 MULTIPLE R: 0.780 SQUARED MULTIPLE R: 0.608
ADJUSTED SQUARED MULTIPLE R: 0.510 STANDARD ERROR OF ESTIMATE: 0.139

VARIABLE	COEFFICIENT	STD ERROR	STD COEF	TOLERANCE	T	P (2 TAIL)
CONSTANT	6.124	1.741	0.000		3.518	0.002
LOGAL	0.760	0.200	0.811	0.431	3.806	0.001
LOGKEV	0.509	0.190	0.381	0.968	2.680	0.014
LOGMS	-0.265	0.097	-0.592	0.418	-2.734	0.013
LOGSEANG	1.448	0.395	0.542	0.894	3.664	0.002
LOGVS	-1.744	0.632	-0.465	0.691	-2.759	0.012

ANALYSIS OF VARIANCE

SOURCE	SUM-OF-SQUARES	DF	MEAN-SQUARE	F-RATIO	P
REGRESSION	0.604	5	0.121	6.213	0.001
RESIDUAL	0.389	20	0.019		

Figure B-1. Regression analysis for all available stage 1 data.

Graphical diagnostics for the residuals in this model are shown as Figure B-2. In Figure B-2a, the Studentized residuals are plotted against the estimated response values. Ideally, one would see only small variation random scatter about the horizontal line representing a Studentized residual of zero. Cases 1 and 25 appear unusual relative to the others. Figure B-2b shows the normal probability plot for the model residuals. The standard normal distribution expected values are plotted against the residuals. Normally distributed residuals would appear as a straight line. However, if one imagines a straight line drawn through the inner majority of the data, case 25 falls to the left and case 1 to the right. This is further indication that cases 1 and 25 might be considered outliers.

Figure B-3 shows the regression analysis with cases 1 and 25 removed. The degree of improvement in the model can be seen several places. First, the SQUARED MULTIPLE R has increased from 0.608 to 0.849. Second, greater significance is listed for all coefficients. Third, and perhaps the best indication of improvement is that the residual mean square error has been reduced from 0.019 to 0.007. The improvement extends to the graphical diagnostics as shown in Figure B-4.

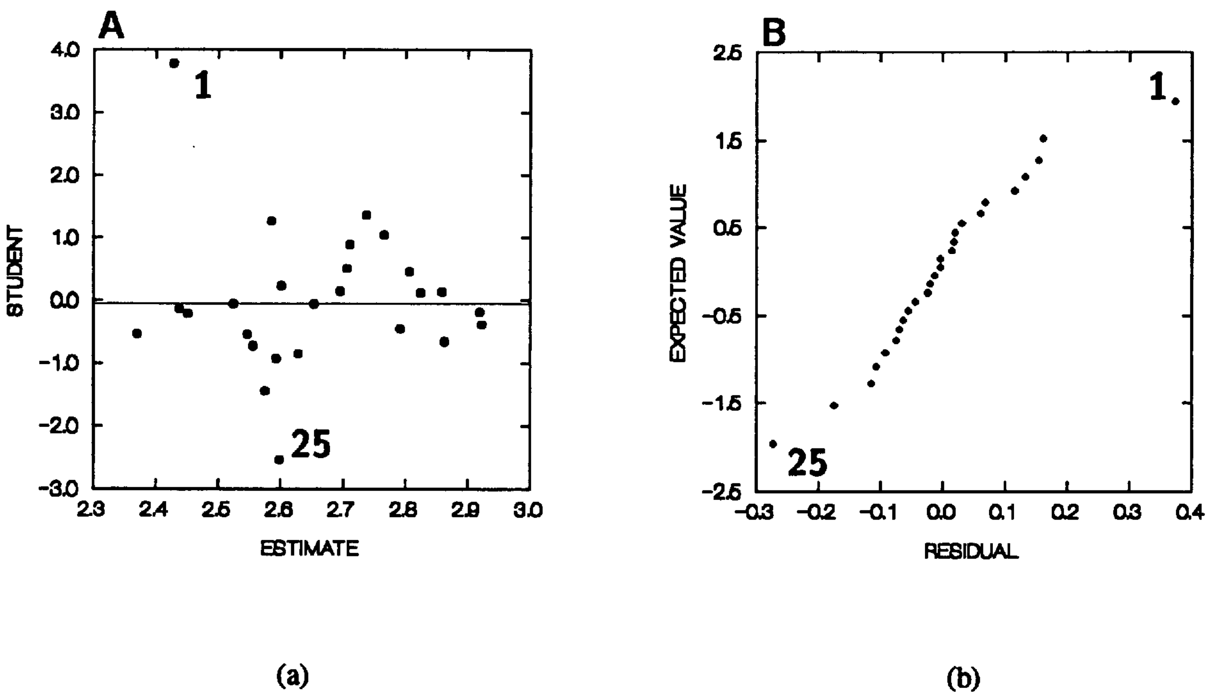


Figure B-2. Graphical regression diagnostics: (a) standard residual plot, and (b) normal probability plot.

DEP VAR: LOGVDIFF N: 24 MULTIPLE R: 0.921 SQUARED MULTIPLE R: 0.849
ADJUSTED SQUARED MULTIPLE R: 0.807 STANDARD ERROR OF ESTIMATE: 0.085

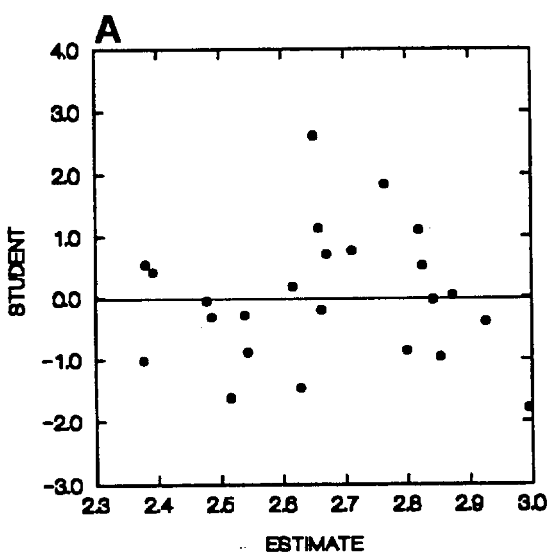
VARIABLE	COEFFICIENT	STD ERROR	STD COEF	TOLERANCE	T	P (2 TAIL)
CONSTANT	5.681	1.064	0.000		5.340	0.000
LOGAL	0.767	0.129	0.824	0.440	5.959	0.000
LOGKEV	0.539	0.120	0.422	0.954	4.498	0.000
LOGMS	-0.354	0.062	-0.816	0.415	-5.730	0.000
LOGSEANG	1.891	0.252	0.728	0.893	7.500	0.000
LOGVS	-1.582	0.388	-0.453	0.681	-4.075	0.001

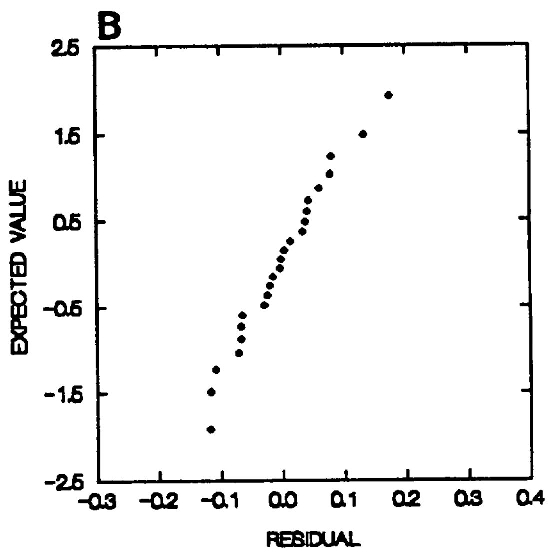
ANALYSIS OF VARIANCE

SOURCE	SUM-OF-SQUARES	DF	MEAN-SQUARE	F-RATIO	P
REGRESSION	0.729	5	0.146	20.181	0.000
RESIDUAL	0.130	18	0.007		

Figure B-3. Regression analysis for stage 1 data, outliers removed.

With a reasonable model in place, we were confident that the form of the model would remain valid with additional data. Stage 2 was used to augment the data base in cells we felt were lacking sufficient information.





(a) (b)

Figure B-4. Graphical regression diagnostics with two outliers removed: (a) standard residual plot, and (b) normal probability plot.

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APPENDIX C:

STAGE 2 ANALYSIS

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Figure C-1 shows the regression analysis results with no values removed except case 23 for which no residual velocity was available.

DEP VAR: LOGVDIFF N: 55 MULTIPLE R: 0.754 SQUARED MULTIPLE R: 0.568
ADJUSTED SQUARED MULTIPLE R: 0.524 STANDARD ERROR OF ESTIMATE: 0.136

VARIABLE	COEFFICIENT	STD ERROR	STD COEF	TOLERANCE	T	P (2 TAIL)
CONSTANT	5.469	0.854	0.000		6.401	0.000
LOGAL	0.795	0.148	0.832	0.367	5.370	0.000
LOGKEV	0.360	0.131	0.270	0.910	2.744	0.008
LOGMS	-0.367	0.065	-0.817	0.416	-5.615	0.000
LOGSEANG	2.155	0.307	0.769	0.736	7.028	0.000
LOGVS	-1.437	0.327	-0.495	0.696	-4.402	0.000

ANALYSIS OF VARIANCE

SOURCE	SUM-OF-SQUARES	DF	MEAN-SQUARE	F-RATIO	P
REGRESSION	1.187	5	0.237	12.906	0.000
RESIDUAL	0.902	49	0.018		

Figure C-1. Regression analysis for complete data set, stage 2.

In Figure C-2a, the Studentized residuals are plotted against the estimated response values. Large Studentized residuals were taken to be those with absolute values in excess of 2. Figure C-2b depicts a normal probability plot with expected normal values on the y-axis. If one can imagine a straight line through most (inner portion) of the values, the residuals are said to be approximately normally distributed. Points to the left or right of that envisioned line are suspected outliers. This explains the rationale for targeting for removal all but case 18.

In addition to looking at the magnitude of residuals, leverage and Cook's distance measures were also considered. Leverage indicates the potential for a design point to influence the parameter estimates in a regression irrespective of the response observed; Cook's distance combines leverage with Studentized residuals, together indicating the real influence made by that design point and its associated response. Case 18, it was determined, had high leverage and relatively high Cook's distance, eschewing significance levels. This can be seen in the examination of Figures C-2c and C-2d. In conferring with the range personnel, it was determined that the velocity for that case, 755 m/s, was the smallest velocity among several actually fired. In no instance was a penetration achieved. Thus, the value 755 m/s is truly a censored value caused by the extreme undermatch conditions.

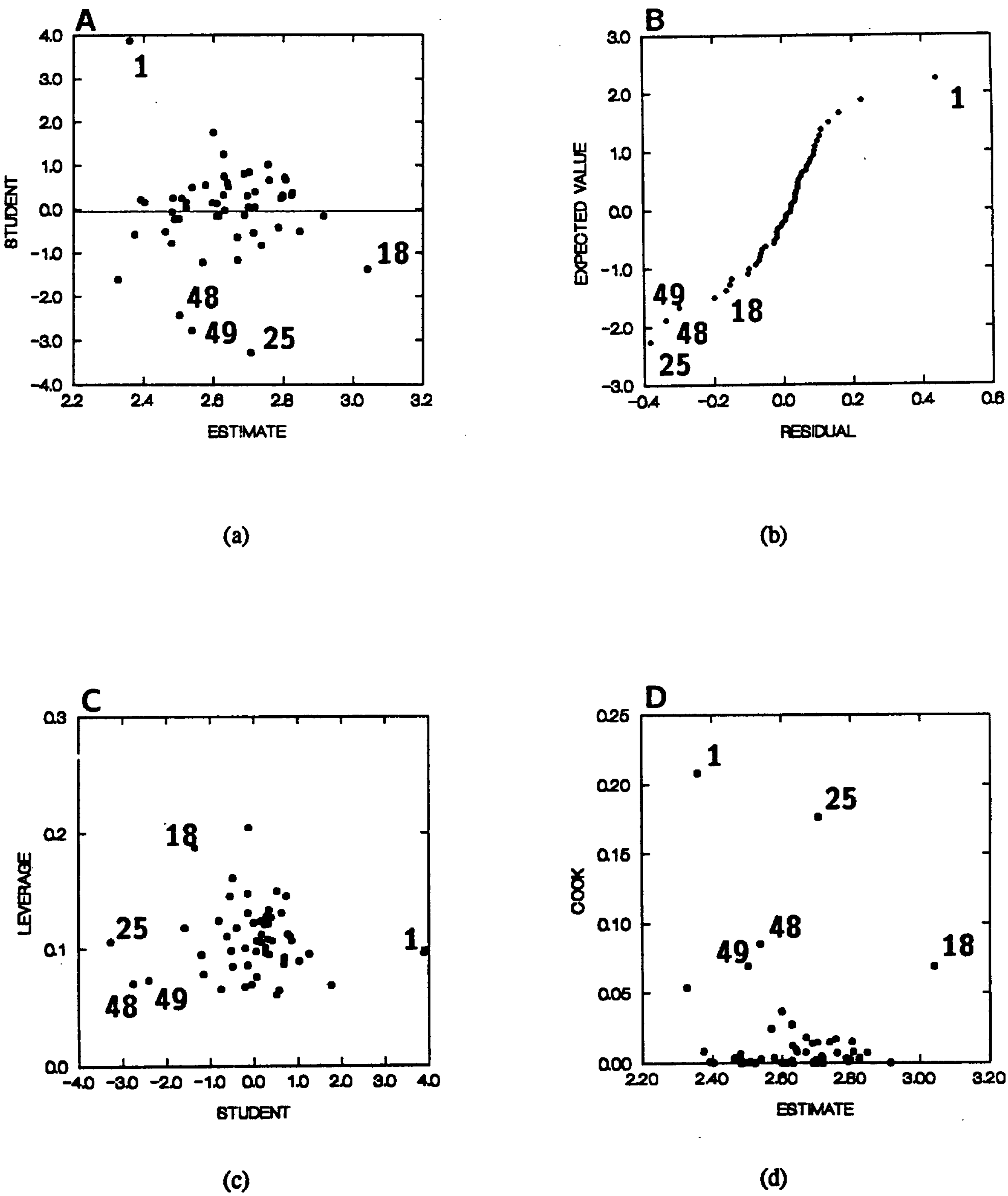

Figure C-2. Graphical regression diagnostics: (a) standard residual plot, (b) normal probability plot, (c) leverage plot, and (d) Cook's distance plot.

Figure C-3 lists the results for the final regression analysis performed after outliers and missing value cases had been removed. The explained variation of the response given the predictor variables is 81%. All coefficients corresponding to the predictor variables were found significant, and all are compatible with regard to the physical setting.

DEP VAR: LOGVDIFF N: 50 MULTIPLE R: 0.902 SQUARED MULTIPLE R: 0.814
ADJUSTED SQUARED MULTIPLE R: 0.793 STANDARD ERROR OF ESTIMATE: 0.080

VARIABLE	COEFFICIENT	STD ERROR	STD COEF	TOLERANCE	T	P (2 TAIL)
CONSTANT	5.859	0.519	0.000		11.297	0.000
LOGAL	0.832	0.096	0.993	0.324	8.695	0.000
LOGKEV	0.493	0.081	0.414	0.925	6.126	0.000
LOGMS	-0.404	0.043	-1.008	0.360	-9.304	0.000
LOGSEANG	2.512	0.199	0.995	0.682	12.637	0.000
LOGVS	-1.641	0.201	-0.661	0.647	-8.180	0.000

ANALYSIS OF VARIANCE

SOURCE	SUM-OF-SQUARES	DF	MEAN-SQUARE	F-RATIO	P
REGRESSION	1.234	5	0.247	38.506	0.000
RESIDUAL	0.282	44	0.006		

Figure C-3. Final regression analysis results after elimination of several outliers.

Figure C-4 shows the residual diagnostics for this final model. A trained eye would detect no serious departures from regression analysis assumptions.

Figure C-5 shows a scatterplot matrix of the Studentized residuals plotted against each term in the model. Ideally, the residuals should have the same pattern of variation over the range of each model term. (For each term, levels increase from top to bottom.) This is true for all but the aluminum thickness term, where it appears that the 31.75-mm target (middle group) generally yielded smaller residuals than did the other two thicknesses. Further analysis was conducted to see if a quadratic term for aluminum thickness could be introduced in the model effectively. We found that it offered no significant improvement.

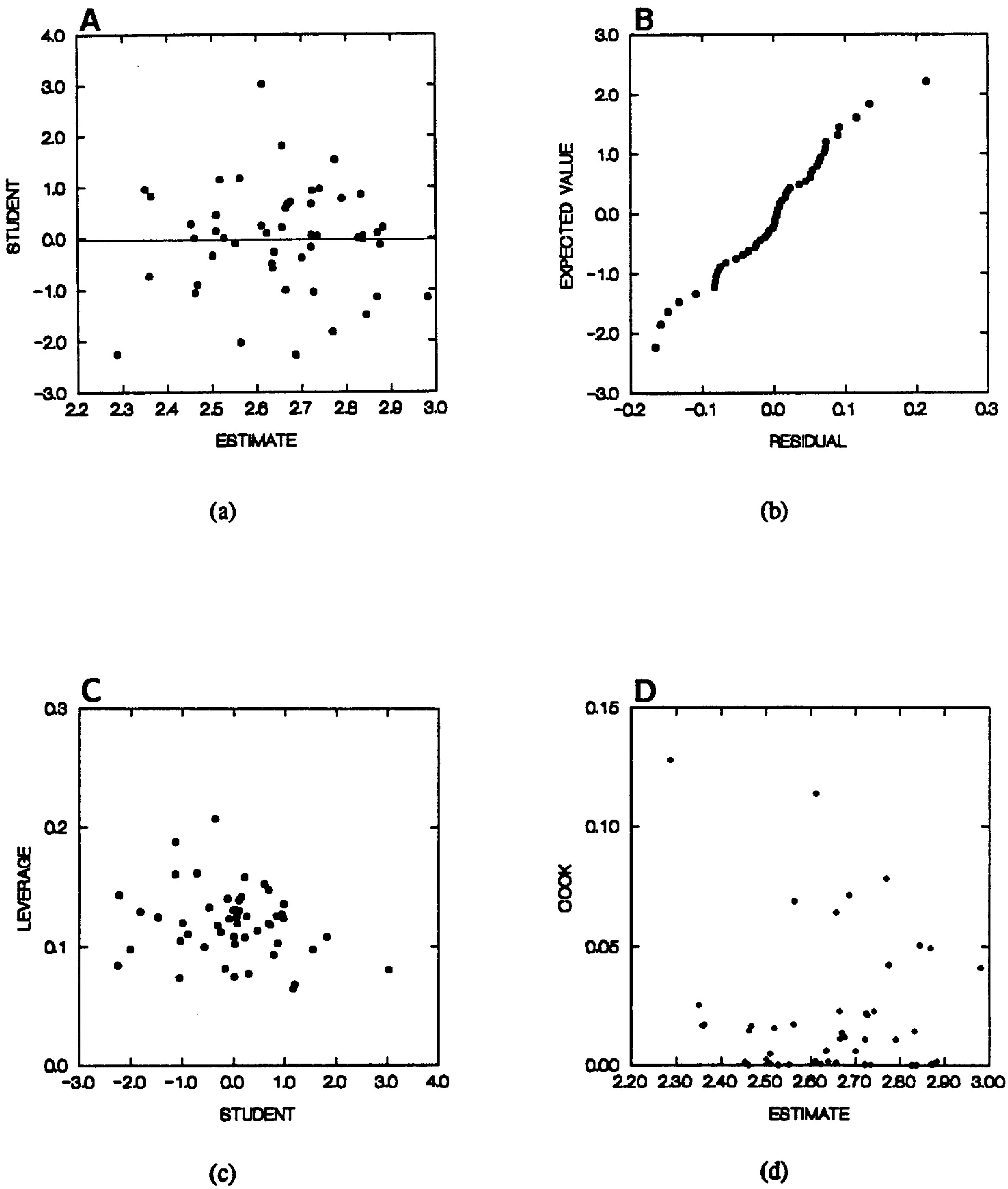

Figure C-4. Graphical regression diagnostics with all outliers removed: (a) standard residual plot, (b) normal probability plot, (c) leverage plot, and (d) Cook's distance plot.

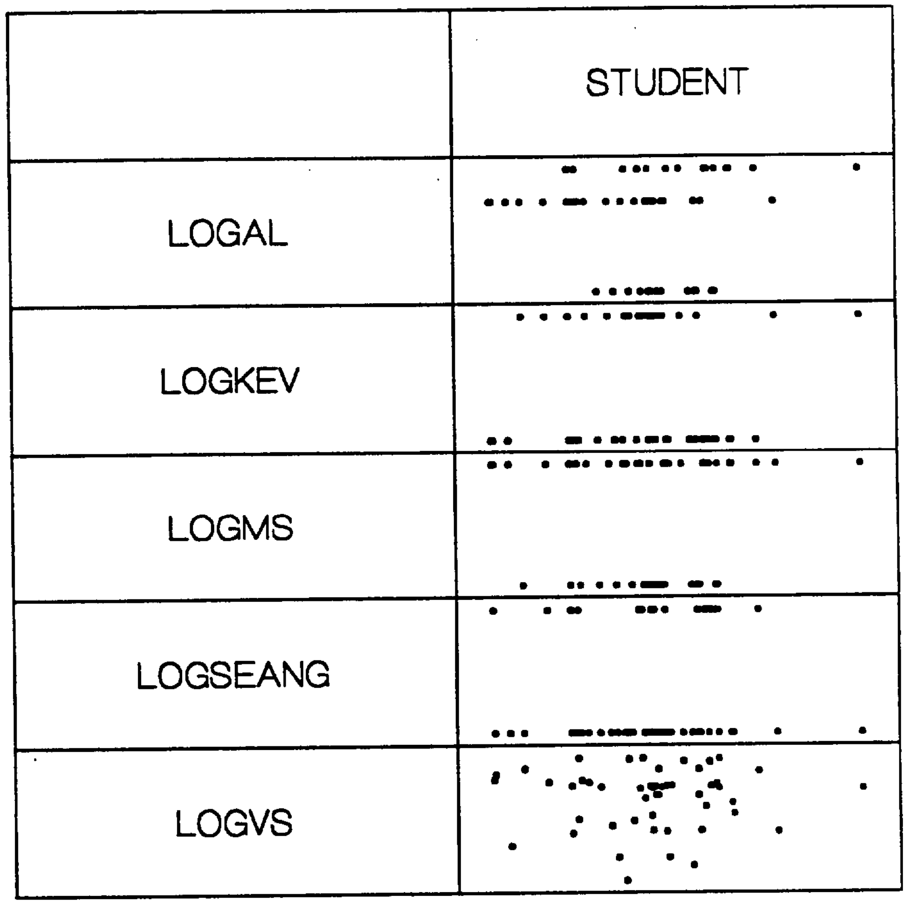

Figure C-5. Scatterplot matrix showing the Studentized residuals for the range of values over each predictor variable.

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