An Analysis of the Requirements Levels and Performance Projection Modules of the Corporate Information Management Requirements System

October 1994

FOR

Joint Logistics Systems Center, Materiel Management Operations Research Group (JLSC-MM/ORG)

and

DLA Materiel Management,
Business Management Office (MMSB)
Headquarters, Defense Logistics Agency
Alexandria, VA 22304

INSIGHT THROUGH ANALYSIS

DORO

An Analysis of the Requirements Levels and Performance Projection Modules of the Corporate Information Management Requirements System

Accesion For	
NTIS CRA&I	X
DTIC TAB	
Unannounced	
Justification	
By	
Distribution /	
Availability Codes	
Dist	Avail and / or Special
A-1	

October 1994

DTIC QUALITY INSPECTED 2

Benedict C. Roberts
MAJ Charles Noll, USA
Jeffrey J. Hobbs
Capt Edward Dawson, USAF
Robert Greiner

DEPARTMENT OF DEFENSE
DEFENSE LOGISTICS AGENCY
Operations Research Office

c/o Defense General Supply Center
8000 Jefferson Davis Highway
Richmond, VA 23297-5082

O'Hare International Airport
P.O. Box 66422
Chicago, IL 60666-0422

DEFENSE LOGISTICS AGENCY
OPERATIONS RESEARCH OFFICE
DORO
c/o DEFENSE GENERAL SUPPLY CENTER
RICHMOND, VIRGINIA 23297-5082

FOREWORD

The Joint Logistics Systems Center (JLSC) is tasked to implement standard wholesale inventory management systems throughout the Department of Defense. The primary purpose of these new systems is to eliminate duplicate systems and systems management while increasing inventory management efficiencies with reduced costs. This goal encompasses many areas of wholesale logistics systems. One of these areas includes the mathematical models that minimize inventory cost while determining when and how much to order for item stock replenishment.

The primary objective of our research under this study effort was to provide management with an evaluation of alternative reorder point and reorder quantity models from the perspective of items managed by the Defense Logistics Agency. While restricting ourselves to models that are currently employed or are planned to be used by other DoD components, we evaluated those that appeared to hold the greatest potential for improved item management efficiencies within DLA.

The analysis presented in this report was conducted under the direction and guidance of the JLSC Materiel Management Operations Research Group (MM/ORG). The ORG is composed of mathematical modelers from all components of the DoD wholesale logistics community.

Gerald F. Wyngaard
GERALD F. WYNGAARD
Colonel, USAF
Chief, DLA Operations Research Office

EXECUTIVE SUMMARY

The primary objective of our research under this study effort for the Joint Logistics Systems Center (JLSC) was to evaluate alternative reorder point and reorder quantity models from the perspective of the Defense Logistics Agency's (DLA) items. While restricting ourselves to models that are currently used or are planned to be used by other DoD components, we evaluated those that appeared to hold the greatest potential for improved item management efficiencies within DLA.

We evaluated four reorder point models. Our research indicated that when using one reorder point model for all items, substantial improvements over the Laplace model currently used by DLA cannot be obtained. We also observed that the holding cost associated with the average on-hand inventory is the primary cost driver and ordering costs play a relatively minor role. This results from on-hand inventories that are much larger than expected from the steady-state mathematical models. These results led us to investigate three analytical excursions: average inventory levels, safety level constraints, and performance projections.

First, we observed that DLA experiences inventory levels that are above what is expected under steady-state inventory theory. This implies that our inventory models are not reflecting accurately the holding costs and are reacting too slowly to over-procurement. Factors that appear to be related to inventory positions being larger than expected include long leadtimes and variable demand rates.

Second, some of the DLA inventory control points indirectly compute negative safety levels. This is accomplished by reducing the forecasted demand for an item. Although the performance advantages gained from this approach are not arguable, more direct methods will be available under the requirements determination standard system. This has the potential to eliminate off-line and ad hoc systems used by some DLA inventory control points.

Finally, our analysis indicates that there is a tendency for the mathematical projection models to over-estimate expected backorders on-hand which also over-estimates response time. The expected backorder projections under the Laplace assumption more closely approximate the actual on-hand backorders. However, it appears that as fill rate increases above 92 percent, the Negative Binomial projections may be more accurate. Both steady-state models projected fill rates that were slightly optimistic.

The secondary objective of our research was to develop some tools and expertise that would permit DLA management to be easily informed on the impacts of utilizing various inventory policies. As DoD standard systems and models become available, DLA needs to be in a position to evaluate these alternatives to improve existing item management policies. In an effort to be in a more responsive position, we developed some tools over the course of meeting JLSC requirements that will serve future analytical needs for evaluating alternative inventory policies.

TABLE OF CONTENTS

TABLE OF CONTENTS (CONTINUED)

LIST OF FIGURES

LIST OF TABLES

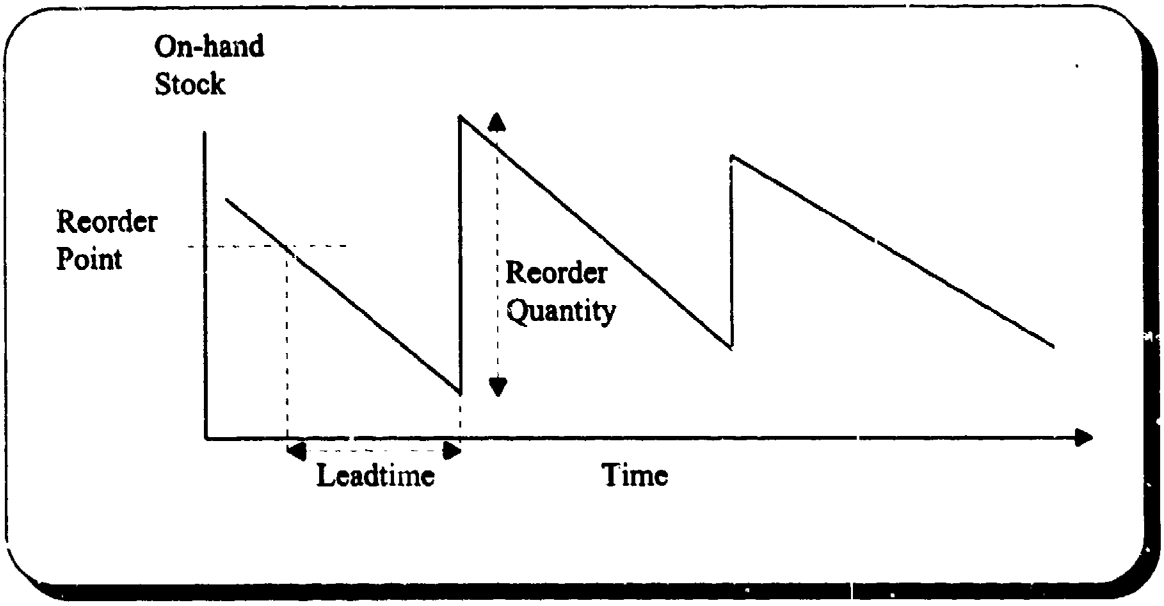
SECTION 1
INTRODUCTION

The Joint Logistics Systems Center (JLSC) is tasked to implement standard wholesale inventory management systems throughout the Department of Defense (DoD). The primary purpose is to eliminate duplicate systems and systems management while increasing inventory management efficiencies. This goal encompasses many areas of wholesale logistics systems. One of these areas includes the mathematical models involved in determining an item's reorder quantity and reorder point.

1.1 BACKGROUND

An item's reorder point is the on-hand stock quantity that signals that a procurement should be initiated. The reorder point tells the inventory manager "when" to buy. As the reorder point is lowered, the risk of a stock-out during the procurement leadtime increases. Conversely, as the reorder point is raised, the costs associated with holding extra inventories increases. The quantity that is procured is called the reorder quantity and it tells the inventory manager "how much" to buy. Ordering small quantities increases the number of procurements which increases the total procurement cost. On the other hand, ordering large quantities increases the costs associated with holding additional stock until it is demanded. The following graph illustrates these basic inventory concepts.

Figure 1-1. Illustration of Basic Inventory Concepts



Both the reorder point and the reorder quantity are set by mathematical models that minimize the expected total variable cost associated with stocking a catalog of items. The total variable cost equation includes ordering costs, holding costs, and backorder costs. The total variable cost equation is provided in Appendix A. There are some important features of the total variable cost equation that is used throughout the DoD.

First, because of procurement leadtime and item demand uncertainties, we base our cost estimates on expected values. This implies that assumptions will be made regarding the leadtime demand distribution. These assumptions lead to different solution methods. Some of the methods have closed-form solutions, while others involve iteration over a bounded solution set.

Second, the cost of backorders is not specified a priori. Instead, the cost is implied from a desired inventory performance goal. For example, setting a supply availability target implies setting a specific backorder cost. By increasing a supply availability goal, the reorder point or the reorder quantity must be increased to reduce the risk of a stock-out. But this process increases the inventory holding costs and the implied backorder costs.

Third, all of the costs are based on the same fixed time horizon of 1 year. The ordering cost is the cost to make one order times the expected number of orders per year. The holding cost is the annual holding cost rate times the dollar value of the expected inventory position. Finally, the backorder cost is the cost of having a requisition on backorder for 1 year times the expected number of requisition-years short. Now, the expected number of requisition-years short is equivalent to the expected number of backorders at a random point in time. Since this relationship may not be obvious, we present an example. Suppose the expected number of requisitions backordered is 10. Then we expect to see 10 requisitions short every time we look at the books. If this average holds true for the year, we would be 10 requisition-years short.

1.2 OBJECTIVES

The primary objective of our research under this study effort is to evaluate alternative reorder point and reorder quantity models with respect to the Defense Logistics Agency's (DLA) items. While restricting ourselves to models that are currently used or are planned to be used by other DoD components, we evaluated those that appeared to hold the greatest potential for improved item management efficiencies within DLA.

The secondary objective of our research was to develop some tools and expertise that would permit DLA management to be easily informed on the impacts of utilizing various inventory policies. As DoD standard systems and models become available, DLA needs to be in a position to evaluate these alternatives to improve existing item management policies.

1.3 REPORT ORGANIZATION

After this introductory section, the methodologies used for our analysis are provided in Section 2. The primary purpose of Section 2 is to describe the simulation model that is employed throughout this analytical effort. In Section 3, we present the historical data that is used to feed the simulation model by providing the procedures that we followed to collect the data and some pertinent characteristics of the data itself.

The core of our analytical efforts is presented in Sections 4 though 5. In Section 4, alternative demand variance estimators are studied with respect to their impact on supply performance and investment. Section 5 focuses on alternative reorder point and reorder quantity models and their impact on supply performance and investment.

Two analytical excursions are presented in Sections 6 and 7. In Section 6, we provide an initial evaluation of the expected inventory position, its relation to actual inventory position, and the use of asset adjustment factors to explain the differences. Section 7 contains an initial evaluation of negative safety levels and their impact on supply performance and investment.

Section 8 begins with a background on the Computational and Research Evaluation System (CARES) and the Supply Performance Analyzer (SPA). This is followed by an evaluation of expected backorder, response time, and fill rate performance projections.

Finally, a report summary can be found in Section 9.

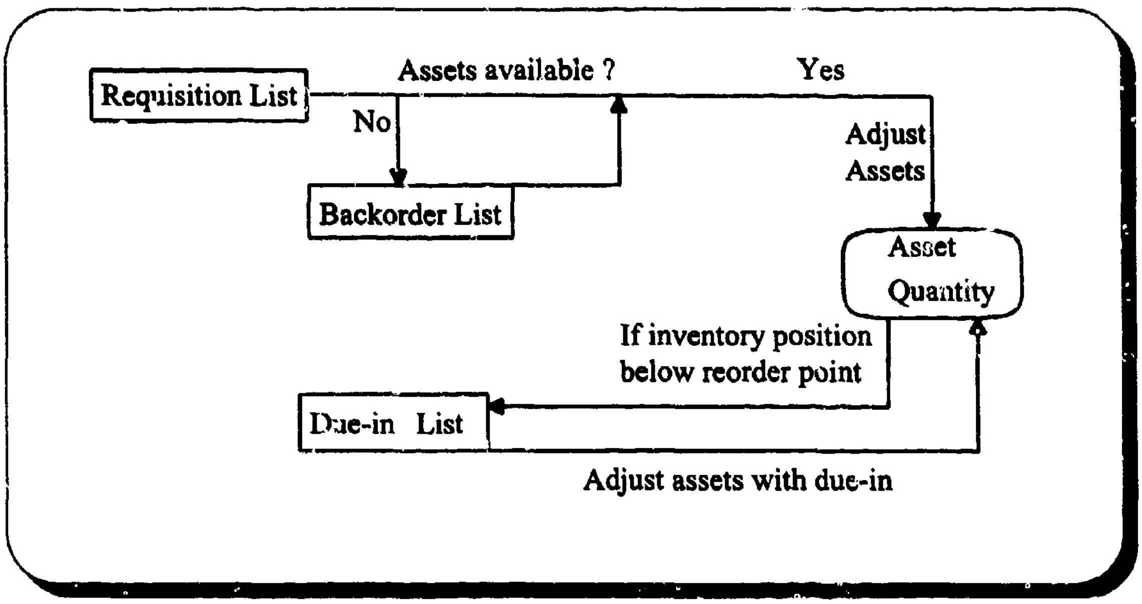
SECTION 2
METHODOLOGY

There exist a variety of reorder point, R, and reorder quantity, Q, models that minimize the total variable costs. This minimization problem is constrained by some performance goal when the backorder cost is not specified a priori. The variety of R/Q models available is primarily due to different assumptions regarding the distributional form of leadtime demand. Since we will be evaluating models that place different assumptions on the leadtime demand process, a historical simulation of various inventory policies was used as the primary evaluation tool.

2.1 SIMULATION DESCRIPTION

The simulation model developed for this effort is fed historical requisitions covering a finite period of time. This provides the capability to "re-play" history under different inventory policies or models. In essence, we can say this is what would have happened had some policy or model been in effect during the item's simulated demand history. The basic components of this simulation are depicted in the following graphic.

Figure 2-1. Basic Components of Inventory Simulation



When a demand requisition hits the system, the demand is recorded. If assets are available, the requisition is filled immediately. For the situation when no assets are available, the requisition is placed on the backorder list or queue.

The system is under continuous review. This means that any time the inventory position, on-hand assets plus due-in assets minus backordered assets, is below the reorder point, a procurement is generated and placed on the due-in list or queue. The procurement is for the reorder quantity plus the reorder point deficit. The procurement quantity will be added to the asset quantity after the item's leadtime expires.

Each period, e.g., monthly or quarterly, the requirements are recomputed. This means the item's demand is reforecasted, the leadtime demand and variances are recomputed, and the reorder quantity and reorder point are recalculated. These periodic recalculations form the basis for the inventory simulation. Different inventory models are applied. These different models generate different requirements and therefore have varying impacts on supply performance and cost. It is these system supply performance and cost measurements that are used to form conclusions about the applicability of various inventory models.

2.2 SIMULATION ENVIRONMENT

As previously stated a major objective for this effort was the development of tools and expertise that would permit DLA management to be easily informed on the impacts of utilizing various inventory policies. Prior to this effort our evaluation tools were limited in scope, difficult to modify, and cumbersome to use. Through the recent acquisition of much improved tools we are able to implement solutions with less effort that are more usable and extendible. This effort presented an opportunity to use these new tools for facilitating management-level reviews of alternative policies.

This inventory problem required the handling of queues for backorders and due-ins, time-oriented events, and complex interactions among the various aspects of the inventory problem. For these reasons, we chose an object-oriented simulation package. The simulation orientation of the package rendered straight forward the representation and manipulation of the queuing and timing aspects while the object-oriented capability allowed for easier and more robust (than previous approaches and tools) handling of the complex interactions among the inventory problem variables. An added benefit of this object-oriented solution approach arose from the inheritance capability. We defined not only the relevant inventory objects and their interrelationships but setup a general framework for an object-oriented representation of a material management system from which the inventory model inherits most of its capability. Hence, the current inventory model requirement is satisfied and a platform to embellish and inherit (reuse) in solving subsequent material management modeling problems is available.

In the future as the JLSC plans are implemented, DoD standard systems will be in use by DLA. These standard systems will have certain valuable features of current service models. If we don't make these models work for DLA, many of the expected benefits will be lost. Making these models work involves knowledge of their assumptions and mechanics. If we gain this knowledge, these models will provide DLA with large pay-offs in terms of better customer support and reduced operation costs. Future DLA analytical studies related to the operation of various service models are inevitable. The DLA analytical community needs the tools to readily conduct these future studies. This object-oriented simulation model with its general framework for material management is such a tool.

2.3 OTHER ANALYTICAL RESOURCES

Since every question cannot be answered by a simulation, other analytical techniques were also used throughout the course of this study. These other evaluation tools included:

1. Technical reviews and evaluations of various inventory models.
2. Descriptive statistical analysis of the current DLA environment.
3. Technical discussions with DLA management and component analysts.
4. Reviews of current DLA supply operations policy manuals.
5. Miscellaneous analytical models for evaluations of key assumptions.

Primarily because of their obvious utility, these evaluation tools will not be discussed in detail. Our purpose for listing them is to alert the reader to the fact that the simulation was just one of several techniques used in this analysis.

SECTION 3
DATA ANALYSIS

Understanding the nature of the data used for our analysis is critical to interpretation of our results. In this section, we provide a description of the types of items and demand patterns that were used.

3.1 ITEM SELECTION

Items chosen for our simulation analysis are currently managed by the Defense General Supply Center (DGSC). We selected these items primarily because of their diversity in demand level and unit price. Items managed by DGSC include the various machine tools and equipment of Federal Supply Group (FSG) 34, lighting fixtures and lamps (FSG 62), and photographic equipment (FSG 67). Not all of the current DGSC-managed items were studied. In particular, we wanted items with some activity during the previous 10 year period. From approximately 600,000 DGSC items, only about 161,000 have been continuously managed for the previous 10 year period. Of these, only about 78,000 experienced a buy within the past 10 years. This information is contained in the following table.

Table 3-1. Item Selection Criteria

Criteria	# Passed
DGSC Managed (as of January 1994)	599,909
Managed for more than 10 years	160,890
Procured within past 10 years	78,266

Since our purpose was to keep the study item population to a maximum, the only other item selection criteria relates to the demand activity over a 10 year time period. In an effort to summarize activity spanning this lengthy period, we looked for a statistical measure that covered the time aspect. In particular, we were interested in the time between requisitions, or interarrival time, and the frequency of requisitions. Since an interarrival time can be computed between each requisition pair, one item may have several interarrivals. These "item-level" interarrivals vary considerably. Our expectation of large variability led us to compute the median interarrival for each item. This information is cross-tabulated by the number of requisitions in the following table.

Table 3-2. Profile of Ten Year Item Demand Activity

Median Interarrival Time	Number of Requisitions	# of Items
A. Less than 30 days	1. Less than 120 / 10 years	14,140
	2. More than 120 / 10 years	17,343
B. Between 30 and 90 days	1. Less than 40 / 10 years	13,634
	2. More than 40 / 10 years	5,967
C. More than 90 days	1. Less than 10 / 10 years	13,774
	2. More than 10 / 10 years	9,860
No interarrival time	Only 1 requisition / 10 years	3,548
Total		78,266

Of those items with a median interarrival time less than 30 days (denoted as category A), approximately 14,000 experienced less than 120 requisitions over 10 years (denoted as sub-category 1) and 17,300 experienced more than 120 total requisitions (denoted as sub-category 2). Since a 30 day interarrival time leads one to expect about 1 requisition each month or 120 for 10 years, the first group (A-1) can be characterized as having requisitions that may be more clustered in a short period of time and the second group (A-2) as having a more dispersed requisitioning pattern. An extreme example of clustered demand is the group A-1 item that had only two requisitions two days apart over the 10 year period. For category B items, we expect about 1 requisition each quarter or 40 for 10 years. Group B-1 has more clustered demand than group B-2. Finally, one can make a similar statement for category C items.

Having most or all of the demand clustered in the first two simulation years would tell us almost nothing about the inventory policy alternatives under study. In a historical simulation of item demand, an analyst desires some demand activity to be dispersed throughout the simulation time horizon. In addition to ensure broad applicability of analytical results, items from all of these groups (A-2, B-2, C-2) will be studied further. For the remainder of this report, the following definitions will be used when referring to these groups of items.

1. Group A-2: High demand items,
2. Group B-2: Medium demand items, and
3. Group C-2: Low demand items.

3.2 ITEM PROFILES

For a more complete picture of the nature of these items, information on the key inventory variables of leadtime (administration leadtime plus production leadtime), standard unit price, quarterly demand value, and annual demand frequency are contained in the following tables.

Table 3-3. Item Profiles According to Key Inventory Variables

Leadtime Range	Number of Items			
	All Items	High Demand	Medium Demand	Low Demand
Less than 90 days	2,195	1,372	361	462
From 91 to 180 days	10,370	4,744	2,171	3,455
From 181 to 365 days	17,251	9,112	2,998	5,141
More than 365 days	3,354	2,115	437	802
Total	33,170	17,343	5,967	9,860

Unit Price Range	Number of Items			
	All Items	High Demand	Medium Demand	Low Demand
Less than $1.00	4,333	2,764	684	885
Between $1 and $10	8,216	4,808	1,405	2,003
Between $11 and $50	8,492	4,470	1,601	2,421
Between $51 and $100	3,450	1,667	625	1,158
Between $101 and $1000	7,136	3,141	1,346	2,649
More than $1000	1,543	493	306	744
Total	33,170	17,343	5,967	9,860

Quarterly Demand Value Range	Number of Items			
	All Items	High Demand	Medium Demand	Low Demand
Less than $100	9,914	2,100	2,488	5,326
Between $101 and $500	8,237	3,774	1,841	2,622
Between $501 and $1000	3,712	2,303	626	783
Between $1001 and $5000	6,888	5,123	817	948
More than $5000	4,419	4,043	195	181
Total	33,170	17,343	5,967	9,860

Item Averages per Year	All Items	High Demand	Medium Demand	Low Demand
Demand Frequency	28.41	51.55	5.27	1.71
Demand Value $	18,458	32,732	5,085	1,443

This information serves an important purpose to the non-DLA reader and to the JLSC because it may be used to compare and contrast the differences between service items and DLA items. Typically, service items have longer leadtimes and higher unit prices. Furthermore, when discussing inventory models, these variables are major factors differentiating the use of alternative models.

SECTION 4
DEMAND VARIANCE ANALYSIS

The primary purpose of this demand variance analysis is to determine an appropriate alternative to the current DLA demand variance estimator. There are three demand variance estimators under consideration in this study. While one of the estimators, the current DLA estimator, is not a planned component of the Materiel Management Standard System, we will include it as a baseline for analytical comparisons.

4.1 BACKGROUND

The three demand variance estimator alternatives, their mathematical representation, and the parameters that were used for our analysis are presented in this section. The first alternative is the current DLA estimator. This estimator is based on the Mean Absolute Deviation (MAD) and is given as,

$\hat{\sigma}_D^2 = (1.25 \cdot MAD_t)^2$, where

$MAD_t = \alpha|e_t| + (1-\alpha)MAD_{t-1}$,

e_i is the forecasting error for period i, and

α is the forecast smoothing constant.

For the purpose of our analytical efforts, we computed a 12 period moving average rather than using α and smoothing the MAD. This permits the same demand base to be used for this variance estimator that is used for the classical estimator which is described next.

The second alternative is the classical estimator for variance and is based on the Sum of the Squared Errors (SSE),

$\hat{\sigma}_D^2 = \Sigma e_i^2/(n-1)$, where

e_i is the forecasting error for period i and

the summation is taken over n periods.

For the purpose of our analytical efforts, we used a 12 period moving average for the squared error, i.e., n = 12.

The third alternative is the percent error table,

$\hat{\sigma}_D^2 = (c \cdot D \cdot Pcer)^2$, where

c is the ratio of the standard deviation to the MAD,

D is the period demand forecast, and

Pcer is the percent error for the item class.

For the purpose of our analytical efforts, we let c = 1.25 since this is the appropriate ratio under normality of forecasting errors. In addition, Pcer is defined according to the annual demand frequency and annual demand value of the item. The following table, given by Roberts, 1994, provides the values of Pcer.

Table 4-1. Percent Error Values

Annual Demand Frequency	Low Demand Value	High Demand Value
0 to 4	1.50	1.00
5 to 8	1.25	0.88
9 to 16	0.95	0.74
17 to 24	0.76	0.63
25 to 32	0.66	0.57
33 to 62	0.55	0.44
63 to 122	0.37	0.23
More than 123	0.18	0.15

4.2 ANALYTICAL RESULTS

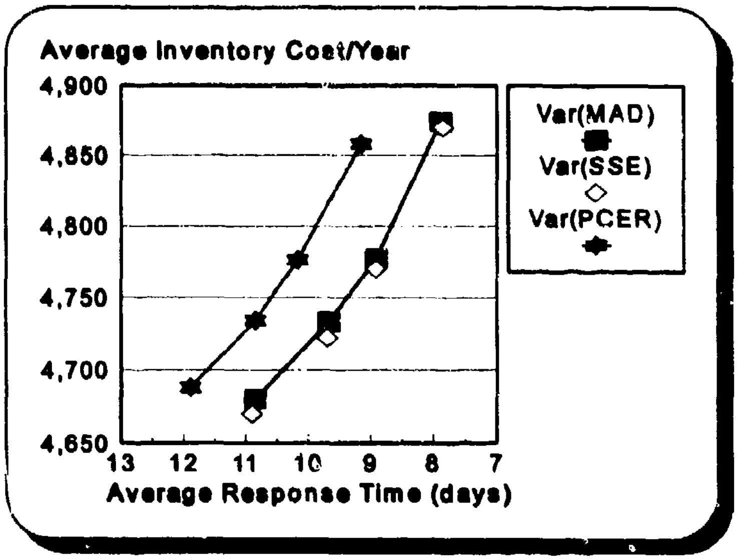
Using the simulation model as described in Section 2 and the historical demands as described in Section 3, we simulated these three demand variance computations. These alternatives and their reference names are summarized in the following table.

Table 4-2. Demand Variance Alternatives and Reference Names

Estimator	Reference Name
DLA estimator based on the MAD	Var(MAD)
Classical estimator based on the SSE	Var(SSE)
Estimator based on Percent Error	Var(PCER)

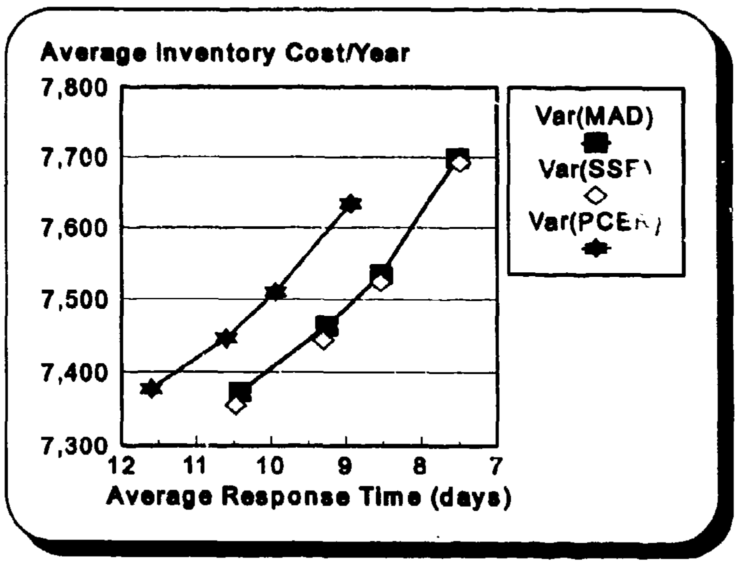
One of our measures of effectiveness includes an inventory cost versus performance curve. Inventory cost is addressed with respect to average annual variable cost. The variable cost includes both procurement and time-weighted holding costs. Performance is addressed with respect to inventory control point requisition response time. The response time was obtained using Little's formula: $L = \lambda \cdot w$, where L is the average number of backorders on-hand, λ is the requisition arrival rate, and w is the wait, or response, time. The response time performance range covers approximately eight to twelve days and is approximately equivalent to an 86 to 92 percent fill rate range. The inventory cost versus performance curves are depicted in the following charts for all items high demand items, medium demand items, and low demand items (please refer to Section 3 for item definitions). For the following graphs, please note that response time decreases from left to right.

Table 4-3. Cost vs. Performance - All Items



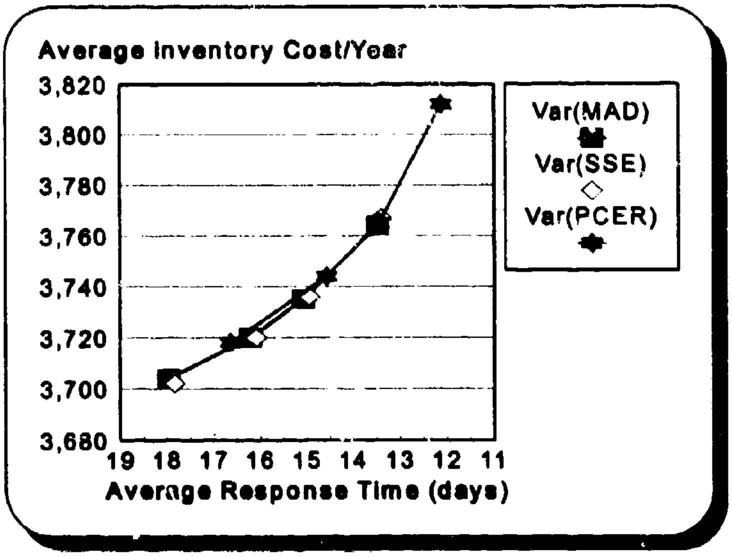
For the same average inventory cost, the Var(MAD) and Var(SSE) estimators offer slightly better performance in terms of response time than the Var(PCER) estimator. This result is evident by the fact that the Var(PCER) cost versus performance curve is always to the left, which indicates a longer response time for the same average inventory cost. Considering an average annual inventory cost of approximately $4,725 with either Var(MAD) or Var(SSE), we could obtain a 10 day response time versus an 11 day response time for Var(PCER).

Table 4-4. Cost vs. Performance - High Demand Items



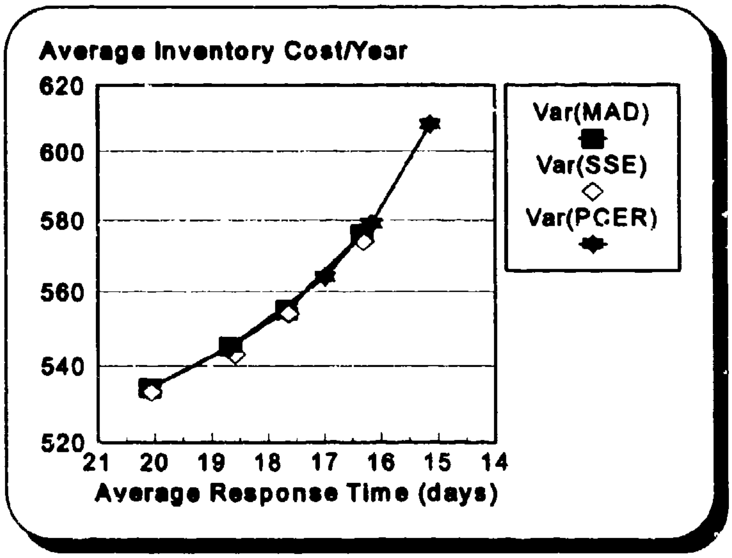
When isolating only the high demand items, the inventory cost versus performance curves are similar to those for all items. The Var(MAD) and Var(SSE) estimators are tracking closely and have a 1 day advantage over the Var(PCER) estimator.

Table 4-5. Cost vs. Performance - Medium Demand Items



When isolating the medium demand items, all three demand variance estimators have similar cost versus performance curves. For a 13 day response time, the average inventory cost would be approximately $3,760 regardless of the demand variance estimator.

Table 4-6. Cost vs. Performance - Low Demand Items



When isolating the low demand items, all three demand variance estimators have similar cost versus performance curves. This conclusion was also obtained for the medium demand items.

The cost versus performance curves indicate that the DLA approach for estimating demand variance behaves similarly to the classical demand variance estimator based on the SSE. Furthermore, either of these two alternatives is appropriate for high demand items. For medium and low demand items the demand variance estimator based on the percent forecasting error could also be used.

We now provide other performance measures associated with using these three alternatives for estimating demand variance. The following tables summarize the fill rate, average time-weighted on-hand inventory by item, average number of buys per year by item, and the average response time by requisition. Fill rate is simply the percent of demands that were satisfied immediately.

Table 4-7. Performance at Approximate 89.0% Fill Rate - All Items

Demand Variance Alternative	Fill Rate (%)	Average On-hand Inventory ($)	Average Number of Buys per Year	Average Response Time (days)
Var(MAD)	88.9	27,344	0.89	9.7
Var(SSE)	88.8	27,283	0.89	9.7
Var(PCER)	89.2	28,088	0.88	9.2

These tables do not indicate which alternative has the best performance and the lowest cost. However, they do provide an indication of the fill rate performance. For example, a system operating at an average response time of 9.7 days is roughly operating at an 88.9 percent fill rate. In addition, we can see that the large on-hand inventories for all three alternatives suggest that the holding cost is the cost driver and that ordering cost plays only a minor role.

Table 4-8. Performance at Approximate 89.0% Fill Rate - High Demand Items

Demand Variance Alternative	Fill Rate (%)	Average On-hand Inventory ($)	Average Number of Buys per Year	Average Response Time (days)
Var(MAD)	89.1	43,225	1.20	9.3
Var(SSE)	89.0	43,114	1.20	9.3
Var(PCER)	89.2	44,246	1.19	8.9

Table 4-9. Performance at Approximate 87.0% Fill Rate - Medium Demand Items

Demand Variance Alternative	Fill Rate (%)	Average On-hand Inventory ($)	Average Number of Buys per Year	Average Response Time (days)
Var(MAD)	87.0	21,737	0.73	13.5
Var(SSE)	87.1	21,755	0.72	13.4
Var(PCER)	87.2	21,749	0.71	13.4

Table 4-10. Performance at Approximate 87.5% Fill Rate - Low Demand Items

Demand Variance Alternative	Fill Rate (%)	Average On-hand Inventory ($)	Average Number of Buys per Year	Average Response Time (days)
Var(MAD)	87.3	3,144	0.44	16.3
Var(SSE)	87.3	3,135	0.44	16.3
Var(PCER)	87.5	3,163	0.43	16.2

4.3 CONCLUSIONS AND IMPACT ASSESSMENT

DLA can move from a MAD-based demand variance estimator to the classical demand variance estimator without any long-term negative impacts on average inventory cost, average inventory, or supply performance. In addition, the demand variance estimator based on percent forecasting error could be used for medium and low demand items.

Since the percent error is based on a collection of items having a similar demand frequency, the table itself may not be constructed to adequately reflect the high demand items. Although improvements could certainly be made in our estimation of the percent error table, our results indicate that DLA should use the MAD-based or the classical demand variance estimator in the near term.

SECTION 5
REORDER POINT ANALYSIS

There are four reorder point models under consideration in this study effort. These models will be part of the Materiel Management Standard System. The primary purpose of the efforts described in this section is to determine if other reorder point models perform better than the current DLA model.

5.1 BACKGROUND

Establishing a reorder point, R, is accomplished by minimizing the total variable cost equation under the requisite assumptions of the reorder point model itself. Each of the reorder point models under study and their pertinent assumptions are discussed next. With the first two approaches, the reorder quantity is assumed to be fixed or known. With the last two approaches, this assumption is relaxed.

The first reorder point model assumes that the leadtime demand is Laplace(μ,σ) and that the reorder quantity, Q, is fixed. The mathematical computation is provided in Appendix A. This model, currently used by DLA, was developed as an approximation to the Normal distribution. This approximation was desirable because it leads to a convenient closed form solution to minimizing the total variable cost.

The second reorder point model assumes that the leadtime demand is Negative Binomial(r,p) and Q is fixed. The mathematical computation is provided in Appendix A. While it is used extensively by other DoD components, DLA has no experience with using this approach for computing the reorder point. By assuming Negative Binomial, a closed form solution to minimizing the total variable cost equation does not exist. We are left with an iterative approach. This is accomplished by starting with the minimum reorder point, checking for optimality, and incrementing the reorder point by one until the optimality conditions are met. The procedure as described here is computationally inefficient, and for actual implementation, we employed a binary search algorithm.

The third reorder point model assumes that the leadtime demand is Laplace(μ,σ) and Q is not fixed. Since Q is not fixed, a priori, a simultaneous search for Q and R is employed. This simultaneous process is required to minimize the total variable cost. An initial Q is obtained using the Wilson EOQ formula (see Appendix A). This initial Q is used to compute an initial R. The total variable cost for this solution set is computed as:

$TVC = \frac{2 \cdot T \cdot D}{Q}P + (R + \frac{Q}{2})I \cdot C + \frac{\lambda}{s}EBO(R,Q)$, where

T = number of forecast periods per year,
D = period demand forecast,
P = procurement cost,

I = holding cost rate,
C = acquisition unit cost,
λ = cost to backorder a requisition for one year,
s = average requisition size, and
EBO(R,Q) = expected backordered units on-hand under Laplace assumption.

If certain stopping conditions (see table below) are not met, a new Q is computed using the expected backorders from the previous solution set. This new Q is described in Appendix A as the iterative Q. The new Q is used to recompute R and the resulting total variable cost is re-evaluated. This iterative process continues until one of the optimality, or stopping, conditions is met.

Table 5-1. Stopping Conditions for the Simultaneous Search for Q and R

Condition
1. Total variable cost increased from previous iteration.
2. Total variable cost changed by less than 1% from previous iteration.
3. Number of iterations greater than 10.

The fourth reorder point model assumes that the leadtime demand is Negative Binomial(r,p). The same simultaneous algorithm for computing R and Q that was employed for the third reorder point model is also used here. The only differences are that the Negative Binomial is assumed for computing R and the expected number of backorders.

5.2 ANALYTICAL RESULTS

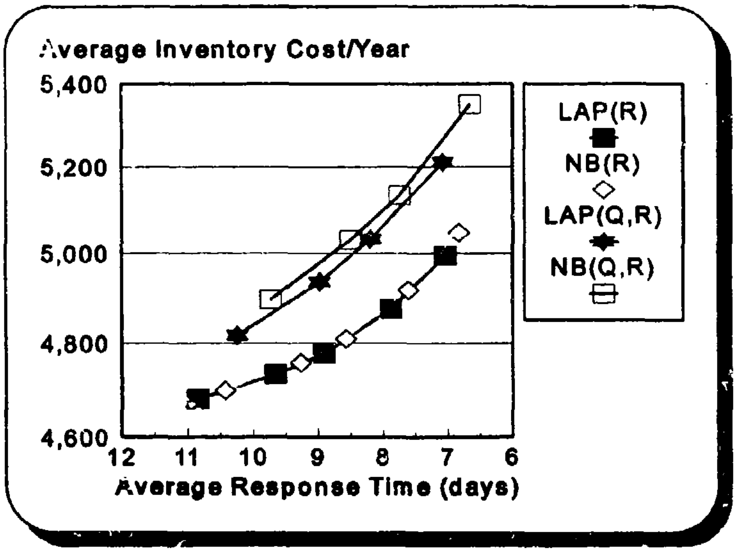
Using the simulation model as described in Section 2 and the historical demands as described in Section 3, we simulated these four reorder point computations. These alternatives and their reference names are summarized in the following table.

Table 5-2. Reorder Point Alternatives and Reference Names

Reorder Point Model	Reference Name
Laplace, Q fixed	LAP(R)
Negative binomial, Q fixed	NB(R)
Laplace, Simultaneous search for Q & R	LAP(Q,R)
Negative binomial, Simultaneous search for Q & R	NB(Q,R)

One of our measures of effectiveness included an inventory cost versus performance curve. Inventory cost is addressed with respect to average annual variable cost. The variable cost includes both procurement and time-weighted holding costs. Performance is addressed with respect to inventory control point requisition response time. The response time was obtained using Little's formula. The response time performance range covers approximately 7 to 11 days and is approximately equivalent to an 86 to 92 percent fill rate range. The inventory cost versus performance curves for each reorder point alternative are depicted in the following chart.

Table 5-3. Cost vs. Performance - All Items



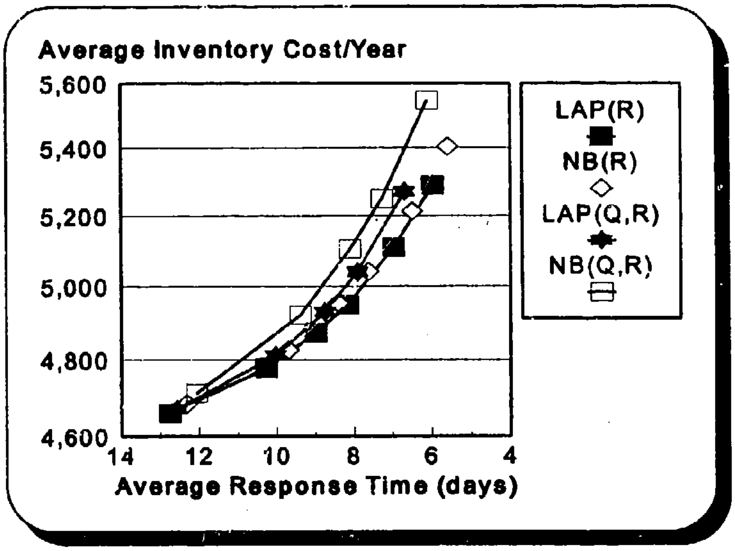
This graph, comparing the inventory costs for all items for the four reorder point models, illustrates two key features. First, the two fixed Q models, LAP(R) and NB(R), have similar inventory cost versus performance curves and the two simultaneous models, LAP(Q,R) and NB(Q,R), also have similar inventory cost versus performance curves.

Second, both fixed Q models outperform their simultaneous counterparts. Although it is accepted that the mathematics behind the simultaneous models is superior to the fixed Q models, problems may be encountered when the reorder point is subject to constraints, in particular, the leadtime demand constraint on the safety level. In an effort to increase performance, the simultaneous search for Q and R results in simply increasing Q since R is typically hitting the upper bound. Furthermore, the known over-projection of expected backorders compounds this situation and increases the iterative Q further. The reader should refer to Section 8 for a discussion on the bias of backorder projections and Appendix A for the iterative Q.

The following graph results from simulation runs without the leadtime demand constraint on the safety level. In addition, the response time performance range was expanded to cover approximately 6 to 12 days and is approximately equivalent to an 85 to 92 percent fill rate range.

Table 5-4. Cost vs. Performance - All Items

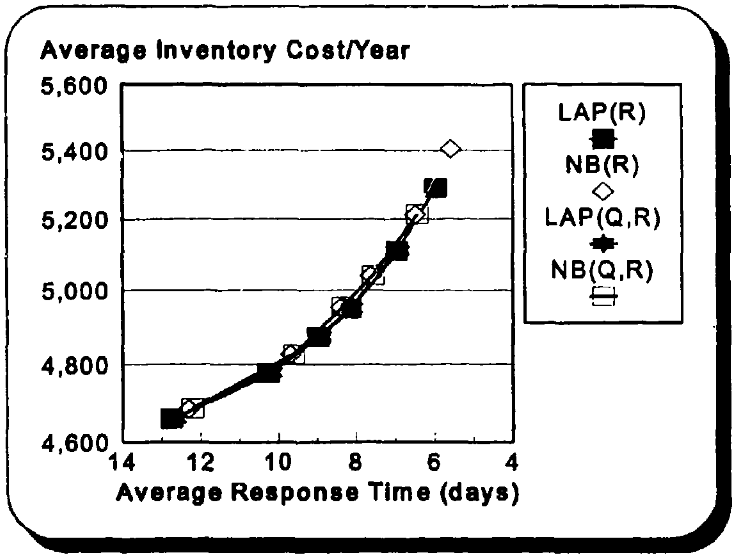
No Safety Level Leadtime Demand Constraint



Without the safety level leadtime demand constraint, all four reorder point models are much more comparable. There is, however, a higher cost associated with the NB(Q,R) model. One explanation may be that the backorder projections associated with the Negative Binomial are higher than both the Laplace projections and the actual backorders (see Section 8 for a comparison of the projections). The larger bias in the Negative Binomial backorder projections leads to a larger, less optimal, iterative reorder quantity.

In an effort to keep the iterative reorder quantity from increasing above the optimal reorder quantity, we incorporated an upper bound on the iterative reorder quantity. This upper bound is based on the Army's research and is referred to as the Inventory Research Office Q, or IRO-Q. See Appendix A for the mathematical description of this upper bound. The following cost versus performance graph results from imposing the IRO-Q upper bound on the iterative reorder quantity and as before removing the safety level leadtime demand constraint.

Table 5-5. Cost vs. Performance - All Items Using IRO-Q as Upper Bound



When using the IRO-Q as an upper bound on the iterative reorder quantity and relaxing the leadtime demand constraint on the safety level, all four reorder point models produce almost equivalent cost versus performance curves.

We now provide other performance measures associated with using these four reorder point models. Using the IRO-Q upper bound on the iterative reorder quantity and removing the safety level leadtime demand constraint, the following table summarizes the fill rate, average time-weighted on-hand inventory by item, average number of buys per year by item, and the average response time by requisition. Note that equal fill rates for all four alternatives could not be obtained since the fill rate is not known until after the simulated time horizon. The response time was obtained using Little's formula.

Table 5-6. Performance at Approximate 90% Fill Rate - All Items

Reorder Point Alternative	Fill Rate (%)	Average On-hand Inventory ($)	Average Number of Buys per Year	Average Response Time (days)
LAP(R)	90.8	28,617	0.89	8.2
NB(R)	90.6	28,639	0.89	8.4
LAP(Q,R)	90.9	28,657	0.83	8.1
NB(Q,R)	90.6	28,682	0.82	8.4

At this approximate 90 percent fill rate, the average on-hand inventory is lower for the fixed Q models but the average number of buys per year is higher. This result is due to the larger order quantities associated with the simultaneous models.

In addition, the large on-hand inventories for all four alternatives suggest that the holding cost is the cost driver and that ordering cost plays only a minor role. This results from on-hand inventories that are much larger than expected from the steady-state mathematical models. This topic is addressed in Section 6.

5.3 NOTES ON THE SIMULTANEOUS (Q,R) MODELS

Our analysis on the simultaneous computation of the reorder point and the reorder quantity, to the best of our knowledge, provides the first large body of empirical results for the DoD. The evidence presented here suggests that the other reorder point models evaluated do not, as a whole, offer substantial benefits to the DLA. However, due to the lack of other published studies, we felt that some additional observations may provide insights for further research.

First, the average number of iterations is 3.5 until one of the stopping condition is met. This seems to suggest that either the optimization process is efficient or a local minimum is achieved. In any event, the stopping criterion of 10 iterations had little impact.

Second, we favored a solution with a smaller reorder point rather than a solution with the smaller reorder quantity when the second stopping condition was met. In other words, when the total variable cost changed by less than 1 percent from the previous iteration and the iterative process ended, the last iteration may or may not have been the most desirable solution. When this situation occurred, we selected the solution with the smaller reorder point rather than the solution with the smaller reorder quantity.

5.4 CONCLUSIONS AND IMPACT ASSESSMENT

Our research indicated that when using one reorder point model for all items, substantial improvements over the Laplace model currently used by DLA cannot be obtained. More research is required to determine what conditions are most favorable to the alternative models studied.

In addition, we observed that the holding cost associated with the average on-hand inventory is the primary cost driver and ordering costs play a relatively minor role. This results from on-hand inventories that are much larger than expected from the steady-state mathematical models. While this topic is addressed in Section 6, it may be a major factor contributing to the small cost differences between the studied alternative reorder point models.

SECTION 6
ASSET ADJUSTMENT FACTORS

Inventory theory leads to certain conclusions about the size of the inventory asset position in relation to the reorder point and reorder quantity. In the real world, these conclusions are not necessarily correct. The use of asset adjustments factors is an attempt to accommodate the real world into the theoretical models.

6.1 BACKGROUND

The inventory position is defined as: on-hand assets + due-in assets - backordered assets. If, as in theory, a due-in for the reorder quantity, Q, is generated each time the inventory position falls below the reorder point, R, then $R \leq$ inventory position $\leq R + Q$. In fact when the demand rate is constant, the expected value of the inventory position is $R + Q/2$.

However, when looking at the average inventory position in the real world where steady-state conditions rarely exist, this conclusion does not hold. To account for this situation, the Army uses asset adjustment factors. These are essentially added costs that more accurately represent the true costs associated with holding materiel. The added costs are applied to the traditional holding cost rate for items that are most susceptible to having an inventory position larger than expected. By increasing the holding cost rate, the asset adjustment factors reduce the reorder quantity and the reorder point. They essentially make it more costly to maintain the traditional steady-state levels and more appropriately reflect the costs associated with holding materiel.

6.2 DATA ANALYSIS

In an effort to determine the extent that DLA items experience inventory levels above that which is expected, we developed some profiles for items managed by the DGSC. We selected items that are stocked as demand-based replenishment at the end of the second quarter of fiscal year 1992. The following table depicts the number of items and their inventory position relative to the reorder point, R, and the reorder quantity, Q.

Table 6-1. Relative Inventory Position

Inventory Position	Number of Items	Percent of Items
Less than R	4,177	4.73
Between R and R+Q/2	7,720	8.74
Between R+Q/2 and R+Q	9,097	10.30
Greater than R+Q	67,300	76.22
Total	88,294	100.00

As is evident from this table, only about 19 percent of the items had an inventory position in the expected range: $R \leq$ inventory position $\leq R + Q$. In addition, this snapshot reveals that over 76 percent of the replenishment items have inventory positions exceeding R+Q, and 5 percent had inventory positions less than R.

There are many external factors that influence this information and the reader should not generalize this as an indication of excess assets. While some of these items may have assets above the requirements objective due to dropping demand rates, material returns, or economic production lots, another reason is budgetary constraints, which force the requirements objective down and have no impact on the actual inventory position. We attempted to isolate some factors that may be contributing to inventory positions outside the expected range. These factors, depicted in the following table, include leadtime, demand frequency, the ratio of returns to demand, and the ratio of the Mean Absolute Deviation (MAD) to demand.

Table 6-2. Factors Impacting the Relative Inventory Position

Inventory Position	Average Leadtime (Days)	Average Annual Demand Frequency	Average Ratio Returns to Demand (%)	Median Ratio MAD to Demand (%)
Less than R	276	31	12	59
Between R and R+Q/2	216	33	16	58
Between R+Q/2 and R+Q	220	32	15	58
Greater than R+Q	254	25	67	69

First, consider the average leadtime. We see that those items having an inventory position below R and those that have an inventory position above R+Q experience longer leadtimes. Based on Roberts, 1994, as an item's leadtime increases so does its leadtime variance. It appears that long leadtimes and hence large leadtime variances contribute to inventory positions being outside of the expected range.

Second, consider average demand frequency. As the number of demands decrease, the demand forecast decreases. This immediately reduces R and Q since they are functions of the demand forecast. However, the inventory position is still indicative of the previous level of demand activity in that it takes longer for the inventory position to "catch up" to the current demand rate. The information contained in the Table 6-2 adds evidence to this conclusion since the items with an inventory position above R+Q also have lower demand frequency rates.

Third, consider material returns. In an effort to describe the impact of returns, we computed the ratio of returns to demand. A high ratio indicates that returns have great importance in the determination of inventory position while a small ratio implies that returns are negligible. For the items with an inventory position above R+Q, returns average 67 percent of the demands. This result appears to play an important part in having inventory positions above R+Q.

Finally, we turn to demand variance. Demand variance is not depicted directly in the previous table, but is represented by the ratio of the MAD to the demand. A high variance to mean ratio would also have a high MAD to demand ratio (see Roberts, 1994, for a summary of the relationship between the demand variance and the MAD). For those items with an inventory position less than R+Q, the median MAD to demand ratio was 58 percent to 59 percent. However, for those items with an inventory position greater than R+Q, the median MAD to demand ratio jumped to 69 percent.

6.3 METHODOLOGY DISCUSSION

While there are other factors, we have identified and discussed four factors that appear to drive an item's inventory position beyond the range that is expected. In an effort to prevent this situation, we need factors or characteristics that give us an indication that the item has the propensity for an average inventory position that is greater than expected.

Three of the four characteristics that were discussed are, in some way, based on demand rates. Consider the impacts of a drop in the level of demand. As soon as the demand forecast begins to reflect decreasing demand, the R and Q will be reduced. The inventory position, however, is still indicative of the previous level of demand activity. In fact as the demand rate lowers, the inventory position reacts more slowly. For example, suppose the demand forecast dropped from 10 units per quarter to 1 unit per quarter. The recomputed R and Q would reflect the low demand rate as soon as the demand forecast changed. The rate at which the inventory position changes is only one unit per quarter since that is the expected demand activity. At this rate, it may take years for the inventory position to correct for the declining demand. Now contrast this with the impact associated with an increase in demand. The demand forecast would increase. This immediately increases R and Q and a buy is generated that increases the inventory position. It is understandable that our inventory systems adapt more readily to increasing demand than to decreasing demand.

The only characteristic that was mentioned that is not based on demand activity is the procurement leadtime. The procurement leadtime could be used as an indicator that future assets may be higher than expected. In fact, the Army currently uses leadtime in its asset adjustment factor computation (Kaplan, 1990). The Army computes a holding cost rate that gets applied to the reorder point and reorder quantity computations, denoted as $I_{applied}$, by increasing the traditional holding cost rate, denoted as $I_{traditional}$. Mathematically, this computation is given as:

$$I_{applied} = I_{traditional} \cdot \left[1 + \frac{L}{2.5} \cdot 0.5\right].$$

From this relationship, one can see that as the leadtime, L, increases, the holding cost rate that gets applied also increases.

6.4 CONCLUSIONS

DLA experiences inventory levels that are above what is expected under our steady-state inventory models. This implies that our inventory models are not reflecting accurately the holding costs and are reacting too slowly to over-procurement. Factors that appear to be related to inventory positions being larger than expected include long leadtimes and variable demand rates. In order to inhibit over-procurement, we recommend more research into applying adjustment factors into DLA reorder point and reorder quantity computations.

One approach for this research may be to consider the projected reorder quantity time-frame and the actual reorder quantity time-frame. For example based on current demand forecasts, we may expect to buy a 24 month supply of materiel. However, this procurement may last for 30 months. This would result in additional holding costs that were not considered when the procurement was made.

SECTION 7
NEGATIVE SAFETY LEVELS

The term negative safety level presents some conceptual problems for many inventory practitioners throughout the Agency. However, the benefits of negative safety levels are known by most practitioners. This apparent dichotomy results from using negative safety levels but referring to it under a another name.

7.1 BACKGROUND

The inventory models that the Agency uses have the ability to provide more safety level investment to some items and less to others. Generally, the more inexpensive and frequently requisitioned items are given more safety level than their expensive and less active counterparts. However, these models operate under safety level constraints. These constraints include forcing the safety level to be non-negative and less than one leadtime demand.

In partial response to these seemingly artificial constraints, DLA supply analysts have used different schemes to force more safety level investment in items that are cheaper and have greater impacts on supply availability. These schemes typically involve setting levels according to altered demand forecasts. This artificially changes the reorder point and effectively by-passes the normal safety level constraints. Consider the example given in the following table.

Table 7-1. Example of By-passing the Safety Level Constraint

Support Level	Quarterly Demand Forecast	Leadtime Demand (3 quarters)	Fixed Safety Level (1 quarter)	Reorder Point	Effective Safety Level
100%	10	30	10	40	10
50%	5	15	5	20	-20

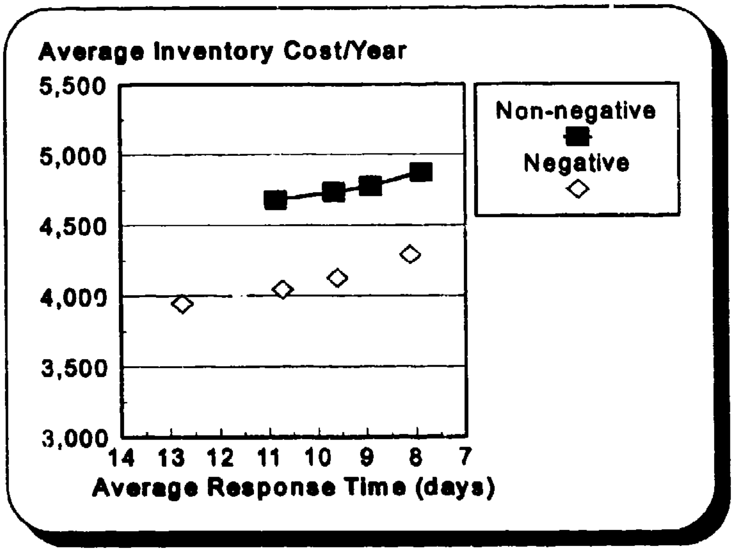
For a 100 percent support level, the demand forecast is not changed and the fixed safety level is 10 units. However under a 50 percent support level, the reorder point is reduced such that the effective safety level when compared to the 100 percent support level is a negative 20 units.

While this trivial example illustrates some of the short-comings of the current environment, the new standard requirements determination system will have the capability to more directly address the issue of negative safety levels. Off-line and ad hoc systems will not be required to provide this capability.

7.2 ANALYSIS

In an effort to address the potential advantages associated with using negative safety levels, we evaluated two alternatives. For the first alternative, we simulated the current DLA restriction that the reorder point must be greater than or equal to the leadtime demand i.e., non-negative safety levels. In the second alternative, we restricted the reorder point to be at least 75 percent of the leadtime demand i.e., allowed negative safety levels. The inventory cost versus performance curve is provided in the following graph.

Table 7-2. An Initial Evaluation of Negative Safety Levels



We can readily see from this graph that allowing for negative safety levels provided for a lower variable inventory cost for the same response time performance. In addition, it is also apparent that as the response time decreases, the advantage gained by permitting negative safety levels diminishes.

7.3 CONCLUSIONS

Some of the DLA inventory control points indirectly compute negative safety levels. This is accomplished by reducing the forecasted demand for an item. Although the performance advantages gained from this approach are not arguable, more direct methods will be available under the requirements determination standard system. This has the potential to eliminate off-line and ad hoc systems used by some DLA inventory control points.

SECTION 8
COMPUTATIONAL AND RESEARCH EVALUATION SYSTEM (CARES) & SUPPLY PERFORMANCE ANALYZER (SPA)

Having mathematical models that are used to compute item requirements, such as reorder quantities and reorder points, is essential for day-to-day item management. However, they do not perform the tasks required of inventory analysts or budget planners. These personnel are interested in projecting system performance and cost. This task is accomplished through CARES/SPA.

8.1 OVERVIEW

CARES/SPA is actually a merger of two supply performance projection systems that were developed independently, one by the Navy and the other by the Army. The Navy supply performance projection system is called the Computational And Research Evaluation System (CARES). The corresponding Army system is called the Supply Performance Analyzer (SPA).

The purpose of CARES/SPA is to relate how investment in inventory levels, such as reorder quantities and reorder points, relates to projected supply performance. This means that CARES/SPA will be used for primarily two purposes:

1. Determine the level of inventory investment required to attain a specific supply performance goal.

2. Determine the supply performance that can be expected with a given inventory investment.

Ultimately it is the implied backorder cost, the cost associated with having a requisition on backorder for 1 year, that relates inventory investment to supply performance. CARES/SPA provides a mechanism to establish this cost.

If the first purpose listed above is used, CARES/SPA will have the capability to use two different supply performance goals in determining the investment level. Alternatively, if the second purpose listed above is used, CARES/SPA will have the capability to estimate two supply performance measurements. These supply performance goals, or measurements, are the fill rate and the requisition response time.

8.2 DETAILS

Since CARES/SPA will project performance during both the apportionment and budget years, it will be used by the inventory control point during the budget formulation or stratification process. At any other time, CARES/SPA can also be used as a supply analyzer to assess the effects of parameter changes on supply performance and costs. This information can be used by inventory control point management staff to provide a "measure of merit" for improving supply performance.

CARES/SPA is actually a computer program that uses both item specific data as well as parameter input from the supply/budget analyst. The CARES/SPA program would supply this input to the reorder quantity and reorder point modules for setting requirements levels and a projection module for computing expected supply performance. The user can modify the parameter inputs based on these projections until the desired level of performance or cost is attained. As previously stated, the implied backorder cost relates investment to performance. It is this parameter that drives the requirements levels up or down. The analyst uses CARES/SPA to determine the appropriate backorder cost.

Unlike the current DLA environment where essentially one backorder cost is used for all items managed by an inventory control point, CARES/SPA allows for grouping items and developing an appropriate backorder cost for a subset of items. For example, suppose a 5 day response time is desired for all weapon system critical items (weapon system essentiality code = 1). However for the rest of the items, a 10 day response time is sufficient. CARES/SPA could be used to establish the backorder cost to achieve a 5 day response time for the critical items and a 10 day response time for the remainder of the items. Of course budgetary constraints may require modification of this response time goal.

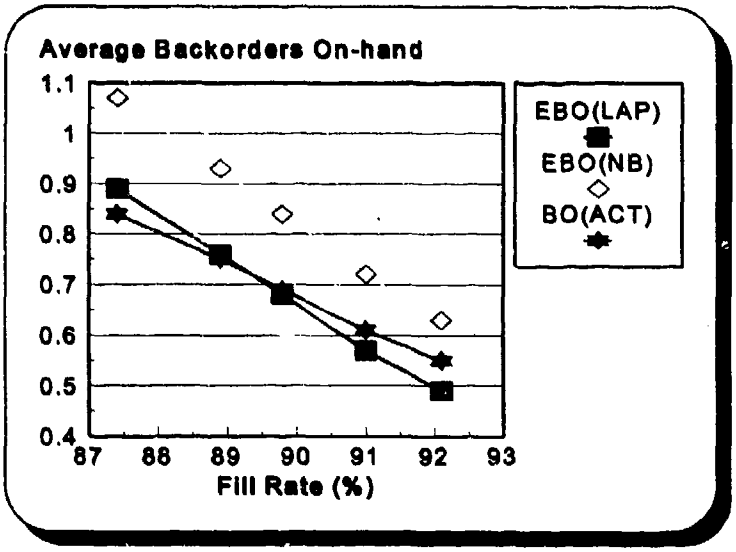
8.3 ANALYSIS

Any performance projection tool makes predictions based on some assumed distributional forms for the leadtime demand. Furthermore, these performance projections typically assume steady-state conditions which probably do not exist in an operational environment. For these reasons, the projections are known to be imperfect.

Despite this fact, these steady-state performance projections can, if used properly, serve as baselines and alert supply analysts to potential performance problems. One of the keys to proper utilization of the projections is understanding any built-in bias. Since the Agency has limited experience in this area, we believe that it is critical to provide some understanding of this built-in bias.

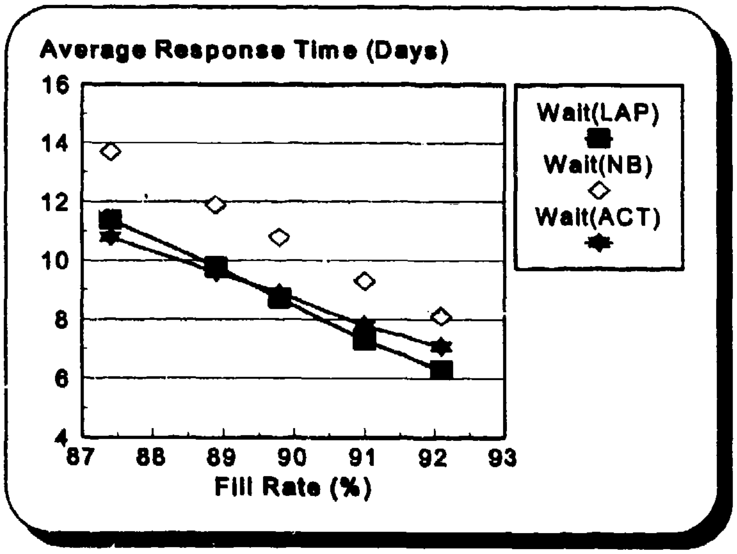
After our inventory simulation model ran 5 years, we computed steady-state performance projections. These projections were developed under the Laplace and Negative Binomial leadtime demand distribution assumptions under various fill rates. The expected backorders under the Laplace assumption are denoted EBO(LAP), under the Negative Binomial assumption they are denoted EBO(NB), and the actual backorders are denoted BO(ACT) in the following graph.

Table 8-1. Performance Projections Versus Actual Performance - Expected Backorders On-hand



As depicted in this graph, the expected backorder projections from the Laplace assumption more closely approximate the actual backorders. However, it appears that as fill rate increases above 92 percent, the Negative Binomial projections may be more accurate. This conclusion is valid when looking at the response time projections in the following graph.

Table 8-2. Performance Projections Versus Actual Performance - Response Time



* The response times are all based on Little's formula.

Fill rate is another inventory performance measure that is commonly projected by steady state inventory models. These projections, under the two distributional assumptions and the actual fill rate, are provided in the following table.

Table 8-3. Fill Rate Projections Versus Actual Performance

Actual	Projected (LAP)	Projected (NB)
87.4	89.2	89.5
88.9	91.0	91.2
89.8	92.1	92.2
91.0	93.6	93.7
92.1	94.7	94.9

The projected fill rates are approximately 1 percent to 3 percent greater than the actual fill rate. For example at a 91 percent actual fill rate, the Laplace based projection is 93.6 percent and the Negative Binomial projection is 93.7 percent. Both projections appear to consistently over-estimate fill rate.

8.4 CONCLUSIONS

In summary, CARES/SPA provides two fundamental benefits:

1. It is a tool for managing requirements levels.
2. It can be used to answer "what-if" questions for research or management purposes.

There is a tendency for the mathematical projection models to over-estimate expected backorders on-hand which also over-estimates response time. The expected backorder projections under the Laplace assumption more closely approximate the actual on-hand backorders. However, it appears that as fill rate increases above 92 percent, the Negative Binomial projections may be more accurate. Both steady-state models projected fill rates that were slightly optimistic. More research in this area is warranted to understand the conditions in which the projection models over-estimate or under-estimate the actual performance.

SECTION 9
FINDINGS AND RECOMMENDATIONS

9.1 SUMMARY OF FINDINGS

The primary objective of our research under this study effort was to evaluate alternative reorder point and reorder quantity models with respect to the Defense Logistics Agency's (DLA) items. While restricting ourselves to models that are currently used or are planned to be used by other DoD components, we evaluated those that appeared to hold the greatest potential for improved item management efficiencies within DLA.

We evaluated three demand variance estimators. Our results indicate that the classical demand variance estimator can be used equally well in lieu of the current DLA estimator which is based on the mean absolute deviation of forecasting error. In addition, the demand variance estimator based on percent forecasting error could be used for low demand items.

In addition, we evaluated four reorder point models. Our research indicated that when using one reorder point model for all items, substantial improvements over the Laplace model currently used by DLA cannot be obtained. We also observed that the holding cost associated with the average on-hand inventory is the primary cost driver with ordering costs playing a relatively minor role. This results from on-hand inventories that are much larger than expected from the steady-state mathematical models. These results led us to investigate three analytical excursions: average inventory levels, safety level constraints, and performance projections.

First, we observed that DLA experiences inventory levels that are above what is expected under steady-state inventory theory. This implies that our inventory models are not reflecting accurately the holding costs and are reacting too slowly to over-procurement. Factors that appear to be related to inventory positions being larger than expected include long leadtimes and variable demand rates.

Second, some of the DLA inventory control points indirectly compute negative safety levels. This is accomplished by reducing the forecasted demand for an item. Although the performance advantages gained from this approach are not arguable, more direct methods will be available under the requirements determination standard system. This has the potential to eliminate off-line and ad hoc systems used by some DLA inventory control points.

Finally, our analysis indicates that there is a tendency for the mathematical projection models to over-estimate expected backorders on-hand which also over-estimates response time. The expected backorder projections under the Laplace assumption more closely approximate the actual on-hand backorders. However, it appears that as fill rate increases above 92 percent, the Negative Binomial projections may be more accurate. Both steady-state models projected fill rates that were slightly optimistic.

The secondary objective of our research was to develop some tools and expertise that would permit DLA management to be easily informed on the impacts of utilizing various inventory policies in a timely manner. As DoD standard systems and models become available, DLA needs to be in a position to evaluate these alternatives to improve existing item management policies. In an effort to be in a more responsive position, we developed some tools that will serve future analytical needs for evaluating alternative inventory policies.

9.2 SPECIFIC RECOMMENDATIONS

- If the MAD-based demand variance estimator is not available, DLA should use the classical demand variance estimator.
- Continue research to determine if there are conditions that favor one reorder point model over another. This may involve an examination of item demand characteristics such as demand frequency, mean, and variance.
- Continue research to determine how asset adjustment factors could be developed for DLA items. These factors would be additives to the holding cost rate and should quantify the risk associated with potential over-procurement.
- DLA should consider the use of negative safety levels rather than making downward adjustments to the demand forecast.
- Continue research to determine the conditions under which backorder projection models over-estimate or under-estimate actual performance. This may involve an examination of item demand characteristics such as demand frequency, mean, and variance.

APPENDIX A
MATHEMATICAL MODELS

TABLE OF CONTENTS

APPENDIX A
MATHEMATICAL MODELS

In this appendix, we provide the reader with background information on various mathematical models that are used in materiel management systems.

A.1 ECONOMIC ORDER QUANTITY

WILSON EOQ

The Wilson EOQ minimizes the annual costs associated with procurement and holding. Both of these costs are assumed to be linear and monotonic. The procurement cost is assumed to monotonically decrease as the size of the procurement increases and the holding cost is assumed to monotonically increase as the size of the procurement increases.

$$Q = \sqrt{\frac{2 \cdot T \cdot D \cdot P}{I \cdot C}}\text{, where}$$

T = number of forecast periods per year,
D = period demand forecast,
P = procurement cost,
I = holding cost rate, and
C = acquisition unit cost.

ITERATIVE EOQ

The iterative EOQ is used in the simultaneous computation for the reorder quantity and the reorder point. The primary difference between this model and the Wilson EOQ is that the expected backorders on-hand are used to increase the costs associated with under procurement.

$$Q = \sqrt{\frac{2 \cdot T \cdot D \cdot (P + \frac{\lambda}{s} EBO)}{I \cdot C}}\text{, where}$$

λ = cost to backorder a requisition for one year,
s = average requisition size, and
EBO = expected backorders on-hand.

IRO-Q

The IRO-Q can be used as an upper bound for the optimal reorder quantity. In this report the IRO-Q was used as an upper bound on the iterative EOQ.

$$Q = \frac{\sigma}{\sqrt{2}\cdot\rho} + \sqrt{\frac{\sigma^2}{2\cdot\rho} + \frac{Q_w^2}{\rho}}\text{, where}$$

σ = standard deviation of leadtime demand,
Q_w = Wilson EOQ, and
$\rho = \frac{1+e^{-\sqrt{2}\cdot\frac{Q_w}{\sigma}}}{1-e^{-\sqrt{2}\cdot\frac{Q_w}{\sigma}}}$.

Deemer & Kruse, 1974.

A.2 REORDER POINT

The reorder point is obtained by minimizing the total variable cost equation subject to a constraint on the expected number of backorders.

ASSUMING LAPLACE

When assuming that the leadtime demand distribution is Laplace(μ,σ), a closed form solution for the reorder point can be obtained (Presutti, 1974).

$$R = \mu + \frac{-1}{\sqrt{2}}\ln\left[\frac{2\sqrt{2}\cdot Q\cdot I\cdot C}{\lambda/s\cdot\sigma\cdot(1-\exp(-\sqrt{2}\cdot Q/\sigma))}\right]\sigma\text{, where}$$

μ = mean leadtime demand,
σ = standard deviation of leadtime demand,
λ = cost to backorder a requisition for one year, and
s = average requisition size.

This reorder point is functionally the same as that used by DLA and is demonstrated by letting

$$\lambda = \frac{I}{\sqrt{2}\,\beta}\sum\sigma_i\cdot C_i\text{, where}$$

β = expected backorders on hand and
the summation, commonly termed the system constant, is taken over a catalog.

ASSUMING NEGATIVE BINOMIAL

When assuming that the leadtime demand distribution is Negative Binomial(r,p), an iterative solution for the reorder point must be obtained. This process involves a search for a reorder point that makes the expected availability greater than the target availability. In other words, find R such that $\alpha(R, Q, r, p) \geq (1 - P_{out})$. See Deemer, 1974 for development and proofs of these results or Kotkin, 1990, for an alternative formulation. The following definitions apply:

$$\alpha(R,Q,r,p) = \frac{1}{Q}\begin{bmatrix} (R+Q)\Pr(R+Q-1) - \frac{r(1-p)}{p}\Pr(R+Q-2,r+1,p) - \\ R\cdot\Pr(R-1,r,p) + \frac{r(1-p)}{p}\Pr(R-2,r+1,p) \end{bmatrix},$$

$P_{out} = \frac{I \cdot C}{\lambda/s}$, and

$\Pr(x, r, p)$ =cumulative distribution function for the Negative Binomial distribution.

Since materiel management systems typically estimate μ and σ the following conversions can be used to determine the estimated parameters for the Negative Binomial:

$p = \frac{\mu}{\sigma^2}$ and

$r = \frac{\mu \cdot p}{1-p}$.

Since the Negative Binomial parameter, r, is estimated from the estimated leadtime demand mean and variance, it may not be an integer. For obtaining the cumulative probability for a Negative Binomial distribution, $\Pr(x,r,p)$, the gamma function arises. Our solution involved using a numerical approximation from Press et. al., 1989.

A.3 EXPECTED BACKORDERS

Since DoD inventory models are based on minimizing the total variable cost equations which include a time-weighted backorder cost, it is important to define the expected number of backorders on hand at a random point in time.

ASSUMING LAPLACE

$$EBO(R,Q,\mu,\sigma) = \frac{\sigma^2}{4 \cdot Q}\left(1 - \exp\left(\frac{-\sqrt{2}\,Q}{\sigma}\right)\right)\exp\left(\frac{-\sqrt{2}\,(R-\mu)}{\sigma}\right),$$

Presutti, 1974.

ASSUMING NEGATIVE BINOMIAL

$$EBO(R,Q,r,p)=\frac{1}{2Q}\left[\begin{array}{c}\left[(R+Q)^2+R+Q\right]\Pr(R+Q,r,p)-R(R+1)\Pr(R,r,p)+\\ \frac{r(r+1)(1-p)^2}{p^2}[\Pr(R+Q-2,r+2,p)-\Pr(R-2,r+2,p)]+\\ 2\frac{r(1-p)}{p}[R\cdot\Pr(R-1,r+1,p)-(R+Q)\Pr(R+Q-1,r+1,p)]+\\ Q\left[2\frac{r(1-p)}{p}-(2R+Q+1)\right]\end{array}\right],$$

Deemer,1974.

A.4 TOTAL VARIABLE COST EQUATION

Both the reorder point and the reorder quantity are set by mathematical models that minimize the expected total variable cost associated with stocking a catalog of items. The total variable cost equation includes ordering costs, holding costs, and backorder costs. The total variable cost is given as:

$$TVC=\frac{2\cdot T\cdot D}{Q}P+(R+\tfrac{Q}{2})I\cdot C+\tfrac{\lambda}{s}EBO(R,Q),\quad \text{where}$$

$EBO(R,Q)$ is computed with respect to an appropriate distributional assumption.

APPENDIX B
BIBLIOGRAPHY

APPENDIX B
BIBLIOGRAPHY

1. Deemer, R. L., A. J. Kaplan, and W. K. Kruse (1974), Application of the Negative Binomial to Inventory Control. Inventory Research Office, U. S. Army Logistics Management Center, Fort Lee, VA.

2. Deemer, R. L. and W. K. Kruse (1974), Evaluation of Several EOQ/VSL Models. Inventory Research Office, U. S. Army Logistics Management Center, Fort Lee, VA.

3. DLAM 4140.2, Supply Operations Manual, Vol II, Part 1, Chapter 55, "Economic Order Quantity and Procurement Cycle."

4. DLAM 4140.2, Supply Operations Manual, Vol II, Part 2, Appendix D-186, "Economic Order Quantity and Procurement Cycle."

5. DLAM 5105.1, Department of Defense Supply Management Reference Book, Section VI, "Secondary Items and Repair Parts Management in the Defense Logistics Agency."

6. Hadley, G. and T. M. Whitin, Analysis of Inventory Systems. Prentice-Hall, Englewood Cliffs, NJ, 1963.

7. Kaplan, A. (1990), Enhancements to the Variable Safety Level / Economic Order Quantity (VSL/EOQ) and Supply Performance Analyzer (SPA). U. S. Army Materiel Systems Analysis Activity, Aberdeen Proving Ground, MD.

8. Kotkin, M. (1990), Computational Improvements in the Application of Negative Binomial Probability to Inventory Control. U. S. Army Materiel Systems Analysis Activity, Aberdeen Proving Ground, MD.

9. Kruse, W. K. (1976), Supply Performance Analyzer. Inventory Research Office, U. S. Army Logistics Management Center, Fort Lee, VA.

10. Press, W. H., B. P. Flannery, S. A. Teukolsky, W. T. Vetterling, Numerical Recipes (FORTRAN). Cambridge University Press, Cambridge, MA, 1989.

11. Presutti, V. J. and R. C. Trepp (1970), "More Ado About Economic Order Quantities (EOQ)," Naval Research Logistics Quarterly, Vol 17, pp 243-252.

12. Naval Supply Systems Command (1993), "Navy Fleet Materiel Support Office Uniform Inventory Control Program System Design Documentation Levels Setting Model," Function Description PD82, REV 2.

13. Naval Supply Systems Command (1991), "Navy Fleet Materiel Support Office Uniform Inventory Control Program System Design Documentation SMA/ADD Projections Model," Function Description PD86 , REV 1.

14. Roberts, B., C. (1994), Forecasting Submodule in the Corporate Information Management (CIM) Requirements System. Defense Logistics Agency Operation Research Office, Defense General Supply Center, Richmond, VA.

15. Silver, E. A. and R. Peterson, Decision Systems for Inventory Management and Production Planning. John Wiley & Sons, New York, 1985.

16. Sims, R. L. and D. L. Zimmerman (1989), Multiple Cost EOQ Study. Synergy, Inc., Washington, DC.

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
October 1994

3. REPORT TYPE AND DATES COVERED
Final

4. TITLE AND SUBTITLE
An Analysis of the Requirements Levels and Performance Projection Modules of the Corporate Information Management Requirements System

5. FUNDING NUMBERS

6. AUTHOR(S)
Benedict C. Roberts, MAJ Charles Noll, USA,
Capt Edward Dawson, USAF, Robert Greiner, Jeffrey J. Hobbs

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Defense Logistics Agency
Operations Research Office (DORO)
c/o Defense General Supply Center
Richmond, VA 23297-5082

8. PERFORMING ORGANIZATION REPORT NUMBER
DLA-94-P30166
DLA-94-P30211

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Joint Logistics Systems Center
Materiel Managment Operations Research Group
JLSC-MM/ORG
1864 Fourth St. Suite 1
Wright-Patterson AFB, OH 45433-7131

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES
This report documents analysis conducted under two DLA project numbers

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Public release; unlimited distribution

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*
The Joint Logistics Systems Center (JLSC) is tasked to implement standard wholesale inventory management systems throughout the Department of Defense. The primary purpose of these new systems is to eliminate duplicate systems and systems management while increasing inventory management efficiencies with reduced costs. This goal encompasses many areas of wholesale logistics systems. One of these areas includes the mathematical models that minimize inventory cost while determining when and how much to order for item stock replenishment. This report focuses on the assumptions, application, and performance of various reorder point and reorder quantity models.

14. SUBJECT TERMS
requirements, levels, reorder point, safety level, reorder quantity wholesale inventory management

15. NUMBER OF PAGES
55

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	

GENERAL INSTRUCTIONS FOR COMPLETING SF 298

The Report Documentation Page (RDP) is used in announcing and cataloging reports. It is important that this information be consistent with the rest of the report, particularly the cover and title page. Instructions for filling in each block of the form follow. It is important to ***stay within the lines*** to meet ***optical scanning requirements***.

Block 1. Agency Use Only *(Leave blank)*.

Block 2. Report Date. Full publication date including day, month, and year, if available (e.g. 1 Jan 88). Must cite at least the year.

Block 3. Type of Report and Dates Covered. State whether report is interim, final, etc. If applicable, enter inclusive report dates (e.g. 10 Jun 87 - 30 Jun 88).

Block 4. Title and Subtitle. A title is taken from the part of the report that provides the most meaningful and complete information. When a report is prepared in more than one volume, repeat the primary title, add volume number, and include subtitle for the specific volume. On classified documents enter the title classification in parentheses.

Block 5. Funding Numbers. To include contract and grant numbers; may include program element number(s), project number(s), task number(s), and work unit number(s). Use the following labels:

C	- Contract	PR	- Project
G	- Grant	TA	- Task
PE	- Program Element	WU	- Work Unit Accession No.

Block 6. Author(s). Name(s) of person(s) responsible for writing the report, performing the research, or credited with the content of the report. If editor or compiler, this should follow the name(s).

Block 7. Performing Organization Name(s) and Address(es). Self-explanatory.

Block 8. Performing Organization Report Number. Enter the unique alphanumeric report number(s) assigned by the organization performing the report.

Block 9. Sponsoring/Monitoring Agency Name(s) and Address(es). Self-explanatory.

Block 10. Sponsoring/Monitoring Agency Report Number. ***(If known)***

Block 11. Supplementary Notes. Enter information not included elsewhere such as: Prepared in cooperation with...; Trans. of...; To be published in.... When a report is revised, include a statement whether the new report supersedes or supplements the older report.

Block 12a. Distribution/Availability Statement. Denotes public availability or limitations. Cite any availability to the public. Enter additional limitations or special markings in all capitals (e.g. NOFORN, REL, ITAR).

- **DOD** - See DoDD 5230.24, "Distribution Statements on Technical Documents."
- **DOE** - See authorities.
- **NASA** - See Handbook NHB 2200.2.
- **NTIS** - Leave blank.

Block 12b. Distribution Code.

- **DOD** - Leave blank.
- **DOE** - Enter DOE distribution categories from the Standard Distribution for Unclassified Scientific and Technical Reports.
- **NASA** - Leave blank.
- **NTIS** - Leave blank.

Block 13. Abstract. Include a brief ***(Maximum 200 words)*** factual summary of the most significant information contained in the report.

Block 14. Subject Terms. Keywords or phrases identifying major subjects in the report.

Block 15. Number of Pages. Enter the total number of pages.

Block 16. Price Code. Enter appropriate price code ***(NTIS only)***.

Blocks 17. - 19. Security Classifications. Self-explanatory. Enter U.S. Security Classification in accordance with U.S. Security Regulations (i.e., UNCLASSIFIED). If form contains classified information, stamp classification on the top and bottom of the page.

Block 20. Limitation of Abstract. This block must be completed to assign a limitation to the abstract. Enter either UL (unlimited) or SAR (same as report). An entry in this block is necessary if the abstract is to be limited. If blank, the abstract is assumed to be unlimited.