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# Assessing the Feasibility of Accelerometer-Only Inertial Measurement Units for Artillery Projectiles

Thomas E. Harkins

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## 1. INTRODUCTION

The term "inertial navigation" is commonly used in the literature to describe the process of obtaining velocity and position information from measurements made with inertial instruments. To accomplish this, an inertial navigation system must contain four basic conceptual elements: a vector accelerometer, an attitude reference, a computer, and a clock (Russell 1962). Russell describes each of these elements as follows (pp. 17-18). A vector accelerometer is a device which measures the nongravitational acceleration of the platform to which it is affixed. Oftentimes this vector is defined by three orthogonal components, each of which is measured by a single-degree-of-freedom (linear) accelerometer. The attitude reference gives the orientation of the accelerometers in the reference frame in which navigation takes place. Gyroscopes have most often been employed to this end. The computer is the conceptual element which solves the acceleration equation by calculating gravitational acceleration, by performing integrations, and by making coordinate transformations as required. The clock is needed to predict the gravitational field and the location and orientation of moving external reference frames (e.g., the earth beneath an aircraft).

It has been shown analytically that it is possible to obtain the information necessary for determining both linear accelerations and angular motions using only measurements from linear accelerometers (Schuler, Grammatikos, and Fegley 1967). Efforts are underway to infer the gravity vector from accelerometer measurements. The goal is to develop signal processing software that can isolate the gravity-induced component of projectile pitch that is present in intra-atmospheric trajectories. Because high-g miniature accelerometers are common but gyros are not, a navigation system which uses only linear accelerometers for inertial measurements would be an attractive candidate for inclusion in artillery projectiles. Such a system could perhaps enable such things as autonomous course corrections and/or, in conjunction with a telemeter, projectile registration.

As a start in investigating such possibilities, equations for the inertial acceleration of an arbitrary point on a flight body have been derived in forms appropriate for inclusion in a computerized six-degree-of-freedom (6dof) trajectory model used at the U. S. Army Research Laboratory (ARL). This trajectory model, called CONTRAJ (Hathaway and Whyte 1991), was then modified to allow up to 12 arbitrarily located 3-axis accelerometers on the projectile. Using appropriate configurations of accelerometer locations and orientations and linear combinations of accelerometer outputs, the components of projectile linear and angular motion can be estimated.

Importantly, it was found that the configurations, orientations, and combinations that successfully estimate the angular motion components for one projectile will sometimes not work for another projectile with different motion characteristics. Specifically, a methodology employed by Schuler, Grammatikos, and Fegley for estimating the components of angular velocity failed due to numerical problems when implemented on the M483A1 artillery projectile. An alternative approach yielded accurate estimates demonstrating that accelerometer-only inertial measurement units need to be tailored to the anticipated flight characteristics of the body on which they will be installed. Appropriate accelerometer configurations and computational algorithms for use in many Army projectiles can be identified using the computer model developed for this study.

## 2. LINEAR ACCELERATION OF A POINT

Define a set of inertially fixed Cartesian axes  $X_0Y_0Z_0$  with origin  $O_0$  and unit vectors  $\vec{i}_0 \vec{j}_0 \vec{k}_0$  and a second set of Cartesian axes  $XYZ$  with origin  $O$  and unit vectors  $\vec{i} \vec{j} \vec{k}$ . Not only may  $O$  move relative to  $O_0$  but the  $XYZ$  axes may rotate as well. Consider a point  $H$  moving relative to both sets of axes. If we denote  $H$ 's position relative to  $O_0$  in the  $X_0Y_0Z_0$  system by  $\vec{H}$ ,  $O$ 's position relative to  $O_0$  in the  $XYZ$  system by  $\vec{O}$ , and  $H$ 's position relative to  $O$  in the  $XYZ$  system by  $\vec{h}$ , then

$$\vec{H} = \vec{O} + \vec{h} \quad (1)$$

and it can be shown (e.g., Page 1952) that inertial velocity and acceleration of  $H$  are given by:

$$\dot{\vec{H}} = \dot{\vec{O}} + \vec{\omega} \times \vec{h} + \dot{\vec{h}}_m \quad (2)$$

$$\ddot{\vec{H}} = \ddot{\vec{O}} + \dot{\vec{\omega}} \times \vec{h} + \vec{\omega} \times (\vec{\omega} \times \vec{h}) + 2(\vec{\omega} \times \dot{\vec{h}}_m) + \ddot{\vec{h}}_m \quad (3)$$

where  $\vec{\omega}$  is the angular velocity of the  $XYZ$  system with respect to the  $X_0Y_0Z_0$  system.

The velocity is the sum of three terms. The first gives the velocity of the origin of the moving axes. The second is the velocity of  $H$  due to the angular velocity  $\vec{\omega}$  of the moving axes. The third  $\dot{\vec{h}}_m$  is the velocity of  $H$  relative to  $O$  as measured in the moving system.

The acceleration is the sum of five terms. The first is the the acceleration of the origin of the moving axes. The second is the linear acceleration due to the angular acceleration  $\vec{\omega}$  of the moving axes. The third is the centripetal acceleration due to the angular velocity  $\vec{\omega}$  of the moving axes. The fourth is the Coriolis acceleration where  $\vec{h}_m$  is the velocity of  $H$  relative to  $O$  as measured in the moving system. The fifth term  $\vec{\dot{h}}_m$  is the acceleration of  $H$  relative to  $O$  as measured in the moving system.

The equations of motion used to compute projectile and rocket trajectories are usually evaluated in either of two right-handed Cartesian coordinate systems. The first, called the body-fixed system, has its origin at the center of mass of the flight body and its  $X$ -axis parallel to the downrange axis of symmetry. The  $Y$  and  $Z$  axes are then oriented so that the products of inertia vanish. The three axes of this system are called the principal axes of inertia of the body. As the name implies, this system is tied to the flight body and translates, orients, and rotates with that body. The second system, called the plane-fixed system, also has its origin at the center of mass and its  $X$ -axis along the axis of symmetry. However, the  $Y$  and  $Z$  axes are not fixed to the projectile. Rather, the  $Y$ -axis is constrained so that it lies in the horizontal plane. A mutually orthogonal  $Z$ -axis then completes this system. Body-fixed coordinates are usually employed for simulating guided flight bodies because the necessary seekers, sensors, and control mechanisms are fixed to the bodies and most easily described in such a system. The plane-fixed system eliminates sensitivity of the integration to projectile roll rate by eliminating the affected component of gravity. For spin-stabilized projectiles, this choice of coordinate systems can result in significant computer savings by not requiring the extremely small integration time steps necessary to avoid smearing of the gravity effect over the roll angle. Though it is possible for a sufficiently large flight body to have a gravitational gradient within the body, CONTRAJ assumes a constant field throughout the body. In CONTRAJ, therefore, the equation of linear motion is  $\vec{\ddot{O}} = \vec{\ddot{A}} + \vec{g}$  where  $\vec{\ddot{A}}$  is the nongravitational acceleration of the center of mass (c.m.) and  $\vec{g}$  is the gravitational acceleration of the c.m.  $\vec{\ddot{A}}$  is also called by some authors the *thrust* acceleration, *sensed* acceleration, or *specific force*.

In the next two subsections, the derivations of the body-fixed and plane-fixed system versions of equations (1)-(3) are summarized. In subsection 2.3, the acceleration components as computed in the plane-fixed system will be rotated into the body-fixed system to allow for comparison with the analogous estimates made for a trajectory computed in the body-fixed system. Uninterested readers may skip these and continue with section 3 without loss of continuity.

## 2.1. Accelerations Computed in a Body-Fixed System.

When computing a trajectory employing the body-fixed coordinate system, the vectors in equations (1)-(3) can be expressed as:

$$\begin{aligned}
 \vec{H} &= \Delta X_0 \vec{i}_0 + \Delta Y_0 \vec{j}_0 + \Delta Z_0 \vec{k}_0 \\
 \vec{O} &= \Delta X \vec{i} + \Delta Y \vec{j} + \Delta Z \vec{k} \\
 \vec{h} &= \Delta x \vec{i} + \Delta y \vec{j} + \Delta z \vec{k} \\
 \vec{\omega} &= p \vec{i} + q \vec{j} + r \vec{k} \\
 \dot{\vec{\omega}} &= \dot{p} \vec{i} + \dot{q} \vec{j} + \dot{r} \vec{k} \\
 \vec{h}_m &= \Delta \dot{x} \vec{i} + \Delta \dot{y} \vec{j} + \Delta \dot{z} \vec{k} \\
 \ddot{\vec{h}}_m &= \Delta \ddot{x} \vec{i} + \Delta \ddot{y} \vec{j} + \Delta \ddot{z} \vec{k}
 \end{aligned} \tag{4}$$

where  $p$  and  $\dot{p}$  are the angular velocity and acceleration of the  $j$ - $k$  axes about the  $i$  axis,  $q$  and  $\dot{q}$  are the velocity and acceleration of the  $i$ - $k$  axes about the  $j$  axis, and  $r$  and  $\dot{r}$  are the velocity and acceleration of the  $i$ - $j$  axes about the  $k$  axis.

Making these substitutions in equation (3), forming the cross products, and collecting terms yields:

$$\begin{aligned}
 \ddot{\vec{H}} &= \ddot{\vec{O}} + \vec{i} \left[ (-q^2 - r^2) \Delta x + (pq - \dot{r}) \Delta y + (pr + \dot{q}) \Delta z + 2(q \Delta \dot{z} - r \Delta \dot{y}) + \Delta \ddot{x} \right] \\
 &+ \vec{j} \left[ (pq + \dot{r}) \Delta x + (-p^2 - r^2) \Delta y + (qr - \dot{p}) \Delta z + 2(r \Delta \dot{x} - p \Delta \dot{z}) + \Delta \ddot{y} \right] \\
 &+ \vec{k} \left[ (pr - \dot{q}) \Delta x + (qr + \dot{p}) \Delta y + (-p^2 - q^2) \Delta z + 2(p \Delta \dot{y} - q \Delta \dot{x}) + \Delta \ddot{z} \right]
 \end{aligned} \tag{5}$$

The components of velocity of the c.m. are typically represented in the moving ( $X Y Z$ ) system as  $u$ ,  $v$ , and  $w$ . If we assume that  $H$  is fixed on the body,  $\Delta x$ ,  $\Delta y$ , and  $\Delta z$  are nonconstant only if the c.m. is not fixed, as it generally would not be if fuel or supplies are consumed during the flight. In such a case, it is useful to make the following substitutions:

$$\begin{aligned}
 u &= u_0 + \mu \\
 v &= v_0 + \eta \\
 w &= w_0 + \xi
 \end{aligned} \tag{6}$$

where  $u_0, v_0,$  and  $w_0$  are the components of velocity of the point that was the initial c.m. and  $\mu, \eta,$  and  $\xi$  are the components of velocity of the c.m. due to the c.m.'s changing location within the projectile.

If we express the velocity of  $O$  in body-fixed coordinates as:

$$\vec{O} = (u_0 + \mu)\vec{i} + (v_0 + \eta)\vec{j} + (w_0 + \xi)\vec{k} \quad (7)$$

it follows from the definition of  $\vec{h}$  (equation [4]) that

$$\vec{O} = (u_0 - \Delta\dot{x})\vec{i} + (v_0 - \Delta\dot{y})\vec{j} + (w_0 - \Delta\dot{z})\vec{k} \quad (8)$$

and after differentiation:

$$\dot{\vec{O}} = (\dot{u}_0 - \Delta\ddot{x})\vec{i} + (\dot{v}_0 - \Delta\ddot{y})\vec{j} + (\dot{w}_0 - \Delta\ddot{z})\vec{k} + \vec{\omega} \times \vec{O} \quad (9)$$

Forming the cross product and collecting terms, we get:

$$\begin{aligned} \dot{\vec{O}} = & \vec{i} \left[ (\dot{u}_0 - \Delta\ddot{x}) + q(w_0 - \Delta\dot{z}) - r(v_0 - \Delta\dot{y}) \right] \\ & + \vec{j} \left[ (\dot{v}_0 - \Delta\ddot{y}) + r(u_0 - \Delta\dot{x}) - p(w_0 - \Delta\dot{z}) \right] \\ & + \vec{k} \left[ (\dot{w}_0 - \Delta\ddot{z}) + p(v_0 - \Delta\dot{y}) - q(u_0 - \Delta\dot{x}) \right] \end{aligned} \quad (10)$$

Finally, substituting (10) into (5), the body-fixed components of  $\vec{H}$  are given by:

$$\vec{H}_i = \left[ \dot{u}_0 - \Delta\ddot{x} + q(w_0 - \Delta\dot{z}) - r(v_0 - \Delta\dot{y}) \right] \quad (11a)$$

$$+ \left[ (-q^2 - r^2)\Delta x + (pq - \dot{r})\Delta y + (pr + \dot{q})\Delta z + 2(q\Delta\dot{z} - r\Delta\dot{y}) + \Delta\ddot{x} \right]$$

$$\vec{H}_j = \left[ (\dot{v}_0 - \Delta\ddot{y}) + r(u_0 - \Delta\dot{x}) - p(w_0 - \Delta\dot{z}) \right] \quad (11b)$$

$$+ \left[ (pq + \dot{r})\Delta x + (-p^2 - r^2)\Delta y + (qr - \dot{p})\Delta z + 2(r\Delta\dot{x} - p\Delta\dot{z}) + \Delta\ddot{y} \right]$$

$$\vec{H}_k = \left[ (\dot{w}_0 - \Delta\ddot{z}) + p(v_0 - \Delta\dot{y}) - q(u_0 - \Delta\dot{x}) \right] \quad (11c)$$

$$+ \left[ (pr - \dot{q})\Delta x + (qr + \dot{p})\Delta y + (-p^2 - q^2)\Delta z + 2(p\Delta\dot{y} - q\Delta\dot{x}) + \Delta\ddot{z} \right]$$

The body-fixed components of the difference in the accelerations of any two points  $H_1$  and  $H_2$  are independent of c.m. motion and given by:

$$\ddot{\vec{H}}_{i_1} - \ddot{\vec{H}}_{i_2} = (-q^2 - r^2)(\Delta x_1 - \Delta x_2) + (pq - \dot{r})(\Delta y_1 - \Delta y_2) + (pr + \dot{q})(\Delta z_1 - \Delta z_2) \quad (12a)$$

$$\ddot{\vec{H}}_{j_1} - \ddot{\vec{H}}_{j_2} = (pq + \dot{r})(\Delta x_1 - \Delta x_2) + (-p^2 - r^2)(\Delta y_1 - \Delta y_2) + (qr - \dot{p})(\Delta z_1 - \Delta z_2) \quad (12b)$$

$$\ddot{\vec{H}}_{k_1} - \ddot{\vec{H}}_{k_2} = (pr - \dot{q})(\Delta x_1 - \Delta x_2) + (qr + \dot{p})(\Delta y_1 - \Delta y_2) + (-p^2 - q^2)(\Delta z_1 - \Delta z_2) \quad (12c)$$

## 2.2. Accelerations Computed in a Plane-Fixed System.

The plane-fixed coordinate system with its origin at the c.m. differs from the body-fixed coordinate system with its origin at the c.m. by the angle between the plane-fixed  $Y$ -axis and the body-fixed  $Y$ -axis. This angle will be designated by  $\Delta\phi$ , and the axes of the two systems will be defined so that they are coincident when  $\Delta\phi = 0$ . Denoting the distance from the  $X$ -axis to  $H$  by  $R$  and the angle between the  $Y$ -axis and the projection of  $\overline{O-H}$  onto the  $Y-Z$  plane by  $\phi_H$ , the vectors in equations (1)-(3) can be expressed as:

$$\begin{aligned} \vec{H} &= \Delta X_0 \vec{i}_0 + \Delta Y_0 \vec{j}_0 + \Delta Z_0 \vec{k}_0 \\ \vec{O} &= \Delta X \vec{i} + \Delta Y \vec{j} + \Delta Z \vec{k} \\ \vec{h} &= \Delta x \vec{i} + R \cos \phi_H \vec{j} + R \sin \phi_H \vec{k} \\ \vec{\omega} &= q \vec{j} + r \vec{k} \\ \dot{\vec{\omega}} &= \dot{q} \vec{j} + \dot{r} \vec{k} \\ \dot{\vec{h}}_m &= \Delta \dot{x} \vec{i} + \left[ \dot{R} \cos \phi_H - R \sin \phi_H \dot{\phi}_H \right] \vec{j} + \left[ \dot{R} \sin \phi_H + R \cos \phi_H \dot{\phi}_H \right] \vec{k} \\ \ddot{\vec{h}}_m &= \Delta \ddot{x} \vec{i} + \left[ -R \left( \sin \phi_H \ddot{\phi}_H + \cos \phi_H \dot{\phi}_H^2 \right) - 2\dot{R} \sin \phi_H \dot{\phi}_H + \ddot{R} \cos \phi_H \right] \vec{j} \\ &\quad + \left[ R \left( \cos \phi_H \ddot{\phi}_H - \sin \phi_H \dot{\phi}_H^2 \right) + 2\dot{R} \cos \phi_H \dot{\phi}_H + \ddot{R} \sin \phi_H \right] \vec{k} \end{aligned} \quad (13)$$

Making these substitutions in (3), forming the cross products, and collecting terms yields:

$$\begin{aligned}
\ddot{\vec{H}} = & \ddot{\vec{O}} + \ddot{\vec{i}} \left[ \Delta x(-q^2 - r^2) + R \cos \phi_H (2q\dot{\phi}_H - \dot{r}) + R \sin \phi_H (2r\dot{\phi}_H + \dot{q}) \right. \\
& \left. + \dot{R} \cos \phi_H (-2r) + \dot{R} \sin \phi_H (2q) + \Delta \ddot{x} \right] \\
& + \ddot{\vec{j}} \left[ \Delta x \dot{r} + \Delta \dot{x} (2r) + R \cos \phi_H (-r^2 - \dot{\phi}_H^2) + R \sin \phi_H (rq - \ddot{\phi}_H) \right. \\
& \left. + \dot{R} \sin \phi_H (-2\dot{\phi}_H) + \ddot{R} \cos \phi_H \right] \\
& + \ddot{\vec{k}} \left[ \Delta x(-q) + \Delta \dot{x}(-2q) + R \cos \phi_H (qr + \ddot{\phi}_H) + R \sin \phi_H (-q^2 - \dot{\phi}_H^2) \right. \\
& \left. + \dot{R} \cos \phi_H (2\dot{\phi}_H) + \ddot{R} \sin \phi_H \right]
\end{aligned} \tag{14}$$

The most convenient way to reconcile the body-fixed (*BF*) and plane-fixed (*FP*) representations of the location of the point *H* is to synchronize the two systems initial conditions. With the coincidence of the two systems ( $\Delta\phi=0$ ), the respective representations are component-wise equal. With  $\Delta x_{BF} = \Delta x_{FP}$ ,  $\Delta y_{BF} = R \cos \phi_0$ ,  $\Delta z_{BF} = R \sin \phi_0$  initially and denoting the cumulative rotation of the flight body by  $\phi$ , the following relations hold:

$$\begin{aligned}
\phi_H &= \phi + \phi_0 \\
\dot{\phi}_H &= \dot{\phi} + \dot{\phi}_0 \\
\ddot{\phi}_H &= \ddot{\phi} + \ddot{\phi}_0 \\
\dot{R} \cos \phi_0 &= \Delta \dot{y} + \Delta z \dot{\phi}_0 \\
\dot{R} \sin \phi_0 &= \Delta \dot{z} - \Delta y \dot{\phi}_0 \\
\ddot{R} \cos \phi_0 &= \Delta \ddot{y} - \Delta y \dot{\phi}_0^2 + 2\Delta \dot{z} \dot{\phi}_0 + \Delta z \ddot{\phi}_0 \\
\ddot{R} \sin \phi_0 &= \Delta \ddot{z} - \Delta z \dot{\phi}_0^2 - 2\Delta y \dot{\phi}_0 - \Delta y \ddot{\phi}_0
\end{aligned} \tag{15}$$

With these substitutions into (14) and sufficient diligence, the components of  $\ddot{\vec{H}}$  are then:

$$\begin{aligned}
\ddot{H}_i = & \ddot{O}_i + \Delta x(-q^2 - r^2) + \Delta \ddot{x} \\
& + \Delta y \left[ \cos \phi (2q\dot{\phi} - \dot{r}) + \sin \phi (2r\dot{\phi} + \dot{q}) \right] + \Delta \dot{y} (2q \sin \phi - 2r \cos \phi) \\
& + \Delta z \left[ \cos \phi (2r\dot{\phi} + \dot{q}) - \sin \phi (2q\dot{\phi} - \dot{r}) \right] + \Delta \dot{z} (2q \cos \phi + 2r \sin \phi)
\end{aligned} \tag{16a}$$

$$\vec{H}_j = \vec{O}_j + \Delta x \dot{r} + \Delta x 2r \quad (16b)$$

$$+ \Delta y \left[ \cos \phi (-r^2 - \dot{\phi}^2) + \sin \phi (rq - \ddot{\phi}) \right] + \Delta y (-2\dot{\phi} \sin \phi) + \Delta y \ddot{\phi} \cos \phi$$

$$+ \Delta z \left[ \cos \phi (rq - \ddot{\phi}) - \sin \phi (-r^2 - \dot{\phi}^2) \right] + \Delta z (-2\dot{\phi} \cos \phi) + \Delta z \ddot{\phi} (-\sin \phi)$$

$$\vec{H}_k = \vec{O}_k + \Delta x (-\dot{q}) + \Delta x (-2q) \quad (16c)$$

$$+ \Delta y \left[ \cos \phi (rq + \ddot{\phi}) + \sin \phi (-q^2 - \dot{\phi}^2) \right] + \Delta y (2\dot{\phi} \cos \phi) + \Delta y \ddot{\phi} \sin \phi$$

$$+ \Delta z \left[ \cos \phi (-q^2 - \dot{\phi}^2) - \sin \phi (rq + \ddot{\phi}) \right] + \Delta z (-2\dot{\phi} \sin \phi) + \Delta z \ddot{\phi} (\cos \phi)$$

Finally, letting  $p = \dot{\phi}$  and  $\dot{p} = \ddot{\phi}$ , the plane-fixed components of the acceleration of  $H$  in the most widely used notation are given by:

$$\vec{H}_i = \vec{O}_i + \Delta x (-q^2 - r^2) + \Delta x \ddot{x} \quad (17a)$$

$$+ \Delta y \left[ \cos \phi (2qp - \dot{r}) + \sin \phi (2rp + \dot{q}) \right] + \Delta y (2q \sin \phi - 2r \cos \phi)$$

$$+ \Delta z \left[ \cos \phi (2rp + \dot{q}) - \sin \phi (2qp - \dot{r}) \right] + \Delta z (2q \cos \phi + 2r \sin \phi)$$

$$\vec{H}_j = \vec{O}_j + \Delta x \dot{r} + \Delta x 2r \quad (17b)$$

$$+ \Delta y \left[ \cos \phi (-r^2 - p^2) + \sin \phi (rq - \dot{p}) \right] + \Delta y (-2p \sin \phi) + \Delta y \ddot{\phi} \cos \phi$$

$$+ \Delta z \left[ \cos \phi (rq - \dot{p}) - \sin \phi (-r^2 - p^2) \right] + \Delta z (-2p \cos \phi) + \Delta z \ddot{\phi} (-\sin \phi)$$

$$\vec{H}_k = \vec{O}_k + \Delta x (-\dot{q}) + \Delta x (-2q) \quad (17c)$$

$$+ \Delta y \left[ \cos \phi (rq + \dot{p}) + \sin \phi (-q^2 - p^2) \right] + \Delta y (2p \cos \phi) + \Delta y \ddot{\phi} \sin \phi$$

$$+ \Delta z \left[ \cos \phi (-q^2 - p^2) - \sin \phi (rq + \dot{p}) \right] + \Delta z (-2p \sin \phi) + \Delta z \ddot{\phi} (\cos \phi)$$

### 2.3. Transformation of Acceleration Components From Plane-Fixed to Body-Fixed System.

Thus far the location of  $H$  has been reconciled in both systems (equation [15]) and expressions have been derived for the components of acceleration at  $H$  in both

systems (equations [11] and [17]). It remains to rotate the components into a common system. Since strapped-down accelerometers' measurements will be at fixed orientations with respect to the flight bodies, the plane-fixed components will be rotated into the body-fixed system using the following:

$$\begin{pmatrix} \ddot{\vec{H}}_{i_{BF}} \\ \ddot{\vec{H}}_{j_{BF}} \\ \ddot{\vec{H}}_{k_{BF}} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \ddot{\vec{H}}_{i_{FP}} \\ \ddot{\vec{H}}_{j_{FP}} \\ \ddot{\vec{H}}_{k_{FP}} \end{pmatrix} \quad (18)$$

Performing the matrix multiplication and equating components gives:

$$\ddot{\vec{H}}_{i_{BF}} = \ddot{\vec{H}}_{i_{FP}} \quad (19a)$$

$$\ddot{\vec{H}}_{j_{BF}} = \ddot{\vec{H}}_{j_{FP}} \cos\phi + \ddot{\vec{H}}_{k_{FP}} \sin\phi \quad (19b)$$

$$\ddot{\vec{H}}_{k_{BF}} = -\ddot{\vec{H}}_{j_{FP}} \sin\phi + \ddot{\vec{H}}_{k_{FP}} \cos\phi \quad (19c)$$

After substituting (17) into (19), collecting terms, and employing several trigonometric identities, the acceleration components in the body-fixed system in terms of the angular motion components in plane-fixed coordinates are given by:

$$\ddot{\vec{H}}_{i_{BF}} = \ddot{\vec{O}}_i + \Delta x(-q^2 - r^2) + \Delta \ddot{x} \quad (20a)$$

$$+ \Delta y [\cos\phi(2qp - \dot{r}) + \sin\phi(2rp + \dot{q})] + \Delta \dot{y}(2q \sin\phi - 2r \cos\phi)$$

$$+ \Delta z [\cos\phi(2rp + \dot{q}) - \sin\phi(2qp - \dot{r})] + \Delta \dot{z}(2q \cos\phi + 2r \sin\phi)$$

$$\ddot{\vec{H}}_{j_{BF}} = \ddot{\vec{O}}_j \cos\phi + \ddot{\vec{O}}_k \sin\phi \quad (20b)$$

$$+ \Delta x(\dot{r} \cos\phi - \dot{q} \sin\phi) + \Delta \dot{x}(2r \cos\phi - 2q \sin\phi)$$

$$+ \Delta y \left[ -p^2 - \left( \frac{r^2 + q^2}{2} \right) - \left( \frac{r^2 - q^2}{2} \right) \cos 2\phi + qr \sin 2\phi \right] + \Delta \ddot{y}$$

$$+ \Delta z \left[ -\dot{p} + \left( \frac{r^2 - q^2}{2} \right) \sin 2\phi + qr \cos 2\phi \right] + \Delta \dot{z}(-2p)$$

$$\begin{aligned}
\ddot{\vec{H}}_{k_{BF}} &= \ddot{\vec{O}}_j (-\sin\phi) + \ddot{\vec{O}}_k (\cos\phi) & (20c) \\
&+ \Delta x (-\dot{r}\sin\phi - \dot{q}\cos\phi) + \Delta \dot{x} (-2r\sin\phi - 2q\cos\phi) \\
&+ \Delta y \left[ \dot{p} + \left( \frac{r^2 - q^2}{2} \right) \sin 2\phi + qr\cos 2\phi \right] + \Delta \dot{y} (2p) \\
&+ \Delta z \left[ -\dot{p}^2 - \left( \frac{r^2 + q^2}{2} \right) - \left( \frac{r^2 - q^2}{2} \right) \cos 2\phi - qr\sin 2\phi \right] + \Delta \ddot{z}
\end{aligned}$$

The body-fixed components of the difference in the accelerations of any two points  $H_1$  and  $H_2$  in terms of the angular motion components in plane-fixed coordinates are given by:

$$\begin{aligned}
\ddot{\vec{H}}_{i_{1BF}} - \ddot{\vec{H}}_{i_{2BF}} &= \left( -q^2 - r^2 \right) (\Delta x_1 - \Delta x_2) & (21a) \\
&+ \left( \cos\phi(2qp - \dot{r}) + \sin\phi(2rp + \dot{q}) \right) (\Delta y_1 - \Delta y_2) \\
&+ \left( \cos\phi(2rp + \dot{q}) - \sin\phi(2qp - \dot{r}) \right) (\Delta z_1 - \Delta z_2)
\end{aligned}$$

$$\begin{aligned}
\ddot{\vec{H}}_{j_{1BF}} - \ddot{\vec{H}}_{j_{2BF}} &= \left( r\cos\phi - \dot{q}\sin\phi \right) (\Delta x_1 - \Delta x_2) & (21b) \\
&+ \left[ -\dot{p}^2 - \left( \frac{r^2 + q^2}{2} \right) - \left( \frac{r^2 - q^2}{2} \right) \cos 2\phi + qr\sin 2\phi \right] (\Delta y_1 - \Delta y_2) \\
&+ \left[ -\dot{p} + \left( \frac{r^2 - q^2}{2} \right) \sin 2\phi + qr\cos 2\phi \right] (\Delta z_1 - \Delta z_2)
\end{aligned}$$

$$\begin{aligned}
\ddot{\vec{H}}_{k_{1BF}} - \ddot{\vec{H}}_{k_{2BF}} &= \left( -\dot{r}\sin\phi - \dot{q}\cos\phi \right) (\Delta x_1 - \Delta x_2) & (21c) \\
&+ \left[ \dot{p} + \left( \frac{r^2 - q^2}{2} \right) \sin 2\phi + qr\cos 2\phi \right] (\Delta y_1 - \Delta y_2) \\
&+ \left[ -\dot{p}^2 - \left( \frac{r^2 + q^2}{2} \right) + \left( \frac{r^2 - q^2}{2} \right) \cos 2\phi - qr\sin 2\phi \right] (\Delta z_1 - \Delta z_2)
\end{aligned}$$

### 3. SIMULATION RESULTS

Though strapped-down linear accelerometers could be installed anywhere on a projectile and oriented arbitrarily, the algebra required to derive rotational motion components from the accelerometer measurements is greatly simplified if the measurement axes of the accelerometers are oriented parallel to the principal axes of inertia of the projectile (Figure 1). An accelerometer so positioned would be directly measuring one of the body-fixed components of nongravitational inertial acceleration at that location. Thus, an accelerometer at  $\Delta x, \Delta y, \Delta z$  from the projectile c.m. oriented in the  $+\vec{i}$  direction would be measuring  $\vec{H}_i - \vec{g}_i$  and similarly for  $+\vec{j}$  and  $+\vec{k}$  orientations. When these components are expressed in terms of the linear and angular motion components in body-fixed coordinates (equation [11]) and the difference between the accelerations at two different locations is similarly expressed (equation [12]), methods for deriving the components of angular velocity and acceleration can be readily demonstrated.

$$\vec{H}_i = \left[ \dot{u}_0 - \Delta \ddot{x} + q(w_0 - \Delta \dot{z}) - r(v_0 - \Delta \dot{y}) \right] \quad (11a)$$

$$+ \left[ (-q^2 - r^2)\Delta x + (pq - \dot{r})\Delta y + (pr + \dot{q})\Delta z + 2(q\Delta \dot{z} - r\Delta \dot{y}) + \Delta \ddot{x} \right]$$

$$\vec{H}_j = \left[ \dot{v}_0 - \Delta \ddot{y} + r(u_0 - \Delta \dot{x}) - p(w_0 - \Delta \dot{z}) \right] \quad (11b)$$

$$+ \left[ (pq + \dot{r})\Delta x + (-p^2 - r^2)\Delta y + (qr - \dot{p})\Delta z + 2(r\Delta \dot{x} - p\Delta \dot{z}) + \Delta \ddot{y} \right]$$

$$\vec{H}_k = \left[ \dot{w}_0 - \Delta \ddot{z} + p(v_0 - \Delta \dot{y}) - q(u_0 - \Delta \dot{x}) \right] \quad (11c)$$

$$+ \left[ (pr - \dot{q})\Delta x + (qr + \dot{p})\Delta y + (-p^2 - q^2)\Delta z + 2(p\Delta \dot{y} - q\Delta \dot{x}) + \Delta \ddot{z} \right]$$

$$\vec{H}_{i_1} - \vec{H}_{i_2} = (-q^2 - r^2)(\Delta x_1 - \Delta x_2) + (pq - \dot{r})(\Delta y_1 - \Delta y_2) + (pr + \dot{q})(\Delta z_1 - \Delta z_2) \quad (12a)$$

$$\vec{H}_{j_1} - \vec{H}_{j_2} = (pq + \dot{r})(\Delta x_1 - \Delta x_2) + (-p^2 - r^2)(\Delta y_1 - \Delta y_2) + (qr - \dot{p})(\Delta z_1 - \Delta z_2) \quad (12b)$$

$$\vec{H}_{k_1} - \vec{H}_{k_2} = (pr - \dot{q})(\Delta x_1 - \Delta x_2) + (qr + \dot{p})(\Delta y_1 - \Delta y_2) + (-p^2 - q^2)(\Delta z_1 - \Delta z_2) \quad (12c)$$

where  $p$  and  $\dot{p}$  are the angular velocity and acceleration of the  $j$ - $k$  axes about the  $i$  axis,  $q$  and  $\dot{q}$  are the velocity and acceleration of the  $i$ - $k$  axes about the  $j$  axis, and  $r$  and  $\dot{r}$  are the velocity and acceleration of the  $i$ - $j$  axes about the  $k$  axis.

Schuler, Grammatikos, and Fegley postulated a configuration of six accelerometers positioned as follows:

$$\begin{aligned}
 H_a &= \Delta x + \gamma_a, \Delta y, \Delta z && \text{aligned with the } \vec{i} \text{ axis} \\
 H_b &= \Delta x + \gamma_b, \Delta y, \Delta z && \text{aligned with the } \vec{i} \text{ axis} \\
 H_c &= \Delta x, \Delta y + \gamma_c, \Delta z && \text{aligned with the } \vec{j} \text{ axis} \\
 H_d &= \Delta x, \Delta y + \gamma_d, \Delta z && \text{aligned with the } \vec{j} \text{ axis} \\
 H_e &= \Delta x, \Delta y, \Delta z + \gamma_e && \text{aligned with the } \vec{k} \text{ axis} \\
 H_f &= \Delta x, \Delta y, \Delta z + \gamma_f && \text{aligned with the } \vec{k} \text{ axis}
 \end{aligned}$$

Denoting the corresponding *sensed* accelerations by  $A_a, A_b, A_c, A_d, A_e,$  and  $A_f,$  they obtained:

$$A_a - A_b = (-q^2 - r^2)(\gamma_a - \gamma_b)$$

or

$$(q^2 + r^2) = \frac{A_b - A_a}{\gamma_a - \gamma_b} \equiv C_1 \quad (22a)$$

similarly

$$(p^2 + r^2) = \frac{A_d - A_c}{\gamma_c - \gamma_d} \equiv C_2 \quad (22b)$$

$$(p^2 + q^2) = \frac{A_f - A_e}{\gamma_e - \gamma_f} \equiv C_3 \quad (22c)$$

and ultimately

$$p^2 = \frac{-C_1 + C_2 + C_3}{2} \quad (23a)$$

$$q^2 = \frac{C_1 - C_2 + C_3}{2} \quad (23b)$$

$$r^2 = \frac{C_1 + C_2 - C_3}{2} \quad (23c)$$

They point out the sign difficulty in obtaining  $p$ ,  $q$ , and  $r$  and suggest that this "difficulty can be resolved through the use of auxiliary devices that may be less accurate and less costly than accelerometers." Implementation of this configuration and computational scheme in a simulated flight of an M483A1 artillery projectile revealed an additional difficulty with this methodology for such projectiles.

### 3.1. Case 1: M483A1 Artillery Projectile.

The M483A1 is a 155-mm-diameter, spin-stabilized projectile that is launched from a rifled tube with a 1/20 twist. When fired from an M109 howitzer with a 7w propelling charge, the M483A1 has a launch velocity of 539 m/s and an initial spin rate of  $\approx 174$  rps. At  $45^\circ$  launcher elevation, under nominal conditions, this projectile has a range of  $\approx 14$  km and a flight time of  $\approx 60$  s. At impact, the spin rate is  $\approx 125$  rps. Thus  $p$ , whose units are radians/s, ranges from  $\approx 1100$  rad/s to  $\approx 785$  rad/s (Figure 2a). At the same time,  $q$  and  $r$  range in magnitude from near zero at launch to  $\approx 0.6$  rad/s at impact (Figures 2b and 2c).

Though the Schuler, Grammatikos, and Fegley method for finding  $p$ ,  $q$ , and  $r$  is analytically correct, it sometimes failed to produce accurate values of  $q$  and  $r$  for simulated flights of this projectile due to numerical difficulties. Whereas  $p$  is on the order of magnitude of  $10^3$ ,  $q$  and  $r$  are on the order of magnitude of  $10^{-1}$  or less. Thus there is an at least four orders of magnitude difference between  $p$  and  $q$  and  $p$  and  $r$  and an eight orders of magnitude difference between their squares. Significant figure limitations then result in estimation errors for the values of  $q$  and  $r$  due to the Schuler, Grammatikos, and Fegley methodology's reliance on accurately determining  $(p^2 + q^2)$  and  $(p^2 + r^2)$ .

An M483A1 trajectory with launch conditions as described above was computed with the known values of  $p$ ,  $q$ , and  $r$  and their Schuler, Grammatikos, and Fegley estimates output to a file at 0.015-s intervals throughout the flight. Additionally, estimates of  $q$  and  $r$  generated using a different combination of accelerometer positions and a different computational algorithm were output. Within the CONTRAJ model as implemented on our branch computer, 64 bits are used to represent reals, 52 bits of which are used for the mantissa, 11 for the exponent, and 1 for the sign. This gives 15 significant figures to the known values of  $p$ ,  $q$ , and  $r$  and the accelerometer outputs.

Figure 3 gives the percentage of the samples for which the errors in the estimates of  $q$  and  $r$  exceeded each of four different levels as a function of the number of

significant figures carried in the computations for both methodologies. Figure 3a gives the percentages of time that the magnitudes of the error in the estimate of  $q$  and  $r$  exceeded 5% of the known magnitudes of  $q$  and  $r$ . Figure 3b gives the percentages for which the error magnitudes exceeded 25%, 3c - 50%, and 3d - 100%. The upper set of curves in each plot is for the errors in the Schuler, Grammatikos, and Fegley estimates of  $q$  and  $r$ . Only at 14 and 15 significant figures are these estimates approximately errorless. The curves for the alternative estimates of  $q$  and  $r$  are effectively colinear and are seen as the lower curve in each plot. These estimates are nonzero only for two significant figure computations. In this case, the estimation error is primarily due to round-off as the estimates are always being compared to the 15 significant figure known values of  $q$  and  $r$ .

Though this alternative method of estimating angular velocity components has the advantage of requiring less significant figures in its computations (and accelerometer data), it has the disadvantage of requiring the output of 10 accelerometers rather than 6. Their configuration is:

$$\begin{aligned}
 H_a &= \Delta x, \Delta y + \gamma_a, \Delta z && \text{aligned with the } \vec{i} \text{ axis} \\
 H_b &= \Delta x, \Delta y + \gamma_b, \Delta z && \text{aligned with the } \vec{i} \text{ axis} \\
 H_c &= \Delta x, \Delta y, \Delta z + \gamma_c && \text{aligned with the } \vec{i} \text{ axis} \\
 H_d &= \Delta x, \Delta y, \Delta z + \gamma_d && \text{aligned with the } \vec{i} \text{ axis} \\
 H_e &= \Delta x + \gamma_e, \Delta y, \Delta z && \text{aligned with the } \vec{j} \text{ axis} \\
 H_f &= \Delta x + \gamma_f, \Delta y, \Delta z && \text{aligned with the } \vec{j} \text{ axis} \\
 H_g &= \Delta x + \gamma_g, \Delta y, \Delta z && \text{aligned with the } \vec{k} \text{ axis} \\
 H_h &= \Delta x + \gamma_h, \Delta y, \Delta z && \text{aligned with the } \vec{k} \text{ axis} \\
 H_i &= \Delta x, \Delta y + \gamma_i, \Delta z && \text{aligned with the } \vec{j} \text{ axis} \\
 H_j &= \Delta x, \Delta y + \gamma_j, \Delta z && \text{aligned with the } \vec{j} \text{ axis}
 \end{aligned}$$

If the corresponding *sensed* accelerations are denoted by  $A_a, A_b, A_c, A_d, A_e, A_f, A_g, A_h, A_i,$  and  $A_j,$  then:

$$pq - \dot{r} = \frac{A_a - A_b}{\gamma_a - \gamma_b} \equiv C_1 \quad (24a)$$

$$pr + \dot{q} = \frac{A_c - A_d}{\gamma_c - \gamma_d} \equiv C_2 \quad (24b)$$

$$pq + \dot{r} = \frac{A_e - A_f}{\gamma_e - \gamma_f} \equiv C_3 \quad (24c)$$

$$pr - \dot{q} = \frac{A_g - A_h}{\gamma_g - \gamma_h} \equiv C_4 \quad (24d)$$

$$p^2 + r^2 = \frac{A_i - A_j}{\gamma_j - \gamma_i} \equiv C_5 \quad (24e)$$

Recalling that  $p^2$  is eight or more orders of magnitude greater than  $r^2$ , the approximation  $p^2 \approx p^2 + r^2$  is made. The estimates of the angular velocity components are then:

$$\bar{p} = \sqrt{C_5} \quad (25a)$$

$$\bar{q} = \frac{C_1 + C_3}{2\sqrt{C_5}} \quad (25b)$$

$$\bar{r} = \frac{C_2 + C_4}{2\sqrt{C_5}} \quad (25c)$$

Also readily available are two analytically exact measures of angular acceleration components:

$$\dot{q} = \frac{C_3 - C_1}{2} \quad (26a)$$

$$\dot{r} = \frac{C_2 - C_4}{2} \quad (26b)$$

### 3.2. Case 2: 2.75-Inch MK66 Rocket.

The preceding computations were also made for a representative trajectory of another projectile with very different kinematics, the 2.75-in MK66 rocket. For this trajectory, the motor was modeled as burning for 1.12 s, at the end of which the projectile speed was  $\approx 720$  m/s. When launched at  $45^\circ$  elevation under nominal conditions, the time of flight was  $\approx 50.6$  s and the range to impact was  $\approx 8050$  m. The known values of the angular velocity components  $p$ ,  $q$ , and  $r$  are seen in Figures 4a, 4b, and 4c respectively. For this trajectory, the differences between the order of magnitude of  $p^2$  and those of  $q^2$  and  $r^2$  are much less than those seen for the M483A1. Plotting the percentages for which the estimation errors exceeded the four levels as

before (Figure 5), the curves for errors in  $q$  and  $r$  are essentially colinear for both methodologies for this projectile. Also, the character of the curves is seen to be like those for the M483A1 albeit at different (lower) values for significant figures. The Schuler, Grammatikos, and Fegley methodology (upper curves) is outperformed by the alternative methodology at all levels for computations maintaining less than eight significant figures. The lower curves, like their counterparts for the M483A1, are constant valued except for the single instance of two significant figures and estimation errors exceeding 5%. It is not apparent because of the scale, but the constant value is not  $\approx$ zero as it was for the M483A1. It is 0.653%. This is due to the greater error in the approximation of  $p^2 \approx p^2 + r^2$ . This approximation can be eliminated if 12 accelerometers are employed rather than 10. With  $H_a$  through  $H_h$ ,  $A_a$  through  $A_h$ , and  $C_1$  through  $C_4$  as before, and

$$\begin{aligned} H_i &= \Delta x, \Delta y, \Delta z + \gamma_i && \text{aligned with the } \vec{j} \text{ axis} \\ H_j &= \Delta x, \Delta y, \Delta z + \gamma_j && \text{aligned with the } \vec{j} \text{ axis} \\ H_k &= \Delta x, \Delta y + \gamma_k, \Delta z && \text{aligned with the } \vec{k} \text{ axis} \\ H_l &= \Delta x, \Delta y + \gamma_l, \Delta z && \text{aligned with the } \vec{k} \text{ axis} \end{aligned}$$

$$\dot{qr - p} = \frac{A_i - A_j}{\gamma_i - \gamma_j} \equiv C_5 \quad (27a)$$

$$\dot{qr + p} = \frac{A_k - A_l}{\gamma_k - \gamma_l} \equiv C_6 \quad (27b)$$

The angular velocity components are then given by:

$$p = \left[ \frac{(C_1 + C_3)(C_2 + C_4)}{2(C_5 + C_6)} \right]^{1/2} \quad (28a)$$

$$q = \frac{C_1 + C_3}{2p} \quad (28b)$$

$$r = \frac{C_2 + C_4}{2p} \quad (28c)$$

and the angular acceleration components by:

$$\dot{p} = \frac{C_6 - C_5}{2} \quad (29a)$$

$$\dot{q} = \frac{C_3 - C_1}{2} \quad (29b)$$

$$\dot{r} = \frac{C_2 - C_4}{2} \quad (29c)$$

In this methodology, the sign difficulty analytically remains only for  $p$ . In practice, this difficulty appears only for projectiles whose spin direction is either unknown or subject to reversal during flight. In these instances, an auxiliary sensor could be used to remove the ambiguity as suggested by Schuler, Grammatikos, and Fegley.

#### 4. SUMMARY

Equations for the inertial acceleration of an arbitrary point on a flight body have been derived in the two coordinate systems most commonly employed in computerized trajectory models, the body-fixed and plane-fixed systems. The equations were constructed in such a form that the location of the point was reconciled in both systems. The acceleration components in the plane-fixed system were projected onto the principal inertial axes of the flight body for comparison with the components as computed in the body-fixed system. Each of the acceleration components given by these equations represent the sum of the acceleration that will be measured by a perfect accelerometer located at that point on the flight body whose measurement axis is oriented parallel to the respective principal axis of inertia and the projection of the gravity vector onto that axis. The equations for these components ([11] and [17]) further show that these accelerations are given by various combinations of the translational and angular velocities and accelerations that constitute the body's kinematics. These equations were included in the CONTRAJ 6dof trajectory model. By appropriately locating and orienting a collection of accelerometers and forming linear combinations of their outputs, solutions for the velocity and acceleration components can be derived.

Implementation of a configuration of accelerometers and a computational algorithm published in a paper by Schuler, Grammatikos, and Fegley revealed a further consideration in the design of accelerometer-only inertial measurement devices. Though the Schuler, Grammatikos, and Fegley method for finding angular velocity components is analytically correct, it sometimes failed to produce accurate estimates of these velocities in a simulated flight of an M483A1 projectile due to numerical difficulties. The successful estimation of these velocities employing an alternative approach demonstrated both the necessity of application driven design of

such systems and the utility of the accelerometer augmented CONTRAJ model in such efforts.

Realistic models of accelerometers need to be added to CONTRAJ as a next step in investigating the possibility of practical accelerometer-only inertial systems. How bias, noise, and other corruptions of the 'perfect' acceleration measurements assumed in this study impact such systems will be the subject of future efforts.

■ ACCELEROMETERS

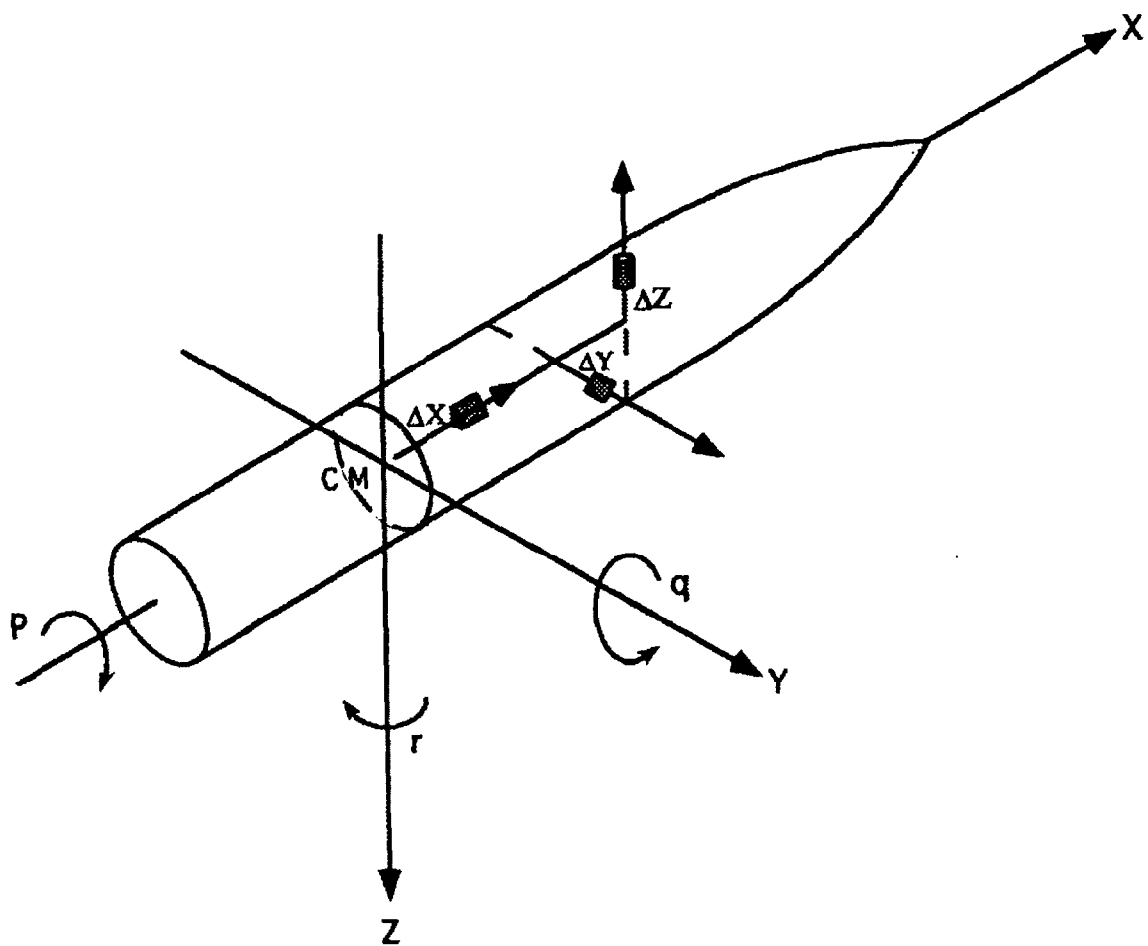
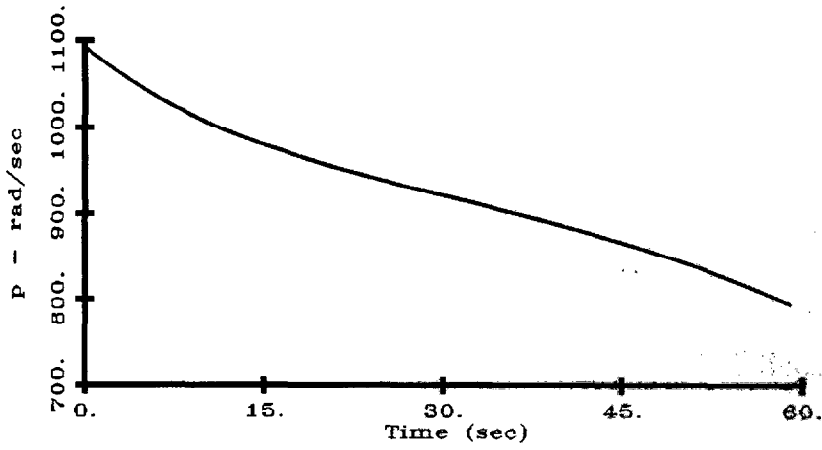
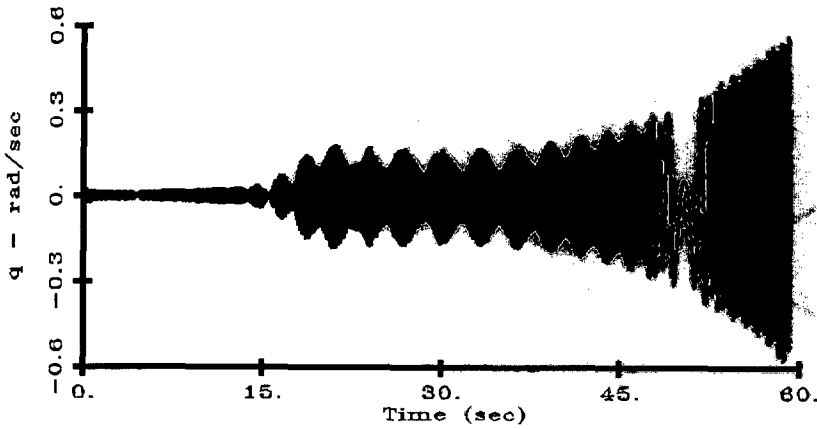


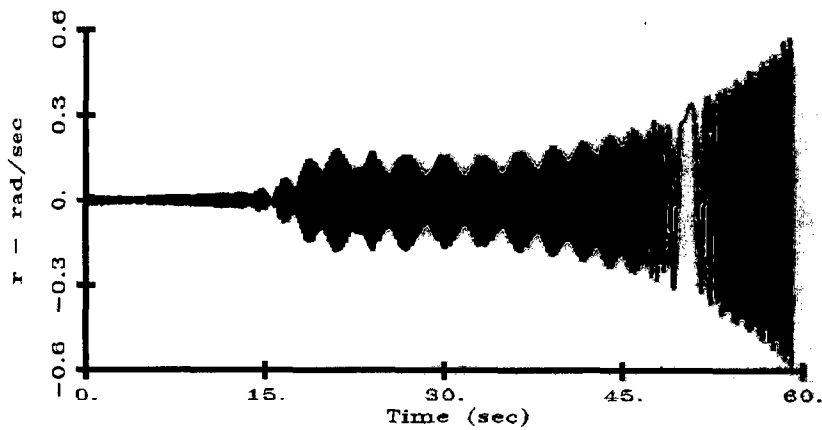
Figure 1. Accelerometer locations and orientations.



a. Angular Velocity ( $p$ ) of j-k Axes About i Axis

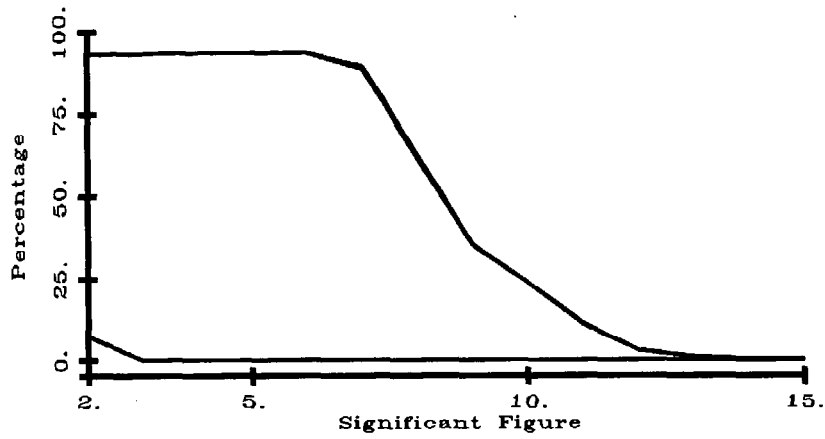


b. Angular Velocity ( $q$ ) of i-k Axes About j Axis

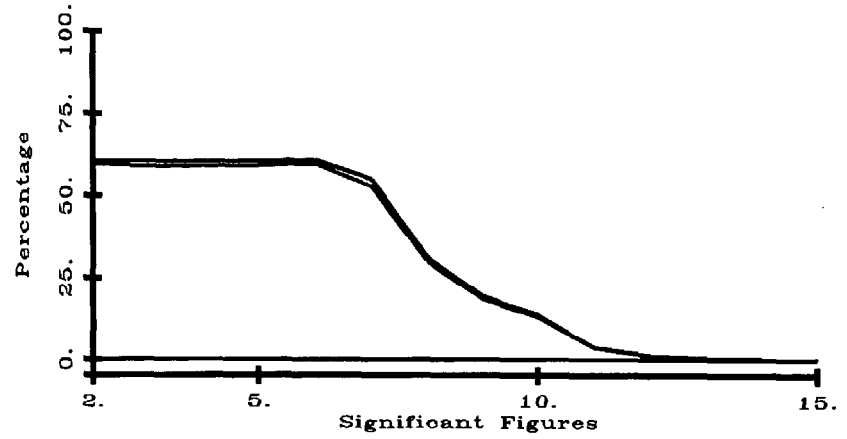


c. Angular Velocity ( $r$ ) of i-j Axes About k Axis

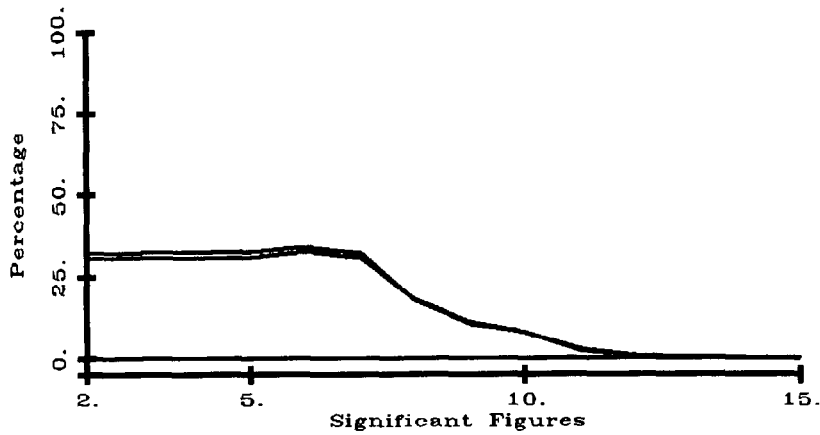
Figure 2. Body-fixed angular velocity components - M483A1 projectile.



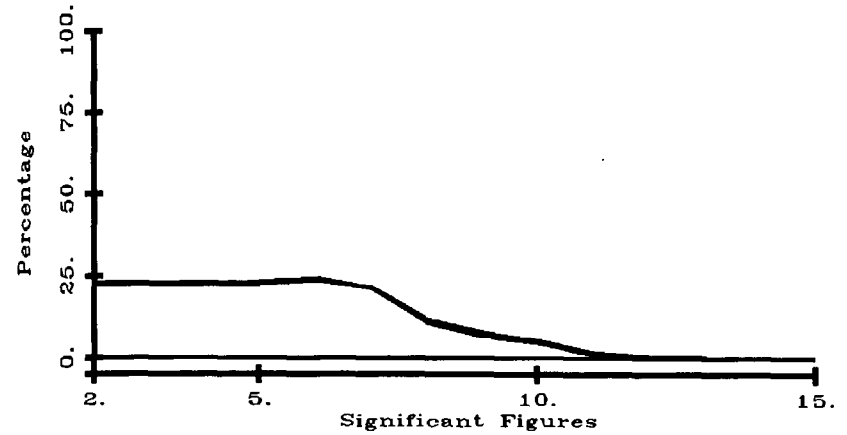
a. Estimate Error Exceeds 5%



b. Estimate Error Exceeds 25%

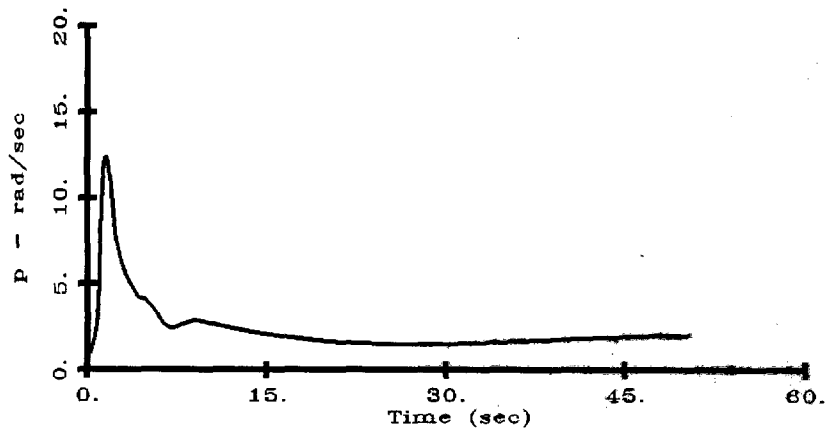


c. Estimate Error Exceeds 50%

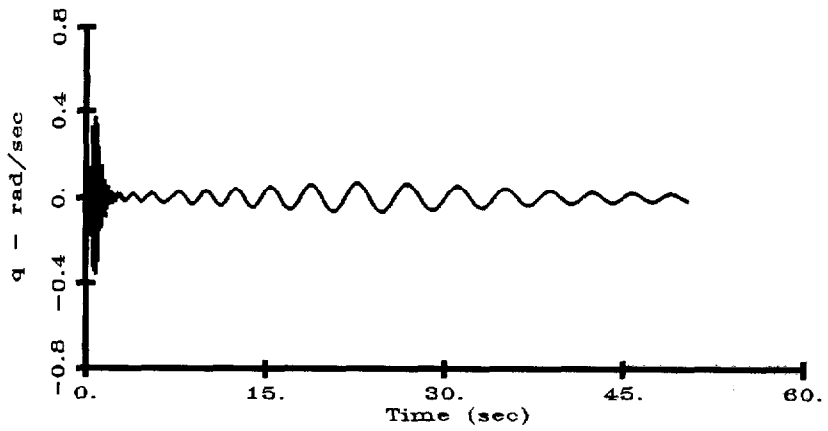


d. Estimate Error Exceeds 100%

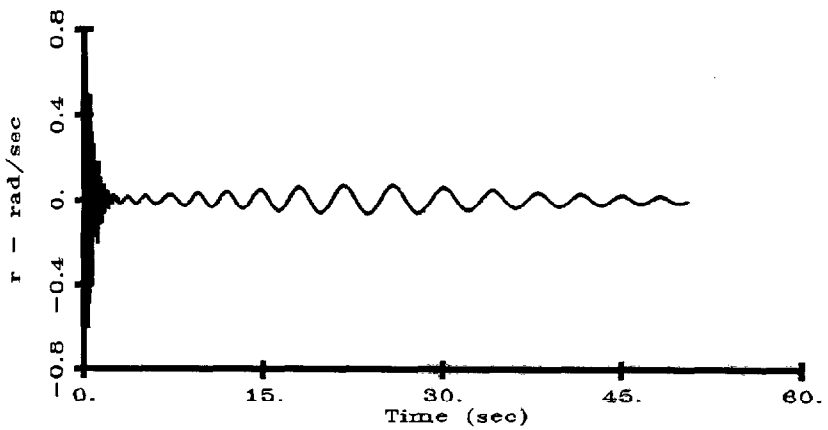
Figure 3. Angular motion estimation errors - M483A1 projectile.



a. Angular Velocity ( $p$ ) of j-k Axes About i Axis

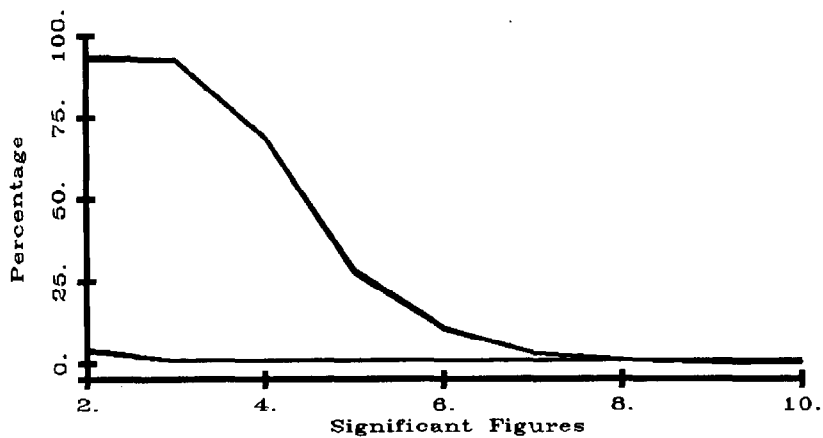


b. Angular Velocity ( $q$ ) of i-k Axes About j Axis

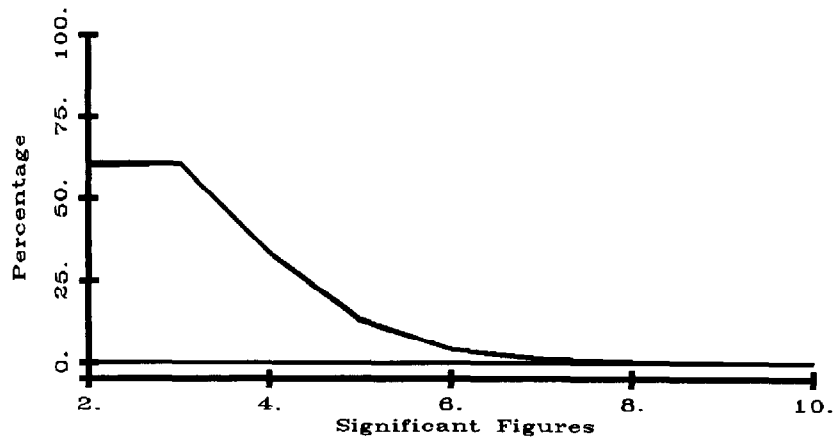


c. Angular Velocity ( $r$ ) of i-j Axes About k Axis

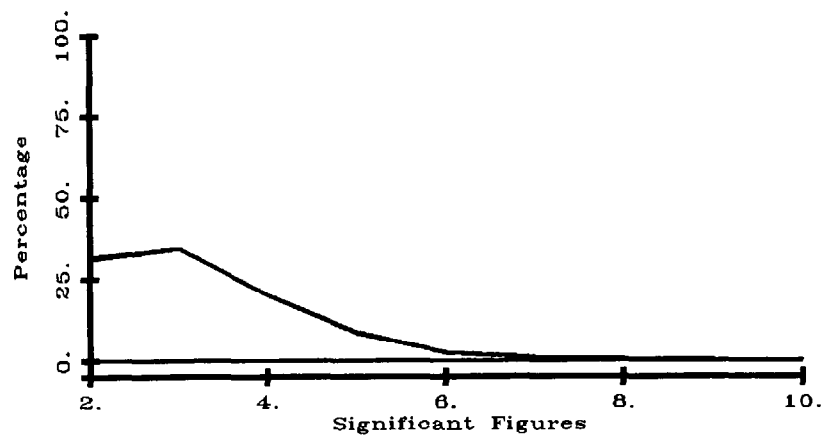
Figure 4. Body-fixed angular velocity components - 2.75-in MK66 rocket.



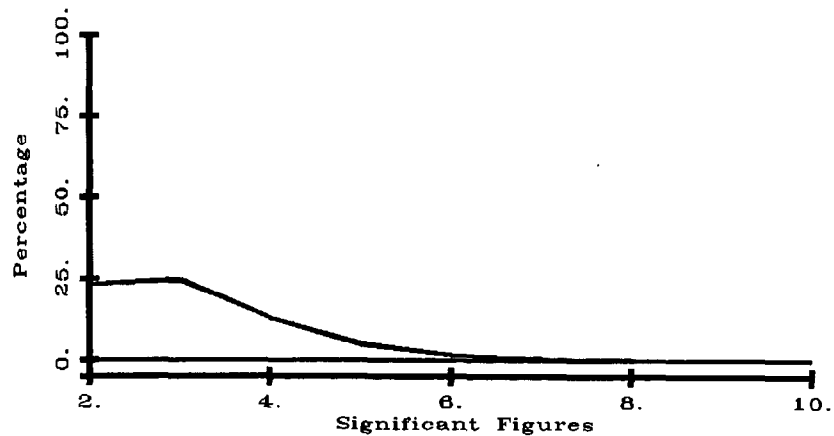
a. Estimate Error Exceeds 5%



b. Estimate Error Exceeds 25%



c. Estimate Error Exceeds 50%



d. Estimate Error Exceeds 100%

Figure 5. Angular motion estimation errors - 2.75-in MK66 rocket.

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