

AD-A286 672



AN
ASME
PUBLICATION

M102523

94-29247



\$1 PER COPY

50¢ TO ASME MEMBERS

The Society shall not be responsible for statements or opinions expressed in papers or in discussion at meetings of the Society or of its Divisions or Sections, or for the publications thereon.

Discussion is printed only if the paper is published in an ASME journal.

Released for general publication upon presentation

THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS
29 West 39th Street, New York 18, N. Y.

PAPER NUMBER

61-AV-56

OCT 16 1961

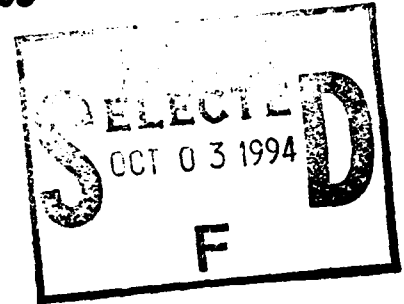
L

COPY 1

Elastic Properties of Orthotropic Monofilament Laminates

J. C. EKVAL

Senior Research Engineer,
Lockheed Aircraft Corporation,
Burbank, Calif.



In order to analyze the structural behavior of orthotropic monofilament laminates, it is necessary to know the elastic properties. In this paper equations are presented for determining the elastic properties of monofilament laminates based upon the properties of the filament and matrix materials. Equations are given for calculating the axial stiffness, shear stiffness and Poisson's ratio in the principal directions for both unidirectional and bidirectional laminates. The effects of the filament shape and spacing on the elastic properties are discussed. Test results for some music wire-epoxy resin laminates are presented which are in good agreement with the theory.

This document has been approved for public release and sale; its distribution is unlimited.

For presentation at the Aviation Conference, Los Angeles, Calif., March 13-16, 1961, of The American Society of Mechanical Engineers. Manuscript received by ASME Headquarters, January 26, 1961.

Written discussion on this paper will be accepted up to April 17, 1961.

Copies will be available until January 1, 1962.

DTIC QUALITY INSPECTED 3

Ma

Elastic Properties of Orthotropic Monofilament Laminates

J. C. EKVALL

NOMENCLATURE

A = cross-sectional area, sq in.
a = length, in.
b = width, in.
c = distance between load point and reaction point along side a, in.
D = filament diameter, in.
 D_{xy} = twisting stiffness of plate, in-lb
d = distance between load point and reaction point along side b, in.
E = modulus of elasticity or axial stiffness, psi
G = shear modulus of elasticity or shear stiffness, psi
 I_{xy} = product of inertia, in.⁴
L = direction of maximum principal stiffness
N = number of layers
P = machine load, lb
R = radius ratio = diameter of filaments divided by spacing between filaments
T = direction of minimum principal stiffness
t = thickness, in.
V = volume ratio = ratio of volume to total volume
x, y, z = orthogonal co-ordinates; z measured normal to plane of plate and x and y parallel to sides of plate, in.
 γ = shear strain, in/in
 Δ = deflection, in.
 ϵ = axial strain, in/in
 θ = angle measured from reference axis to plane of minimum shear stiffness

$$\bar{\Phi} = \int_0^{\pi/2} \frac{\sin\theta \, d\theta}{\frac{G_m}{G_f} + \sin^2\theta \left(1 - \frac{G_m}{G_f}\right)}$$
 μ = Poisson's ratio
 σ = normal stress, psi
 τ = shear stress, psi

Subscripts and Superscripts

c = composite composed of more than one layer
f = filament
H = head deflection
L = co-ordinate direction
m = matrix
n = nth layer
T = co-ordinate direction
x, y, z = co-ordinate direction
1, 2 = laminate layer number

* = corrected value to account for variations in laminate thickness

INTRODUCTION

In this paper, methods are presented for determining the elastic properties of orthotropic laminates composed of filaments bonded together with a matrix. The elastic properties are the basic properties needed to analyze the behavior of a composite under any condition of load. The equations developed in this paper enable the designer to determine the elastic properties of orthotropic laminates composed of any combination of materials that can be bonded together. Also, it is possible to design a monofilament laminate to have some given elastic properties.

SUMMARY

The elastic properties were found to be dependent upon the shape of the filament as well as the spacing and properties of the constituents. Equations are derived for two shapes of filaments, i.e., square and round cross section. The procedures outlined in this paper could be used to determine the elastic properties of other shapes of filaments as well.

Some steel wire and epoxy-resin laminates were made to verify the equations developed in this paper. The methods for calculating the elastic properties E_L , E_T , μ_{LT} , μ_{TL} , and G_{LT} agree with the tests results within the experimental accuracy of the measurements. It was found that the shear stiffness was quite sensitive to variations in laminate thickness. Also the shear deformations of the laminates should be included in the calculations for the bending stiffness if there is an appreciable amount of low-modulus material between layers, or if the majority of the layers are aligned transverse to the direction of bending. Further research in this field should do much toward obtaining a better understanding of the factors that affect the properties of orthotropic monofilament laminates.

THEORETICAL CONSIDERATIONS

Filament-wound materials with filaments aligned in mutually perpendicular directions as

A-1

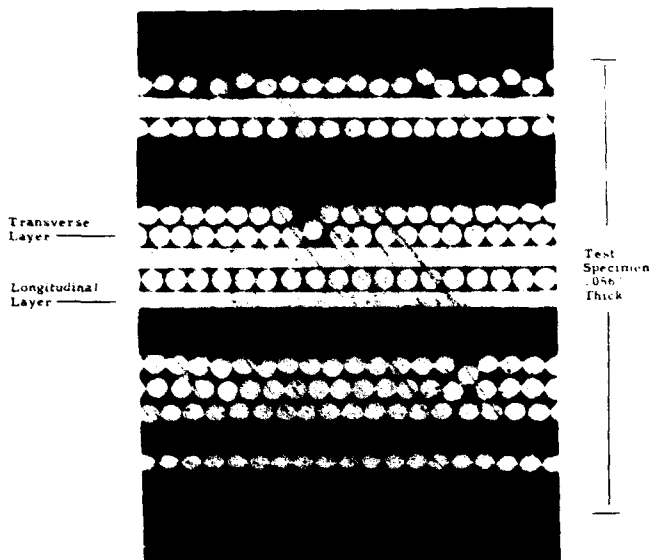


Fig. 1 Cross section of a 19-layer bidirectional monofilament laminate

shown in Fig.1 can be analyzed by the theory of elasticity for orthotropic materials. In order to solve the stress-strain relations for the case of plane stress, five elastic constants must be known. These constants are the axial stiffnesses in the direction of the filaments and transverse to the direction of the filaments as well as the Poisson's ratios and the shear stiffness associated with these directions. There are, however, only four independent elastic constants because the following relationship holds

$$\frac{\mu_{LT}}{E_L} = \frac{\mu_{TL}}{E_T} \quad (1)$$

In order to analyze a laminate such as shown in Fig.1, the following assumptions are made:

- 1 The laminate is composed of unidirectional layers, one filament diameter thick.
- 2 The filaments in each layer are evenly spaced.
- 3 The filament and matrix material are bonded together and act as a composite.
- 4 The two materials obey Hooke's law.
- 5 There is no interaction between the two materials that would change their material properties. In other words, the elastic properties of the materials in the laminates are same as the elastic properties of the materials in bulk form.

At first these assumptions may seem overly restrictive; however, most laminates made by winding single filaments over a mandrel will conform reasonably close to these assumptions. The effect of variations of laminate thickness on the properties will be discussed in a subsequent section.

To analyze the properties of a bidirectional laminate, first the properties of the unidirectional layers must be obtained. In this paper, methods for determining the properties of unidirectional layers will be discussed in some detail. Methods for determining the properties of laminates based on the properties of each layer have been covered quite thoroughly (1,2,3)¹ and will be discussed only briefly here.

In the subsequent sections equations are derived for determining the five elastic constants E_L , E_T , μ_{LT} , μ_{TL} and G_{LT} for unidirectional layers and for bidirectional laminates. The equations will be derived for filaments of both square or rectangular and round cross section. In the former case it can be assumed that the laminate is homogeneous through its thickness and therefore, the stress distribution across the thickness is uniform. For round or other shaped filaments the distribution of the materials through the thickness must be considered.

ELASTIC CONSTANTS FOR UNIDIRECTIONAL LAMINATES

Longitudinal Properties

First consider the axial stiffness for loads applied in the direction of the filaments. To simplify the analysis, the composite material can be considered as a solid piece of filament material bonded to a solid piece of matrix material. In this case the strain in the matrix, filament, and composite are all equal in the direction of the applied load. Therefore, the amount that each material contributes to the axial stiffness is simply the volume per cent of that material times its modulus of elasticity. The axial stiffness of the composite is then given by the following equation (4,5):

$$E_L = V_f E_f + V_m E_m \quad (2)$$

For similar reasons the amount that each material contributes to the Poisson's ratio of the composite is directly proportional to the amount of material in the composite. The equation for Poisson's ratio for the composite is similar to the equation for the axial stiffness and is given by the following equation (4):

$$\mu_{LT} = V_f \mu_f + V_m \mu_m \quad (3)$$

Both equations (2) and (3) apply to composites with any shape filaments. This is true because the strain in the filaments or the matrix is independent of the shape of the filaments or the distribution of the materials as long as the distribution of materials are continuous end to end.

¹ Underlined numbers in parentheses designate References at the end of the paper.

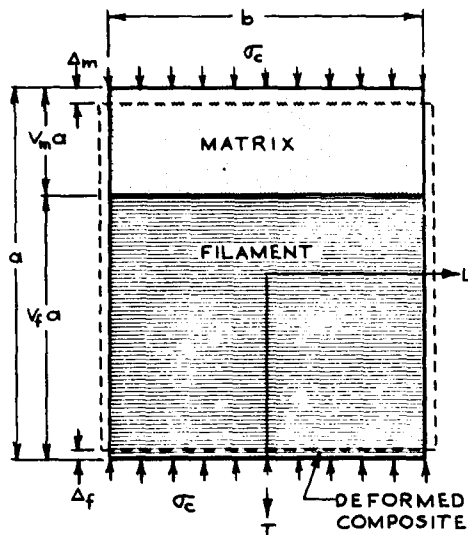


Fig. 2 Equivalent composite for analyzing the transverse stiffness

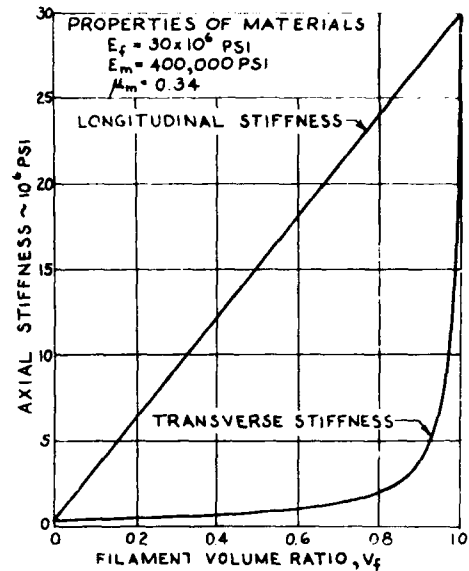


Fig. 3 Axial stiffness comparisons for steel-epoxy composites with square or rectangular filaments

Transverse Properties

The axial stiffness in a direction transverse to the filaments can also be determined by considering an equivalent composite as shown in Fig. 2. In this case it is first assumed that the filaments are square or rectangular so that changes in stiffness in the z direction can be neglected. The strains and stresses in the filament and matrix materials can be determined by equating the loads and strains in the L and T directions as follows:

$$\begin{aligned}
 \sigma_{TA} &= \sigma_{mTA} \\
 \sigma_{TA} &= \sigma_{fTA} \\
 \sigma_{mL} A_{mL} + \sigma_{fL} A_{fL} &= 0 \\
 \epsilon_{mT} &= \frac{\sigma_{mT}}{E_m} - \mu_m \frac{\sigma_{mL}}{E_m} \\
 \epsilon_{fT} &= \frac{\sigma_{fT}}{E_f} - \mu_f \frac{\sigma_{fL}}{E_f} \\
 \epsilon_{mL} &= \frac{\sigma_{mL}}{E_m} - \mu_m \frac{\sigma_{mT}}{E_m} \\
 \epsilon_{fL} &= \frac{\sigma_{fL}}{E_f} - \mu_f \frac{\sigma_{fT}}{E_f} \\
 \epsilon_{mL} = \epsilon_{fL} = \epsilon_{cL}
 \end{aligned}
 \tag{4}$$

The strain in the composite in the T direction can be obtained by noting that

$$\epsilon_T = \epsilon_{fT} v_f + \epsilon_{mT} v_m \tag{5}$$

By substituting the values of ϵ_{fT} and ϵ_{mT} from (4) into (5) and rearranging terms, the equation for the transverse stiffness is obtained

$$\frac{1}{E_T} = \frac{v_m}{E_m} + \frac{v_f}{E_f} - \frac{v_f}{E_f} \frac{\left(\frac{E_f}{E_m} \mu_m - \mu_f\right)^2}{\left(\frac{v_f E_f}{v_m E_m} + 1\right)} \tag{6}$$

Poisson's ratio can also be obtained from (4) and (5).

By definition Poisson's ratio is the negative ratio of ϵ_{cL} divided by ϵ_{cT} . Using the expressions for ϵ_{cL} and ϵ_{cT} from (4) and (5) and reducing Poisson's ratio to its simplest terms gives

$$\mu_{TL} = \mu_{LT} E_T/E_L \tag{7}$$

Equation (7) agrees with the condition given by equation (1) and indicates that the equations for E_T , E_L , and μ_{LT} are consistent.

For laminates with plastic resin as the matrix, equation (6) can be simplified since μ_T is usually small compared to $\mu_m E_f/E_m$ and 1 is small compared to $v_f E_f/v_m E_m$. Eliminating μ_T and 1 in equation (6) yields

$$E_T = \frac{E_f E_m}{v_f E_m + v_m E_f (1 - \mu_m^2)} \tag{8}$$

If Poisson's ratios in equation (6) or (8) can be neglected without much loss in accuracy, the transverse stiffness is simply

$$E_T = \frac{E_f E_m}{v_f E_m + v_m E_f} \tag{9}$$

A comparison between the transverse stiffness as given by equation (8) and the longitudinal stiffness is shown in Fig. 3 for laminates made with steel wire and epoxy resin. For this partic-

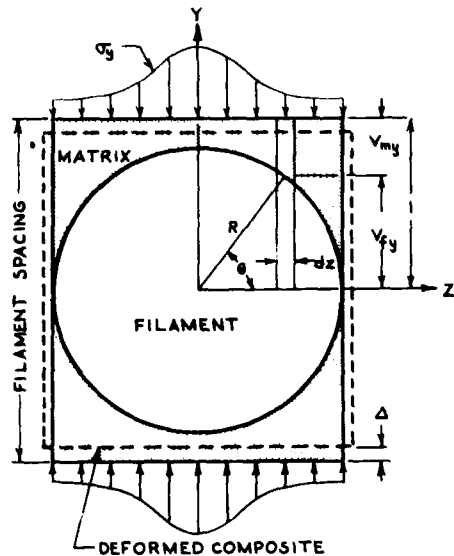


Fig. 4 Typical element for analyzing the transverse stiffness

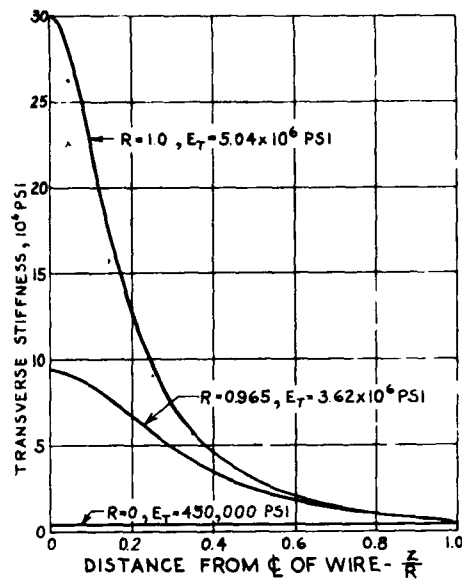


Fig. 5 Transverse stiffness of steel-epoxy composites for several filament spacings

ular combination of materials, the longitudinal stiffness is considerably larger than the transverse stiffness even for filament volume ratios as high as 0.9. This indicates that epoxy resin is a poor material for carrying load and that even a small amount of resin between the filaments reduces the stiffness and hence the load carrying ability considerably.

For laminates with round filaments, the load distribution across the thickness of the laminate must be considered. In this case a typical unit is selected for analytical purposes as shown in Fig. 4. Because of the round shape of the filament, the stiffness varies across the width of the laminate. And since the deflections must be the same at every point, the stress distribution is nonuniform as shown. The axial stiffness is determined by considering the stiffness of the differential element $dz \times 1 \times 1$. Assuming that equation (9) applies, the stiffness of the differential element is given by

$$E_y = \frac{E_f}{v_{my} E_f/E_m + v_{fy}} \quad (10)$$

Integrating equation (10) from 0 to R and dividing by R gives the average stiffness. R here is not the radius of the filament but the ratio of the filament diameter to the filament spacing

$$E_T = \frac{1}{R} \int_0^R E_y dz = \frac{1}{R} \int_0^R \frac{E_f dz}{v_{my} E_f/E_m + v_{fy}} \quad (11)$$

Putting equation (11) in terms of R and θ and changing the variable to $d\theta$ gives

$$E_T = E_f \int_0^{\pi/2} \frac{\sin \theta d\theta}{E_f/E_m + R \sin \theta [1 - E_f/E_m]} \quad (12)$$

Equation (12) cannot be reduced any further and must be integrated graphically or by computer.

Equation (12) was developed neglecting the effect of Poisson's ratio. When Poisson's ratio cannot be neglected, equation (6) would have to be used for the stiffness of the element. In this case it would be better to plot the stiffness E_T as a function of the radius ratio R, and then calculate the average stiffness graphically.

For laminates which have a large E_f/E_m ratio, like epoxy resin and steel wire composites, equation (8) can be used for the differential element. Then the equation for the transverse stiffness is given by

$$E_T = E_f \int_0^{\pi/2} \frac{\sin \theta d\theta}{E_f/E_m (1 - \mu_m^2) + R \sin \theta \left[1 - \frac{E_f}{E_m} (1 - \mu_m^2) \right]} \quad (13)$$

Equation (13) was evaluated for epoxy-steel composites for several filament spacings as shown in Fig. 5. As these curves illustrate, most of the load is carried by the center half of the laminate. The maximum value of E_T when adjacent wires are touching is 5.04×10^6 psi. By increasing the spacing $3\frac{1}{2}$ per cent, the transverse stiffness is reduced to 3.62×10^6 psi or a decrease of 27 per cent. Because of the low modulus of elasticity of

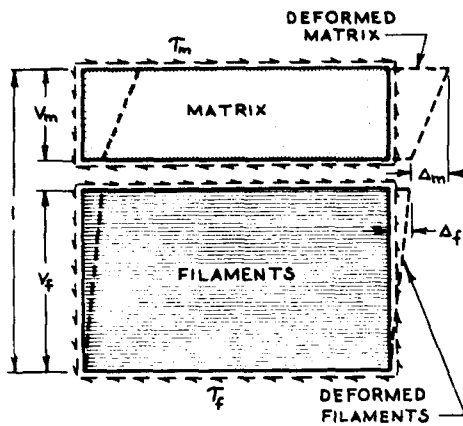


Fig. 6 Equivalent composite for analyzing the shear stiffness

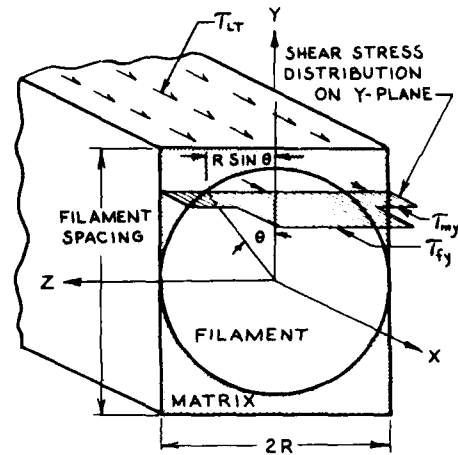


Fig. 7 Typical element for analyzing shear stiffness

the resin, even a small amount between the steel filaments drastically reduces the transverse stiffness. It also shows that for material combinations such as this, the filament volume ratios must be known very accurately when the filaments are closely spaced, in order to determine the transverse stiffness accurately.

Shear Stiffness

For determining the shear stiffness of a uni-directional laminate with square or rectangular filaments, an equivalent composite is selected as shown in Fig. 6. As before, the monofilament laminate will be considered as a solid piece of filament material bonded to a solid piece of matrix material. Because the shear load is uniform around the edge, it doesn't matter in which direction the filaments are oriented. The shear stiffness of the composite is given by

$$G_{LT} = \frac{T_{LT}}{\Delta_f + \Delta_m} \quad (14)$$

Substituting for the values of the deflection in terms of the properties of the filament and matrix material gives

$$G_{LT} = \frac{T_{LT}}{\frac{V_f T_f}{G_f} + \frac{V_m T_m}{G_m}} \quad (15)$$

In order for the matrix and filaments to be in equilibrium, the shear stress of the filaments, matrix, and composite must be equal so that equation (15) can be reduced to

$$G_{LT} = \frac{G_f G_m}{V_m G_f + V_f G_m} \quad (16)$$

The shear stiffness for a laminate with round filaments can be determined by considering the shear stiffness of a typical element as shown in

Fig. 7. First it is assumed that the shear deformation of all \$dz\$-elements across the width of the laminate are equal; that is, there is no warping of the laminate in the \$yz\$-plane. Then the shear stress distribution on any plane \$y\$ distant from the center line of the filament is as shown in Fig. 7. Equating the summation of the forces on this plane, to the applied shear force gives

$$T_{LT} = \tau_{fy} \sin \theta + \tau_{my} (1 - \sin \theta) \quad (17)$$

Since on any \$y\$-plane the shear deformation in the matrix and filament are equal, the relation between the shear stresses is given by

$$\tau_{my} = \frac{G_m}{G_f} \tau_{fy} \quad (18)$$

Substituting the value for the shear stress in the matrix into equation (18) and solving for \$\tau_{fy}\$ gives

$$\tau_{fy} = \frac{T_{LT}}{\sin \theta + \frac{G_m}{G_f} (1 - \sin \theta)} \quad (19)$$

Therefore, the shear stress in the filament will vary from \$\tau_{LT}\$ at \$y = 0\$ to \$(G_m/G_f) \tau_{LT}\$ at \$y = R\$ and the shear stress in the matrix will vary from \$(G_m/G_f) \tau_{LT}\$ at \$y = 0\$ to \$\tau_{LT}\$ at \$y = R\$. The shear stress in the matrix between the filaments will be equal to \$\tau_{LT}\$.

To determine the shear stiffness, the shear deformations along the center of the laminate are considered as shown in Fig. 8. Actually the deformations are the same for all \$dz\$-elements; however, the center element is selected to simplify the integration.

The shear deformation can be divided into two parts. The first part is the shear deformation \$\Delta_{x1}\$ across the filament and the second part \$\Delta_{x2}\$

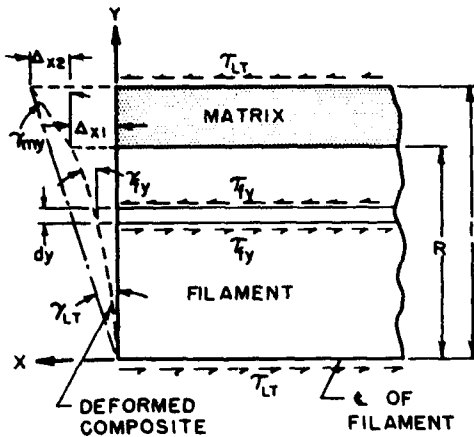


Fig. 8 Shear deformation of typical element shown in Fig. 7

across the matrix. The shear deformation of the filament is given by

$$\Delta x_1 = \int_0^R \gamma_{fy} dy = \int_0^R \frac{\tau_{fy}}{G_f} dy \quad (20)$$

Substituting the value for τ_{fy} from equation (19) and changing the variables from dy to $d\theta$ gives

$$\Delta x_1 = \frac{\tau_{LT} R}{G_f} \int_0^{\pi/2} \frac{\sin \theta d\theta}{\frac{G_m}{G_f} + \sin \theta (1 - \frac{G_m}{G_f})} \quad (21)$$

For each particular combination of materials the value of the integral could be evaluated. For steel-wire filaments and epoxy-resin matrix, the value of the integral was found to be 1.52. Let the integral in equation (21) equal to Φ , then

$$\Delta x_1 = \frac{\tau_{LT} R \Phi}{G_f} \quad (22)$$

The shear deformation of the matrix can be found in a similar manner. However, it is not necessary to integrate since the shear stress is constant and equal to τ_{LT} . The shear deformation across the matrix then is

$$\Delta x_2 = \frac{\tau_{LT}}{G_m} (1 - R) \quad (23)$$

The shear strain is the sum of the two deformations or

$$\gamma_{LT} = \frac{\Delta x_1 + \Delta x_2}{1} = \frac{\tau_{LT} R \Phi}{G_f} + \frac{\tau_{LT} (1 - R)}{G_m} \quad (24)$$

By rearranging terms in equation (24) the shear stiffness of the composite is

$$G_{LT} = \frac{G_m G_f}{R \Phi G_m + (1 - R) G_f} \quad (25)$$

Comparing equation (25) with equation (16)

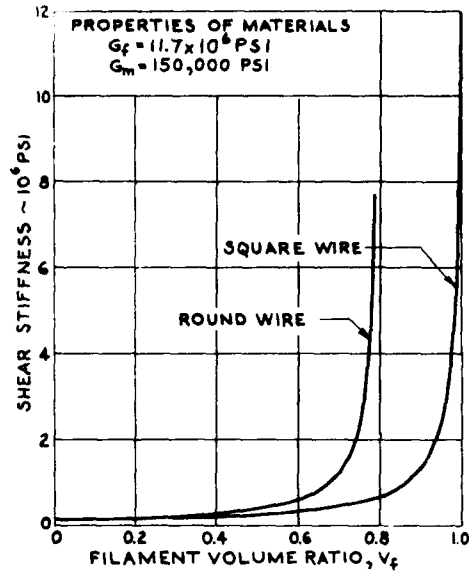


Fig. 9 Shear stiffness comparisons for steel-epoxy composites

for square filaments it can be seen that the round filaments can be considered as square filaments with an equivalent filament volume ratio V_f equal to $R \Phi$ and a matrix volume ratio equal to $(1 - R)$.

To illustrate the effect of filament shape on the shear stiffness, equations (16) and (25) are plotted in Fig. 9 for steel wire-epoxy resin laminates. This shows that for a given filament volume ratio, the round wire laminates are stiffer than square wire laminates especially for filament volume ratios above 0.70.

In practice it would be difficult to obtain filament volume ratios close to one for the square wire composites. It also would probably be undesirable, because then there would be very little matrix material available to distribute the loads within the composite. However, a filament volume ratio close to the theoretical maximum value of 0.785 for round wire composites would not be too difficult to obtain, and there would be sufficient matrix material available to facilitate load distribution. Therefore to obtain a laminate with a high shear stiffness, the round cross section would be the better choice for the filament shape.

Corrections for Laminate Thickness

The equations derived in the previous sections were based on the assumption that the layers were one filament diameter thick. This is usually not the case; however, the previous equations can still be used and the calculations modified to account for any thickness differences.

The equations for determining the longitudinal properties do not need to be modified as they

are not a function of the filament spacing. The transverse properties, on the other hand, are a function of R and variations in laminate thickness must be accounted for. If the layer is thicker than one filament diameter, the layer can be considered as a two-layer laminate. One layer is assumed to be one filament diameter thick with the elastic properties determined by the appropriate equation for E_T . The other layer would be the extra thickness with the properties of the matrix. The transverse stiffness of the combination would simply be

$$E_T^* = \frac{D}{t} E_T + \frac{(t-D)E_m}{t} \quad (26)$$

For laminates with several adjacent transverse layers, one layer may overlap another so that the average thickness of each layer is less than one filament diameter. Across the thickness that two layers overlap the stiffness would be higher than given by equation (6) because the filaments from both layers contribute to the stiffness. The stiffness could be calculated by determining the stiffness of elements dz wide across the laminate from the center of one layer to the center of the next layer. The average stiffness would then be determined as before by integration. This, however, is quite a lengthy computation for a small correction. A simpler method was found to account for this effect which gave approximately the same results as by the more laborious and lengthy method discussed above. The equations derived in the previous sections for the transverse stiffness are used, but the radius ratio R is modified to account for the increase in stiffness. The radius ratio to use for the calculations is determined by assuming that the laminate is composed of layers one filament diameter thick and spaced so that the filament volume ratio of the typical unit in Fig.4 is equal to the filament volume ratio of the laminate. Based on this assumption the radius ratio is given by

$$R = \frac{hV_f}{\pi t} \quad (27)$$

Poisson's ratio μ_{TL} in the transverse direction is determined as before from equation (1), after having found E_L , E_T , and μ_{LT} .

The shear stiffness of layers that are thicker than one filament diameter can be modified in the same way as the transverse properties. In this case the equation for the shear stiffness is

$$G_{LT}^* = \frac{D}{t} G_{LT} + \frac{(t-D)}{t} G_m \quad (28)$$

The method of modifying the radius ratio does not apply to the shear stiffness for laminates with overlapped layers. The shear stiffness is

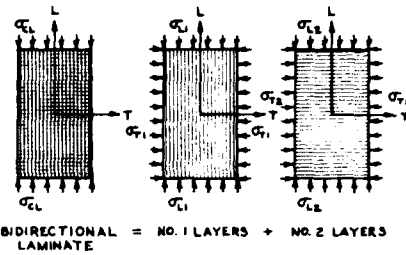


Fig.10 Equivalent composite for analyzing bidirectional laminate

much more sensitive to variations in R than the transverse modulus, and by using a modified R as given by equation (27) gives an unconservative value for G_{LT}^* . For the laminates made in this program the following procedure was used for calculating the shear stiffness. It was assumed that the stiffness of the layer across the thickness that does not overlap with an adjacent layer was equal to that given by equation (25). In the overlapped area it was assumed that each layer contributed to the stiffness so that the total stiffness was twice as much. The shear stiffness then is given by

$$G_{LT}^* = \frac{t G_{LT} + (D-t)G_{LT}}{t} \quad (29)$$

Simplifying equation (29) gives

$$G_{LT}^* = \frac{D}{t} G_{LT} \quad (30)$$

This method of correcting for overlapped layers is probably on the conservative side, but should be sufficiently accurate for most engineering calculations providing the degree of overlap is not too great.

BIDIRECTIONAL LAMINATES

Knowing the properties of each layer, the properties of the laminate can be determined by considering each layer as a homogeneous material. Methods for determining the properties of laminates are discussed quite thoroughly in (1,2,3). In this section the equations derived for evaluating the elastic properties of the bidirectional laminates made in this test program will be presented. The bidirectional laminates considered in this section are those which are made by cross laminating unidirectional layers orthotropically. It is assumed that each unidirectional layer has the same properties in the principal directions, however, some layers will have the filaments aligned in the L-direction and some of the filaments will be aligned in the T-direction. The properties of the bidirectional laminate will then

TABLE 1. DIMENSIONAL DATA FOR BIDIRECTIONAL LAMINATES

Specimen No.	Thickness In.	Density lb/in. ³	Radius Ratio, R	Filament Volume Ratio, V _f	Layup Configuration 0 Transv. Layer - 1 Long. Layer
1-1	0.0576	1.136	0.946	0.639	0101110010100111010
1-2	0.0665	0.260	0.947	0.652	
2-1	62	0.225	0.941	0.759	1010001110100111010
2-2	0.3572	0.222	0.942	0.747	
3-1	0.0571	0.221	0.938	0.745	01100111111111100110
3-2	0.0558	0.225	0.938	0.760	
4-1	0.0565	0.225	0.943	0.758	00111111111111111100
4-2	0.0559	0.226	0.942	0.764	

be determined in terms of the properties of the unidirectional layers.

A bidirectional laminate can be considered to consist of two layers, one layer with the fibers aligned in the direction of the load and the other layer with the fibers aligned in a direction perpendicular to the load as shown in Fig. 10. Each layer is assumed to be a thickness equal to the sum of the thicknesses of the individual layers of the unidirectional laminates regardless of the order of layup. Because of the difference in Poisson's ratio between the two layers, some stresses will be developed in the T-direction.

The equations for the bidirectional laminates can be determined by equating the loads and strains in the L and T-directions. These equations are

$$\begin{aligned}
 \sigma_{cL} A_c &= \sigma_{L1} A_1 + \sigma_{L2} A_2 \\
 0 &= \sigma_{T1} A_1 + \sigma_{T2} A_2 \\
 \epsilon_{L1} &= \frac{\sigma_{L1}}{E_{L1}} - \frac{\mu_{TL1} \sigma_{T1}}{E_{T1}} \\
 \epsilon_{L2} &= \frac{\sigma_{L2}}{E_{L2}} - \frac{\mu_{TL2} \sigma_{T2}}{E_{T2}} \\
 \epsilon_{T1} &= \frac{\sigma_{T1}}{E_{T1}} - \frac{\mu_{LT1} \sigma_{L1}}{E_{L1}} \\
 \epsilon_{T2} &= \frac{\sigma_{T2}}{E_{T2}} - \frac{\mu_{LT2} \sigma_{L2}}{E_{L2}}
 \end{aligned} \tag{31}$$

Two additional equations can be obtained by noting that the strains in the two layers must be the same in both directions or

$$\begin{aligned}
 \epsilon_{L1} &= \epsilon_{L2} = \epsilon_{cL} \\
 \epsilon_{T1} &= \epsilon_{T2} = \epsilon_{cT}
 \end{aligned} \tag{32}$$

The foregoing eight equations can be solved simul-

taneously for the stresses and strains in the laminate. To simplify equations (31), the following relationships can be substituted

$$\begin{aligned}
 \frac{\mu_{LT1}}{E_{L1}} &= \frac{\mu_{LT2}}{E_{L2}} = \frac{\mu_{TL1}}{E_{T1}} = \frac{\mu_{TL2}}{E_{T2}} \\
 E_{L1} &= E_{L2} \\
 E_{T1} &= E_{T2}
 \end{aligned} \tag{33}$$

Solution of the foregoing equations for the axial stiffness and Poisson's ratio gives

$$\frac{E_{T1}}{E_{cL}} = \frac{\left(\frac{E_{L1}}{E_{T1}} v_2 + v_1 \right) - \mu_{LT1}^2 - \mu_{LT1} \mu_{TL1} \left[1 - \left(v_2 + \frac{E_{L1}}{E_{T1}} v_1 \right) \right]}{\left(\frac{E_{L1}}{E_{T1}} v_1 + v_2 \right) \left(\frac{E_{L1}}{E_{T1}} v_2 + v_1 \right) - \mu_{LT1}^2} \tag{34}$$

$$\frac{\mu_{cLT}}{\mu_{cLT}} = \frac{\left(\frac{E_{L1}}{E_{T1}} v_2 + v_1 \right) - \mu_{LT1}^2 - \mu_{LT1} \mu_{TL1} \left[1 - \left(v_2 + \frac{E_{L1}}{E_{T1}} v_1 \right) \right]}{1 - \mu_{LT1} \mu_{TL1}} \tag{35}$$

The terms containing products of Poisson's ratio can usually be neglected for most laminates. In these cases equations (34) and (35) simplify to

$$E_{cL} = v_1 E_{L1} + v_2 E_{L2} \tag{36}$$

$$\mu_{cLT} = \frac{\mu_{LT1} E_{T1}}{v_1 E_{T1} + v_2 E_{T2}} \tag{37}$$

For a multilayered laminate equation (36) can be generalized to give

$$E_{cL} = \sum_{n=1}^{n=N} v_n E_{Ln} \tag{38}$$

The equations for the properties of the bidirectional laminate in the T direction can be

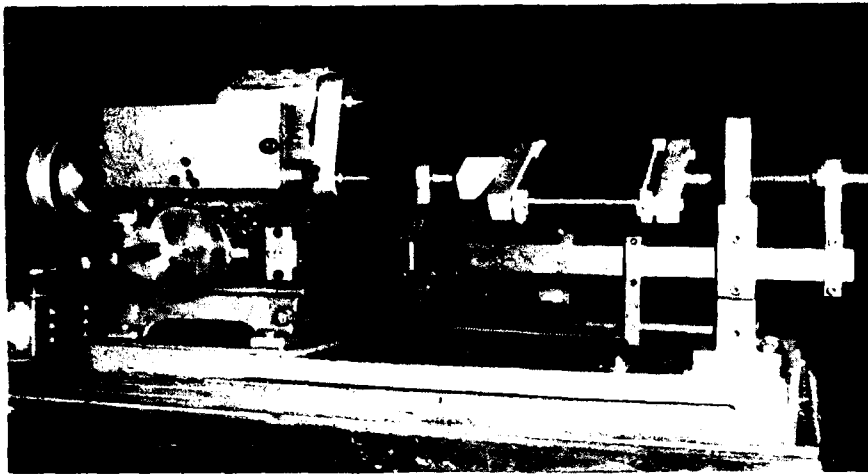


Fig. 11 Universal coil winding machine with bidirectional mandrel in place

written directly from the equations for the properties in the L-direction. By rotating the laminate the volume ratio V_2 is in the direction of the applied load instead of V_1 . Therefore, the elastic stiffness E_{cT} and Poisson's ratio ν_{cTL} are given by equations (34) through (38) with V_2 and V_1 interchanged.

Since the shear stiffness for a unidirectional laminate is independent of the direction of the filaments, the shear stiffness for a bidirectional laminate is the same as the shear stiffness of the unidirectional layers it is made from. If the layers have different shear stiffnesses, the shear stiffness can be determined by considering the laminate to be made up of homogeneous layers. In this case the shear strain of each layer must be the same so that the amount each layer contributes to the laminate stiffness is the ratio of the layer thickness to the total thickness times the shear stiffness of the layer. For a multilayered laminate, then, the shear stiffness is given by

$$G_{cLT} = \sum_{n=1}^{n=N} \frac{t_n}{t_c} G_{LTn} \quad (39)$$

TEST SPECIMENS

To verify the equations developed in the previous sections, some test specimens were made using 0.00305-in-diam music wire for the filament material and epoxy resin for the matrix material. This combination of materials was selected because of their availability, reliability of properties, and good bonding characteristics. It was also desirable to have a large difference between the stiffness of the two materials so that the stiff-

ness of the laminates would be sensitive to small changes in the filament spacing.

The laminates were made by winding the wire over a flat mandrel on a Stevens universal coil-winding machine as shown in Fig.11. This machine advances the mandrel at a predetermined rate so that a uniform layer of closely spaced wire is obtained. The wire spacing can be varied by changes in the gear train. A 10:1 reductor was installed between the motor and the coil winding machine so that winding speeds could be reduced to approximately 1-3 rps.

The mandrel shown in Fig.11 was designed for making bidirectional laminates by providing means for rotating the mandrel orthogonally. The cam-operated mechanism could advance the mandrel up to 2 in. By inserting a spacer block between the mandrel and the chuck, an additional 1.5 in. of advance could be obtained. With this mandrel, thin flat laminates up to $3\frac{1}{2}$ in. square could be made.

A schematic arrangement of the winding process is shown in Fig.12. The wire spool was mounted in the universal unrolling tensioner and the wire passed over two pulleys which maintained a constant tension on the wire. The unrolling tensioner was adjusted to maintain a tension of approximately 90 grams, which is about 8 per cent of the ultimate strength of the wire. The wire was cleaned by passing it through two baths which were ultrasonically vibrated. From the second bath the wire passed over the stationary wire guide assembly and then onto the mandrel. Resin was applied to the mandrel during the winding process. The fluidity of the resin was maintained by the radiant-heat lamp mounted directly over the mandrel. After the winding process was complete, the mandrel was pressed at 50 psi for 2 hr at a

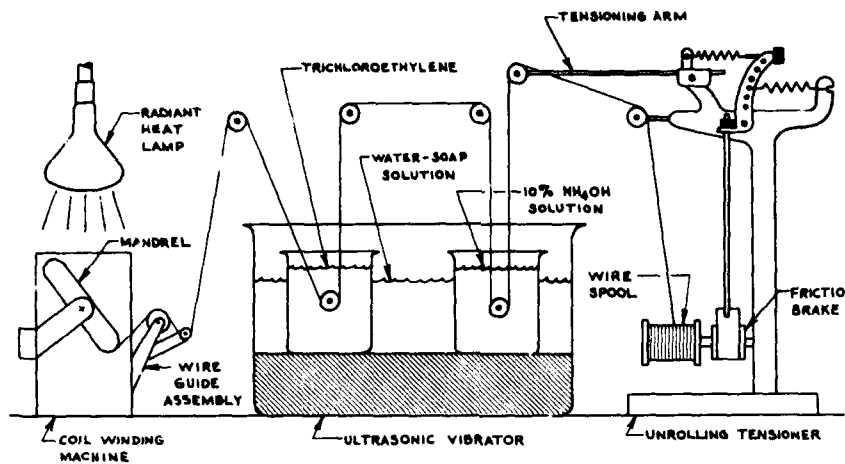


Fig. 12 Schematic diagram of winding process

TABLE 2 MATERIAL PROPERTIES

Material Property	Musis Wire	Epoxy Resin
Modulus of elasticity, psi	30×10^6	405,000
Poisson's Ratio	0.28	0.34
Shear modulus of elasticity, psi	11.7×10^6	150,000
Density, lbs./in. ³	0.283	0.043

temperature of 285 F and then finish-cured in an oven at 285 F for 14 hr. After the finish cure, a 3.5 x 3.5-in. laminate was cut from each side of the mandrel.

Four bidirectional specimens (eight 3.5 x 3.5-in. laminates) were made with different layup configurations. The dimensional data for the eight laminates are shown in Table 1. The layers of the specimens were arranged so that the laminates would have approximately the same bending stiffness in both directions but with various percentages of filaments in the longitudinal and transverse directions. Specimen No. 1 was thicker than 2, 3 or 4 because the wire in some of the layers crossed over each other during the winding process and thus the laminate could not be compacted as much. Special precautions were taken with Specimens 2, 3 and 4 to prevent wire cross-overs. The two laminates of each configuration were not of the same thickness as the wires compacted more on one side of the mandrel than on the other.

It was necessary to measure the thickness quite accurately in order to get good results, particularly for the bending-stiffness and shear-stiffness calculations, since the cube of the thickness enters the equations. The "miked"

thickness was not accurate enough because of the variation in thickness due to the unevenness of the surface. Therefore, specimens were cut from the laminates and weighed in both air and water. Then the thickness was determined based on the length and width measurements and the calculated volume of the displaced water.

TEST PROCEDURE

Twisting Tests

To determine the shear stiffness, a twisting test was conducted on each laminate as it was cut from the mandrel. The test setup for the twisting tests is shown in Fig. 13. Small aluminum pads were bonded to the corners of the laminate so that the loads could be applied directly at the corners. The head deflection was measured with a microdeflectometer and recorded on an autograph recorder. The specimens were loaded continuously up to about 10 lb and unloaded. In order to get a linear-load-deflection curve it was necessary to load the specimens twice. The slope obtained from the second load-deflection curve was used in the analysis. After the first series of tests, the laminates were rotated 90 deg and retested.

In reference (6) it was shown that the twisting stiffness can be determined from the twist test by the following equation:

$$D_{xy} = \frac{P c^2 d^2}{4 \Delta_H (bc + ad)} \quad (40)$$

For these tests the distance between the support points was equal to 3.49 inches. Also the relation between the twisting stiffness and shear stiffness for a thin plate is given by

$$D_{xy} = 2G_{LT}I_{xy} = \frac{G_{LT}t^3}{6} \quad (41)$$

TABLE 3 CALCULATED ELASTIC PROPERTIES
OF
INDIVIDUAL LAYERS OF SPECIMENS

Specimen	E_L 10 ⁶ psi	E_T 10 ⁶ psi	μ_{LT}	μ_{TL}	G_{LT}
1-1	19.3	2.90	0.302	0.045	1.79
1-2	19.7	2.90	0.301	0.044	1.86
2-1	22.9	3.62	0.294	0.046	2.04
2-2	22.5	3.08	0.295	0.040	2.00
3-1	22.4	2.93	0.296	0.039	1.90
3-2	22.9	3.65	0.284	0.045	1.94
4-1	23.7	3.57	0.294	0.044	2.05
4-2	23.0	3.80	0.294	0.049	2.05

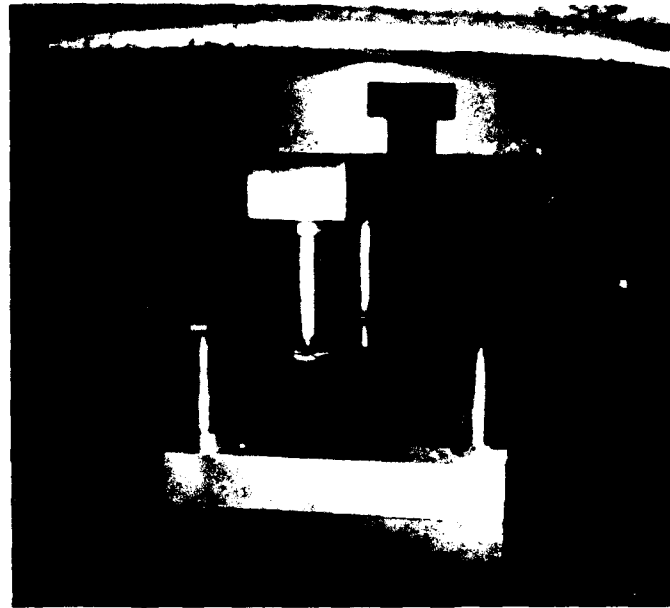


Fig. 13 Test setup for twisting tests

Making the above substitutions in equation (40), the shear rigidity is given by

$$G_{LT} = \frac{63.6 P}{\Delta H^3 c^3 (a+b)} \quad (42)$$

The shear stiffness of the laminates was evaluated by the foregoing equation.

Compression Tests

After the twisting tests were completed, each laminate was cut up into four specimens approximately 3/4 in. by 3.5 in. long. Two of the specimens were tested in compression, the other two specimens were tested in bending. The specimens from the two laminates with the same layup configuration were cut in opposite directions so that both longitudinal and transverse properties could be determined. The ends of the compression coupons were reinforced with 1/32-in. micarta sheet to prevent delamination of the filaments.

The compression coupons were tested in a compression jig which was supported by side blocks to

prevent the specimens from buckling. The space between micarta reinforcing sheets had to be shimmed to make continuous contact. The longitudinal strain was measured with a 2-in-gage-length extensometer that attached to the specimen through holes in the side blocks. SR-4 strain gages were mounted on some of the specimens on the side opposite the extensometer for the purpose of measuring the transverse strain.

Bending Tests

The bending coupons were tested according to Federal Specification L-P-406b. The center deflection was measured on a 1-in. span with about a

TABLE 4 ELASTIC PROPERTY COMPARISONS

Direction Bending and Compression Coupons were Tested	Specimen	Modulus of Elasticity, 10 ⁶ psi		Poisson's Ratio		Shear Modulus of Elasticity, 10 ⁶ psi		Bending Modulus of Elasticity, 10 ⁶ psi	
		Calc.	Test	Calc.	Test	Calc.	Test	Calc.	Test
Transverse	1-1	10.8	11.8 10.1	0.075	0.093 0.073	1.79	1.52 1.56	11.1	11.5 14.0
Longitudinal	1-2	11.7	11.4 11.3	0.074		1.86	1.62 1.59	11.2	8.9 9.1
Transverse	2-1	12.8	15.5 13.1	0.078	0.117	2.04	1.86 2.07	13.2	12.5 12.4
Longitudinal	2-2	13.3	14.8 13.9	0.074	0.096	2.00	1.82 1.82	11.8	11.7 11.3
Transverse	3-1	9.1	7.2 9.2	0.054	0.073	1.90	1.86 1.94	12.6	12.6 11.7
Longitudinal	3-2	16.9	19.1 17.1	0.107	0.165	1.94	1.98 2.40	13.3	13.1 12.8
Transverse	4-1	7.8	5.0 9.6	0.054	0.014	2.05	2.16 2.39	13.8	10.9 6.9
Longitudinal	4-2	19.0	20.8 18.1	0.143		2.05	2.29 2.58	13.2	12.3 11.6

1¹/₄-in. overhang. The deflection was measured with a microdeflectometer and recorded on an autograph recorder. The slope of the load-deflection curve was used to calculate the bending stiffness.

COMPARISON BETWEEN TEST DATA AND THEORY

The elastic properties of each laminate were calculated according to the equations developed in the previous sections. The material properties used in the calculation are given in Table 2. The properties for music wire were taken from reference and the epoxy-resin properties were determined by tests.

First the elastic properties of the individual layers were determined. These calculated values are tabulated in Table 3. Using the values from Table 3, the laminate properties were determined as described in the section on bidirectional laminates. These values are summarized in Table 4. The modulus of elasticity in bending was calculated by methods described in (1, 2) and neglecting the effect of shear deformations on the stiffness. In the sections that follow the comparison and accuracy of the results will be discussed.

Axial Stiffness and Poisson's Ratio

The values for the elastic stiffness agree very well with the theoretical values. Specimens 4-1 show the largest disagreement. The two outside layers of this specimen were loose when it was made and, therefore, the compression coupons may not have loaded up properly. The average of the test values and the calculated values agree within 11 per cent which is within the experimental accuracy.

The agreement between the values for Poisson's ratio is not as good. This is probably due to curvature of the specimens. The laminates had some curvature when they were taken off the mandrel. The side load applied to the specimens tended to straighten the specimens but no specimen was perfectly straight. The bending stresses induced by curvature in the specimens would give high strain readings on one side of the specimen and low readings on the other side. Because Poisson's ratio was determined from strains measured on opposite sides of the specimen, the error due to this effect would be twice as large as the discrepancy in the stiffness values. This can be seen by comparing the stiffness values and Poisson's ratio values for each specimen. With the exception of Specimens 3-1, the discrepancy in Poisson's ratio is in the same direction as the discrepancy in the stiffness values and more than twice as large. To get more accurate results the

strain should be read on both sides of the specimen.

Shear Stiffness

The shear stiffness is probably the most difficult property to measure accurately. For one reason, any errors in thickness measurements would be greatly magnified because the thickness is cubed in equation (42). Also because of the small size of the specimens and the low loads used in the tests the error in measurements were significant. The error in measurements can be estimated by comparing the values for successive tests for the same specimen. Variation in tests results was as high as 20 per cent for some specimens. If the error due to thickness measurements is included, the difference between calculated and theoretical results could be as high as 30 per cent. All the results fell within this degree of accuracy, the greatest difference being 26 per cent for Specimen 4-2.

The calculated values for the shear stiffness were based on the measured values for the radius ratio. For these specimens, the calculated values of the shear stiffness are quite sensitive to small variations in R. For a radius ratio of 0.94, a deviation of ± 1 per cent in R gives a ± 14 per cent deviation in the value for the shear stiffness. Since the value for R is no more accurate than 1 per cent, the calculated values agree with the experimental values within the degree of accuracy of the measurements.

Comparison of the results also indicates that the shear stiffness is quite sensitive to variations in layer thickness. The best agreement between test values and theory was for the laminates with the least amount of correction. This indicates that a more accurate method of accounting for variations in laminate thickness should be developed.

Bending Stiffness

In the calculations the shear deformations were neglected, therefore the calculated values should be higher in general than the test values as was the case. The largest discrepancy was for Specimens 1-2 and 4-1. Both these specimens would be expected to have large shear deformations; Specimen 1-2 because of the high resin content and Specimen 4-1 because the majority of the layers were aligned transverse to the bending direction. The shear stiffness of layers aligned transverse to the load would be much lower than longitudinal layers.

Except for Specimens 1-2 and 4-1, the agreement between test data and theory was within 10 per cent or less. This is a good check on the

theory since the bending test is less subject to testing inaccuracies than either the compression test or the twisting test.

CONCLUSIONS

Based on the test results of this program the following conclusions are made:

1 The methods for calculating the elastic properties E_L , E_T , μ_{LT} , μ_{TL} , and G_{LT} agree with the test results within the experimental accuracy of the measurements. These equations should be applicable for calculating the elastic properties of any two combinations of materials.

2 The shear stiffness was quite sensitive to variation in laminate thickness. Some better method of calculating the shear stiffness of layers that are thicker or thinner than one filament diameter thick should be developed.

3 The shear deflections of the laminates must be included in the calculations for the bending stiffness if there is an appreciable amount of resin between individual layers or if the majority of the layers are aligned transverse to the direction of bending. Some means of calculating the shear stiffness of layers with filaments aligned transverse to the direction of bending should be developed.

ACKNOWLEDGMENTS

Sincere appreciation is expressed to Mr. Jack Rebman for checking the equations, to Mr. E. L.

Giachino for making the laminates, and to personnel of Lockheed Aircraft Corporation in the Electrical Instrumentation Group and Physical Test Laboratory for instrumenting the specimens and conducting the tests. I also wish to thank Mr. Warren Gilmour for his assistance in developing the techniques for making the test specimens.

References

- 1 N. J. Hoff, "Engineering Laminates," chapter 1, edited by A. G. H. Dietz, John Wiley and Sons, Inc., New York, N. Y., 1949.
- 2 Wilhelm Thielman, "Contribution to the Problem of Buckling of Orthotropic Plates, with Special Reference to Plywood," NACA TM 1263, 1950.
- 3 A. G. H. Dietz, "Design Theory of Reinforced Plastics," chapter 9 of Fiberglass Reinforced Plastics by Ralph H. Sonneborn, Reinhold Publishing Corp., New York, N. Y., 1954.
- 4 J. R. Tinklepaugh, B. R. Gross, J. J. Swica and W. R. Haskyns, "Metallic Fiber Reinforced Ceramics," Progress Report No. 6, March 1, 1959 to May 31, 1959, State University of New York, College of Ceramics at Alfred University, Alfred, N. Y.
- 5 J. C. Ekvall, "Determination of the Elastic Properties of Orthotropic Laminates in the Principal Directions," SRM 442, Lockheed Aircraft Corp., Burbank, Calif., March 28, 1960.
- 6 R. S. Jusko and J. C. Ekvall, "Stiffness Parameters of 17-7PH (RH950) Unicore Panels," SRM 418, Lockheed Aircraft Corporation, Burbank, Calif., August 22, 1958.