



# INTEGRATION OF A CONSTITUTIVE EQUATION BASED FINITE ELEMENT FOR COMPOSITE MATERIALS INTO A COMMERCIAL FINITE ELEMENT SOFTWARE CODE

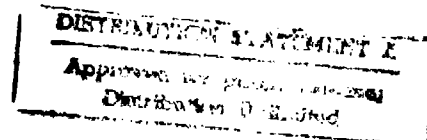
**FINAL REPORT**  
SwRI Project No. 06-7171

**Prepared by:**  
**Mark Jones**

**Southwest Research Institute**

**April 1996**

**Sponsoring Organization:**  
**The Advanced Research Projects Agency**



**96-02475**



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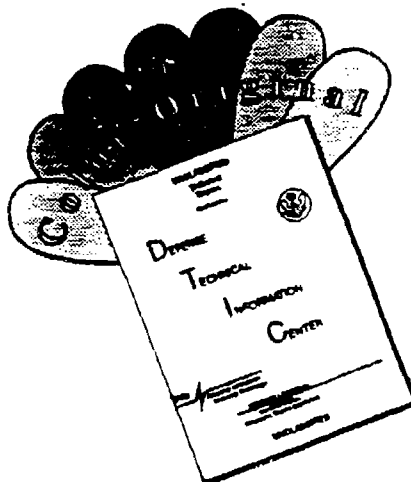


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# INTEGRATION OF A CONSTITUTIVE EQUATION BASED FINITE ELEMENT FOR COMPOSITE MATERIALS INTO A COMMERCIAL FINITE ELEMENT SOFTWARE CODE

## FINAL REPORT

SwRI Project No. 06-7171

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April 1996

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The Contractor, Southwest Research Institute, hereby certifies that, to the best of its knowledge and belief, the technical data delivered herewith under Contract No. N00167-95-C-0064 is complete, accurate, and complies with all requirements of the contract.

April 3, 1996  
Date

  
Geoffrey Dearnaley, Vice President  
Materials and Structures Division

## EXECUTIVE SUMMARY

This report discusses the integration of a novel finite element for analyzing composite material structures into a commercially available finite element software package. This element is a constitutive equation based formulation that predicts individual layer strains and stresses in a computationally efficient manner. Implementing this element into a commercial code will improve its efficiency for predicting the structural response of composite material structures.

The work discussed in this report involved utilizing algorithms previously developed at Southwest Research Institute (SwRI) that are based on the constitutive equations and integrating them with the ANSYS finite element code. An example problem using this method is presented that predicts the laminate free edge stress distribution that is important for examining potential delaminations at design features such as holes, cutouts, and joints.

The results of this study show that the element is accurate for determining the stress distribution at the laminate free edge. Recommendations for enhancements to the element are made to improve its modeling efficiency even further.

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## 1.0 INTRODUCTION

This report discusses the integration of a novel finite element for analyzing composite material structures into a commercial finite element software code. The work described within was Task 3.0 of the U.S. Government contract N00167-95-C-0064, and is intended as a stand-alone document. The government report numbered CD/NSWC MSSPO (102) CR-95/02 and entitled "Development of Large, Thick Laminate Structures for Offshore Applications", discusses the rest of the tasks associated with this program.

The thrust of this effort was to develop a computationally effective method to analyze the through-thickness stress and strain distribution of thick composite parts as applied to offshore structures. However this method can be used on a variety of composite material applications for structures.

This report is organized into the following sections: background information and the rationale for using this method are presented in Section 2.0. Section 3.0 discusses the technical approach for integrating the constitutive based model into a commercial finite element program. The results of an example problem based on the free edge stress distribution problem are presented in Section 4.0. Section 5.0 discusses the conclusions of using this method. Recommendations for future efforts are discussed in Section 6.0. Appendix A presents the source code listing for integrating the model into the commercial finite element code. Results from the example problem are presented in Appendix B.



## 2.0 BACKGROUND

The application of composite structures for offshore structures will require parts that are thick (relative to typical aerospace structures), and have loads that are applied through the thickness plane of the structure. Particular attention must be paid to potential delamination of the composite material near design details such as holes, cutouts, and both composite-to-composite and steel-to-composite joint regions.

Therefore an analytical method that can determine the structural response of these laminated parts must be used to ensure their soundness when placed into service. Due to the complex geometry and loading conditions acting on this parts, the finite element method must be employed to adequately analyze their structural behavior. Moreover, the method used must be computationally efficient to permit analyzing these structures in a timely and cost-effective manner.

There are many methods available to describe the mechanical response of layered composite materials. Typically these methods are based on classical laminated plate theory with estimates of the bulk mechanical response for thick composite sections. Classical laminate plate theory is very effective in modeling the response of thin laminates when there are no significant loads applied in the thickness direction of the laminate. However, when this is not the case, some other material description must be used to determine the through-thickness mechanical response.

One common approach to describe the through thickness material properties is to replace the layer by layer characteristics with an average set of orthotropic constitutive properties that are used to model many layers of composite. This method is adequate for large sections of composite, but fail to determine the layer-by-layer behavior that is critical information near design details. One means to get around this problem is to employ "Global-Local" modeling, where a very refined layer-by-layer finite model mesh is used around the local design details, and a coarser mesh with effective properties is used for the bulk response of the structure. Two major problems with this method are that the analyst must know beforehand what design features are important for the bulk mechanical response of the structure, and if there are many design features of interest then the refined layer-by-layer mesh portions of the model can grow to the point where computation becomes prohibitively expensive. It is clear that what is needed is a reliable three dimensional material description that can be implemented into a finite element that captures the essential features of a layered composite without the computational expense of a layer-by-layer approach.

There are many methods proposed in the literature for the description of the three dimensional "effective" properties of a laminated composite. However, these methods generally have very complex mathematical descriptions of these properties and require extensive numerical matrix manipulations. These characteristics result in finite element solutions that are computationally prohibitive for modeling composite structures of any great size or detail.

A method that overcomes these obstacles is a generalized averaging procedure for the description of the mechanical properties of a layered composite that was proposed by

Christensen [1]. The main point of this procedure is the separation of the fiber-dominated properties from the matrix dominated properties. The matrix dominated properties are described by two independent constants that are the result of this averaging procedure, and are used in a manner that is similar to Hooke's Law. The fiber dominated properties are then averaged into one constant that provides an additive term to the constitutive formulation in the direction of the fibers in that layer of the composite.

This constitutive equation method was adapted into a displacement-based three dimensional finite element code by Patton [2]. This code, which will be referred to as the SWRI code, is a "university-grade" code, meaning it does not have a pre- and post processor and has limited functionality. In order to utilize this method in a cost-effective manner, it is necessary to implement it into a commercial grade FEA code.

### 3.0 TECHNICAL APPROACH

The technical approach of this effort was to use portions of the SwRI code specific to the single element formulation and integrate it with a commercial finite element program. There are several factors that must be considered before deciding on which software package to use.

The most important characteristic for choosing which finite element code to integrate the constitutive equation based element is that the commercial code have the capability to accept user written elements. Also the present formulation of the constitutive equation based element requires that the finite element program have a displacement based wavefront solver. This is the most common means of solving finite element models, so there are a large number of codes available. Another requirement is that the program preferably have an integrated pre- and post processor, or at least be able to interface with a third-party pre- and post processing program such as PATRAN or IDEAS. A highly desirable requirement is that the program is already in place within the organization and has an experienced user base.

There are a number of finite element codes that meet these requirements. NASTRAN, ANSYS, and ABAQUS among the better known products. There is no one program that is the best choice, the decision being based on cost, familiarity, and of course whether or not if one already possess one of these codes. The best choice for the Marine Technology Department at SwRI was ANSYS 5.1 coded to run a HP 700 series UNIX Workstation. It should be noted that the user element interface capability is not available on the PC-DCS version of this software package.

An interface to the ANSYS code is provided by the use of User Programmable Features (UPFs) [3]. UPFs are a set of FORTRAN routines that allow the user to write custom elements, loading conditions, material properties, and other user specific applications.

This particular application required the use of the user element subroutines. These subroutines provide the interface between the user element and the ANSYS code. The user is responsible for developing the entire element formulation including the load vector, stiffness matrix, and material matrix. The user must also address issues with ANSYS variable passing, input/output switches, and any options for the element.

The portions specific to single element formulations in the SwRI code were used in the ANSYS user routine. Because the SwRI code was developed on a DOS based PC with Microsoft FORTRAN V5, some of the code had to be changed to successfully port it to the HP Workstation. The source code listing for this element is presented in Appendix A.

## 4.0 RESULTS

An example problem was tested to determine the accuracy and utility of implementing the constitutive formulation with ANSYS. The geometry, material properties and boundary conditions for the example problem were identical to the one solved by Pipes and Pagano [4]. This problem is a free edge stress problem that consists of a 4-ply laminate under uniform axial extension. This is one of the most demanding solutions for laminate analyses, in that there are interlaminar stresses at the free edge of the laminate that are not accounted for by classical laminate theory. Additionally, this same problem was solved by the existing SwRI code and compared to the results from the ANSYS model.

### 4.1 Model Description

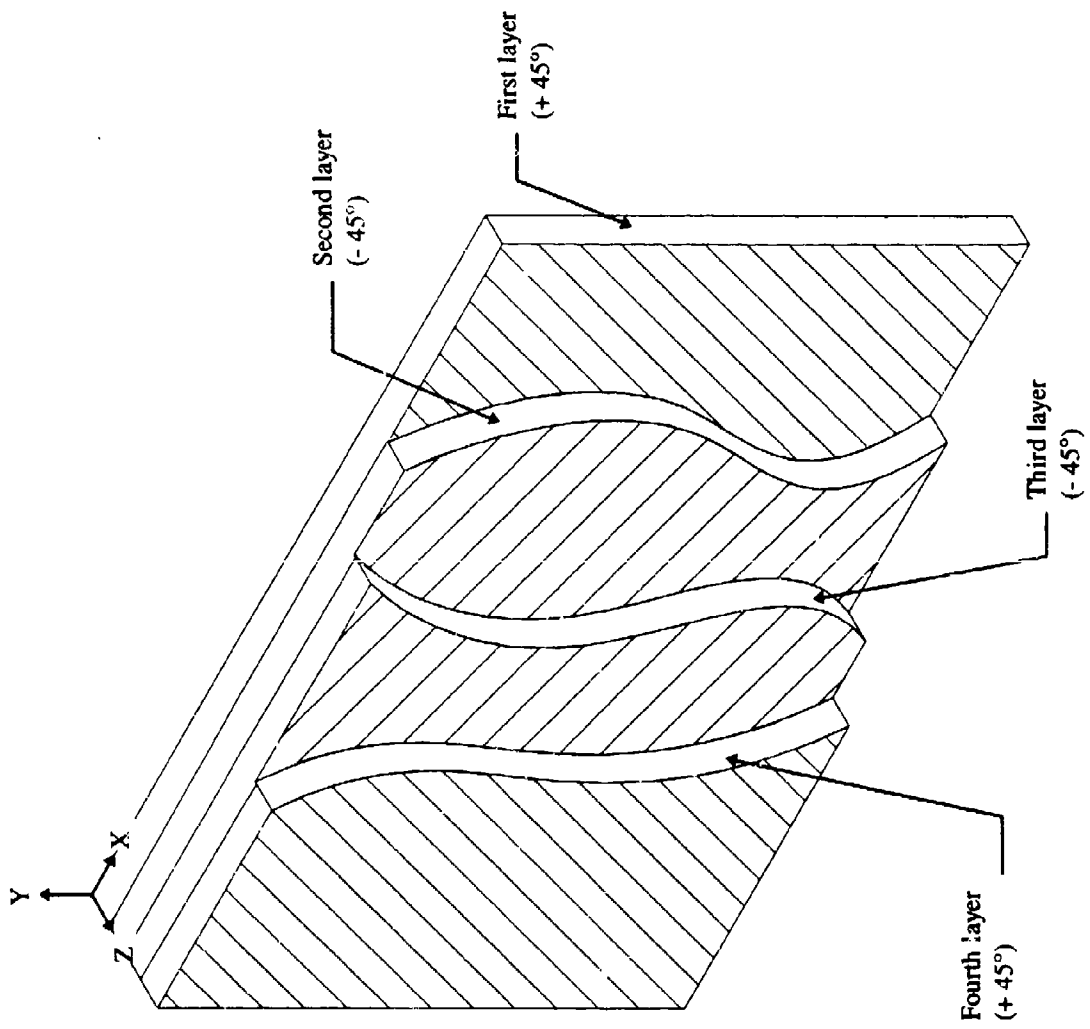
The geometry and corresponding finite element mesh was created in the interactive "mode" of ANSYS, which is a graphical interface. This model is a flat plate that is 1.2 inches long, by 0.8 inches wide by 0.2 inches thick. As shown in Figure 1, it is a +45/-45/+45/-45 composite structure. The finite element model consists of 96 8-noded brick elements and is shown in Figure 2. There are two symmetry planes, one along the x-axis, and one along the y-axis. The loading is a uni-axial extension applied by displacements along one surface of the plate in the x-direction. The value of these displacements was 0.075 inch at each node of the surface.

The material properties for this problem are shown in Table 1. These values are input using the ANSYS MP command via the command line.

TABLE 1- MATERIAL PROPERTIES

| Material Property                                | ANSYS Variable | Value                  |
|--------------------------------------------------|----------------|------------------------|
| Young's Modulus along fiber direction            | EX             | $2.1 \times 10^7$ psi  |
| Young's Modulus perpendicular to fiber direction | EY             | $2.0 \times 10^6$ psi  |
| Poisson's Ratio                                  | NUXY           | 0.21                   |
| Shear Modulus in xy plane                        | GXY            | $0.85 \times 10^6$ psi |
| Shear Modulus in xz plane                        | GXZ            | 0.0                    |
| Density                                          | RHO            | 0.0                    |
| Coefficient of thermal expansion in x direction  | ALPHAX         | 0.0                    |
| Coefficient of thermal expansion in y direction  | ALPHAY         | 0.0                    |
| Coefficient of thermal expansion in z direction  | ALPHAZ         | 0.0                    |

The individual ply layer properties are input using ANSYS real constants. It was found that the parameters for the layers must be entered with two material properties through the lamina



**FIGURE 1 - Ply Orientation of Plate**

ANSYS 5.1 34

MAR 18 1996

14:02:07

ELEMENTS

TYPE NUM

XV =1

YV =2

ZV =3

DIST=0.660916

XF =0.6

YF =-0.4

ZF =0.1

CENTROID HIDDEN

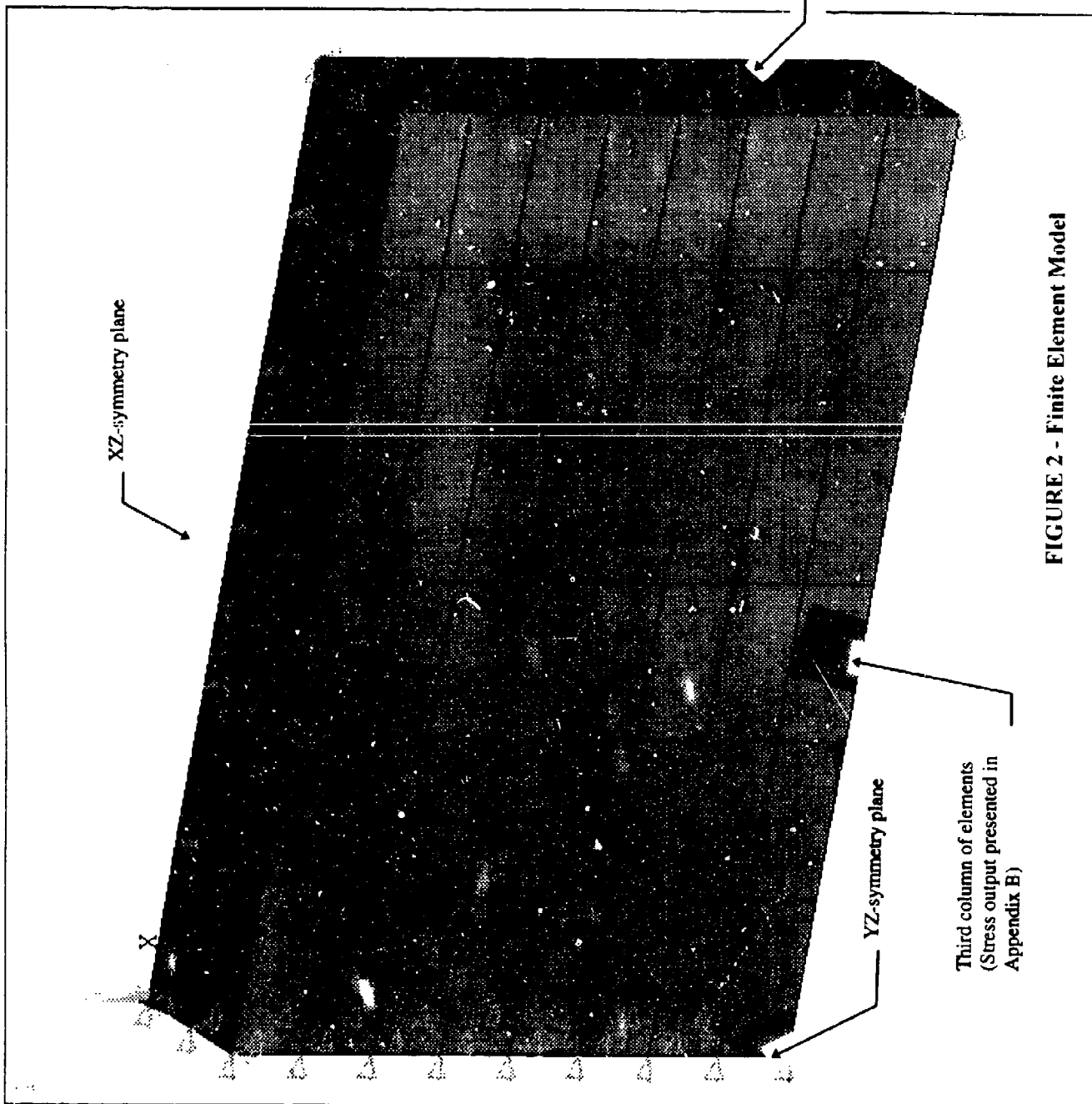


FIGURE 2 - Finite Element Model

thickness. This requires that there are two elements through the thickness, each with a different ANSYS material property ID number. However, the material properties are identical for both material sets. It is suspected that this condition is caused by the formulation of the material matrix in the SwRI code and will eventually need to be rectified. Therefore a separate real constant is required for each set of elements through the laminate thickness.

The real constant data consists of the number of layers through the element, layer material numbers, fiber orientations, and layer thickness. This data is entered using the ANSYS commands R and RMORE via the command line. The format for entering the real constants is based on an existing ANSYS composite element, SOLID 46, and is shown below:

*R,N1,NL,R2,R3,R4,R5,R6*

Where: N1 is the real constant ID set  
NL is the number of layers in this set  
R2 - R6 are not used

*RMORE,R7,R8,R9,R10,R11,R12*

Where: R7 - R12 are not used

*RMORE,R13,R14,R15,R16,R17,R18*

Where: R13 is the material ID for the first layer in this set  
R14 is the fiber direction from element x-axis for first layer in this set  
R15 is the lamina thickness of the first layer in this set  
R16 is the material ID for the second layer in this set  
R17 is the fiber direction from element x-axis for second layer in this set  
R18 is the lamina thickness of the second layer in this set

The command line data entry for this problem is as follows:

(First real constant set, layers 1-2)

*R,1,2,,,,,*  
*RMORE,,,,,*  
*RMORE,1,45,0.05,1,-45,0.05*

(Second real constant set, layers 3-4)

*R,2,2,,,,,*  
*RMORE,,,,,*  
*RMORE,2,-45,0.05,2,45,0.05*

## 4.2 Results

The contour plot for the direction of the axial displacements is shown in Figure 3. The first item to note is how the displacements behave at the xz-symmetry plane. Instead of acting like the adjoining row of elements, the displacements increase at the symmetry plane. This behavior should not happen at a symmetry plane, and can be explained by the fact that a symmetry plane implies isotropy, which is not the case for lamina with fiber orientations. The fiber angles for each layer are mirrored by the boundary conditions as shown in Figure 4, which results in a discontinuity. At this boundary the fiber angles are reversed which is why the displacements are reversed in reference to the free edge at the other side of the plate.

The second feature of the displacement results show difference in displacements between the  $+45^\circ$  and the  $-45^\circ$  layers through the plate thickness. This can be seen in Figure 3, and Figure 5 which shows the contour plot for displacements perpendicular to the load direction. This is expected behavior for the given laminate ply orientations.

The stress output was not incorporated into the ANSYS post-processor due to time constraints. The stress values for each layer is available in tabular form, and is presented in Appendix B for the third column of elements as shown in Figure 2. A graph of the stresses across the plate width for the second lamina layer is shown in Figure 6.

The results of Pipes and Pagano revealed that at the free edge of a laminate the interlaminar stresses ( $\tau_{xz}$ ) grow, while the in-plane shear stresses ( $\tau_{xy}$ ) diminish to zero. Also there is a decrease in the stresses acting along the load direction ( $\sigma_x$ ) at the free edge. The results shown in Figure 6 follow this behavior, but with some discrepancies. The first discrepancy takes place from the xz-symmetry plane to  $y/b=0.3$ . Here the stresses are effected by the presence of the symmetry plane and can be discounted. The second discrepancy occurs near the free edge, in that  $\tau_{xy}$  does not diminish to zero, and that  $\tau_{xz}$  does not increase as much as predicted by Pipes and Pagano. This phenomena can be explained by the fact that an 8-noded element has a linear displacement distribution and can not fully capture the rapidly changing structural response at the free edge. Two methods to capture this response is to use a finer mesh at the free edge and/or use 20-noded brick elements.



ANSYS 5.1 34  
 MAR 18 1996  
 14:14:22  
 NODAL SOLUTION  
 STEP=1  
 SUB =1  
 TIME=1  
 UX  
 RSYS=0  
 DMX =0.083811  
 SMX =0.075

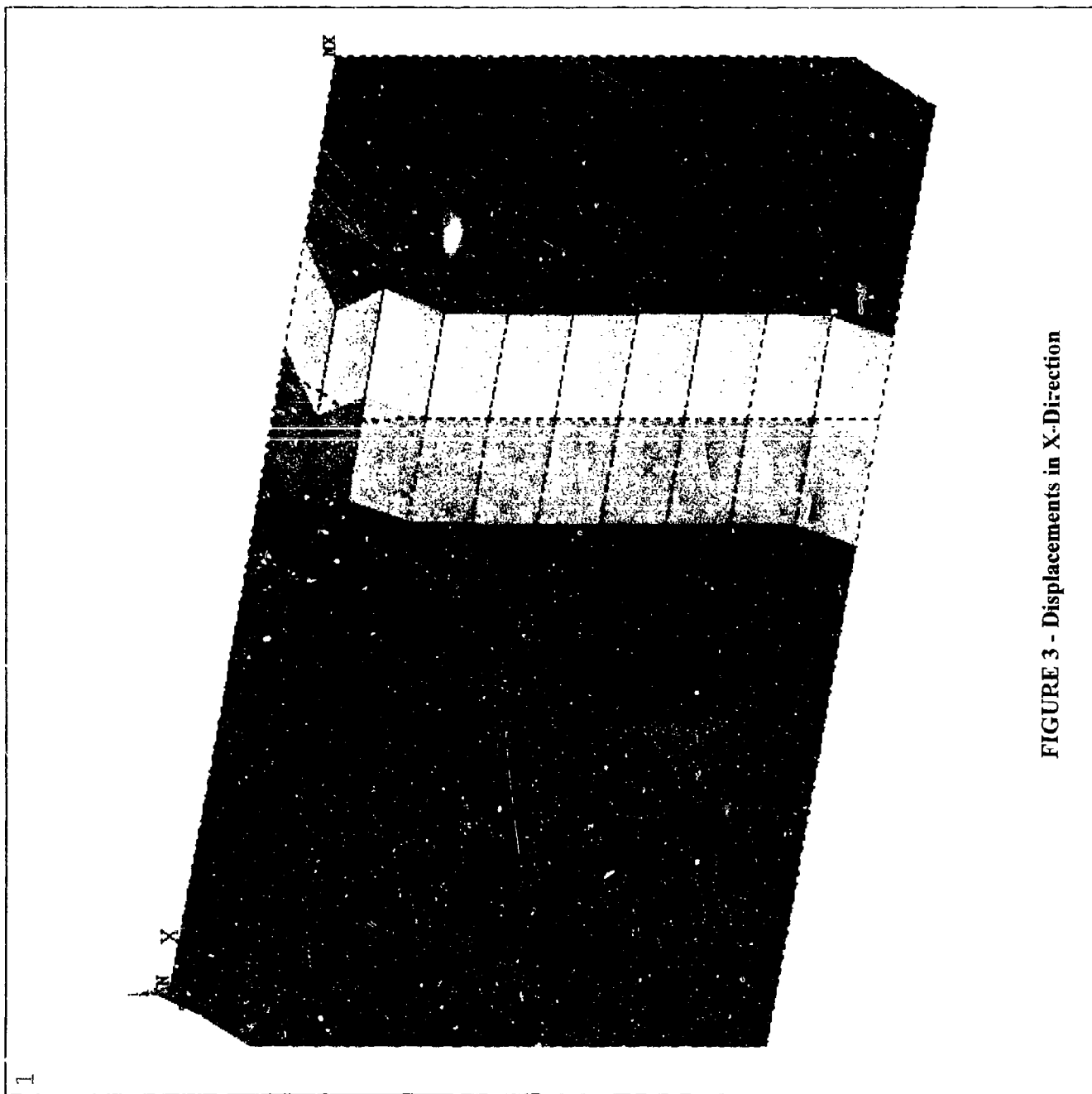
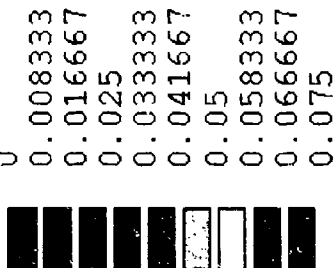
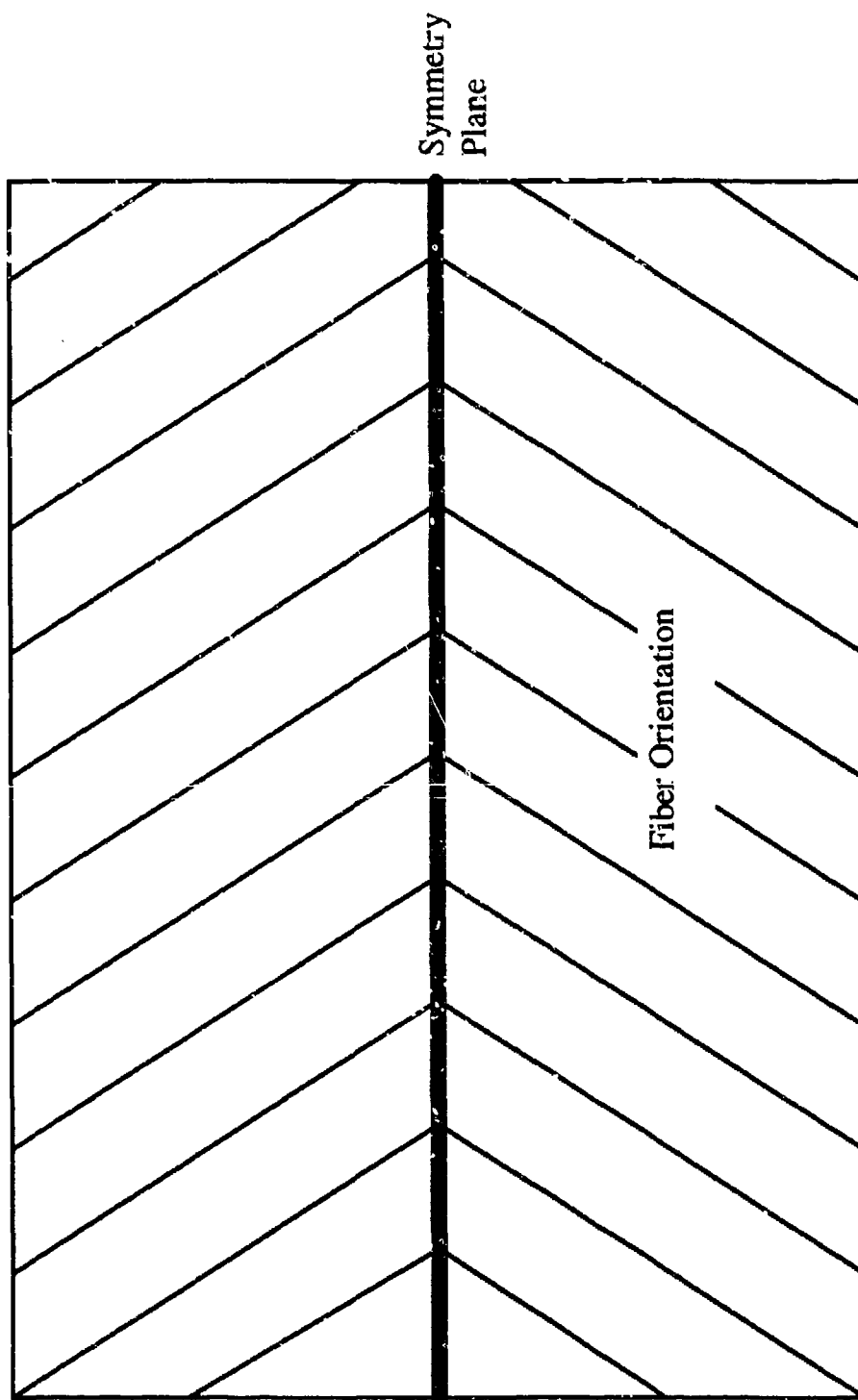


FIGURE 3 - Displacements in X-Direction



**FIGURE 4- Physical Interpretation of Symmetry Plane**

ANSYS 5.1 34

MAR 18 1996

14:20:24

NODAL SOLUTION

STEP=1

SUB =1

TIME=1

UY

RSYS=0

DMX =0.083811

SMX =0.037433

0

0.004159

0.008319

0.012478

0.016637

0.020796

0.024956

0.029115

0.033274

0.037433

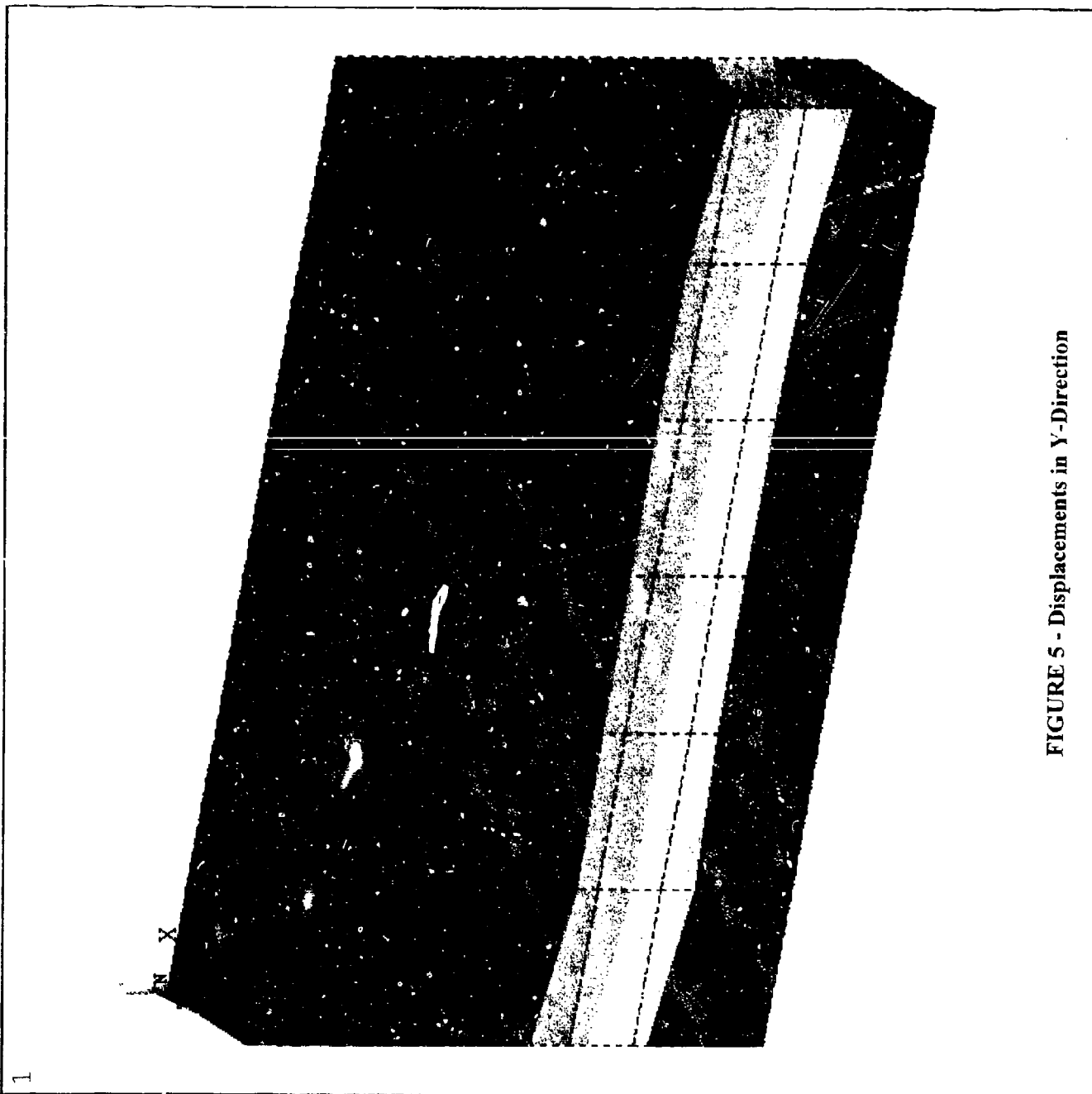


FIGURE 5 - Displacements in Y-Direction

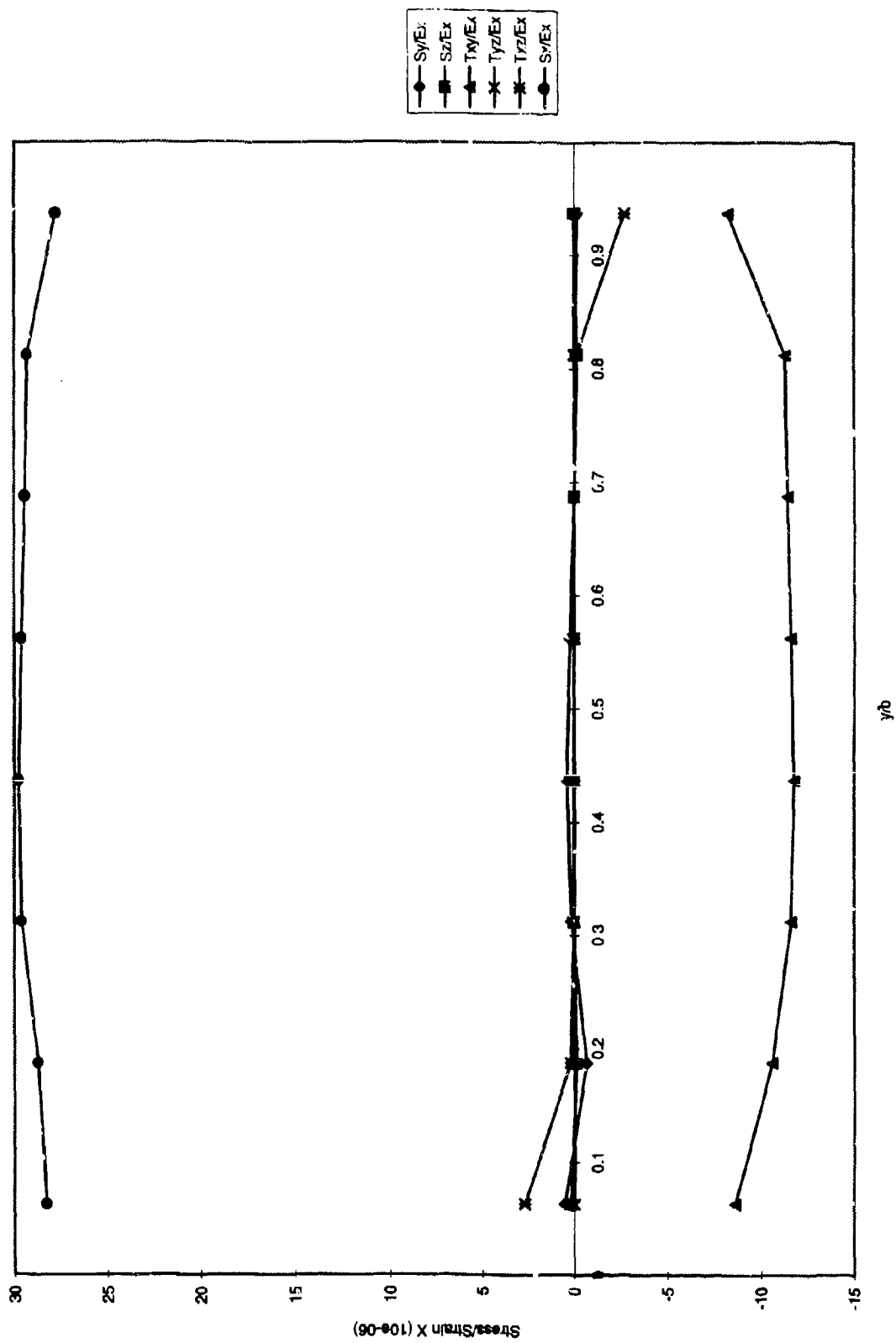


FIGURE 6 - Stress Across Plate Width

## 5.0 CONCLUSIONS

The Christensen constitutive equation based finite element method has successfully been integrated into ANSYS 5.1. The results follow the stress distribution at the laminate free edge according to the results of Pipes and Pagano. Therefore with proper modeling the element has the accuracy for examining potential delaminations at design features. Proper modeling techniques include not using symmetry planes and using a fine mesh at the laminate free edge.

The problem with using symmetry planes is that it implies isotropy, which was shown not to be the case for the element formulation. For this reason, symmetry planes are not recommended for modeling. Further development of the element formulation algorithms is needed to take advantage of symmetry planes.

The element formulation requires that there are two material sets through the lamina thickness. Using only one material set gives inaccurate results. Again further development of the element formulation algorithms are needed to correct this deficiency.

The present element formulation is an 8-noded isoparametric brick. To accurately capture the rapidly changing stress field near the laminate free edge, the mesh will have to be finer than the rest of model. Another way to better capture this stress field is to incorporate a 20-noded brick element with the ANSYS code.

Writing software code that interfaces with an existing program always presents challenges and results in a product that is not as tightly coupled as desired. The greatest difficulty in writing the user element was dealing with the input/output with ANSYS. Determining how to bring in variables from ANSYS and storing results was not intuitive and poorly documented. ANSYS offers a short course on writing user programmable features and in retrospect this would have been a valuable course for developing this element UPF.

## 6.0 RECOMMENDATIONS

The efficiency of using this element could be improved by more development to the algorithms and tighter integration with ANSYS. Specific enhancements are allowing the user to use the graphical input for material properties and real constants, adding algorithms to take advantage of symmetry planes, refining the algorithms so that only one material property is needed, incorporating 20-node brick elements, and writing the code to view the strain and stress output in graphical form.

The best way to implement these enhancements would be to improve the element formulations and incorporate them into the existing ANSYS composite element SOLID 46. This approach would give the tightest coupling between the Christensen formulation and would result in the most efficient use of this element.

## 7.0 REFERENCES

1. Christensen, R.M., and Zywicki, E., "A Three Dimensional Constitutive Theory for Fiber Composite Laminates," ASME Paper 90-WA/APM-32, presented at the Winter Annual Meeting of the ASME, Nov. 25-30, 1990, Dallas, TX.
2. Patton, E.M., "Implementation of an Efficient Description of Elastic Properties of a Layered Composite in a Finite Element," Computers in Engineering, Vol 2, pp 91-94, ASME 1992.
3. Swanson Analysis Systems, Inc., "ANSYS Programmer's Manual for Revision 5.0," Houston, PA, March 15, 1993.
4. Pipes, R.B., and Pagano, N.J., "Interlaminar Stresses in Composite Laminates Under Uniform Axial Extension," Journal of Composite Materials, V.4, pp.538-548, 1970.

**APPENDIX A**



```

*DECK UEC102  PARALLEL      USER  2/23/90      PCK
SUBROUTINE UEC102 (ELCDN,IELC,KERR)
C   *** THIS SUBROUTINE DEFINES THE CHARACTERISTICS OF US1 R102.
C   **** SEE UEC100 FOR DESCRIPTIVE COMMENTS **
C
  INCLUDE 'IMPCOM'
  INCLUDE 'ECHPRM'
C
  EXTERNAL NMINFO,ALTINF,ANSERR,ERINQR,WRINQR
  INTEGER ERINQR,WRINQR,IOTT
  INTEGER IELC(*),I,KERR,KY2
  CHARACTER*28 ELCDN
C
C   *** ANSYS(R) COPYRIGHT(C) 1971,78,82,83,85,87,89,92
C   *** SWANSON ANALYSIS SYSTEMS, INC.
C
C   **** DEFINE ELEMENT NAME ***
  CALL NMINFO (IELC(1),'USER102 ')
  CALL ALTINF (IELC(1),' ')
  ELCDN = 'USER ELEMENT 102'
C
C   **** ELEMENT TYPE CHARACTERISTICS:
C
C   **** BASED ON SOLID46 ELEMENT
C
  IELC(KDIM ) = 3
  IELC(ISHAP ) = JBRICK
  IELC(IDEGEN) = 3
  IELC(MSHLIM) = 1
  IELC(KELSTO) = 12
  IELC(KDOFS ) = UX + UY + UZ
  KY2 = IELC(KYOP2 )
C
  IF (KY2 .LE. 1) THEN
    IELC(NMLAYS) = -1
    IF (KY2 .EQ. 1) IELC(NMLAYS) = -2
    IELC(LCANGL) = 7
    IELC(NMDRLC) = 312
    IELC(MATRQD) = 1
    IELC(MATMUL) = 1
  ELSE
    IELC(NMLAYS) = -3
    IELC(LCANGL) = 128
    IELC(NMDRLC) = 128
  ENDIF
  IELC(NMNDMX) = 8
  IELC(MATRXS) = STIFM + MASSM + SSTIFM
  IELC(NMPRES) = 6
  IELC(NMTEMP) = 8
  RETURN
END

```

```

SUBROUTINE UEL102 (ELEM,IELC,ELMDAT,EOMASK,NODES,LOCSVR,KELREQ,
X KELFIL,NR,XYZ,U,KELOUT,ZS,ZASS,DAMP,GSTIF,ZSC,ZSCNR,ELVOL,ELMASS,
X CENTER,ELENER,EDINDX,LCEST)
C --- SEE EL100 AND EL101 FOR DOCUMENTATION ---
C
C
C *** ANSYS(R) COPYRIGHT(C) 1971,78,82,83,85,87,89,92
C *** SWANSON ANALYSIS SYSTEMS, INC.
C
C INPUT ARGUMENTS:
C ELEM (INT,SC,IN) - ELEMENT LABEL (NUMBER)
C IELC (INT,AR(1),IN) - ARRAY OF ELEMENT TYPE CHARACTERISTICS
C ELMDAT (INT,AR(10),IN) - ARRAY OF ELEMENT DATA
C EOMASK (INT,SC,IN) - BIT PATTERN FOR ELEMENT OUTPUT
C (SEE OUTPCM)
C NODES (INT,AR(NNOD),IN) - ARRAY OF ELEMENT NODE NUMBERS
C LOCSVR (INT,SC,IN) - LOCATION OF THE SAVED VARIABLES
C ON FILE ESAV FOR THIS ELEMENT
C KELREQ (INT,AR(10),IN) - MATRIX AND LOAD VECTOR FORM REQUESTS
C (INDICES FOR KELREQ ARE GIVEN WITH OUTPUT
C ARGUMENTS BELOW)
C KELFIL (INT,AR(10),IN) - KEYS INDICATING INCOMING MATRICES AND
C LOAD VECTORS (INDICES FOR KELFIL ARE THE
C SAME AS GIVEN FOR KELREQ WITH OUTPUT
C ARGUMENTS BELOW)
C NR (INT,SC,IN) - MATRIX AND LOAD VECTOR SIZE
C XYZ (DP,AR(6,NNOD),IN) - NODAL COORDINATES (ORIG) AND ROTATION ANGLE
C U (DP,AR(NR,5),IN) - ELEMENT NODAL SOLUTION VALUES
C
C OUTPUT ARGUMENTS:
C KELOUT (INT,AR(10),OUT) - KEYS INDICATING CREATED MATRICES AND
C LOAD VECTORS (INDICES FOR KELOUT
C ARE THE SAME AS FOR KELREQ BELOW)
C ZS (DP,AR(NR,NR),INOUT)- K MATRIX (KELREQ(1))
C ZASS (DP,AR(NR,NR),INOUT)- M MATRIX (KELREQ(2))
C DAMP (DP,AR(NR,NR),INOUT)- C MATRIX (KELREQ(3))
C GSTIF (DP,AR(NR,NR),INOUT)- S MATRIX (KELREQ(4))
C ZSC (DP,AR(NR),OUT) - APPLIED F VECTOR (KELREQ(5))
C ZSCNR (DP,AR(NR),OUT) - N-R RESTORING F VECTOR (KELREQ(6))
C OR IMAGINARY F VECTOR (KELREQ(7))
C ELVOL (DP,SC,OUT) - ELEMENT VOLUME
C ELMASS (DP,SC,OUT) - ELEMENT MASS
C CENTER (DP,AR(3),OUT) - CENTROID LOCATION
C ELENER (DP,AR(5),OUT) - ELEMENT ENERGIES
C EDINDX (INT,AR(20),OUT) - ELEMENT RESULT DATA FILE INDEXES
C LCEST (INT,SC,INOUT) - POSITION ON RESULT FILE
C
C ---- START OF COMDECKS
C
C INCLUDE 'IMPCOM'
C INCLUDE 'ECHPRM'
C INCLUDE 'ELPARM'
C INCLUDE 'ELUCOM'
C

```

```

C ---- END OF COMDECKS
C
EXTERNAL TRACK,ANSERR,SVGIDX,SVRGET,RVRGET,WRINQR,PROPEI,
X VZERO
C
C --- ARGUMENTS
INTEGER ELEM,IELC(IELCSZ),ELMDAT(10),EOMASK,NODES(8),LOCSVR,
X KELREQ(10),KELFIL(10),NR,KELOUT(10),EDINDX(20),LCERST,WRINQR
C
C --- ARGUMENTS
DOUBLE PRECISION
X XYZ(6,8),U(NR,5),ZS(NR,NR),ZASS(NR,NR),DAMP(NR,NR),GSTIF(NR,NR),
X ZSC(NR),ZSCNR(NR),ELVOL,ELMASS,CENTER(3),ELENER(5)
C
C --- LOCAL
C
INTEGER
X SVINDX(10),
X PMAT,IREAL,PREAL,NRVR,NSSVR,I,J,IOT,T,
X K,KY2,NUMLAYERS,KP1,KP2
C
C
C
C --- PROPERTIES
C
DOUBLE PRECISION
X PROP(10),PROPO,
X THK(100)
C
C --- RVR,SSVR
C
DOUBLE PRECISION
X RVR(312),SSVR(10),SVIDNX(1),TK
C
INTEGER ANGFIB(100,100),MATNUM(250),THETA
DOUBLE PRECISION PROPS(10)
C
CALL TRACK (5,'UEL102')
C
C --- DEFINE INITIAL DATA
C
MATNUM(ELEM) = ELMDAT(PMAT)
IREAL = ELMDAT(PREAL)
C
C --- IELC POINTERS DEFINED IN ECHPRM AND ELCCMT
C
NRVR = IELC(NMTRLC)
NSSVR = IELC(NMSSVR)
KY2 = IELC(KYOP2)
C
C DEFINE KELOUT SWITCHES FOR STIFFNESS MATRIX
C
KELOUT(1)=1
C
C DEFINE KELREQ SWITCHES FOR LOAD VECTOR

```

```

C
C   KELREQ(5) = 1
C
C   --- GET THE SVR INDEX VECTOR
C
C   CALL SVGIDX (LOCSVR,SVINDX)
C
C   --- GET THE ELEMENT REAL CONSTANT DATA
C
C   REAL CONSTANT DATA IS INPUTTED USING SAME FORMAT
C   AS SOLID46 ELEMENT WITH KEYOPT(2) = 0
C   NL,LSYM,LP1,LP2,(BLANKS(2)),KREF,(BLANKS(5)),
C   MAT,THETA,TK FOR LAYER 1
C   MAT,THETA,TK FOR LAYER 2
C   ETC. UP TO LAYER NL
C
C   FOR THIS USER ELEMENT HOWEVER, THE VARIABLES
C   LSYM,LP1,LP2,BLANKS(2),KREF,BLANKS(5)
C   WILL NOT BE USED, AND WILL INPUTTED AS BLANKS
C
C
C   CALL RVRGET (ELEM,IREAL,IELC(1),NRVR,RVR)C
C
C   IF (KY2.EQ. 0) THEN
C     NUMLAYERS = RVR(1)
C     K = 13
C     KP1 = 14
C     KP2 = 15
C     DO 10 I = 1, NUMLAYERS
C       MATNUM(I) = RVR(K)
C       THETA = RVR(KP1)
C       TK = RVR(KP2)
C
C   C   CONVERT ANSYS VARIABLES TO CHRISTENSEN VARIABLES
C
C     ANGFI(I,MATNUM(I)) = THETA
C     THK(I) = TK
C     K = K + 3
C     KP1 = KP1 + 3
C     KP2 = KP2 + 3
C   10 CONTINUE
C   ELSE
C     WRITE(IOTT,1)
C   1   FORMAT(/10X,'***** TAPERED LAYER OR MATRIX INPUT NOT,
C   X /17X,'YET IMPLEMENTED ***** '/')
C     RETURN
C   END IF
C
C   --- SET UP MATERIAL PROPERTIES
C
C   E11
C
C   CALL PROPE1(ELEM,MATNUM(ELEM),1,0.0,PROPO)
C   PROPS(1) = PROPO
C

```

```

C E22
C
CALL PROPE1(ELEM,MATNUM(ELEM),2,0.0,PROPO)
PROPS(2) = PROPO
C
C NU12
C
CALL PROPE1(ELEM,MATNUM(ELEM),4,0.0,PROPO)
PROPS(3) = PROPO
C
C G12
C
CALL PROPE1(ELEM,MATNUM(ELEM),7,0.0,PROPO)
PROPS(4) = PROPO
C
C G13
C
CALL PROPE1(ELEM,MATNUM(ELEM),9,0.0,PROPO)
PROPS(5) = PROPO
C
C DENSITY
C
CALL PROPE1(ELEM,MATNUM(ELEM),13,0.0,PROPO)
PROPS(6) = PROPO
C
C CHRIST.FOR VARIABLE P7 (LAMINA THICKNESS) IS NOW
C INPUTTED AS A REAL CONSTANT IN ANSYS
C THEREFORE PROPS(7) WILL BE SKIPPED
C AND SET TO ZERO
C
PROPS(7) = 0.0D0
C
C ALPHAX
C
CALL PROPE1(ELEM,MATNUM(ELEM),10,0.0,PROPO)
PROPS(8) = PROPO
C
C ALPHAY
C
CALL PROPE1(ELEM,MATNUM(ELEM),11,0.0,PROPO)
PROPS(9) = PROPO
C
C ALPHAZ
C
CALL PROPE1(ELEM,MATNUM(ELEM),12,0.0,PROPO)
PROPS(10) = PROPO
C
C CALCULATE THE STIFFNESS MATRIX
C
CALL BRICK (ELEM,ZS,XYZ,NUMLAYERS,ANGFIB,PROPS,MATNUM)
C
C WRITE OUT RESULTS
C
CALL STRBRI(U,ELEM,XYZ,PROPS,NR,NODES,NUMLAYERS,MATNUM,ANGFIB,
X EDINDX,LCERST)

```

```

C
CALL TRACK (15, 'UEL102')
  RETURN
  END
C
C *****
C
SUBROUTINE BRICK(IELEM,ESTIF,COORD,NLAYERS,IALPHA,PROPS,MATNO)
C
C  CALCULATES THE STIFFNESS MATRIX FROM THE INPUT DATA
C
DOUBLE PRECISION ESTIF(24,24),BMATX(6,42),DMATX(6,6)
DOUBLE PRECISION SHAPE(14),DERIV(3,14),DBMAT(6,42),CARTD(3,14)
DOUBLE PRECISION ELCOD(6,8),COORD(6,8),TSTIF(33,33),TEMP
DOUBLE PRECISION POSGP(3),WEIGP(3),DVOLU
COMMON /CONTROL/ MPOIN,NPOIN,NELEM,NNODE,NDOFN,NDIME,NSTRE,
! NPROP,NMATS,NVFIX,NEVAB,NINTR
COMMON /FIXEDBC/ PRESC(100,3),NOFIX(100),IFPRE(100,3)
DOUBLE PRECISION PROPS(10)
COMMON /MESH/ LNODS(6,8),
! ISNODE(750),ISELEM(500)
EXTERNAL TRACK,WRINQR
INTEGER WRINQR,NLAYERS,IALPHA(100,100),MATNO(250),IELEM
INTEGER NGAUS
CALL TRACK (6,'BRICK')
IOTT = WRINQR(2)
C
C
C  EVALUATE THE COORDINATES OF THE ELEMENT NODAL POINTS
C
C  --- SETS UP PARAMETERS FOR 8-NODED BRICK
C
NNODE = 8
NDIME = 3
C
C  --- SETS UP CHRIST.FOR ONLY VARIABLES
C
NINTR = 1
NEVAB = 24
NGAUS = 3
NSTRE = 6
NDOFN = 3
NPROP = 13
C
C  --- ZERO OUT VARIABLES
C
EXISP = 0.0D0
ETASP = 0.0D0
RHOSP = 0.0D0
DVOLU = 0.0D0
C
C  --- SET UP GAUSSIAN INTEGRATION CONSTRAINTS
C
CALL GAUSSQ(NGAUS,POSGP,WEIGP)
C

```

```

DO 10 INODE = 1,NNODE
DO 10 IDIME = 1,NDIME
ELCOD(IDIME,INODE) = COORD(IDIME,INODE)
10 CONTINUE
C
C INITIALIZE THE ELEMENT STIFFNESS MATRIX
C
DO 20 IEVAB = 1,NEVAB+NINTR*9
DO 20 JEVAB = 1,NEVAB+NINTR*9
TSTIF(IEVAB,JEVAB) = 0.0D0
20 CONTINUE
KGASP = 0
C
C ENTER LOOPS FOR AREA NUMERICAL INTEGRATION, STARTING WITH THE
C LOOP FOR EACH LAYER IN THE ELEMENT
C
DO 50 ILAYER = 1,NLAYERS
C
C EVALUATE THE D-MATRIX IN THIS LAYER
C
CALL DCHRIST(IELEM,ILAYER,IALPHA,PROPS,MATNO,DMATX)
C
C NOW THE GAUSSIAN INTEGRATION OF X AND Y IN THIS LAYER
C
DO 50 IGAUS = 1,NGAUS
DO 50 JGAUS = 1,NGAUS
KGASP = KGASP+1
EXISP = POSGP(IGAUS)
ETASP = POSGP(JGAUS)
RHOSP = -1.+(2.*FLOAT(ILAYER)-1.)/FLOAT(NLAYERS)
C
C EVALUATE THE SHAPE FUNCTIONS, ELEMENTAL VOLUME, ETC. AT THIS
C SAMPLING POINT
C
CALL SFBRIK(EXISP,ETASP,RHOSP,SHAPE,DERIV)
CALL JACBRIK(IELEM,DJACB,CARTD,DERIV,ELCOD)
DVOLU = DJACB*WEIGP(IGAUS)*WEIGP(JGAUS)
! /FLOAT(NLAYERS)
C EVALUATE THE B AND BXD MATRICES
C
CALL BBRICK(BMATX,CARTD)
CALL DBE(BMATX,DMATX,DBMAT)
C
C CALCULATE THE ELEMENT STIFFNESSES
C
DO 30 IEVAB = 1,NEVAB+NINTR*9
DO 30 JEVAB = IEVAB,NEVAB+NINTR*9
DO 30 ISTRE = 1,NSTRE
TSTIF(IEVAB,JEVAB) = TEMP +
! BMATX(ISTRE,IEVAB)*DBMAT(ISTRE,JEVAB)*DVOLU
30 CONTINUE
50 CONTINUE
C
C CONSTRUCT THE LOWER TRIANGLE OF THE STIFFNESS MATRIX
C

```

```

DO 60 IEVAB = 1,NEVAB+NINTR*9
DO 60 JEVAB = 1,NEVAB+NINTR*9
TSTIF(JEVAB,IEVAB) = TSTIF(IEVAB,JEVAB)
60 CONTINUE
C
C   NOW DO THE STATIC REDUCTION OF TSTIF TO ESTIF
C
C   CALL REDUCE (TSTIF,ESTIF,NINTR)
C
C   ALL DONE
C
CALL TRACK(16,'BRICK')
RETURN
END
C
C *****
C
SUBROUTINE REDUCE (TSTIF,ESTIF,NINTR)
C
C   PERFORMS A STATIC REDUCTION OF THE TWO INTERNAL DEGREES OF
C   FREEDOM IN THE 8-NODE BRICK THAT HAVE BEEN ADDED TO CAPTURE
C   THE THROUGH-THICKNESS DISPLACEMENT FIELD IN A LAMINATED
C   COMPOSITE MATERIAL
C
EXTERNAL WRINQR
INTEGER WRINQR
DOUBLE PRECISION TSTIF(33,33),ESTIF(24,24),RSTIF(24,24),INDX(22)
DOUBLE PRECISION STAU(22,24),STUA(24,22),STAA(22,22)
DOUBLE PRECISION SINVAA(22,22),SINT(22,24)
IOTT = WRINQR(2)
C
C   FIRST, GET THE PART OF THE MATRIX THAT HAS JUST THE
C   ADDED INTERNAL DEGREES OF FREEDOM
C
NADD = 9*NINTR
C
C   --- INITIALIZE ESTIF
C
DO 90 I=1,24
DO 90 J=1,24
90 ESTIF(I,J) = 0.0D0
C
C   --- INITIALIZE STAA
C
DO 5 I = 1,NADD
DO 5 J = 1,NADD
STAA(I,J) = 0.0D0
5 CONTINUE
IF (NADD.GT.0) THEN
DO 10 IAA = 1,NADD
DO 10 KAA = 1,NADD
STAA(IAA,KAA) = TSTIF(IAA+24,KAA+24)
10 CONTINUE
C
C   NOW GET THE INVERSE OF THIS MATRIX FOR THE STATIC REDUCTION

```



```

C   EQUATION - NUMERICAL RECIPES IN FORTRAN - P. 38
C
DO 20 IA = 1,NADD
DO 15 JA = 1,NADD
    SINVAA(IA,JA) = 0.0D0
15  CONTINUE
    SINVAA(IA,IA) = 1.0D0
20  CONTINUE
    CALL LUDCMP(STAA,NADD,22,INDX,D)
DO 25 JA = 1,NADD
    CALL LUBKSB(STAA,NADD,22,INDX,SINVAA(1,JA))
25  CONTINUE
C
C   NOW DO THE MATRIX MULTIPLICATION OF [KUA] X [KAA INVERSE] X
C
C   FIRST GET [KUA] AND [KAU]
C
DO 30 IROW = 1,NADD
DO 30 ICOL = 1,24
    STAU(IROW,ICOL) = TSTIF(IROW+24,ICOL)
30  CONTINUE
DO 40 IROW = 1,24
DO 40 ICOL = 1,NADD
    STUA(IROW,ICOL) = TSTIF(IROW,ICOL+24)
40  CONTINUE
C
C   NOW MULTIPLY OUT THE MATRICES
C
DO 50 IROW = 1,NADD
DO 50 ICOL = 1,24
    SINT(IROW,ICOL) = 0.0D0
DO 50 JROW = 1,NADD
    SINT(IROW,ICOL) = SINT(IROW,ICOL)
    ! +SINVAA(IROW,JROW)*STAU(JROW,ICOL)
50  CONTINUE
DO 60 IROW = 1,24
DO 60 ICOL = 1,24
    RSTIF(IROW,ICOL) = 0.0D0
DO 60 J = 1,NADD
    RSTIF(IROW,ICOL) = RSTIF(IROW,ICOL)
    ! +STUA(IROW,J)*SINT(J,ICOL)
60  CONTINUE
C
C   NOW SUBTRACT RSTIF FROM TSTIF TO GET ESTIF
C
DO 70 IROW = 1,24
DO 70 ICOL = 1,24
    ESTIF(IROW,ICOL) = TSTIF(IROW,ICOL)-RSTIF(IROW,ICOL)
70  CONTINUE
ELSE
DO 80 IROW = 1,24
DO 80 ICOL = 1,24
    ESTIF(IROW,ICOL) = TSTIF(IROW,ICOL)
80  CONTINUE
ENDIF

```

```

C
C   ALL DONE
C
C   RETURN
C   END
C
C   *****
C
C   SUBROUTINE LUDCMP(A,N,NP,INDX,D)
C
C   ROUTINE EXTRACTED FROM "NUMERICAL RECIPES IN FORTRAN", PP. 35-36
C
C   PARAMETER (NMAX=100,TINY=1.D-20)
C   DOUBLE PRECISION A(NP,NP),INDX(NP),VV(NMAX),AAMAX
C   EXTERNAL WRINQR
C   INTEGER WRINQR,NP
C   IOTT = WRINQR(2)
C   D = 1.0D0
DO 12 I=1,N
    AAMAX = 0.0D0
    DO 11 J = 1,N
        IF (ABS(A(I,J)).GT.AAMAX) AAMAX = ABS(A(I,J))
    11    CONTINUE
    IF (AAMAX.EQ.0.0D0) THEN
        WRITE (*,*) ' YOU HAVE A SINGULAR MATRIX'
        WRITE (8,*) A
        STOP
    ENDIF
    VV(I) = 1./AAMAX
12    CONTINUE
    DO 19 J = 1,N
        DO 14 I = 1,J-1
            SUM = A(I,J)
            DO 13 K = 1,I-1
                SUM = SUM - A(I,K)*A(K,J)
            13    CONTINUE
            A(I,J) = SUM
        14    CONTINUE
        AAMAX = 0.0D0
        DO 16 I=J,N
            SUM = A(I,J)
            DO 15 K = 1,J-1
                SUM = SUM - A(I,K)*A(K,J)
            15    CONTINUE
            A(I,J) = SUM
            DUM = VV(I)*ABS(SUM)
            IF (SUM.GE.AAMAX) THEN
                IMAX = I
                AAMAX = DUM
            ENDIF
        16    CONTINUE
        IF (J.NE.IMAX) THEN
            DO 17 K = 1,N
                DUM = A(IMAX,K)
                A(IMAX,K) = A(J,K)

```

```

      A(J,K) = DUM
17      CONTINUE
      D = -D
      VV(IMAX) = VV(J)
      ENDIF
      INDX(J) = IMAX
      IF (A(J,J).EQ.0.0D0) A(J,J) = TINY
      IF (J.NE.N) THEN
          DUM = 1./A(J,J)
          DO 18 I=J+1,N
              A(I,J) = A(I,J)*DUM
18          CONTINUE
      ENDIF
19  CONTINUE
      RETURN
      END

C
C *****
C
      SUBROUTINE LUBKSB(A,N,NP,INDX,B)
C
C  EXTRACTED FROM "NUMERICAL RECIPES IN FORTRAN"
C  DOES THE BACK SUBSTITUTION AFTER THE LU DECOMPOSITION
C
      DOUBLE PRECISION A(NP,NP),INDX(NP),B(NP)
      INTEGER NP
C
      II = 0
      DO 12 I = 1,N
          LL = INDX(I)
          SUM = B(LL)
          B(LL) = B(I)
          IF (II.NE.0) THEN
              DO 11 J = II,I-1
                  SUM = SUM - A(I,J)*B(J)
11          CONTINUE
          ELSEIF (SUM.NE.0.0D0) THEN
              II = I
          ENDIF
          B(I) = SUM
12  CONTINUE
      DO 14 I = N,1,-1
          SUM = B(I)
          IF (I.LT.N) THEN
              DO 13 J = I+1,N
                  SUM = SUM - A(I,J)*B(J)
13          CONTINUE
          ENDIF
          B(I) = SUM/A(I,I)
14  CONTINUE
      RETURN
      END

C
C *****
C

```

```

SUBROUTINE DBE(BMATX,DMATX,DBMAT)
C
C   CALCULATES D X B
C
DOUBLE PRECISION BMATX(6,42),DMATX(6,6),DBMAT(6,42)
COMMON /CONTROL/ MPOIN,NPOIN,NELEM,NNODE,NDOFN,NDIME,NSTRE,
!   NPROP,NMATS,NVFIX,NEVAB,NINTR
C
DO 10 ISTR = 1,NSTRE
DO 10 IEVAB = 1,NEVAB+9*NINTR
DBMAT(ISTR,IEVAB) = 0.0D0
DO 10 JSTR = 1,NSTRE
DBMAT(ISTR,IEVAB) = DBMAT(ISTR,IEVAB)+
!   DMATX(ISTR,JSTR)*EMATX(JSTR,IEVAB)
10 CONTINUE
RETURN
END
C
C   *****
C
SUBROUTINE SFBRIK(S,T,R,SHAPE,DERIV)
C
C   CALCULATES SHAPE FUNCTIONS AND THEIR DERIVATIVES FOR
C   BRICK STRESS AND STRAIN 2-D ELEMENTS
C
DOUBLE PRECISION SHAPE(14),DERIV(3,14)
EXTERNAL WRINQR
INTEGER WRINQR
C
C   --- INITIALIZE SHAPE FUNCTIONS AND DERIVATIVES
C
IOTT=WRINQR(2)
DO 10 I = 1,14
10  SHAPE(I) = 0.0D0
DO 20 I = 1,3
DO 20 J = 1,14
20  DERIV(I,J)=0.0D0
C
C
SHAPE(1) = 0.125*(1.-T)*(1.-S)*(1.-R)
SHAPE(2) = 0.125*(1.-T)*(1.+S)*(1.-R)
SHAPE(3) = 0.125*(1.+T)*(1.+S)*(1.-R)
SHAPE(4) = 0.125*(1.+T)*(1.-S)*(1.-R)
SHAPE(5) = 0.125*(1.-T)*(1.-S)*(1.+R)
SHAPE(6) = 0.125*(1.-T)*(1.+S)*(1.+R)
SHAPE(7) = 0.125*(1.+T)*(1.+S)*(1.+R)
SHAPE(8) = 0.125*(1.+T)*(1.-S)*(1.+R)
SHAPE(9) = 1.-S**2
SHAPE(10) = 1.-T**2
SHAPE(11) = 1.-R**2
SHAPE(12) = S-S**3
SHAPE(13) = T-T**3
SHAPE(14) = R-R**3
C

```

C AND THEIR DERIVATIVES

C

DERIV(1,1) = -.125\*(1.-T)\*(1.-R)  
DERIV(1,2) = .125\*(1.-T)\*(1.-R)  
DERIV(1,3) = .125\*(1.+T)\*(1.-R)  
DERIV(1,4) = -.125\*(1.+T)\*(1.-R)  
DERIV(1,5) = -.125\*(1.-T)\*(1.+R)  
DERIV(1,6) = .125\*(1.-T)\*(1.+R)  
DERIV(1,7) = .125\*(1.+T)\*(1.+R)  
DERIV(1,8) = -.125\*(1.+T)\*(1.+R)

DERIV(1,9) = -2.\*S

DERIV(1,10) = 0.0D0

DERIV(1,11) = 0.0D0

DERIV(1,12) = 1.-3.\*S\*\*2

DERIV(1,13) = 0.0D0

DERIV(1,14) = 0.0D0

DERIV(2,1) = -.125\*(1.-S)\*(1.-R)

DERIV(2,2) = -.125\*(1.+S)\*(1.-R)

DERIV(2,3) = .125\*(1.+S)\*(1.-R)

DERIV(2,4) = .125\*(1.-S)\*(1.-R)

DERIV(2,5) = -.125\*(1.-S)\*(1.+R)

DERIV(2,6) = -.125\*(1.+S)\*(1.+R)

DERIV(2,7) = .125\*(1.+S)\*(1.+R)

DERIV(2,8) = .125\*(1.-S)\*(1.+R)

DERIV(2,9) = 0.0D0

DERIV(2,10) = -2.\*T

DERIV(2,11) = 0.0D0

DERIV(2,12) = 0.0D0

DERIV(2,13) = 1.-3.\*T\*\*2

DERIV(2,14) = 0.0D0

DERIV(3,1) = -.125\*(1.-S)\*(1.-T)

DERIV(3,2) = -.125\*(1.+S)\*(1.-T)

DERIV(3,3) = -.125\*(1.+S)\*(1.+T)

DERIV(3,4) = -.125\*(1.-S)\*(1.+T)

DERIV(3,5) = .125\*(1.-S)\*(1.-T)

DERIV(3,6) = .125\*(1.+S)\*(1.-T)

DERIV(3,7) = .125\*(1.+S)\*(1.+T)

DERIV(3,8) = .125\*(1.-S)\*(1.+T)

DERIV(3,9) = 0.0D0

DERIV(3,10) = 0.0D0

DERIV(3,11) = -2.\*R

DERIV(3,12) = 0.0D0

DERIV(3,13) = 0.0D0

DERIV(3,14) = 1.-3.\*R\*\*2

C

C ALL DONE

C

RETURN

END

C

C

C

SUBROUTINE JACBRIK(IELEM,DJACB,CARTD,DERIV,ELCOD)

C

C CALCULATES THE JACOBIAN MATRIX AND DETERMINANT AND INVERSE

```

C   FOR 3-D ELEMENTS
C
DOUBLE PRECISION XM(3,3),XI(3,3),CARTD(3,14)

C
COMMON /CONTROL/ MPOIN,NPOIN,NELEM,NNODE,NDOFN,NDIME,NSTRE,
!   NPROP,NMATS,NVFIX,NEVAB,NINTR

DOUBLE PRECISION DERIV(3,14),ELCOD(6,8)
INTEGER IELEM
EXTERNAL WRINQR
INTEGER WRINQR
IOTT = WRINQR(2)

C
C --- INITIALIZE XI MATRIX
C
DO 10 I = 1, NDIME
DO 10 J = 1, NDIME
XM(I,J) = 0.0D0
10 XI(I,J) = 0.0D0
DJACB = 0.0D0

C
C CREATE JACOBIAN MATRIX
C
DO 20 IDIME = 1,NDIME
DO 20 JDIME = 1,NDIME
DO 20 INODE = 1,NNODE
XM(IDIME,JDIME) = XM(IDIME,JDIME)+
!   DERIV(IDIME,INODE)*ELCOD(JDIME,INODE)
20 CONTINUE

C
C NOW THE DETERMINANT AND INVERSE OF THE JACOBIAN
C
DJACB = XM(1,1)*(XM(2,2)*XM(3,3)-XM(3,2)*XM(2,3))
!   -XM(1,2)*(XM(2,1)*XM(3,3)-XM(3,1)*XM(2,3))
!   +XM(1,3)*(XM(2,1)*XM(3,2)-XM(3,1)*XM(2,2))
IF (DJACB.GT.0.) GO TO 30
WRITE (IOTT,999) IELEM
999 FORMAT (//, 'ELEMENT NUMBER ',I5,' HAS ZERO OR ',
!   'NEGATIVE AREA - PLEASE CHECK INPUT',///)
STOP
30 XI(1,1) = (XM(2,2)*XM(3,3)-XM(3,2)*XM(2,3))/DJACB
XI(1,2) = (XM(3,2)*XM(1,3)-XM(1,2)*XM(3,3))/DJACB
XI(1,3) = (XM(1,2)*XM(2,3)-XM(2,2)*XM(1,3))/DJACB
XI(2,1) = (XM(3,1)*XM(2,3)-XM(2,1)*XM(3,3))/DJACB
XI(2,2) = (XM(1,1)*XM(3,3)-XM(3,1)*XM(1,3))/DJACB
XI(2,3) = (XM(2,1)*XM(1,3)-XM(1,1)*XM(2,3))/DJACB
XI(3,1) = (XM(2,1)*XM(3,2)-XM(3,1)*XM(2,2))/DJACB
XI(3,2) = (XM(3,1)*XM(1,2)-XM(1,1)*XM(3,2))/DJACB
XI(3,3) = (XM(1,1)*XM(2,2)-XM(2,1)*XM(1,2))/DJACB

C
C CALCULATE CARTESIAN DERIVATIVES
C
DO 50 I=1,6
DO 50 J=1,42

```

```

50  CARTD(I,J) = 0.0D0
    DO 40 IDIME = 1,NDIME
      DO 40 INODE = 1,NNODE+NINTR*3
        C  CARTD(IDIME,INODE) = 0.0D0
          DO 40 JDIME = 1,NDIME
            CARTD(IDIME,INODE) = CARTD(IDIME,INODE)+
              !  XI(JIME,JDIME)*DERIV(JDIME,INODE)
          40 CONTINUE
        C
        C  ALL DONE
        C
        RETURN
        END
      C
      C  *****
      C
      SUBROUTINE BBRICK(BMATX,CARTD)
      C
      C  CALCULATES THE STRAIN MATRIX B FOR THE BRICK ELEMENT
      C
      DOUBLE PRECISION BMATX(6,42),CARTD(3,14)
      COMMON /CONTROL/ MPOIN,NPOIN,NELEM,NNODE,NDOFN,NDIME,NSTRE,
        !  NPROP,NMATS,NVFIX,NEVAB,NINTR
      C
      DO 5 I=1,6
        DO 5 J = 1,42
          5  BMATX(I,J) = 0.0D0
          DO 10 INODE = 1,NNODE+NINTR*3
            MGASH = (INODE-1)*3+1
            NGASH = (INODE-1)*3+2
            LGASH = (INODE-1)*3+3
            BMATX(1,MGASH) = CARTD(1,INODE)
            BMATX(1,NGASH) = 0.0D0
            BMATX(1,LGASH) = 0.0D0
            BMATX(2,MGASH) = 0.0D0
            BMATX(2,NGASH) = CARTD(2,INODE)
            BMATX(2,LGASH) = 0.0D0
            BMATX(3,MGASH) = 0.0D0
            BMATX(3,NGASH) = 0.0D0
            BMATX(3,LGASH) = CARTD(3,INODE)
            BMATX(4,MGASH) = CARTD(2,INODE)
            BMATX(4,NGASH) = CARTD(1,INODE)
            BMATX(4,LGASH) = 0.0D0
            BMATX(5,MGASH) = 0.0D0
            BMATX(5,NGASH) = CARTD(3,INODE)
            BMATX(5,LGASH) = CARTD(2,INODE)
            BMATX(6,MGASH) = CARTD(3,INODE)
            BMATX(6,NGASH) = 0.0D0
            BMATX(6,LGASH) = CARTD(1,INODE)
          10 CONTINUE
        C
        C  ALL DONE
        C
        RETURN
        END

```

```

C *****
C
C SUBROUTINE DCHRIST(IELEM,ILAYER,IALPHA,PROPS,MATNO,DMATX)
C
C ROUTINE TO EVALUATE ELASTICITY MATRIX FOR THE ITH LAYER OF
C THE ELEMENT, ACCORDING TO CHRISTENSEN (1990) EQUATION 25
C
C COMMON /CONTROL/ MPOIN,NPOIN,NELEM,NNODE,NDOFN,NDIME,NSTRE,
C ! NPROP,NMATS,NVFIX,NEVAB,NINTR
C COMMON /FIXEDBC/ PRESC(100,3),NOFIX(100),IFPRE(100,3)
C INTEGER IALPHA(100,100),MATNO(250),IELEM
C DOUBLE PRECISION PROPS(10),AVEMU,AVELAM,AVEMOD,A1,A2
C COMMON /MESH/ LNODS(6,8),
C ! ISNODE(750),ISELEM(500)
C
C DOUBLE PRECISION DMATX(6,6)
C
C MATERIAL PROPERTIES INPUT AS FOLLOWS
C
C P1 = E11
C P2 = E22
C P3 = NU12
C P4 = G12
C P5 = G13
C P6 = DENSITY
C P7 = LAMINA THICKNESS
C P8 = THERMAL EXPANSION - X
C P9 = THERMAL EXPANSION - Y
C P10 = THERMAL EXPANSION - Z
C
C CALCULATE THE MATRIX DOMINATED PROPERTIES AND THE AXIAL PROPERTY
C
C --- ZERO OUT VARIABLES
C
C AVEMU = 0.0D0
C AVELAM = 0.0D0
C AVEMOD = 0.0D0
C A1 = 0.0D0
C A2 = 0.0D0
C
C IF (PROPS(5).LE.1.) THEN
C     AVEMU = (PROPS(2)*(1.-PROPS(3)))/(2.*(1.-PROPS(2)/
C ! PROPS(1)*PROPS(3)**2))+PROPS(4))/2.
C ELSE
C     AVEMU = (PROPS(2)*(1.-PROPS(3)))/(1.-PROPS(2)/PROPS(1)*
C ! PROPS(3)**2)+2.*PROPS(4)+PROPS(5))/5.
C ENDIF
C AVELAM = AVEMU*2.*PROPS(3)/(1.-2.*PROPS(3))
C AVEMOD = PROPS(1)-AVEMU*2.*(1.+PROPS(3))
C
C ZERO OUT THE D-MATRIX
C

```



```

DO 10 ISTRE = 1,NSTRE
DO 10 JSTRE = 1,NSTRE
DMATX(ISTRE,JSTRE) = 0.0D0
10 CONTINUE
C
C   EVALUATE THE DIRECTION COSINES FOR THIS LAYER
C
CONVER = 3.1415926536/180.
A1 = COS(CONVER*FLOAT(IALPHA(ILAYER,MATNO(IELEM))))
A2 = SIN(CONVER*FLOAT(IALPHA(ILAYER,MATNO(IELEM))))
C   NOW THE COMPONENTS OF THE D-MATRIX FOR THIS LAYER
C
DMATX(1,1) = AVELAM*(1.-PROPS(3))/PROPS(3) + AVEMOD*A1**4
DMATX(1,2) = AVELAM + AVEMOD*A1**2*A2**2
DMATX(1,3) = AVELAM
DMATX(1,4) = AVEMOD*A1**3*A2
DMATX(2,1) = DMATX(1,2)
DMATX(2,2) = AVELAM*(1.-PROPS(3))/PROPS(3) + AVEMOD*A2**4
DMATX(2,3) = AVELAM
DMATX(2,4) = AVEMOD*A1*A2**3
DMATX(3,1) = DMATX(1,3)
DMATX(3,2) = DMATX(2,3)
DMATX(3,3) = AVELAM*(1.-PROPS(3))/PROPS(3)
DMATX(4,1) = DMATX(1,4)
DMATX(4,2) = DMATX(2,4)
DMATX(4,4) = AVEMU + AVEMOD*A1**2*A2**2
DMATX(5,5) = AVEMU
DMATX(6,6) = AVEMU
C
C   ALL DONE
C
RETURN
END
C
C   *****
C
SUBROUTINE GAUSSQ(NGAUS,POSGP,WEIGP)
C
C   ROUTINE TO SET UP SAMPLING POINTS AND WEIGHTING FACTORS
C   FOR THE ELEMENT NUMERICAL INTEGRATIONS
C
COMMON /CONTROL/ MPOIN,NPOIN,NELEM,NNODE,NDOFN,NDIME,NSTRE,
!   NPROP,NMATS,NVFIX,NEVAB,NINTR
DOUBLE PRECISION POSGP(3),WEIGP(3)
INTEGER KGAUS,JGAUS,JGAUS
C
EXTERNAL TRACK,WRINQR
INTEGER WRINQR,NGAUS
IOTT = WRINQR(2)
C
CALL TRACK(7,'GAUSSQ')
IF (NGAUS.GT.2) GO TO 10
POSGP(1) = -.577350269189626
WEIGP(1) = 1.0D0
GO TO 20

```

```

10 POSGP(1) = .774596669241483
C
C CHANGE BY M. JONES ON 1/15/96
C CHANGED POSGP(3) TO POSGP(2) AND WEIGP(3) TO WEIGP(2)
C TO REFLECT GUASS-QUAD SAMPLING POINTS AND WEIGHTS
C WHEN KGUAS = 3
C
  POSGP(2) = 0.0D0
  WEIGP(1) = .555555555555556
  WEIGP(2) = 0.0D0
20 KGAUS = NGAUS/2
  DO 30 IGASH = 1,KGAUS
    JGASH = NGAUS+1-IGASH
    POSGP(JGASH) = -POSGP(IGASH)
    WEIGP(JGASH) = WEIGP(IGASH)
30 CONTINUE
  CALL TRACK(17,'GAUSSQ')
  RETURN
  END
C
C
C
C *****
C
  SUBROUTINE STRBRI(ASDIS,IELEM,COORD,PROPS,NR,LNODS,NLAYERS,
X MATNO,IALPHA,LCERST,EDINDX)
C
C ROUTINE TO CALCULATE STRESSES IN BRICK ELEMENTS AFTER
C DISPLACEMENTS HAVE BEEN CALCULATED
C
C LOGICAL THERMAL
C DOUBLE PRECISION STRIN(6,8),STRAN(6,8)
  EXTERNAL WRINQR,UEP102,TRACK,ELDWRT
  INTEGER WRINQR,NLAYERS,IALPHA(100,100),MATNO(250),IELEM,NR
  INTEGER NGAUS,SVINDX(1)
  DOUBLE PRECISION ELDIS(3,8),STRES(6),STEMP(27,6),STRAI(6),
X SRTMP(27,6),SMATX(6,24,27),SRMAT(6,24,27),BMATX(6,42),
X DMATX(6,6)
  DOUBLE PRECISION DBMAT(6,42),CARTD(3,14),COORD(6,8),ELCOD(6,8),
  DOUBLE PRECISION SHAPE(14),DERIV(3,14),PROPS(10)
  DOUBLE PRECISION ASDIS(NR,5)
  DOUBLE PRECISION POSGP(3),WEIGP(3)
  INTEGER EDINDX(20),LCERST
  INTEGER LNODS(8)
  INCLUDE 'ELPARM'
C
  COMMON /CONTROL/ MPOIN,NPOIN,NELEM,NNODE,NDOFN,NDIME,NSTRE,
! NPROP,NMATS,NVFIX,NEVAB,NINTR
  COMMON /FIXEDBC/ PRES(100,3),NOFIX(100),IFPRE(100,2),
! ISNODE(750),ISELEM(500)
C KGAST = 0
C THERMAL = .FALSE.
C
  NNODE = 8
  NDIME = 3

```

```

NINTR = 1
NEVAB = 24
NGAUS = 3
NSTRE = 6
NDOFN = 3
NPROP = 13
C  WRITE HEADER
C
IOTT = WRINQR(2)
WRITE (IOTT,*) ' CALCULATING STRESSES'
WRITE (6,2)
2  FORMAT (/,' ELEM',2X,
!      ' X   Y   Z', ' SX   SY   SZ',
!      ' SXY  SYZ  SXZ')
C
C  CALCULATE AND OUTPUT STRESSES FOR EACH ELEMENT
C
      KGAST = 0
C
C  EVALUATE THE COORDINATES OF THE ELEMENT NODES
C
C
C
DO 80 INODE = 1,NNODE
C  LNODE = IABS(LNODS(IELEM,INODE))
DO 80 IDIME = 1,NDIME
      ELCOD(IDIME,INODE) = COORD(IDIME,INODE)
80  CONTINUE
      XCENT = 0.0D0
      YCENT = 0.0D0
C
C  --- Z-HEIGHT OF ELEMENT
C
      DZELEM = ELCOD(3,5)-ELCOD(3,1)
      Z1 = ELCOD(3,1)
DO 9 INODE = 1,NNODE
      XCENT = XCENT + ELCOD(1,INODE)*.125
      YCENT = YCENT + ELCOD(2,INODE)*.125
9  CONTINUE
C
C  IDENTIFY THE DISPLACEMENTS OF THE ELEMENT NODAL POINTS
C
      NPOSN = 0
DO 10 INODE = 1,NNODE
DO 10 IDOFN = 1,NDOFN
      NPOSN = NPOSN+1
      ELDIS(IDOFN,INODE) = ASDIS(NPOSN,1)
10  CONTINUE
C
C  CALCULATE THE STRESS AND STRAIN MATRIX FOR THE ELEMENT
C
      DO 90 ILAYER = 1,NLAYERS
      RHOSP = -1.+(2.*FLOAT(ILAYER)-1.)/FLOAT(NLAYERS)
      ZCNT = Z1 + .5*(1.+RHOSP)*DZELEM
C

```

```

C   ZERO OUT THE TEMPORARY STRESSES
C
  NGASP = NGAUS*NGAUS
  DO 5 JSTRE = 1,NSTRE
    DO 5 IGASP = 1,NGASP
      STEMP(IGASP,JSTRE) = 0.
      SRTMP(IGASP,JSTRE) = 0.
  5  CONTINUE
C
C   EVALUATE THE D-MATRIX
C
  CALL DCHRIST(IELEM,ILAYER,IALPHA,PROPS,MATNO,DMATX)
C
  KGASP = 0
  DO 40 IGAUS = 1,NGAUS
    DO 40 JGAUS = 1,NGAUS
      KGASP = KGASP+1
      EXISP = POSGP(IGAUS)
      ETASP = POSGP(JGAUS)
C
C   EVALUATE THE SHAPE FUNCTIONS, ELEMENTAL VOLUME, ETC.
C
  CALL SFBRIK(EXISP,ETASP,RHOSP,SHAPE,DERIV)
  CALL JACBRIK(IELEM,DJACB,CARTD,DERIV,ELCOD)
C
C   EVALUATE THE B AND BXD MATRICES
C
  CALL BBRICK(BMATX,CARTD)
  CALL DBE(BMATX,DMATX,DBMAT)
C
C   STORE THE COMPONENTS OF THE STRESS AND STRAIN MATRICES
C
  DO 15 ISTRE = 1,NSTRE
    DO 15 IEVAB = 1,NEVAB
      SMATX(ISTRE,IEVAB,KGASP) = DBMAT(ISTRE,IEVAB)
      SRMAT(ISTRE,IEVAB,KGASP) = BMATX(ISTRE,IEVAB)
  15  CONTINUE
  40  CONTINUE
C
C   NOW TO CALCULATE STRESSES
C
  KGASP = 0
C
C   ENTER LOOPS OVER EACH GAUSS POINT IN THE LAYER
C
  DO 50 IGAUS = 1,NGAUS
    DO 50 JGAUS = 1,NGAUS
      KGAST = KGAST+1
      KGASP = KGASP+1
C
C   COMPUTE THE STRESS AND STRAIN COMPONENTS AT THE SAMPLING
C   POINTS
C
  DO 20 ISTRE = 1,NSTRE
    KGASH = 0

```

```

DO 20 INODE = 1,NNODE
DO 20 IDOFN = 1,NDOFN
KGASH = KGASH+1
STEMP(KGASP,ISTRE) = STEMP(KGASP,ISTRE)+
! SMATX(ISTRE,KGASH,KGASP)*ELDIS(IDOFN,INODE)
SRTMP(KGASP,ISTRE) = SRTMP(KGASP,ISTRE)+
! SRMAT(ISTRE,KGASH,KGASP)*ELDIS(IDOFN,INODE)
20 CONTINUE
C
C FOR THERMAL LOADING, ADD ON THE INITIAL THERMAL STRESS
C
C IF (THERMAL) THEN
C READ (1,REC=IELEM) STRIN,STRAN
C DO 30 ISTR1 = 1,NSTRE
C STEMP(KGASP,ISTR1) = STEMP(KGASP,ISTR1)+STRIN(ISTR1,KGASP)
C SRTMP(KGASP,ISTR1) = SRTMP(KGASP,ISTR1)+STRAN(ISTR1,KGASP)
C 30 CONTINUE
C ENDIF
50 CONTINUE
3 FORMAT (2I4,3F10.4,2(/,1X,6G12.5))
C
C OUTPUT STRESSES FOR THIS LAYER
C
C
C --- AT THIS POINT UEP102 SHOULD BE CALLED FOR OUTPUT
C
NGASP = NGAUS*NGAUS
DGASP = 2.*FLOAT(NGASP)
DO 60 LSTRE=1,NSTRE
STRES(LSTRE) = 0.
STRAI(LSTRE) = 0.
60 CONTINUE
DO 70 KSTRE = 1,NSTRE
DO 70 IGASP = 1,NGASP
STRES(KSTRE) = STRES(KSTRE)+STEMP(IGASP,KSTRE)/DGASP
STRAI(KSTRE) = STRAI(KSTRE)+SRTMP(IGASP,KSTRE)/DGASP
70 CONTINUE
90 WRITE (IOTT,3) IELEM,ILAYER,XCENT,YCENT,ZCENT,(STRES(ISTRE),
! ISTR=1,NSTRE),(STRAI(ISTRE),ISTR=1,NSTRE)
CALL ELDWRT(IELM,EDENS,LCERST,EDINDX(1),NSTRE,STRES)
RETURN
END

```

## **APPENDIX B**

First Row of Elements (45,-45 layers)

ELMDAT( 1) = 1 for Element 45  
CALCULATING STRESSES

| ELEM | X      | Y       | Z      | SX          | SY           | SZ           | SXY          | SYZ          | SXZ         |
|------|--------|---------|--------|-------------|--------------|--------------|--------------|--------------|-------------|
| 45 1 | 0.5000 | -0.0500 | 0.0250 | 85665.      | -487.81      | 481.11       | 24702.       | 82.856       | 8328.7      |
|      |        |         |        | 0.30951E-01 | -0.20127E-01 | -0.26677E-02 | -0.44800E-02 | 0.98246E-04  | 0.98758E-02 |
| 45 2 | 0.5000 | -0.0500 | 0.0750 | 87187.      | 1504.4       | 521.47       | -26619.      | -69.370      | 8306.2      |
|      |        |         |        | 0.30844E-01 | -0.19955E-01 | -0.26677E-02 | 0.41761E-02  | -0.82256E-04 | 0.98491E-02 |

ELMDAT( 1) = 1 for Element 39  
CALCULATING STRESSES

| ELEM | X      | Y       | Z      | SX          | SY           | SZ           | SXY          | SYZ          | SXZ         |
|------|--------|---------|--------|-------------|--------------|--------------|--------------|--------------|-------------|
| 39 1 | 0.5000 | -0.1500 | 0.0250 | 91559.      | 687.80       | -355.99      | 35803.       | 49.614       | 563.02      |
|      |        |         |        | 0.30953E-01 | -0.22922E-01 | -0.22899E-02 | -0.48002E-04 | 0.58830E-04  | 0.66760E-03 |
| 39 2 | 0.5000 | -0.1500 | 0.0750 | 89013.      | -2093.8      | -402.32      | -32834.      | -116.77      | 559.10      |
|      |        |         |        | 0.30985E-01 | -0.23030E-01 | -0.22899E-02 | 0.54083E-03  | -0.13846E-03 | 0.66295E-03 |

ELMDAT( 1) = 1 for Element 33  
CALCULATING STRESSES

| ELEM | X      | Y       | Z      | SX          | SY           | SZ           | SXY          | SYZ          | SXZ         |
|------|--------|---------|--------|-------------|--------------|--------------|--------------|--------------|-------------|
| 33 1 | 0.5000 | -0.2500 | 0.0250 | 91228.      | -503.58      | 43.513       | 34929.       | -43.803      | 50.369      |
|      |        |         |        | 0.31176E-01 | -0.23210E-01 | -0.20985E-02 | -0.15644E-03 | -0.51939E-04 | 0.59726E-04 |
| 33 2 | 0.5000 | -0.2500 | 0.0750 | 92541.      | 511.04       | 46.429       | -36300.      | -38.825      | 45.634      |
|      |        |         |        | 0.31266E-01 | -0.23296E-01 | -0.20985E-02 | -0.95467E-04 | -0.46037E-04 | 0.54111E-04 |

ELMDAT( 1) = 1 for Element 27  
CALCULATING STRESSES

| ELEM | X           | Y            | Z            | SX           | SY          | SZ           | SXY     | SYZ    | SXZ      |
|------|-------------|--------------|--------------|--------------|-------------|--------------|---------|--------|----------|
| 27 1 | 0.5000      | -0.3500      | 0.0250       | 92021.       | 192.20      | 60.987       | 35598.  | 15.948 | -0.80817 |
|      | 0.31249E-01 | -0.23195E-01 | -0.21144E-02 | -0.10541E-03 | 0.18911E-04 | -0.95829E-06 |         |        |          |
| 27 2 | 0.5000      | -0.3500      | 0.0750       | 93056.       | 1234.2      | 69.668       | -36795. | 24.362 | -10.832  |
|      | 0.31254E-01 | -0.23186E-01 | -0.21144E-02 | -0.10709E-03 | 0.28887E-04 | -0.12844E-04 |         |        |          |

ELMDAT( 1) = 1 for Element 21  
CALCULATING STRESSES

| ELEM | X           | Y            | Z            | SX          | SY          | SZ           | SXY     | SYZ    | SXZ     |
|------|-------------|--------------|--------------|-------------|-------------|--------------|---------|--------|---------|
| 21 1 | 0.5000      | -0.4500      | 0.0250       | 92654.      | 999.15      | 69.529       | 36388.  | 18.344 | -1.2282 |
|      | 0.31210E-01 | -0.23130E-01 | -0.21176E-02 | 0.20634E-04 | 0.21751E-04 | -0.14564E-05 |         |        |         |
| 21 2 | 0.5000      | -0.4500      | 0.0750       | 92285.      | 732.56      | 48.222       | -36100. | 12.680 | -12.178 |
|      | 0.31162E-01 | -0.23117E-01 | -0.21176E-02 | 0.40114E-05 | 0.15035E-04 | -0.14440E-04 |         |        |         |

ELMDAT( 1) = 1 for Element 15  
CALCULATING STRESSES

| ELEM | X           | Y            | Z            | SX          | SY          | SZ           | SXY     | SYZ    | SXZ     |
|------|-------------|--------------|--------------|-------------|-------------|--------------|---------|--------|---------|
| 15 1 | 0.5000      | -0.5500      | 0.0250       | 92654.      | 1148.9      | 82.563       | 36468.  | 27.817 | -2.3970 |
|      | 0.31167E-01 | -0.23085E-01 | -0.21125E-02 | 0.33844E-04 | 0.32984E-04 | -0.28422E-05 |         |        |         |
| 15 2 | 0.5000      | -0.5500      | 0.0750       | 91328.      | -39.729     | -12.260      | -35374. | 29.173 | -9.7503 |
|      | 0.31048E-01 | -0.23121E-01 | -0.21125E-02 | 0.40684E-04 | 0.34592E-04 | -0.11561E-04 |         |        |         |



ELMDAT( 1) = 1 for Element 9  
CALCULATING STRESSES

| ELEM                                                                    | X       | Y       | Z       | SX      | SY      | SZ | SXY | SYZ | SXZ |
|-------------------------------------------------------------------------|---------|---------|---------|---------|---------|----|-----|-----|-----|
| 9 1                                                                     | 0.5000  | -0.6500 | 0.0250  |         |         |    |     |     |     |
| 91468.                                                                  | 411.51  | -311.82 | 35112.  | -172.14 | -443.69 |    |     |     |     |
| 0.31105E-01-0.22880E-01-0.23220E-02-0.34010E-03-0.20412E-03-0.52610E-03 |         |         |         |         |         |    |     |     |     |
| 9 2                                                                     | 0.5000  | -0.6500 | 0.0750  |         |         |    |     |     |     |
| 90467.                                                                  | -266.98 | -471.66 | -34788. | 3.1402  | -438.84 |    |     |     |     |
| 0.30879E-01-0.22916E-01-0.23220E-02 0.18055E-03 0.37235E-05-0.52036E-03 |         |         |         |         |         |    |     |     |     |

ELMDAT( 1) = 1 for Element 3  
CALCULATING STRESSES

| ELEM                                                                    | X       | Y       | Z       | SX      | SY      | SZ | SXY | SYZ | SXZ |
|-------------------------------------------------------------------------|---------|---------|---------|---------|---------|----|-----|-----|-----|
| 3 1                                                                     | 0.5000  | -0.7500 | 0.0250  |         |         |    |     |     |     |
| 86481.                                                                  | 279.66  | 250.74  | 25474.  | -189.82 | -8152.0 |    |     |     |     |
| 0.31016E-01-0.20091E-01-0.27950E-02-0.44209E-02-0.22508E-03-0.96663E-02 |         |         |         |         |         |    |     |     |     |
| 3 2                                                                     | 0.5000  | -0.7500 | 0.0750  |         |         |    |     |     |     |
| 85291.                                                                  | -459.24 | 51.317  | -25167. | 33.788  | -8122.8 |    |     |     |     |
| 0.30719E-01-0.20120E-01-0.27950E-02 0.42034E-02 0.40064E-04-0.96317E-02 |         |         |         |         |         |    |     |     |     |

Second Row of Elements (-45,45 layers)

ELMDAT( 2) = 2 for Element 93  
CALCULATING STRESSES

| ELEM                                                                    | X       | Y       | Z       | SX      | SY      | SZ | SXY | SYZ | SXZ |
|-------------------------------------------------------------------------|---------|---------|---------|---------|---------|----|-----|-----|-----|
| 93 1                                                                    | 0.5000  | -0.0500 | 0.1250  |         |         |    |     |     |     |
| 87187.                                                                  | 1504.4  | 521.47  | -26619. | 69.370  | -8306.2 |    |     |     |     |
| 0.30844E-01-0.19955E-01-0.26677E-02 0.41761E-02 0.82256E-04-0.98491E-02 |         |         |         |         |         |    |     |     |     |
| 93 2                                                                    | 0.5000  | -0.0500 | 0.1750  |         |         |    |     |     |     |
| 85665.                                                                  | -487.81 | 481.11  | 24702.  | -82.856 | -8328.7 |    |     |     |     |
| 0.30951E-01-0.20127E-01-0.26677E-02-0.44800E-02-0.98246E-04-0.98758E-02 |         |         |         |         |         |    |     |     |     |

ELMDAT( 2) = 2 for Element 87  
CALCULATING STRESSES

| ELEM                                                                    | X       | Y       | Z       | SX      | SY      | SZ | SXY | SYZ | SXZ |
|-------------------------------------------------------------------------|---------|---------|---------|---------|---------|----|-----|-----|-----|
| 87 1                                                                    | 0.5000  | -0.1500 | 0.1250  |         |         |    |     |     |     |
| 89013.                                                                  | -2093.8 | -402.32 | -32834. | 116.77  | -559.10 |    |     |     |     |
| 0.30985E-01-0.23030E-01-0.22899E-02 0.54083E-03 0.13846E-03-0.66295E-03 |         |         |         |         |         |    |     |     |     |
| 87 2                                                                    | 0.5000  | -0.1500 | 0.1750  |         |         |    |     |     |     |
| 91559.                                                                  | 687.80  | -355.99 | 35803.  | -49.614 | -563.02 |    |     |     |     |
| 0.30953E-01-0.22922E-01-0.22899E-02-0.48002E-04-0.58830E-04-0.66760E-03 |         |         |         |         |         |    |     |     |     |

ELMDAT( 2) = 2 for Element 81  
CALCULATING STRESSES

| ELEM                                                                    | X       | Y       | Z       | SX     | SY      | SZ | SXY | SYZ | SXZ |
|-------------------------------------------------------------------------|---------|---------|---------|--------|---------|----|-----|-----|-----|
| 81 1                                                                    | 0.5000  | -0.2500 | 0.1250  |        |         |    |     |     |     |
| 92541.                                                                  | 511.04  | 46.429  | -36300. | 38.825 | -45.634 |    |     |     |     |
| 0.31266E-01-0.23296E-01-0.20985E-02-0.96467E-04 0.45037E-04-0.54111E-04 |         |         |         |        |         |    |     |     |     |
| 81 2                                                                    | 0.5000  | -0.2500 | 0.1750  |        |         |    |     |     |     |
| 91228.                                                                  | -503.58 | 43.513  | 34929.  | 43.803 | -50.369 |    |     |     |     |
| 0.31176E-01-0.23210E-01-0.20985E-02-0.15644E-03 0.51939E-04-0.59726E-04 |         |         |         |        |         |    |     |     |     |

ELMDAT( 2) = 2 for Element 75  
CALCULATING STRESSES

| ELEM                                                                    | X      | Y       | Z       | SX      | SY      | SZ | SXY | SYZ | SXZ |
|-------------------------------------------------------------------------|--------|---------|---------|---------|---------|----|-----|-----|-----|
| 75 1                                                                    | 0.5000 | -0.3500 | 0.1250  |         |         |    |     |     |     |
| 93056.                                                                  | 1234.2 | 69.668  | -36795. | -24.362 | 10.832  |    |     |     |     |
| 0.31254E-01-0.23186E-01-0.21144E-02-0.10709E-03-0.28837E-04 0.12844E-04 |        |         |         |         |         |    |     |     |     |
| 75 2                                                                    | 0.5000 | -0.3500 | 0.1750  |         |         |    |     |     |     |
| 92021.                                                                  | 192.20 | 60.987  | 35598.  | -15.948 | 0.80817 |    |     |     |     |
| 0.31249E-01-0.23195E-01-0.21144E-02-0.10541E-03-0.18911E-04 0.95829E-06 |        |         |         |         |         |    |     |     |     |

ELMDAT( 2) = 2 for Element 69  
CALCULATING STRESSES

| ELEM                                                                    | X      | Y       | Z      | SX     | SY     | SZ     | SXY     | SYZ     | SXZ    |
|-------------------------------------------------------------------------|--------|---------|--------|--------|--------|--------|---------|---------|--------|
| 69 1                                                                    | 0.5000 | -0.4500 | 0.1250 | 92285. | 732.56 | 48.222 | -36100. | -12.680 | 12.178 |
| 0.31162E-01-0.23117E-01-0.21176E-02 0.40114E-05-0.15035E-04 0.14440E-04 |        |         |        |        |        |        |         |         |        |
| 69 2                                                                    | 0.5000 | -0.4500 | 0.1750 | 92654. | 999.15 | 69.529 | 36388.  | -18.344 | 1.2282 |
| 0.31210E-01-0.23130E-01-0.21176E-02 0.20634E-04-0.21751E-04 0.14564E-05 |        |         |        |        |        |        |         |         |        |

ELMDAT( 2) = 2 for Element 63  
CALCULATING STRESSES

| ELEM                                                                    | X      | Y       | Z      | SX     | SY      | SZ      | SXY     | SYZ     | SXZ    |
|-------------------------------------------------------------------------|--------|---------|--------|--------|---------|---------|---------|---------|--------|
| 63 1                                                                    | 0.5000 | -0.5500 | 0.1250 | 91328. | -39.729 | -12.260 | -35374. | -29.173 | 9.7503 |
| 0.31048E-01-0.23121E-01-0.21125E-02 0.40684E-04-0.34592E-04 0.11561E-04 |        |         |        |        |         |         |         |         |        |
| 63 2                                                                    | 0.5000 | -0.5500 | 0.1750 | 92654. | 1148.9  | 82.563  | 36468.  | -27.817 | 2.3970 |
| 0.31167E-01-0.23085E-01-0.21125E-02 0.33844E-04-0.32984E-04 0.28422E-05 |        |         |        |        |         |         |         |         |        |

ELMDAT( 2) = 2 for Element 57  
CALCULATING STRESSES

| ELEM                                                                    | X      | Y       | Z      | SX     | SY      | SZ      | SXY     | SYZ     | SXZ    |
|-------------------------------------------------------------------------|--------|---------|--------|--------|---------|---------|---------|---------|--------|
| 57 1                                                                    | 0.5000 | -0.6500 | 0.1250 | 90467. | -266.98 | -471.66 | -34788. | -3.1402 | 438.84 |
| 0.30879E-01-0.22916E-01-0.23220E-02 0.18055E-03-0.37235E-05 0.52036E-03 |        |         |        |        |         |         |         |         |        |
| 57 2                                                                    | 0.5000 | -0.6500 | 0.1750 | 91468. | 411.51  | -311.82 | 35112.  | 172.14  | 443.69 |
| 0.31105E-01-0.22880E-01-0.23220E-02-0.34010E-03 0.20412E-03 0.52610E-03 |        |         |        |        |         |         |         |         |        |

ELMDAT( 2) = 2 for Element 51  
CALCULATING STRESSES

| ELEM | X           | Y            | Z            | SX           | SY           | SZ          | SXY     | SYZ     | SXZ    |
|------|-------------|--------------|--------------|--------------|--------------|-------------|---------|---------|--------|
| 51 1 | 0.5000      | -0.7500      | 0.1250       | 85291.       | -459.24      | 51.317      | -25167. | -33.788 | 8122.8 |
|      | 0.30719E-01 | -0.20120E-01 | -0.27950E-02 | 0.42034E-02  | -0.40064E-04 | 0.96317E-02 |         |         |        |
| 51 2 | 0.5000      | -0.7500      | 0.1750       | 86481.       | 279.66       | 250.74      | 25474.  | 189.82  | 8152.0 |
|      | 0.31016E-01 | -0.20091E-01 | -0.27950E-02 | -0.44209E-02 | 0.22508E-03  | 0.96663E-02 |         |         |        |

## APPENDIX C