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13. ABSTRACT *(Maximum 200 words)*

This is the final report for the ONR grant. This report summarizes the results obtained in graph theory that are related to funded research. The report divides the publications into topics or areas and briefly lists results obtained in each area. A list of papers published, accepted, and submitted is also given.

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Final Report
on
Diametral Path Graphs and Incremental Distance Sequences

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Summary of Research

Our research efforts funded by ONR have produced excellent results in several different areas of graph theory and its applications.

Hamiltonian Graphs: In Hamiltonian graphs our approach was twofold. On one hand we investigated sufficient degree and edge conditions for balanced bipartite graphs to be hamiltonian. On the other hand, we developed algorithms for finding Hamiltonian paths and cycles in permutation and cocomparability graphs. It may be noted that Hamiltonian cycle problem was well known open problem since 1985. We developed $O(n^2)$ algorithm for permutation graphs and $O(n^3)$ algorithm for cocomparability graphs. In addition, toughness properties of permutation graphs and cocomparability graphs were also investigated. These results contribute significantly to the understanding of Hamiltonian properties.

Line Graphs and their generalizations: Line graphs provide a way of studying the graph by concentrating attention on edges without regard to vertices. We generalized the notion of line graphs to super line graphs and obtained several results about their properties. Our approach studies line graphs combinatorially, by looking at sets of edges of a given cardinality. Several interresting new parameters related to the notion of super line graphs have been introduced and studied. This study contributes significantly to the generalizations of the line graph transformation.

Diametral Path Graphs We developed the concept of Diametral Path graphs. Some properties of diametral path graphs were investigated. Characterization of chordal diametral path graphs as well as weakly triangulated diametral path graphs in terms forbidden subgraph was developed.

Integrity and its Generalizations: We have introduced the concept of Pure Integrity, which is a measure of vulnerability for graphs and networks. Pure integrity is important not only because it extends earlier work on integrity but also because it is related to Integrity through the notion of line graphs. Applications of integrity to distributed and multiprocessor systems were also investigated.

Ranking of Graphs: We have studied the algorithmic complexity of the VERTEX RANKING and EDGE RANKING problems. We characterize those graphs where the vertex ranking number χ_r and the chromatic number χ coincide on all induced subgraphs. In addition, we develop vertex ranking algorithms for permutation graphs and discuss how the technique developed can be applied for finding vertex rankings of interval graphs,

circular arc graphs, trapezoid graphs and cocomparability graphs of bounded dimension.

Visibility Graphs: Visibility properties of certain geometric objects have been studied in computational geometry and its applications to robotics, and path planning in the presence of obstacles. Visibility graphs provide a very good vehicle for the underlying graph theoretic properties of the structures determined by the given geometric objects. We investigated visibility graphs of segments in the plane, and proved results regarding the bounds and realizability of all values between these bounds for the sizes of these visibility graphs.

Tournaments: Packing and covering problems in tournaments were investigated, and in particular, we proved results for cyclic and transitive triples.

Papers (Published or Accepted)

1. "On the Pure Edge Integrity of Graphs," J. Bagga and J. S. Deogun. to appear in *Graph Theory, Combinatorics, and Algorithms: Proceedings of the Seventh Quadrennial International Conference on the Theory and Applications of Graphs.*

2. "Super Line Graphs, " J. Bagga, L. Beineke and B. Varma. to appear in *Graph Theory, Combinatorics, and Algorithms: Proceedings of the Seventh Quadrennial International Conference on the Theory and Applications of Graphs.*

3. "Permutation Graphs: Hamiltonian Paths," C. Riedesel, and J. S. Deogun. Accepted to appear in *Journal of Combinatorial Mathematics and Combinatorial Computing.*

4. "The Line Completion Number of a Graph," J. Bagga, L. Beineke and B. Varma, to appear in *Graph Theory, Combinatorics, and Algorithms: Proceedings of the Seventh Quadrennial International Conference on the Theory and Applications of Graphs.*

5. "Ranking of graphs," H. Bodlaender, J. S. Deogun, K. Jansen, T. Kloks, D. Kratsch, H. M"uller, and Zs. Tuza. Accepted to appear in *Proceedings of International Workshop on Graph-Theoretic Concepts in Computer Science*, Germany.

6. "On the Sizes of Some Classes of Visibility Graphs," J. Bagga, M. McGrew, J. Emert, and W. Toll. to appear in Congressus Numerantium, vol. 100-104.

8. "Polynomial Algorithms for Hamiltonian Cycles in Cocomparability Graphs," J. S. Deogun, and G. Steiner. SIAM *J. on Computing*, Vol. 23, No.3, pp. 520-552, June 1994.

9. "On vertex Ranking for Permutation and other Graphs ," J. S. Deogun, T. Kloks, D. Kratsch, and H. Müller. Proceedings of the 11^{th} Symposium on Theoretical Aspects of Computer Science (STACS'94), Lecture Notes in Computer Science, #775, Springer-Verlag, February, 1994, Caen, France, pp. 747-758.

10. "The Super Line Graph $\mathcal{L}_2$ for Hypercubes," J. Bagga and M. R. Vasquez. *Congressus Numerantium*, vol 93, December 1993, pp 111-113.

11. "On the Size of Minimal Visibility Graphs," J. Bagga, M. McGrew, J. Emert, and W. Toll. Department of Mathematical Sciences Technical Report #87, Ball State University, April 1993.

12. "Two Problems on Coloring Tournaments," J. Bagga, L. Beineke and F. Harary. *Vishwa International Journal of Graph Theory*, vol 1, December 1992.

13. "A Variation on the Edge-Integrity," J. Bagga and J. S. Deogun. *Congressus Numerantium*, vol 91, 1992, pp207-211.

14. "Hamiltonian Cycle is Polynomial on Cocomparability Graphs," J. S. Deogun, and G. Steiner. *Discrete Applied Mathematics*, vol. 39, 1992, pp. 165-172.

15. "Finding Hamiltonian Paths in Cocomparability Graphs Using The Bump Number Algorithm," P. Damaschke, J. S. Deogun, D. Kratsch, and G. Steiner. *The Order* Journal, 8:383 - 391, 1992.

16. "Degree Conditions and Long Cycles in Bipartite Graphs," J. Bagga and B. Varma. *Congressus Numerantium*, vol 85, 1991, pp 123-128.

Papers Submitted

17 "Super Line Graphs and Their Properties," J. Bagga, L. Beineke and B. Varma. submitted to the Proceedings of the Third China-USA International Conference on Graph Theory, Algorithms, and Applications.

18 "Toughness and Hamiltonicity in Permutation Graphs," C. Riedesel, and J. S. Deogun. Submitted to *Journal of Combinatorial Mathematics and Combinatorial Computing*, 1994.

19 "Efficient Algorithms for Mapping A Special Class of Task Graphs onto Linear Array Microprocessors," S. Ray, H. Jiang, and J. S. Deogun. Submitted to *Journal of Computer and Software Engineering* , 1994.

20 "Diametral Path Graphs," J. S. Deogun, and D. Kratsch. Submitted to *Discrete Applied Mathematics*, 1994.

21 "A Measurement of Vulnerability for Distributed Computing Systems: Weighted Integrity and its Applications," S. Ray, H. Jiang, and J. S. Deogun. Submitted to *Journal of Computer and Software Engineering* , 1994.

22 "1-tough cocomparability graphs are hamiltonian," J. S. Deogun, D. Kratsch, and G. Steiner. Submitted to *Discrete Applied Mathematics*, 1994.