

ON A PARTICULAR FOURIER INVERSION  
by  
Dr. Robert A. Granger  
EW-19-94



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19941214 039

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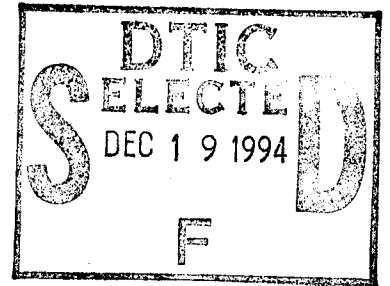
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A function  $f(x)$  is the Fourier transform of  $g(x)$  if

$$f(x) = (1/2\pi)^{0.5} \int_{-\infty}^{\infty} g(t) e^{itx} dt \quad \dots 1)$$

Under suitable conditions on  $g(x)$ , it then follows that

$$g(x) \doteq (1/2\pi)^{0.5} \int_{-\infty}^{\infty} f(t) e^{-itx} dt \quad \dots 2)$$

where the value of the right member is

$$\lim_{h \rightarrow 0} \frac{1}{2} [g(x+h) + g(x-h)] \quad \dots 3)$$

if  $g(x)$  is of bounded variation in the neighborhood of  $x$ . Such functions  $f(x)$  and  $g(x)$  are sometimes said to be a pair of Fourier transforms.

Inversion formulae are solutions of the integral equation 2). Conditions must be imposed on  $f(t)$  and on the path of integration of the contour integrals.

In formulating a solution to a particular problem, it becomes necessary to evaluate the Fourier inversion expression

$$f(x) = (1/2\pi)^{0.5} \int_{-\infty}^{\infty} F(u) e^{-iux} du \quad \dots 4)$$

which is identical to equation 2) save for the dummy variable of integration, where the integrand  $F(u)$  is given by

$$F(u) = u G(u) \ln(u+a). \quad \dots 5)$$

We rewrite equation 5) as

$$F(u) = u^m (u+a) \bar{G}(u) \bar{H}(u) \quad \dots 6)$$

where

$$\bar{H}(u) = [1/(u+a)] \ln(u+a) \quad \dots 7)$$

and  $\bar{G}\bar{H}$  is the transform of a product. According to Bateman, Ref. 1, the inversion of this part is

$$[i^m / (2\pi)^{0.5}] \frac{\partial^m}{\partial x^m} (i \frac{\partial}{\partial x} + a) \int_{-\infty}^{\infty} G(\xi) H(x-\xi) d\xi \quad \dots 8)$$

The function  $H(x)$  is obtained from Titchmarsh, Ref. 2, as

$$H(x) = [1/(2\pi)^{0.5}] \int_{-\omega + iU''}^{\infty + iU''} \bar{H}(u) e^{ux} dx \quad \dots 9)$$

Let

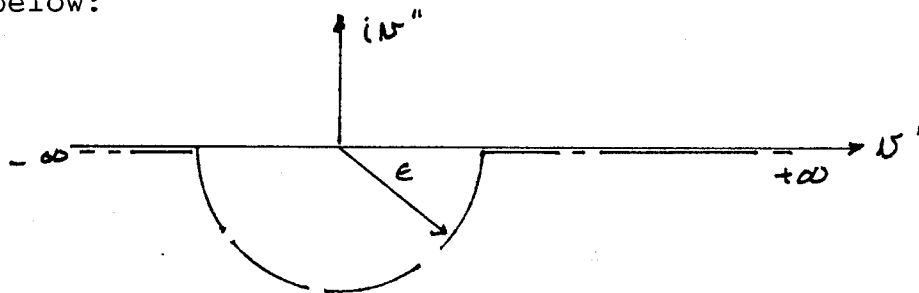
$$u + a = v \quad \dots 10)$$

$$v = v' + iv'' \quad \dots 11)$$

Then

$$H(x) = [1/(2\pi)^{0.5}] e^{iax} \int_{-\omega + iU''}^{\infty + iU''} (1/v) \ln v e^{iv'x} dv \quad \dots 12)$$

Since the only pole of the integral is situated at  $v = 0$ , it is possible to choose the path of integration according to the figure below:



and then let  $\epsilon \rightarrow 0$ . For  $\log v$ , the branch is chosen which is purely real along the positive abscissa to make  $\log v$  single valued. For  $H(x)$  this results with  $v = \epsilon e^{i\theta}$  along the semi-circle.

#### References

1. Bateman, Harry, Higher Transcendental Functions, McGraw Hill Book Co., NY, 1953
2. Titchmarsh, E.C., Theory of Functions, Oxford University Press, 1932