

# REPORT DOCUMENTATION PAGE

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6. AUTHOR(S) <b>Gregory Beylkin</b>				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) <b>Program in Applied Mathematics University of Colorado, Boulder Campus Box 526 Boulder, CO 80309-0526</b>			8. PERFORMING ORGANIZATION REPORT NUMBER  <b>1534646</b>	
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13. ABSTRACT (Maximum 200 words)  <p>During the past three years there have been 7 publications, two of which are chapters in books, in addition to 8 Program in Applied Mathematics Reports (preprints) which have been submitted for publication. Also, the PI has been invited to give ~20 lectures at universities and conferences in the US and abroad. The research program has been productive. A brief overview is given below; for the full details we refer to the publications.</p> <p style="font-size: 2em; font-weight: bold; margin-top: 20px;">19941227 022</p> <p style="text-align: right; font-weight: bold; margin-top: 10px;">BRIEF QUALITY INSPECTED 2</p>				
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**ONR Grant: N00014-91-J-4037**

**Nonlinear Problems in Fluid Dynamics and Inverse Scattering**

**Principle Investigator: Gregory Beylkin**

**Subtitle: Propagation and capturing of singularities in problems of fluid dynamics and inverse scattering**

**Program Manager: Dr. Reza Malek-Madani**

## **FINAL REPORT**

### **Overview:**

Research investigations supported under this grant addressed problems of solving PDE's using orthonormal bases with controlled localization in the time-frequency domain (e.g. wavelets). We have developed a number of fast algorithms and started investigations in several new areas. Among the results is the Unequally Spaced Fast Fourier Transform (USFFT) algorithm and a new approach to numerical homogenization.

### **Research Activities:**

With James Keiser we have developed an adaptive approach for solving non-linear evolution equations in wavelet bases. The analysis provides a fundamental understanding of how to develop algorithms of this kind. This work is part of Keiser's thesis.

Using the Battle-Lemarie scaling function we have constructed a fast algorithm to evaluate the Fourier transform of generalized functions. In particular, we have developed an algorithm for the Unequally Spaced Fast Fourier Transform and tested its performance in dimensions one and two. Also we have a new algorithm to evaluate the Fourier Transform of an image taking into account the discontinuities within the image. Our approach also has implications for numerical solutions of PDE's which we plan to investigate further.

Jointly with Mary E. Brewster, we have constructed a numerical scheme for homogenization of linear differential and difference equations via multiresolution analysis. This work has potential for significant impact by permitting us to construct the equations for computing the projection on a sparse scale of the solution of equations given on the very fine scales.

Jointly with John Dunn and David Gines we developed a practical hybrid method for solving static and quasi-static problems in electromagnetics using wavelets.

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Jointly with Bruno Torresani we developed a method of applying operators in wavelet bases. This method is different from both standard and non-standard forms that have been developed previously and is very efficient for convolution operators, e.g. the Hilbert transform.

### **Work with graduate students:**

This work was supported via an AASERT Grant. I am working with two students, James Keiser and Robert Cramer.

The work with James Keiser on solving partial differential equations in wavelet bases is almost finished and James will be graduating soon. The results will be submitted for publication.

The work with Robert Cramer on the multidimensional algorithms for the fast evaluation of a class of integral operators on functions is continuing. We expect to release some of the results shortly.

### Books

Co-Editor, *Wavelets and Their Applications*, Jones and Bartlett, 1992  
ISBN 0-86720-225-4

### Published Papers:

"Wavelets in Numerical Analysis", G. Beylkin, R.R.Coifman and V. Rokhlin in *Wavelets and Their Applications*, Jones and Bartlett, 1992

"On the representation of operators in bases of compactly supported wavelets", G. Beylkin, *SIAM Journal on Numerical Analysis.*, 29, (6), pp. 1716-1740, 1992.

"Wavelets and Fast Numerical algorithms", G. Beylkin, *Proceedings of Symposia in Applied Mathematics*, vol. 47, 1993

"Wavelet-Like Bases for the Fast Solution of Second-Kind Integral Equation", B. Alpert, G. Beylkin, R.R. Coifman and V. Rokhlin, *SIAM Journal on Scientific and Statistical Computing*, v. 14, (1), pp. 159-184, 1993

"Multiresolution representations using the auto-correlation functions of compactly supported wavelets", N. Saito and G. Beylkin, , *IEEE Transactions on Signal Processing*, v.41, Dec.1993, pp.3584-3590.

"On the fast algorithm for multiplication of functions in the wavelet bases", *Proceedings of the conference on application of wavelets in Toulouse*, June 1992. (PAM Report #152, August 1992).

"On wavelet-based algorithms for solving differential equations",  
*Wavelets: Mathematics and Applications*, Ed. J.J.Benedetto & M.W.Frazier, CRC Press,  
1994 ( PAM Report #153, Dec. 1992)

Preprints:

"Wavelets and Fast Numerical Algorithms", Short Course, AMS-93, April 1993

"Inversion and Application of the Generalized Radon Transform", to  
appear in Proceedings of Conference "75 years of Radon Transform",  
Vienna, Austria, Aug. 30 - Sept.5, 1992. PAM Report 154, Jan. 1993.

"On Factored FIR Approximation of IIR Filters", G. Beylkin, November 1993, PAM  
Report #181

"Implementation of Operators Via Filter Banks, Autocorrelation Shell and Hardy  
Wavelets", G. Beylkin and B. Torresani, February 1994, PAM Report #185

"A Multiresolution Strategy for Numerical Homogenization", M.E. Brewster and G.  
Beylkin, March 1994, PAM Report #187

"Order N Static and Quasi-Static Computations in Electromagnetics Using Wavelets", G.  
Beylkin, J. Dunn, and D. Gines, April 1994, PAM Report #189

"On the Fast Fourier Transform of Functions With Singularities", G. Beylkin, June 1994,  
PAM Report #195

"Fast and Accurate Computation of Fourier Transform of an Image", G. Beylkin, July  
1994, PAM Report #206

Lectures:

"Wavelets and Fast Numerical Algorithms", Jan. 11-12, 1993, AMS Short Course"  
ICASE/LaRC Short Course, 4 lectures, on , Feb. 1993

ONR Workshop on "Methods of shock capturing and image processing", "Adaptive Algorithms  
for Numerical Solution of PDE's", April 30 : May 1, 1993

Physics Computing 93 Conference, 'Wavelet Transforms and Fast Multiresolution Algorithms',  
June 3, 1993

3rd AMS-SIAM Summer Seminar, Mt. Holyoke College, South Hardley, Mass"The Mathematics  
of Tomography, Impedence Imaging, and Integral Geometry", talk: "Mathematical Framework  
for linearized inverse scattering problems in acoustics and elasticity", June 1993

SPIE's Annual Meeting July 15 and 16, 1993 at San Diego, coauthor, talk given by R. Jenkins ,  
Techniques for detecting densely wavelength-multiplexed solitons"

Martin Marietta workshop on Advanced Topics in Wavelets and Adapted Waveform Analysis,  
"An approach to compression and fast processing of SAR data", October 93

Automatic Target Recognition 93 Conference, 'On Factored FIR Approximation Of IIR Filters  
and Implementation Of Operators Via Filter Banks', Nov 1 - Nov 4, 1993 at MIT Lincoln Labs,  
Lexington, MA

Talk at Tel-Aviv University, Israel, "A simple algorithm for solving two-point boundary value  
problem for differential operators", Dec. 1993

"Wavelets and their applications", Jan. 27, 1992, Carl Heiland Lecture, CSM.

"Wavelets in Numerical Analysis", AAAS Meeting, Chicago, Feb. 7, 1992

"Multiresolution representations using the auto-correlation functions of compactly supported  
wavelets", AFIT, March 1992

"On the fast algorithm for multiplication of functions in the wavelet bases", International  
Conference Wavelets and Applications, Toulouse, France, June 1992

"On wavelet-based algorithms for solving differential equations", AMS Conference on  
Wavelets, June 30, 1992

"Inversion and Applications of the Generalized Radon Transform", Conference "75 years of  
Radon Transform", Vienna, Austria, Aug. 30 - Sept.5, 1992

"Wavelets and Fast Numerical Algorithms", Fourth Annual Symposium on Frontiers of Science,  
National Academy Of Science, Newport, CA Nov. 5-7, 1992

"Wavelets, their autocorrelation functions, and multiresolution representation of signals", Meeting  
of SPIE, Boston, Nov. 15- Nov. 20, 1992

" Wavelets in Numerical Analysis", Princeton University, 2/18/91

"Wavelets in Numerical Analysis", Rice University, 3/4/91

" Applications of Wavelets", Fermilab, 3/6/91

Series of four lectures at Colloque "Ondelettes et Analyse Numerique", CPT, CNRS-Luminy,  
Marseilles, France, 4/91

"Representations of Differential Operators and Fast Numerical Algorithms", ICIAM -91, 7/91