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13. ABSTRACT *(Maximum 200 words)*

Research investigations on the interaction of surface waves, wind and currents in stratified ocean layers have been carried out. The research program involves the development of appropriate models for general physical situations (e.g., systems of coupled nonlinear partial differential equations) as well as the analysis of these models for certain specific problems such as Langmuir circulations and double-diffusive convection. During the period of the report, six papers were published and two papers were accepted for publication. Some of the results obtained and the methods used in these papers are outlined in this report.

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Subtitle: **Langmuir Circulation and Spiral Flows**

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I. Summary of Research Activities.

(1) Landau type amplitude equations for the small-gap Taylor problem were derived and analyzed in [2] (see the list of publications in II). The equations obtained are global and are more complete than those determined by a formal two-timing analysis. Transition solutions (i.e., heteroclinic orbits) connecting the trivial Couette flow with bifurcating steady flows were obtained by solving singular evolution equations in infinite-dimensional spaces. The results obtained lead to the first analytic description of the difference between primary and secondary flows of a viscous fluid and provide a more detailed description of the transition solutions than that obtained by the use of formal two-timing methods.

The work in (2)–(6) is joint with George H. Knightly of the University of Massachusetts. The work in (2)–(4) was preliminary work on spiral flows that led to the work in (5) and (6) on wind-driven convection in ocean surface layers.

(2) The bifurcation and stability properties for rotating plane Couette flow problems were obtained in [3]. These are the first results on the bifurcation and stability properties of viscous spiral flows.

(3) Periodic waves were shown to exist and their expansions obtained in [1;4] for various Couette–Poiseuille problems. The existence of periodic waves in such problems has been conjectured but never proved.

(4) A continuum of periodic waves bifurcating supercritically from the basic spiral flow was obtained in [5;6] for rotating plane Couette flow. Results of this type provide a

simple explanation for the occurrence of turbulent-like flows in spiral flow problems and also the first analytic description of such turbulent-like flows. The usual Hopf bifurcation theory does not apply to such problems and new perturbation methods using singular operators were developed in [5;7] to solve the problem and a more general approach using center manifolds was developed in [6].

(5) The basic equations for Langmuir circulations in ocean surface layers when the Stokes drift has a cross-wind component were developed in [8], the set of partial differential equations are derived from a rational model that describes the development of mean currents and Langmuir circulations as a single system driven by a prescribed wind stress and a prescribed surface wave field. In physical situations where there is no cross-wind component of the Stokes drift and one seeks only roll-like solutions independent of the wind direction, the equations in [8] reduce to those derived previously by other investigators. The equations in [8] are derived by the use of multiple time scales related to the interaction of surface waves, wind and currents. The results in [8] provide the only rational theoretical explanation to date of observed phenomena in Langmuir circulations such as cross-drift of Langmuir cells and the rapid breakdown of Langmuir cells under a change in wind direction.

(6) A mathematical analysis of the equations described in (5) is also developed in [8]. The usual methods of Hopf bifurcation provide only partial results here and new perturbation methods related to those in [6] were developed to treat Langmuir circulation problems. Such methods provide a more detailed description of the resultant periodic waves than those obtained by the use of formal two-timing methods. Other problems for Langmuir circulations involving phenomena such as changes in the number of cells and patterns other than rolls also can be treated using the basic model equations derived in [8]. Such problems will be considered in a series of papers in preparation as part of N00014–94–1–0194.

II. Publications.

The following papers were completed and accepted for publication during the period of the grant. Reprints have been forwarded to ONR.

[1] "Time-periodic states in problems containing a structure parameter" (with George H. Knightly), Progress in Partial Differential Equations (J. Bandle, et al., eds.), Pitman Research Notes in Mathematics Series, John Wiley & Sons, Inc., New York, 1992, pp. 34–42.

[2] "Transition solutions in the Taylor problem," Arch. Rational Mech. Anal. **121** (1992), 267–301.

[3] "Periodic waves in rotating plane Couette flow" (with George H. Knightly), Z. Angew. Math. Phys. 44 (1993), 1–16.

[4] *Symmetry in rotating plane Couette–Poiseuille flow* (with George H. Knightly), Lectures in Applied Mathematics, Vol. 29, Exploiting Symmetry in Applied and Numerical Analysis, (E. Allgower, et al., eds.), American Mathematical Society, Providence, 1993, 299–306 (refereed article).

[5] *Singular Hopf bifurcation problems and rotating-sliding spiral flows* (with G. Knightly), Inequalities and Applications, (R. Agarwal, ed.), World Scientific Publishing Co., Singapore, 1994, 345–360.

The following papers were initiated during the period of the grant and completed and accepted for publication under N00014–94–1–0194. Reprints and Preprints have been forwarded to ONR.

[6] *Continua of periodic waves in rotating plane Couette flow* (with G. Knightly), European J. Mech. B/Fluids, **13** (1994), 511–526.

[7] *Reduction of a singular equation of Navier–Stokes type to a regular Hopf bifurcation problem* (with G. Knightly), Proceedings Volume for the International Conference on Advances in Geometric Analysis and Continuum Mechanics (K. Lancaster, ed.), International Press, 1994, (in press).

[8] *Langmuir circulations when the Stokes drift has a cross-wind component* (with G. Knightly), European J. Mech. B/Fluids, to appear,(37 manuscript pages).

III. Symposia Lectures.

(1) Summer Seminar in Applied Mathematics, "Exploiting Symmetry in Applied and Numerical Analysis," sponsored by AMS–SIAM, Fort Collins, July 26–August 1, 1992.

(2) Chapman Conference on "Double–Diffusive Convection," sponsored by the American Geophysical Union, Phoenix, November 3–6, 1993.