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13. ABSTRACT *(Maximum 200 words)*

This is the final report for the research project entitled "The Stroh formalism for anisotropic elasticity with applications to composite materials". Ten papers have been published under this research project. Important findings of the results are outlined, and potential applications to composite materials are indicated.

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A. STATEMENT OF THE PROBLEM STUDIED

Anisotropic elasticity has been an active research topic since the need of high strength, light weight composites in the aerospace industry became apparent. A composite material consists of two or more materials that are in general anisotropic. The oldest theory of two-dimensional anisotropic elasticity is due to Lekhnitskii. The Lekhnitskii formalism is outdated and inefficient. A newer theory, originally due to Stroh and further developed by others and myself is very powerful and elegant. We have extended the Stroh formalism by presenting new identities and sum rules that allow one to convert a complex form expression into a real form. Using these identities and sum rules some previously unsolved problems are solved and solutions that are available but are in a complex form are converted into a real form. As a result, more physical insights have been gained, and a better understanding of the physical problems has been achieved.

B. SUMMARY OF THE MOST IMPORTANT RESULTS

The list of publications under this project is given in Section C. Brief description of each paper is summarized below:

The first paper [1] is a review paper which is the content of an invited keynote speech at an Army Solid Mechanics Symposium. It shows how mathematics and mechanics can be blended together to solve anisotropic elasticity problems effectively. Many interesting and unexpected phenomena are pointed out in the paper.

The second paper [2] offers an insight look at the structure of Green's functions for anisotropic elastic media. It also provides the solution for cases when the material is degenerate, of which isotropic materials are a special case. This is useful in employing Green's functions for solving more complicated problems in applications

The third paper [3] deals with the motion of one-component surface waves. One-component surface waves exist only in certain anisotropic elastic solids. It does not exist, for example, in isotropic elastic solids.

The motion of one-component surface waves is rather unique in that it is a plane strain deformation as well as a plane stress deformation. This unusual property is very puzzling.

The Stroh formalism is most elegant when the boundary conditions are prescribed in terms of displacement only, or in terms of traction only. For mixed boundary value problems such as those applied to a slippery boundary, the boundary conditions are prescribed in terms of a mixture of displacement components and traction components. As a consequence, the elegant mathematical formulation of Stroh is destroyed. This explains that very few mixed boundary value problems for anisotropic materials have been solved in the literature. We have overcome the difficulty associated with the mixed boundary conditions by introducing a generalized Stroh formalism in paper [4]. The result is applied to study the stress decay in an semi-infinite anisotropic elastic strip [5]. The generalized boundary conditions allow 8 different boundary conditions each at the upper and lower sides of the strip and 8 different boundary conditions at the end of the strip. This results in $8^3=512$ sets of boundary conditions. They are all encompassed in one formulation. This is particularly useful in applications.

The image force on a line dislocation in a half space of isotropic material with a fixed boundary is well-known. The solution for the corresponding problem for anisotropic half space has eluded many investigators. With the Stroh formalism, and with several identities discovered during this investigation, the problem has been solved successfully in [6]. The physical meaning of the image force is interesting. It is shown that while the image force for the traction free boundary always tends to attracts the dislocation to the boundary, the image force for the fixed boundary is repulsive. Moreover, the magnitude of this repulsive force is always equal or larger than that of the attractive force associated with a free boundary.

The seventh paper [7] is concerned with a long standing unanswered question: Are there anisotropic materials whose three Stroh eigenvalues are identical and are equal to $\sqrt{-1}$, as are the cases for isotropic materials? It has been believed that there are such anisotropic materials but no concrete examples have been given. The paper shows that there are infinitely many such anisotropic materials.

The eighth paper [8] is an invited paper for a special volume entitled *Recent Advances in Elasticity, Inelasticity, and Viscoelasticity.* Here we restricted the deformations to simple antiplane deformations of anisotropic elastic materials. Even with this specialized deformation, it shows that anisotropic elasticity is a fascinating subject. Many of the solutions presented here are new, and can be used for comparison with the solutions for more general deformations.

In a bimaterial that consists of two isotropic elastic materials bonded together along their interface, there are two elastic constants each in the two materials. The stress solution naturally depends on these four elastic constants, but it can be reduced to three by a dimensional analysis. Dundurs has proved that the stress in the bimaterial depends on two composite elastic constants (known as Dundurs constants) if the boundary conditions are prescribed in terms of traction. The geometry of the interface can be arbitrary. This is a very important result in the analysis and design of a composite. In paper [9] we have shown that the same applies to anisotropic bimaterials. It is expected that this result would have important impact on composite research.

The three Barnett–Lothe tensors appear frequently in the solutions to anisotropic elasticity problems. Explicit expressions of the Barnett–Lothe tensors are known only for certain special anisotropic materials. In paper [10] explicit solutions are given for general anisotropic materials. Needless to say, this is an important contribution in anisotropic elasticity.

C. PUBLICATIONS UNDER THIS PROJECT

[1] Ting, T. C. T. (1992) "Synergism of mechanics, mathematics and anisotropic elastic materials," *Proc. 12th Army Symposium on solid Mechanics*, S. C. Chou, ed., U.S. Army Materials Technology Laboratory, Watertown, MA, 17–35.

[2] Ting, T. C. T. (1992) "Anatomy of Green's functions for line forces and dislocations in anisotropic media and degenerate materials," in *The Jens Lothe Symposium Volume, Physica Scripta* T44, 137–144.

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[4] Ting, T. C. T., and Wang, M. Z. (1992) "Generalized Stroh formalism for anisotropic elasticity for general boundary conditions," *Acta Mechanica Sinica* **8**, 193–207.

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[7] Ting, T. C. T. (1994) "On anisotropic elastic materials that possess three identical Stroh eigenvalues as do isotropic materials," *Q. Appl. Math.* **52**, 363-375.

[8] Ting, T. C. T. (1994b) "Antiplane deformations of anisotropic elastic materials," in *Recent Advances in Elasticity, Inelasticity, and Viscoelasticity*, ed. by K. R. Rajagopal, World Scientific Pub. Co. New Jersey. In print.

[9] Ting, T. C. T. (1995a) "Generalized Dundurs constants for anisotropic bimaterials," *Int. J. Solids Structures* **32**, in press.

[10] Wei, Lixin, and Ting, T. C. T. (1995) "Explicit expressions of Barnett–Lothe tensors for anisotropic materials," *J. Elasticity*. In print.

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