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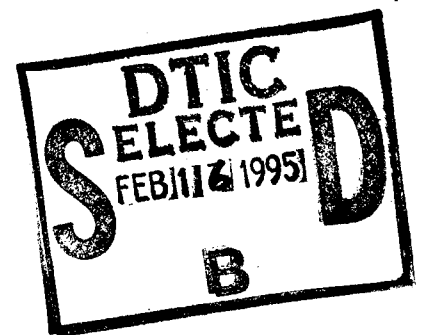
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**COMPLETE ANALYSIS OF ELECTRODE-
TERMINATED CABLES IN TWO-LAYER
CONDUCTOR BOUNDED ABOVE BY AIR:
CURRENT DENSITY, MAGNETIC FIELD,
AND MAGNETIC FIELD GRADIENT**

BY W. M. WYNN

COASTAL RESEARCH AND TECHNOLOGY DEPARTMENT

DECEMBER 1994



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**Coastal Systems Station, Dahlgren Division
Naval Surface Warfare Center**

PANAMA CITY, FLORIDA 32407-7001

CAPT D. C. STEERE, USN
Commanding Officer

MR. TED C. BUCKLEY
Executive Director

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13. ABSTRACT (Maximum 200 words) A complete analysis is given for the current density, magnetic field, and magnetic field gradient of a point current source located in the upper layer of a two-layer conductor that is bounded above by air. This problem has a long history, and treatments of varying degrees of completeness and/or rigor are scattered throughout the older geophysical literature. The intent here is to do a complete, self-contained analysis based on potential theory and Ampere's Law; treat all possible cases for the relative conductivity of the media, including the special cases of a lower medium that is an insulator or a perfect conductor; and provide expressions for the magnetic field vector, the current density vector, and the magnetic field gradient tensor in both conducting layers. The effect of performing gradient measurements within an enclosure also is discussed. The principle new results are the inclusion of expressions for the current density and magnetic field in the lower conducting layer, expressions for the magnetic field gradient tensor in both conducting layers, and the analysis of the case of a perfectly conducting lower layer. To complete the set of tools needed to describe a cable terminated by point electrodes, compact general expressions are given for the magnetic field vector and magnetic gradient tensor for a straight current element with arbitrary end points. These are based on the Biot-Savart Law and can be used to calculate the field and gradient contribution from an arbitrary cable joining two or more electrodes by means of superposition. The end results are applicable to such diverse problems as long-wire, electro-prospecting the current distribution around sheet piling protected by active electrodes, and the distributed currents of grounded direct current power systems.			
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INTRODUCTION

The geometry for the following analysis is shown in Figure 1. A point source of current I is located on the z axis at z coordinate h . Space is divided into three regions with electrical conductivities as follows:

$$\sigma = 0, -\infty < z \leq 0 \quad (\text{region 0}) \tag{1a}$$

$$\sigma = \sigma_1, 0 < z \leq d \quad (\text{region 1}) \tag{1b}$$

$$\sigma = \sigma_2, d < z < \infty \quad (\text{region 2}) \tag{1c}$$

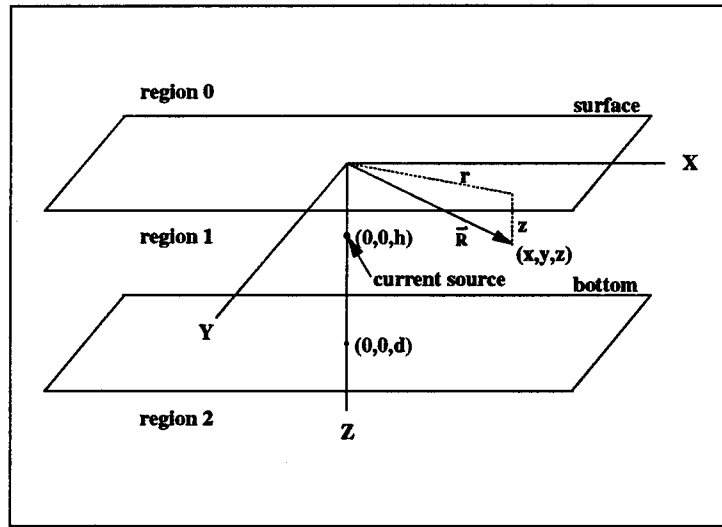


FIGURE 1. GEOMETRY FOR POINT CURRENT SOURCE FIELDS

The source is located in the middle region. It has a potential of the form

$$\Phi_p = \frac{I}{4\pi\sigma_1\sqrt{r^2 + (z-h)^2}} \tag{2}$$

which has the integral representation

$$\Phi_p = \frac{I}{4\pi\sigma_1} \int_0^\infty d\lambda J_0(\lambda r) e^{-\lambda|z-h|}. \tag{3}$$

Elsewhere, the potential function satisfies Laplace's equation

$$\nabla^2\Phi(r, \theta, z) = 0, \tag{4}$$

which has the axially symmetric solution in cylindrical coordinates

$$\Phi_H = \int_0^{\infty} d\lambda J_0(\lambda r) [a(\lambda)e^{\lambda z} + b(\lambda)e^{-\lambda z}]. \quad (5)$$

The potential in the three regions then has the physically acceptable forms

$$\Phi_0 = \int_0^{\infty} d\lambda J_0(\lambda r) a_0(\lambda) e^{\lambda z} \quad (-\infty < z \leq 0) \quad (6)$$

$$\Phi_1 = \int_0^{\infty} d\lambda J_0(\lambda r) \left[\frac{I}{4\pi\sigma_1} e^{-\lambda z - h} + a_1(\lambda) e^{\lambda z} + b_1(\lambda) e^{-\lambda z} \right] \quad (0 < z \leq d) \quad (7)$$

and

$$\Phi_2 = \int_0^{\infty} J_0(\lambda r) b_2(\lambda) e^{-\lambda z} \quad (d < z < \infty). \quad (8)$$

BOUNDARY CONDITIONS

The boundary conditions require continuity of the normal component of the current density and the tangential component of the electric field at each boundary. The electric field and current density are given by

$$\mathbf{E} = -\nabla\Phi \quad \text{and} \quad \mathbf{J} = \sigma\mathbf{E}. \quad (9)$$

Continuity of the normal current density at each boundary leads to the equations

$$a_1(\lambda) - b_1(\lambda) = -\frac{I}{4\pi\sigma_1} e^{-\lambda h} \quad (10)$$

and

$$a_1(\lambda)e^{\lambda d} - b_1(\lambda)e^{-\lambda d} + \epsilon b_2(\lambda)e^{-\lambda d} = \frac{I}{4\pi\sigma_1} e^{-\lambda(d-h)} \quad (11)$$

where

$$\epsilon = \sigma_2/\sigma_1. \quad (12)$$

Continuity of the tangential electric field at the bottom gives the equation

$$a_1(\lambda)e^{\lambda d} + b_1(\lambda)e^{-\lambda d} - b_2(\lambda)e^{-\lambda d} = \frac{I}{4\pi\sigma_1} e^{-\lambda(d-h)}. \quad (13)$$

Equations (11), (12), and (13) are sufficient to determine $a_1(\lambda)$, $b_1(\lambda)$, and $b_2(\lambda)$. This allows the computation of all fields in the middle and lower regions, which are of primary interest here. The function $a_0(\lambda)$ can be determined from continuity of the tangential electric field at the air-conductor interface, if needed.

With the introduction of the parameter $Q = (1 - \epsilon)/(1 + \epsilon)$, the solutions for the coefficients in the upper and lower conducting regions can be written

$$a_1(\lambda) = \frac{I}{4\pi\sigma_1} \frac{Qe^{-2\lambda d}}{1 - Qe^{-2\lambda d}} (e^{\lambda h} + e^{-\lambda h}) \quad (14)$$

$$b_1(\lambda) = \frac{I}{4\pi\sigma_1} \left[e^{-\lambda h} + \frac{Qe^{-2\lambda d}}{1 - Qe^{-2\lambda d}} (e^{\lambda h} + e^{-\lambda h}) \right] \quad (15)$$

and

$$b_2(\lambda) = \frac{I}{4\pi\sigma_2} \frac{(1 - Q)(e^{\lambda h} + e^{-\lambda h})}{1 - Qe^{-2\lambda d}}. \quad (16)$$

The parameter Q has the range of values $-1 \leq Q \leq 1$, where $Q = 1$ corresponds to region 2 being a perfect insulator, $Q = 0$ corresponds to regions 1 and 2 being identical, and $Q = -1$ corresponds to region 2 being a perfect conductor.

POTENTIAL AND CURRENT DENSITY

The potential for the middle and lower regions is given by

$$\begin{aligned} \Phi_1 = & \frac{I}{4\pi\sigma_1} \int_0^\infty d\lambda J_0(\lambda r) \{ e^{-\lambda|z-h|} + e^{-\lambda(z+h)} \\ & + \frac{Qe^{-2\lambda d}}{1 - Qe^{-2\lambda d}} [e^{\lambda(z+h)} + e^{-\lambda(z+h)} + e^{\lambda(z-h)} + e^{-\lambda(z-h)}] \} \end{aligned} \quad (17)$$

and

$$\Phi_2 = \frac{I}{4\pi\sigma_2} (1 - Q) \int_0^\infty d\lambda J_0(\lambda r) \left[\frac{e^{-\lambda(z-h)} + e^{-\lambda(z+h)}}{1 - Qe^{-2\lambda d}} \right]. \quad (18)$$

Integrals involving the bessel functions are notoriously difficult to perform numerically. However, a geometric series expansion valid for $|Q| < 1$

$$\frac{1}{1 - Qe^{-2\lambda d}} = \sum_{n=0}^{\infty} Q^n e^{-2n\lambda d} \quad (19)$$

and the integral representations exemplified by Equations (2) and (3) can be used to write the potential in the upper and lower conductors as

$$\Phi_1 = \frac{I}{4\pi\sigma_1} \left\{ \frac{1}{\sqrt{r^2 + (z-h)^2}} + \frac{1}{\sqrt{r^2 + (z+h)^2}} + \sum_{n=1}^{\infty} Q^n \left[\frac{1}{\sqrt{r^2 + (2nd - z - h)^2}} \right. \right. \quad (20)$$

$$\left. \left. + \frac{1}{\sqrt{r^2 + (2nd + z + h)^2}} + \frac{1}{\sqrt{r^2 + (2nd - z + h)^2}} + \frac{1}{\sqrt{r^2 + (2nd + z - h)^2}} \right] \right\}$$

and

$$\Phi_2 = \frac{I}{4\pi\sigma_2} (1-Q) \left\{ \frac{1}{\sqrt{r^2 + (z-h)^2}} + \frac{1}{\sqrt{r^2 + (z+h)^2}} \right. \quad (21)$$

$$\left. + \sum_{n=1}^{\infty} Q^n \left[\frac{1}{\sqrt{r^2 + (2nd + z - h)^2}} + \frac{1}{\sqrt{r^2 + (2nd + z + h)^2}} \right] \right\}.$$

By combining Equation (9) with Equations (20) and (21), expressions for the cartesian components of the current density in the middle and lower regions can be derived. They are

$$J_{1x(y)} = \frac{x(y)I}{4\pi} \left\{ \frac{1}{\sqrt{(r^2 + (z-h)^2)^3}} + \frac{1}{\sqrt{(r^2 + (z+h)^2)^3}} + \sum_{n=1}^{\infty} Q^n \left[\frac{1}{\sqrt{(r^2 + (2nd - z - h)^2)^3}} \right. \right. \quad (22)$$

$$\left. \left. + \frac{1}{\sqrt{(r^2 + (2nd + z + h)^2)^3}} + \frac{1}{\sqrt{(r^2 + (2nd - z + h)^2)^3}} + \frac{1}{\sqrt{(r^2 + (2nd + z - h)^2)^3}} \right] \right\}$$

$$J_{1z} = \frac{I}{4\pi} \left\{ \frac{|z-h|}{\sqrt{(r^2 + (z-h)^2)^3}} + \frac{z+h}{\sqrt{(r^2 + (z+h)^2)^3}} + \sum_{n=1}^{\infty} Q^n \left[-\frac{2nd - z - h}{\sqrt{(r^2 + (2nd - z - h)^2)^3}} \right. \right. \quad (23)$$

$$\left. \left. + \frac{2nd + z + h}{\sqrt{(r^2 + (2nd + z + h)^2)^3}} - \frac{2nd - z + h}{\sqrt{(r^2 + (2nd - z + h)^2)^3}} + \frac{2nd + z - h}{\sqrt{(r^2 + (2nd + z - h)^2)^3}} \right] \right\}.$$

$$J_{2x(y)} = \frac{x(y)I}{4\pi} (1-Q) \sum_{n=0}^{\infty} Q^n \left[\frac{1}{\sqrt{(r^2 + (2nd + z - h)^2)^3}} + \frac{1}{\sqrt{(r^2 + (2nd + z + h)^2)^3}} \right] \quad (24)$$

and

$$J_{2z} = \frac{I}{4\pi} (1-Q) \sum_{n=0}^{\infty} Q^n \left[\frac{2nd + z - h}{\sqrt{(r^2 + (2nd + z - h)^2)^3}} + \frac{2nd + z + h}{\sqrt{(r^2 + (2nd + z + h)^2)^3}} \right] \quad (25).$$

MAGNETIC FIELD DUE TO DISTRIBUTED CURRENT

The magnetic field of the distributed current can be constructed by means of a careful application of Ampere's Law, applied to a properly constructed system; that is, one in which charge conservation holds. To accomplish this, the point current source is imagined to be fed by an insulated wire that enters region 1 perpendicular to the boundaries, and extends into medium 0 for a great distance, ultimately returning to a point current sink in region 1 at a great lateral distance from the current source. Application of the Biot-Savart Law to the wire shows that if the wire extends far enough from the boundary in the z direction and returns to the medium sufficiently far away, the magnetic field of the wire segment is indistinguishable from that of a semi-infinite wire extending in the $-z$ direction. In addition, if the current sink is sufficiently far away, its fields are negligible at the current source, and axial symmetry about the incoming wire can be applied. This is illustrated in Figure 2.

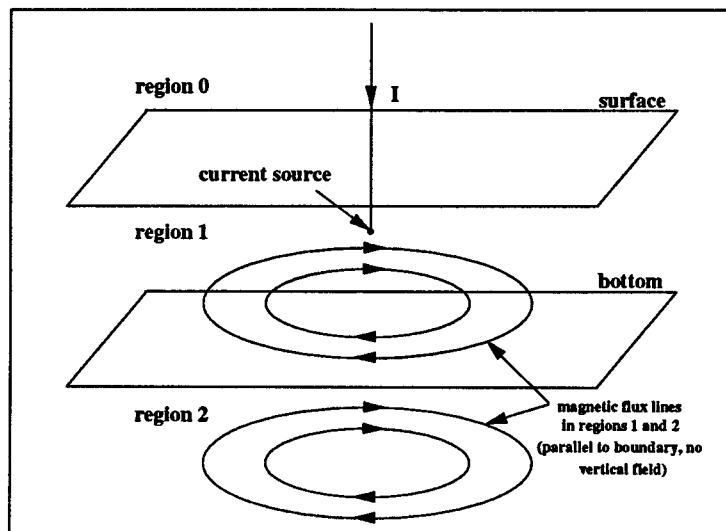


FIGURE 2. MAGNETIC FIELD OF WIRE AND DISTRIBUTED CURRENTS*

Ampere's Law can now be applied to the current distribution. Ampere's Law takes the form

$$\oint \mathbf{H} \cdot d\mathbf{l} = \int \mathbf{J} \cdot d\mathbf{A}, \quad (26)$$

which states that the line integral of the magnetic field around a closed path is equal to the integral of the normal current density over the enclosed surface; that is, the net current through the loop. In the present application, symmetry about the wire axis can be used to write

$$\oint \mathbf{H} \cdot d\mathbf{l} = 2\pi r H_{\theta} \quad (27)$$

*Note that the magnetic field is entirely horizontal, there is no vertical field from the wire current or the distributed currents from the point source.

where r is the radius of a circle centered on the wire axis and H_θ is the tangential component of magnetic field on the circle, which is the same everywhere, independent of θ . Now, if the integral on the right hand side of Equation (26) can be evaluated, the magnetic field in regions 1 and 2 can be determined directly. To accomplish this, region 2, $d \leq z < \infty$, is first examined, with reference to the notation of Figure 1. The right-hand rule is used to assign the sense of the currents and fields, and then

$$\int \mathbf{J}_2 \cdot d\mathbf{A} = \int J_{2z} dA = \int_0^{2\pi} d\theta \int_0^r duu J_{2z} = 2\pi \int_0^r duu J_{2z} \quad (28)$$

where J_{2z} is independent of θ .

Substitution of Equation (25) into Equation (28) and application of Equations (26) and (27) yields

$$H_{2\theta} = \frac{I}{4\pi r} \left\{ 2 - (1 - Q) \left(\frac{z-h}{\sqrt{r^2 + (z-h)^2}} + \frac{z+h}{\sqrt{r^2 + (z+h)^2}} \right) + \sum_{n=1}^{\infty} Q^n \left[\frac{2nd+z-h}{\sqrt{r^2 + (2nd+z-h)^2}} + \frac{2nd+z+h}{\sqrt{r^2 + (2nd+z+h)^2}} \right] \right\} \quad (29)$$

$(d \leq z < \infty)$

Repeating this analysis for region 1, for $h < z \leq d$, and using Equation (23) instead of Equation (25),

$$H_{1\theta} = \frac{I}{4\pi r} \left\{ 2 - \frac{z-h}{\sqrt{r^2 + (z-h)^2}} - \frac{z+h}{\sqrt{r^2 + (z+h)^2}} + \sum_{n=1}^{\infty} Q^n \left[\frac{2nd-z-h}{\sqrt{r^2 + (2nd-z-h)^2}} - \frac{2nd+z+h}{\sqrt{r^2 + (2nd+z+h)^2}} + \frac{2nd-z+h}{\sqrt{r^2 + (2nd-z+h)^2}} - \frac{2nd+z-h}{\sqrt{r^2 + (2nd+z-h)^2}} \right] \right\} \quad (30)$$

$(h < z \leq d)$.

Next, region 1 is examined for the case $0 \leq z < h$. Consistent application of Ampere's Law for this case requires that the contribution of the line current be included, and then

$$2\pi r H_\theta = I + 2\pi \int_0^r duu J_z \quad (31)$$

and this time substitution from Equation (23) with careful attention to the relative values of z and h , yields

$$H_{1\theta} = \frac{I}{4\pi r} \left\{ 2 - \frac{z-h}{\sqrt{r^2+(z-h)^2}} - \frac{z+h}{\sqrt{r^2+(z+h)^2}} + \sum_{n=1}^{\infty} Q^n \left[\frac{2nd-z-h}{\sqrt{r^2+(2nd-z-h)^2}} \right. \right. \\ \left. \left. - \frac{2nd+z+h}{\sqrt{r^2+(2nd+z+h)^2}} + \frac{2nd-z+h}{\sqrt{r^2+(2nd-z+h)^2}} - \frac{2nd+z-h}{\sqrt{r^2+(2nd+z-h)^2}} \right] \right\} \quad (32)$$

$(0 \leq z < h),$

which is identical in form to Equation (30), and the magnetic field is continuous at the electrode ($z = h$) plane, as it must be to be physical.

Now, to isolate the magnetic field due to the electrode distributed currents alone, the Biot-Savart magnetic field of the wire alone is constructed and it is subtracted from the expressions in Equations (29), (30), and (32). Then

$$H_{\theta}^{bs} = \frac{I}{4\pi r} \left[1 - \frac{(z-h)}{\sqrt{r^2+(z-h)^2}} \right] \quad (33)$$

and the electrode magnetic field alone, everywhere in region 1, is given by

$$H_{1\theta}^e = \frac{I}{4\pi r} \left\{ 1 - \frac{z+h}{\sqrt{r^2+(z+h)^2}} + \sum_{n=1}^{\infty} Q^n \left[\frac{2nd-z-h}{\sqrt{r^2+(2nd-z-h)^2}} \right. \right. \\ \left. \left. - \frac{2nd+z+h}{\sqrt{r^2+(2nd+z+h)^2}} + \frac{2nd-z+h}{\sqrt{r^2+(2nd-z+h)^2}} - \frac{2nd+z-h}{\sqrt{r^2+(2nd+z-h)^2}} \right] \right\} \quad (34)$$

$(0 \leq z \leq d),$

and in region 2, the field is

$$H_{2\theta}^e = \frac{I}{4\pi r} \left\{ 1 + \frac{z-h}{\sqrt{r^2+(z-h)^2}} - (1-Q) \sum_{n=0}^{\infty} Q^n \left[\frac{2nd+z-h}{\sqrt{r^2+2nd+z-h^2}} + \frac{2nd+z+h}{\sqrt{r^2+2nd+z+h^2}} \right] \right\}. \quad (35)$$

Then the cartesian components in either region are given by

$$H_x^e = -\frac{y}{r} H_{\theta}^e \quad \text{and} \quad H_y^e = \frac{x}{r} H_{\theta}^e. \quad (36)$$

NON-CONDUCTING BOTTOM ($Q=1$)

Region 1

In the limit of a non-conducting bottom, $Q = 1$, and the series solutions for the fields diverge. To deal with the fields in region 1, the integral expressions defining them will be revisited. The current density is determined from Equations (9) and (17) and has the form

$$\begin{aligned}
 J_{1x(y)} = & \frac{Ix(y)}{4\pi r} \int_0^{\infty} d\lambda \lambda J_1(\lambda r) \{e^{-\lambda|z-h|} + e^{-\lambda(z+h)} \\
 & + \frac{e^{-2\lambda d}}{1 - e^{-2\lambda d}} [e^{\lambda(z+h)} + e^{-\lambda(z+h)} + e^{\lambda(z-h)} + e^{-\lambda(z-h)}]\}
 \end{aligned} \quad (37)$$

and

$$\begin{aligned}
 J_{1z} = & \frac{I}{4\pi} \int_0^{\infty} d\lambda \lambda J_0(\lambda r) \{ \text{sgn}(z-h) e^{-\lambda|z-h|} + e^{-\lambda(z+h)} \\
 & + \frac{e^{-2\lambda d}}{1 - e^{-2\lambda d}} [-e^{\lambda(z+h)} + e^{-\lambda(z+h)} - e^{\lambda(z-h)} + e^{-\lambda(z-h)}] \}.
 \end{aligned} \quad (38)$$

The case $z > h$ is explicitly selected. The end result is the same for any case. The expression for the electrode magnetic field is then obtained by combining Equations (26) through (28), (33), and (38). The result is

$$\begin{aligned}
 H_{1\theta}^e = & \frac{I}{4\pi r} \left(-1 + \frac{z-h}{\sqrt{r^2 + (z-h)^2}} + r \int_0^{\infty} d\lambda J_1(\lambda r) \{ e^{-\lambda(z-h)} + e^{-\lambda(z+h)} \right. \\
 & \left. + \frac{e^{-2\lambda d}}{1 - e^{-2\lambda d}} [-e^{\lambda(z+h)} + e^{-\lambda(z+h)} - e^{\lambda(z-h)} + e^{-\lambda(z-h)}] \} \right).
 \end{aligned} \quad (39)$$

In each of Equations (37) through (39) a common denominator is used and terms are combined into hyperbolic functions. Then, the fields have the forms

$$J_{1x(y)} = \frac{Ix(y)}{4\pi r} \int_0^{\infty} d\lambda \lambda J_1(\lambda r) \left\{ \frac{\cosh[\lambda(d-z+h)] + \cosh[\lambda(d-z-h)]}{\sinh(\lambda d)} \right\} \quad (40)$$

$$J_{1z} = \frac{I}{4\pi} \int_0^{\infty} d\lambda \lambda J_0(\lambda r) \left\{ \frac{\sinh[\lambda(d-z+h)] + \sinh[\lambda(d-z-h)]}{\sinh(\lambda d)} \right\} \quad (41)$$

and

$$\begin{aligned}
 H_{1\theta}^e = & \frac{I}{4\pi r} \left(-1 + \frac{z-h}{\sqrt{r^2 + (z-h)^2}} \right. \\
 & \left. + r \int_0^{\infty} d\lambda J_1(\lambda r) \left\{ \frac{\sinh[\lambda(d-z+h)] + \sinh[\lambda(d-z-h)]}{\sinh(\lambda d)} \right\} \right).
 \end{aligned} \quad (42)$$

There are three integral types; they are

$$v_1 = \int_0^{\infty} d\lambda \lambda J_1(\lambda r) \frac{\cosh(\lambda a)}{\sinh(\lambda d)} \quad (43)$$

$$v_2 = \int_0^{\infty} d\lambda \lambda J_0(\lambda r) \frac{\sinh(\lambda a)}{\sinh(\lambda d)} \quad (44)$$

and

$$v_3 = \int_0^{\infty} d\lambda J_1(\lambda r) \frac{\sinh(\lambda a)}{\sinh(\lambda d)}. \quad (45)$$

Contour Integration-Residue Theorem. If the integrands of v_1 , v_2 , and v_3 are analytically continued into the complex plane $\lambda \rightarrow w = u + iv$, the denominator $\sinh(wd)$ will have zeros for $wd = in\pi$, where $n = 0, \pm 1, \pm 2, \dots$. This suggests that the required integrals be related to integrals over a closed contour containing these zeros, and the Cauchy residue theorem be exploited. An appropriate contour is shown in Figure 3. The contour is closed in the upper half plane, and the zeros involved are those for $n > 0$.

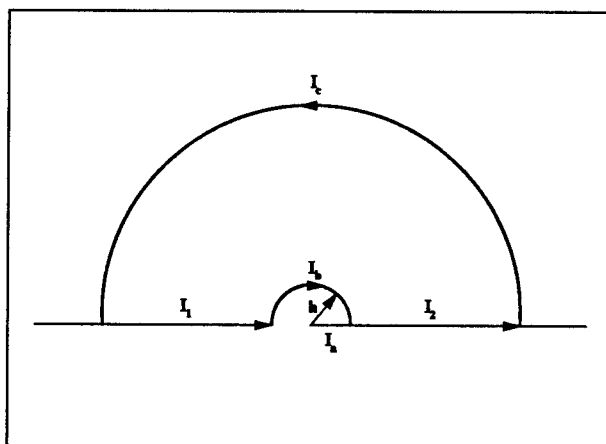


FIGURE 3. INTEGRATION CONTOUR IN w PLANE

The integrals of interest are taken over the interval I_2 in Figure 3, in the limit that I_a goes to 0, and I_2 becomes infinite. This integral will be related to the closed contour integral over the successive intervals I_1 , I_b , I_2 , and I_c . To this end, the integral over I_2 will be expressed in terms of an integral over both I_1 and I_2 . To accomplish this, Bessel functions J_n will be expressed in terms of Hankel functions. This relationship is analogous to the representation of the cosine function by means of complex exponentials, which is a trick used in Fourier transform theory to extend integrals from the positive real axis to the entire real axis. The Hankel functions are defined by

$$H_n^{(1)}(z) = J_n(z) + iY_n(z) \quad (46)$$

$$H_n^{(2)}(z) = J_n(z) - iY_n(z)$$

where Y_n is the bessel function irregular at the origin. The bessel function J_n , which is regular at the origin, now can be expressed as

$$J_n(z) = \frac{1}{2}[H_n^{(1)}(z) + H_n^{(2)}(z)]. \quad (47)$$

Next, it is noted that the hankel functions have the symmetry properties

$$H_0^{(1)}(-z) = -H_0^{(2)}(z) \quad (48)$$

$$H_1^{(1)}(-z) = H_1^{(2)}(z).$$

Inserting Equation (47) in Equation (43), for example, and using the symmetry properties of all the factors in the integrand, gives

$$\begin{aligned} & \int_{I_2} d\lambda \lambda J_1(\lambda r) \frac{\cosh(\lambda a)}{\sinh(\lambda d)} \quad (49) \\ &= \frac{1}{2} \int_{I_2} d\lambda \lambda H_1^{(1)}(\lambda r) \frac{\cosh(\lambda a)}{\sinh(\lambda d)} + \frac{1}{2} \int_{I_2} d\lambda \lambda H_1^{(2)}(\lambda r) \frac{\cosh(\lambda a)}{\sinh(\lambda d)} \\ &= \frac{1}{2} \int_{I_2} d\lambda \lambda H_1^{(1)}(\lambda r) \frac{\cosh(\lambda a)}{\sinh(\lambda d)} + \frac{1}{2} \int_{-I_2} d(-\lambda) (-\lambda) H_1^{(2)}(-\lambda r) \frac{\cosh(-\lambda a)}{\sinh(-\lambda d)} \\ &= \frac{1}{2} \int_{I_1 \& I_2} d\lambda \lambda H_1^{(1)}(\lambda r) \frac{\cosh(\lambda a)}{\sinh(\lambda d)} \end{aligned}$$

where the relation has been used for the intervals $-(-I_2) = I_1$. Thus, the integral over I_2 is replaced by an integral over I_1 and I_2 , with J_1 replaced by $H_1^{(1)}/2$. A similar operation can be done for the integrals in Equations (44) and (45).

Now, using a shorthand notation, relations between the integrals over the various contour segments can be written, where it is understood that the equations hold in the limit that the inner semicircle shrinks to zero, and the outer semicircle recedes to infinity. Then

$$v = \int_{I_1 \& I_2} + \int_{I_a} \quad (50)$$

and

$$\oint = \int_{I_1 \& I_2} + \int_{I_b} + \int_{I_c} \quad (51)$$

where \oint is the integral around the closed contour. Thus

$$1 = \oint - \int_{I_b} - \int_{I_c} + \int_{I_a}. \quad (52)$$

The following generic results hold for the three integrals of interest.

$$\oint \rightarrow 2\pi i \sum_{n=1}^{\infty} S_n \quad (53)$$

$$\int_{I_a} \rightarrow 0, \quad (\text{all cases}) \quad (54)$$

$$\int_{I_c} \rightarrow 0, \quad (\text{all cases}) \quad (55)$$

$$\int_{I_b} \rightarrow (\text{simple limit}) \quad (56)$$

Here, the S_n are the residues, defined by

$$\oint dw f(w) = \oint dw \frac{f(w)(w - w_0)}{w - w_0} = \oint dw \frac{g(w)}{w - w_0}, \quad (57)$$

$$S_n = \lim_{w \rightarrow w_0} g(w) |_{w_0 = in\pi d}.$$

To establish the limiting values of the various integrals, the limiting forms of the functions involved are examined;

$$J_0(u) \rightarrow 1, \quad J_1(u) \rightarrow u, \quad u \rightarrow 0 \quad (58)$$

$$\cosh(w) \rightarrow 1, \quad \sinh(w) \rightarrow w, \quad |w| \rightarrow 0 \quad (59)$$

$$H_0^{(1)}(w) \rightarrow \frac{2i}{\pi} [\ln(w/2) + \gamma], \quad H_1^{(1)}(w) \rightarrow -\frac{2i}{\pi w}, \quad |w| \rightarrow 0 \quad (60)$$

($\gamma = 0.5772156649 \dots$ Euler's constant)

and

$$H_0^{(1)}(w) \rightarrow \frac{(1-i)e^{iw}}{\sqrt{\pi w}}, \quad H_1^{(1)}(w) \rightarrow -\frac{(1+i)e^{iw}}{\sqrt{\pi w}}, \quad |w| \rightarrow \infty. \quad (61)$$

Applying Equations (58) and (59) to the integrands of ι_1, ι_2 and ι_3 shows that the integrands approach 0 as λ approaches 0, establishing Equation (54). For the integrals on contour $I_c, w = Re^{i\phi}$, and the hankel function factor in the integrands has the behavior

$$\frac{e^{-rR \sin \phi}}{\sqrt{rR}} \rightarrow 0, \quad R \rightarrow \infty. \quad (62)$$

The other factors are all proportional to the limiting form

$$e^{(a-d)R|\cos \phi|}, \quad R \rightarrow \infty. \quad (63)$$

For $\phi = \pi/2$, the associated factors are purely oscillatory, and are dominated by the decaying hankel function factor. For other angles

$$a-d = d-z+h-d = -(z-h) < 0, \quad \text{or} \quad a-d = d-z-h-d = -(z+h) < 0 \quad (64)$$

and the factor vanishes as $R \rightarrow \infty$. This establishes Equation (55).

To determine the limiting forms for the integrals over the contour I_b , let $w = he^{i\phi}$, and explicitly list the integrals. They are

$$\frac{1}{2} \int_{\pi}^0 d\phi i h^2 e^{2i\phi} \frac{-2i}{\pi h r e^{i\phi} h d e^{i\phi}} = \frac{1}{\pi r d} \int_{\pi}^0 d\phi \rightarrow -\frac{1}{rd}, \quad h \rightarrow 0 \quad (65)$$

$$\frac{1}{2} \int_{\pi}^0 d\phi i h^2 e^{2i\phi} \frac{2i}{\pi} \left[\ln \left(\frac{h}{2} e^{i\phi} \right) + \gamma \right] \frac{h a e^{i\phi}}{h d e^{i\phi}} \rightarrow 0, \quad h \rightarrow 0 \quad (66)$$

and

$$\frac{1}{2} \int_{\pi}^0 d\phi i h e^{i\phi} \frac{-2i}{\pi h r e^{i\phi} h d e^{i\phi}} \frac{h a e^{i\phi}}{h d e^{i\phi}} = \frac{a}{\pi r d} \int_{\pi}^0 d\phi \rightarrow -\frac{a}{rd}, \quad h \rightarrow 0. \quad (67)$$

The residues are constructed using a simple limit process, as indicated in Equation (57). Some useful expressions for the evaluations are

$$\sinh(dw) = \cosh(dw_n) d(w - w_n) + \dots, \quad w \rightarrow w_n \quad (68)$$

where dw_n is a zero of $\sinh(dw)$,

$$\cosh(iu) = \cos(u) \quad (69)$$

$$\sinh(iu) = i \sin(u)$$

and

$$H_0^{(1)}(iu) = -\frac{2i}{\pi} K_0(u) \quad (70)$$

$$H_1^{(1)}(iu) = -\frac{2}{\pi} K_1(u)$$

where the K_n are the modified bessel functions regular at infinity. Using these results, the residues can be written as

$$\begin{aligned} \frac{1}{2} w H_1^{(1)}(rw) \frac{\cosh(aw)}{\sinh(dw)} (w - in\pi/d) &\rightarrow \frac{in\pi}{2d} H_1^{(1)}(in\pi r/d) \frac{\cosh(in\pi a/d)}{(-1)^n d} \\ &= -\frac{i}{d^2} n (-1)^n K_1(n\pi r/d) \cos(n\pi a/d) \end{aligned} \quad (71)$$

$$\begin{aligned} \frac{1}{2} w H_0^{(1)}(rw) \frac{\sinh(aw)}{\sinh(dw)} (w - in\pi/d) &\rightarrow \frac{in\pi}{2d} H_0^{(1)}(in\pi r/d) \frac{\sinh(in\pi a/d)}{(-1)^n d} \\ &= \frac{i}{d^2} n (-1)^n K_0(n\pi r/d) \sin(n\pi a/d) \end{aligned} \quad (72)$$

and

$$\begin{aligned} \frac{1}{2} H_1^{(1)}(rw) \frac{\sinh(aw)}{\sinh(dw)} (w - in\pi/d) &\rightarrow \frac{1}{2} H_1^{(1)}(in\pi r/d) \frac{\sinh(in\pi a/d)}{(-1)^n d} \\ &= -\frac{i}{\pi d} (-1)^n K_1(n\pi r/d) \sin(n\pi a/d). \end{aligned} \quad (73)$$

Now the explicit forms for the fields can be constructed. In doing so, the arguments of the trigonometric functions are expanded and integral multiples of π are eliminated. Now

$$v = \oint - \int_b \quad (74)$$

and Equations (40) through (42), Equation (53), Equations (65) through (67), and Equations (71) through (73) are combined. The results are

$$J_{1x(y)} = \frac{Ix(y)}{2\pi d r^2} \left(1 + \frac{\pi r}{d} \sum_{n=1}^{\infty} n K_1(n\pi r/d) [\cos\{n\pi(z-h)/d\} + \cos\{n\pi(z+h)/d\}] \right) \quad (75)$$

$$J_{1z} = \frac{I}{2d^2} \sum_{n=1}^{\infty} n K_0(n\pi r/d) [\sin\{n\pi(z-h)/d\} + \sin\{n\pi(z+h)/d\}] \quad (76)$$

and, subtracting the Biot-Savart field (Equation (33)),

$$H_{1\theta}^e = \frac{I}{4\pi r} \left(1 + \frac{z-h}{\sqrt{r^2 + (z-h)^2}} - \frac{2z}{d} \right) + \frac{\pi r}{d} \sum_{n=1}^{\infty} K_1(n\pi r/d) [\sin\{n\pi(z-h)/d\} + \sin\{n\pi(z+h)/d\}]. \quad (77)$$

Region 2

Inspection of Equations (24), (25), and (35) shows that the fields reduce to the simple forms

$$J_{2x(y)} = 0 \quad (78)$$

$$J_{2z} = 0 \quad (79)$$

and

$$H_{2\theta}^e = \frac{I}{4\pi r} \left[1 + \frac{z-h}{\sqrt{r^2 + (z-h)^2}} \right]. \quad (80)$$

PERFECTLY CONDUCTING BOTTOM (Q=-1, REGION 1 ONLY)

Region 1

In this case, when the various terms in region 1 are combined to give hyperbolic functions, the resulting integrals are

$$J_{1x(y)} = \frac{Ix(y)}{4\pi r} \int_0^{\infty} d\lambda \lambda J_1(\lambda r) \left\{ \frac{\sinh[\lambda(d-z+h)] + \sinh[\lambda(d-z-h)]}{\cosh(\lambda d)} \right\} \quad (81)$$

$$J_{1z} = \frac{I}{4\pi} \int_0^{\infty} d\lambda \lambda J_0(\lambda r) \left\{ \frac{\cosh[\lambda(d-z+h)] + \cosh[\lambda(d-z-h)]}{\cosh(\lambda d)} \right\} \quad (82)$$

and

$$H_{1\theta}^e = \frac{I}{4\pi} \int_0^{\infty} d\lambda J_1(\lambda r) \left\{ \frac{\cosh[\lambda(d-z+h)] + \cosh[\lambda(d-z-h)]}{\cosh(\lambda d)} \right\}. \quad (83)$$

The three types of integral now are

$$I_1 = \int_0^{\infty} d\lambda \lambda J_1(\lambda r) \frac{\sinh(\lambda a)}{\cosh(\lambda d)} \quad (84)$$

$$I_2 = \int_0^{\infty} d\lambda \lambda J_0(\lambda r) \frac{\cosh(\lambda a)}{\cosh(\lambda d)} \quad (85)$$

and

$$I_3 = \int_0^{\infty} d\lambda J_1(\lambda r) \frac{\cosh(\lambda a)}{\cosh(\lambda d)}. \quad (86)$$

Again the construct of Equation (49) can be used to introduce the hankel function, and analytic continuation and the residue theorem can be used, where now the singularities occur for

$$z = i(n + 1/2)\pi/d, \quad n = 0, \pm 1, \pm 2, \dots \quad (87).$$

Again,

$$I = \oint - \int_{I_b} - \int_{I_c} + \int_{I_a} \quad (88)$$

and, again

$$I_a \rightarrow 0 \quad \text{and} \quad I_c \rightarrow 0. \quad (89)$$

Now, for I_b

$$\frac{1}{2} \int_{\pi}^0 d\phi i h^2 e^{2i\phi} \frac{-2i a h e^{i\phi}}{\pi h r e^{i\phi} 1} \rightarrow 0 \quad (90)$$

$$\frac{1}{2} \int_{\pi}^0 d\phi i h^2 e^{2i\phi} \left[\frac{2i}{\pi} \ln \left(\frac{h r e^{i\phi}}{2} \right) + \gamma \right] \frac{1}{1} \rightarrow 0 \quad (91)$$

$$\frac{1}{2} \int_{\pi}^0 d\phi i h e^{i\phi} \frac{-2i}{\pi h r e^{i\phi} 1} \rightarrow -\frac{1}{r} \quad (92)$$

and for the residues S_n

$$\frac{1}{2}wH_1^{(1)}(rw)\frac{\sinh(aw)}{\cosh(dw)}[w - i(n + 1/2)\pi/d] \quad (93)$$

$$\begin{aligned} &\rightarrow \frac{1}{2}[i(n + 1/2)\pi/d]H_1^{(1)}[i(n + 1/2)\pi r/d] \sinh[i(n + 1/2)\pi a/d] \frac{1}{i(-1)^n d} \\ &= \frac{i(n + 1/2)(-1)^{n+1}}{d^2} K_1[(n + 1/2)\pi r/d] \sin[(n + 1/2)\pi a/d] \end{aligned}$$

$$\frac{1}{2}wH_0^{(1)}(rw)\frac{\cosh(aw)}{\cosh(dw)}[w - i(n + 1/2)\pi/d] \quad (94)$$

$$\begin{aligned} &\rightarrow \frac{1}{2}[(n + 1/2)\pi/d]H_0^{(1)}[i(n + 1/2)\pi r/d] \cosh[i(n + 1/2)\pi a/d] \frac{1}{(-1)^n d} \\ &= \frac{i(n + 1/2)(-1)^{n+1}}{d^2} K_0[(n + 1/2)\pi r/d] \cos[(n + 1/2)\pi a/d] \end{aligned}$$

$$\frac{1}{2}H_1^{(1)}(rw)\frac{\cosh(aw)}{\cosh(dw)}[w - i(n + 1/2)\pi/d] \quad (95)$$

$$\begin{aligned} &\rightarrow \frac{1}{2}H_1^{(1)}[i(n + 1/2)\pi r/d] \cosh[i(n + 1/2)\pi a/d] \frac{1}{i(-1)^n d} \\ &= \frac{i(-1)^n}{\pi d} K_1[(n + 1/2)\pi r/d] \cos[(n + 1/2)\pi a/d]. \end{aligned}$$

Then, using

$$v = \oint -I_b = 2\pi i \sum_{n=1}^{\infty} S_n - I_b \quad (96)$$

gives

$$J_{1x(y)} = \frac{Ix(y)}{2rd^2} \sum_{n=0}^{\infty} (n + 1/2)K_1[(n + 1/2)\pi r/d] \quad (97)$$

$$\{\cos[(n + 1/2)\pi(z - h)/d] + \cos[(n + 1/2)\pi(z + h)/d]\}$$

$$J_{1z} = \frac{I}{2d^2} \sum_{n=0}^{\infty} (n + 1/2)K_0[(n + 1/2)\pi r/d] \quad (98)$$

$$\{\sin[(n + 1/2)\pi(z - h)/d] + \sin[(n + 1/2)\pi(z + h)/d]\}$$

and, again subtracting the Biot-Savart field (Equation (33)),

$$H_{1\theta} = \frac{I}{4\pi r} \left(1 + \frac{z-h}{\sqrt{r^2 + (z-h)^2}} - \frac{2r}{d} \sum_{n=0}^{\infty} K_1[(n+1/2)\pi r/d] \right. \\ \left. \{ \sin[(n+1/2)\pi(z-h)/d] + \sin[(n+1/2)\pi(z+h)/d] \} \right) \quad (99)$$

Region 2

In region 2, the integral forms for the fields are

$$J_{2x(y)} = \frac{x(y)I}{2\pi r} \int_0^{\infty} d\lambda \lambda J_1(\lambda r) \left[\frac{e^{-\lambda(z+h)} + e^{-\lambda(z-h)}}{1 + e^{-2\lambda d}} \right] \quad (100)$$

$$J_{2z} = \frac{I}{2\pi} \int_0^{\infty} d\lambda \lambda J_0(\lambda r) \left[\frac{e^{-\lambda(z+h)} + e^{-\lambda(z-h)}}{1 + e^{-2\lambda d}} \right] \quad (101)$$

and

$$H_{2\theta}^e = \frac{I}{4\pi r} \left\{ -1 + \frac{z-h}{\sqrt{r^2 + (z-h)^2}} \right. \\ \left. + 2r \int_0^{\infty} d\lambda J_1(\lambda r) \left[\frac{e^{-\lambda(z+h)} + e^{-\lambda(z-h)}}{1 + e^{-2\lambda d}} \right] \right\}. \quad (102)$$

In these integrals, the denominator can be converted to a hyperbolic cosine, but the numerators will be linear combinations of hyperbolic sine and cosine functions. Some of the terms can be rewritten using a construct similar to Equation (49), and can be treated with contour integration techniques. However, the remaining terms do not have the appropriate symmetry to accomplish this. These remaining terms have symmetry such that the integral can be extended over the real axis while retaining the Bessel function form, and there is no singularity at the origin as there is with the Hankel function. However, when the integrands of these terms are analytically continued into the complex plane, the Bessel functions diverge in either half plane, and the outer contour cannot be closed. Thus, there is no way to explicitly evaluate these integrals. As this particular case is more of a curiosity without practical significance, it will not be pursued further.

DISTRIBUTED CURRENT MAGNETIC FIELD GRADIENT

The magnetic field gradients due to the electrode distributed currents can be expressed in terms of the r and z derivatives of the magnetic field expressions already obtained in Equations (34), (35), (77), (80), and (99). First

$$H_x^e = -H_\theta^e \sin \theta = -\frac{y}{r} H_\theta^e \quad (103)$$

and

$$H_y^e = H_\theta^e \cos \theta = \frac{x}{r} H_\theta^e. \quad (104)$$

Then, for the diagonal gradients

$$\frac{\partial H_x^e}{\partial x} = \frac{xy}{r^3} - \frac{y}{r} \frac{\partial H_\theta^e}{\partial x} = \frac{xy}{r^2} \left(\frac{H_\theta^e}{r} - \frac{\partial H_\theta^e}{\partial r} \right) \quad (105)$$

and

$$\frac{\partial H_y^e}{\partial y} = -\frac{\partial H_x^e}{\partial x}, \quad (106)$$

the latter following from $\nabla \cdot \mathbf{H}^e = 0$ and $H_z^e = 0$ ($\nabla \cdot \mathbf{H} = 0$ and below, it is shown that $\nabla \cdot \mathbf{H}^{BS} = 0$).

For the off-diagonal gradients, the non-zero terms are

$$\frac{\partial H_x^e}{\partial z} = -\frac{y}{r} \frac{\partial H_\theta^e}{\partial z} \quad (107)$$

$$\frac{\partial H_y^e}{\partial z} = \frac{x}{r} \frac{\partial H_\theta^e}{\partial z} \quad (108)$$

$$\frac{\partial H_x^e}{\partial y} = -\left[\frac{x^2}{r^3} H_\theta^e + \frac{y^2}{r^2} \frac{\partial H_\theta^e}{\partial r} \right] \quad (109)$$

and

$$\frac{\partial H_y^e}{\partial x} = \left[\frac{y^2}{r^3} H_\theta^e + \frac{x^2}{r^2} \frac{\partial H_\theta^e}{\partial r} \right]. \quad (110)$$

CONDUCTING BOTTOM

Region 1

From Equation (34)

$$\begin{aligned} \frac{\partial H_{i\theta}^e}{\partial r} = & -\frac{H_{i\theta}^e}{r} + \frac{I}{4\pi} \left[\frac{z+h}{\sqrt{(r^2+(z+h)^2)^3}} + \sum_{n=1}^{\infty} Q^n \left\{ -\frac{2nd-z-h}{\sqrt{(r^2+(2nd-z-h)^2)^3}} \right. \right. \\ & \left. \left. + \frac{2nd+z+h}{\sqrt{(r^2+(2nd+z+h)^2)^3}} - \frac{2nd-z+h}{\sqrt{(r^2+(2nd-z+h)^2)^3}} + \frac{2nd+z-h}{\sqrt{(r^2+(2nd+z-h)^2)^3}} \right\} \right]. \end{aligned} \quad (111)$$

and

$$\begin{aligned} \frac{\partial H_{10}^e}{\partial z} = \frac{I}{4\pi r} & \left[\frac{(z+h)^2}{\sqrt{(r^2+(z+h)^2)^3}} - \frac{1}{\sqrt{r^2+(z+h)^2}} + \sum_{n=1}^{\infty} Q^n \left\{ \frac{(2nd-z-h)^2}{\sqrt{(r^2+(2nd-z-h)^2)^3}} \right. \right. \\ & + \frac{(2nd+z+h)^2}{\sqrt{(r^2+(2nd+z+h)^2)^3}} + \frac{(2nd-z+h)^2}{\sqrt{(r^2+(2nd-z+h)^2)^3}} + \frac{(2nd+z-h)^2}{\sqrt{(r^2+(2nd+z-h)^2)^3}} \\ & \left. \left. - \frac{1}{\sqrt{r^2+(2nd-z-h)^2}} - \frac{1}{\sqrt{r^2+(2nd+z+h)^2}} - \frac{1}{\sqrt{r^2+(2nd-z+h)^2}} - \frac{1}{\sqrt{r^2+(2nd+z-h)^2}} \right\} \right] \end{aligned} \quad (112)$$

Region 2

From Equation (35)

$$\begin{aligned} \frac{\partial H_{20}^e}{\partial r} = -\frac{H_{20}^e}{r} + \frac{I}{4\pi} & \left\{ -\frac{z-h}{\sqrt{(r^2+(z-h)^2)^3}} \right. \\ & \left. + (1-Q) \sum_{n=0}^{\infty} Q^n \left[\frac{2nd+z-h}{\sqrt{(r^2+2nd+z-h^2)^3}} + \frac{2nd+z+h}{\sqrt{(r^2+2nd+z+h^2)^3}} \right] \right\} \end{aligned} \quad (113)$$

and

$$\begin{aligned} \frac{\partial H_{20}^e}{\partial z} = \frac{I}{4\pi r} & \left\{ -\frac{(z-h)^2}{\sqrt{(r^2+(z-h)^2)^3}} \right. \\ & \left. + (1-Q) \sum_{n=0}^{\infty} Q^n \left[\frac{(2nd+z-h)^2}{\sqrt{(r^2+2nd+z-h^2)^3}} + \frac{(2nd+z+h)^2}{\sqrt{(r^2+2nd+z+h^2)^3}} \right] \right\}. \end{aligned} \quad (114)$$

NON-CONDUCTING BOTTOM (Q=1)

Region 1

From Equation (77)

$$\begin{aligned} \frac{\partial H_{10}^e}{\partial r} = \frac{I}{4\pi} & \left(-\frac{1}{r^2} \left[1 + \frac{z-h}{\sqrt{r^2+(z-h)^2}} - \frac{2z}{d} \right] - \frac{z-h}{\sqrt{(r^2+(z-h)^2)^3}} \right. \\ & \left. + \frac{\pi^2}{d^2} \sum_{n=1}^{\infty} n K_0(n\pi r/d) [\sin\{n\pi(z-h)/d\} + \sin\{n\pi(z+h)/d\}] \right) \end{aligned} \quad (115)$$

and

$$\frac{\partial H_{10}^c}{\partial z} = \frac{I}{4\pi r} \left(\frac{1}{\sqrt{r^2 + (z-h)^2}} - \frac{(z-h)^2}{\sqrt{(r^2 + (z-h)^2)^3}} - \frac{2}{d} \right. \quad (116)$$

$$\left. + r \frac{\pi^2}{d^2} \sum_{n=1}^{\infty} n K_1(n\pi r/d) [\cos\{n\pi(z-h)/d\} + \cos\{n\pi(z+h)/d\}] \right).$$

Region 2

From Equation (80)

$$\frac{\partial H_{20}^c}{\partial r} = -\frac{H_{20}^c}{r} - \frac{I}{4\pi} \left[\frac{z-h}{\sqrt{(r^2 + (z-h)^2)^3}} \right]. \quad (117)$$

and

$$\frac{\partial H_{20}^c}{\partial z} = -\frac{I}{4\pi r} \left[\frac{(z-h)^2}{\sqrt{(r^2 + (z-h)^2)^3}} \right]. \quad (118)$$

PERFECTLY CONDUCTING BOTTOM (Q=-1, REGION 1 ONLY)

From Equation (99)

$$\frac{\partial H_{10}^c}{\partial r} = \frac{I}{4\pi} \left(-\frac{z-h}{r^2 \sqrt{r^2 + (z-h)^2}} - \frac{z-h}{\sqrt{(r^2 + (z-h)^2)^3}} - \frac{2\pi}{d^2} \sum_{n=0}^{\infty} (n+1/2) K_1[(n+1/2)\pi r/d] \right. \quad (119)$$

$$\left. \{\sin[(n+1/2)\pi(z-h)/d] + \sin[(n+1/2)\pi(z+h)/d]\} \right)$$

and

$$\frac{\partial H_{10}^c}{\partial z} = \frac{I}{4\pi r} \left(-\frac{(z-h)^2}{\sqrt{(r^2 + (z-h)^2)^3}} + \frac{1}{\sqrt{r^2 + (z-h)^2}} - \frac{2\pi r}{d^2} \sum_{n=0}^{\infty} (n+1/2) K_1[(n+1/2)\pi r/d] \right. \quad (120)$$

$$\left. \{\cos[(n+1/2)\pi(z-h)/d] + \cos[(n+1/2)\pi(z+h)/d]\} \right).$$

**MAGNETIC FIELD/FIELD GRADIENT GENERATED
BY A CABLE TERMINATED IN REGION 1:
CABLE-ONLY CONTRIBUTION**

The magnetic field/field gradient of a cable carrying a current I , with each end shorted to the medium in region 1, can be constructed as the superposition of the magnetic field/field gradient of an electrode at each cable end point and the Biot-Savart magnetic field/field gradient of the cable current. The configuration is shown in Figure 4.

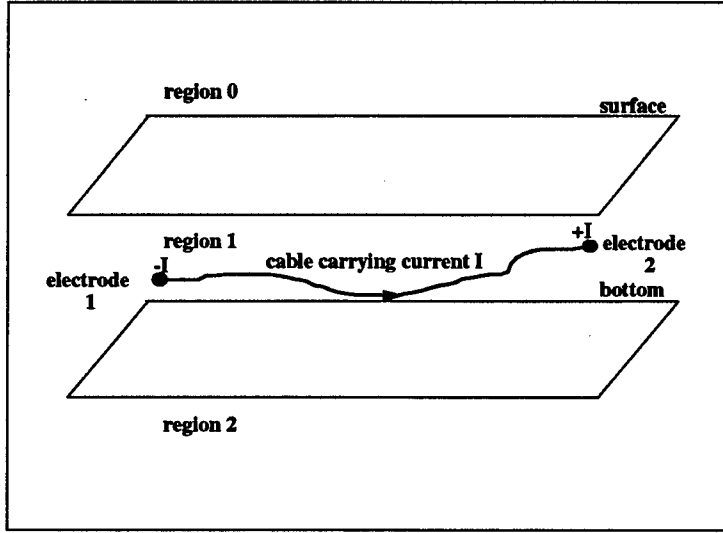


FIGURE 4. CURRENT-CARRYING CABLE/ELECTRODE CONFIGURATION

The electrode contributions to the magnetic field are constructed by means of Equations (34) through (36) with appropriate assignments of geometry and with current signs as indicated in Figure 4. The electrode contributions to the magnetic field gradient tensor are detailed in Equations (105) through (120). The contribution of the cable is obtained by means of a line integral along the cable. In practice, the line integral is approximated as a sum of line integrals over short linear segments. A general representation is now given of the magnetic field vector and gradient tensor for such a segment.

BIOT-SAVART FIELD AND GRADIENT OF A STRAIGHT CABLE SEGMENT

The geometry for the construction is shown in Figure 5. Vector quantities there are indicated with superposed arrows, but in the text they are indicated with boldface type. The various vectors involved are

$\mathbf{R}_1, \mathbf{R}_2$ locate cable segment end points (1 \rightarrow 2 positive current flow)

\mathbf{R} locates the field point

$\mathbf{R}_2 - \mathbf{R}_1$ is along the cable segment

$$\hat{\mathbf{i}} = \frac{\mathbf{R}_2 - \mathbf{R}_1}{|\mathbf{R}_2 - \mathbf{R}_1|}, \quad \hat{\mathbf{u}}_1 = \frac{\mathbf{R}_1 - \mathbf{R}}{|\mathbf{R}_1 - \mathbf{R}|}, \quad \text{and} \quad \hat{\mathbf{u}}_2 = \frac{\mathbf{R}_2 - \mathbf{R}}{|\mathbf{R}_2 - \mathbf{R}|} \quad \text{are unit vectors.}$$

With these definitions, the general expression for the magnetic field of the cable segment can be given. This expression is the result of analytically performing the line integral

$$\mathbf{H} = \frac{I}{4\pi} \int_{\mathbf{R}_1}^{\mathbf{R}_2} \frac{d\mathbf{l}' \times (\mathbf{R} - \mathbf{R}')}{|\mathbf{R} - \mathbf{R}'|^3} = \frac{I}{4\pi} \hat{\mathbf{l}} \times (\mathbf{R} - \mathbf{R}_1) I_1 \quad (121)$$

$$= \frac{I}{4\pi} \hat{\mathbf{l}} \times (\mathbf{R} - \mathbf{R}_1) \int_0^{|\mathbf{R}_1 - \mathbf{R}_2|} \frac{du}{(\sqrt{u^2 - 2\hat{\mathbf{l}} \cdot (\mathbf{R} - \mathbf{R}_1)u + |\mathbf{R} - \mathbf{R}_1|^2})^3}$$

over the straight segment, and using some results from vector geometry. The result is

$$I_1 = \frac{1}{|\mathbf{R} - \mathbf{R}_1|^2} \left[\frac{\hat{\mathbf{l}} \cdot (\hat{\mathbf{u}}_2 - \hat{\mathbf{u}}_1)}{1 - (\hat{\mathbf{l}} \cdot \hat{\mathbf{u}}_1)^2} \right] \quad (122)$$

and

$$\mathbf{H} = \frac{I}{4\pi |\mathbf{R}_1 - \mathbf{R}|} \hat{\mathbf{u}}_1 \times \hat{\mathbf{l}} \left[\frac{\hat{\mathbf{l}} \cdot (\hat{\mathbf{u}}_2 - \hat{\mathbf{u}}_1)}{1 - (\hat{\mathbf{u}}_1 \cdot \hat{\mathbf{l}})^2} \right]. \quad (123)$$

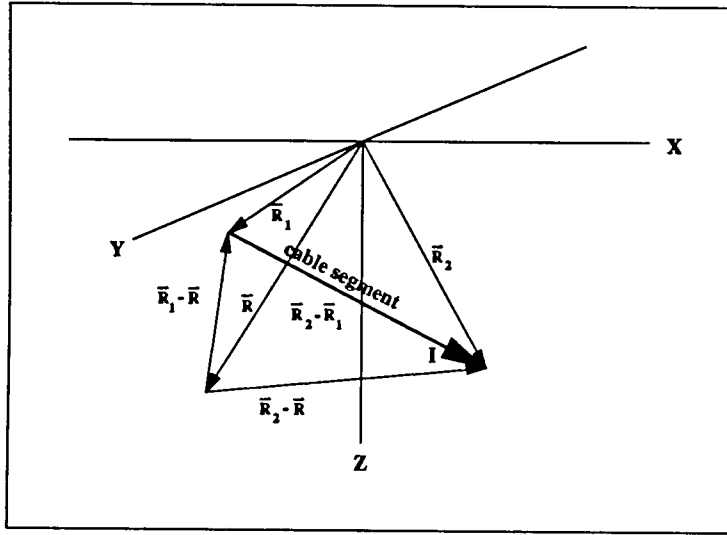


FIGURE 5. CABLE SEGMENT GEOMETRY

The field gradient tensor is best constructed by writing out Equation (121) in component form and taking the appropriate derivatives. Then

$$H_i = \frac{I}{4\pi} \epsilon_{imn} \hat{l}_m (R_n - R_{1n}) \int_0^{|\mathbf{R}_1 - \mathbf{R}_2|} \frac{du}{(\sqrt{u^2 - 2\hat{\mathbf{l}} \cdot (\mathbf{R} - \mathbf{R}_1)u + |\mathbf{R} - \mathbf{R}_1|^2})^3} \quad (124)$$

where the Levi-Civita tensor ϵ_{imn} has the properties

$$\epsilon_{imn} = 1 \quad i,m,n \text{ an even permutation of } 1,2,3 \quad (125)$$

$$\epsilon_{imn} = -1 \quad i,m,n \text{ an odd permutation of } 1,2,3$$

$$\epsilon_{imn} = 0 \quad \text{otherwise.}$$

This gives the gradient tensor components

$$\frac{\partial H_i}{\partial R_j} = \frac{I}{4\pi} \{ \epsilon_{imn} \hat{m}_m \delta_{nj} I_1 - 3 [\hat{\mathbf{I}} \times (\mathbf{R} - \mathbf{R}_1)]_i [(R_j - R_{1j}) I_2 - \hat{I}_j I_3] \} \quad (126)$$

where

$$I_2 = \int_0^{|\mathbf{R}_1 - \mathbf{R}_2|} \frac{du}{(\sqrt{u^2 - 2\hat{\mathbf{I}} \cdot (\mathbf{R} - \mathbf{R}_1)u + |\mathbf{R} - \mathbf{R}_1|^2})^5} \quad (127)$$

and

$$I_3 = \int_0^{|\mathbf{R}_1 - \mathbf{R}_2|} \frac{duu}{(\sqrt{u^2 - 2\hat{\mathbf{I}} \cdot (\mathbf{R} - \mathbf{R}_1)u + |\mathbf{R} - \mathbf{R}_1|^2})^5}. \quad (128)$$

Explicitly, define

$$J = \frac{1}{|\mathbf{R}_1 - \mathbf{R}|^2 [1 - (\hat{\mathbf{I}} \cdot \hat{\mathbf{u}}_1)^2]} \left[\frac{2\hat{\mathbf{I}} \cdot (\hat{\mathbf{u}}_2 - \hat{\mathbf{u}}_1)}{1 - (\hat{\mathbf{I}} \cdot \hat{\mathbf{u}}_1)^2} + \hat{\mathbf{I}} \cdot (\alpha_{12}^2 \hat{\mathbf{u}}_2 - \hat{\mathbf{u}}_1) \right] \quad (129)$$

$$\mathcal{K} = \frac{1}{|\mathbf{R}_1 - \mathbf{R}|^2} - \frac{\alpha_{12}}{|\mathbf{R}_2 - \mathbf{R}|^2} - (\hat{\mathbf{I}} \cdot \hat{\mathbf{u}}_1) J \quad (130)$$

with

$$\alpha_{12} = \frac{|\mathbf{R}_1 - \mathbf{R}|}{|\mathbf{R}_2 - \mathbf{R}|}. \quad (131)$$

Then

$$I_2 = \frac{J}{3|\mathbf{R}_1 - \mathbf{R}|^2} \quad (132)$$

$$I_3 = \frac{\mathcal{K}}{3|\mathbf{R}_1 - \mathbf{R}|} \quad (133)$$

and the gradient tensor components can all be written in the compact form

$$\frac{\partial H_i}{\partial R_j} = \frac{I}{4\pi} [\epsilon_{ijn} \hat{u}_m I_m - (\hat{\mathbf{1}} \times \hat{\mathbf{u}}_1)_i (\hat{\mathbf{1}}_j \mathcal{K} + \hat{\mathbf{u}}_{1j} \mathcal{J})]. \quad (134)$$

Here, it is noted that

$$\nabla \cdot \mathbf{H} = -\frac{I}{4\pi} \{ (\hat{\mathbf{1}} \times \hat{\mathbf{u}}_1) \cdot \hat{\mathbf{1}} \mathcal{K} + (\hat{\mathbf{1}} \times \hat{\mathbf{u}}_1) \cdot \hat{\mathbf{u}}_1 \mathcal{J} \} \equiv 0. \quad (135)$$

GRADIENT MEASUREMENTS: HOUSING EFFECTS

The presence of a distributed current causes the undisturbed gradient tensor to be asymmetric via the relation $\nabla \times \mathbf{H} = \mathbf{J}$. However, any practical gradiometer will be housed in a container that excludes the current from the measurement volume. This will have a profound effect on the measurement, in that it may modify the magnetic field in the enclosure relative to the original field, and it forces the measured gradient tensor to be symmetric, possibly in a way that may be difficult to determine.

Joseph, et. al.,^{1,2,3} have analyzed the effects of enclosures to determine the relationship between the measured field vector and gradient tensor components and those that exist in the absence of the enclosure. They have given detailed results for spherical and axisymmetric enclosures under the assumption that there are no other boundaries, and that the undisturbed current density \mathbf{J}^0 is uniform over the volume occupied by the enclosure.

SPHERICAL ENCLOSURE

For a spherical enclosure, the interior field has the form

$$\mathbf{H} = \mathbf{H}^0 + \mathbf{H}' \quad (136)$$

where \mathbf{H}^0 is the field in the absence of the enclosure, and \mathbf{H}' is the distortion field, which is related to the current \mathbf{J}^0 via

$$\mathbf{H}' = \frac{1}{2} (\mathbf{r} \times \mathbf{J}^0) \quad (137)$$

From this expression, it is clear that the enclosure has no effect on the field at the center of the sphere. Away from the center, there is a nonzero correction term. The expression also can be used to show that the effect of the enclosure on the gradient tensor is to select the symmetric part of the tensor; that is,

$$G_{ij} = \frac{1}{2} (G_{ij}^0 + G_{ji}^0) \quad (138)$$

an expression that holds *everywhere within the enclosure*. Thus, the measured gradient tensor may be easily calculated in terms of the undistorted tensor.

AXISYMMETRIC ENCLOSURE

In the axisymmetric case, the enclosure affects the field in the following way. On the symmetry axis, the axial distortion field is zero, while the field perpendicular to (athwart) the axis has nontrivial corrections. Off the symmetry axis, both components of the field have nontrivial corrections.

For the gradient tensor, on the symmetry axis, the diagonal components are undistorted, and those involving derivatives with respect to the coordinates perpendicular to the symmetry axis are symmetrized as in the spherical case. Those components involving derivatives with respect to the axial coordinate have corrections that depend on both the ambient current density and the axial enclosure profile, and these corrections may be as large as the undisturbed gradients.

GENERAL ENCLOSURE

In the general nonsymmetric enclosure case, all the gradient tensor components have nontrivial corrections depending on the current density vector and the enclosure shape. The determination of the corrections involves the solution of the system of equations³

$$\mathbf{H}'(\mathbf{r}) = \oint_S dS' \frac{1}{4\pi|\mathbf{r}-\mathbf{r}'|} \hat{\mathbf{n}}' \times (\mathbf{J}'_s - \sigma \nabla' \phi') \quad (139)$$

$$\nabla'^2 \phi' = 0 \quad (140)$$

$$\phi' \rightarrow 0 \quad (141)$$

and

$$\sigma \hat{\mathbf{n}} \cdot \nabla \phi' = \hat{\mathbf{n}} \cdot \mathbf{J}'_s \quad (142)$$

where ϕ is the potential due to the presence of the enclosure in the ambient (uniform) current distribution, σ is the medium conductivity, and $\hat{\mathbf{n}}$ is the outward normal to the surface of the enclosure.

The computation of enclosure corrections is beyond the scope of this report. It will be limited to the case of a spherical enclosure. For this case the measured gradient tensor from the electrode-cable combination will have the form

$$\mathbf{G}^s = \begin{pmatrix} G_{xx}^e + G_{xx}^c & \frac{1}{2}(G_{xy}^e + G_{yx}^e + G_{xy}^c + G_{yx}^c) & \frac{1}{2}(G_{xz}^e + G_{zx}^c + G_{zx}^c) \\ \frac{1}{2}(G_{xy}^e + G_{yx}^e + G_{xy}^c + G_{yx}^c) & (G_{yy}^c - G_{xx}^e) & \frac{1}{2}(G_{yz}^e + G_{yz}^c + G_{zy}^c) \\ \frac{1}{2}(G_{xz}^e + G_{zx}^c + G_{zx}^c) & \frac{1}{2}(G_{yz}^e + G_{yz}^c + G_{zy}^c) & -(G_{xx}^c + G_{yy}^c) \end{pmatrix}. \quad (143)$$

REFERENCES

1. Joseph, R. I.; Thomas, M. E.; and Allen, K. R., *Magnetic Field and Field Gradient Corrections within a Nonconducting Sensor Enclosure in a Conducting Fluid-Part I: Potential Flow*, IEEE Trans. Geosci. Rem. Sens., **GE-21**, #4, Oct 1983.
2. Joseph, R. I.; Thomas, M. E., *Magnetic Field and Field Gradient Corrections within a Nonconducting Sensor Enclosure in a Conducting Fluid-Part II: Vorticity*, IEEE Trans. Geosci. Rem. Sens., **GE-22**, #2, Mar 1984.
3. Joseph, R. I., *Magnetic Field and Field Gradient Corrections within a Nonconducting Sensor Enclosure in a Conducting Fluid-Part III: Current Exclusion Contribution for an Arbitrary Axisymmetric Enclosure*, IEEE Trans. Geosci. Rem. Sens., **GE-22**, #4, Jul 1984.

APPENDIX A

TRANSERA HIGH TECH BASIC CODES FOR REGION 1

CURRENT DENSITY CODE

```

10 SUB Jelectrode(R(*),Re(*),Db,S,Sb,I,Jay(*),Err)
20 X=R(1)
30 Y=R(2)
40 Z=R(3)
50 Xe=Re(1)
60 Ye=Re(2)
70 Ze=Re(3)
80 Dx=X-Xe
90 Dy=Y-Ye
100 IF SQR((Dx*Dx+Dy*Dy)/(X*X+Y*Y+Xe*Xe+Ye*Ye))<1.E-12 THEN
110   Jay(1)=0
120   Jay(2)=0
130   Jay(3)=0
140   GOTO 1560
150 END IF
160 !
170 !This program calculates the current density within the upper
180 !layer of a two-layer conductor, bounded above by air, due to
190 !an electrode injecting a current I into that layer. X,Y, and
200 !Z are the coordinates (meters) of the field point in a coordi-
210 !nate system whose origin is at the surface. Xe, Ye, and Ze are
220 !the coordinates (meters) of the electrode. Db is the depth to
230 !the conductor-conductor boundary below the air-conductor bound-
240 !ary. S is the conductivity (Siemens/meter) of the upper conduct-
250 !ing layer and Sb is that for the lower layer(for Sb=0, corre-
260 !sponding to an insulating bottom, or Sb<0, a perfectly conducting
270 !bottom, alternate series expressions are used). Jay(*) is the
280 !three-component field vector in amperes/meter^2. Err is the size
290 !of the summand relative to the sum when summation ceases. Note
300 !that I is positive if current enters the medium from the electrode
310 !and negative otherwise.
320 !
330 OPTION BASE 1
340 IF Sb>0 THEN
350   Q=(S-Sb)/(S+Sb)
360 ELSE
370   IF Sb=0 THEN
380     Q=1
390   ELSE
400     Q=-1
410   END IF
420 END IF
430 X2=(X-Xe)*(X-Xe)
440 Y2=(Y-Ye)*(Y-Ye)
450 R0=SQR(X2+Y2)
460 R03=R0*R0*R0

```

```

470 Rm=SQR(R0*R0+(Z-Ze)*(Z-Ze))
480 Rm3=Rm*Rm*Rm
490 Rp=SQR(R0*R0+(Z+Ze)*(Z+Ze))
500 Rp3=Rp*Rp*Rp
510 IF ABS(Q)<1 THEN
520   Smxy=1/Rm3+1/Rp3
530   Smz=(Z-Ze)/Rm3+(Z+Ze)/Rp3
540   Qn=Q
550   N=1
560   Zmm=2*N*Db-Z-Ze
570   Zpp=2*N*Db+Z+Ze
580   Zmp=2*N*Db-Z+Ze
590   Zpm=2*N*Db+Z-Ze
600   Rmm=SQR(R0*R0+Zmm*Zmm)
610   Rpp=SQR(R0*R0+Zpp*Zpp)
620   Rmp=SQR(R0*R0+Zmp*Zmp)
630   Rpm=SQR(R0*R0+Zpm*Zpm)
640   Rmm3=Rmm*Rmm*Rmm
650   Rpp3=Rpp*Rpp*Rpp
660   Rmp3=Rmp*Rmp*Rmp
670   Rpm3=Rpm*Rpm*Rpm
680   Intz=Qn*(-Zmm/Rmm3+Zpp/Rpp3-Zmp/Rmp3+Zpm/Rpm3)
690   Intxy=Qn*(1/Rmm3+1/Rpp3+1/Rmp3+1/Rpm3)
700   Smz=Smz+Intz
710   Smxy=Smxy+Intxy
720   IF N>4 THEN
730     IF ABS(Intz/Smz)<Err THEN
740       IF ABS(Intxy/Smxy)<Err THEN
750         GOTO 820
760       END IF
770     END IF
780   END IF
790   N=N+1
800   Qn=Q*Qn
810   GOTO 560
820   Jay(3)=I*Smz/4/PI
830   Jay(1)=(X-Xe)*I*Smxy/4/PI
840   Jay(2)=(Y-Ye)*I*Smxy/4/PI
850 ELSE
860   Zm=Z-Ze
870   Zp=Z+Ze
880   Smxy=0
890   Smz=0
900   Fxy=0
910   Fz=0
920   IF Q=1 THEN
930     N=1
940     CALL K0(N*PI*R0/Db,K0)
950     CALL K1(N*PI*R0/Db,K1)
960     Intxy=N*K1*(COS(N*PI*Zm/Db)+COS(N*PI*Zp/Db))
970     Intz=N*K0*(SIN(N*PI*Zm/Db)+SIN(N*PI*Zp/Db))
980     Smxy=Smxy+Intxy

```

```

990   Smz=Smz+Intz
1000  IF N>4 THEN
1010   IF ABS(Smxy)=0 THEN
1020     Fxy=1
1030   ELSE
1040     IF ABS(Intxy)/ABS(Smxy)<Err THEN
1050       Fxy=1
1060     END IF
1070   END IF
1080  IF ABS(Smz)=0 THEN
1090     Fz=1
1100   ELSE
1110     IF ABS(Intz)/ABS(Smz)<Err THEN
1120       Fz=1
1130     END IF
1140   END IF
1150  IF Fxy+Fz>1 THEN GOTO 1190
1160  N=N+1
1170  GOTO 940
1180  END IF
1190  Jay(3)=I/2/Db/Db*Smz
1200  Jfac=I/(2*PI*R0*R0*Db)*(1+PI*R0/Db*Smxy)
1210  Jay(1)=(X-Xe)*Jfac
1220  Jay(2)=(Y-Ye)*Jfac
1230  ELSE
1240    N=0
1250    CALL K0((N+.5)*PI*R0/Db,K0)
1260    CALL K1((N+.5)*PI*R0/Db,K1)
1270    Intxy=(N+.5)*K1*(COS((N+.5)*PI*Zm/Db)+COS((N+.5)*PI*Zp/Db))
1280    Intz=(N+.5)*K0*(SIN((N+.5)*PI*Zm/Db)+SIN((N+.5)*PI*Zp/Db))
1290    Smxy=Smxy+Intxy
1300    Smz=Smz+Intz
1310  IF N>4 THEN
1320    IF ABS(Smxy)=0 THEN
1330      Fxy=1
1340    ELSE
1350      IF ABS(Intxy)/ABS(Smxy)<Err THEN
1360        Fxy=1
1370      END IF
1380    END IF
1390  IF ABS(Smz)=0 THEN
1400    Fz=1
1410  ELSE
1420    IF ABS(Intz)/ABS(Smz)<Err THEN
1430      Fz=1
1440    END IF
1450  END IF
1460  IF Fxy+Fz>1 THEN GOTO 1500
1470  N=N+1
1480  GOTO 1250
1490  END IF
1500  Jay(3)=I/2/Db/Db*Smz

```

```

1510   Jfac=I/(2*R0*Db*Db)
1520   Jay(1)=(X-Xe)*Jfac*Smxy
1530   Jay(2)=(Y-Ye)*Jfac*Smxy
1540   END IF
1550   END IF
1560   SUBEND

```

ELECTRODE MAGNETIC FIELD CODE

```

10  SUB Helectrode(R(*),Re(*),Db,S,Sb,I,Aitch(*),Err)
20  X=R(1)
30  Y=R(2)
40  Z=R(3)
50  Xe=Re(1)
60  Ye=Re(2)
70  Ze=Re(3)
80  Dx=X-Xe
90  Dy=Y-Ye
100 IF SQR((Dx*Dx+Dy*Dy)/(X*X+Y*Y+Xe*Xe+Ye*Ye))<1.E-12 THEN
110   Aitch(1)=0
120   Aitch(2)=0
130   Aitch(3)=0
140   GOTO 1080
150 END IF
160 !
170 !This program calculates the magnetic field within the upper
180 !layer of a two-layer conductor, bounded above by air, due to
190 !an electrode injecting a current I into that layer. X,Y, and
200 !Z are the coordinates (meters) of the field point in a coordi-
210 !nate system whose origin is at the surface. Xe, Ye, and Ze are
220 !the coordinates (meters) of the electrode. Db is the depth to
230 !the conductor-conductor boundary below the air-conductor bound-
240 !ary. S is the conductivity (Siemens/meter) of the upper conduct-
250 !ing layer and Sb is that for the lower layer(for Sb=0, corre-
260 !sponding to an insulating bottom, or Sb<0, a perfectly conducting
270 !bottom, alternate series expressions are used). Aitch(*) is
280 !the three-component field vector(amperes/meter). Err is the size
290 !of the summand relative to the sum when summation ceases. Note
300 !that I is positive if current enters the medium from the electrode
310 !and negative otherwise.
320 !
330 OPTION BASE 1
340 IF Sb>0 THEN
350   Q=(S-Sb)/(S+Sb)
360 ELSE
370   IF Sb=0 THEN
380     Q=1
390   ELSE
400     Q=-1
410   END IF

```

```

420 END IF
430 X2=(X-Xe)*(X-Xe)
440 Y2=(Y-Ye)*(Y-Ye)
450 R0=SQR(X2+Y2)
460 R03=R0*R0*R0
470 Rp=SQR(R0*R0+(Z+Ze)*(Z+Ze))
480 R3=Rp*Rp*Rp
490 IF ABS(Q)<1 THEN
500   Sm=1-(Z+Ze)/Rp
510   Qn=Q
520   N=1
530   Zmm=2*N*Db-Z-Ze
540   Zpp=2*N*Db+Z+Ze
550   Zmp=2*N*Db-Z+Ze
560   Zpm=2*N*Db+Z-Ze
570   Rmm=SQR(R0*R0+Zmm*Zmm)
580   Rpp=SQR(R0*R0+Zpp*Zpp)
590   Rmp=SQR(R0*R0+Zmp*Zmp)
600   Rpm=SQR(R0*R0+Zpm*Zpm)
610   Int=Qn*(Zmm/Rmm-Zpp/Rpp+Zmp/Rmp-Zpm/Rpm)
620   Sm=Sm+Int
630   IF N>2 THEN
640     IF ABS(Int/Sm)<Err THEN
650       Sm=I*Sm/(4*PI*R0)
660       GOTO 1050
670     END IF
680   END IF
690   N=N+1
700   Qn=Q*Qn
710   GOTO 530
720 ELSE
730   Sm=0
740   Zm=Z-Ze
750   Zp=Z+Ze
760   Rm=SQR(R0*R0+Zm*Zm)
770   IF Q=1 THEN
780     N=1
790     CALL K1(N*PI*R0/Db,K1)
800     Int=K1*(SIN(N*PI*Zm/Db)+SIN(N*PI*Zp/Db))
810     Sm=Sm+Int
820     IF N>=2 THEN
830       IF ABS(Int)/ABS(Sm)<Err THEN 880
840     ELSE
850       N=N+1
860       GOTO 790
870     END IF
880     Sm=1-2*Z/Db+Zm/Rm+PI*R0/Db*Sm
890     Sm=I*Sm/(4*PI*R0)
900   ELSE
910     N=0
920     CALL K1((N+.5)*PI*R0/Db,K1)
930     Int=K1*(SIN((N+.5)*PI*Zm/Db)+SIN((N+.5)*PI*Zp/Db))

```

```

940   Sm=Sm+Int
950   IF N>=2 THEN
960     IF ABS(Int)/ABS(Sm)<Err THEN 1010
970   ELSE
980     N=N+1
990     GOTO 920
1000  END IF
1010  Sm=1+Zm/Rm-2*R0/Db*Sm
1020  Sm=I*Sm/(4*PI*R0)
1030  END IF
1040  END IF
1050  Aitch(1)=(Ye-Y)*Sm/R0
1060  Aitch(2)=(X-Xe)*Sm/R0
1070  Aitch(3)=0
1080  SUBEND

```

ELECTRODE MAGNETIC FIELD GRADIENT CODE

```

10  SUB Gelectrode(R(*),Re(*),Db,S,Sb,I,G(*),Err)
20  OPTION BASE 1
30  DIM H(3)
40  X=R(1)
50  Y=R(2)
60  Z=R(3)
70  Xe=Re(1)
80  Ye=Re(2)
90  Ze=Re(3)
100 !
110 !This program calculates the magnetic field gradient within the upper
120 !layer of a two-layer conductor, bounded above by air, due to
130 !an electrode injecting a current I into that layer. X,Y, and
140 !Z are the coordinates (meters) of the field point in a coordi-
150 !nate system whose origin is at the surface. Xe, Ye, and Ze are
160 !the coordinates (meters) of the electrode. Db is the depth to
170 !the conductor-conductor boundary below the air-conductor bound-
180 !ary. S is the conductivity (Siemens/meter) of the upper conduct-
190 !ing layer and Sb is that for the lower layer(for Sb=0, corre-
200 !sponding to an insulating bottom, or Sb<0, a perfectly conducting
210 !bottom, alternate series expressions are used). G(*) is the
220 !3x3 magnetic gradient tensor(amperes/meter^2). Err is the size
230 !of the summand relative to the sum when summation ceases. Note
240 !that I is positive if current enters the medium from the electrode
250 !and negative otherwise.
260 !
270 IF Sb>0 THEN
280   Q=(S-Sb)/(S+Sb)
290 ELSE
300   IF Sb=0 THEN
310     Q=1
320   ELSE

```

```

330   Q=-1
340   END IF
350   END IF
360   X2=(X-Xe)*(X-Xe)
370   Y2=(Y-Ye)*(Y-Ye)
380   Xy=(X-Xe)*(Y-Ye)
390   R0=SQR(X2+Y2)
400   R02=R0*R0
410   R03=R0*R0*R0
420   Rp=SQR(R0*R0+(Z+Ze)*(Z+Ze))
430   R3=Rp*Rp*Rp
440   IF ABS(Q)<1 THEN
450     Smtr=(Z+Ze)/R3
460     Smtz=(Z+Ze)*(Z+Ze)/R3-1/Rp
470     Qn=Q
480     N=1
490     Zmm=2*N*Db-Z-Ze
500     Zmm2=Zmm*Zmm
510     Zpp=2*N*Db+Z+Ze
520     Zpp2=Zpp*Zpp
530     Zmp=2*N*Db-Z+Ze
540     Zmp2=Zmp*Zmp
550     Zpm=2*N*Db+Z-Ze
560     Zpm2=Zpm*Zpm
570     Rmm=SQR(R0*R0+Zmm*Zmm)
580     Rmm3=Rmm*Rmm*Rmm
590     Rpp=SQR(R0*R0+Zpp*Zpp)
600     Rpp3=Rpp*Rpp*Rpp
610     Rmp=SQR(R0*R0+Zmp*Zmp)
620     Rmp3=Rmp*Rmp*Rmp
630     Rpm=SQR(R0*R0+Zpm*Zpm)
640     Rpm3=Rpm*Rpm*Rpm
650     Intr=Qn*(-Zmm/Rmm3+Zpp/Rpp3-Zmp/Rmp3+Zpm/Rpm3)
660     Inttz=Qn*(Zmm2/Rmm3+Zpp2/Rpp3+Zmp2/Rmp3+Zpm2/Rpm3)
670     Inttz=Inttz+Qn*(-1/Rmm-1/Rpp-1/Rmp-1/Rpm)
680     Smtr=Smtr+Intr
690     Smtz=Smtz+Inttz
700   IF N>2 THEN
710     IF ABS(Intr/Smtr)<Err THEN
720       IF ABS(Inttz/Smtz)<Err THEN
730         Smtr=I*Smtr/(4*PI)
740         Smtz=I*Smtz/(4*PI*R0)
750         GOTO 1290
760       END IF
770     END IF
780   END IF
790   N=N+1
800   Qn=Q*Qn
810   GOTO 490
820 ELSE
830   Smtr=0
840   Smtz=0

```

```

850  Zm=Z-Ze
860  Zp=Z+Ze
870  Rm=SQR(R0*R0+Zm*Zm)
880  Rm3=Rm*Rm*Rm
890  IF Q=1 THEN
900    N=1
910    CALL K0(N*PI*R0/Db,K0)
920    Intr=N*K0*(SIN(N*PI*Zm/Db)+SIN(N*PI*Zp/Db))
930    Smtr=Smtr+Intr
940    CALL K1(N*PI*R0/Db,K1)
950    Intz=N*K1*(COS(N*PI*Zm/Db)+COS(N*PI*Zp/Db))
960    Smtz=Smtz+Intz
970    IF N>=2 THEN
980      IF ABS(Intr)/ABS(Smtr)<Err THEN
990        IF ABS(Intz)/ABS(Smtz)<Err THEN
1000         GOTO 1060
1010        END IF
1020      END IF
1030    END IF
1040    N=N+1
1050    GOTO 910
1060    Smtr=I*(-(1+Zm/Rm-2*Z/Db)/R02-Zm/Rm3+PI*PI/Db/Db*Smtr)/4/PI
1070    Smtz=I*(-2/Db-Zm*Zm/Rm3+1/Rm+PI*PI*R0*Smtz/Db/Db)/(4*PI*R0)
1080  ELSE
1090    N=0
1100    CALL K0((N+.5)*PI*R0/Db,K0)
1110    Intr=(N+.5)*K0*(SIN((N+.5)*PI*Zm/Db)+SIN((N+.5)*PI*Zp/Db))
1120    Smtr=Smtr+Intr
1130    CALL K1((N+.5)*PI*R0/Db,K1)
1140    Intz=(N+.5)*K1*(COS((N+.5)*PI*Zm/Db)+COS((N+.5)*PI*Zp/Db))
1150    Smtz=Smtz+Intz
1160    IF N>=2 THEN
1170      IF ABS(Intr)/ABS(Smtr)<Err THEN
1180        IF ABS(Intz)/ABS(Smtz)<Err THEN
1190          GOTO 1250
1200        END IF
1210      END IF
1220    END IF
1230    N=N+1
1240    GOTO 1100
1250    Smtr=-I*(Zm/Rm/R02+Zm/Rm3+2*PI*Smtr/Db/Db)/4/PI
1260    Smtz=I*(-Zm*Zm/Rm3+1/Rm-2*PI*R0*Smtz/Db/Db)/(4*PI*R0)
1270  END IF
1280 END IF
1290 CALL Helectrode(R(*),Re(*),Db,S,Sb,I,H(*),Err)
1300 Ht=(X-Xe)*H(2)/R0-(Y-Ye)*H(1)/R0
1310 IF ABS(Q)<1 THEN
1320   Smtr=Smtr-Ht/R0
1330 END IF
1340 G(1,1)=Xy*(Ht-R0*Smtr)/R03
1350 G(2,2)=-G(1,1)
1360 G(3,3)=0

```

```

1370 G(3,1)=0
1380 G(3,2)=0
1390 G(1,2)=- (X2*Ht+R0*Y2*Smtz)/R03
1400 G(2,1)= (Y2*Ht+R0*X2*Smtz)/R03
1410 G(1,3)= -(Y-Ye)*Smtz/R0
1420 G(2,3)= (X-Xe)*Smtz/R0
1430 SUBEND

```

BIOT-SAVART MAGNETIC FIELD CODE

```

10 SUB Hbiotsavrt(I,R1(*),R2(*),R(*),H(*))
20 OPTION BASE 1
30 DIM X1(3),X2(3),X12(3),L(3),U1(3),U2(3)
40 !
50 !This program calculates the magnetic field H(*) (amperes/meter)
60 !at the point R(*) (meters), due to a straight wire segment
70 !whose end points are R1(*) and R2(*) (meters), and in which
80 !the current flows from R1(*) to R2(*).
90 !
100 X1(1)=R1(1)-R(1)
110 X1(2)=R1(2)-R(2)
120 X1(3)=R1(3)-R(3)
130 X2(1)=R2(1)-R(1)
140 X2(2)=R2(2)-R(2)
150 X2(3)=R2(3)-R(3)
160 X12(1)=R2(1)-R1(1)
170 X12(2)=R2(2)-R1(2)
180 X12(3)=R2(3)-R1(3)
190 R1m=SQR(DOT(X1,X1))
200 R2m=SQR(DOT(X2,X2))
210 R12m=SQR(DOT(X12,X12))
220 MAT L= (1/R12m)*X12
230 MAT U1= (1/R1m)*X1
240 MAT U2= (1/R2m)*X2
250 N=DOT(L,U1)-DOT(L,U2)
260 D=1-DOT(L,U1)*DOT(L,U1)
270 Fac=(I*N)/(4*PI*R1m*D)
280 H(1)=L(2)*U1(3)-L(3)*U1(2)
290 H(2)=L(3)*U1(1)-L(1)*U1(3)
300 H(3)=L(1)*U1(2)-L(2)*U1(1)
310 MAT H= (Fac)*H
320 SUBEND

```

BIOT-SAVART MAGNETIC FIELD GRADIENT CODE

```

10 SUB Gbiotsavrt(I,R1(*),R2(*),R(*),G(*))
20 OPTION BASE 1
30 DIM X1(3),X2(3),X12(3),L(3),U1(3),U2(3)

```

```

40      !
50      !This program calculates the magnetic field gradient tensor
60      !G(*) (ampere/meter^2) at the point R(*) (meters), due to a
70      !straight wire segment whose end points are R1(*) and R2(*)
80      !(meters) and in which positive current I flows from R1(*)
90      !to R2(*).
100     !
110    X1(1)=R1(1)-R(1)
120    X1(2)=R1(2)-R(2)
130    X1(3)=R1(3)-R(3)
140    X2(1)=R2(1)-R(1)
150    X2(2)=R2(2)-R(2)
160    X2(3)=R2(3)-R(3)
170    X12(1)=R2(1)-R1(1)
180    X12(2)=R2(2)-R1(2)
190    X12(3)=R2(3)-R1(3)
200    R1m=SQR(DOT(X1,X1))
210    R2m=SQR(DOT(X2,X2))
220    R12m=SQR(DOT(X12,X12))
230    MAT L=(1/R12m)*X12
240    MAT U1=(1/R1m)*X1
250    MAT U2=(1/R2m)*X2
260    N=DOT(L,U1)-DOT(L,U2)
270    D=1-DOT(L,U1)*DOT(L,U1)
280    I1=-N/D/R1m/R1m
290    A12=R1m/R2m
300    I2=(-2*N/D+A12*A12*DOT(L,U2)-DOT(L,U1))/D/R1m/R1m
310    I3=1/R1m/R1m-A12/R2m/R2m-DOT(L,U1)*I2
320    Gfac=I/4/PI
330    G(1,1)=Gfac*(-L(2)*U1(3)+L(3)*U1(2))*(L(1)*I3+U1(1)*I2)
340    G(2,2)=Gfac*(-L(3)*U1(1)+L(1)*U1(3))*(L(2)*I3+U1(2)*I2)
350    G(3,3)=-G(1,1)-G(2,2)
360    G(1,2)=Gfac*(-L(3)*I1+(-L(2)*U1(3)+L(3)*U1(2))*(L(2)*I3+U1(2)*I2))
370    G(2,1)=Gfac*(L(3)*I1+(-L(3)*U1(1)+L(1)*U1(3))*(L(1)*I3+U1(1)*I2))
380    G(1,3)=Gfac*(L(2)*I1+(-L(2)*U1(3)+L(3)*U1(2))*(L(3)*I3+U1(3)*I2))
390    G(3,1)=Gfac*(-L(2)*I1+(-L(1)*U1(2)+L(2)*U1(1))*(L(1)*I3+U1(1)*I2))
400    G(2,3)=Gfac*(-L(1)*I1+(-L(3)*U1(1)+L(1)*U1(3))*(L(3)*I3+U1(3)*I2))
410    G(3,2)=Gfac*(L(1)*I1+(-L(1)*U1(2)+L(2)*U1(1))*(L(2)*I3+U1(2)*I2))
420  SUBEND

```

MODIFIED BESSEL FUNCTION CODES

```

10  SUB I0(X,I0)
20  !
30  !calculates the modified bessel function I0(x) for real x>=0
40  !
50  OPTION BASE 1
60  DIM A(9),T(9)
70  MAT A=(0)
80  MAT T=(0)

```

```

90  U=X/3.75
100 IF U<=1 THEN
110  A(1)=1
120  A(2)=3.5156229
130  A(3)=3.0899424
140  A(4)=1.2067492
150  A(5)=.2659732
160  A(6)=.0360768
170  A(7)=.0045813
180  T(1)=1
190  T(2)=U*U
200  T(3)=T(2)*T(2)
210  T(4)=T(2)*T(3)
220  T(5)=T(3)*T(3)
230  T(6)=T(2)*T(5)
240  T(7)=T(4)*T(4)
250  I0=DOT(A,T)
260  ELSE
270  A(1)=.39894228
280  A(2)=.01328592
290  A(3)=.00225319
300  A(4)=-.00157565
310  A(5)=.00916281
320  A(6)=-.02057706
330  A(7)=.02635537
340  A(8)=-.01647633
350  A(9)=.00392377
360  T(1)=1
370  T(2)=1/U
380  T(3)=T(2)*T(2)
390  T(4)=T(2)*T(3)
400  T(5)=T(3)*T(3)
410  T(6)=T(2)*T(5)
420  T(7)=T(4)*T(4)
430  T(8)=T(2)*T(7)
440  T(9)=T(5)*T(5)
450  I0=EXP(X)/SQR(X)*DOT(A,T)
460  END IF
470  SUBEND

480  SUB I1(X,I1)
490  !
500  !calculates the modified bessel function I1(x) for real x>=0
510  !
520  OPTION BASE 1
530  DIM A(9),T(9)
540  MAT A=(0)
550  MAT T=(0)
560  U=X/3.75
570  IF U<=1 THEN
580  A(1)=.5
590  A(2)=.87890594

```

```

600  A(3)=.51498869
610  A(4)=.15084934
620  A(5)=.02658733
630  A(6)=.00301532
640  A(7)=.00032411
650  T(1)=1
660  T(2)=U*U
670  T(3)=T(2)*T(2)
680  T(4)=T(2)*T(3)
690  T(5)=T(3)*T(3)
700  T(6)=T(2)*T(5)
710  T(7)=T(4)*T(4)
720  I1=X*DOT(A,T)
730  ELSE
740  A(1)=.39894228
750  A(2)=-.03988024
760  A(3)=-.00362018
770  A(4)=.00163801
780  A(5)=-.01031555
790  A(6)=.02282967
800  A(7)=-.02895312
810  A(8)=.01787654
820  A(9)=-.00420059
830  T(1)=1
840  T(2)=1/U
850  T(3)=T(2)*T(2)
860  T(4)=T(2)*T(3)
870  T(5)=T(3)*T(3)
880  T(6)=T(2)*T(5)
890  T(7)=T(4)*T(4)
900  T(8)=T(2)*T(7)
910  T(9)=T(5)*T(5)
920  I1=EXP(X)/SQR(X)*DOT(A,T)
930  END IF
940  SUBEND
950  SUB K0(X,K0)
960  !
970  !calculates the modified bessel function K0(x) for real x>=0
980  !
990  OPTION BASE 1
1000 DIM A(7),T(7)
1010 MAT A=(0)
1020 MAT T=(0)
1030 U=X/2
1040 IF U<=1 THEN
1050  A(1)=-.57721566
1060  A(2)=.42278420
1070  A(3)=.23069756
1080  A(4)=.03488590
1090  A(5)=.00262698
1100  A(6)=.00010750
1110  A(7)=.00000740

```

```

1120 T(1)=1
1130 T(2)=U*U
1140 T(3)=T(2)*T(2)
1150 T(4)=T(2)*T(3)
1160 T(5)=T(3)*T(3)
1170 T(6)=T(2)*T(5)
1180 T(7)=T(4)*T(4)
1190 CALL I0(X,I)
1200 K0=-LOG(U)*I+DOT(A,T)
1210 ELSE
1220 A(1)=1.25331414
1230 A(2)=-.07832358
1240 A(3)=.02189568
1250 A(4)=-.01062446
1260 A(5)=.00587872
1270 A(6)=-.00251540
1280 A(7)=.00053208
1290 T(1)=1
1300 T(2)=1/U
1310 T(3)=T(2)*T(2)
1320 T(4)=T(2)*T(3)
1330 T(5)=T(3)*T(3)
1340 T(6)=T(2)*T(5)
1350 T(7)=T(4)*T(4)
1360 K0=EXP(-X)/SQR(X)*DOT(A,T)
1370 END IF
1380 SUBEND
1390 SUB K1(X,K1)
1400 !
1410 !calculates the modified bessel function K1(x) for real x>=0
1420 !
1430 OPTION BASE 1
1440 DIM A(7),T(7)
1450 MAT A=(0)
1460 MAT T=(0)
1470 U=X/2
1480 IF U<=1 THEN
1490 A(1)=1
1500 A(2)=.15443144
1510 A(3)=-.67278579
1520 A(4)=-.18156897
1530 A(5)=-.01919402
1540 A(6)=-.00110404
1550 A(7)=-.00004686
1560 T(1)=1
1570 T(2)=U*U
1580 T(3)=T(2)*T(2)
1590 T(4)=T(2)*T(3)
1600 T(5)=T(3)*T(3)
1610 T(6)=T(2)*T(5)
1620 T(7)=T(4)*T(4)
1630 CALL I1(X,I)

```

```
1640 K1=LOG(U)*I+DOT(A,T)/X
1650 ELSE
1660 A(1)=1.25331414
1670 A(2)=-.23498619
1680 A(3)=-.03655620
1690 A(4)=-.01504268
1700 A(5)=-.00780353
1710 A(6)=-.00325614
1720 A(7)=-.00068245
1730 T(1)=1
1740 T(2)=1/U
1750 T(3)=T(2)*T(2)
1760 T(4)=T(2)*T(3)
1770 T(5)=T(3)*T(3)
1780 T(6)=T(2)*T(5)
1790 T(7)=T(4)*T(4)
1800 K1=EXP(-X)/SQR(X)*DOT(A,T)
1810 END IF
1820 SUBEND
```

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