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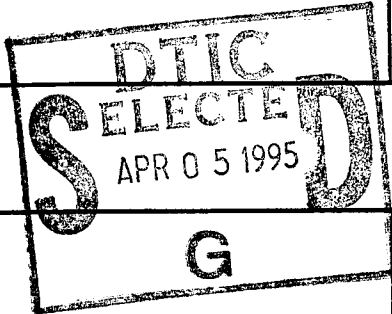
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13. ABSTRACT (Maximum 200 words)

This final report summarizes research accomplishments supported by AFOSR Grant F49620-92-J-0002. Over the three-year period of support under this grant we have had considerable success in each of the several components of our research program, namely the development of multiresolution statistical approaches to problems of image analysis the analysis of singular systems with applications in efficient processing of multidimensional data, large-scale estimation and computation in remote sensing and space-time data assimilation, multiresolution and wavelet-based methods for the detection and classification of abrupt changes in signals, and data fusion and inversion using multiresolution and wavelet-based methods. In this report we outline our accomplishments in each of these areas and also include a complete list of reports and publications describing this work.

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LABORATORY FOR INFORMATION AND DECISION SYSTEMS  
Massachusetts Institute of Technology  
Cambridge, Massachusetts 02139

*Final Technical Report for*  
Grant F49620-92-J-0002

**MULTIRESOLUTION SIGNAL AND SYSTEM ANALYSIS AND  
THE ANALYSIS AND CONTROL OF DISCRETE-EVENT  
DYNAMIC SYSTEMS**

*for the period*  
1 October 1991 through 30 September 1994

*Submitted to:* Drs.. Marc Q. Jacobs and Jon Sjogren  
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## I. Summary

In this final report we summarize our accomplishments in the research program supported by Grant AFOSR-92-J-0002 over the period from October 1, 1991 to September 30, 1994. The basic scope of this research program was to carry out fundamental research in the analysis, estimation, processing, and control of complex signals and systems with particular emphasis on (a) the development of theories and processing methodologies for signals possessing multiple resolution descriptions and for systems with multiple time scales using concepts related to wavelet transforms and pyramidal processing structures for the former and singular perturbation methods for discrete-state systems for the latter; and (b) the analysis of complex, discrete-event systems, providing a more qualitative control-theoretic counterpart to the analytic perturbation methods under (a). The principal investigator for this effort is Professor Alan S. Willsky, and Dr. W. C. Karl is co-principal investigator. Professor Willsky and Dr. Karl have been assisted by several graduate research assistants as well as additional thesis students not requiring stipend or tuition support from this grant.

We feel that our work under this grant has been highly successful not only in producing a significant number of results and publications but also in both uncovering new research questions and directions for future investigations and in providing advanced technology of direct relevance to Air Force missions and programs. Indeed discussions with Air Force personnel and with other organizations working on Air Force projects have provided significant motivation and direction for the research we have carried out and that we plan for the future. At the end of this report we have included a list of the publications supported by Grant F49620-92-J-002. In total the publications resulting from research supported by Grant F49620-92-J-002 include 23 papers that have appeared or been submitted to journals, 9 journal papers presently in preparation, 25 papers presented at conferences, 2 book chapters, 3 S.M. theses, and 5 Ph.D. theses. We anticipate that several additional papers will also be written.

In addition, our work has received considerable national and international attention and recognition. In particular:

Professor Willsky was one of the Guest Editors (along with Prof. Stephane Mallat and Dr. Ingrid Daubechies) of the *IEEE Transactions on Information Theory* special issue on wavelet transforms and multiresolution signal analysis, March 1992.

Professor Willsky was invited to give the keynote address at the inaugural workshop for the Centre for Robust and Adaptive Systems, Canberra, Australia, February, 1992. The official center opening was performed by The Hon. Ross

Free, M.P. and Minister of Science and Technology. Prof. Willsky gave lectures on both of the principal components of the research supported by this grant, namely multiresolution signal and image processing and modeling and detection of abrupt changes in systems and signals and related problems of discrete event systems. In addition, Prof. Willsky gave a speech on the role of national research centers and on the issues involved in technology transfer and partnerships between universities, industry, and government.

Prof. Willsky was asked to give the principal lecture on "Multiresolution Methods in Statistical Image Analysis," at the Tri-Service Workshop on Statistical Methods in Image Processing, Harry Diamond Lab, Maryland, May 1992.

Dr. Mark Luetngen a Ph.D. student with Prof. Willsky presented an invited talk on "Multiscale Signal Processing" at the American Mathematics Society Conference on Wavelets and Applications, S. Hadley, MA, June 1992.

Prof. Willsky delivered a plenary address at the SIAM Conference on Control and Its Applications, Minneapolis, September 1992. The subject of this address was systems and control challenges in image analysis, with particular emphasis on multiresolution models and methods.

Prof. Willsky gave one of the principal plenary lectures at the INRIA (Institut National de Recherche en Informatique et en Automatique) Twenty-Fifth Anniversary Symposium, held at the French Ministry of Research, Paris, France, December 1992.

Dr. Karl was asked to organize an invited session on geometric and stochastic methods in image analysis and reconstruction.

Prof. Willsky gave the opening keynote address at the IEEE Image and Multidimensional Signal Processing Workshop, Cannes, France, Sept. 1993.

Dr. Karl has been asked to serve on the organizing committee for the upcoming Workshop on Wavelets in Medical Image Processing to be held at Johns Hopkins.

Prof. Willsky was asked to give a plenary lecture on estimation and statistical inference problems in image analysis at the American Statistical Association Meeting in Toronto in August 1994.

In addition, our work has also been directly recognized by the Air Force as evidenced by the following:

Our work was cited in the 1991 AFOSR Research Highlights publication.

Prof. Willsky was one of the participants in the March 1992 AFOSR-sponsored Workshop on Applications of Wavelet Transforms in Signal Processing, held at The Air Force Institute of Technology, Wright-Patterson Air Force Base, Ohio.

Prof. Willsky delivered an invited talk on Multiresolution Signal Analysis in Rome, New York, sponsored both by the local chapter of the IEEE and by personnel at Rome Laboratory (Mr. John Graneiro and Mr. Vincent Vannicola).

Mr. Eric Miller, one of Prof. Willsky's Ph.D. students working on multiresolution methods, was awarded an Air Force Doctoral Scholarship, championed by the signal processing and surveillance professionals at Rome Laboratories.

Furthermore, there has been considerable interaction with Air Force personnel, a number of examples of technology transfer, and a variety of research problems that have been inspired by discussions with Air Force personnel or that have been motivated by Air Force missions and needs:

Our work on multiresolution image processing has formed the basis for several projects in surveillance applications. In particular, an AFOSR-sponsored Phase II SBIR project on multiresolution image fusion (being performed by Alphatech, Inc.) is a direct outgrowth of our MIT research. In addition, several additional SBIR projects involving the transition of our research are currently being pursued with Rome Laboratory (in remote sensing and surveillance).

In March 1992, Prof. Willsky met with Drs. S. Banda and P. Chandler of Wright Laboratories to discuss problems in robust failure detection. Prof. Willsky's previous work in failure detection has already found its way into flight-tested systems, and he is recognized as one of the leading authorities in this area. The limitations of existing methods has led us again to formulate new research problems (see Section V).

Prof. Willsky and Dr. W.C. Karl have been engaged in extensive discussions and collaboration with engineers at Lincoln Laboratory concerning the incorporation of advanced multiresolution methods into integrated automatic target recognition systems. Lincoln Lab serves as the Center of Excellence for this ARPA's program in this area, and Prof. Willsky has established a continuing relationship with Lincoln. As part of this interaction, we have recently demonstrated that our multiresolution image analysis methods can significantly enhance performance of existing ATR algorithms such as the one that has been developed over a number of years at Lincoln.

In addition, Prof. Willsky has established contact with Air Force efforts in ATR, most notably at Wright Laboratory with Mr. E. Zelnio and his group. In particular, in June 1994, Prof. Willsky visited Mr. Zelnio's group together with Dr. Charles Holland and Dr. Abe Waksman in order to engage in extensive technical discussions and to review ongoing research activities at Wright Lab.

Prof. Willsky participated in discussions with Drs. C. Holland and N. Glassman of AFOSR in order to assist them in defining a new initiative in discrete event systems (in which they were ultimately successful).

Prof. Willsky and Dr. Karl have also recently initiated discussions with researchers at Phillips Laboratory (in particular, we have recently met with Dr. S. Cusumano) dealing with issues related to optical tracking for the High-Energy Laser program.

Prof. Willsky, Dr. Karl, and graduate student Mr. M. Bhatia have recently initiated interactions with other MIT researchers (led by Dr. R. Lanza of MIT's Laboratory for Nuclear Science) involved in advanced methods of nondestructive evaluation for applications such as corrosion detection in aging aircraft.

## II. Research Progress Description

In this section we briefly describe the research accomplishments we have achieved over the three-year period of our grant. We limit ourselves here to a succinct summary of these results and refer to the publications listed at the end of this report for detailed developments. Our research can roughly be divided several parts which we describe in the following paragraphs.

### Multiresolution approaches to image analysis

The research described in this section is reported in a number of papers and reports [16-32, 34, 41-44,49-50,54,62]. The basic idea behind our work is the use of scale-recursive stochastic models to describe signals, images, and phenomena in 1, 2, or higher dimension (i.e., where not only the process values but also the independent variable may be multidimensional--e.g., to describe spatially-distributed phenomena). More precisely, the models that we have developed evolve on pyramidal structures, namely trees, where each level in the tree corresponds to a particular resolution of representation and where the process is described by scale-to-scale dynamics. For the most part the dynamics that we have used are linear, although, as we discuss shortly, the framework can readily accommodate nonlinear and/or discrete-valued dynamics and variables. More precisely, the dynamics we have considered are of the form

$$x(t) = A(t) x(t\gamma) + B(t) w(t) \quad (1)$$

$$y(t) = C(t) x(t) + v(t) \quad (2)$$

where  $t$  is the index of nodes on a multiresolution tree over which our model is defined. In particular,  $t$  should be thought of as indexing a pair (scale, location) where "scale" indexes the level on the tree (i.e., the resolution) corresponding to node  $t$  and "location" indexes the spatial or temporal location to which this node corresponds. The node  $t$  is connected to a number of child nodes at the next finer scale representing finer scale descriptions of the phenomenon and a finer scale partitioning of the region corresponding to node  $t$ . While the number of children is arbitrary in general, the standard cases usually considered are that of a dyadic tree (where each node has 2 children) for 1-D processes and a quad-tree (4 children) for 2-D processes (although it is certainly possible to represent either 1-D or 2-D processes with trees with any number of children and in fact with space and scale-varying numbers of children)

In general the process  $x(t)$  is a vector process capturing all of the relevant information about the phenomenon of interest at that particular scale and location so that given  $x(t)$ , finer-resolution detail at this location is independent of the behavior of the process outside this local region. Also, we use the notation  $t\gamma$  to denote the "parent" of node  $t$ , namely the next coarser-scale node covering the region corresponding to node  $t$ . Thus what (1) states is that the finer-scale representation  $x(t)$  of our phenomenon at node  $t$  is obtained by interpolating the coarser-scale description  $x(t\gamma)$  at parent node  $t\gamma$  and then adding some independent finer scale detail, captured by the term  $B(t)w(t)$ . Equation (2) then models the available data, which may be available at multiple scales of resolution. An important fact here is that the model parameters ( $A$ ,  $B$ ,  $C$ , and the noise intensities) may vary from node to node. This allows us to capture a number of important features, including scale-varying statistics (which, for example, allows us to capture scaling laws used to describe fractal processes such as those with  $1/f$  - like spectra) and the availability of irregularly sampled data in both space and resolution (allowing us to capture frequently encountered situations in sensor fusion, remote sensing, and other contexts in which complete coverage data may not be available at any scale but there may be an irregular pattern of overlaps of coverage of sensors with different resolutions)

In our work we have been able to demonstrate the considerable power of this modeling framework. In particular, some of the important results of our work are the following:

1. The simplest example of a model of the form of (1), (2) is the so-called Haar model, in which  $x(t)$  simply represents the average value of the phenomenon of interest over an interval of length  $2^{-m}$  (for 1-D processes using a dyadic tree and where the scale of  $t$  is denoted by  $m$ ) or over a square with side of length  $2^{-m}$  (for 2-D processes using a quad-tree). However, the framework described by (1), (2) is in fact far richer than this simple example. In particular, as described in [26], *any* Markov process in 1-D (and in fact any process in the somewhat larger class of reciprocal processes or 1-D Markov random fields) can be exactly represented as in (1), (2) using a generalization of the so-called midpoint deflection construction of Brownian motion. In particular, for any such process, given the value at one point, the values to its left and right are independent, and this fact then leads directly to a method for constructing the process that is exactly of the form of (1), (2). Moreover, it is also shown in [26] that in principle any Markov random field (MRF) in 2-D may also be exactly modeled using a multiresolution model as in (1), (2). The idea here is a generalization of that used in 1-D. Specifically, if we wish to represent an MRF over a spatial region, then, given the values of the field over any line separating the region into two disjoint regions, then the values of the field in these two regions are independent. This again leads to a method for constructing the MRF exactly as in (1), (2). The problem, of course, in 2-D is that the memory needed to decorrelate the behavior of the field over the two regions is an entire *line* of values of the field (and not simply the value of the process at a point), so that the "state" dimension of  $x(t)$  depends on the size of the region of interest--*that is if we truly wish to model the MRF exactly*. In particular, since MRF models are idealizations themselves, we were led to

the idea of using an approximate multiscale representation for such a field, where at each scale we assume that the two disjoint subregions of the region corresponding to node  $t$  are independent when we are given a coarse approximation of the values of the field along the boundary between the two subregions. Specifically, if we imagine taking the wavelet transform of the values of the field along the 1-D boundary between the two regions, then our approximate models consist of assuming that the two regions are independent given only a certain knowledge of only *some* of that transform. By varying how much "some" is we can trade off model complexity (in terms of the dimensionality of the wavelet coefficients that must be captured in  $x(t)$ ) with accuracy in approximating the MRF (where we have an exact representation if all of the wavelet coefficients are kept). Note that as we move to finer scales, the size of the regions being considered (which are continually subdivided as we move from scale to scale) become smaller, so that the "coarse" representation that we are keeping in fact becomes rather fine (and eventually is exact). While a rational and mathematically precise methodology for determining when this method is useful (in that acceptably accurate models are obtained with acceptably low dimensionality) is yet to be developed, we have demonstrated [26] that this approach does yield surprisingly good models of very low dimensionality for a variety of MRF's. Since MRF's are widely accepted as a very rich modeling framework, what we have demonstrated is that our multiresolution modeling framework is at least as rich--and indeed richer, since it provides a much more direct way in which to capture correlation structures at different scales than can be captured using MRF's. Moreover, as the next point makes clear, these models have additional and very significant advantages.

2. One of the great strengths of the modeling framework provided by (1), (2) is that it leads to extremely powerful and efficient optimal processing algorithms. In particular, as described in our papers (see in particular [25]), the optimal estimation problem for (1), (2)--i.e., the construction of the optimal estimate of  $x(t)$  at each node (and thus at every scale and location) based on all of the data given in (2) has a very efficient solution, which involves the scale-recursive generalization of the powerful time-recursive estimation methods of systems and control, namely Kalman filtering and Rauch-Tung-Striebel smoothing. In particular, the optimal estimator consists of a fine-to-coarse Kalman filtering sweep in which at each node we compute the best estimate at that node given all of the data available at nodes in the subtree beneath that node, followed by a coarse-to-fine smoothing sweep, again following the topology of the tree. The end result is an algorithm that produces optimal estimates and corresponding error covariances at each scale and location with computational complexity per data point at the fine scale that is constant independent of the size of the data region being considered.<sup>1</sup> While this is not surprising for 1-D signals (where the usual Kalman filter does this), it represents a major breakthrough for 2-D processes. In particular, solving optimal estimation problems for MRF models roughly corresponds to solving elliptic partial differential equations, which generally have complexity that grows with the size of the domain being considered. Moreover, when such models are used, one essentially never computes error covariances, since the complexity of that calculation dwarfs that of the calculation of the estimates. With our models, we not only get the estimates and the error covariances with constant computational complexity per image point, but we also get both of these at multiple scales of resolution and, as shown in [54] we actually get a complete multiscale model for the estimation errors so that we in essence have a complete description of the correlation between estimation errors at any locations and scales. These features are of profound importance for many of the applications of practical

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<sup>1</sup>This follows since (a) each node in the tree is visited twice--once on the fine-to-coarse sweep and once on the coarse-to-fine sweep--and the total number of nodes on the tree is proportional to the number of nodes at the finest level (roughly twice as many for the dyadic tree and 4/3 as many for the quad-tree).

interest in which either real-time image processing is required or the extent of the spatial region of interest is enormous.

3. We have successfully applied this estimation methodology to the problem of optical flow estimation in image sequences, as described in [25]. In particular, in the field of computer vision a "smoothness penalty" is generally used to regularize a variety of problems in image analysis including the problem of estimating the motion from one frame in an image sequence to the next. By interpreting this penalty as a "fractal prior" we were able to replace it with an analogous multiscale fractal model, allowing us to use our extremely fast optimal estimation algorithms not only to produce the optimal estimates but also to produce these at multiple resolutions and to produce the corresponding multiresolution error covariances with an algorithm that was shown to be anywhere from 1 to 2 orders of magnitude faster than previously known algorithms for  $512 \times 512$  images. In addition, as shown in [25], these multiscale error statistics can be used to identify the optimal scale of resolution for the optimal estimates at each point in the image.

4. As described in [44], we have also shown that the model (1), (2) admits an extremely efficient whitening algorithm which can be used for the efficient calculation of likelihood functions for these models. What is interesting mathematically about this algorithm is the novel twist that is required as compared to the whitening approach for standard temporal state space models. In particular, for temporal models, the Kalman filter directly produces a whitened version of the measurements in the form of the innovations process, the fine-to-coarse scale-recursive Kalman filter only whitens the measurement at any particular node with respect to the data in the subtree below that node. Consequently, there is still the task of whitening each measurement with respect to the data outside its corresponding descendent subtree. As shown in [44], it is possible to do this in a conceptually pleasing and algorithmically effective manner, with calculations that can also be organized as a coarse-to-fine step (although different from the one used in computing the smoothed estimates). The result is again an algorithm with constant computational complexity per node, for any set of measurement data--i.e., this algorithm performs whitening and likelihood calculation for problems in which we have irregularly sampled measurements at one or more resolutions. This is again in stark contrast to other models for random fields, such as MRF's for which the calculation of likelihoods (including the so-called partition function) is extremely complex computationally, especially if the model or data have spatially-varying characteristics (e.g., if the data are irregularly sampled, have gaps, or if the parameters of the model are spatially varying, as they are in most image processing applications such as the optical flow problem considered in [25]).

5. We have demonstrated the potential of our multiresolution likelihood function calculation algorithm by using it, together with our approximate models of MRF's, for texture discrimination. In particular, using our computationally efficient approach we achieve nearly the same performance for distinguishing MRF textures, as the prohibitively complex optimal algorithm (using the exact likelihood calculation for the MRF model) and substantially outperform well-known suboptimal methods that have been developed in order to overcome this computational obstacle. Since, as we have stated, MRF's are an idealization to begin with, these results argue once again for the attractiveness of our modeling framework, as it not only is very rich in terms of the random fields that it can faithfully represent but also leads to significantly superior algorithms.

6. In addition, as mentioned in the previous section, we have recently demonstrated the potential of our multiresolution modeling and discrimination techniques

in the context of Automatic Target Recognition. In particular, in this work we formed multiresolution SAR images, used our methodology to construct and validate models for clutter and man-made objects, and then used these models as the basis for likelihood calculations in order to distinguish natural clutter from objects of interest. The results by themselves were extremely encouraging, and, when combined with the feature-based ATR system developed by Lincoln, our method enhanced overall performance (e.g. as measured by false alarm rate) considerably.

### **Analysis and estimation of singular systems with applications in efficient processing of multidimensional data**

Our work in this area, which is described in [10-16, 35-36, 39-41, 49] has grown out of our study of singular or descriptor systems and, in particular the rich class of noncausal models that can be represented by so-called boundary-value descriptor systems. Our early work in this area established a system theory for such systems, while our recent work has built on this in order to develop efficient and highly parallelizable processing algorithms for spatially-distributed phenomena (such as imagery). In particular we have developed a general theory for the estimation for singular systems-- which is described in its most complete and general form in the recently completed paper [12]--and have also developed a number of new parallel algorithms for the processing of one-dimensional (i.e., time series) data and for 2-D data sets [11,13-15, 49].

There are two particularly key ideas that we have identified and exploited in our work in this direction. The first is that in dealing with implicitly specified systems--e.g. partial difference equations with boundary conditions--the distinction between measurements and dynamic equations becomes indistinct. In particular, both of these can be viewed as particular types of *noisy constraints or relations* on the variables that we wish to estimate, and the key to efficient estimation algorithm development is the identification of the structure of this set of constraints (i.e., how they couple together in providing joint constraints on overlapping sets of variables) and exploiting this structure by incorporating constraints recursively in as computationally efficient a way as possible. In particular, in [12] we have shown that the relationships among noisy constraints and the sets of variables that they constrain can always be organized into a tree structure, which in turn directly leads to a recursive estimation structure as variables and constraints are merged as we progress through the tree. This concept turns out to unify many apparently disparate approaches to solving estimation and least squares problems, ranging from standard Kalman filtering to so-called nested dissection methods to the radial/angular recursions that we have developed in our work in [15, 49]

Specifically, the second idea we have exploited in [15, 49] is that one of the problems in multidimensional estimation and processing is that instead of dealing with initial conditions, as in recursive filtering for time series, we must deal with boundary conditions in higher dimensions. An implication of this observation is that algorithm complexity in higher dimensions depends not only on model or filter order but also on the dimensions of the data field to be processed. This has led us to the idea of partitioning data into manageable-sized subregions, having one processor assigned to the processing of data within each subregion, and then combining the results via interprocessor communication. In addition, we have developed a novel notion of recursion for the processing within each subregion, namely that of *radial* recursion. Furthermore, by exploiting the estimation-theoretic interpretations of the sequential steps in this recursion we have been able to identify methods for approximating the exact radial recursions that yield near-optimal results for many estimation problems with dramatically reduced computational loads. In particular, each step in the radial recursion can be viewed as an optimal estimation problem for a 1-D process consisting of the values of the 2-D process along the curve of points corresponding to the current radius in the radial recursion. Thus by modeling this as a 1-D estimation problem in *angle* around this radial shell, we can use 1-D recursive estimation methods to develop efficient and near-optimal angle-recursive implementations for each step in our radial recursion. In addition to yielding excellent performance for many 2-D estimation problems, we also believe that these ideas will be of value in constructing efficient preconditioners for many other problems such as for the solution of certain partial differential equations.

### **Large-scale estimation and computation in remote sensing and space-time data assimilation**

One of the important characteristics of the multiresolution methods that we have developed is that they lead to algorithms offering considerable computational advantages as compared to other estimation formalisms. This suggests, of course, that it should be possible to adapt and apply these methods to extremely large-scale estimation problems. Indeed the work we have performed to date demonstrates this potential and also points to several other important problems in large-scale estimation and computation that are worthy of research. In particular, the work that we have performed that is relevant to this portion of our research is described in [11-16, 35-36, 39-41, 53, 58, 64]. The results of note are the following:

1. In [53, 64] we present the preliminary results of our first effort in applying our multiscale estimation results to remote sensing problems of considerable size. In particular, in this work we describe the processing of TOPEX/Poseidon satellite altimetry data over the North Pacific, a particularly large-scale estimation problem for which we were able to get a substantial amount of real data. This example is indicative of the capabilities of our methodology, as we have now demonstrated that we can produce estimates and associated error covariance and error correlation information for extremely large remote sensing problems (in this case we in essence solve 1 million dimensional least squares estimation problems *and* characterize the error covariance for the estimated field in 5 seconds on a Sparc 10). In addition, we have recently begun collaboration with researchers involved in atmospheric sensing problems in which we believe that precisely the same types of models and algorithms will be applicable with similar results.

2. We have also performed some research on the problem of space-time estimation of random fields that evolve in time. In particular, in [14, 35-36, 39-40] we consider dynamic problems in computer vision, including the problems of space-time surface reconstruction and the tracking optical flow over time, using a prior model that corresponds to smoothness in both space and in time of the optical flow field. Roughly speaking, if we view an entire optical flow field at any point in time as the state of a dynamical system, then for a 512 x 512 image what we have is a Kalman filtering problem of dimension somewhat larger than 500,000!! The real problem here, of course, is in propagating the error covariance information, since this is both needed in order to fuse data effectively over time and impossible to compute or store given its enormous size. The key observation used in our work is that the update step of the Kalman filter can be thought of as using the innovations at that time--i.e., the difference between the measurements and our best prediction of them based on past data--to estimate the error in the prediction of the optical flow field based on previous data. This is nothing more than a static random field estimation problem, where the random field to be estimated consists of the one-step prediction errors in the optical flow.

Solving this static problem, of course, requires a specification of the statistical structure of the error field and it is precisely this statistical structure that is captured explicitly in the error covariance matrix. What is done in [14, 35-36, 39-40] is to propose the idea of using an implicit and approximate specification for these statistics in terms of an MRF model for the error field. With such a model, then each Kalman filter update corresponds to the solution of an MRF estimation problem, which, as we have said, is equivalent to solving an elliptic partial differential equation in space. In addition, however, we need a method for propagating the error statistics over time--i.e., instead of using the Riccati equation to propagate the error covariance, we need an alternative procedure that propagates the parameters of the MRF model for the error field, accounting for the effects of temporal dynamics and the assimilation of new data at each point in time. The success in our work is directly attributable to the fact that we were able to derive a procedure for propagating such MRF models that accurately captures the statistics of the error field.

3. As we have pointed out on several occasions, the solution of MRF estimation problems correspond to the solution of elliptic partial differential equations. In addition, we have also shown that MRF models can be well-approximated by multiresolution models, so that estimation algorithms that are far superior computationally can be used. Putting these two facts together, we are led to the idea of using our multiresolution methods for the approximate solution of elliptic partial differential equations or, in signal processing contexts, to the approximation of 2-D filters described by 2-D difference equations. This idea is the subject of several parts of our work. In particular, in [11, 15, 49, 58] we have shown that there is a close connection between our approach to

multiresolution modeling and direct approaches to solving partial differential equations such as nested dissection, the critical difference being that we use approximate models at each stage of such procedures. For example, as we have seen, rather than keeping all of the values of an MRF along a separating boundary in order to exactly decorrelate the values in two subregions, we keep a coarse resolution approximation to the boundary values and neglect the residual correlation. That is, we develop an approximate and comparatively low-dimensional description of the linear constraints coupling these two regions through the boundary. In most of our work we have demonstrated this approximation through estimation problems, but in [58] we have shown that accurate approximations can be obtained for simple partial differential equations not explicitly tied to estimation problems. In addition, in [25] in which we consider replacing the smoothness constraint in optical flow estimation by a multiresolution prior, we also consider the use of the resulting output of the multiresolution estimator as an initial guess for the iterative solution of the vector Poisson differential equation that must be solved if we wish to solve the estimation problem using the MRF smoothness-based prior.

### **Multiresolution and wavelet-based methods for the detection of abrupt changes in signals**

In this section we turn our attention to the use of wavelets and multiresolution methods for signals and systems that evolve in time rather than in space. Once again the work that we propose builds directly on our recent research results, which are reported in [8,9,38,48,55, 57, 65, 66]. The overall emphasis of the work in these papers and reports is on the development of a statistically sound basis for the use of wavelets in several different estimation and detection contexts. Specifically, our work to date has yielded the following:

1. We have developed, implemented and successfully tested a technique for the design of algorithms for the classification of transient waveforms based on the use of wavelet packet transforms [8,9,38]. In particular, the idea behind this approach was to take advantage of the fact that wavelet packets provide a very flexible framework in which to identify the underlying structure of a signal in terms of the way in which it is best focused using wavelet packet bases. The wavelet packet transform provides a hierarchy of waveform bases, where at the top of the hierarchy the original signal appears, thus providing maximal resolution in time and no resolution of the frequency content of the signal. As we move down the hierarchy we obtain increasingly resolution in frequency and a commensurate decrease in resolution in time, culminating at the bottom level with what is in effect the windowed Fourier transform of the signal, providing maximal frequency resolution and no time resolution (over the considered window of data). The concept behind our work was to perform what we think of as partially coherent, adaptive detection, where a completely coherent approach would correspond to a bank of matched filters, computing correlations with a fixed set of signals (one per class to be identified) followed by a comparison to choose the largest value. Such a coherent approach provides maximum focusing of information, but, of course, requires a reasonably accurate model for each signal class. Indeed if such a model is not available or if the variability of the signals within each class are such that no such simple focusing is possible, the use of a completely coherent approach will in general fail. Our objective was to develop a robust method for signal classification in just such cases.

The approach that we took was to use a training set of data to identify the level in the rich wavelet packet library at which we could effectively focus information--i.e., to identify those parts of the wavelet packet tableau in which signal energy focused. Since calculating each wavelet packet coefficient corresponds exactly to a correlation or matched filtering operation, if we were fortunate enough to find that the energy always focused in exactly the same small sets of coefficients, then we could indeed use a completely coherent matched filtering approach. If this were not the case, then we would need to follow the calculation of the wavelet coefficient tableau by the incoherent summing of energies over sets of coefficients. In the specific approach that we adopted, we summed energies over entire sets of coefficients: at the top of the tableau this corresponds simply to the calculation of the overall energy in the signal, at the next level it corresponds to calculation of energies over two orthogonal signals corresponding to (crude) low-pass and high-pass versions of the signal, etc., where at the bottom we are simply calculating the squared-magnitude of the Fourier transform at each individual frequency.

Using training sets of data for each signal class, we then (a) used SVD analysis to identify the regions in this energy tableau in which energy focused for each signal class; and (b) used the dominant singular vectors of energy distribution in each class to determine the critical differences in these energy distributions from one class to the next, thereby providing the basis for robust signal classification. This approach was tested on real sonar data for the detection and classification of marine biological sounds, and decision rules using a remarkably small number of wavelet packet energy statistics resulted in excellent performance when tested on other data sets.

2. We have also recently completed a project [55, 65] on the use of the continuous wavelet transform (CWT--i.e., the set of multiscale signals obtained by convolving the original signal with scaled versions of the wavelet but without the subsampling that is done to form the orthonormal wavelet transform) for the detection of abrupt changes in signals, in this case based on the use of the CWT extrema processing methods proposed by Mallat. The motivation for Mallat's methodology stems from two facts: (a) although not true in certain pathological cases, a signal can usually be completely characterized and reconstructed from the knowledge of the locations and values of the extrema of its CWT at all dyadic scales; and (b) the set of extrema across scale that correspond to a particular point in the original signal characterize the nature of the singularity of the signal at that point. In particular, the magnitudes of these extrema vary geometrically across scale, with an exponent determined by the Lipschitz exponent of the original signal at that point. For example, discontinuities have Lipschitz exponent of 0 (so that the extrema should in principle have constant value across scale), and "noise-like" or impulsive behavior will have Lipschitz exponent of -1, corresponding to decaying extrema magnitudes as we move to coarser scales (at which such high frequency effects are blurred out). Thus, as Mallat argued, if one can "chain" the extrema across scale--i.e. identify which extremum at one scale goes with which at the next, and so on--then we can estimate Lipschitz exponents, discard those with negative values as being noise, and reconstruct a "denoised" version of the signal.

In our work we have taken a rather different look at the use of these extrema. One of our objectives has been to begin to provide a statistical justification for this type of method. Our conjecture has been that these methods, while certainly not optimal for any obvious specific choice of models for signal and noise (e.g., such as deterministic signals in Gaussian white noise), may very well be extremely robust to the detailed behavior of the statistics of the signal and noise. The context in which we have chosen to investigate this is the detection of abrupt changes in signals, in particular, the robust identification of knots in spline functions observed in noise. This is a prototypical abrupt change detection problem of interest in a variety of applications including failure detection. If we know that the measurement noise is Gaussian, then the generalized

likelihood ratio method provides the optimal algorithm for detecting and locating the spline knots. However, it is well known that such a method can degrade considerably if the real noise has "heavy tails". The approach that we have taken, then, is to look at CWT extrema as the basis for detecting and locating these knots.

Moreover, we have now shown that there is a very novel way in which the chaining of extrema across scale and the location of the knots can be folded into an optimal estimation framework. In particular, depending upon the specific wavelet chosen and the order of the spline (i.e., the order of the derivative that is discontinuous at the knot location), the noise-free locations of the knots across scale is a completely deterministic trajectory. This, together with the fact that the same is true for the magnitudes of these extrema suggest *the adaptation of techniques for multitarget tracking and data association to the chaining of extrema across scale and the detection and localization of knots across scale*. For example, if we focus on linear splines (with discontinuities in first derivatives) and use a symmetric wavelet (e.g., the second derivative of a Gaussian), then the extrema corresponding to a knot should occur at exactly the same location at each scale (i.e., a straight line trajectory across scale) and should double in size at each successively coarser scale. Thus for each hypothesized knot we have a 2-D state model across scale and then can use optimal multiple hypothesis testing methods in order to chain the extrema, discard extraneous ones or ones corresponding to non-spline-like behavior, and identify the knot locations and magnitudes of derivative discontinuities. Our experimental results show that this technique is exceedingly robust, both in the presence of Gaussian noise and in the presence of heavy-tailed noises.

## Data fusion and inversion

We next describe research directions that involve the processing of measurement data that is characterized by the fact that (a) they come from several different measurement sources with different resolutions; and/or (b) the measurement mechanisms used involve "probing" of the phenomenon to be imaged--i.e., they provide non-local measurements of the medium to be imaged. Problems of this type are the rule rather than the exception in many signal processing, estimation, and detection applications including nondestructive evaluation and radar imaging (such as synthetic aperture radar processing) and other remote sensing problems (in which the probing energy is applied at a distance and therefore yields nonlocal data).

Our work in this area is described in a number of papers and reports [33, 37, 45-48, 50-52, 59-61, 63]. The basic idea behind this work, as it was for the work described in the preceding section, is to develop multiscale methods for data fusion and inversion. In this case, however, our work has focused directly on the use of wavelet transform ideas rather than on the use of pyramidal scale-recursive models defined on trees.

Inverse and multiresolution fusion problems present a number of important challenges, and our work in this area has been to investigate the use of multiresolution methods to provide a framework to deal with these. In the first place, by their very nature, inverse and multiresolution fusion problems are confronted with a mismatch

between the domain of the data--which is nonlocal or multiresolution and the desired domain of the reconstruction. This mismatch manifests itself in serious computational and conceptual problems if attacked by brute force methods. For example, if we try to regularize such problems by using a smoothness or related prior MRF model directly in the domain in which we wish to perform reconstruction, the resulting variational problem is extraordinarily complex--in general an elliptic partial differential-integral equation (where the differential part comes from the MRF model and the integral part from the nonlocal nature of the measurements). Not only is this complex computationally, but it also makes the computation of error statistics prohibitively complex and, further, obscures questions such as determining the relative value of data from different sources, identifying the best resolution for reconstruction (which may be space-varying if we have irregularly and differently sampled data sources), etc.

Our work has had issues such as these in mind as we have explored the use of wavelets for these problems. What wavelets in essence provide is a mechanism for translating both our data *and* in some cases our reconstruction domain as well to a common setting *in which descriptions of both measurements and our prior knowledge are local*. In this way we then can address many important questions such as those mentioned above with comparative ease and with tools that greatly facilitate our insight.

In particular, in our work to date we have accomplished the following:

1. In part of our work [33, 37, 46, 52, 60, 61] we have examined several specific multisensor inverse problems, namely 1-D deconvolution problems and 2-D linearized electromagnetic inverse scattering problems, described by linear integral equations. In this work we have developed wavelet-based data fusion and inversion algorithms that (i) are efficient in that they take advantage of the properties of wavelets to compress integral operators as in; (ii) they allow us to use fractal or other multiresolution prior models described directly in the wavelet coefficient domain (as described, for example, in [17, 22, 28]); to regularize the inversion and fusion operations without increasing computational complexity; (iii) they allow us to calculate error statistics and thus to identify, at every point in space, the relative importance of each measurement source (a useful tool for measurement placement and design) and the optimal scale for reconstruction. In particular by applying what may be different wavelet transforms to each data source and to the reconstruction domain itself, we identify a framework in which the prior statistics and measurements are both local. Of course for deconvolution problems with complete data--i.e., with densely sampled and complete-coverage measurements--we could simply apply FFT's to accomplish the same thing as long as the prior model that is used is that of a stationary random field. What is particularly interesting about the approach we have developed is that (a) it allows nonstationarity of the prior model (of critical importance in many nondestructive evaluation problems in which we expect differing levels of fine scale behavior in different regions); (b) it allows incomplete and/or sparse data which may differ from sensor to sensor<sup>2</sup>; (c) it deals with

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<sup>2</sup>This is another innovation of our approach as compared to other recent attempts to use wavelets for inverse problems (e.g., [W])--i.e., what is particular to our approach is not only the use of the methods of optimal estimation to provide error statistics and detailed analyses based on these (as in Fig. 5) but also the

nonconvolutional kernels as well; and (d) it produces the detailed multiresolution statistical information mentioned previously.

2. In the other portion of our work [45-48, 51, 59, 63] we have focused on problems of tomographic reconstruction, i.e., on problems in which the measurement data are line integrals through the object to be imaged. These measurements are usually organized as "projections", i.e., as sets of parallel line integrals, which can then be viewed as samples of the 1-D "projection" of the function onto an axis perpendicular to the lines of integration. Regularization of tomographic reconstruction problems, either when the measurement data are noisy or sparse is a notoriously difficult problem, due to the severe nonlocality of the measurements and thus to the mismatch between measurement and object domain. What we have done in our work is to overcome this problem by transforming both the data and the object to a common intermediate domain in which we can then develop wavelet based methods. In particular, as in, we note that the exact inversion from complete tomographic measurement data to object domain, that is the inverse Radon transform, can be viewed as the composition of a 1-D filtering operation, namely the so-called "ramp" filter, on each projection, followed by a so-called back-projection, which is nothing more than the representation of the object in a nonstandard basis, termed the "natural pixel" basis in. What we have done in essence is to use this basis as a common starting point and then have further applied 1-D wavelet transforms to both the projection data and to the representation of the object in the natural pixel basis. What this does is (a) provide a model for our object in what we have termed the "natural wavelet basis"; and (b) essentially diagonalizes the ramp filtering operation--i.e., the map from wavelet-transformed data to natural wavelet coefficients is essentially diagonal. An important point here is that this approach is quite different from other approaches to the use of wavelets in tomography, since our approach uses 1-D wavelet transforms for both the measurement data and for the object representation (thanks to our use of natural pixels). This has profound consequences both in terms of the simplification it brings to the tomographic inversion problem (which again greatly facilitates estimation, error analysis, identifying scales for reconstruction, etc.) and in terms of our ability to deal with situations in which projections are available over a limited range of angles or are sparsely or irregularly sampled in angle).

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fact that we have been able to accommodate sparse and incompletely sampled data by tailoring possibly different wavelet transforms to each data source.

## Personnel

The following is a list of individuals who have worked during the past year on research projects supported in whole or in part by the Air Force Office of Scientific Research under Grant AFOSR-92-J-0002:

Prof. Alan S. Willsky, professor of electrical engineering, MIT

Dr. William C. Karl, research scientist, MIT Lab. for Information and Decision Systems

Dr. Mark Luetngen, graduate student (finished Ph.D. this year)

Mr. Michael Daniel, graduate student

Mr. William Irving, graduate student

Mr. Mickey Bhatia, graduate student

Ms. Rachel Learned, graduate student

Mr. Eric Miller, graduate student

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## Publications

The publications listed below represent papers, reports, and theses supported in whole or in part by the Air Force Office of Scientific Research under Grant AFOSR-92-J-0002:

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