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David L. Colton & Peter B. Monk

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13. ABSTRACT (Maximum 200 words)

This research project was concerned with the inverse scattering problem for time harmonic electromagnetic waves. The main accomplishments were 1) the introduction of modified far field operators to eliminate unwanted eigenvalues in the dual space method for solving the inverse problem, 2) the numerical analysis of a combined finite element-boundary element method for solving the direct scattering problem, 3) an analysis of the spectral properties of the far field operator and the physical meaning of the distribution of eigenvalues, and 4) the adaptation of the dual space method to the problem of the detection and monitoring of leukemia.

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Final Report
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November 1, 1991 – October 31, 1994

Summary: This research project was concerned with the inverse scattering problem for time harmonic electromagnetic waves. The main accomplishments were 1) The introduction of modified far field operators to eliminate unwanted eigenvalues in the dual space method for solving the inverse problem, 2) The numerical analysis of a combined finite element-boundary element method for solving the direct scattering problem, 3) An analysis of the spectral properties of the far field operator and the physical meaning of the distribution of eigenvalues and 4) the adaptation of the dual space method to the problem of the detection and monitoring of leukemia.

Professional Personnel

Dr. David Colton Professor of Mathematics

University of Delaware

Co-Principal Investigator

Dr. Peter Monk Professor of Mathematics

University of Delaware

Co-Principal Investigator

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Report on Research

The research of AFOSR Grant F49620-92-J-0048 was concerned with the development of new methods to solve the inverse scattering problem for time harmonic electromagnetic waves. The main problem under consideration was the determination of the permittivity and conductivity of an isotropic inhomogeneous medium from a knowledge of the electric far field data of the scattered wave. Our approach to this problem has been through the use of Herglotz wave functions and finite element methods which yield what we call the *dual space method* to solve the inverse problem. The main advantage of this method is that by means of an averaging procedure the number of unknowns in the nonlinear optimization scheme for determining the index of refraction is reduced by roughly 75%.

Prior to this research project, a major problem with the numerical implementation of the dual space method was that it failed at certain values of the wave number called transmission eigenvalues. Initial attempts to resolve this problem did so by introducing a new (but more manageable) eigenvalue problem A complete solution to this problem was provided in [4], [12] and [14] where the far field operator was modified in such a way as to avoid any problems with undesirable eigenvalues. This approach was extended to near field data and an absorbing host medium in [15]. With these modifications, the dual space method is now applicable for any positive value of the wave number for both absorbing and non-absorbing host media.

A second problem with the numerical implementation of the dual space method was that it was originally formulated in terms of a volume integral equation over an inhomogeneous region (subject to a constraint arising from the averaging procedure applied to the far field data). A closely related volume integral equation is also often used in generating synthetic far field data to test the efficiency of various approaches for solving the inverse problem. In our opinion, a more efficient method for implementing the dual space method as well as for solving scattering problems in an inhomogeneous medium is through the use of coupled finite element-boundary element methods or coupled finite-element-spectral methods. These approaches were analyzed in [17] and [18] and incorporated into the numerical algorithm which implements the dual space method.

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The canonical model for inverse scattering theory uses the far field pattern as data. Hence, for both mathematical and practical reasons, it is of interest to determine what information about the scattering object can be obtained from specialized knowledge of either the far field pattern or the far field operator (the integral operator defined over the unit sphere with far field pattern as kernel). In [16] it was shown that the far field pattern uniquely determines the scattering object and in [8] and [9] the spectral theory of the far field operator for a conducting medium was investigated. In these papers it was shown that the eigenvalues of the far field operator exist and lie inside a disk in the complex plane. It was further shown that the radius of this disk gives information on the magnitude of the permittivity and conductivity of the scattering object.

In the past year, particular concern has been given to the adaptation of the dual space method to the problem of the detection or monitoring of leukemia. As suggested to us by Dr. Richard Albanese of the Mathematical Products Division of the USAF Armstrong Laboratory, increases in the cellular population of bone marrow are reflected as increases or decreases in the permittivity and conductivity respectively. Hence, this change can potentially be detected by electromagnetic imaging. Using a two dimensional model (e.g. applicable to the monitoring of bone marrow in the upper leg) and synthetic data, we have shown that this approach is potentially viable ([13], [15]). Research in this direction is continuing.

A survey of our work on inverse scattering theory up to October 1992 under Air Force support has appeared as Volume 93 of the Springer-Verlag Series on Applied Mathematics ([7]).

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