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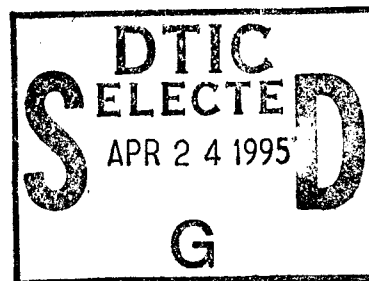
**DIELECTRIC PROPERTIES OF MAGNETOPLASMAS**

**Francis S. Johnson**

University of Texas/Dallas  
Box 830688  
Richardson, TX 75083-0688

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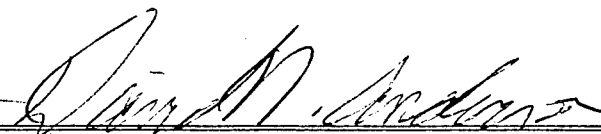
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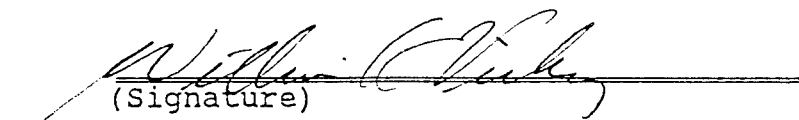
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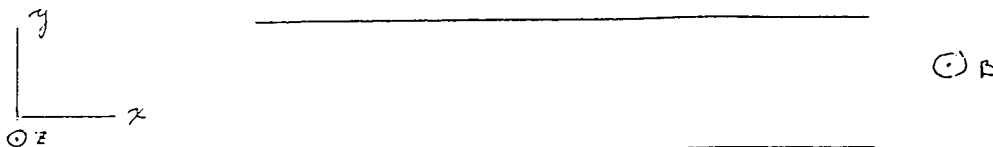
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DIELECTRIC PROPERTIES OF MAGNETOPLASMAS  
(dielectric, 11/20/89)

Consider a slab of plasma bounded by capacitor plates and permeated by a uniform, constant, magnetic field in the z direction. Apply a potential



difference across the plates so that E is in the +y direction. Then the plasma drift is in the +x direction and  $v_x = E/B$ . This agrees with the first term of Schmidt's equation (2-166),

$$v_p = \frac{E \times B}{B^2} + \frac{m \dot{E}_p}{q B^2}.$$

The second term is in the +y direction if E is increasing with time. Schmidt identifies this term as the polarization drift and notes that it is associated with the conversion of electric field energy to plasma kinetic energy; thus the polarization current is in the direction of E, and  $J_p \cdot E > 0$  for  $\dot{E} > 0$ .

If we consider the plasma to be initially at rest (and cold) and accelerate it by increasing E slowly, how much electrical energy must be supplied to the capacitor to do this? When the electric field reaches the value E, the energy of the system is

$$\frac{\epsilon_0 E^2}{2} + \frac{Nm v^2}{2}.$$

The y component of motion of the guiding center is given by the second term in Schmidt's equation, and the displacement of the guiding center is

$$\int \frac{m}{q B^2} \frac{dE}{dt} dt = \frac{mE}{q B^2}.$$

The energy extracted from the electric field is

$$\int NqE \frac{m}{q B^2} \frac{dE}{dt} dt = \frac{NmE^2}{2B^2} = \frac{Nm v^2}{2},$$

which is just the kinetic energy imparted to the plasma.

The polarization current is

$$Nq \frac{m}{q B^2} \frac{dE}{dt} = \frac{Nm}{B^2} \frac{dE}{dt}.$$

In dielectric terms, this contributes to the polarization P of the medium,

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where

$$P = \frac{NmE}{B^2} = \frac{Nm}{\epsilon_0 B^2} \epsilon_0 E.$$

Thus the electrical susceptibility is

$$\chi_e = \frac{Nm}{\epsilon_0 B^2} = \frac{Nm v^2}{\epsilon_0 E^2}$$

and the relative permittivity is

$$\epsilon_r = 1 + \frac{Nm v^2}{\epsilon_0 E^2}.$$

The electric field energy density, expressed in terms of the dielectric properties, is

$$\frac{\epsilon_0 \epsilon_r E^2}{2} = \frac{\epsilon_0 E^2}{2} + \frac{Nm v^2}{2},$$

just the total energy mentioned above. The kinetic energy in the medium is recovered as electrical energy when the capacitor is discharged.

It is useful to look at the magnitude of some parameters typical of space plasmas:

$$\begin{array}{ll} B = 10 \text{ nT} = 10^{-8} \text{ T} & N = 10^7 \text{ m}^{-3} \\ m = 1.67 \times 10^{-27} \text{ kg} & q = 1.6 \times 10^{-19} \text{ C} \\ \epsilon_0 = 8.85 \times 10^{-12} & v = 10^5 \text{ m/s} \\ E = Bv = 10^{-3} \text{ V/m} & \epsilon_r = 1.9 \times 10^7 \\ P = \chi_e \epsilon_0 E = 1.68 \times 10^{-7} = Nq\Delta y; \quad \Delta y = 1.05 \times 10^5 \text{ m} \\ \text{Proton gyro radius} = mv/qB = 1 \times 10^3 \text{ m (smaller if thermal velocity is used).} \end{array}$$

Thus the displacement due to polarization is of the order of 100 gyro radii or more. The charge density that must be applied to the capacitor plates to produce a velocity of  $10^5 \text{ m/s}$  is

$$\sigma_f = \epsilon_0 \epsilon_r E = 1.68 \times 10^{-7} \text{ C/m}^2;$$

this would produce a field of  $1.9 \times 10^4 \text{ V/m}$  except for the polarization of the plasma. This also shows that  $\epsilon_0 E^2/2$  is generally negligible by comparison with  $Nmv^2/2$ , the importance of the electric field in the problem notwithstanding.

### Remove the Capacitor Plates

If a slab of plasma is moving through a uniform B field with velocity v, there must be surface charges sufficient to produce an electric field within the plasma that corresponds to its drift velocity. These surface charges are usually attributed to  $\mathbf{v} \times \mathbf{B}$  forces. It seems natural enough to call this polarization of the medium, but this inevitably leads to confusion with the polarization current discussed above, which is in the opposite direction. To avoid this confusion, we will refer to the surface charge density as the induced

surface charge density  $\sigma_i$ , whose magnitude is the same as the free charge density that had to be placed on the capacitor plates in the previous discussion. Note that  $\sigma_i$  is positive on the lower surface and negative on the upper, whereas the surface charge density  $\sigma_p$  associated with the polarization is negative on the lower surface and positive on the upper, so the two tend to cancel.

$$\begin{aligned}\sigma_i &= \epsilon_0 \epsilon_r E = (1 + \chi_e) \epsilon_0 E \\ \sigma_p &= \chi_e \epsilon_0 E\end{aligned}$$

Thus, for space plasmas, the two cancel to within about one part in  $10^7$ , but it is just this small difference between them that is responsible for the electric field within the plasma.

### Now Add a Gravity Field

(1) Start with stationary plasma between capacitor plates, with the plates connected together so that there is no electric field. To this add a gravity field



in the  $+x$  direction; this will produce a current in the  $-y$  direction, with

$$\mathbf{J} \times \mathbf{B} = -Nmg, \text{ or } \mathbf{J} = -Nmg/B.$$

The current in the plasma produces a force on the plasma that cancels the force of gravity. The current is continuous but dissipationless, as the potential difference between the plates is zero.

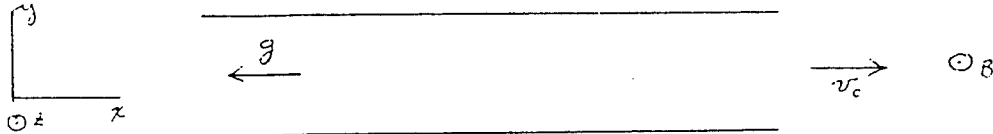
(2) Now, remove the connection between the capacitor plates. Positive charge will accumulate on the lower plate, leading to the development of an  $E$  field in the  $+y$  direction and consequent plasma drift in the  $+x$  direction. There will also be a polarization current in the  $+y$  direction, and it will almost cancel the current flow caused by the presence of gravity. We can consider the current flow due to gravity as all accumulating on the capacitor plates, with the force of gravity being totally cancelled by the current. But then we must recognize that there is a polarization current in the plasma that accelerates the plasma in the  $+g$  direction, and that this acceleration is smaller than  $g$  by only about 1 part in  $10^7$ .

What must actually happen is that the two currents largely cancel one another, and the plasma is nearly in free fall. A very small part of the work done by the gravity field produces electric field energy  $\epsilon_0 E^2/2$ , and the predominant remainder goes into the production of kinetic energy.

(3) It is not necessary to imagine the presence of the capacitor plates at all. A slab of plasma in a uniform field  $B$  will accelerate at almost the rate  $g$ , while accumulating net charge density at its surface  $\sigma = \sigma_i - \sigma_p$ , electric field in the  $+y$  direction  $E = \sigma/\epsilon_0$ , and electric field energy density  $\epsilon_0 E^2/2$  in addition to its kinetic energy density.

For typical space plasmas, we noted that charges were displaced something like 100 km due to polarization. In the case of a plasma slab accelerated by gravity to 100 km/s, the net surface charge density is approximately  $\sigma_p/\chi_e$ , and the charge displacements required to produce the electric field is only about 1 cm.

(4) Now let  $g$  act to slow down a slab of plasma that is initially moving with velocity  $v_0$  in the  $+x$  direction. From the preceding, the net surface



charge initially is  $\sigma = \epsilon_0 E = \epsilon_0 B v_0$ . This can be thought of in terms of  $\sigma = \sigma_i - \sigma_p$ , or  $\sigma_i = \sigma + \sigma_p$ , where  $\sigma_p = P = \chi_e \epsilon_0 E$ ; i.e.,  $\sigma_i = \epsilon_0 E + \chi_e \epsilon_0 E = \epsilon_0 \epsilon_r E$ . (This is just the charge that could be withdrawn from the capacitor plates in an earlier discussion.)

Now the  $g$  field causes a current to flow in the  $+y$  direction, and the  $J \times B$  force is in the  $+x$  direction, again opposing gravity. Suppose for the sake of argument that it exactly cancels gravity. The current flow now acts to decrease the surface charge, and would reduce it to zero in a time

$$\sigma_i/J = \frac{\epsilon_r \epsilon_0 E}{Nmg/B}$$

(at which time the  $E$  field would be reduced to zero and the plasma would be stationary, but we must still show this).

The tendency of the current  $Nmg/B$  to decrease the surface charge is largely counteracted by the decreasing polarization of the medium. As the charge  $\epsilon_0 \epsilon_r E$  is delivered to the upper boundary by the current that cancels the force of gravity on the plasma, the polarization charge on the upper surface is reduced from  $\chi_e \epsilon_0 E$  to zero, and the depolarization current associated with this accelerates the plasma in the  $-x$  direction, producing an acceleration almost equal to  $g$ . It is  $\frac{\chi_e}{1+\chi_e} g$ , where  $\chi_e \approx 10^7$ . The fact that the plasma does not slow down quite as fast as it would under action of gravity alone reflects the fact that electric field energy  $\epsilon_0 E^2/2$  is eliminated in the process, contributing to the maintenance of velocity in the  $+x$  direction.

Under action of the acceleration produced by the  $J_p \times B$  force, the plasma stops in time  $\frac{\epsilon_r \epsilon_0 EB}{Nmg}$ , and then reverses its direction of flow and the direction of the electric field within it.

### Alternate Treatment Not Involving Dielectric Properties

It is probably preferable to analyze the response of a slab of plasma to a gravitational field without invoking, or developing, the dielectric properties of the magnetoplasma. As before, consider a slab of plasma in the xz plane,



with gravity acting in the +x direction. Consider a current  $J$  in the +y direction;  $J$  will turn out to be negative so as to produce a  $J \times B$  force on the plasma that opposes gravity. Later on, we will solve for  $J$ . Newton's Law gives for the acceleration of the plasma

$$Nma = Nmg + JB, \text{ or } a = g + JB/Nm.$$

The drift velocity must be consistent with the electric field that results from the surface charges produced by current  $J$ . The surface charge on the upper surface of the slab is

$$\sigma = \int J dt$$

and  $E = -\sigma/\epsilon_0$  ( $\epsilon_r = 1$ , as no dielectric properties are involved in this analysis). For consistency, it is necessary that  $v = E/B = \int a dt$ , or that

$$-J/\epsilon_0 B = g + JB/Nm.$$

Therefore

$$-J = g \left( \frac{1}{\epsilon_0 B} + \frac{B}{Nm} \right) = \frac{Nmg}{B} \frac{1}{Nm v^2 / \epsilon_0 E^2 + 1},$$

and

$$a = -\frac{J}{\epsilon_0 B} = (Nmg/\epsilon_0 B^2) \frac{1}{Nm v^2 / \epsilon_0 E^2 + 1}$$

$$= g Nm v^2 \frac{1}{Nm v^2 + \epsilon_0 E^2}$$

$$= \frac{g}{1 + \epsilon_0 E^2 / Nm v^2}.$$

This is smaller than  $g$  by about 1 part in  $10^7$ . It is the same expression as  $\frac{\chi_e}{1+\chi_e} g$  derived earlier making use of the dielectric properties of the plasma.