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DYNAMIC MAINTENANCE SCHEDULING FOR A STOCHASTIC TELECOMMUNICATION NETWORK: DETERMINATION OF PERFORMANCE FACTORS

THESIS

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THESIS

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Preface

This research effort developed analytical approaches to access maintenance facility location and communication network performance measurement subject to dynamic maintenance scheduling. This effort is the result of nearly eight months of research into the areas of network flow estimation, multiple-criteria decision making, and stochastic location modeling.

Since this thesis is the culmination of my master's program, there are many individuals who helped to make this successful and to whom I owe my thanks. First, my wife Hallie, her patience and devotion made the effort possible and rewarding. She has constantly helped me to realize what is best in life. Second, my thesis advisor, Dr. Chan, for his technical expertise and helpful guidance. Third, my thesis reader, LtCol Dennis Dietz, for keeping the edits to a minimum and struggling through my grammar. Finally, the thesis sponsor for proposing the research project and for taking the time to answer my questions.

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Abstract

This research proposes an analytical approach to access the relationship between maintenance facility location and communication network performance measurement using a selected dynamic maintenance scheduling protocol. There were three objectives established for this effort. The first objective was the determination of an upper-bound upon the level of performance for a telecommunication network using dynamically scheduled maintenance to evaluate maintenance depot location. This was achieved by using a two-stage algorithm, first locate a maintenance depot by using stochastic algorithms, and then to measure the resulting impact upon performance with a multi-commodity network flow model. The second objective was to develop the metrics by which network performance should be measured. This was accomplished by comparing multiple criteria using a constraint conversion technique. The third objective created the mathematical models necessary to evaluate network operations. The models were created within a least-cost multi-commodity network flow environment. The approaches proposed in this research are offered as initial investigations toward the long-term goal of automated maintenance scheduling for a stochastic communication network.

Dynamic Maintenance Scheduling for a Stochastic Telecommunication Network: Determination of Performance Factors

I. Introduction

The efficient and effective allocation of maintenance resources to support today's large-scale telecommunications networks is critical if the highest level of support to the users of these networks is to be maintained. This chapter first provides background on telecommunications networks and the need for maintenance scheduling. Second, this chapter presents the thesis research objectives with a description of the scope and specific research topics, followed by basic assumptions about the telecommunications network.

1.1 Background

Telecommunications networks have experienced tremendous growth in the last twenty years. This is particularly true for Department of Defense (DoD) agencies, where the need for fast reliable communications is critical to decision makers and operations personnel. This growth has made these communications networks extremely complicated and difficult to maintain. While there has been a tremendous amount of research into maximizing the flows across a communications network, (which increases network efficiency and throughput) the area of maintenance scheduling has not received this same level of attention.

The increases in network complexity dictate the reliance upon automated control structures for minute-to-minute operations and monitoring of a communications network. For example, the physical routing of a call and call loading [defined to be the number of calls upon any link] are handled via automated control systems. Maintenance scheduling is one area where automated controls have not been developed. In fact, network custodians currently have no ability to assess the effect of the maintenance schedule on network performance.

Maintenance schedules are created by the Telecommunications Maintenance Office (TMO), which schedules three types of maintenance operations: (1) Corrective Maintenance (CM), which includes repair of equipment after it has failed or is operating in a degraded state; (2) Preventative Maintenance (PM), which is routine maintenance that must be performed on the network to prevent failure and degradation from occurring; (3) Adaptive Maintenance (AM), which includes updates to network equipment or even whole new systems that must be installed. PM and AM are included in the maintenance schedule, while the first type, CM, must be reactionary and thus is one of the principal reasons a maintenance schedule must be changed. Logically, other events also create the need to change maintenance schedules. For example, unavailability of replacement parts and unavailability of maintenance personnel (due to sickness, improper training, etc.) are only two of many. The changes that occur in the maintenance schedule will then affect other maintenance tasks depending upon their priorities, thus creating a ripple effect through the rest of the schedule.

TMO personnel can change a maintenance schedule by hand. However, they are currently unable to assess whether they are creating the "best" maintenance schedule possible since the TMO currently does not have any measures of network performance or models upon which to evaluate performance measures if some were chosen. While the TMO has several Customer Level of Support Agreements (LOSAs) with some of its customers, the TMO managers are not certain that these LOSAs provide the best possible metric. Additionally, while supporting the customers is of primary importance, TMO managers must also be concerned with an efficient allocation of their maintenance equipment and maintenance personnel, since today's tumultuous budgets and limited resources could create a situation where not enough maintenance personnel and equipment are available to meet a contingency or even day-to-day operations.

The limits upon maintenance personnel and equipment also play a role in the ability of the TMO to meet its own maintenance schedules. Personnel can become sick or injured and unable to perform scheduled maintenance items or maintenance equipment itself can break with the same result.

All of these factors lead to the concept of "dynamic maintenance scheduling", which is defined to be the continuously changing process of scheduling maintenance functions for telecommunications operations. While on the surface the changes in the maintenance schedule appear to be driven by CM needs, the truth is that each type of maintenance (along with the limits upon maintenance personnel and equipment) affect this constantly changing process.

1.2 Research Objectives

1.2.1 Scope - This thesis is the first step in the development of an automated dynamic maintenance scheduling system for the Telecommunications Maintenance Office of a DoD agency. Specifically, this thesis:

- determines an upper-bound upon the level of performance for a telecommunications network using dynamically scheduled maintenance to evaluate maintenance depot location;
- develops the metrics by which network performance should be measured; and,
- creates the mathematical models necessary for network evaluation.

This thesis delivers to the sponsoring agency three things:

- a set of insights showing the effect of maintenance depot location upon network performance and maintenance scheduling;
- a model formulation for measuring network performance with any parametricly created network maintenance schedule; and,
- a case study of a telecommunication network examining various network maintenance schedules.

This thesis does not develop a completely automated maintenance scheduling software package. This is left for later extensions.

1.2.2 Topics - The scope of this research requires focusing upon several research topics: network reliability and availability theory; capacitated multi-commodity networks that assume stochastic link availability and demands; and automated network solving software.

First, network reliability and availability theory will be used to help determine the metrics for evaluation of a telecommunications network. Some of the variables that are possible candidates in the formulation and modeling process include system reliability, degradation, data loss rates, data delay times, call throughput rates, and availability of service.

Second, capacitated multi-commodity network theory will be used to develop the mathematical models of network operations and maintenance functions. For example, these models will optimize the location of maintenance depots and then show the impact of maintenance depot location upon network performance. Additionally, the model formulations will need to address equity issues among the different customers since each class of customer will need to receive the same level of service unless otherwise directed by agency managers.

Finally, the working computer simulations of the network will be implementable upon a standard network solving software, such as: MICROSOLVE, NETWORK-FLOW SOLVER, NETSIDE, or SAS/OR [6].

1.3 Assumptions

The basic assumptions that carry throughout this research include:

- The basic network is a multi-path network with multiple O-D pairs representing the users of the system;
- The network structure has a circuit switched design rather than packet-switching, meaning that once a route is established it will continue with allowance for rerouting as necessary;
- The flow across the network is controlled by a shortest path algorithm that serves as the message routing control structure and as a congestion control;
- Network components are either operational or failed (a component requiring corrective maintenance action can be considered to be failed);
- Maintenance operations are handled on a first-come-first-serve (FIFO) basis;
- Maintenance personnel and resources do the "best" job possible in performance of their tasks. This allows for direct comparison between maintenance operations at competing locations, and limits the research area to location and performance factors.

II. Literature Review

This chapter reviews topics related to the development of an automated dynamic maintenance scheduling system for a telecommunications network including: locating the maintenance facilities supporting a network operation, performance evaluation models of stochastic networks, and the principal tenets of multiple-criteria decision making (MCDM). First, I present the network notation and symbols to be used throughout this thesis. Next, I cover facility location literature and minimal-cost network flow models, to include definitions of performance metrics. Finally, I provide a review of MCDM topics and their possible application to facility location and network performance.

2.1 Network Model Notation

A network or directed graph G = (N,A) consists of the distinct node set, N = {1, 2, ... , n} and the set of m distinct arcs (links), A ={(i,j), (k,l), ..., (s,t)} represented by directed node pair combinations going from node *i* to node *j*. [2] The two sets, also called network components, have associated numerical values representing costs, capacities, supply, demand, etc. Specifically, c_{ij} represents the arc cost associated with flow from node *i* to node *j*, while arc capacities, u_{ij}, indicate the maximum allowable flow upon the arc. Additionally, an external flow value is indicated by b_i which has a positive, negative, or zero value if the node is a source (supply), sink (demand), or intermediate node [25]. Additionally, a network can have several types of external flow representing different commodities, where each commodity has a single source and supply node [16].

Obviously, each of these parameters have a physical interpretation in a telecommunications network. The nodes represent banks of multiplexors and switching centers, while the arcs are the communications links between these centers that carry simultaneous messages. For this specific problem, each node is a separate DoD facility within the communications network, and each arc is the communications link to another facility [18]. The associated arc costs can be an number of things, including: Euclidean distances (e.g. miles); units of time, (e.g. nano-seconds); or facility usage charges (e.g. the dollar cost to use a dedicated satellite-communication link). Arc capacities simply indicate how many messages, calls, data packets, etc. can flow simultaneously over a link. The external flow value indicates how many different message types each node either sends or receives from another node.

The representation of a minimum cost network flow problem as a mathematical program has an objective function designed to minimize the cost (time, delay, lost calls, distance, etc) and a set of constraints representing the feasible region [3]. The constraints enforce nodal conservation of flow and arc capacity limits. An example of a single commodity unconstrained minimum cost network flow model is Figure 2.1:

$$\text{Minimize } \sum_{i=1}^{m}\sum_{j=1}^{m} c_{ij}x_{ij}$$

$$\text{Subject to } \sum_{j=1}^{m} x_{ij} - \sum_{k=1}^{m} x_{ki} = b_i \quad i = 1,\dots,m$$

$$x_{ij} \geq 0 \quad i,j = 1,\dots,m$$

where x_{ij} = message flow on the arc between node i and j

Figure 2.1 Unconstrained Minimum Cost Flow Model

2.2 Facility Location Models

The basic goal of a facility location problem is to place *p* facilities and/or servers upon a network to provide service to meet a set of demands. The service in this case will be maintenance operations performed at each communication site (node) upon a link.

There has been a tremendous amount of research into the area of facility location by members of the public and private sectors [**5**]. Facility location models can generally be divided into two categories: deterministic and stochastic. The system to be modeled and its simplifying assumptions usually determine the modeling approach to use. While deterministic approaches have the advantage of simplicity and ease of calculation, stochastic models sometimes offer a better representation of reality. Both types will be reviewed below.

2.2.1 Deterministic Models. One of the most fundamental of deterministic models developed to date is the *p-median location model* developed by Hakimi [12] in 1964 . The p-median approach determines optimal facility location when a certain set of demands must be met by minimizing the total transportation cost, which can be measured as distance or time. The basic model developed by Hakimi is

$$\text{Minimize } \sum_{i \in I} \sum_{j \in I} f_i d_{ij} x_{ij}$$

$$\text{Subject To: } \sum_{j \in I} x_{ij} \geq 1 \quad \forall i \in I \quad \text{(Service Provided)}$$

$$\sum_{j \in I} x_{jj} = p \quad \text{(Number of Facilities)}$$

$$x_{jj} - x_{ij} \geq 0 \quad \forall i, j \in I, i \neq j \quad \text{(Facility Servicing)}$$

$$x_{ij} \geq 0 \quad \text{(Non - negativity)}$$

$$x_{jj} = \{1,0\} \quad \text{(Facility Location)}$$

where: f_i = frequency of maintenance operation (as rate);
d_{ij} = distance to node;
x_{jj} = node where facility is located;
p = number of facilities to be located;
I = set of nodes.

Figure 2.2 *P-Median* Location Model

This simple *p-median* model assumes that each service facility handles an exclusive set of demands with no overlap between service facilities. Chan [5] proposed a relaxation of this limit by allowing the different service regions to overlap. In his formulation, a node could belong to more than one service region, thus allowing service to be provided by any one of the supporting service centers depending upon the state of the network. An important assumption of both the simple *p-median* and Chan formulations is disallowance of fractional servicing by several sites (where several sites combine efforts to work on the affected node). This assumption is driven by the type of service to be performed, for instance, service calls may be handled by an individual person and would therefore be impossible to split out. Obviously, this assumption can be relaxed if necessary.

Love et al. [22] proposed another deterministic model that allows for interaction between service centers. This type of situation can occur when the service calls are interrelated. For example, traditional telecommunication maintenance operations are set up along functional lines that do not allow for overlap and each site will only have technicians capable of dealing with certain problems [4]. When a service call arrives it may require more than one type of technician.

The solution techniques for the deterministic models discussed vary. When demands are paired with the facilities, the simple p-median model can be represented as a bipartite graph as demonstrated by Chan [5]. The unique structure of bipartite graphs allows them to be solved by several different algorithms including: (1) bipartite preflow-push; (2) double scaling [2].

In solving the overlapping service region model, Chan [5] proposes using a solution dubbed the *network-with-gains* algorithm as explained in his book. Chan also experimented with simple Mixed Integer Programming (MIP) and the node-arc-incidence-matrix representation using side constraints. Chan concludes, "Of the three solution procedures, the *network-with-gains* algorithm appears to be the most promising." [5]

2.2.2 Stochastic Models. These types of location models remove some of the simplifying assumptions of the deterministic p-median models. As highlighted by Odoni [24] in 1987, arc travel times will vary and the number of service calls waiting to be serviced can build up depending upon the arrival rate, thus creating a queuing situation.

Obviously, it is easier to deal with the random arc travel times than it is with the second relaxed assumption. Since the network will operate over a long period and can thus be assumed to have reached steady-state, it is allowable to use the 'average' long-run travel times for each arc. This yields the same results as the p-median approach. The second assumption requires use of stochastic queuing models. Chan points out that these types of models become analytically complex when the number of facilities goes beyond one facility to locate[5].

Ahituv and Berman [1] proposed an approach for modeling this situation involving partitioning a network down into smaller subnetworks, each capable of independent operations. Once partitioned, a maintenance facility could be located within the new subnetwork using a stochastic facility location algorithm for a single facility.

2.2.2.1 Network Partitioning Algorithm. Garfinkel and Nemhauser [9] created an elaborate partitioning algorithm to handle political redistricting. This work served as a guide only until Ahituv and Berman [1] were able to adapt the model. Their adaptation considers three key ideas in network division:

(1) equity (demand for maintenance services should be equal across the larger network);

(2) contiguity (each node of the subnetwork can be reached without having to pass through a node assigned to another subnetwork); and,

(3) compactness (the distance between nodes of any subnetwork is constrained). Their adaptation of the model takes the form shown in Figure 2.3.

The purpose of constraint (1) is to ensure the required number of subnetworks. The second constraint (2) fulfills the collectively exhaustive and mutually exclusive requirements.

The real problem with this model lies in the fact that it is an exhaustive enumeration technique that requires a large number of constraints to implement. For a simple network of only five nodes, this formulation creates 120 possible network partitions to evaluate. Since most communications networks have several hundred to thousands of nodes this technique requires a large amount of computer time to operate.

$$\text{Min} \sum_{j=1,S} C_j \; X_j$$

$$\text{Subject To:} \quad \sum_{j=1,S} X_j = M \qquad (1)$$

$$\sum_{j=1,S} a_{i,j} \; X_j = 1 \quad i=1,\ldots,n \qquad (2)$$

where: $X_j = 1$ if subnetwork j is selected
$= 0$ otherwise;
$a_{i,j} = 1$ if node i is an element of subnetwork j
$= 0$ otherwise;
M = number of subnetworks;
n = number of nodes;

$$C_j = \frac{\left| \sum h_i - 1/M \right|}{\alpha/M} \qquad \sum \text{ is over } i \in j;\ 0<\alpha<1$$

h_i = node i fraction of demand.

Figure 2.3 Network Partitioning Model

Kumar and Babu [20] developed another partitioning technique for communications networks using a stochastic search method called evolutionary programming to search for a globally optimal partition based upon minimization of communications cost. Evolutionary programming techniques mimic the fundamental aspects of evolution and have been shown to converge asymptotically to the global optimum [20].

2.2.2.2 Stochastic Location Algorithm. Once a network is partitioned into p subnetworks, each subnetwork needs to have a maintenance facility placed within the confines of the subnetwork. The development of the stochastic location algorithm comes from [1, 5]:

$$\text{Min } TRj(X) \quad \forall \ j \in I$$

where the minimum TRj is computed by taking the first derivative for each possible location j within the set I. The expected response time (TR) is the sum of the mean-queuing-delay ($\overline{Q}$) and the mean travel time($\bar{t}$), highlighted as:

$$TR(X) = \overline{Q} + \bar{t}$$

$\overline{Q}$ is further defined as:

$$\overline{Q} = \frac{\lambda S^2(X)}{2(1 - \lambda S(X))}$$

X is the facility location, λ is the failure arrival rate, S(X) is the mean total service time (first moment of service time), and $S^2(X)$ is the second moment of the total service times.

This formulation is easy to solve provided the maintenance depot is placed at a node in the communications network. This assumption is a reasonable given the assumption maintenance operations occur at the nodes and additionally, in all likelihood the network managers would want the facilities to be co-located [18].

2.3 Minimum Cost Network Flow Models

2.3.1 Basic Formulation. A multicommodity minimal cost flow (MMCF) problem, as explained by [16] in 1978, attempts to determine the minimal cost flow of multiple commodities through a network subject to (1) supply and demand requirements, (2) arc capacity limits, and (3) flow conservation at transshipment nodes. The mathematical representation of a MMCF can have a node-arc formulation or an arc-path formulation. The node-arc formulation is:

$$\text{Min } Z = \sum_{p \in O} \sum_{q \in D} c_u{}^{pq} X_u^{pq}$$

Subject To:

$$\begin{aligned}
\text{Capacity:}\quad & \sum_{p \in O} \sum_{q \in D} X_u^{pq} \le u_u \quad \forall \ \mathrm{u} \in \mathrm{M} \\
\text{Flow Conservation:}\quad & \sum_{u \in \Gamma(i)} \mathrm{X}_{\mathrm{u}}^{\mathrm{pq}} - \sum_{u \in \Gamma-(i)} \mathrm{X}_{\mathrm{u}}^{\mathrm{pq}} = d^{pq}, \text{ if } \mathrm{i} = \mathrm{p} \\
& \sum_{u \in \Gamma(i)} \mathrm{X}_{\mathrm{u}}^{\mathrm{pq}} - \sum_{u \in \Gamma-(i)} \mathrm{X}_{\mathrm{u}}^{\mathrm{pq}} = -d^{pq}, \text{ if } \mathrm{i} = \mathrm{q} \\
& \sum_{u \in \Gamma(i)} \mathrm{X}_{\mathrm{u}}^{\mathrm{pq}} - \sum_{u \in \Gamma-(i)} \mathrm{X}_{\mathrm{u}}^{\mathrm{pq}} = 0 \text{ otherwise} \\
\text{Non Negativity:}\quad & \mathrm{X}_{\mathrm{u}}^{\mathrm{pq}} \ge 0 \ \ \forall \ \mathrm{u} \in \mathrm{M},\ \mathrm{p} \in \mathrm{O},\ \mathrm{q} \in \mathrm{D}
\end{aligned}$$

where	$\mathrm{X}_{\mathrm{u}}^{\mathrm{pq}}$	Traffic flow of O - D (p = origin & q = destination using arc u);
	u_{u}	Capacity of arc u;
	d^{pq}	Origin - destination offered load;
	$\Gamma(i)$	Set of arcs whose origin node is i;
	$\Gamma-(i)$	Set of arcs whose terminal node is i;
	O	Origin node set;
	D	Destination node set;
	N	Node set for network;
	M	Arc (link) set for network.

Figure 2.4 Node-Arc Formulation of Network Flow Model

The formulation of the arc-path model will not be presented since the node-arc model forms the basis of the performance evaluation model described in the next chapter.

This multicommodity minimum cost model considers only a single time period, or the system it represents can be considered to be at steady state. This poses a problem for network maintenance schedulers when attempting to use this type of a model to determine the impact upon network performance of a dynamic maintenance schedule. In a soon to be published paper, Haghani and Oh [11] have developed a dynamic network flow model representing multiple time periods as well. Their work focused on minimizing the vehicular, commodity, supply/demand, and transfer costs for a transportation network. Their work is useful in realizing the necessary structure for dynamic network model that considers many time periods. The obvious problem with such a model is the linear system of equations created would be tremendously large, and the advantage of special network structures would probably break down.

2.3.2 Network Routing. Routing protocols perform the job of determining the "best" possible path through a network for each O-D pair [15]. There are two primary routing algorithms in use today: shortest path and delay. Shortest path algorithms, also called *distance-vector algorithms*, can be based upon physical distances or the number of "hops" [15]. The goal of this protocol is to minimize the number of "hops" between source to sink, where a "hop" is defined as passage through an intermediate node.

The second protocol, delay, is simply the amount of time that it takes a message to reach its destination from its origin [15]. At first inspection, this could be considered as the shortest path, but this is not the case. The amount of time it takes a message to go from source to sink is also influenced by congestion effects upon each link [23:278]. Li and Silvester [21] present simple techniques for estimating the impact on network performance using Kleinrock's average network delay formula [17]:

$$Delay = \frac{1}{\gamma}\sum_{i,j}\frac{\lambda_{i,j}}{\mu C_{i,j} - \lambda_{i,j}}$$

where: γ is total network throughput;
$1/\mu$ is average message length;
$\lambda_{i,j}$ is flow upon link i, j

This delay function can then be used to minimize average network delay through each link and hence the "best" path is achieved for each message based upon network congestion.

2.3.3 Solution Algorithms. Multicommodity network flow problems are linear programs and could thus be solved using the simplex method, but large networks can lead to extremely large problems. Network solution algorithms are designed to take advantage of special structures [3:419, 16:219]. The choice of network solution algorithms for capacitated multicommodity network flow problems is affected by the special structure of the problem. Unlike a single commodity problem, the multicommodity problem has an additional set of constraints (side constraints) that make the solution algorithm more complicated.

Kennington states there are three basic approaches for solution: (1) price-directive decomposition; (2) resource-directive decomposition; and, (3) partitioning methods [16]. One solution algorithm that uses the first decomposition technique is the specialized Dantzig-Wolfe decomposition algorithm [3:320-349, 16:221]. Readers interested in more solution algorithms for multicommodity flow problems can turn to Kennington's authoritative review [16] presented in 1978.

2.4 Multiple Criteria Decision Making

In the last few decades there has been a realization within the operations research community that in most situations there are several criteria that usually need to be examined and compared in order to determine the "best" solution to a problem [30, 10, 19]. Quite often these criteria must be compared against each other in some fashion that usually involves a compromise between each criteria's "ideal" value in order to find a set of nondominated alternatives.

The mathematical expression of a p-dimensional multi-criteria problem has the form [10]:

$$\begin{aligned} &\text{Max } \mathbf{z}(\mathbf{x}) = [\mathrm{z}_1(x), z_2(x), \ldots, z_p] \\ &\text{Subject To: } \mathrm{g}_\mathrm{i}(\mathbf{x}) \leq 0 \quad \mathrm{i} = 1, \ldots, \mathrm{m} \\ &\qquad\qquad\qquad \mathrm{x}_\mathrm{j} \geq 0 \quad \mathrm{j} = 1, \ldots, \mathrm{n} \end{aligned}$$

where $g_i(\mathbf{x})$ is the feasible region with the goal of this formulation being to find the set of non-dominated solutions. The non-dominated solution set can then be used to determine the Pareto (more is better) optimal based upon a known or revealed preference structure [8:8, 28:14].

The network flow model I develop in the next chapter is an example of a multicriteria decision. This model must balance a network relaibility measure against network availability and flow routing.

2.4.1 User Vs. Operator Trade-offs. The above formulation for multi-criteria decisions describes possible trade-offs within a model, but there is another aspect to this type of decision making. Quite often the decision maker must make comparisons between two different models. For example, the locating of a maintenance facility using the minimum-time-to-respond to a service call as the performance metric is an operator's perspective while the network flow model measuring network performance is a measure of a network user's perspective.

Clearly, the two perspectives come into conflict if a solution for one results in a sub-optimal solution for the other. Idealy, I would like to have a win-win situation where both models reach maximum "happiness". As Manheim indicates, this is not always possible since the optimal maintenance facility location for a system operator might not result in optimal network performance from a system user's perspective [23:191]. If a win-win situation does not exist then I am faced with another multicriteria decision problem to compare the two models.

2.4.2 Solution Approaches. Ross and Soland [26] discuss several different solution approaches to find the Pareto optimal for deterministic, multi-criteria problems: value functions, efficient solution sets (frontiers), and interactive algorithms. Using the notation introduced in Chan [7], let Y be the outcome space, where $\mathbf{y} = f(\mathbf{x})$ represents the vector of *p* criterion functions discussed previously, and y^1 is the outcome of alternative 1.

2.4.2.1 Value Functions. Value functions assume the analyst has some prior knowledge of the decision maker's underlying preference structure [25]. The value function $v(y)$ is defined such that $y^1 \succ y^2$, if and only if $v(y^1) \succ v(y^2)$ [5]. The obvious difficulty with this approach is in determining the value function v. This value function for the performance model is definitely unknown at this time since the sponsor is still trying to decide upon what is important to measure.

2.4.2.2 Interactive Algorithms. The interactive algorithm method requires the analyst and decision maker to work together as the solution space is refined and broken out, incorporating the decision maker's preferences as each step is accomplished. According to Ross and Soland [26] this solution method can lead to the selection of a non-efficient solution.

2.4.2.3 Efficient Frontier. The efficient frontier is the set of efficient solutions. This approach relies upon generating the set of solutions that the decision maker should consider [10].

Goicoechea describes four methods to arrive at an efficient frontier: (1) weighting method; (2) constraint method; (3) Phillip's linear multiobjective method; (4) Zeleny's linear multiobjective method [10]. Hsu and Tseng [14] have developed a new method combining the first two methods, called the CONWEIGHT algorithm. While this method is intriguing, I will use the constraint method in the next chapter to generate the non-dominated set for the performance evaluation model.

III. Model Formulation

This chapter describes the mathematical models and approaches used to model the communication network. First, we present a problem summary and research objectives that the models are designed to achieve. The next two sections discuss the two major optimization perspectives and the development of the network partitioning, facility location, and performance measurement models. The last section presents the complete solution algorithm incorporating the models used as a solution procedure and the software packages used for solution.

3.1 Problem Summary

The long-term research problem has an ultimate goal of automating the scheduling process of a telecommunication network while seeking to utilize the maintenance resources as efficiently as possible and at the same time provide the highest level of service possible to the customer. The writing of a simple computer program to do automated scheduling would be fairly simple, but it would not necessarily guarantee the "best" possible solution, particularly when we are not yet even sure of the criteria that should be used in determining "best" or even how to compare them. Hence, this specific thesis research goal is the development of models to evaluate the impact of changes in the dynamic maintenance schedule of a telecommunication network upon selected network performance measures. This statement creates several questions that must be addressed before each of the research objectives can be accomplished: (1) What performance measures need to be considered? (2) Should the system be optimized from the operator perspective or from the user perspective, or does it even matter?

How each of these questions is answered in the next few sections determines how each of the three research objectives are met. Recall that the research objectives are:

- determine an upper-bound upon the level of performance for a telecommunications network by optimal basing of maintenance facilities using dynamically scheduled maintenance;
- develop the metrics by which network performance should be measured; and,
- create the mathematical models necessary for network evaluation.

3.2 Operator Vs. User Perspective

The first issue of maintenance schedule optimization that must be addressed before proceeding into model development is system optimization perspective. The classical approach is to optimize a system from either the user perspective or the operator perspective [23:198]. Quite often the results achieved from the user perspective will be quite different than those yielded by the operator perspective. For example, the classic Braes Paradox transportation network solution yields two different optimal solutions, depending upon which perspective is used as the basis for optimization.

I decided to approach the research problem sequentially using results of operator optimization in creating the results for the user optimization. The goal of this approach is to demonstrate the affect maintenance depot location has upon network performance when both perspectives are incorporated into maintenance scheduling operations.

The modeling approach used for this type of formulation requires breaking the problem down into a model to optimize operator performance and one to optimize user performance. These models are then run sequentially to develop an algorithm for the complete picture.

3.3 Performance Models

Before discussing the two models, it is important to state specific assumptions used in both perspectives:

(1) Network communication links (arcs) are either up (operational) or down for maintenance (broadly referred to as failed). While the links fail, the actual maintenance activities occur at a node. It is important to realize that I am attempting to model network maintenance procedures and the effect these procedures have upon network operations, not just straight data or message flow within any generic telecommunications system. I think it is important to take a moment and explain this further.

Each link (arc) between two sites (nodes) within a telecommunication network usually contains many individual lines. My observation of the sponsor's outage reports for its telecommunication network indicate outages are constantly occurring on individual lines between two sites, yet the link continues to operate in a degraded state. According to the sponsor these outages do not always need maintenance personnel to take any action. The outages could be due to weather, sun-spot activity, or many other phenomenon that eventually clear themselves. [18] At this time, attempting to model each link's operations in a degraded state would be extremely difficult and unnecessary since I am interested in the effect maintenance has upon the network.

There are three types of network maintenance: Adaptive (AM), Preventative (PM), and Corrective (CM). The reliability value, a_{ij}, attached to each link in the model represents the proportion of time over the long run that the link is operational and, 1-a, is the proportion of time the link is down for all three types of maintenance. This approach will also necessitate a solution technique using networks with gains. Hence this approach does not make a distinction between the types of maintenance. It only assumes that maintenance occurs and that any type of maintenance activity would take the link down for some variable period of time. The bi-modal assumption allows me to model the effect of maintenance operations upon network performance. An advantage of using network reliability rates is the elimination of the need to solve the network many times sequentially to simulate multiple time periods.

(2) Failure rates (the rate at which calls are made on the service organization) will be modeled according to a Poisson process. [1:64]

(3) The servicing organization must handle all network failures (maintenance calls) since it is the only entity that repairs inoperative nodes. Hence, we are modeling an infinite capacity queue. This is a logical assumption since if the network is not repaired customer service levels would not be maintained.

(4) The network structure assumes continuous 24-hour operations. It is also assumed to be at steady state.

(5) The server must always return to the service depot node after completing a job and before starting on a new job. [1:64]

(6) Each service (maintenance) call is handled in a first-in-first-out (FIFO) queue discipline.

(7) We must assume that the time required to complete a repair job is stochastic in nature, since some breakdowns are worse than others. There are four factors forming the maintenance process for each job:

(1) Travel time from the depot node to the failed node (Deterministic).

(2) Time required to repair the facility (Normally distributed).

(3) Return time to depot node (Deterministic).

(4) Regeneration time in preparation for next service call. (Normally distributed). [1:64]

(8) The nodes are assumed to be 100% reliable. [18] Nodes subject to failure could me modeled by representing the site as two nodes connected by a dummy arc.

(9) The length of a message carried on the network is constant or deterministic.

Using the previous assumptions, it is possible to model this servicing facility as a queuing system of the form $M / G / 1 / \infty$, where M indicates that the failures arrive according to a Poisson process, G indicates there is a general distribution of service times, 1 indicates a single server, and ∞ indicates a queue with infinite capacity. [1:64]

3.3.1 Operator Performance Model. The operator perspective performance model was developed around the goal of optimizing maintenance depot locations supporting the telecommunication network. The previous chapter presented the two types of facility location models and solution techniques (stochastic and deterministic) that have been developed. I chose the stochastic modeling approach from that discussion which necessitates breaking the location model down into a partitioning model and then a subnetwork location model.

The performance measure chosen to be optimized was minimum time-to-respond (TR) to a maintenance call from a centralized maintenance depot. My reasons for using this metric include: there has been some amount of research into this subject by Ahituv and Berman [1] and follow-on work performed by Chan [5]; additionally, the total expected response time does encompass some of the criterion currently used in the LOSAs: reliability and availability; in that the faster a problem (failure or other maintenance operation) within the network is fixed, the higher the network's reliability and availability of service should be.

3.3.1.1. Network Partitioning Algorithm. As stated in the previous chapter, the partitioning model considers three principal factors: equity, contiguity, and compactness. The detailed explanation for each factor is presented next.

Equity as defined by Ahituv and Berman, "...the concept of equity asserts that the entire population of potential clients [each node of the communications network] be treated as equally as possible in terms of the quality of service they get. In other words, subpopulations of customers shall not be deprived by the service provider." [1:24] This appears to be a logical and necessary standard since the maintenance office as the provider of the service (maintenance) should not be the determiner of who gets service. He must ensure that everyone gets equal service unless directed to do otherwise by superiors. Obviously, there can be different classes of customers to be considered since national decision makers will have a higher priority for maintenance service than will an administrative support user during a time of national crisis, but this can be included as a weighting factor. These priorities would still be set by the decision makers and system users, not by system operators.

The quantitative formulation of equity comes from [1:26]:

M = desired number of subnetworks;

$\bar{h} = 1 / M$; if each subnetwork obtains exactly $\bar{h}$ fraction of maintenance service, we would have perfect equity. Since it is not always feasible to have perfect equity, there must be a margin on either side (called α) that is acceptable (ie, 10%). Overall, a proposed subnetwork, G_i, is feasible if

$$\left| \sum_{j \in G_i} h_j - \bar{h} \right| \leq a\,\bar{h}$$

There are two other principle that I will include in the model: contiguity (T) and compactness (P). Contiguity for our purpose is the ability to travel from each node in the subnetwork to every other node within the subnetwork without having to use links of a different subnetwork. You may cross another network, but you can not use its links. Compactness means simply that we want the nodes of our subnetworks to be close together and not spread out. This is accomplished by simply including a maximum distance constraint upon the subnetwork. [1:26-8]

The actual model I will use is based upon the work of Garfinkel and Nemhauser, whose algorithm was developed for use primarily in political rezoning and district structuring. Their model is based upon enumeration and was adapted for network topology by Ahivut and Berman [1]:

$$\text{Min} \sum_{j=1,S} C_j X_j$$

$$\text{Subject To:} \quad \sum_{j=1,S} X_j = M \qquad (1)$$

$$\sum_{j=1,S} a_{i,j} X_j = 1 \quad i = 1,\ldots,n \qquad (2)$$

where: $X_j = 1$ if subnetwork j is selected;

$= 0$ otherwise;

$a_{i,j} = 1$ if node i is an element of subnetwork j;

$= 0$ otherwise;

$M =$ number of subnetworks;

$n =$ number of nodes;

$$C_j = \frac{\left| \sum_{i \in j} h_i - \frac{1}{M} \right|}{\alpha / M}, \quad 0 < \alpha < 1, \quad h_i = \text{ fraction of demand at node } i$$

Figure 3.1 Network Partitioning Model

The purpose of constraint (1) is to ensure that we get the required number of subnetworks. Constraint (2) fulfills the collectively exhaustive and mutually exclusive requirements.

The algorithm consists of two different phases: **Phase I** determines all feasible subnetworks within the larger network, and **Phase II** determines the final subnetworks based upon our equity objective function. Contiguity and compactness will be bounding constraints for the first phase. One final requirement is that the M subnetworks must be collectively exhaustive and mutually exclusive. In other words, every node must be within one and only one subnetwork. This is accounted for in Phase II. [1:34]

PHASE I: [1:34] Using a "tree search" algorithm we find the feasible set by picking the smallest number node and connecting contiguous nodes while enforcing the compactness requirement until the combined demand becomes. Attention must be paid to not creating separate enclaves, which are node(s) that are incapable of being separate subnetworks and can not be connected to other subnetworks without going through a previously defined subnetwork. This will prevent impossible solutions.

PHASE II: [1:38-41] The algorithm for node partitioning was developed by Garfinkel and Nemhauser in 1970. The following notation is needed:

D is the set of fixed variables;
N(D) is the number of fixed variables;
U is the set of nodes in zones of D;
Y is the set of zones in the current partial solution;
T are the nodes in the zones of Y.

The steps are briefly outlined below:

Step 1: *Initialization.* Set L=0, Y=D, T=U
Step 2: *Choose next list.* Pick the smallest numbered node not in T
Step 3: *Adding a zone to Y.*
Step 4: *Backtracking.* If L=0 stop, else L=L-1
Step 5: *Test for a solution.* Test L = M - N(D)
Step 6: *Solution is found.* Largest cost of district in Y.

My final goal was to automate the entire process into a spreadsheet. This would have the effect of allowing the user to enter a model and then break it into as many pieces as desired, but I was unable to automate the first phase which included the contiguity and compactness constraints. My problem with Phase I was that I could not form an approach for the tree search algorithm using the spreadsheet. This will be left for a later extension. I think that the Phase I can eventually be converted to some sort of linear programming model similar to the Phase II model, which I was able to implement in a spreadsheet.

3.3.1.2. Facility Location Algorithm. The next step is to determine where the maintenance depot should be placed within the network. The optimal location is determined using the minimized expected response time for a maintenance call. The development of the stochastic location algorithm comes from [5, 1]:

$$\text{Min TRj}(X) \quad \forall \;\; j \in I$$

where the expected response time (TR) is the sum of the mean-queuing-delay ($\overline{Q}$) and the mean travel time($\bar{t}$), highlighted as:

$$TR(X) = \overline{Q} + \bar{t}$$

$\overline{Q}$ is further defined as:

$$\overline{Q} = \frac{\lambda S^2(X)}{2(1-\lambda S(X))}$$

X is the facility location, λ is the failure arrival rate, S(X) is the mean total service time (first moment of service time), $S^2(X)$ is the second moment of the total service times.

This formulation is easy to solve provided the maintenance depot is placed at a node in the communications network. This assumption is reasonable given I am using the assumption that maintenance operations occur at the nodes and, additionally, the network managers would probably want the facilities to be co-located [18].

Chan [5] presents a very important observation about this formulation: the time to repair a facility and the regeneration time before a next call are zero. This assumption can be made since this amount of time will be treated as a constant when the objective function for TR is minimized by taking its first derivative, and it will thus be eliminated. The objective function is defined as:

$$\bar{t}(X) = \sum_{j=1}^{I} h_j d(X,j)$$

where I is the total number of nodes, h_j is the demand proportion at each node, and d(X,j) is the shortest distance between X and node j.

3.3.1.3 Model Significance. The two previous algorithms are used sequentially to form a model determining optimal facility location from the network operator perspective. The result of this model is the creation of an outage record and more importantly a dynamic maintenance schedule that is then used to drive the network performance model evaluating user perspective.

3.3.2 User Performance Model. The mathematical representation of the model as adapted from Sanso, et al. is shown in Figure 3.2 [27]. My extensions to this model include: the third objective function for average link delay; and, the inclusion of link availability rates to represent maintenance scheduling.

$$\text{Min } Z1 = \sum_{p\in O}\sum_{q\in D} w^{pq} Y^{pq} \qquad \text{(Number of Lost Calls)}$$

$$\text{Min } Z2 = \sum_{u\in M}\sum_{p\in O}\sum_{q\in D} l_u X_u^{pq} \qquad \text{(Shortest Path)}$$

$$\text{Min } Z3 = \sum_{u\in M}\left[\frac{\sum_{p\in O}\sum_{q\in D} X_u^{pq}}{C_u - \sum_{p\in O}\sum_{q\in D} X_u^{pq}}\right] \qquad \text{(Data Delay)}$$

Subject To:

$$\text{Capacity: } \sum_{p\in O}\sum_{q\in D} X_u^{pq} \le C_u \quad \forall\ u\in M$$

$$\text{Flow Conservation: } \sum_{u\in\Gamma(i)} a_u X_u^{pq} - \sum_{u\in\Gamma-(i)} a_u X_u^{pq} = d^{pq} - Y^{pq}, \text{ if } i = p$$

$$\sum_{u\in\Gamma(i)} a_u X_u^{pq} - \sum_{u\in\Gamma-(i)} a_u X_u^{pq} = -d^{pq} + Y^{pq}, \text{ if } i = q$$

$$\sum_{u\in\Gamma(i)} a_u X_u^{pq} - \sum_{u\in\Gamma-(i)} a_u X_u^{pq} = 0 \text{ otherwise}$$

$$\text{Non Negativity: } X_u^{pq},\ Y_u^{pq} \ge 0 \ \ \forall\ u\in M,\ p\in O,\ q\in D$$

where	X_u^{pq}	Traffic flow of O - D (p = origin & q = destination using arc u);
	C_u	Capacity of arc u;
	d^{pq}	Origin - destination offered load;
	Y^{pq}	Origin - destination lost traffic;
	$\Gamma(i)$	Set of arcs whose origin node is i;
	$\Gamma-(i)$	Set of arcs whose terminal node is i;
	a_u	Availability rate of link as a proportion;
	w^{pq}	Weighting priority given to the p - q commodity ;
	l_u	Length of arc between nodes;
	O	Origin node set;
	D	Destination node set;
	N	Node set for network;
	M	Arc (link) set for network.

Figure 3.2: User Performance Model

As the reader can see, this model has three separate objective functions (criterion) to evaluate, but this is not all that unusual. These criteria must be compared against each other in some fashion that usually involves a compromise between each criteria's "ideal" value. The last two objective functions are where the many potential trade-offs would have to occur since each represent a separate routing protocol governing the network. This model's flexibility is the ability to remain a valid network representation with only one of the routing protocols included. I discuss this possibility further in the last chapter.

3.3.2.1 Number of Lost Calls Objective Function. The first objective function attempts to minimize the number of lost calls. This metric is the direct opposite of the call throughput metric measured in my early attempts at development of a network performance model. The number of lost calls metric is a form of network reliability.

The definition of reliability has always been difficult to define and evaluate, but traditionally, reliability has been based upon some measure of network connectivity [27,13]. Sanso, Soumis, and Gendreau in 1991 proposed a new measure of network reliability based upon the expected number of lost calls due to failures within the telecommunication network [27].

I think this measure of reliability can do a better job in assessing network performance than simple connectivity measures. This stems from the fact that today's communications networks have extremely high connectivity rates, usually greater than 99% [27]. Additionally, telling the user, "Last month, twenty-three phone calls were disrupted and eighteen of the calls were of the highest national priority, supporting national policy makers during a contingency operation," will certainly have a greater meaning to the customer and service provider than saying, "Your calls were connected 99.99% of the time last month".

The determination of priority for a particular commodity is accomplished by the constant w^{pq}. This weight would need to be assigned by the network operators, users, and senior policy makers.

3.3.2.2 Shortest Path Objective Function. The shortest path objective is a routing discipline for the model. The shortest path protocol can be based upon minimum distance or upon the minimum number of hops (number of nodes that each message must traverse in reaching its destination) [15:142]. The shortest path routing discipline in either formulation also helps to prevent some blocking and rerouting problems that can occur when a link is down for maintenance. [27]

Logically, the shortest path criterion could conflict with the objective function (minimizing data delay) since the shortest path requirement could force flow into an already crowded link and consequently increase network delay. The resolution of this problem is addressed in the next section.

3.3.2.3 Data Delay Objective Function. The data delay measure can also be referred to as network availability. My creation of the measure for data delay stemmed from adapting the formula created by Kleinrock [17,21]:

$$Delay = \frac{1}{\gamma} \sum_{u} \frac{\sum_{p \in O} \sum_{q \in D} X_{u}^{pq}}{\mu C_{u} - \sum_{p \in O} \sum_{q \in D} X_{u}^{pq}}$$

where: γ is total network throughput;

$1/\mu$ is average message length

By making the assumption that call duration (γ) and average call length (μ) are both one, the formula is transformed into the formula used for the objective function presented in the model.

The nice feature of this formulation is that it forces equity in proportion of flow over each link. This occurs since in taking the minimum of the function these proportions will be keep as small as possible. Additionally, each link is prevented from reaching its capacity since the delay tends toward infinity at capacity. This fact is somewhat artificial, but for large capacity network it is not usually a problem. Additionally, this problem could easily be over come by adding a one to the capacity in the denominator of the measure if the link tended towards complete saturation.

The other problem created by this objective function is that it is non-linear. While there certainly are software packages available that can solve formulations that have linear constraints and non-linear objective functions [6], we would lose the advantage of the special network structure for ease of formulation and solution. Therefore, it would be better to have a linear version of this objective function.

This can be accomplished by creating an approximation of the function for delay. Simply removing the flow variable from the denominator of the objective function and altering the ratio accomplishes this. The linear approximation to the original delay objective function:

$$Delay = \frac{1}{\gamma}\sum_{u}\frac{\sum_{p\in O}\sum_{q\in D} X_{u}^{pq}}{\mu C_{u}}$$

where: γ is total network throughput;

$1/\mu$ is average message length

This new objective function has the advantage of being linear, without being a major conceptual extension of the original delay formula. Additionally, the problem of the denominator going to zero in the original ratio is eliminated. Kleinrock's formula had the distinct disadvantage of never allowing a link to reach capacity. Obviously, a link must be able to operate at full capacity.

The constraint method solution approach for this objective function would be to fold the objective function itself into the constraint set using a known delay factor as an upper bound on delay. A potential source for the upper bound on delay is the sponsor's original request letter stating that average data delay for the network must be less than .1 seconds. This figure comes from old LOSA's between network users and operators. The disadvantage of such an approach is that the solution for the system would need to be recalculated in order to see the effect of each data delay factor the sponsor considers.

The constraint method usually adds another constraint to the constraint set. However, as the case study shows in the next chapter, this is not the situation with this model formulation. When the network delay objective function is incorporated as a set of constraints to represent link delay for each link, the link capacity constraints become redundant or the new data delay constraints are themselves redundant depending upon the size of the right-hand-side. Thus, the size of the overall constraint set does not increase.

While I will include data delay in the case study, my research has indicated that data delay probably does not need to be included in a final performance evaluation model. Such routing protocols have fallen out of favor and shortest path formulations based upon minimum number of hops (node crossings) have become more widely accepted as the better protocol [15:142-3].

3.3.2.4 Network Constraint Set. The network model constraints were formulated by Sanso, et al. [27]. There are two principal kinds of constraints for the network: limits upon link capacity and conservation of flow for the nodes.

The link capacity constraints have the job of limiting the amount of flow upon each link of the network. This is an obvious requirement since in real life there is a physical limit to the amount of flow possible upon a communications network. Assuming simple circuit switching, a link only has a finite number of data lines within in it.

I have added an additional variable to the link capacity constraints, that being the "a" value as discussed previously. My purpose of this variable is to represent the amount of time long-term that the link is down for maintenance. This method is not without problems however. The proportion does not actually tell the user when during the interval the link is unavailable, but instead gives an average. If the maintenance scheduling office wanted to determine the effect upon network operations of taking a link down for maintenance in a specific time period, a better approach would be to set the capacity of a link determined to be down for maintenance in a time period to zero and then solve the network over several time periods, creating a new constraint set for each time period. This would require the addition of a new time subscript "t" in the model, as shown in Figure 3.3 Time-Interval Performance Network.

The conservation of flow constraints for both models are different from most network flow models since each node can experience "loss" of calls. Traditionally, network models require flow in to equal flow out of a node, but this may not always be the case when dealing with communication networks. In this case we are interested in the number of calls that are lost due to link maintenance. This problem of imbalanced flow is eliminated by the inclusion of Y^{pq} variable to count the number of lost calls. This variable serves the purpose of letting the lost call "flow out" of the node, and maintain nodal conservation.

$$\text{Min } Z1 = \sum_{t \in T} \sum_{p \in O} \sum_{q \in D} w^{pqt} Y^{pqt} \qquad \text{(Number of Lost Calls)}$$

$$\text{Min } Z2 = \sum_{t \in T} \sum_{u \in M} \sum_{p \in O} \sum_{q \in D} l_u X_u^{pqt} \qquad \text{(Shortest Path)}$$

$$\text{Min } Z3 = \sum_{t \in T} \sum_{u \in M} \left[\frac{\sum_{p \in O} \sum_{q \in D} X_u^{pqt}}{C_u - \sum_{p \in O} \sum_{q \in D} X_u^{pqt}} \right] \qquad \text{(Data Delay)}$$

Subject To:

Capacity: $\sum_{p \in O} \sum_{q \in D} X_u^{pqt} \leq C_u^t \quad \forall \; u \in M, \; t \in T$

Flow Conservation: $\sum_{u \in \Gamma(i)} X_u^{pqt} - \sum_{u \in \Gamma-(i)} X_u^{pqt} = d^{pqt} - Y^{pqt}$, if i = p

$\sum_{u \in \Gamma(i)} X_u^{pqt} - \sum_{u \in \Gamma-(i)} X_u^{pqt} = -d^{pqt} + Y^{pqt}$, if i = q

$\sum_{u \in \Gamma(i)} X_u^{pq} - \sum_{u \in \Gamma-(i)} X_u^{pq} = 0$ otherwise

Non Negativity: $X_u^{pqt}, \; Y_u^{pqt} \geq 0 \;\; \forall \; u \in M, \; p \in O, \; q \in D$

where	X_u^{pqt}	Traffic flow of O - D in period t (p = origin & q = destination using ar
	C_u^t	Capacity of arc u in time period t;
	d^{pqt}	Origin - destination offered load in time period t;
	Y^{pqt}	Origin - destination lost traffic in time period t;
	$\Gamma(i)$	Set of arcs whose origin node is i;
	$\Gamma-(i)$	Set of arcs whose terminal node is i;
	w^{pqt}	Weighting priority given to the p - q commodity for period t;
	l_u	Length of arc between nodes;
	O	Origin node set;
	D	Destination node set;
	N	Node set for network;
	M	Arc (link) set for network.
	T	Set of time intervals t

Figure 3.3 Time - Interval Performance Network

These formulations are not without problems. As pointed out by Sanso, et al, the first model as formulated needs $O(n^3)$ constraints and $O(n^4)$ variables, where n is the number of nodes in the network [27]. Even a small network of ten nodes would therefore have 1,000 constraints and 10,000 variables. Additionally, the second model formulation increases the number of variables to $O(tn^4)$. Which means the small ten node network requires approximately 240,000 variables. Obviously, it is better if a network solver package of some kind can be used that takes advantage of the special structure a network tableau creates. Even today's best commercial linear programming packages would get bogged down by a hundred node network, which is still a small communications network by today's standards.

3.4 Solution Algorithm

The previous sections highlight how each part of the problem is solved. The next step is to show how each part is linked together and iterated to find a solution. The solution algorithm for the entire model is presented in Appendix E.

Step (1): Optimize operator perspective

(1.1) Choose number of maintenance depots (M) to be located.

(1.2) Network partitioning algorithm to create M subnetworks among the N nodes, where N is the number of nodes.

(1.3) Subnetwork location algorithm to locate a single facility within the M subnetworks. RESULTS: S = optimal location solution.

The optimal solutions is chosen according to the minimum time-to-respond objective.

Step (2): Optimize user perspective

(2.1) Create network maintenance schedule for solution from Step 1.3

(2.2) Run network user performance model for solution from Step 1.3 without network delay factor.

(2.3) **IF** (Delay Factor Desired) **THEN** Create efficient frontier incorporating by varying the network delay factor over the desired range.

The next chapter will discuss the application of this solution algorithm and any necessary extensions that can be made to improve upon it.

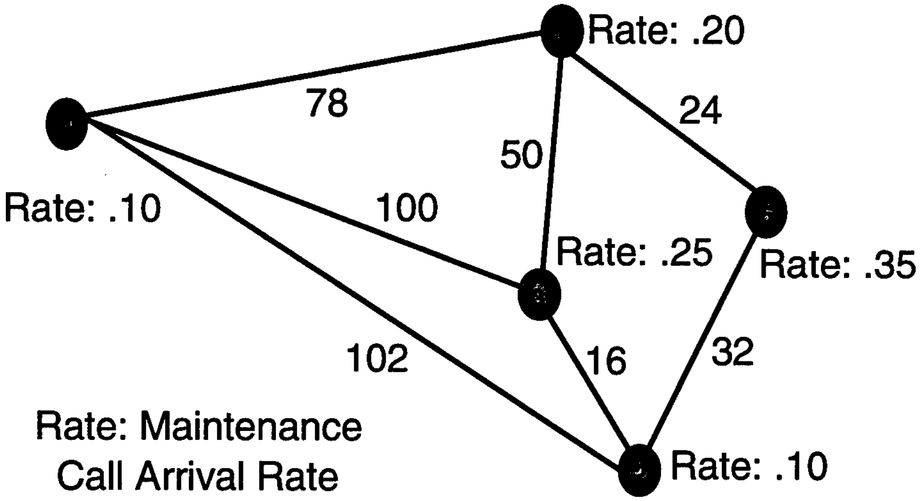
IV. Case Study

This chapter contains a case study to demonstrate the solution algorithm for improving maintenance depot location and network operations. The experiment solves a small-sized network similar in configuration and commodity flow to that which the sponsoring agency uses.

4.1 Location Algorithm

The network this case study solves is shown in Figure 4.1. This network consists of five nodes and seven arcs. The distance (d_{ij}) from node i to node j is shown along each arc in the figure. The service call arrival rates for each node are also shown.

Figure 4.1. Small Network



The network topology for the location algorithm and for the flow model are the same. This assumption means the message flow through the network will use the same arcs as the maintenance service providers.

The application of the location model requires several steps as described in Chapter 3.

Step # 1.1: Choose the number (M) of maintenance depots to be located within the communications network. I choose M = 2. Therefore, the $\bar{h}$ = .50 and the tolerance (α) is set to .10.

Step # 1.2: Partitioning algorithm. This algorithm is broken into two phases, I and II. Phase I is a complete enumeration of the possible subnetworks. The maximum distance for compactness is arbitrarily set at P = 150 (which is arbitrarily chosen to demonstrate the elimination of a possible subnetwork because of proximity concerns, that being a subnetwork consisting of Nodes 1-2-4). The Phase I subnetwork set is shown in Appendix A, page 1.

The second phase implements the model described in section *3.3.1.1.* The model is formulated and solved using the ***EXCEL*** Spreadsheet. The formulation and solution are shown in Appendix A, pages 2-3, respectfully. The results of this application are two subnetworks in which the maintenance depots are to be located. The best partition possible consists of Nodes 1-3 and 2-4-5. This means that one depot is located at node 1 or 3, while the other is located at node 2, 4, or 5.

Step # 1.3: Location algorithm. The number of possible solutions for this problem is 6. This stems from the combination of possible locations for a maintenance depot within each of the subnetworks: (1 & 2), (1 & 4), (1 & 5), (3 & 2), (3 & 4), and (3 & 5).

The location calculations are accomplished in Appendix A, pages 4-6 using MATHCAD. Table 4.1 shows the possible combination of locations and the associated service call response times for each location in preferred order.

Table 4.1 Location Combinations

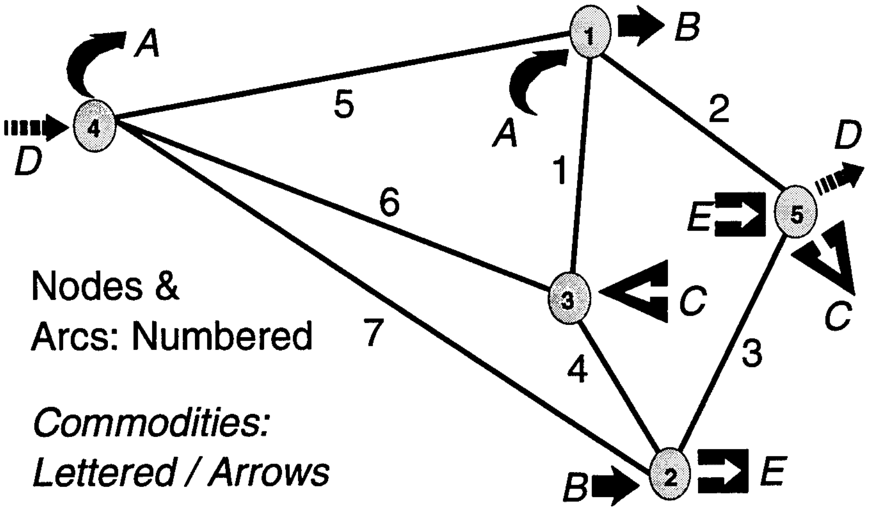
Depot Locations; Nodes i & j	Min-Time-to-Respond (Hours)
3 & 5	**0.924 & 3.715**
1 & 5	**1.263 & 3.715**
3 & 2	**0.924 & 4.877**
1 & 2	**1.263 & 4.877**
3 & 4	**0.924 & Infinite**
1 & 4	**1.263 & Infinite**

Obviously, locating the maintenance depot at node 4 in the second subnetwork would be a major mistake since the network would eventually be unable to perform maintenance operations upon node 2, 4, and 5 due to an infinite queue building up waiting for service calls. This implies that no messages would be able to flow through these nodes. Therefore, it is possible to conclude that these two solutions are dominated and there are four possible solutions (S = 4) that must be examined for flow optimality.

4.2 Flow Model.

The network representation depicting message flows is shown in Figure 4.2. Each flow arrow represents a different type of multi-commodity flow.

Figure 4.2. Multi-Commodity Flow Orientation



Step # 2.1 Create Network Maintenance Schedule for Step 1.3 Solution.

This application of the solution algorithm uses the reliability factor (a_{ij}) and the model depicted in section *3.3.2 User Performance Model.* Table 4.2(a) shows the impact in reliability measures and decreased capacity for each link that results from using the optimal solution set in Table 4.1 (depots located at node 3 & 5) to derive a maintenance schedule.

Table 4.2(a) Solution #1 Schedule Impact (Location 3 & 5)

Node i	From	To	Reliability (a_{ij})	Previous Capacity	New Capacity
1	1	3	96.2%	50	48
2	1	5	80.7%	50	40
3	5	2	84.5%	50	42
4	2	3	80.7%	50	40
5	1	4	80.7%	50	40
6	3	4	80.7%	50	40
7	2	4	84.5%	50	42

The calculation of the reliability measure and hence the impact on the schedule is shown in Appendix B, page 1-5 for all four possible solutions. In general, the creation of a real maintenance schedule is important for actual operations and the long-term goal of the sponsor. However, for this exercise, the assumptions that create a given maintenance schedule are not critical. It is more important that the assumptions and calculations remain consistent for each solution schedule so that a reasonable basis for comparison can be made. The assumptions are:

(1) the schedule is based upon continuous 24-clock and operations, where each schedule represents a single 24-hour period;

(2) the average-time-to-respond calculated in the location calculation is translated directly into a reliability measure for scheduling purposes; and,

(3) the affect upon links between each subnetwork are cumulative (incorporating both maintenance depots and both average-time-to-respond approximations). For example, link #2 from Table 4.2.a. is impacted by both subnetworks since it connects two nodes of different subnetworks.

Step # 2.2 Run Network User Performance Model. The message flow model is implemented and solved using **SAS/OR** and the **NETFLOW** procedure. The case study network can be seen in Appendix B, pages 6-8, in the proper **SAS** implementation code.

The SAS/OR model formulation requires two alterations to the *3.3.2 User Performance Model*, presented previously in order to run in the **SAS/OR** environment: (1) the delay objective function must be incorporated as a constraint; and, (2) the number of lost calls objective function is incorporated as a slack external flow (which accomplishes the objective function and maintains network conservation of flow). These changes are reflected in the formulation shown in Appendix B, pages 6 - 8. The **SAS** software package solves networks of the following form:

$$\min \mathbf{c}^T\mathbf{x} + \mathbf{d}^T\mathbf{z}$$

subject to:

$$\mathbf{F}\mathbf{x} \quad = \mathbf{b}$$
$$\mathbf{H}\mathbf{x} + \mathbf{Q}\mathbf{z} \ \le = \ge \mathbf{r}$$
$$\mathrm{l} \le \mathbf{x} \le \mathrm{u}$$
$$\mathrm{m} \le \mathbf{z} \le \mathrm{v}$$

where:

c is the a x 1 objective function cost vector
x is the a x 1 arc variable value vector
d is the g x 1 objective function coefficient vector of nonarc variables
z is the g x 1 nonarc variable value vector
F is the totally unimodular n x a node-arc incidence matrix of the network
b is the n x 1 node supply/demand vector
H is the k x a side constraint coefficient matrix for arc variables
Q is the k x g side constraint coefficient matrix for nonarc variables
l is the a x 1 arc lower flow bound vector
u is the a x 1 arc upper flow bound vector
m is the g x 1 nonarc variable lower bound
v is the g x 1 nonarc variable upper bound

Figure 4.3 SAS/OR NETFLOW Implementation Format

The solution generated by the SAS/OR software package is shown in Appendix B, pages 9 - 10. The solution set traces out the path each commodity takes between origin and destination. Table 4.2(b) depicts the solution set by the five different commodity types. This table indicates that all flows made it through the network except commodity type C, which lost 20 calls.

Table 4.2(b) Solution #1: Lost Calls

Comm-odity	Origin i	Destin j	Number Sent	Number Delivered	Number Lost
A	1	4	30	30	0
B	2	1	30	30	0
C	3	5	30	10	20
D	4	5	30	30	0
E	5	2	30	30	0
		Total	150	130	20

Step #2.3 Create Delay Efficient Frontier. The creation of the efficient frontier involves the selection of different delay factor values. These values are used to create the right-hand-side for the new set of link delay constraints. The new RHS calculations are accomplished in MATHCAD and shown in appendix B, pages 11 - 13.

The selected values for data delay and the resulting impact upon RHS values are shown in Table 4.3. This table indicates that the effect of the average link delay does not start to impact the model solution until the allowable delay is less than .05 seconds. The table also shows the dramatic impact a network delay of less than .01 seconds has upon link capacity. This impact translates into 123 lost calls, as Appendix B, pages 17 - 18 show.

Table 4.3 Delay Constraint RHS Values

Link Capacity	Data Delay (seconds)			
Node i to j	None	.01	.05	.10
1 - 3	48	10	51	102
1 - 4	40	8	42	85
1 - 5	40	8	42	85
2 - 3	40	8	42	85
2 - 4	42	9	45	90
2 - 5	42	9	45	90
3 - 4	40	8	42	85

As shown in Figure 4.4, the efficient frontier for data delay at these three link delay factors is a set containing three points. The original solution from Step #2.2 provides two points (since the delay constraints are redundant for the delay values of .05 and .10 seconds). The third point is the solution generated by using the new capacity limits imposed by the delay factor of .01 seconds. Obviously, if more points are desired for the decision maker's evaluation, it is necessary to solve the model using more delay factor values less than .05 seconds.

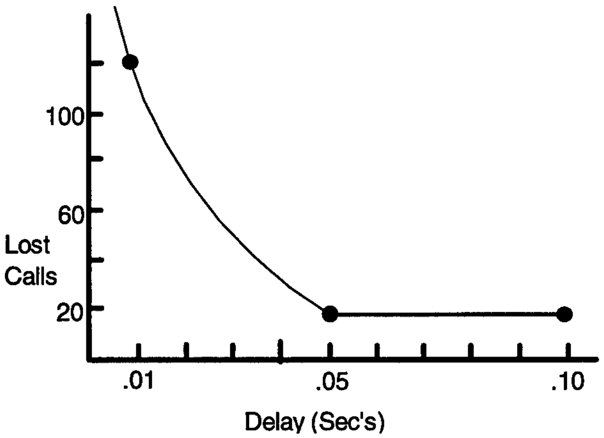


Figure 4.4 Efficient Frontier with Data Delay

This demonstrates the complete algorithm and its application to a small-sized network. However, one thing must still be proven: existence of a win-win situation between the operator and user model.

4.3 Win-Win Situation

The win-win situation occurs if the optimum operator location for the maintenance depot(s) also creates the optimum performance for the user network flow model. The key assumption for the win-win situation is the FIFO maintenance queue. The idea that while one maintenance operation is being acted upon, any other maintenance service calls that arrive go into a maintenance queue. The maintenance service calls receive service in the order in which they arrive to the queue. If this maintenance scheduling protocol is not followed and some other queuing discipline is utilized, the win-win situation is not guaranteed.

The win-win case intuitively appears to be true. The minimum-time-to-respond objective for the operator perspective selects the maintenance depot location that allows maintenance operations to be performed as quickly as possible. This allows the network to operate for a longer proportion of time, and hence each link has a higher reliability measure. A higher reliability measure translates into less capacity lost.

The user performance model attempts to minimize the number of lost calls, take the shortest path possible, and induce as little delay as possible upon network flow. Total network and individual link capacities are the major factors for all three of these objectives (assuming, of course, the network is capacitated). Obviously, anything that decreases capacity in a capacitated network will increase the number of lost calls, possibly require longer paths between origin and destination, and definitely increase delay due to increases in link volume.

The concept of a win-win situation existing between the optimum operator value and the optimum user value is essential to the success and usefulness of the designed algorithm. If a win-win situation does not exist, the algorithm is forced to enumerate each possible location and combination of locations for the maintenance depot(s) within the network. This type of an algorithm is inefficient and requires an extreme amount of time to complete even for a small network. The next two subsections offer a demonstration of the win-win for the other five possible location combinations from the five node network and a mathematical proof of the win-win concept.

4.3.1 Case Study: Win-Win Example. The case study creates six possible location combinations for the maintenance depots (see Table 4.1, Location Combinations). Ignoring the two locations containing node 4 (since the queue builds to an infinite size) and using the remaining three location combinations, the following schedule tables help illustrate the win-win example (calculation for these tables is shown in Appendix B, pages 1-5:

Table 4.4(a) Solution #2 Schedule Impact (Location 1 & 5)

Node i	From	To	Reliability (a_{ij})	Previous Capacity	New Capacity
1	1	3	94.7%	50	47
2	1	5	79.2%	50	39
3	5	2	84.5%	50	42
4	2	3	79.2%	50	39
5	1	4	79.2%	50	39
6	3	4	79.2%	50	39
7	2	4	84.5%	50	42

Table 4.5(a) Solution #3 Schedule Impact (Location 3 & 2)

Node i	From	To	Reliability (a_{ij})	Previous Capacity	New Capacity
1	1	3	96.2%	50	48
2	1	5	75.9%	50	37
3	5	2	79.7%	50	39
4	2	3	75.9%	50	37
5	1	4	75.9%	50	37
6	3	4	75.9%	50	37
7	2	4	79.7%	50	39

Table 4.6(a) Solution #4 Schedule Impact (Location 1 & 2)

Node i	From	To	Reliability (a_{ij})	Previous Capacity	New Capacity
1	1	3	94.7%	50	47
2	1	5	74.4%	50	37
3	5	2	79.7%	50	39
4	2	3	74.4%	50	37
5	1	4	74.4%	50	37
6	3	4	74.4%	50	37
7	2	4	79.7%	50	39

Using the SAS/OR software package and repeating Step #2.2 again for each solution, (see Appendix C, page 1-9 for SAS coded networks) the results are displayed in the next three tables (4.4(b), 4.5(b), and 4.6(b)). These solutions highlight existence of a win-win situation very well. Each solution shows an increasing number of lost calls as the minimum-time-to-respond increases. Appendix C, pages 10-15, also contain the solution in **SAS** form. (The **SAS** output is useful for path determination.)

Table 4.4(b) Solution #2: Lost Calls

Commodity	Origin i	Destin j	Number Sent	Number Delivered	Number Lost
A	1	4	30	30	0
B	2	1	30	30	0
C	3	5	30	8	22
D	4	5	30	30	0
E	5	2	30	30	0
		Total	150	128	22

Table 4.5(b) Solution #3: Lost Calls

Commodity	Origin i	Destin j	Number Sent	Number Delivered	Number Lost
A	1	4	30	30	0
B	2	1	30	30	0
C	3	5	30	4	26
D	4	5	30	30	0
E	5	2	30	30	0
		Total	150	124	26

Table 4.6(b) Solution #4: Lost Calls

Commodity	Origin i	Destin j	Number Sent	Number Delivered	Number Lost
A	1	4	30	30	0
B	2	1	30	30	0
C	3	5	30	3	27
D	4	5	30	30	0
E	5	2	30	30	0
		Total	150	123	27

These tables indicate an increasing number of lost calls as we move down the location list. These results are consistent with the win-win theory. But, these numbers only show it for the without delay metric included. For there to be a total win-win it must hold for both metrics.

The delay constraint development and calculation for the three locations is shown in Appendix D, pages 1-9. These calculations show the effect of the different link delay factors. Tables 4.7(a), (b), and (c) indicates the delay constraint values for each possible solution location:

Table 4.7(a) Delay Constraint RHS Values (Location 1 & 5)

Link Capacity	Data Delay (seconds)			
Node i to j	None	.01	.05	.10
1 - 3	47	10	51	102
1 - 4	39	8	42	85
1 - 5	39	8	42	85
2 - 3	39	8	42	85
2 - 4	42	9	45	90
2 - 5	42	9	45	90
3 - 4	39	8	42	85

Table 4.7(b) Delay Constraint RHS Values (Location 3& 2)

Link Capacity	Data Delay (seconds)			
Node i to j	None	.01	.05	.10
1 - 3	48	10	51	102
1 - 4	37	7	39	79
1 - 5	37	7	39	79
2 - 3	37	7	39	79
2 - 4	39	8	41	83
2 - 5	39	8	41	83
3 - 4	37	7	39	79

Table 4.7(c) Delay Constraint RHS Values (Location 1 & 2)

Link Capacity	Data Delay (seconds)			
Node i to j	None	.01	.05	.10
1 - 3	47	10	51	102
1 - 4	37	7	39	79
1 - 5	37	7	39	79
2 - 3	37	7	39	79
2 - 4	39	8	41	83
2 - 5	39	8	41	83
3 - 4	37	7	39	79

These tables show the same pattern as the data from the first solution. The delay factor does not come into play until the delay is set to less than .05 seconds. Table 4.8 shows the resulting impact to lost calls. The solutions are generated as before in SAS and shown in Appendix D, pages 10-15.

Table 4.8 All Solutions: Number of Lost Calls (With Delay)

	Locations (Values Indicate Number of Lost Calls)			
Delay	3 & 5	1 & 5	3 & 2	1 & 2
None	20	22	26	27
.10	20	22	26	27
.05	20	22	26	27
.01	123	123	125	125

If a win-win situation exists a plot of the different location values should create a graph similar to Figure 4.5. Basically, the different location contours should not cross each other. Each contour line should start at the maximum number of lost calls possible (total throughput) and bottom out into a horizontal line as allowable link delay is increased to a breakpoint where it no longer changes network capacity constraints. (Remember, network capacity constraints and link delay constraints are redundant.) If the lines do not cross one of the contours would thus dominate all of the others by achieving a lower number of lost calls at a smaller link delay value.

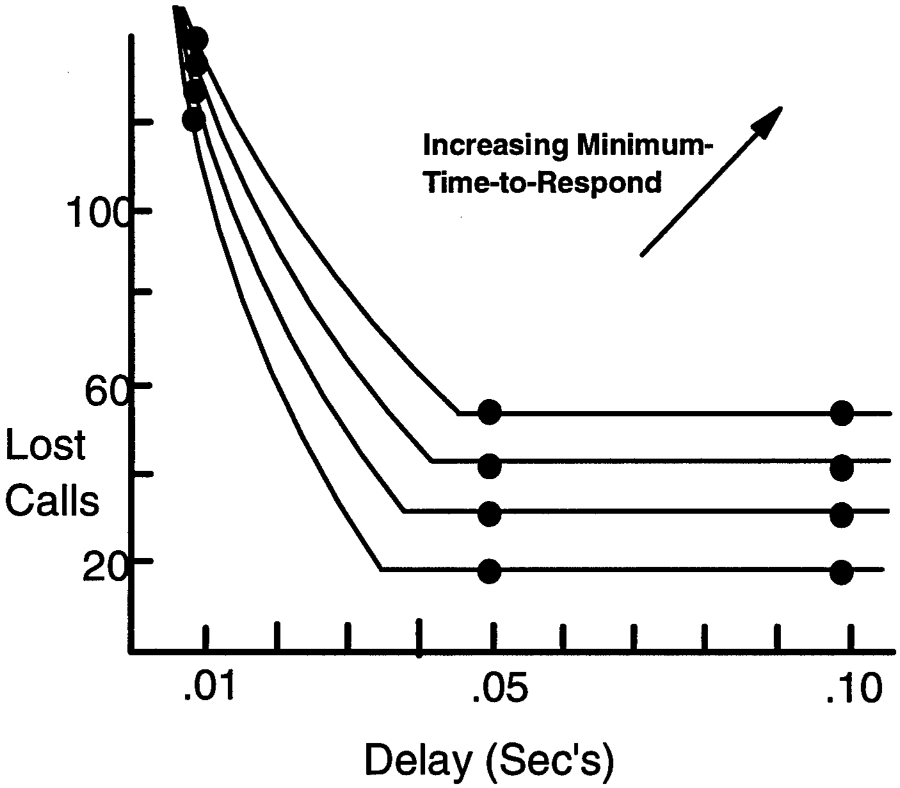


Figure 4.5 Theoretical Efficient Frontier for Win-Win Situation

The data from Table 4.8 translate into Figure 4.6. This figure presents the win-win situation in dramatic fashion. Each contour line represents a different location solution. As the graph indicates these line follow the expected pattern very closely. The reason two sets of these contours touch at the delay value of .01 is due to the level of the inputs used in the test. This point requires further explanation since it would seem to indicate that the win-win condition described above might be violated.

Figure 4.6 Case Study Actual Efficient Frontier

Table 4.8 indicates that the number of lost calls at the data delay of .01 seconds did not change between location (3 & 5) and (1 & 5). This is due to the assumption I use when calculating capacities: truncation of all link capacities (rounding down). I use this assumption so that only complete calls can get through (e.g. no fractional calls).

For example, suppose the calculation of delay RHS values for Location (3 & 5) versus (1 & 5) yielded a link capacity value of 39.996 and 39.002, respectfully. Both values are rounded down to 39. This has the effect of changing the number of lost calls for the two model runs and subsequently a slight alteration of the slope for each contour line. Now imagine if the inputs for number of calls and initial link capacities had been one or more orders of magnitude larger (which is quite reasonable given the size and capacity of present communication networks) the truncation effect would not be so pronounced, and it would be possible to observe the real differences in the results. Obviously, the truncation error becomes more pronounced as the size of the inputs is decreased.

4.3.2 Win-Win Proof. One way to prove the win-win situation is by showing that the operator and user models' objective functions are based upon similar factors. This proof will show that both models are optimized by achieving maximum available arc capacity. This proof assumes a capacitated network.

Operator (Location) Objective Function:

Optimization factor = Minimum-time-to-respond (MTR) to service call

$$\text{Period Link Reliability} = \frac{\text{MTR}}{\text{Time Units / Period}}$$

Period Link Capacity = Period Link Reliability * Maximum Link Capacity

User (Flow) Performance Objective Function: (Per Individual Link)

Objective 1: Number of Lost Calls

Max Number Calls = Period Link Capacity

Number Lost Calls = Total Calls Required on Link - Max Number Calls

Objective 2: Shortest Path

The shortest path is chosen from the set of possible paths **P** that connect origin i to destination j, where a path is defined as the number of links required to span from i to j. A path is defined if there is capacity remaining for each link (Period Link Capacity) within the path. Therefore, as the capacity of the shortest path in each set **P** is consumed a longer path from the set **P** is chosen.

Objective 3: Network Delay

$$Delay = \frac{1}{\gamma} \frac{\sum_{p \in O} \sum_{q \in D} X_u^{pq}}{\mu C_u}$$

where: γ is total network throughput;

$1/\mu$ is average message length

As the link delay formula shows, as (period) link capacity, C_u, increases the value of Delay will decrease, all other variables remaining constant.

This proof shows that each objective function in the user performance model depends directly upon the time period link capacity generated from the operator location model. Therefore, as MTR is minimized, period link capacity is increased, and all three objective functions are optimized.

The example and the proof demonstrate the important feature of this algorithm: the operator versus user trade-off can be complimentary (win-win). The use of the FIFO queuing system for dynamic scheduling of maintenance operations is the necessary protocol to insure the win-win situation.

V. Conclusions and Recommendations

This chapter presents the research conclusions and recommendations for future research efforts.

5.1 Conclusions

Even though this research effort was centered upon a small piece of a new and large problem for the sponsor, I was able to make several observations.

First, the network performance model measuring message "flow" makes it possible to assess the impact to network operations caused by changes in the maintenance schedule. One of the problems the sponsor faces is the inability to quantify the impact to network operations when the maintenance schedule changes. The creation of a network performance model to measure different metrics (forms of reliability and availability) makes this assessment possible. Two separate performance models were created for this purpose. The first model (demonstrated in the case study) observes the network from an aggregate level, while the second model implements a specific maintenance schedule for a specified length of time. The two models should prove useful for evaluating "what-if" situations and specific maintenance schedules. The ability to quantify the impact of maintenance schedule changes is the first important step toward the goal of creating an automated maintenance scheduling system.

Second, the performance metric of total number of lost calls is probably a better metric than measuring average network delay. The number of lost calls metric allows both system operators and users to better assess network performance. This metric goes beyond a simple measure of reliability since it shows which messages would be lost. The delay metric may not be very meaningful to the network evaluators since a time delay of a few hundredths of a second is not dramatic. However, the incorporation of the delay metric as a constraint can have large impact upon network performance. As the case study results demonstrate there is a trade-off between the number of lost calls and average link delay (as allowable link delay is decreased the number of lost calls will increase). The level at which average link delay begins to affect the number of lost calls will be different for each network analyzed.

Third, the location of maintenance depots using a metric geared toward the system's operators can produce positive benefits for the user of a system. I found that under the right set of circumstances (first-in-first-out dynamic maintenance protocols, steady-state operations, and no priority maintenance) an optimal location of maintenance service depots based upon the minimum-time-to-respond can improve network performance by reducing the time required to perform maintenance operations.

5.2 Recommendations

Since this was the beginning of a much longer research effort for the sponsor, there are a tremendous number of areas needing further research. I will confine myself to a few of the important ones that stem directly from this research:

(1) Creation and development of a large-scale telecommunication network that accurately depicts network operations. This network would need to accommodate a few

hundred commodities and forty nodes. This would allow testing of the network measurement metrics (lost calls, delay, and any others). It may be necessary to move to a simulation environment since the size of this type of network might very well go beyond even the best network solving software packages and become largely intractable. A simulation approach has several advantages if underpinned with a good theoretical development.

(2) Development of metrics for measuring network performance levels which are useful to both the operator and user. Metrics supporting the operator should be able to tell the operator if maintenance resources are being utilized efficiently, or if all necessary support agreements (LOSAs) are being met. Good user metrics should be able to tell if LOSAs are being satisfied. But, more importantly, these new metrics need to give the users specific information about different messages and data. (For example, whose messages are lost or interrupted, what were the priority levels of the messages, etc.)

(3) Extension of the model to eliminate some of the simplifying assumptions. Since this was a new topic area, I had to make many simplifying assumptions in order to prove basic concepts. Now there is an opportunity to go beyond these assumptions in order to improve the model. An example of a simplifying assumption that needs to be altered is the current FIFO maintenance protocol. The inclusion of more detail is important if the model is going to have the fidelity necessary to handle real world operations.

(4) Development of an interactive scheduling tool. This tool should allow the operator to create a maintenance schedule and put it into a data structure that the network performance model can utilize to test how good the schedule is. This would then allow

the operator to parametrically alter the schedule and evaluate the impact of schedule changes in both quantitative and qualitative metrics. The tool would achieve the ultimate objective of performing dynamic maintenance scheduling for the telecommunication system.

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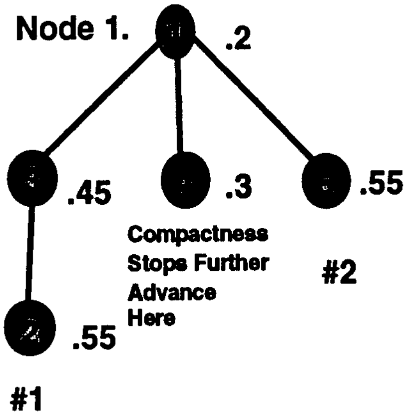
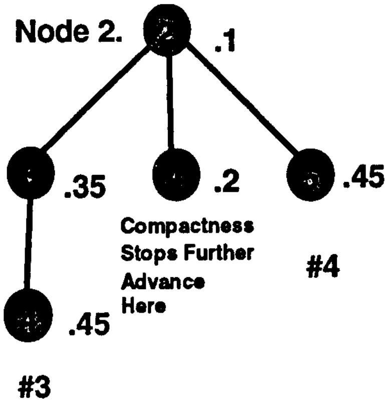
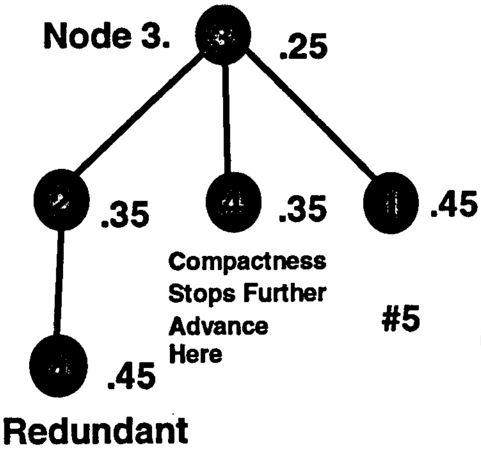
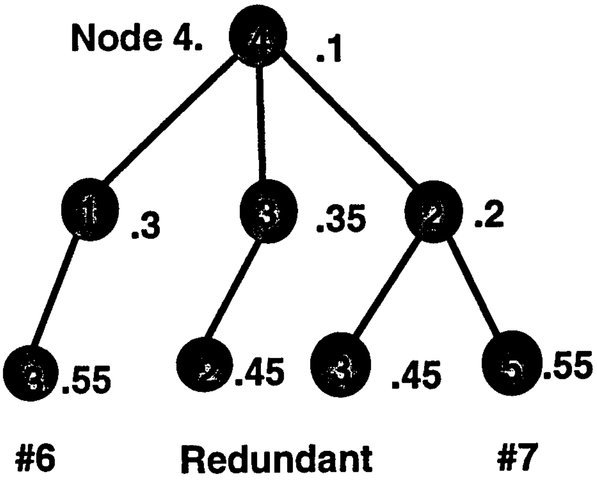
Appendix A. Location Model

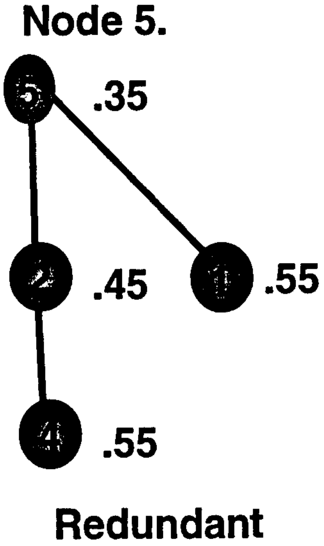
A.1 Partitioning Phase I











A.2 Partitioning Model Setup

M =	2	2	2	2	2	2	2
α =	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Objective							
Node Proportion							
Node 1 =	0.2	0.2	0.2	0.2	0.2	0.2	0.2
Node 2 =	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Node 3 =	0.25	0.25	0.25	0.25	0.25	0.25	0.25
Node 4 =	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Node 5 =	0.35	0.35	0.35	0.35	0.35	0.35	0.35

Link Connection	Sub 1	Sub 2	Sub 3	Sub 4	Sub 5	Sub 6	Sub 7	Used Links	Link RHS Limits
Node 1	1	1	0	0	1	1	0	0	1
Node 2	1	0	1	1	0	0	1	0	1
Node 3	1	0	1	0	1	1	0	0	1
Node 4	0	0	1	0	0	1	1	0	1
Node 5	0	1	0	1	0	0	1	0	1

								# of Subs Chosen	Required # Subnetworks
Mutual exclusivity Constraint	1	1	1	1	1	1	1	0	2
Link Variable	0	0	0	0	0	0	0		

								Total Value of Function
Objective	1	1	1	1	1	1	1	0

A.3 Partitioning Model Setup

M =	2	2	2	2	2	2	2
α =	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Objective							

Node Proportion							
Node 1 =	0.2	0.2	0.2	0.2	0.2	0.2	0.2
Node 2 =	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Node 3 =	0.25	0.25	0.25	0.25	0.25	0.25	0.25
Node 4 =	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Node 5 =	0.35	0.35	0.35	0.35	0.35	0.35	0.35

Link Connection	Sub 1	Sub 2	Sub 3	Sub 4	Sub 5	Sub 6	Sub 7	Used Links	Link RHS Limits
Node 1	1	1	0	0	1	1	0	1	1
Node 2	1	0	1	1	0	0	1	1	1
Node 3	1	0	1	0	1	1	0	1	1
Node 4	0	0	1	0	0	1	1	1	1
Node 5	0	1	0	1	0	0	1	1	1

								# of Subs Chosen	Required # of Subnetworks
Mutual exclusivity Constraint	1	1	1	1	1	1	1	2	2
Link Variable	0	0	0	0	1	0	1		

								Total Value of Function
Objective	1	1	1	1	1	1	1	2

A.4 Location Calculations

Simplifying Assumptions:

(1) I will assume that the travel time to and from a node to the maintenance depc equal. This means thaβ = 2.

(2) I will also assume a constant travel speed over the network, v = 55 miles per hour.

(3) The distance between nodes is assumed to be miles, and is defined below as ve

$$\beta := 2 \qquad v := 55 \qquad d := (50\ \ 24\ \ 32\ \ 16\ \ 78\ \ 100\ \ 102)$$

Subnetwork 1: Containing nodes 1 & 3.

$$j := 1..2 \qquad h := \begin{pmatrix} .2 \\ .25 \end{pmatrix} \qquad \lambda := \sum_{j=1}^{2} h_j \qquad \lambda = 0.45$$

NODE 1:

$$t_{bar} := \frac{h_2}{\lambda} \cdot \frac{d_{1,1}}{v} \qquad t_{bar} = 0.505$$

$$S_{bar} := \beta \cdot t_{bar} \qquad S_{bar} = 1.01$$

$$S2_{bar} := \frac{h_2}{\lambda} \cdot \left(\frac{\beta \cdot d_{1,1}}{v} \right)^2 \qquad S2_{bar} = 1.837$$

$$TR_X := \frac{\lambda \cdot S2_{bar}}{2 \cdot \left(1 - \lambda \cdot S_{bar}\right)} + t_{bar} \qquad TR_X = 1.263$$

NODE 3: $t_{bar} := \frac{h_1}{\lambda} \cdot \frac{d_{1,1}}{v}$ $t_{bar} = 0.404$

$S_{bar} := \beta \cdot t_{bar}$ $S_{bar} = 0.808$

$S2_{bar} := \frac{h_1}{\lambda} \cdot \left(\frac{\beta \cdot d_{1,1}}{v} \right)^2$ $S2_{bar} = 1.469$

$TR_X := \frac{\lambda \cdot S2_{bar}}{2 \cdot \left(1 - \lambda \cdot S_{bar}\right)} + t_{bar}$ $TR_X = 0.924$

This calculation shows the best location to be at node 3, since it will have a sma to respon versus at node 1.

Subnetwork 2: Containing nodes 2,4, & 5.

$j := 1..3$ $h := \begin{pmatrix} .1 \\ .1 \\ .35 \end{pmatrix}$ $\lambda := \sum_{j=1}^{3} h_j$ $\lambda = 0.55$

NODE 2: $t_{bar} := \frac{h_2}{\lambda} \cdot \frac{d_{1,7}}{v} + \frac{h_3}{\lambda} \cdot \frac{d_{1,3}}{v}$ $t_{bar} = 0.707$

$S_{bar} := \beta \cdot t_{bar}$ $S_{bar} = 1.415$

$S2_{bar} := \frac{h_2}{\lambda} \cdot \left(\frac{\beta \cdot d_{1,7}}{v} \right)^2 + \frac{h_3}{\lambda} \cdot \left(\frac{\beta \cdot d_{1,3}}{v} \right)^2$ $S2_{bar} = 3.363$

$TR_X := \frac{\lambda \cdot S2_{bar}}{2 \cdot \left(1 - \lambda \cdot S_{bar}\right)} + t_{bar}$ $TR_X = 4.877$

NODE 4: $t_{bar} := \frac{h_1}{\lambda} \cdot \frac{d_{1,7}}{v} + \frac{h_3}{\lambda} \cdot \frac{d_{1,3} + d_{1,7}}{v}$ $t_{bar} = 1.888$

$S_{bar} := \beta \cdot t_{bar}$ $S_{bar} = 3.775$

$S2_{bar} := \frac{h_1}{\lambda} \cdot \left(\frac{\beta \cdot d_{1,7}}{v} \right)^2 + \frac{h_3}{\lambda} \cdot \left[\frac{\beta \cdot \left(d_{1,3} + d_{1,7}\right)}{v} \right]^2$ $S2_{bar} = 17.611$

$TR_X := \frac{\lambda \cdot S2_{bar}}{2 \cdot \left(1 - \lambda \cdot S_{bar}\right)} + t_{bar}$ $TR_X = -2.612$

$S_{bar} \cdot \lambda = 2.076$

As this result shows a negative value we can also check the requirement Sbarλ)<1. Since this value is greater than 1 we know that the wait time in the queue is infinite which clearly shows that on this network you would not want to put the maintenance depot at NODE #4.

NODE 5:

$$t_{bar} := \frac{h_2}{\lambda} \cdot \frac{d_{1,3}}{v} + \frac{h_1}{\lambda} \cdot \frac{d_{1,3} + d_{1,7}}{v} \qquad t_{bar} = 0.549$$

$$S_{bar} := \beta \cdot t_{bar} \qquad S_{bar} = 1.098$$

$$S2_{bar} := \frac{h_2}{\lambda} \cdot \left(\frac{\beta \cdot d_{1,3}}{v} \right)^2 + \frac{h_1}{\lambda} \cdot \left[\frac{\beta \cdot \left(d_{1,3} + d_{1,7} \right)}{v} \right]^2 \qquad S2_{bar} = 4.563$$

$$TR_X := \frac{\lambda \cdot S2_{bar}}{2 \cdot \left(1 - \lambda \cdot S_{bar} \right)} + t_{bar} \qquad TR_X = 3.715$$

The best location for the second subnetwork is at node 5, then node 2, but ne node 4 since the network would eventually build to an infinite queue and henc breakdown.

Appendix B. Network Flow Model

B.1 Reliability Calculations

The reliability values are calculated using the concepts explained in Chapter 4 (based a 24-hour period).

The minimum-time-to-respond for each location was calculated previously in Appendi pages 4 - 6. The values for each subnetwork are:

Subnetwork #1	Subnetwork #2
$Loc_3 := .924$	$Loc_2 := 4.877$
$Loc_1 := 1.263$	$Loc_5 := 3.715$

The flow capacity for each link is the same: $Capacity := 50$

Solution #1: Using Location 3 & 5

Arc 1: $Reliab_1 := \left(1 - \frac{Loc_3}{24}\right) \cdot 100$ $\quad Capac_1 := floor\left(\frac{Reliab_1}{100} \cdot Capacity\right)$

$Reliab_1 = 96.2$ $\quad Capac_1 = 48$

Arc 2: $Reliab_2 := \left(1 - \frac{Loc_3 + Loc_5}{24}\right) \cdot 100$ $\quad Capac_2 := floor\left(\frac{Reliab_2}{100} \cdot Capacity\right)$

$Reliab_2 = 80.7$ $\quad Capac_2 = 40$

Arc 3: $Reliab_3 := \left(1 - \frac{Loc_5}{24}\right) \cdot 100$ $\quad Capac_3 := floor\left(\frac{Reliab_3}{100} \cdot Capacity\right)$

$Reliab_3 = 84.5$ $\quad Capac_3 = 42$

Arc 4: $Reliab_4 := \left(1 - \frac{Loc_3 + Loc_5}{24}\right) \cdot 100$ $\quad Capac_4 := floor\left(\frac{Reliab_4}{100} \cdot Capacity\right)$

$Reliab_4 = 80.7$ $\quad Capac_4 = 40$

Node 5: $\mathrm{Reliab}_5 := \left(1 - \frac{\mathrm{Loc}_3 + \mathrm{Loc}_5}{24}\right) \cdot 100$ $\mathrm{Capac}_5 := \mathrm{floor}\left(\frac{\mathrm{Reliab}_5}{100} \cdot \mathrm{Capacity}\right)$

$\mathrm{Reliab}_5 = 80.7$ $\mathrm{Capac}_5 = 40$

Node 6: $\mathrm{Reliab}_6 := \left(1 - \frac{\mathrm{Loc}_3 + \mathrm{Loc}_5}{24}\right) \cdot 100$ $\mathrm{Capac}_6 := \mathrm{floor}\left(\frac{\mathrm{Reliab}_6}{100} \cdot \mathrm{Capacity}\right)$

$\mathrm{Reliab}_6 = 80.7$ $\mathrm{Capac}_6 = 40$

Node 7: $\mathrm{Reliab}_7 := \left(1 - \frac{\mathrm{Loc}_5}{24}\right) \cdot 100$ $\mathrm{Capac}_7 := \mathrm{floor}\left(\frac{\mathrm{Reliab}_7}{100} \cdot \mathrm{Capacity}\right)$

$\mathrm{Reliab}_7 = 84.5$ $\mathrm{Capac}_7 = 42$

Solution #2: Using Location 1 & 5

Node 1: $\mathrm{Reliab}_1 := \left(1 - \frac{\mathrm{Loc}_1}{24}\right) \cdot 100$ $\mathrm{Capac}_1 := \mathrm{floor}\left(\frac{\mathrm{Reliab}_1}{100} \cdot \mathrm{Capacity}\right)$

$\mathrm{Reliab}_1 = 94.7$ $\mathrm{Capac}_1 = 47$

Node 2: $\mathrm{Reliab}_2 := \left(1 - \frac{\mathrm{Loc}_1 + \mathrm{Loc}_5}{24}\right) \cdot 100$ $\mathrm{Capac}_2 := \mathrm{floor}\left(\frac{\mathrm{Reliab}_2}{100} \cdot \mathrm{Capacity}\right)$

$\mathrm{Reliab}_2 = 79.3$ $\mathrm{Capac}_2 = 39$

Node 3: $\mathrm{Reliab}_3 := \left(1 - \frac{\mathrm{Loc}_5}{24}\right) \cdot 100$ $\mathrm{Capac}_3 := \mathrm{floor}\left(\frac{\mathrm{Reliab}_3}{100} \cdot \mathrm{Capacity}\right)$

$\mathrm{Reliab}_3 = 84.5$ $\mathrm{Capac}_3 = 42$

Node 4: $\mathrm{Reliab}_4 := \left(1 - \frac{\mathrm{Loc}_1 + \mathrm{Loc}_5}{24}\right) \cdot 100$ $\mathrm{Capac}_4 := \mathrm{floor}\left(\frac{\mathrm{Reliab}_4}{100} \cdot \mathrm{Capacity}\right)$

$\mathrm{Capac}_4 = 39$

$\mathrm{Reliab}_4 = 79.3$

Node 5: $\mathrm{Reliab}_5 := \left(1 - \frac{\mathrm{Loc}_1 + \mathrm{Loc}_5}{24}\right) \cdot 100$ $\mathrm{Capac}_5 := \mathrm{floor}\left(\frac{\mathrm{Reliab}_5}{100} \cdot \mathrm{Capacity}\right)$

$\mathrm{Reliab}_5 = 79.3$ $\mathrm{Capac}_5 = 39$

Node 6: $\mathrm{Reliab}_6 := \left(1 - \frac{\mathrm{Loc}_1 + \mathrm{Loc}_5}{24}\right) \cdot 100$ $\mathrm{Capac}_6 := \mathrm{floor}\left(\frac{\mathrm{Reliab}_6}{100} \cdot \mathrm{Capacity}\right)$

$\mathrm{Reliab}_6 = 79.3$ $\mathrm{Capac}_6 = 39$

Node 7: $\mathrm{Reliab}_7 := \left(1 - \frac{\mathrm{Loc}_5}{24}\right) \cdot 100$ $\mathrm{Capac}_7 := \mathrm{floor}\left(\frac{\mathrm{Reliab}_7}{100} \cdot \mathrm{Capacity}\right)$

$\mathrm{Reliab}_7 = 84.5$ $\mathrm{Capac}_7 = 42$

Solution #3: Using Location 3 & 2

Node 1: $\mathrm{Reliab}_1 := \left(1 - \frac{\mathrm{Loc}_3}{24}\right) \cdot 100$ $\mathrm{Capac}_1 := \mathrm{floor}\left(\frac{\mathrm{Reliab}_1}{100} \cdot \mathrm{Capacity}\right)$

$\mathrm{Reliab}_1 = 96.2$ $\mathrm{Capac}_1 = 48$

Node 2: $\mathrm{Reliab}_2 := \left(1 - \frac{\mathrm{Loc}_3 + \mathrm{Loc}_2}{24}\right) \cdot 100$ $\mathrm{Capac}_2 := \mathrm{floor}\left(\frac{\mathrm{Reliab}_2}{100} \cdot \mathrm{Capacity}\right)$

$\mathrm{Reliab}_2 = 75.8$ $\mathrm{Capac}_2 = 37$

Node 3: $\mathrm{Reliab}_3 := \left(1 - \frac{\mathrm{Loc}_2}{24}\right) \cdot 100$ $\mathrm{Capac}_3 := \mathrm{floor}\left(\frac{\mathrm{Reliab}_3}{100} \cdot \mathrm{Capacity}\right)$

$\mathrm{Reliab}_3 = 79.7$ $\mathrm{Capac}_3 = 39$

Node 4: $\mathrm{Reliab}_4 := \left(1 - \frac{\mathrm{Loc}_3 + \mathrm{Loc}_2}{24}\right) \cdot 100$ $\mathrm{Capac}_4 := \mathrm{floor}\left(\frac{\mathrm{Reliab}_4}{100} \cdot \mathrm{Capacity}\right)$

$\mathrm{Capac}_4 = 37$

$\mathrm{Reliab}_4 = 75.8$

Node 5: $\text{Reliab}_5 := \left(1 - \frac{\text{Loc}_3 + \text{Loc}_2}{24}\right) \cdot 100$ $\text{Capac}_5 := \text{floor}\left(\frac{\text{Reliab}_5}{100} \cdot \text{Capacity}\right)$

$\text{Reliab}_5 = 75.8$ $\text{Capac}_5 = 37$

Node 6: $\text{Reliab}_6 := \left(1 - \frac{\text{Loc}_3 + \text{Loc}_2}{24}\right) \cdot 100$ $\text{Capac}_6 := \text{floor}\left(\frac{\text{Reliab}_6}{100} \cdot \text{Capacity}\right)$

$\text{Reliab}_6 = 75.8$ $\text{Capac}_6 = 37$

Node 7: $\text{Reliab}_7 := \left(1 - \frac{\text{Loc}_2}{24}\right) \cdot 100$ $\text{Capac}_7 := \text{floor}\left(\frac{\text{Reliab}_7}{100} \cdot \text{Capacity}\right)$

$\text{Reliab}_7 = 79.7$ $\text{Capac}_7 = 39$

Solution #4: Using Location 1 & 2

Node 1: $\text{Reliab}_1 := \left(1 - \frac{\text{Loc}_1}{24}\right) \cdot 100$ $\text{Capac}_1 := \text{floor}\left(\frac{\text{Reliab}_1}{100} \cdot \text{Capacity}\right)$

$\text{Reliab}_1 = 94.7$ $\text{Capac}_1 = 47$

Node 2: $\text{Reliab}_2 := \left(1 - \frac{\text{Loc}_1 + \text{Loc}_2}{24}\right) \cdot 100$ $\text{Capac}_2 := \text{floor}\left(\frac{\text{Reliab}_2}{100} \cdot \text{Capacity}\right)$

$\text{Reliab}_2 = 74.4$ $\text{Capac}_2 = 37$

Node 3: $\text{Reliab}_3 := \left(1 - \frac{\text{Loc}_2}{24}\right) \cdot 100$ $\text{Capac}_3 := \text{floor}\left(\frac{\text{Reliab}_3}{100} \cdot \text{Capacity}\right)$

$\text{Reliab}_3 = 79.7$ $\text{Capac}_3 = 39$

Node 4: $\text{Reliab}_4 := \left(1 - \frac{\text{Loc}_1 + \text{Loc}_2}{24}\right) \cdot 100$ $\text{Capac}_4 := \text{floor}\left(\frac{\text{Reliab}_4}{100} \cdot \text{Capacity}\right)$

$\text{Capac}_4 = 37$

$\text{Reliab}_4 = 74.4$

Node 5: $\mathrm{Reliab}_5 := \left(1 - \frac{\mathrm{Loc}_1 + \mathrm{Loc}_2}{24}\right) \cdot 100$

$\mathrm{Reliab}_5 = 74.4$

$\mathrm{Capac}_5 := \mathrm{floor}\left(\frac{\mathrm{Reliab}_5}{100} \cdot \mathrm{Capacity}\right)$

$\mathrm{Capac}_5 = 37$

Node 6: $\mathrm{Reliab}_6 := \left(1 - \frac{\mathrm{Loc}_1 + \mathrm{Loc}_2}{24}\right) \cdot 100$

$\mathrm{Reliab}_6 = 74.4$

$\mathrm{Capac}_6 := \mathrm{floor}\left(\frac{\mathrm{Reliab}_6}{100} \cdot \mathrm{Capacity}\right)$

$\mathrm{Capac}_6 = 37$

Node 7: $\mathrm{Reliab}_7 := \left(1 - \frac{\mathrm{Loc}_2}{24}\right) \cdot 100$

$\mathrm{Reliab}_7 = 79.7$

$\mathrm{Capac}_7 := \mathrm{floor}\left(\frac{\mathrm{Reliab}_7}{100} \cdot \mathrm{Capacity}\right)$

$\mathrm{Capac}_7 = 39$

B.2 SAS/OR Case Study Network #1

```
OPTIONS LINESIZE=72;
TITLE 'COMMUNICATIONS NETWORK - CASE 1: SOLUTION 1';
TITLE2 'MULTI-COMMODITY TELECOMMUNICATION NETWORK';
TITLE3 'DELAY OBJECTIVE FUNCTION INCLUDED AS CONSTRAINT';
DATA NODE0;
        INPUT _NODE_ $ _SUPDEM_;
        CARDS;
N1_1 30
N4_1 -30
N2_2 30
N1_2 -30
N3_3 30
N5_3 -30
N4_4 30
N5_4 -30
N5_5 30
N2_5 -30
;
DATA ARC0;
        INPUT _TAIL_ $ _HEAD_ $ _COST_ _CAPAC_ _LO_ _NAME_$6.;
        CARDS;
N1_1 N3_1 1 48 . A131
N1_1 N4_1 1 40 . A141
N1_1 N5_1 1 40 . A151
N3_1 N4_1 1 40 . A341
N3_1 N2_1 1 40 . A321
N2_1 N3_1 1 40 . A231
N5_1 N2_1 1 42 . A521
N2_1 N4_1 1 42 . A241
N2_2 N3_2 1 40 . A232
N2_2 N4_2 1 42 . A242
N2_2 N5_2 1 42 . A252
N3_2 N4_2 1 40 . A342
N3_2 N1_2 1 48 . A312
N4_2 N3_2 1 40 . A432
N5_2 N1_2 1 40 . A512
N4_2 N1_2 1 40 . A412
N3_3 N1_3 1 48 . A313
N3_3 N2_3 1 40 . A323
N3_3 N4_3 1 40 . A343
N4_3 N2_3 1 42 . A423
N4_3 N1_3 1 40 . A413
N1_3 N5_3 1 40 . A153
N2_3 N5_3 1 42 . A253
N4_4 N1_4 1 40 . A414
N4_4 N2_4 1 42 . A424
N4_4 N3_4 1 40 . A434
N3_4 N1_4 1 48 . A314
N3_4 N2_4 1 40 . A324
N2_4 N3_4 1 40 . A234
N1_4 N3_4 1 48 . A134
```

```
N2_4 N5_4 1 42 . A254
N1_4 N5_4 1 40 . A154
N5_5 N1_5 1 40 . A515
N5_5 N2_5 1 42 . A525
N1_5 N3_5 1 48 . A135
N1_5 N4_5 1 40 . A145
N3_5 N2_5 1 40 . A325
N3_5 N4_5 1 40 . A345
N4_5 N2_5 1 42 . A425
N4_5 N3_5 1 40 . A435
N1_1 N6_1 100 30 . LC1
N6_1 N4_1 1 30 . D1
N2_2 N6_2 100 30 . LC2
N6_2 N1_2 1 30 . D2
N3_3 N6_3 100 30 . LC3
N6_3 N5_3 1 30 . D3
N4_4 N6_4 100 30 . LC4
N6_2 N5_4 1 30 . D4
N5_5 N6_5 100 30 . LC5
N6_5 N2_5 1 30 . D5
;
DATA COND0;
        INPUT _COLUMN_ $ _ROW1 $ _COEF1 ;
        CARDS;
A131 CON1 1
A141 CON5 1
A151 CON2 1
A341 CON6 1
A321 CON4 1
A231 CON4 1
A521 CON3 1
A241 CON7 1
A232 CON4 1
A242 CON7 1
A252 CON3 1
A342 CON6 1
A312 CON1 1
A432 CON6 1
A512 CON2 1
A412 CON5 1
A313 CON1 1
A323 CON4 1
A343 CON6 1
A423 CON7 1
A413 CON5 1
A153 CON2 1
A253 CON3 1
A414 CON5 1
A424 CON7 1
A434 CON6 1
A314 CON1 1
A324 CON4 1
A234 CON4 1
A134 CON1 1
```

```
A254 CON3 1
A154 CON2 1
A515 CON2 1
A525 CON3 1
A135 CON1 1
A145 CON5 1
A325 CON4 1
A345 CON6 1
A425 CON7 1
A435 CON6 1
_TYPE_ CON1 -1
_TYPE_ CON2 -1
_TYPE_ CON3 -1
_TYPE_ CON4 -1
_TYPE_ CON5 -1
_TYPE_ CON6 -1
_TYPE_ CON7 -1
_RHS_ CON1 48
_RHS_ CON2 40
_RHS_ CON3 42
_RHS_ CON4 40
_RHS_ CON5 40
_RHS_ CON6 40
_RHS_ CON7 42
;
PROC NETFLOW
        SCDATA
        NODEDATA=NODE0
        ARCDATA=ARC0
        CONDATA=COND0
        CONOUT=SOLUTION;
RUN;
PROC PRINT DATA=SOLUTION;
        SUM _FCOST_;
        SUM _DEMAND_;
RUN;
ENDSAS;
```

B.3 SAS/OR Case Study #1 Solution

```
            COMMUNICATIONS NETWORK - CASE 1: SOLUTION 1              1
             MULTI-COMMODITY TELECOMMUNICATION NETWORK
          DELAY OBJECTIVE FUNCTION INCLUDED AS CONSTRAINT
                                     00:52 Wednesday, January 25, 1995

                            _  _                           _
                  _         S  D        _   _  _  _        S
    _    _     _  C   _     U  E   _    F   R  A  T        T
    T    H     C  A   N     P  M   F    C   C  N  N        A
    A    E     O  P _ A     P  A   L    O   O  U  U        T
 O  I    A     S  A L M     L  N   O    S   S  M  M        U
 B  L    D     T  C O E     Y  D   W    T   T  B  B        S
 S  _    _     _  _ _ _     _  _   _    _   _  _  _        _

 1 N3_2 N1_2   1 48 0 A312  .  30 30   30   . 15  7 KEY_ARC     BASIC
 2 N5_2 N1_2   1 40 0 A512  .  30  0    0   1 16  9 LOWERBD NONBASIC
 3 N4_2 N1_2   1 40 0 A412  .  30  0    0   1 17  8 LOWERBD NONBASIC
 4 N6_2 N1_2   1 30 0 D2    .  30  0    0   0 18 27 LOWERBD NONBASIC
 5 N3_3 N1_3   1 48 0 A313 30   . 10   10   . 19 11 KEY_ARC     BASIC
 6 N4_3 N1_3   1 40 0 A413  .   .  0    0   2 20 14 LOWERBD NONBASIC
 7 N4_4 N1_4   1 40 0 A414 30   . 18   18   . 27 16 KEY_ARC     BASIC
 8 N3_4 N1_4   1 48 0 A314  .   .  0    0   0 28 19 LOWERBD NONBASIC
 9 N5_5 N1_5   1 40 0 A515 30   .  0    0   . 37 21 KEY_ARC     BASIC
10 N3_1 N2_1   1 40 0 A321  .   .  0    0   .  8  2 KEY_ARC     BASIC
11 N5_1 N2_1   1 42 0 A521  .   .  0    0   1  9  4 LOWERBD NONBASIC
12 N3_3 N2_3   1 40 0 A323 30   .  0    0   . 21 11 KEY_ARC     BASIC
13 N4_3 N2_3   1 42 0 A423  .   .  0    0   1 22 14 LOWERBD NONBASIC
14 N4_4 N2_4   1 42 0 A424 30   . 12   12   . 29 16 KEY_ARC     BASIC
15 N3_4 N2_4   1 40 0 A324  .   .  0    0   1 30 19 LOWERBD NONBASIC
16 N5_5 N2_5   1 42 0 A525 30  30 30   30   . 38 21 KEY_ARC     BASIC
17 N3_5 N2_5   1 40 0 A325  .  30  0    0   1 39 24 LOWERBD NONBASIC
18 N4_5 N2_5   1 42 0 A425  .  30  0    0   2 40 25 LOWERBD NONBASIC
19 N6_5 N2_5   1 30 0 D5    .  30  0    0   0 41 30 LOWERBD NONBASIC
20 N1_1 N3_1   1 48 0 A131 30   .  8    8   .  1  1 KEY_ARC     BASIC
21 N2_1 N3_1   1 40 0 A231  .   .  0    0   2  2  5 LOWERBD NONBASIC
22 N2_2 N3_2   1 40 0 A232 30   . 30   30   . 10  6 KEY_ARC     BASIC
23 N4_2 N3_2   1 40 0 A432  .   .  0    0   1 11  8 LOWERBD NONBASIC
24 N4_4 N3_4   1 40 0 A434 30   .  0    0   . 31 16 KEY_ARC     BASIC
25 N2_4 N3_4   1 40 0 A234  .   .  0    0   1 32 18 LOWERBD NONBASIC
26 N1_4 N3_4   1 48 0 A134  .   .  0    0 200 33 17 LOWERBD NONBASIC
27 N1_5 N3_5   1 48 0 A135  .   .  0    0   . 42 22 KEY_ARC     BASIC
28 N4_5 N3_5   1 40 0 A435  .   .  0    0   2 43 25 LOWERBD NONBASIC
29 N1_1 N4_1   1 40 0 A141 30  30 22   22   .  3  1 KEY_ARC     BASIC
30 N3_1 N4_1   1 40 0 A341  .  30  8    8   .  4  2 NONKEY ARC BASIC
31 N2_1 N4_1   1 42 0 A241  .  30  0    0   1  5  5 LOWERBD NONBASIC
32 N6_1 N4_1   1 30 0 D1    .  30  0    0   0  6 26 LOWERBD NONBASIC
33 N2_2 N4_2   1 42 0 A242 30   .  0    0   . 12  6 KEY_ARC     BASIC
34 N3_2 N4_2   1 40 0 A342  .   .  0    0   1 13  7 LOWERBD NONBASIC
35 N3_3 N4_3   1 40 0 A343 30   .  0    0   . 23 11 KEY_ARC     BASIC
36 N1_5 N4_5   1 40 0 A145  .   .  0    0   . 44 22 KEY_ARC     BASIC
37 N3_5 N4_5   1 40 0 A345  .   .  0    0   0 45 24 LOWERBD NONBASIC


                            _  _                           _
                  _         S  D        _   _  _  _        S
    _    _     _  C   _     U  E   _    F   R  A  T        T
    T    H     C  A   N     P  M   F    C   C  N  N        A
    A    E     O  P _ A     P  A   L    O   O  U  U        T
 O  I    A     S  A L M     L  N   O    S   S  M  M        U
 B  L    D     T  C O E     Y  D   W    T   T  B  B        S
 S  _    _     _  _ _ _     _  _   _    _   _  _  _        _
```

```
  38 N1_1 N5_1    1 40 0 A151 30    .  0    0   .  7  1 KEY_ARC    BASIC
  39 N2_2 N5_2    1 42 0 A252 30    .  0    0   . 14  6 KEY_ARC    BASIC
  40 N1_3 N5_3    1 40 0 A153  .   30 10   10   . 24 12 KEY_ARC    BASIC
  41 N2_3 N5_3    1 42 0 A253  .   30  0    0   1 25 13 LOWERBD NONBASIC
  42 N6_3 N5_3    1 30 0 D3    .   30 20   20   . 26 28 NONKEY ARC BASIC
  43 N2_4 N5_4    1 42 0 A254  .   30 12   12   . 34 18 KEY_ARC    BASIC
  44 N1_4 N5_4    1 40 0 A154  .   30 18   18   . 35 17 NONKEY ARC BASIC
  45 N6_2 N5_4    1 30 0 D4    .   30  0    0   . 36 27 KEY_ARC    BASIC
  46 N1_1 N6_1  100 30 0 LC1   30   .  0    0   . 46  1 KEY_ARC    BASIC
47 N2_2 N6_2 100 30 0 LC2   30   .  0    0   . 47  6 KEY_ARC    BASIC
  48 N3_3 N6_3  100 30 0 LC3   30   . 20 2000   . 48 11 KEY_ARC    BASIC
  49 N4_4 N6_4  100 30 0 LC4   30   .  0    0   . 49 16 KEY_ARC    BASIC
  50 N5_5 N6_5  100 30 0 LC5   30   .  0    0   . 50 21 KEY_ARC    BASIC
                                  ===      ====
                                  540      2228
```

B.4 Delay Constraint Development

The development of the network delay constraint from the objective function for network delay required the understanding of what each element of the original work by Kleinrock entailed. Th important thing to realize was the meaning and units associated with each variable or constant the formula. The formula for approximating delay within an individual link is presented below:

$$\frac{Delay}{Link} = \frac{1}{\gamma} \cdot \frac{\sum\sum_{p \varepsilon O q \varepsilon D} X_u^{p,q}}{\mu \cdot C_u}$$

where: γ = Total network throughput (Messages/Time Unit)
μ = Average call length (Time Unit)
$_u$ C= Link capacity (Messages/Time Unit)
$_u$ X= Number of messages (Messages)
Link = Number of links in network

The time units involved in the development of this formulation are not that significant as lon you are consistent throughout. I used seconds for this development since the originial LOSAs been written with a maximum delay of .1 seconds per message.

I made the following assumptions: γ = 150 Messages/Second
μ = 1 Second
$_{1-3}$ = 48 C
$_{1-4}$ = 40 C
$_{1-5}$ = 40 C
$_{2-3}$ = 40 C
$_{2-4}$ = 42 C
$_{3-4}$ = 40 C
$_{2-5}$ = 42 C

Additionally, I will create three points for consideration in the efficient frontier. These p depend upon the delay factor. The three delay factors are: Delay < .05, .10, and .20

Simplifying the previous equation into the constraint form yields:

$$\sum\sum_{p \varepsilon O q \varepsilon D} X_u^{p,q} = \frac{Delay \cdot \gamma \cdot \mu \cdot C_u}{Link}$$

Since there are only three different capacities in the list of links, I only solve for the th

$\gamma := 150$ $\mu := 1$ $Link := 7$ $C_{13} := 48$ $C_{24} := 42$ $C_{14} := 40$

Solution 1: (Delay < .01) $\mathrm{Delay} := .01$

Arc 1-3 (C=48): $\sum_{p\varepsilon O}\sum_{q\varepsilon D} X_u^{p,q} \le \frac{\gamma \cdot \mathrm{Delay} \cdot \mu \cdot C_{13}}{\mathrm{Link}}$ $\frac{\gamma \cdot \mathrm{Delay} \cdot \mu \cdot C_{13}}{\mathrm{Link}} = 10.286$

$$\sum_{p\varepsilon O}\sum_{q\varepsilon D} X_u^{p,q} = 10$$

Arc 1-4 (C=40): $\sum_{p\varepsilon O}\sum_{q\varepsilon D} X_u^{p,q} \le \frac{\gamma \cdot \mathrm{Delay} \cdot \mu \cdot C_{14}}{\mathrm{Link}}$ $\frac{\gamma \cdot \mathrm{Delay} \cdot \mu \cdot C_{14}}{\mathrm{Link}} = 8.571$

$$\sum_{p\varepsilon O}\sum_{q\varepsilon D} X_u^{p,q} = 8$$

Arc 2-4 (C=42): $\sum_{p\varepsilon O}\sum_{q\varepsilon D} X_u^{p,q} \le \frac{\gamma \cdot \mathrm{Delay} \cdot \mu \cdot C_{24}}{\mathrm{Link}}$ $\frac{\gamma \cdot \mathrm{Delay} \cdot \mu \cdot C_{24}}{\mathrm{Link}} = 9$

$$\sum_{p\varepsilon O}\sum_{q\varepsilon D} X_u^{p,q} = 9$$

The resulting network delay constraints are now more restrictive than the capacity const
and must be substituted for the capacity constraints.

Solution 2: (Delay < .05) $\mathrm{Delay} := .05$

Arc 1-3 (C=48): $\sum_{p\varepsilon O}\sum_{q\varepsilon D} X_u^{p,q} \le \frac{\gamma \cdot \mathrm{Delay} \cdot \mu \cdot C_{13}}{\mathrm{Link}}$ $\frac{\gamma \cdot \mathrm{Delay} \cdot \mu \cdot C_{13}}{\mathrm{Link}} = 51.429$

$$\sum_{p\varepsilon O}\sum_{q\varepsilon D} X_u^{p,q} = 51$$

Arc 1-4 (C=40): $\sum\sum_{p\varepsilon O q\varepsilon D} X_u^{p,q} \le \frac{\gamma\cdot \text{Delay}\cdot\mu\cdot C_{14}}{\text{Link}}$ $\frac{\gamma\cdot \text{Delay}\cdot\mu\cdot C_{14}}{\text{Link}} = 42.857$

$$\sum\sum_{p\varepsilon O q\varepsilon D} X_u^{p,q} \blacksquare 42$$

Arc 2-4 (C=42): $\sum\sum_{p\varepsilon O q\varepsilon D} X_u^{p,q} \le \frac{\gamma\cdot \text{Delay}\cdot\mu\cdot C_{24}}{\text{Link}}$ $\frac{\gamma\cdot \text{Delay}\cdot\mu\cdot C_{24}}{\text{Link}} = 45$

$$\sum\sum_{p\varepsilon O q\varepsilon D} X_u^{p,q} \blacksquare 45$$

This solution does not create a more restrictive delay constraint set and is therefore
The same solution set that was used in Step 2.2 is used.

Solution 3: (Delay < .10) Delay := .10

Arc 1-3 (C=48): $\sum\sum_{p\varepsilon O q\varepsilon D} X_u^{p,q} \le \frac{\gamma\cdot \text{Delay}\cdot\mu\cdot C_{13}}{\text{Link}}$ $\frac{\gamma\cdot \text{Delay}\cdot\mu\cdot C_{13}}{\text{Link}} = 102.857$

$$\sum\sum_{p\varepsilon O q\varepsilon D} X_u^{p,q} \blacksquare 51$$

Arc 1-4 (C=40): $\sum\sum_{p\varepsilon O q\varepsilon D} X_u^{p,q} \le \frac{\gamma\cdot \text{Delay}\cdot\mu\cdot C_{14}}{\text{Link}}$ $\frac{\gamma\cdot \text{Delay}\cdot\mu\cdot C_{14}}{\text{Link}} = 85.714$

$$\sum\sum_{p\varepsilon O q\varepsilon D} X_u^{p,q} \blacksquare 85$$

Arc 2-4 (C=42): $\sum_{p \varepsilon O} \sum_{q \varepsilon D} X_u^{p,q} \leq \frac{\gamma \cdot \text{Delay} \cdot \mu \cdot C_{24}}{\text{Link}}$ $\frac{\gamma \cdot \text{Delay} \cdot \mu \cdot C_{24}}{\text{Link}} = 90$

$$\sum_{p \varepsilon O} \sum_{q \varepsilon D} X_u^{p,q} \leq 90$$

This solution does not create a more restrictive delay constraint set and is therefore red
The same solution set that was used in Step 2.2 is used.

B.5 SAS/OR Case Study Network #1 (Delay < .01)

```
OPTIONS LINESIZE=72;
TITLE 'COMMUNICATIONS NETWORK - CASE 1: SOLUTION 2';
TITLE2 'MULTI-COMMODITY TELECOMMUNICATION NETWORK';
TITLE3 'DELAY OBJECTIVE FUNCTION INCLUDED AS CONSTRAINT';
DATA NODE0;
      INPUT _NODE_ $ _SUPDEM_;
      CARDS;
N1_1 30
N4_1 -30
N2_2 30
N1_2 -30
N3_3 30
N5_3 -30
N4_4 30
N5_4 -30
N5_5 30
N2_5 -30
;
DATA ARC0;
      INPUT _TAIL_ $ _HEAD_ $ _COST_ _CAPAC_ _LO_ _NAME_$6.;
      CARDS;
N1_1 N3_1 1 10 . A131
N1_1 N4_1 1  8 . A141
N1_1 N5_1 1  8 . A151
N3_1 N4_1 1  8 . A341
N3_1 N2_1 1  8 . A321
N2_1 N3_1 1  8 . A231
N5_1 N2_1 1  9 . A521
N2_1 N4_1 1  9 . A241
N2_2 N3_2 1  8 . A232
N2_2 N4_2 1  9 . A242
N2_2 N5_2 1  9 . A252
N3_2 N4_2 1  8 . A342
N3_2 N1_2 1 10 . A312
N4_2 N3_2 1  8 . A432
N5_2 N1_2 1  8 . A512
N4_2 N1_2 1  8 . A412
N3_3 N1_3 1 10 . A313
N3_3 N2_3 1  8 . A323
N3_3 N4_3 1  8 . A343
N4_3 N2_3 1  9 . A423
N4_3 N1_3 1  8 . A413
N1_3 N5_3 1  8 . A153
N2_3 N5_3 1  9 . A253
N4_4 N1_4 1  8 . A414
N4_4 N2_4 1  9 . A424
N4_4 N3_4 1  8 . A434
N3_4 N1_4 1 10 . A314
N3_4 N2_4 1  8 . A324
N2_4 N3_4 1  8 . A234
N1_4 N3_4 1 10 . A134
N2_4 N5_4 1  9 . A254
N1_4 N5_4 1  8 . A154
N5_5 N1_5 1  8 . A515
N5_5 N2_5 1  9 . A525
N1_5 N3_5 1 10 . A135
N1_5 N4_5 1  8 . A145
N3_5 N2_5 1  8 . A325
N3_5 N4_5 1  8 . A345
N4_5 N2_5 1  9 . A425
N4_5 N3_5 1  8 . A435
```

```
N1_1 N6_1 100 30 . LC1
N6_1 N4_1 1 30 . D1
N2_2 N6_2 100 30 . LC2
N6_2 N1_2 1 30 . D2
N3_3 N6_3 100 30 . LC3
N6_3 N5_3 1 30 . D3
N4_4 N6_4 100 30 . LC4
N6_4 N5_4 1 30 . D4
N5_5 N6_5 100 30 . LC5
N6_5 N2_5 1 30 . D5
;
DATA COND0;
      INPUT _COLUMN_ $ _ROW1 $ _COEF1 ;
      CARDS;
A131 CON1 1
A141 CON5 1
A151 CON2 1
A341 CON6 1
A321 CON4 1
A231 CON4 1
A521 CON3 1
A241 CON7 1
A232 CON4 1
A242 CON7 1
A252 CON3 1
A342 CON6 1
A312 CON1 1
A432 CON6 1
A512 CON2 1
A412 CON5 1
A313 CON1 1
A323 CON4 1
A343 CON6 1
A423 CON7 1
A413 CON5 1
A153 CON2 1
A253 CON3 1
A414 CON5 1
A424 CON7 1
A434 CON6 1
A314 CON1 1
A324 CON4 1
A234 CON4 1
A134 CON1 1
A254 CON3 1
A154 CON2 1
A515 CON2 1
A525 CON3 1
A135 CON1 1
A145 CON5 1
A325 CON4 1
A345 CON6 1
A425 CON7 1
A435 CON6 1
_TYPE_ CON1 -1
_TYPE_ CON2 -1
_TYPE_ CON3 -1
_TYPE_ CON4 -1
_TYPE_ CON5 -1
_TYPE_ CON6 -1
_TYPE_ CON7 -1
_RHS_ CON1 10
_RHS_ CON2  8
_RHS_ CON3  9
```

```
_RHS_ CON4   8
_RHS_ CON5   8
_RHS_ CON6   8
_RHS_ CON7   9
;
PROC NETFLOW
      SCDATA
      NODEDATA=NODE0
      ARCDATA=ARC0
      CONDATA=COND0
      CONOUT=SOLUTION;
RUN;
PROC PRINT DATA=SOLUTION;
      SUM _FCOST_;
      SUM _DEMAND_;
RUN;
ENDSAS;
```

B.6 SAS/OR Case Study #1 (Delay < .01) Solution

```
             COMMUNICATIONS NETWORK - CASE 1: SOLUTION 2               1
              MULTI-COMMODITY TELECOMMUNICATION NETWORK
           DELAY OBJECTIVE FUNCTION INCLUDED AS CONSTRAINT
                                       17:28 Thursday, February 2, 1995

                            _  _                            _
                            S  D                            S
                  _         U  E         _   _  _  _        T
                  C         U  E         F   R  A  T        T
    _    _     _      _            _
    T    H     C  A   N     P  M   F     C   C  N  N        A
    A    E     O  P _ A     P  A   L     O   O  U  U        T
 O  I    A     S  A L M     L  N   O     S   S  M  M        U
 B  L    D     T  C O E     Y  D   W     T   T  B  B        S
 S  _    _     _  _ _ _     _  _   _     _   _  _  _        _

 1 N3_2 N1_2   1 10 0 A312  .  30  2     2   . 15  7 KEY_ARC    BASIC
 2 N5_2 N1_2   1  8 0 A512  .  30  0     0   1 16  9 LOWERBD NONBASIC
 3 N4_2 N1_2   1  8 0 A412  .  30  0     0   1 17  8 LOWERBD NONBASIC
 4 N6_2 N1_2   1 30 0 D2    .  30 28    28   . 18 27 KEY_ARC    BASIC
 5 N3_3 N1_3   1 10 0 A313 30   .  0     0   . 19 11 KEY_ARC    BASIC
 6 N4_3 N1_3   1  8 0 A413  .   .  0     0   2 20 14 LOWERBD NONBASIC
 7 N4_4 N1_4   1  8 0 A414 30   .  0     0   1 27 16 LOWERBD NONBASIC
 8 N3_4 N1_4   1 10 0 A314  .   .  0     0   1 28 19 LOWERBD NONBASIC
 9 N5_5 N1_5   1  8 0 A515 30   .  0     0   . 37 21 KEY_ARC    BASIC
10 N3_1 N2_1   1  8 0 A321  .   .  0     0   1  8  2 LOWERBD NONBASIC
11 N5_1 N2_1   1  9 0 A521  .   .  0     0   .  9  4 KEY_ARC    BASIC
12 N3_3 N2_3   1  8 0 A323 30   .  0     0   1 21 11 LOWERBD NONBASIC
13 N4_3 N2_3   1  9 0 A423  .   .  0     0   2 22 14 LOWERBD NONBASIC
14 N4_4 N2_4   1  9 0 A424 30   .  0     0   1 29 16 LOWERBD NONBASIC
15 N3_4 N2_4   1  8 0 A324  .   .  0     0   2 30 19 LOWERBD NONBASIC
16 N5_5 N2_5   1  9 0 A525 30  30  9     9   . 38 21 NONKEY ARC BASIC
17 N3_5 N2_5   1  8 0 A325  .  30  0     0   . 39 24 KEY_ARC    BASIC
18 N4_5 N2_5   1  9 0 A425  .  30  0     0   . 40 25 KEY_ARC    BASIC
19 N6_5 N2_5   1 30 0 D5    .  30 21    21   . 41 30 KEY_ARC    BASIC
20 N1_1 N3_1   1 10 0 A131 30   .  8     8   .  1  1 KEY_ARC    BASIC
21 N2_1 N3_1   1  8 0 A231  .   .  0     0   1  2  5 LOWERBD NONBASIC
22 N2_2 N3_2   1  8 0 A232 30   .  2     2   . 10  6 NONKEY ARC BASIC
23 N4_2 N3_2   1  8 0 A432  .   .  0     0   1 11  8 LOWERBD NONBASIC
24 N4_4 N3_4   1  8 0 A434 30   .  0     0   . 31 16 KEY_ARC    BASIC
25 N2_4 N3_4   1  8 0 A234  .   .  0     0   . 32 18 KEY_ARC    BASIC
26 N1_4 N3_4   1 10 0 A134  .   .  0     0 199 33 17 LOWERBD NONBASIC
27 N1_5 N3_5   1 10 0 A135  .   .  0     0   1 42 22 LOWERBD NONBASIC
28 N4_5 N3_5   1  8 0 A435  .   .  0     0   1 43 25 LOWERBD NONBASIC
29 N1_1 N4_1   1  8 0 A141 30  30  8     8   .  3  1 NONKEY ARC BASIC
30 N3_1 N4_1   1  8 0 A341  .  30  8     8   0  4  2 UPPERBD NONBASIC
31 N2_1 N4_1   1  9 0 A241  .  30  0     0   .  5  5 KEY_ARC    BASIC
32 N6_1 N4_1   1 30 0 D1    .  30 14    14   .  6 26 KEY_ARC    BASIC
33 N2_2 N4_2   1  9 0 A242 30   .  0     0   . 12  6 KEY_ARC    BASIC
34 N3_2 N4_2   1  8 0 A342  .   .  0     0   1 13  7 LOWERBD NONBASIC
35 N3_3 N4_3   1  8 0 A343 30   .  0     0   . 23 11 KEY_ARC    BASIC
36 N1_5 N4_5   1  8 0 A145  .   .  0     0   2 44 22 LOWERBD NONBASIC

                            _  _                            _
                            S  D                            S
                  _         U  E         _   _  _  _        T
                  C         U  E         F   R  A  T        T
    _    _     _      _            _
    T    H     C  A   N     P  M   F     C   C  N  N        A
    A    E     O  P _ A     P  A   L     O   O  U  U        T
 O  I    A     S  A L M     L  N   O     S   S  M  M        U
 B  L    D     T  C O E     Y  D   W     T   T  B  B        S
 S  _    _     _  _ _ _     _  _   _     _   _  _  _        _

37 N3_5 N4_5   1  8 0 A345  .   .  0     0   1 45 24 LOWERBD NONBASIC
```

```
38 N1_1 N5_1   1  8 0 A151 30    .  0     0   2  7  1 LOWERBD NONBASIC
39 N2_2 N5_2   1  9 0 A252 30    .  0     0   . 14  6 KEY_ARC    BASIC
40 N1_3 N5_3   1  8 0 A153  .  30  0     0   0 24 12 LOWERBD NONBASIC
41 N2_3 N5_3   1  9 0 A253  .  30  0     0   . 25 13 KEY_ARC    BASIC
42 N6_3 N5_3   1 30 0 D3    .  30 30    30   . 26 28 KEY_ARC    BASIC
43 N2_4 N5_4   1  9 0 A254  .  30  0     0   0 34 18 LOWERBD NONBASIC
44 N1_4 N5_4   1  8 0 A154  .  30  0     0   . 35 17 KEY_ARC    BASIC
45 N6_4 N5_4   1 30 0 D4    .  30 30    30   . 36 29 KEY_ARC    BASIC
46 N1_1 N6_1 100 30 0 LC1  30    . 14  1400   . 46  1 KEY_ARC    BASIC
47 N2_2 N6_2 100 30 0 LC2  30    . 28  2800   . 47  6 KEY_ARC    BASIC
48 N3_3 N6_3 100 30 0 LC3  30    . 30  3000   . 48 11 KEY_ARC    BASIC
49 N4_4 N6_4 100 30 0 LC4  30    . 30  3000   . 49 16 KEY_ARC    BASIC
50 N5_5 N6_5 100 30 0 LC5  30    . 21  2100   . 50 21 KEY_ARC    BASIC
                              ===    =====
                              540    12460
```

Appendix C. Case Study Win-Win Example

C.1 SAS/OR Case Study Network #2

```
OPTIONS LINESIZE=72;
TITLE 'COMMUNICATIONS NETWORK - CASE 1: SOLUTION 2';
TITLE2 'MULTI-COMMODITY TELECOMMUNICATION NETWORK';
TITLE3 'DELAY OBJECTIVE INCLUDED AS CONSTRAINT: DELAY = .10';
DATA NODE0;
        INPUT _NODE_ $ _SUPDEM_;
        CARDS;
N1_1 30
N4_1 -30
N2_2 30
N1_2 -30
N3_3 30
N5_3 -30
N4_4 30
N5_4 -30
N5_5 30
N2_5 -30
;
DATA ARC0;
        INPUT _TAIL_ $ _HEAD_ $ _COST_ _CAPAC_ _LO_ _NAME_$6.;
        CARDS;
N1_1 N3_1 1 47 . A131
N1_1 N4_1 1 39 . A141
N1_1 N5_1 1 39 . A151
N3_1 N4_1 1 39 . A341
N3_1 N2_1 1 39 . A321
N2_1 N3_1 1 39 . A231
N5_1 N2_1 1 42 . A521
N2_1 N4_1 1 42 . A241
N2_2 N3_2 1 39 . A232
N2_2 N4_2 1 42 . A242
N2_2 N5_2 1 42 . A252
N3_2 N4_2 1 39 . A342
N3_2 N1_2 1 47 . A312
N4_2 N3_2 1 39 . A432
N5_2 N1_2 1 39 . A512
N4_2 N1_2 1 39 . A412
N3_3 N1_3 1 47 . A313
N3_3 N2_3 1 39 . A323
N3_3 N4_3 1 39 . A343
N4_3 N2_3 1 42 . A423
N4_3 N1_3 1 39 . A413
N1_3 N5_3 1 39 . A153
N2_3 N5_3 1 42 . A253
N4_4 N1_4 1 39 . A414
N4_4 N2_4 1 42 . A424
N4_4 N3_4 1 39 . A434
```

```
N3_4 N1_4 1 47 . A314
N3_4 N2_4 1 39 . A324
N2_4 N3_4 1 39 . A234
N1_4 N3_4 1 47 . A134
N2_4 N5_4 1 42 . A254
N1_4 N5_4 1 39 . A154
N5_5 N1_5 1 39 . A515
N5_5 N2_5 1 42 . A525
N1_5 N3_5 1 47 . A135
N1_5 N4_5 1 39 . A145
N3_5 N2_5 1 39 . A325
N3_5 N4_5 1 39 . A345
N4_5 N2_5 1 42 . A425
N4_5 N3_5 1 39 . A435
N1_1 N6_1 100 30 . LC1
N6_1 N4_1 1 30 . D1
N2_2 N6_2 100 30 . LC2
N6_2 N1_2 1 30 . D2
N3_3 N6_3 100 30 . LC3
N6_3 N5_3 1 30 . D3
N4_4 N6_4 100 30 . LC4
N6_2 N5_4 1 30 . D4
N5_5 N6_5 100 30 . LC5
N6_5 N2_5 1 30 . D5
;
DATA COND0;
        INPUT _COLUMN_ $ _ROW1 $ _COEF1 ;
        CARDS;
A131 CON1 1
A141 CON5 1
A151 CON2 1
A341 CON6 1
A321 CON4 1
A231 CON4 1
A521 CON3 1
A241 CON7 1
A232 CON4 1
A242 CON7 1
A252 CON3 1
A342 CON6 1
A312 CON1 1
A432 CON6 1
A512 CON2 1
A412 CON5 1
A313 CON1 1
A323 CON4 1
A343 CON6 1
A423 CON7 1
A413 CON5 1
A153 CON2 1
A253 CON3 1
A414 CON5 1
A424 CON7 1
A434 CON6 1
```

```
A314 CON1 1
A324 CON4 1
A234 CON4 1
A134 CON1 1
A254 CON3 1
A154 CON2 1
A515 CON2 1
A525 CON3 1
A135 CON1 1
A145 CON5 1
A325 CON4 1
A345 CON6 1
A425 CON7 1
A435 CON6 1
_TYPE_ CON1 -1
_TYPE_ CON2 -1
_TYPE_ CON3 -1
_TYPE_ CON4 -1
_TYPE_ CON5 -1
_TYPE_ CON6 -1
_TYPE_ CON7 -1
_RHS_ CON1 47
_RHS_ CON2 39
_RHS_ CON3 42
_RHS_ CON4 39
_RHS_ CON5 39
_RHS_ CON6 39
_RHS_ CON7 42
;
PROC NETFLOW
        SCDATA
        NODEDATA=NODE0
        ARCDATA=ARC0
        CONDATA=COND0
        CONOUT=SOLUTION;
RUN;
PROC PRINT DATA=SOLUTION;
        SUM _FCOST_;
        SUM _DEMAND_;
RUN;
ENDSAS;
```

C.2 SAS/OR Case Study Network #3

```
OPTIONS LINESIZE=72;
TITLE 'COMMUNICATIONS NETWORK - CASE 1: SOLUTION 3';
TITLE2 'MULTI-COMMODITY TELECOMMUNICATION NETWORK';
TITLE3 'DELAY OBJECTIVE INCLUDED AS CONSTRAINT: DELAY = .10';
DATA NODE0;
        INPUT _NODE_ $ _SUPDEM_;
        CARDS;
N1_1 30
N4_1 -30
N2_2 30
N1_2 -30
N3_3 30
N5_3 -30
N4_4 30
N5_4 -30
N5_5 30
N2_5 -30
;
DATA ARC0;
        INPUT _TAIL_ $ _HEAD_ $ _COST_ _CAPAC_ _LO_ _NAME_$6.;
        CARDS;
N1_1 N3_1 1 48 . A131
N1_1 N4_1 1 37 . A141
N1_1 N5_1 1 37 . A151
N3_1 N4_1 1 37 . A341
N3_1 N2_1 1 37 . A321
N2_1 N3_1 1 37 . A231
N5_1 N2_1 1 39 . A521
N2_1 N4_1 1 39 . A241
N2_2 N3_2 1 37 . A232
N2_2 N4_2 1 39 . A242
N2_2 N5_2 1 39 . A252
N3_2 N4_2 1 37 . A342
N3_2 N1_2 1 48 . A312
N4_2 N3_2 1 37 . A432
N5_2 N1_2 1 37 . A512
N4_2 N1_2 1 37 . A412
N3_3 N1_3 1 48 . A313
N3_3 N2_3 1 37 . A323
N3_3 N4_3 1 37 . A343
N4_3 N2_3 1 39 . A423
N4_3 N1_3 1 37 . A413
N1_3 N5_3 1 37 . A153
N2_3 N5_3 1 39 . A253
N4_4 N1_4 1 37 . A414
N4_4 N2_4 1 39 . A424
N4_4 N3_4 1 37 . A434
N3_4 N1_4 1 48 . A314
N3_4 N2_4 1 37 . A324
N2_4 N3_4 1 37 . A234
N1_4 N3_4 1 48 . A134
```

```
N2_4 N5_4 1 39 . A254
N1_4 N5_4 1 37 . A154
N5_5 N1_5 1 37 . A515
N5_5 N2_5 1 39 . A525
N1_5 N3_5 1 48 . A135
N1_5 N4_5 1 37 . A145
N3_5 N2_5 1 37 . A325
N3_5 N4_5 1 37 . A345
N4_5 N2_5 1 39 . A425
N4_5 N3_5 1 37 . A435
N1_1 N6_1 100 30 . LC1
N6_1 N4_1 1 30 . D1
N2_2 N6_2 100 30 . LC2
N6_2 N1_2 1 30 . D2
N3_3 N6_3 100 30 . LC3
N6_3 N5_3 1 30 . D3
N4_4 N6_4 100 30 . LC4
N6_2 N5_4 1 30 . D4
N5_5 N6_5 100 30 . LC5
N6_5 N2_5 1 30 . D5
;
DATA COND0;
        INPUT _COLUMN_ $ _ROW1 $ _COEF1 ;
        CARDS;
A131 CON1 1
A141 CON5 1
A151 CON2 1
A341 CON6 1
A321 CON4 1
A231 CON4 1
A521 CON3 1
A241 CON7 1
A232 CON4 1
A242 CON7 1
A252 CON3 1
A342 CON6 1
A312 CON1 1
A432 CON6 1
A512 CON2 1
A412 CON5 1
A313 CON1 1
A323 CON4 1
A343 CON6 1
A423 CON7 1
A413 CON5 1
A153 CON2 1
A253 CON3 1
A414 CON5 1
A424 CON7 1
A434 CON6 1
A314 CON1 1
A324 CON4 1
A234 CON4 1
A134 CON1 1
```

```
A254 CON3 1
A154 CON2 1
A515 CON2 1
A525 CON3 1
A135 CON1 1
A145 CON5 1
A325 CON4 1
A345 CON6 1
A425 CON7 1
A435 CON6 1
_TYPE_ CON1 -1
_TYPE_ CON2 -1
_TYPE_ CON3 -1
_TYPE_ CON4 -1
_TYPE_ CON5 -1
_TYPE_ CON6 -1
_TYPE_ CON7 -1
_RHS_ CON1 48
_RHS_ CON2 37
_RHS_ CON3 39
_RHS_ CON4 37
_RHS_ CON5 37
_RHS_ CON6 37
_RHS_ CON7 39
;
PROC NETFLOW
        SCDATA
        NODEDATA=NODE0
        ARCDATA=ARC0
        CONDATA=COND0
        CONOUT=SOLUTION;
RUN;
PROC PRINT DATA=SOLUTION;
        SUM _FCOST_;
        SUM _DEMAND_;
RUN;
ENDSAS;
```

C.3 SAS/OR Case Study Network #4

```
OPTIONS LINESIZE=72;
TITLE 'COMMUNICATIONS NETWORK - CASE 1: SOLUTION 4';
TITLE2 'MULTI-COMMODITY TELECOMMUNICATION NETWORK';
TITLE3 'DELAY OBJECTIVE INCLUDED AS CONSTRAINT: DELAY = .01';
DATA NODE0;
        INPUT _NODE_ $ _SUPDEM_;
        CARDS;
N1_1 30
N4_1 -30
N2_2 30
N1_2 -30
N3_3 30
N5_3 -30
N4_4 30
N5_4 -30
N5_5 30
N2_5 -30
;
DATA ARC0;
        INPUT _TAIL_ $ _HEAD_ $ _COST_ _CAPAC_ _LO_ _NAME_$6.;
        CARDS;
N1_1 N3_1 1 47 . A131
N1_1 N4_1 1 37 . A141
N1_1 N5_1 1 37 . A151
N3_1 N4_1 1 37 . A341
N3_1 N2_1 1 37 . A321
N2_1 N3_1 1 37 . A231
N5_1 N2_1 1 39 . A521
N2_1 N4_1 1 39 . A241
N2_2 N3_2 1 37 . A232
N2_2 N4_2 1 39 . A242
N2_2 N5_2 1 39 . A252
N3_2 N4_2 1 37 . A342
N3_2 N1_2 1 47 . A312
N4_2 N3_2 1 37 . A432
N5_2 N1_2 1 37 . A512
N4_2 N1_2 1 37 . A412
N3_3 N1_3 1 47 . A313
N3_3 N2_3 1 37 . A323
N3_3 N4_3 1 37 . A343
N4_3 N2_3 1 39 . A423
N4_3 N1_3 1 37 . A413
N1_3 N5_3 1 37 . A153
N2_3 N5_3 1 39 . A253
N4_4 N1_4 1 37 . A414
N4_4 N2_4 1 39 . A424
N4_4 N3_4 1 37 . A434
N3_4 N1_4 1 47 . A314
N3_4 N2_4 1 37 . A324
N2_4 N3_4 1 37 . A234
N1_4 N3_4 1 47 . A134
```

```
N2_4 N5_4 1 39 . A254
N1_4 N5_4 1 37 . A154
N5_5 N1_5 1 37 . A515
N5_5 N2_5 1 39 . A525
N1_5 N3_5 1 47 . A135
N1_5 N4_5 1 37 . A145
N3_5 N2_5 1 37 . A325
N3_5 N4_5 1 37 . A345
N4_5 N2_5 1 39 . A425
N4_5 N3_5 1 37 . A435
N1_1 N6_1 100 30 . LC1
N6_1 N4_1 1 30 . D1
N2_2 N6_2 100 30 . LC2
N6_2 N1_2 1 30 . D2
N3_3 N6_3 100 30 . LC3
N6_3 N5_3 1 30 . D3
N4_4 N6_4 100 30 . LC4
N6_2 N5_4 1 30 . D4
N5_5 N6_5 100 30 . LC5
N6_5 N2_5 1 30 . D5
;
DATA COND0;
        INPUT _COLUMN_ $ _ROW1 $ _COEF1 ;
        CARDS;
A131 CON1 1
A141 CON5 1
A151 CON2 1
A341 CON6 1
A321 CON4 1
A231 CON4 1
A521 CON3 1
A241 CON7 1
A232 CON4 1
A242 CON7 1
A252 CON3 1
A342 CON6 1
A312 CON1 1
A432 CON6 1
A512 CON2 1
A412 CON5 1
A313 CON1 1
A323 CON4 1
A343 CON6 1
A423 CON7 1
A413 CON5 1
A153 CON2 1
A253 CON3 1
A414 CON5 1
A424 CON7 1
A434 CON6 1
A314 CON1 1
A324 CON4 1
A234 CON4 1
A134 CON1 1
```

```
A254 CON3 1
A154 CON2 1
A515 CON2 1
A525 CON3 1
A135 CON1 1
A145 CON5 1
A325 CON4 1
A345 CON6 1
A425 CON7 1
A435 CON6 1
_TYPE_ CON1 -1
_TYPE_ CON2 -1
_TYPE_ CON3 -1
_TYPE_ CON4 -1
_TYPE_ CON5 -1
_TYPE_ CON6 -1
_TYPE_ CON7 -1
_RHS_ CON1 47
_RHS_ CON2 37
_RHS_ CON3 39
_RHS_ CON4 37
_RHS_ CON5 37
_RHS_ CON6 37
_RHS_ CON7 39
;
PROC NETFLOW
        SCDATA
        NODEDATA=NODE0
        ARCDATA=ARC0
        CONDATA=COND0
        CONOUT=SOLUTION;
RUN;
PROC PRINT DATA=SOLUTION;
        SUM _FCOST_;
        SUM _DEMAND_;
RUN;
ENDSAS;
```

C.4 SAS/OR Case Study #2 Solution

```
               COMMUNICATIONS NETWORK - CASE 1: SOLUTION 2                 1
                 MULTI-COMMODITY TELECOMMUNICATION NETWORK
            DELAY OBJECTIVE INCLUDED AS CONSTRAINT: DELAY = .10
                                          17:35 Thursday, February 2, 1995

                             _ _                       _
                   _         S D       _   _  _  _     S
     _    _     _  C   _     U E   _   F   R  A  T     T
     T    H     C  A   N     P M   F   C   C  N  N     A
     A    E     O  P _ A     P A   L   O   O  U  U     T
  O  I    A     S  A L M     L N   O   S   S  M  M     U
  B  L    D     T  C O E     Y D   W   T   T  B  B     S
  S  _    _     _  _ _ _     _ _   _   _   _  _  _     _

  1 N3_2 N1_2   1 47 0 A312  . 30 30  30   . 15  7 KEY_ARC    BASIC
  2 N5_2 N1_2   1 39 0 A512  . 30  0   0   1 16  9 LOWERBD NONBASIC
  3 N4_2 N1_2   1 39 0 A412  . 30  0   0   1 17  8 LOWERBD NONBASIC
  4 N6_2 N1_2   1 30 0 D2    . 30  0   0   0 18 27 LOWERBD NONBASIC
  5 N3_3 N1_3   1 47 0 A313 30  .  8   8   . 19 11 KEY_ARC    BASIC
  6 N4_3 N1_3   1 39 0 A413  .  .  0   0   2 20 14 LOWERBD NONBASIC
  7 N4_4 N1_4   1 39 0 A414 30  . 18  18   . 27 16 KEY_ARC    BASIC
  8 N3_4 N1_4   1 47 0 A314  .  .  0   0   0 28 19 LOWERBD NONBASIC
  9 N5_5 N1_5   1 39 0 A515 30  .  0   0   . 37 21 KEY_ARC    BASIC
 10 N3_1 N2_1   1 39 0 A321  .  .  0   0   .  8  2 KEY_ARC    BASIC
 11 N5_1 N2_1   1 42 0 A521  .  .  0   0   1  9  4 LOWERBD NONBASIC
 12 N3_3 N2_3   1 39 0 A323 30  .  0   0   . 21 11 KEY_ARC    BASIC
 13 N4_3 N2_3   1 42 0 A423  .  .  0   0   1 22 14 LOWERBD NONBASIC
 14 N4_4 N2_4   1 42 0 A424 30  . 12  12   . 29 16 KEY_ARC    BASIC
 15 N3_4 N2_4   1 39 0 A324  .  .  0   0   1 30 19 LOWERBD NONBASIC
 16 N5_5 N2_5   1 42 0 A525 30 30 30  30   . 38 21 KEY_ARC    BASIC
 17 N3_5 N2_5   1 39 0 A325  . 30  0   0   1 39 24 LOWERBD NONBASIC
 18 N4_5 N2_5   1 42 0 A425  . 30  0   0   2 40 25 LOWERBD NONBASIC
 19 N6_5 N2_5   1 30 0 D5    . 30  0   0   0 41 30 LOWERBD NONBASIC
 20 N1_1 N3_1   1 47 0 A131 30  .  9   9   .  1  1 KEY_ARC    BASIC
 21 N2_1 N3_1   1 39 0 A231  .  .  0   0   2  2  5 LOWERBD NONBASIC
 22 N2_2 N3_2   1 39 0 A232 30  . 30  30   . 10  6 KEY_ARC    BASIC
 23 N4_2 N3_2   1 39 0 A432  .  .  0   0   1 11  8 LOWERBD NONBASIC
 24 N4_4 N3_4   1 39 0 A434 30  .  0   0   . 31 16 KEY_ARC    BASIC
 25 N2_4 N3_4   1 39 0 A234  .  .  0   0   1 32 18 LOWERBD NONBASIC
 26 N1_4 N3_4   1 47 0 A134  .  .  0   0 200 33 17 LOWERBD NONBASIC
 27 N1_5 N3_5   1 47 0 A135  .  .  0   0   . 42 22 KEY_ARC    BASIC
 28 N4_5 N3_5   1 39 0 A435  .  .  0   0   2 43 25 LOWERBD NONBASIC
 29 N1_1 N4_1   1 39 0 A141 30 30 21  21   .  3  1 KEY_ARC    BASIC
 30 N3_1 N4_1   1 39 0 A341  . 30  9   9   .  4  2 NONKEY ARC BASIC
 31 N2_1 N4_1   1 42 0 A241  . 30  0   0   1  5  5 LOWERBD NONBASIC
 32 N6_1 N4_1   1 30 0 D1    . 30  0   0   0  6 26 LOWERBD NONBASIC
 33 N2_2 N4_2   1 42 0 A242 30  .  0   0   . 12  6 KEY_ARC    BASIC
 34 N3_2 N4_2   1 39 0 A342  .  .  0   0   1 13  7 LOWERBD NONBASIC
 35 N3_3 N4_3   1 39 0 A343 30  .  0   0   . 23 11 KEY_ARC    BASIC
 36 N1_5 N4_5   1 39 0 A145  .  .  0   0   . 44 22 KEY_ARC    BASIC

                             _ _                       _
                   _         S D       _   _  _  _     S
     _    _     _  C   _     U E   _   F   R  A  T     T
     T    H     C  A   N     P M   F   C   C  N  N     A
     A    E     O  P _ A     P A   L   O   O  U  U     T
  O  I    A     S  A L M     L N   O   S   S  M  M     U
  B  L    D     T  C O E     Y D   W   T   T  B  B     S
  S  _    _     _  _ _ _     _ _   _   _   _  _  _     _
```

```
37 N3_5 N4_5   1 39 0 A345  .    .  0    0   0 45 24 LOWERBD NONBASIC
38 N1_1 N5_1   1 39 0 A151 30    .  0    0   .  7  1 KEY_ARC    BASIC
39 N2_2 N5_2   1 42 0 A252 30    .  0    0   . 14  6 KEY_ARC    BASIC
40 N1_3 N5_3   1 39 0 A153  .   30  8    8   . 24 12 KEY_ARC    BASIC
41 N2_3 N5_3   1 42 0 A253  .   30  0    0   1 25 13 LOWERBD NONBASIC
42 N6_3 N5_3   1 30 0 D3    .   30 22   22   . 26 28 NONKEY ARC BASIC
43 N2_4 N5_4   1 42 0 A254  .   30 12   12   . 34 18 KEY_ARC    BASIC
44 N1_4 N5_4   1 39 0 A154  .   30 18   18   . 35 17 NONKEY ARC BASIC
45 N6_2 N5_4   1 30 0 D4    .   30  0    0   . 36 27 KEY_ARC    BASIC
46 N1_1 N6_1 100 30 0 LC1  30    .  0    0   . 46  1 KEY_ARC    BASIC
47 N2_2 N6_2 100 30 0 LC2  30    .  0    0   . 47  6 KEY_ARC    BASIC
48 N3_3 N6_3 100 30 0 LC3  30    . 22 2200   . 48 11 KEY_ARC    BASIC
49 N4_4 N6_4 100 30 0 LC4  30    .  0    0   . 49 16 KEY_ARC    BASIC
50 N5_5 N6_5 100 30 0 LC5  30    .  0    0   . 50 21 KEY_ARC    BASIC
                                   ===     ====
                                   540     2427
```

C.5 SAS/OR Case Study #3 Solution

```
            COMMUNICATIONS NETWORK - CASE 1: SOLUTION 3                1
               MULTI-COMMODITY TELECOMMUNICATION NETWORK
         DELAY OBJECTIVE INCLUDED AS CONSTRAINT: DELAY = .10
                                  17:36 Thursday, February 2, 1995

                            _   _                                _
                  _         S   D       _    _  _  _             S
    _    _     _  C   _     U   E  _    F    R  A  T             T
    T    H     C  A   N     P   M  F    C    C  N  N             A
    A    E     O  P _ A     P   A  L    O    O  U  U             T
 O  I    A     S  A L M     L   N  O    S    S  M  M             U
 B  L    D     T  C O E     Y   D  W    T    T  B  B             S
 S  _    _     _  _ _ _     _   _  _    _    _  _  _             _

 1 N3_2 N1_2   1 48 0 A312  .  30 30   30    . 15  7 KEY_ARC    BASIC
 2 N5_2 N1_2   1 37 0 A512  .  30  0    0    1 16  9 LOWERBD NONBASIC
 3 N4_2 N1_2   1 37 0 A412  .  30  0    0    1 17  8 LOWERBD NONBASIC
 4 N6_2 N1_2   1 30 0 D2    .  30  0    0    0 18 27 LOWERBD NONBASIC
 5 N3_3 N1_3   1 48 0 A313 30   .  4    4    . 19 11 KEY_ARC    BASIC
 6 N4_3 N1_3   1 37 0 A413  .   .  0    0    2 20 14 LOWERBD NONBASIC
 7 N4_4 N1_4   1 37 0 A414 30   . 21   21    . 27 16 KEY_ARC    BASIC
 8 N3_4 N1_4   1 48 0 A314  .   .  0    0    0 28 19 LOWERBD NONBASIC
 9 N5_5 N1_5   1 37 0 A515 30   .  0    0    . 37 21 KEY_ARC    BASIC
10 N3_1 N2_1   1 37 0 A321  .   .  0    0    .  8  2 KEY_ARC    BASIC
11 N5_1 N2_1   1 39 0 A521  .   .  0    0    1  9  4 LOWERBD NONBASIC
12 N3_3 N2_3   1 37 0 A323 30   .  0    0    . 21 11 KEY_ARC    BASIC
13 N4_3 N2_3   1 39 0 A423  .   .  0    0    1 22 14 LOWERBD NONBASIC
14 N4_4 N2_4   1 39 0 A424 30   .  9    9    . 29 16 KEY_ARC    BASIC
15 N3_4 N2_4   1 37 0 A324  .   .  0    0    1 30 19 LOWERBD NONBASIC
16 N5_5 N2_5   1 39 0 A525 30  30 30   30    . 38 21 KEY_ARC    BASIC
17 N3_5 N2_5   1 37 0 A325  .  30  0    0    1 39 24 LOWERBD NONBASIC
18 N4_5 N2_5   1 39 0 A425  .  30  0    0    2 40 25 LOWERBD NONBASIC
19 N6_5 N2_5   1 30 0 D5    .  30  0    0    0 41 30 LOWERBD NONBASIC
20 N1_1 N3_1   1 48 0 A131 30   . 14   14    .  1  1 KEY_ARC    BASIC
21 N2_1 N3_1   1 37 0 A231  .   .  0    0    2  2  5 LOWERBD NONBASIC
22 N2_2 N3_2   1 37 0 A232 30   . 30   30    . 10  6 KEY_ARC    BASIC
23 N4_2 N3_2   1 37 0 A432  .   .  0    0    1 11  8 LOWERBD NONBASIC
24 N4_4 N3_4   1 37 0 A434 30   .  0    0    . 31 16 KEY_ARC    BASIC
25 N2_4 N3_4   1 37 0 A234  .   .  0    0    1 32 18 LOWERBD NONBASIC
26 N1_4 N3_4   1 48 0 A134  .   .  0    0  200 33 17 LOWERBD NONBASIC
27 N1_5 N3_5   1 48 0 A135  .   .  0    0    . 42 22 KEY_ARC    BASIC
28 N4_5 N3_5   1 37 0 A435  .   .  0    0    2 43 25 LOWERBD NONBASIC
29 N1_1 N4_1   1 37 0 A141 30  30 16   16    .  3  1 KEY_ARC    BASIC
30 N3_1 N4_1   1 37 0 A341  .  30 14   14    .  4  2 NONKEY ARC BASIC
31 N2_1 N4_1   1 39 0 A241  .  30  0    0    1  5  5 LOWERBD NONBASIC
32 N6_1 N4_1   1 30 0 D1    .  30  0    0    0  6 26 LOWERBD NONBASIC
33 N2_2 N4_2   1 39 0 A242 30   .  0    0    . 12  6 KEY_ARC    BASIC
34 N3_2 N4_2   1 37 0 A342  .   .  0    0    1 13  7 LOWERBD NONBASIC
35 N3_3 N4_3   1 37 0 A343 30   .  0    0    . 23 11 KEY_ARC    BASIC
36 N1_5 N4_5   1 37 0 A145  .   .  0    0    . 44 22 KEY_ARC    BASIC

                            _   _                                _
                  _         S   D       _    _  _  _             S
    _    _     _  C   _     U   E  _    F    R  A  T             T
    T    H     C  A   N     P   M  F    C    C  N  N             A
    A    E     O  P _ A     P   A  L    O    O  U  U             T
 O  I    A     S  A L M     L   N  O    S    S  M  M             U
 B  L    D     T  C O E     Y   D  W    T    T  B  B             S
 S  _    _     _  _ _ _     _   _  _    _    _  _  _             _

37 N3_5 N4_5   1 37 0 A345  .   .  0    0    0 45 24 LOWERBD NONBASIC
```

```
38 N1_1 N5_1   1 37 0 A151 30   .  0    0   .  7  1 KEY_ARC    BASIC
39 N2_2 N5_2   1 39 0 A252 30   .  0    0   . 14  6 KEY_ARC    BASIC
40 N1_3 N5_3   1 37 0 A153  .  30  4    4   . 24 12 KEY_ARC    BASIC
41 N2_3 N5_3   1 39 0 A253  .  30  0    0   1 25 13 LOWERBD NONBASIC
42 N6_3 N5_3   1 30 0 D3    .  30 26   26   . 26 28 NONKEY ARC BASIC
43 N2_4 N5_4   1 39 0 A254  .  30  9    9   . 34 18 KEY_ARC    BASIC
44 N1_4 N5_4   1 37 0 A154  .  30 21   21   . 35 17 NONKEY ARC BASIC
45 N6_2 N5_4   1 30 0 D4    .  30  0    0   . 36 27 KEY_ARC    BASIC
46 N1_1 N6_1 100 30 0 LC1  30   .  0    0   . 46  1 KEY_ARC    BASIC
47 N2_2 N6_2 100 30 0 LC2  30   .  0    0   . 47  6 KEY_ARC    BASIC
48 N3_3 N6_3 100 30 0 LC3  30   . 26 2600   . 48 11 KEY_ARC    BASIC
49 N4_4 N6_4 100 30 0 LC4  30   .  0    0   . 49 16 KEY_ARC    BASIC
50 N5_5 N6_5 100 30 0 LC5  30   .  0    0   . 50 21 KEY_ARC    BASIC
                              ===    ====
                              540    2828
```

```
            COMMUNICATIONS NETWORK - CASE 1: SOLUTION 4              1
             MULTI-COMMODITY TELECOMMUNICATION NETWORK
        DELAY OBJECTIVE INCLUDED AS CONSTRAINT: DELAY = .01
                                      17:53 Thursday, February 2, 1995

                            _  _                            _
                  _         S  D        _   _  _  _         S
    _    _     _  C   _     U  E   _    F   R  A  T         T
    T    H     C  A   N     P  M   F    C   C  N  N         A
    A    E     O  P _ A     P  A   L    O   O  U  U         T
 O  I    A     S  A L M     L  N   O    S   S  M  M         U
 B  L    D     T  C O E     Y  D   W    T   T  B  B         S
 S  _    _     _  _ _ _     _  _   _    _   _  _  _         _

 1 N3_2 N1_2   1 47 0 A312  .  30 30    30   . 15  7 KEY_ARC    BASIC
 2 N5_2 N1_2   1 37 0 A512  .  30  0     0   1 16  9 LOWERBD NONBASIC
 3 N4_2 N1_2   1 37 0 A412  .  30  0     0   1 17  8 LOWERBD NONBASIC
 4 N6_2 N1_2   1 30 0 D2    .  30  0     0   0 18 27 LOWERBD NONBASIC
 5 N3_3 N1_3   1 47 0 A313 30   .  3     3   . 19 11 KEY_ARC    BASIC
 6 N4_3 N1_3   1 37 0 A413  .   .  0     0   2 20 14 LOWERBD NONBASIC
 7 N4_4 N1_4   1 37 0 A414 30   . 21    21   . 27 16 KEY_ARC    BASIC
 8 N3_4 N1_4   1 47 0 A314  .   .  0     0   0 28 19 LOWERBD NONBASIC
 9 N5_5 N1_5   1 37 0 A515 30   .  0     0   . 37 21 KEY_ARC    BASIC
10 N3_1 N2_1   1 37 0 A321  .   .  0     0   .  8  2 KEY_ARC    BASIC
11 N5_1 N2_1   1 39 0 A521  .   .  0     0   1  9  4 LOWERBD NONBASIC
12 N3_3 N2_3   1 37 0 A323 30   .  0     0   . 21 11 KEY_ARC    BASIC
13 N4_3 N2_3   1 39 0 A423  .   .  0     0   1 22 14 LOWERBD NONBASIC
14 N4_4 N2_4   1 39 0 A424 30   .  9     9   . 29 16 KEY_ARC    BASIC
15 N3_4 N2_4   1 37 0 A324  .   .  0     0   1 30 19 LOWERBD NONBASIC
16 N5_5 N2_5   1 39 0 A525 30  30 30    30   . 38 21 KEY_ARC    BASIC
17 N3_5 N2_5   1 37 0 A325  .  30  0     0   1 39 24 LOWERBD NONBASIC
18 N4_5 N2_5   1 39 0 A425  .  30  0     0   2 40 25 LOWERBD NONBASIC
19 N6_5 N2_5   1 30 0 D5    .  30  0     0   0 41 30 LOWERBD NONBASIC
20 N1_1 N3_1   1 47 0 A131 30   . 14    14   .  1  1 KEY_ARC    BASIC
21 N2_1 N3_1   1 37 0 A231  .   .  0     0   2  2  5 LOWERBD NONBASIC
22 N2_2 N3_2   1 37 0 A232 30   . 30    30   . 10  6 KEY_ARC    BASIC
23 N4_2 N3_2   1 37 0 A432  .   .  0     0   1 11  8 LOWERBD NONBASIC
24 N4_4 N3_4   1 37 0 A434 30   .  0     0   . 31 16 KEY_ARC    BASIC
25 N2_4 N3_4   1 37 0 A234  .   .  0     0   1 32 18 LOWERBD NONBASIC
26 N1_4 N3_4   1 47 0 A134  .   .  0     0 200 33 17 LOWERBD NONBASIC
27 N1_5 N3_5   1 47 0 A135  .   .  0     0   . 42 22 KEY_ARC    BASIC
28 N4_5 N3_5   1 37 0 A435  .   .  0     0   2 43 25 LOWERBD NONBASIC
29 N1_1 N4_1   1 37 0 A141 30  30 16    16   .  3  1 KEY_ARC    BASIC
30 N3_1 N4_1   1 37 0 A341  .  30 14    14   .  4  2 NONKEY ARC BASIC
31 N2_1 N4_1   1 39 0 A241  .  30  0     0   1  5  5 LOWERBD NONBASIC
32 N6_1 N4_1   1 30 0 D1    .  30  0     0   0  6 26 LOWERBD NONBASIC
33 N2_2 N4_2   1 39 0 A242 30   .  0     0   . 12  6 KEY_ARC    BASIC
34 N3_2 N4_2   1 37 0 A342  .   .  0     0   1 13  7 LOWERBD NONBASIC
35 N3_3 N4_3   1 37 0 A343 30   .  0     0   . 23 11 KEY_ARC    BASIC
36 N1_5 N4_5   1 37 0 A145  .   .  0     0   . 44 22 KEY_ARC    BASIC

                            _  _                            _
                  _         S  D        _   _  _  _         S
    _    _     _  C   _     U  E   _    F   R  A  T         T
    T    H     C  A   N     P  M   F    C   C  N  N         A
    A    E     O  P _ A     P  A   L    O   O  U  U         T
 O  I    A     S  A L M     L  N   O    S   S  M  M         U
 B  L    D     T  C O E     Y  D   W    T   T  B  B         S
 S  _    _     _  _ _ _     _  _   _    _   _  _  _         _

37 N3_5 N4_5   1 37 0 A345  .   .  0     0   0 45 24 LOWERBD NONBASIC
```

```
38 N1_1 N5_1   1 37 0 A151 30   .  0    0   .  7  1 KEY_ARC    BASIC
39 N2_2 N5_2   1 39 0 A252 30   .  0    0   . 14  6 KEY_ARC    BASIC
40 N1_3 N5_3   1 37 0 A153  .  30  3    3   . 24 12 KEY_ARC    BASIC
41 N2_3 N5_3   1 39 0 A253  .  30  0    0   1 25 13 LOWERBD NONBASIC
42 N6_3 N5_3   1 30 0 D3    .  30 27   27   . 26 28 NONKEY ARC BASIC
43 N2_4 N5_4   1 39 0 A254  .  30  9    9   . 34 18 KEY_ARC    BASIC
44 N1_4 N5_4   1 37 0 A154  .  30 21   21   . 35 17 NONKEY ARC BASIC
45 N6_2 N5_4   1 30 0 D4    .  30  0    0   . 36 27 KEY_ARC    BASIC
46 N1_1 N6_1 100 30 0 LC1  30   .  0    0   . 46  1 KEY_ARC    BASIC
47 N2_2 N6_2 100 30 0 LC2  30   .  0    0   . 47  6 KEY_ARC    BASIC
48 N3_3 N6_3 100 30 0 LC3  30   . 27 2700   . 48 11 KEY_ARC    BASIC
49 N4_4 N6_4 100 30 0 LC4  30   .  0    0   . 49 16 KEY_ARC    BASIC
50 N5_5 N6_5 100 30 0 LC5  30   .  0    0   . 50 21 KEY_ARC    BASIC
                              ===    ====
                              540    2927
```

Appendix D. Case Study: Extension

The development of the network delay constraints was previously accomplished in Appendix B.4. The development was for the general case and then showed the specific calculations for maintenance depots located at nodes 3 and 5. The next three sections will present the calculations for delay for the other three possible locations.

D.1 Delay Constraints @ Location 1 & 5

I will continue to use three points for consideration in the efficient frontier. These points be the same delay factors from previously: Delay < .01, .05, and .10

The constraint form is:

$$\sum_{p\varepsilon O}\sum_{q\varepsilon D} X_u^{p,q} = \frac{\mathrm{Delay}\cdot\gamma\cdot\mu\cdot C_u}{\mathrm{Link}}$$

Since there are only three different capacities in the list of links, I only solve for the three

$\gamma := 150$ $\mu := 1$ $\mathrm{Link} := 7$ $C_{13} := 47$ $C_{24} := 42$ $C_{14} := 39$

Solution 1: (Delay < .01) $\mathrm{Delay} := .01$

Arc 1-3 (C=48):

$$\sum_{p\varepsilon O}\sum_{q\varepsilon D} X_u^{p,q} \le \frac{\gamma\cdot\mathrm{Delay}\cdot\mu\cdot C_{13}}{\mathrm{Link}} \qquad \frac{\gamma\cdot\mathrm{Delay}\cdot\mu\cdot C_{13}}{\mathrm{Link}} = 10.071$$

$$\sum_{p\varepsilon O}\sum_{q\varepsilon D} X_u^{p,q} = 10$$

Arc 1-4 (C=40):

$$\sum_{p\varepsilon O}\sum_{q\varepsilon D} X_u^{p,q} \le \frac{\gamma\cdot\mathrm{Delay}\cdot\mu\cdot C_{14}}{\mathrm{Link}} \qquad \frac{\gamma\cdot\mathrm{Delay}\cdot\mu\cdot C_{14}}{\mathrm{Link}} = 8.357$$

$$\sum_{p\varepsilon O}\sum_{q\varepsilon D} X_u^{p,q} = 8$$

Arc 2-4 (C=42): $\sum\sum_{p\varepsilon O q\varepsilon D} X_u^{p,q} \le \frac{\gamma\cdot \text{Delay}\cdot\mu\cdot C_{24}}{\text{Link}}$ $\frac{\gamma\cdot \text{Delay}\cdot\mu\cdot C_{24}}{\text{Link}} = 9$

$$\sum\sum_{p\varepsilon O q\varepsilon D} X_u^{p,q} = 9$$

The resulting network delay constraints are now more restrictive than the capa constraints and must be substituted for the capacity constraints.

Solution 2: (Delay < .05)

$\text{Delay} := .05$

Arc 1-3 (C=48): $\sum\sum_{p\varepsilon O q\varepsilon D} X_u^{p,q} \le \frac{\gamma\cdot \text{Delay}\cdot\mu\cdot C_{13}}{\text{Link}}$ $\frac{\gamma\cdot \text{Delay}\cdot\mu\cdot C_{13}}{\text{Link}} = 50.357$

$$\sum\sum_{p\varepsilon O q\varepsilon D} X_u^{p,q} = 51$$

Arc 1-4 (C=40): $\sum\sum_{p\varepsilon O q\varepsilon D} X_u^{p,q} \le \frac{\gamma\cdot \text{Delay}\cdot\mu\cdot C_{14}}{\text{Link}}$ $\frac{\gamma\cdot \text{Delay}\cdot\mu\cdot C_{14}}{\text{Link}} = 41.786$

$$\sum\sum_{p\varepsilon O q\varepsilon D} X_u^{p,q} = 42$$

Arc 2-4 (C=42): $\sum\sum_{p\varepsilon O q\varepsilon D} X_u^{p,q} \le \frac{\gamma\cdot \text{Delay}\cdot\mu\cdot C_{24}}{\text{Link}}$ $\frac{\gamma\cdot \text{Delay}\cdot\mu\cdot C_{24}}{\text{Link}} = 45$

$$\sum\sum_{p\varepsilon O q\varepsilon D} X_u^{p,q} = 45$$

This solution does not create a more restrictive delay constraint set and is therefore redun The same solution set that was used in Step 2.2 is used.

Solution 3: (Delay < .10)

$\text{Delay} := .10$

Arc 1-3 (C=48):

$$\sum_{p\varepsilon O}\sum_{q\varepsilon D} X_u^{p,q} \le \frac{\gamma\cdot\text{Delay}\cdot\mu\cdot C_{13}}{\text{Link}} \qquad \frac{\gamma\cdot\text{Delay}\cdot\mu\cdot C_{13}}{\text{Link}} = 100.714$$

$$\sum_{p\varepsilon O}\sum_{q\varepsilon D} X_u^{p,q} = 51$$

Arc 1-4 (C=40):

$$\sum_{p\varepsilon O}\sum_{q\varepsilon D} X_u^{p,q} \le \frac{\gamma\cdot\text{Delay}\cdot\mu\cdot C_{14}}{\text{Link}} \qquad \frac{\gamma\cdot\text{Delay}\cdot\mu\cdot C_{14}}{\text{Link}} = 83.571$$

$$\sum_{p\varepsilon O}\sum_{q\varepsilon D} X_u^{p,q} = 85$$

Arc 2-4 (C=42):

$$\sum_{p\varepsilon O}\sum_{q\varepsilon D} X_u^{p,q} \le \frac{\gamma\cdot\text{Delay}\cdot\mu\cdot C_{24}}{\text{Link}} \qquad \frac{\gamma\cdot\text{Delay}\cdot\mu\cdot C_{24}}{\text{Link}} = 90$$

$$\sum_{p\varepsilon O}\sum_{q\varepsilon D} X_u^{p,q} = 90$$

This solution does not create a more restrictive delay constraint set and is therefor redundant. The same solution set that was used in Step 2.2 is used.

D.2 Delay Constraints @ Location 3 & 2

I will continue to use three points for consideration in the efficient frontier. These points be the same delay factors from previously: Delay < .01, .05, and .10

The constraint form is:

$$\sum\sum_{p\varepsilon O q\varepsilon D} X_u^{p,q} = \frac{Delay \cdot \gamma \cdot \mu \cdot C_u}{Link}$$

Since there are only three different capacities in the list of links, I only solve for the three

$\gamma := 150$ $\mu := 1$ $Link := 7$ $C_{13} := 48$ $C_{24} := 39$ $C_{14} := 37$

Solution 1: (Delay < .01) $Delay := .01$

Arc 1-3 (C=48):

$$\sum\sum_{p\varepsilon O q\varepsilon D} X_u^{p,q} \le \frac{\gamma \cdot Delay \cdot \mu \cdot C_{13}}{Link} \qquad \frac{\gamma \cdot Delay \cdot \mu \cdot C_{13}}{Link} = 10.286$$

$$\sum\sum_{p\varepsilon O q\varepsilon D} X_u^{p,q} = 10$$

Arc 1-4 (C=40):

$$\sum\sum_{p\varepsilon O q\varepsilon D} X_u^{p,q} \le \frac{\gamma \cdot Delay \cdot \mu \cdot C_{14}}{Link} \qquad \frac{\gamma \cdot Delay \cdot \mu \cdot C_{14}}{Link} = 7.929$$

$$\sum\sum_{p\varepsilon O q\varepsilon D} X_u^{p,q} = 7$$

Arc 2-4 (C=42):

$$\sum\sum_{p\varepsilon O q\varepsilon D} X_u^{p,q} \le \frac{\gamma \cdot Delay \cdot \mu \cdot C_{24}}{Link} \qquad \frac{\gamma \cdot Delay \cdot \mu \cdot C_{24}}{Link} = 8.357$$

$$\sum\sum_{p\varepsilon O q\varepsilon D} X_u^{p,q} = 8$$

The resulting network delay constraints are now more restrictive than the capacity constraints and must be substituted for the capacity constraints.

Solution 2: (Delay < .05) Delay := .05

Arc 1-3 (C=48):

$$\sum\sum_{p\varepsilon O q\varepsilon D} X_u^{p,q} \le \frac{\gamma\cdot \mathrm{Delay}\cdot \mu\cdot C_{13}}{\mathrm{Link}} \qquad \frac{\gamma\cdot \mathrm{Delay}\cdot \mu\cdot C_{13}}{\mathrm{Link}} = 51.429$$

$$\sum\sum_{p\varepsilon O q\varepsilon D} X_u^{p,q} = 51$$

Arc 1-4 (C=40):

$$\sum\sum_{p\varepsilon O q\varepsilon D} X_u^{p,q} \le \frac{\gamma\cdot \mathrm{Delay}\cdot \mu\cdot C_{14}}{\mathrm{Link}} \qquad \frac{\gamma\cdot \mathrm{Delay}\cdot \mu\cdot C_{14}}{\mathrm{Link}} = 39.643$$

$$\sum\sum_{p\varepsilon O q\varepsilon D} X_u^{p,q} = 39$$

Arc 2-4 (C=42):

$$\sum\sum_{p\varepsilon O q\varepsilon D} X_u^{p,q} \le \frac{\gamma\cdot \mathrm{Delay}\cdot \mu\cdot C_{24}}{\mathrm{Link}} \qquad \frac{\gamma\cdot \mathrm{Delay}\cdot \mu\cdot C_{24}}{\mathrm{Link}} = 41.786$$

$$\sum\sum_{p\varepsilon O q\varepsilon D} X_u^{p,q} = 41$$

This solution does not create a more restrictive delay constraint set and is therefore redun
The same solution set that was used in Step 2.2 is used.

Solution 3: (Delay < .10) $\text{Delay} := .10$

Arc 1-3 (C=48): $$\sum_{p\varepsilon O}\sum_{q\varepsilon D} X_u^{p,q} \le \frac{\gamma \cdot \text{Delay} \cdot \mu \cdot C_{13}}{\text{Link}} \qquad \frac{\gamma \cdot \text{Delay} \cdot \mu \cdot C_{13}}{\text{Link}} = 102.857$$

$$\sum_{p\varepsilon O}\sum_{q\varepsilon D} X_u^{p,q} = 102$$

Arc 1-4 (C=40): $$\sum_{p\varepsilon O}\sum_{q\varepsilon D} X_u^{p,q} \le \frac{\gamma \cdot \text{Delay} \cdot \mu \cdot C_{14}}{\text{Link}} \qquad \frac{\gamma \cdot \text{Delay} \cdot \mu \cdot C_{14}}{\text{Link}} = 79.286$$

$$\sum_{p\varepsilon O}\sum_{q\varepsilon D} X_u^{p,q} = 79$$

Arc 2-4 (C=42): $$\sum_{p\varepsilon O}\sum_{q\varepsilon D} X_u^{p,q} \le \frac{\gamma \cdot \text{Delay} \cdot \mu \cdot C_{24}}{\text{Link}} \qquad \frac{\gamma \cdot \text{Delay} \cdot \mu \cdot C_{24}}{\text{Link}} = 83.571$$

$$\sum_{p\varepsilon O}\sum_{q\varepsilon D} X_u^{p,q} = 83$$

This solution does not create a more restrictive delay constraint set and is therefor redundant. The same solution set that was used in Step 2.2 is used.

D.3 Delay Constraints @ Location 1 & 2

I will continue to use three points for consideration in the efficient frontier. These points be the same delay factors from previously: Delay < .01, .05, and .10

The constraint form is:

$$\sum_{p\varepsilon O q\varepsilon D}\sum X_u^{p,q} = \frac{\text{Delay}\cdot\gamma\cdot\mu\cdot C_u}{\text{Link}}$$

Since there are only three different capacities in the list of links, I only solve for the three

$\gamma := 150$ $\mu := 1$ $\text{Link} := 7$ $C_{13} := 47$ $C_{24} := 39$ $C_{14} := 37$

Solution 1: (Delay < .01) $\text{Delay} := .01$

Arc 1-3 (C=48):

$$\sum_{p\varepsilon O q\varepsilon D}\sum X_u^{p,q} \le \frac{\gamma\cdot\text{Delay}\cdot\mu\cdot C_{13}}{\text{Link}} \qquad \frac{\gamma\cdot\text{Delay}\cdot\mu\cdot C_{13}}{\text{Link}} = 10.071$$

$$\sum_{p\varepsilon O q\varepsilon D}\sum X_u^{p,q} = 10$$

Arc 1-4 (C=40):

$$\sum_{p\varepsilon O q\varepsilon D}\sum X_u^{p,q} \le \frac{\gamma\cdot\text{Delay}\cdot\mu\cdot C_{14}}{\text{Link}} \qquad \frac{\gamma\cdot\text{Delay}\cdot\mu\cdot C_{14}}{\text{Link}} = 7.929$$

$$\sum_{p\varepsilon O q\varepsilon D}\sum X_u^{p,q} = 7$$

Arc 2-4 (C=42):

$$\sum_{p\varepsilon O q\varepsilon D}\sum X_u^{p,q} \le \frac{\gamma\cdot\text{Delay}\cdot\mu\cdot C_{24}}{\text{Link}} \qquad \frac{\gamma\cdot\text{Delay}\cdot\mu\cdot C_{24}}{\text{Link}} = 8.357$$

$$\sum_{p\varepsilon O q\varepsilon D}\sum X_u^{p,q} = 8$$

The resulting network delay constraints are now more restrictive than the capacity constraints and must be substituted for the capacity constraints.

Solution 2: (Delay < .05) Delay := .05

Arc 1-3 (C=48):

$$\sum_{p\varepsilon O}\sum_{q\varepsilon D} X_u^{p,q} \le \frac{\gamma \cdot \mathrm{Delay} \cdot \mu \cdot C_{13}}{\mathrm{Link}} \qquad \frac{\gamma \cdot \mathrm{Delay} \cdot \mu \cdot C_{13}}{\mathrm{Link}} = 50.357$$

$$\sum_{p\varepsilon O}\sum_{q\varepsilon D} X_u^{p,q} = 50$$

Arc 1-4 (C=40):

$$\sum_{p\varepsilon O}\sum_{q\varepsilon D} X_u^{p,q} \le \frac{\gamma \cdot \mathrm{Delay} \cdot \mu \cdot C_{14}}{\mathrm{Link}} \qquad \frac{\gamma \cdot \mathrm{Delay} \cdot \mu \cdot C_{14}}{\mathrm{Link}} = 39.643$$

$$\sum_{p\varepsilon O}\sum_{q\varepsilon D} X_u^{p,q} = 39$$

Arc 2-4 (C=42):

$$\sum_{p\varepsilon O}\sum_{q\varepsilon D} X_u^{p,q} \le \frac{\gamma \cdot \mathrm{Delay} \cdot \mu \cdot C_{24}}{\mathrm{Link}} \qquad \frac{\gamma \cdot \mathrm{Delay} \cdot \mu \cdot C_{24}}{\mathrm{Link}} = 41.786$$

$$\sum_{p\varepsilon O}\sum_{q\varepsilon D} X_u^{p,q} = 41$$

This solution does not create a more restrictive delay constraint set and is therefore redun
The same solution set that was used in Step 2.2 is used.

Solution 3: (Delay < .10) $\text{Delay} := .10$

Arc 1-3 (C=48):

$$\sum\sum_{p\varepsilon O q\varepsilon D} X_u^{p,q} \le \frac{\gamma\cdot\text{Delay}\cdot\mu\cdot C_{13}}{\text{Link}} \qquad \frac{\gamma\cdot\text{Delay}\cdot\mu\cdot C_{13}}{\text{Link}} = 100.714$$

$$\sum\sum_{p\varepsilon O q\varepsilon D} X_u^{p,q} = 100$$

Arc 1-4 (C=40):

$$\sum\sum_{p\varepsilon O q\varepsilon D} X_u^{p,q} \le \frac{\gamma\cdot\text{Delay}\cdot\mu\cdot C_{14}}{\text{Link}} \qquad \frac{\gamma\cdot\text{Delay}\cdot\mu\cdot C_{14}}{\text{Link}} = 79.286$$

$$\sum\sum_{p\varepsilon O q\varepsilon D} X_u^{p,q} = 79$$

Arc 2-4 (C=42):

$$\sum\sum_{p\varepsilon O q\varepsilon D} X_u^{p,q} \le \frac{\gamma\cdot\text{Delay}\cdot\mu\cdot C_{24}}{\text{Link}} \qquad \frac{\gamma\cdot\text{Delay}\cdot\mu\cdot C_{24}}{\text{Link}} = 83.571$$

$$\sum\sum_{p\varepsilon O q\varepsilon D} X_u^{p,q} = 83$$

This solution does not create a more restrictive delay constraint set and is therefor redundant. The same solution set that was used in Step 2.2 is used.

D.4 SAS/OR Network Locations Code (Delay < .01)

The SAS/OR code is the same for solutions associated with locations (1 & 2) and (3 & 2). This code is presented next. Since the delay values did not change for location (1 & 5), that code is not presented.

```
OPTIONS LINESIZE=72;
TITLE 'COMMUNICATIONS NETWORK - CASE 1: SOLUTION 2';
TITLE2 'MULTI-COMMODITY TELECOMMUNICATION NETWORK';
TITLE3 'DELAY OBJECTIVE FUNCTION INCLUDED AS CONSTRAINT';
DATA NODE0;
      INPUT _NODE_ $ _SUPDEM_;
      CARDS;
N1_1 30
N4_1 -30
N2_2 30
N1_2 -30
N3_3 30
N5_3 -30
N4_4 30
N5_4 -30
N5_5 30
N2_5 -30
;
DATA ARC0;
      INPUT _TAIL_ $ _HEAD_ $ _COST_ _CAPAC_ _LO_ _NAME_$6.;
      CARDS;
N1_1 N3_1 1 10 . A131
N1_1 N4_1 1  7 . A141
N1_1 N5_1 1  7 . A151
N3_1 N4_1 1  7 . A341
N3_1 N2_1 1  7 . A321
N2_1 N3_1 1  7 . A231
N5_1 N2_1 1  8 . A521
N2_1 N4_1 1  8 . A241
N2_2 N3_2 1  7 . A232
N2_2 N4_2 1  8 . A242
N2_2 N5_2 1  8 . A252
N3_2 N4_2 1  7 . A342
N3_2 N1_2 1 10 . A312
N4_2 N3_2 1  7 . A432
N5_2 N1_2 1  7 . A512
N4_2 N1_2 1  7 . A412
N3_3 N1_3 1 10 . A313
N3_3 N2_3 1  7 . A323
N3_3 N4_3 1  7 . A343
N4_3 N2_3 1  8 . A423
N4_3 N1_3 1  7 . A413
N1_3 N5_3 1  7 . A153
N2_3 N5_3 1  8 . A253
N4_4 N1_4 1  7 . A414
```

```
N4_4 N2_4 1  8 . A424
N4_4 N3_4 1  7 . A434
N3_4 N1_4 1 10 . A314
N3_4 N2_4 1  7 . A324
N2_4 N3_4 1  7 . A234
N1_4 N3_4 1 10 . A134
N2_4 N5_4 1  8 . A254
N1_4 N5_4 1  7 . A154
N5_5 N1_5 1  7 . A515
N5_5 N2_5 1  8 . A525
N1_5 N3_5 1 10 . A135
N1_5 N4_5 1  7 . A145
N3_5 N2_5 1  7 . A325
N3_5 N4_5 1  7 . A345
N4_5 N2_5 1  8 . A425
N4_5 N3_5 1  7 . A435
N1_1 N6_1 100 30 . LC1
N6_1 N4_1 1 30 . D1
N2_2 N6_2 100 30 . LC2
N6_2 N1_2 1 30 . D2
N3_3 N6_3 100 30 . LC3
N6_3 N5_3 1 30 . D3
N4_4 N6_4 100 30 . LC4
N6_4 N5_4 1 30 . D4
N5_5 N6_5 100 30 . LC5
N6_5 N2_5 1 30 . D5
;
DATA COND0;
      INPUT _COLUMN_ $ _ROW1 $ _COEF1 ;
      CARDS;
A131 CON1 1
A141 CON5 1
A151 CON2 1
A341 CON6 1
A321 CON4 1
A231 CON4 1
A521 CON3 1
A241 CON7 1
A232 CON4 1
A242 CON7 1
A252 CON3 1
A342 CON6 1
A312 CON1 1
A432 CON6 1
A512 CON2 1
A412 CON5 1
A313 CON1 1
A323 CON4 1
A343 CON6 1
A423 CON7 1
A413 CON5 1
A153 CON2 1
```

```
A253 CON3 1
A414 CON5 1
A424 CON7 1
A434 CON6 1
A314 CON1 1
A324 CON4 1
A234 CON4 1
A134 CON1 1
A254 CON3 1
A154 CON2 1
A515 CON2 1
A525 CON3 1
A135 CON1 1
A145 CON5 1
A325 CON4 1
A345 CON6 1
A425 CON7 1
A435 CON6 1
_TYPE_ CON1 -1
_TYPE_ CON2 -1
_TYPE_ CON3 -1
_TYPE_ CON4 -1
_TYPE_ CON5 -1
_TYPE_ CON6 -1
_TYPE_ CON7 -1
_RHS_ CON1 10
_RHS_ CON2  7
_RHS_ CON3  8
_RHS_ CON4  7
_RHS_ CON5  7
_RHS_ CON6  7
_RHS_ CON7  8
;
PROC NETFLOW
      SCDATA
      NODEDATA=NODE0
      ARCDATA=ARC0
      CONDATA=COND0
      CONOUT=SOLUTION;
RUN;
PROC PRINT DATA=SOLUTION;
      SUM _FCOST_;
      SUM _DEMAND_;
RUN;
ENDSAS;
```

D.5 SAS/OR Network Locations Code (Delay < .01) Solution

```
              COMMUNICATIONS NETWORK - CASE 1: SOLUTION 2             1
              MULTI-COMMODITY TELECOMMUNICATION NETWORK
           DELAY OBJECTIVE FUNCTION INCLUDED AS CONSTRAINT
                                        13:17 Sunday, February 19, 1995

                            _  _                            _
                            S  D                            S
                  _                      _   _  _  _
                  C         U  E         F   R  A  T        T
    _    _     _     _            _
    T    H     C  A   N     P  M   F     C   C  N  N        A
    A    E     O  P _ A     P  A   L     O   O  U  U        T
 O  I    A     S  A L M     L  N   O     S   S  M  M        U
 B  L    D     T  C O E     Y  D   W     T   T  B  B        S
 S
    _    _     _  _ _ _     _  _   _     _   _  _  _        _

 1 N3_2 N1_2   1 10 0 A312  .  30   3    3   . 15  7 KEY_ARC    BASIC
 2 N5_2 N1_2   1  7 0 A512  .  30   0    0   1 16  9 LOWERBD NONBASIC
 3 N4_2 N1_2   1  7 0 A412  .  30   0    0   1 17  8 LOWERBD NONBASIC
 4 N6_2 N1_2   1 30 0 D2    .  30  27   27   . 18 27 KEY_ARC    BASIC
 5 N3_3 N1_3   1 10 0 A313 30   .   0    0   . 19 11 KEY_ARC    BASIC
 6 N4_3 N1_3   1  7 0 A413  .   .   0    0   2 20 14 LOWERBD NONBASIC
 7 N4_4 N1_4   1  7 0 A414 30   .   0    0   1 27 16 LOWERBD NONBASIC
 8 N3_4 N1_4   1 10 0 A314  .   .   0    0   1 28 19 LOWERBD NONBASIC
 9 N5_5 N1_5   1  7 0 A515 30   .   0    0   . 37 21 KEY_ARC    BASIC
10 N3_1 N2_1   1  7 0 A321  .   .   0    0   1  8  2 LOWERBD NONBASIC
11 N5_1 N2_1   1  8 0 A521  .   .   0    0   .  9  4 KEY_ARC    BASIC
12 N3_3 N2_3   1  7 0 A323 30   .   0    0   1 21 11 LOWERBD NONBASIC
13 N4_3 N2_3   1  8 0 A423  .   .   0    0   2 22 14 LOWERBD NONBASIC
14 N4_4 N2_4   1  8 0 A424 30   .   0    0   1 29 16 LOWERBD NONBASIC
15 N3_4 N2_4   1  7 0 A324  .   .   0    0   2 30 19 LOWERBD NONBASIC
16 N5_5 N2_5   1  8 0 A525 30  30   8    8   . 38 21 NONKEY ARC BASIC
17 N3_5 N2_5   1  7 0 A325  .  30   0    0   . 39 24 KEY_ARC    BASIC
18 N4_5 N2_5   1  8 0 A425  .  30   0    0   . 40 25 KEY_ARC    BASIC
19 N6_5 N2_5   1 30 0 D5    .  30  22   22   . 41 30 KEY_ARC    BASIC
20 N1_1 N3_1   1 10 0 A131 30   .   7    7   .  1  1 KEY_ARC    BASIC
21 N2_1 N3_1   1  7 0 A231  .   .   0    0   1  2  5 LOWERBD NONBASIC
22 N2_2 N3_2   1  7 0 A232 30   .   3    3   . 10  6 NONKEY ARC BASIC
23 N4_2 N3_2   1  7 0 A432  .   .   0    0   1 11  8 LOWERBD NONBASIC
24 N4_4 N3_4   1  7 0 A434 30   .   0    0   . 31 16 KEY_ARC    BASIC
25 N2_4 N3_4   1  7 0 A234  .   .   0    0   . 32 18 KEY_ARC    BASIC
26 N1_4 N3_4   1 10 0 A134  .   .   0    0 199 33 17 LOWERBD NONBASIC
27 N1_5 N3_5   1 10 0 A135  .   .   0    0   1 42 22 LOWERBD NONBASIC
28 N4_5 N3_5   1  7 0 A435  .   .   0    0   1 43 25 LOWERBD NONBASIC
29 N1_1 N4_1   1  7 0 A141 30  30   7    7   .  3  1 NONKEY ARC BASIC
30 N3_1 N4_1   1  7 0 A341  .  30   7    7   0  4  2 UPPERBD NONBASIC
31 N2_1 N4_1   1  8 0 A241  .  30   0    0   .  5  5 KEY_ARC    BASIC
32 N6_1 N4_1   1 30 0 D1    .  30  16   16   .  6 26 KEY_ARC    BASIC
33 N2_2 N4_2   1  8 0 A242 30   .   0    0   . 12  6 KEY_ARC    BASIC
34 N3_2 N4_2   1  7 0 A342  .   .   0    0   1 13  7 LOWERBD NONBASIC
35 N3_3 N4_3   1  7 0 A343 30   .   0    0   . 23 11 KEY_ARC    BASIC
36 N1_5 N4_5   1  7 0 A145  .   .   0    0   2 44 22 LOWERBD NONBASIC
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              COMMUNICATIONS NETWORK - CASE 1: SOLUTION 2             2
              MULTI-COMMODITY TELECOMMUNICATION NETWORK
           DELAY OBJECTIVE FUNCTION INCLUDED AS CONSTRAINT
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    A    E     O  P _ A     P  A   L     O   O  U  U        T
 O  I    A     S  A L M     L  N   O     S   S  M  M        U
 B  L    D     T  C O E     Y  D   W     T   T  B  B        S
 S  _    _     _  _ _ _     _  _   _     _   _  _  _        _

37 N3_5 N4_5   1  7 0 A345  .   .  0     0   1 45 24 LOWERBD NONBASIC
38 N1_1 N5_1   1  7 0 A151 30   .  0     0   2  7  1 LOWERBD NONBASIC
39 N2_2 N5_2   1  8 0 A252 30   .  0     0   . 14  6 KEY_ARC    BASIC
40 N1_3 N5_3   1  7 0 A153  .  30  0     0   0 24 12 LOWERBD NONBASIC
41 N2_3 N5_3   1  8 0 A253  .  30  0     0   . 25 13 KEY_ARC    BASIC
42 N6_3 N5_3   1 30 0 D3    .  30 30    30   . 26 28 KEY_ARC    BASIC
43 N2_4 N5_4   1  8 0 A254  .  30  0     0   0 34 18 LOWERBD NONBASIC
44 N1_4 N5_4   1  7 0 A154  .  30  0     0   . 35 17 KEY_ARC    BASIC
45 N6_4 N5_4   1 30 0 D4    .  30 30    30   . 36 29 KEY_ARC    BASIC
46 N1_1 N6_1 100 30 0 LC1  30   . 16  1600   . 46  1 KEY_ARC    BASIC
47 N2_2 N6_2 100 30 0 LC2  30   . 27  2700   . 47  6 KEY_ARC    BASIC
48 N3_3 N6_3 100 30 0 LC3  30   . 30  3000   . 48 11 KEY_ARC    BASIC
49 N4_4 N6_4 100 30 0 LC4  30   . 30  3000   . 49 16 KEY_ARC    BASIC
50 N5_5 N6_5 100 30 0 LC5  30   . 22  2200   . 50 21 KEY_ARC    BASIC
                              ===    =====
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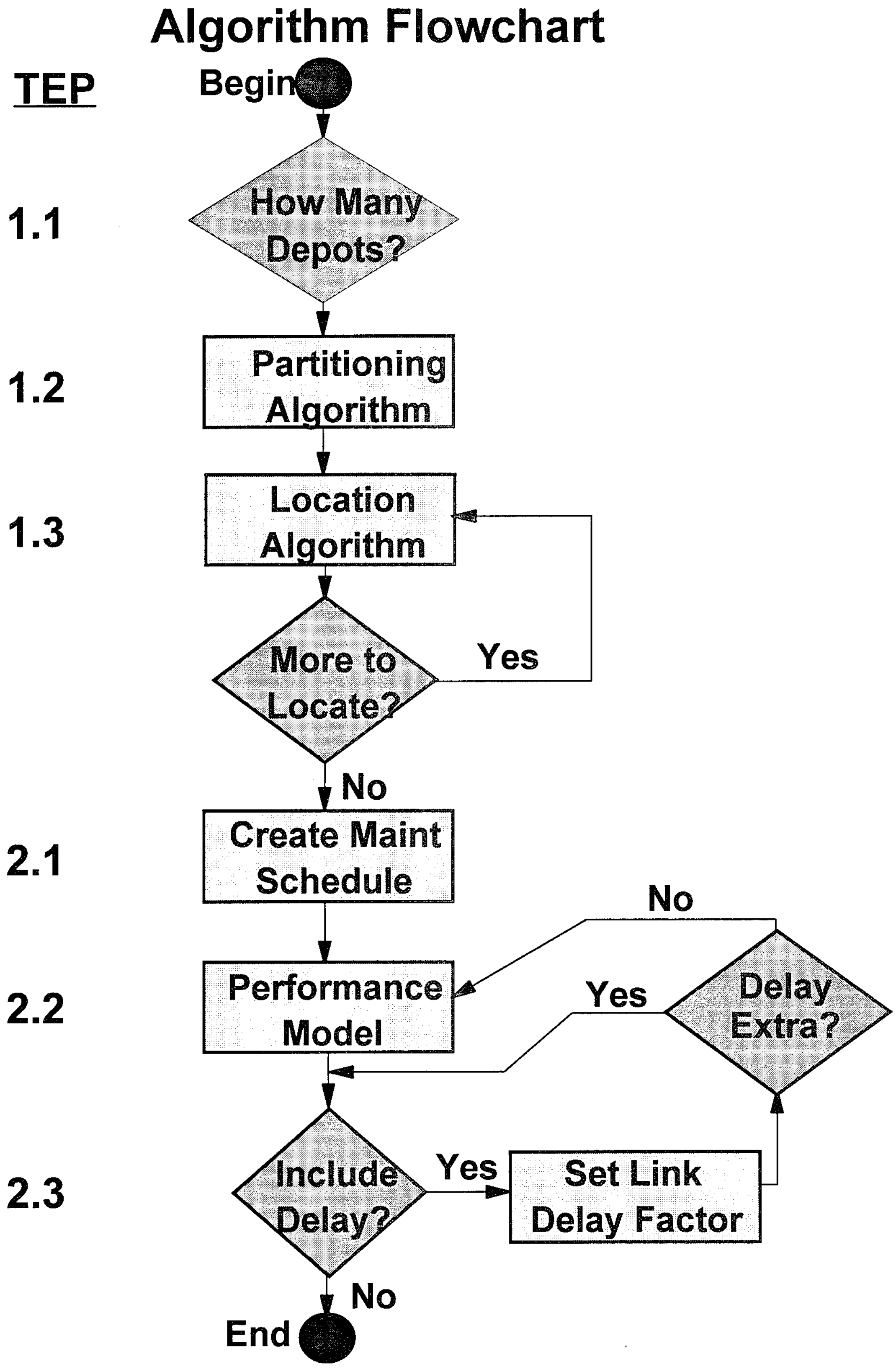
Appendix E. Algorithm Flowchart



Vita

Captain Todd Patterson was born on 30 March, 1967 in Moline, Illinois. He graduated from Ottumwa High School in Ottumwa, Iowa. He attended the United States Air Force Academy in Colorado Springs, Colorado, from which he received the degree of Bachelor of Science in Operations Research with a minor in the Arabic language. He was commissioned in May 1989 upon graduation. He then attended the Intelligence Officer Training Program at Goodfellow AFB, Texas. His first assignment after completion of technical training in August 1990, was Chief of Intelligence, at Onizuka AFB, California. He entered the School of Engineering, Air Force Institute of Technology in August 1993.

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This research proposes an analytical approach to access the relationship between maintenance facility location and communication network performance measurement using a selected dynamic maintenance scheduling protocol. There were three objectives established for this effort. The first objective was the determination of an upper-bound upon the level of performance for a telecommunication network using dynamically scheduled maintenance to evaluate maintenance depot location. This was achieved by using a two-stage algorithm, first locate a maintenance depot by using stochastic algorithms, and then to measure the resulting impact upon performance with a multi-commodity network flow model. The second objective was to develop the metrics by which network performance should be measured. This was accomplished by comparing multiple criteria using a constraint conversion technique. The third objective created the mathematical models necessary to evaluate network operations. The models were created within a least-cost multi-commodity network flow environment. The approaches used in this research are offered as initial investigations toward the long-term goal of automated maintenance scheduling for a stochastic communication network.

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