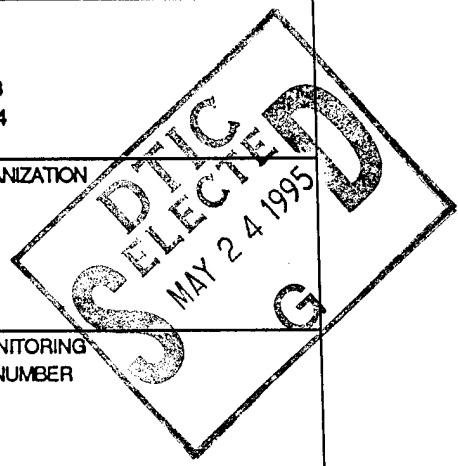


# REPORT DOCUMENTATION PAGE

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302 and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE 14-Mar-95	3. REPORT TYPE AND DATES COVERED Final Technical Report 9/30/94	
4. TITLE AND SUBTITLE Air Sea Exchanges of Energy and Momentum under Well-Developed Sea Conditions: Theory and Experiment			5. FUNDING NUMBERS N00014-92-J-1343 N00014-94-1-0574	
6. AUTHOR(S) V.E. Zakharov			8. PERFORMING ORGANIZATION REPORT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Department of Mathematics University of Arizona Tucson, Arizona 85721				
9. SPONSORING / MONITORING AGENCY NAMES(S) AND ADDRESS(ES) Department of the Navy Office of the Chief of Naval Research Arlington, VA 22217-5000			10. SPONSORING/MONITORING AGENCY REPORT NUMBER	
11. SUPPLEMENTARY NOTES The view, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official agency position, policy, or decision, unless so designated by other documentation.				
12a. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution unlimited.			12b. DISTRIBUTION CODE or SI d	
13. ABSTRACT (Maximum 200 words) The research group studied the following problems in the field of the theory of hydrodynamics: 1. Integrability of free surface hydrodynamics and five-wave interaction for gravity waves; 2. Singularity formation on the free surface of an ideal fluid; 3. Wave dynamics and collapse in shear-flows; 4. Turbulent Kolmogorov spectra for capillary waves.				
			Distribution /	
			Availability Codes	
			Dist	Avail and/or Special
			A-1	
DTIC QUALITY INSPECTED 5				
14. SUBJECT TERMS			15. NUMBER OF PAGES 65	
			16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT UNCLASSIFIED	18. SECURITY CLASSIFICATION UNCLASSIFIED	19. SECURITY CLASSIFICATION OF ABSTRACT UNCLASSIFIED	20. LIMITATION OF ABSTRACT UL	



Standard Form 298 (Rev. 2-89)  
Prescribed by ANSI Std. Z39-18  
298-102

# 19950519 055

ONR Contract No. N00014-92-J-1343/N00014-94-1-0574  
"Air-Sea Exchanges of Energy and Momentum under Well-Developed Sea Conditions: Theory and  
Experiment"

V. Zakharov, Principal Investigator  
Department of Mathematics  
University of Arizona  
Tucson, AZ 85721

and

Landau Institute for Theoretical Physics  
Moscow, Former Soviet Union

Final Report

The research group studied the following problems in the field of the theory of the hydrodynamics. Results of the collaboration by Dr. R. E. Glazman at the Jet Propulsion Laboratory are found in Appendix I.

### 1. Integrability of Free Surface Hydrodynamics and Five-Wave Interaction for Gravity Waves

It is shown in (1) that the set of equation for the gravity wave propagation on the surface of the fluid of infinite depth is integrable one up to the fourth order in the perturbation series in the powers of complex normal variables. Based on the Hamiltonian approach, we have shown that the scattering matrix for four-wave interaction of the gravity waves is identically equal to zero for the one-dimensional case. We have also made a great deal of progress in understanding the importance of the five-wave interaction for gravity wave turbulence. The elegant form of the scattering matrix for that interaction is calculated for the one-dimensional case (2).

As a side effect of that activity we have shown that two approaches for statistical description of gravity wave turbulence introduced by K.Hasselmann and V.Zakharov result in the same kinetic equation for the second order correlator (3).

### 2. Singularity Formation On the Free Surface of an Ideal Fluid

The method of conformal mapping is used for the description of the nonlinear dynamics of an ideal fluid with a free surface. Unlike the commonly-used Birkhoff-Rott equations, based on the Lagrangian approach, the equations obtained are pseudo-differential ones, containing only the Hilbert transform on the straight line only. These (exact) equations are much simpler and better adjusted for numerical simulations (4). The numerical code was also developed.

We have shown that the equations of motion of an ideal fluid with a free surface in the absence of both gravitational and capillary forces can be effectively solved in the approximation of small surface angles (5). It is done by analytical continuation for both the velocity potential of the surface and its elevation. For most initial conditions, the system evolves to form singularities in a finite time. The types of different singularities are classified. All of them correspond to the moving branch points in the complex plane, and singularity appears when such a branch point hits the real axis which corresponds to the physical space.

### 3. Wave Dynamics and Collapse in Shear-Flows

The sheared variables, corresponding to the amplitudes of the convective modes, are shown (6) to diagonalize the quadratic Hamiltonian, i.e., to be normal variables of the Hamiltonian equations while the convective modes are the normal ones in the Hamiltonian dynamics as the basis for the nonlinear theory.

In the framework of the generalized two-dimensional Benjamin-Ono equation, we have studied both analytically and numerically the nonlinear evolution of "quasi" one-dimensional perturbations of the shear flow. We have shown that in the limit of small viscosity the instability of the one-dimensional soliton,

perturbed in the transverse direction, results in the self-focusing and collapse. It leads to the separation of a one-dimensional soliton into individual clusters and to the subsequent self-focussing of each cluster. With the help of the embedding inequalities, an upper boundary for the noncollapsing sector is found. Both the theoretical and numerical results obtained are in a qualitative agreement with experimental data on coherent structures in the boundary layer.

#### 4. Turbulent Kolmogorov Spectra for Capillary Waves

The behavior of an ensemble of capillary waves on the surface of incompressible fluid of infinite depth was studied numerically (8). Driving force (external pressure) localized at small wavenumbers and an energy absorption was provided by the viscosity dumping acting at high wavenumbers. It is found that the picture of turbulence depends dramatically on the nature of long-wave driving force, i.e., whether the driving force is in or out of resonance with local linear frequency of the system. Resonant or non-resonant pumping defines correspondingly two different regimes of turbulence: first, one without energy flux to the damping region of small scales (interpreted as KAM regime) and second, one with the finite energy flux. In the latter, the formation of power-like Kolmogorov spectra was observed. Index of this spectra corresponds to that predicted by "weak turbulence" approach.

For the mean field approximation for the Euler equation the Kolmogorov constant for constant energy flux was calculated (9). The results of (1), (4), (5), and (8) were reported at ONR "Ocean Waves" Workshop, 1994, Tucson. (Appendix II). The results of (1), (2), (4), (5) and (7) were reported at "New achievements in the Nonlinear Schroedinger Equation"-Workshop, Landau Institute for Theoretical Physics, Moscow, Russia, 1994.

#### BIBLIOGRAPHY

- (1) A.I.Dyachenko, V.E.Zakharov., Is the Free Surface Hydrodynamics is an Integrable System?, Phys. Lett. A, 190, pp144--148, (1994)
- (2) A.I.Dyachenko, Y.V.L'vov and V.E.Zakharov, Five-wave Interaction on the Surface of Deep Fluid, Physica D, (Accepted)
- (3) A.I.Dyachenko and Y.V.L'vov, On the Hasselmann's and Zakharov's Approaches to the Kinetic Equations for the Gravity Waves. Interaction On the Surface of Deep Fluid, J. Physical Oceanography, (Submitted)
- (4) E.A.Kuznetsov, M.D.Spector, and V.E.Zakharov, Analytical Description of the Free Surface Dynamics of Ideal Fluid, (Submitted to Phys.Rev. E.)
- (5) E.A.Kuznetsov, M.D.Spector, and V.E.Zakharov, Formation of Singularities on the Free Surface of an ideal Fluid, Phys.Rev.E , 48, 1283-1290 (1994)
- (6) S.V.Nazarenko S.V. and V.E.Zakharov, The Role of the Convective Modes and Sheared Variables in the Hamiltonian Dynamics of Uniform-Shear-Flow Perturbations, Phys.Lett.A, 191, pp.403-408, (1994)
- (7) E.A.Kuznetsov and A.I.Dyachenko, Two-dimensional Wave Collapse in the Boundary Layer, Physica D, (Accepted)
- (8) A.N.Pushkarev and V.E.Zakharov, Numerical Simulation of the Capillary Kolmogorov Spectra, J. Fluid Mech. (Submitted)
- (9) A.Balk, A.Pushkarev and V.Zakharov, Kolmogov Constant for Hydrodynamics Turbulence in Mean Field Theory in Clebsh Variables, (To Be Submitted)

# APPENDIX I

R. E. Glazman  
Jet Propulsion Laboratory  
Pasadena, CA

## I. Theory

1. We now understand the fundamental role of surface tension in explaining the presence of two regimes of a wind-driven sea surface. For the wind speed  $U$  less than a critical value [1], [2], [9],

$$U_0 \sim \left( \frac{\rho_w}{\rho_a} \right)^{1/2} (\sigma g)^{1/4}, \quad (1)$$

the surface tension dominated wrinkling of the surface can absorb the direct energy cascade and the surface stays smooth. For  $U > U_0$ , however, the Kolmogorov equilibrium spectrum

$$E_1(k) \sim k_0^4 (\eta k^4) = \left( \frac{P}{P_0} \right)^{1/3} \left( \frac{k_0}{k} \right)^{7/2} \quad (2)$$

corresponding to a constant energy flux  $P$  ( $P = \left( \frac{\rho_a}{\rho_w} \right)^{3/2} U^3$ ,  $k_0 = (g/\sigma)^{1/2}$ ), meets the Phillips spectrum

$$E_3(k) = \left( \frac{k_0}{k} \right)^4 \quad (3)$$

at a value  $k_1 < k_0$ , that is at wavelengths much larger than those at which surface tension effects come into play. The result is that the surface develops singular derivatives and in order to create sufficient surface area to absorb the energy flux, the surface breaks and issues forth a cloud of water droplets or foam. A first attempt to describe some of the characteristics of this new phase (foam height, water droplet size) in terms of the parameters  $P$ ,  $P_0 = \left( \frac{\rho_w}{\rho_a} \right)^{3/2} (\sigma g)^{3/4}$ ,  $k_0$  is given in [2].

2. If surface tension effects and three wave interactions can be neglected, then the first closure of the moment equations at order  $\epsilon^4$  gives rise to the well known Hasselmann kinetic equation for a system of random dispersive waves which share energy via four wave resonances. It is important to study also the situation at the next order, for two reasons. First, over longer times than  $\epsilon^{-4}$ , the five wave resonances are important. Second, and equally interesting, is the fact that at this order, the wave action  $\int n_k d\vec{k}$  is no longer a motion constant. This can have important ramifications for the inverse cascade and the understanding of the aging process, namely the presence of waves with phase speeds greater than the wind speed. Using Feynmann diagram techniques, this kinetic equation has been derived by V. P. Krazitzky and V. E. Zakharov [3], [4]. Weak turbulent Kolmogorov-like spectra for multiwave interactions have been found recently by Glazman [6].
3. We have made some progress towards understanding the nature of wave breaking. We consider that the wave breaking results from the spontaneous formation of angle-type singularities of the free fluid surface. A self-similar theory of such singularities was developed in the paper [1] and a more advanced theory is contained in [10].

## II. Analysis of in situ data

This analysis was performed by R. Glazman and published in the articles [5, 7-8]. The analysis includes measurements of wave spectra in the open sea by buoy-installed devices, studying of duration and fetch dependence of the characteristic wavelength and wave energy. The results are compared with the predictions of the weak turbulent theory. Measurements have quantitatively confirmed the existence of direct and inverse cascades of energy. The shape of the spectrum in the direct cascade area is very well approximated by the weak turbulence formula  $I(\omega) \simeq \omega^{-4}$ . In the inverse cascade area ( $\omega \simeq \frac{g}{u}$ ) measurements give  $I(\omega) \simeq \omega^{-s}$  and  $s$  is 3. The weak-turbulence prediction 11/3. The fetch dependence of the characteristic wavelength is power-like

$$S = Cx^c$$

and  $c$  varies from 0.29 to 0.33. The weak turbulence theory predicts  $c = \frac{3}{10} \simeq 0.22$ . Sizable deviations from the weak turbulence theory can perhaps

be explained by the facts that long wave damping due to friction with the turbulent air is neglected and that the interaction with the shear flow near the surface is ignored.

### III. Plans for the next stage

1. It is now clear that a whole new theory of surface wave theory must be developed in which the sea surface is no longer a discontinuity dividing air and water but consists of a finitely thick layer of a new phase, a foam mixture of air and water. Our first attempts to describe some of the characteristics of this phase in [2] were very elementary and what is needed is a new dynamical theory which takes account of water droplets in air, air bubbles contained in water (here compressibility may be important). We must also include nonpotential flow effects as the momentum absorbed by the water layer from the waves will create a surface shear flow.
2. We plan to develop a numerical simulation of the existing water wave equations (with nonbroken surface) much in the same vein as we studied optical turbulence by integrating the nonlinear Schrödinger equation [11]. The results should be compatible with the results of simulations of the kinetic equation but will allow us also to understand the transition regimes between waves dominated by gravity and those dominated by surface tension. The key step in this work is to formulate the water wave equations as a local evolution of some appropriately defined variables of  $x$  and  $y$ , the surface coordinates. We have some good ideas. This part of the work will be carried out by Drs. S. Dyachenko and A. Pushkarev who will be at Arizona from July 1993 to August 1994.
3. We shall also be studying models for explaining the observed angular distribution of energy in long waves. The distribution is more directed along the wind direction than would be predicted by present theories.

## References

- [1] V. E. Zakharov, *Direct and inverse cascade in wind-driven sea and wave breaking*, Proceedings of the IUTAM symposium on wave breaking, Sidney, 1965. Springer-Verlag, 1992, ed. by M. Banner.
- [2] A. C. Newell and V. E. Zakharov, *Rough sea foam*, Phys. Rev Letters, 63, 1149-1151, August, 1992.
- [3] V. P. Krazitzky, *Five-wave interaction Hamiltonian for gravity surface waves*, submitted to the Journ. of Fluid Mechanics.
- [4] V. P. Krazitzky and V. E. Zakharov, *Five-wave kinetic equation for surface gravity waves*. To be presented at the Air-Sea Interface Symposium, Marseille, France, June 29-30, 1993.
- [5] R. E. Glazman and V. E. Zakharov, *Within and Beyond the Weak Turbulence Theory of Wind Generated Waves*, in "Nonlinear Dynamics of Ocean Waves". Proceeding of symposium at the John Hopkins University, Applied Physics Laboratory, 1991. World Scientific Publication.
- [6] R. E. Glazman, *Multi-wave interaction theory for wind-generated surface gravity waves*, J. Fluid. Mech. , Vol. 243, pp. 623-635, 1992.
- [7] R. E. Glazman, *Surface gravity waves at equilibrium with a steady wind*. Submitted to the Journal of Geophysical Research, Jan. 1993.
- [8] R. E. Glazman, *Wave spectra of developed seas*, to appear in "Nonlinear waves and weak turbulence". N. Fitzmaurice, D. Gurerie, F. McCaughan, W. A. Woyczynski, eds. Birkhauser-Boston, 1993.
- [9] V. E. Zakharov, *Kolmogorov spectra in weak and strong turbulence*, Ibid.
- [10] E. Kuznetsov, M. Spectra and V. Zakharov, *On singularity formation on the surface of ideal fluid*, submitted to the Phys. Letters A, April, 1993.
- [11] S. Dyachenko, A.C. Newell, A. Pushkarev and V.E. Zakharov, *Optical Turbulence: Weak Turbulence, Condensates and Collapsing Filaments in the Nonlinear Schrödinger Equation*, Physica D 57, 96-160, 1992.

## APPENDIX II

**WELCOME TO**  
**THE**  
**OCEAN WAVES WORKSHOP**

**SPONSORED BY**

**THE OFFICE OF NAVAL RESEARCH**

**MARCH 16 - 18, 1994**

**DEPARTMENT OF MATHEMATICS**

**UNIVERSITY OF ARIZONA**

**TUCSON ARIZONA 85721**