

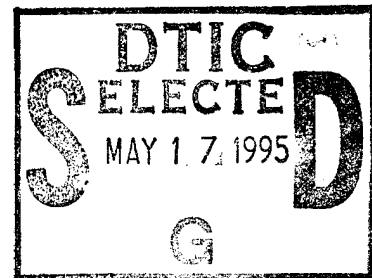
PENETRABLE WEDGE ANALYSIS

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PREFACE

Two complementary analyses of the time-harmonic scattering by a penetrable wedge are presented. The distance from the apex (appropriately scaled by the wavenumber in the exterior region) of the exciting line source is the single length scale in this infinite-domain boundary value problem. The work summarized herein represents two mathematical approaches (among a series of candidates) to solve this important scattering problem and to visualize the wave physics.

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ACOUSTICAL SCATTERING BY A PENETRABLE WEDGE

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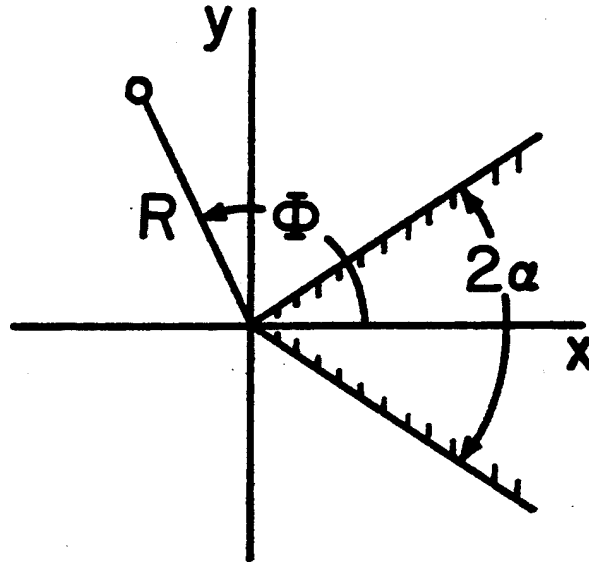


Figure 1: Wedge Geometry

1 Introduction

Consider the two-dimensional scattering of a time-harmonic sound wave generated by a line source and incident upon a penetrable wedge of angle 2α which, to simplify the presentation, is assumed in the main text to be such that π/α is an integer m (Figure 1). The wave speeds in the interior and exterior of the wedge are distinct and the radiation condition of only outgoing waves at infinity is applied in all directions. At the boundary of the wedge there is a pair of transmission conditions which ensure continuity of the acoustic pressure and normal velocity. Additional simplification is achieved by assuming that only one side of the wedge is directly 'lit' by the source. Such a transmission problem can be formulated, by use of

the free space Green's function in each region, in terms of a pair of coupled integral equations for two unknowns, the pressure and normal velocity, over the total boundary of the wedge. The details are given by Kleinman and Martin (1988) for the finite transmitting body.

However, by using suitably modified Green's functions and considering separately the symmetric and antisymmetric parts of the pressure field with respect to the center plane of the wedge, a pair of disjoint integral equations of the first kind can be obtained for the two parts of the normal velocity on just one face of the wedge. Transformation to equations of the second kind is then achieved by using a technique for solving integral equations with Hankel function kernels (Porter, 1983). The procedure described here can be regarded as a perturbation from the case of the impenetrable hard wedge and this context is adopted for the sake of extra simplification of the presentation.

2 Formulation of the Transmission Problem

A sound wave generated by a z -directed source of period $2\pi/\omega$ is incident on a penetrable wedge which, in terms of cylindrical polar coordinates (r, ϕ, z) , occupies the region $r > 0, -\alpha < \phi < \alpha$ (Figure 1). With the time factor $e^{i\omega t}$ suppressed, the induced exterior and interior pressure fields, u_e and u_i respectively, satisfy the wave equations

$$(\nabla^2 + k_e^2)u_e = 0, \quad (|\phi| > \alpha), \quad (1)$$

$$(\nabla^2 + k_i^2)u_i = 0, \quad (|\phi| < \alpha), \quad (2)$$

while the incident field, with the source at (R, Φ) , is given by

$$u_{inc} = \frac{1}{4}iH_0^{(2)}[k_e(r^2 + R^2 - 2rR\cos(\phi - \Phi))^{1/2}], \quad (3)$$

where $\alpha < \Phi < \pi - \alpha$, in accordance with the assumption that only one side of the wedge is directly 'lit' by the source. The transmission conditions at the interfaces are

$$u = u_i, \quad \frac{\partial u}{\partial \phi} = \rho \frac{\partial u_i}{\partial \phi} \quad (\phi = \pm\alpha), \quad (4)$$

where

$$u = u_e + u_{inc} \quad (5)$$

denotes the total pressure field in the exterior region. At infinity, all fields contain only outgoing waves. The wavenumbers k_e, k_i and the density

ratio ρ are given real, positive constants with $k_i > k_e$. Now introduce the symmetric and antisymmetric parts of u_{inc} , u_e , u_i by writing, for each field

$$u^{(\pm)}(r, \phi) = \frac{1}{2}[u(r, \phi) \pm u(r, -\phi)] \quad (6)$$

and, similarly, set

$$f^{(\pm)}(r) = \frac{1}{r} \frac{\partial u_i^{(\pm)}}{\partial \phi}(r, \alpha) \quad (r > 0). \quad (7)$$

Then the exterior and interior regions are reduced to $\alpha < \phi < \pi$ and $0 < \phi < \alpha$ respectively, with the differential equations (1) and (2) satisfied therein and the transmission conditions (4) applicable at the sole interface $\phi = \alpha$. The additional hard and soft conditions are

$$\begin{aligned} \frac{\partial u_e^{(+)}}{\partial \phi} = 0, \quad u_e^{(-)} = 0 \quad (\phi = \pi), \\ \frac{\partial u_i^{(+)}}{\partial \phi} = 0, \quad u_i^{(-)} = 0 \quad (\phi = 0). \end{aligned} \quad (8)$$

Let $G_e^{(\pm)}(r, \phi; r', \phi')$ ($\alpha < \phi, \phi' < \pi$) be two Green's functions defined in the exterior region where they satisfy (1) except at the singularity in the prescribed term $\frac{1}{4}iH_0^{(2)}[k_e(r^2 + r'^2 - 2rr' \cos(\phi - \phi'))^{1/2}]$. Similarly, let $G_i^{(\pm)}(r, \phi; r', \phi')$ ($0 < \phi, \phi' < \alpha$) be two Green's functions defined in the interior region where they satisfy (2) except at the singularity in the prescribed term $\frac{1}{4}iH_0^{(2)}[k_i(r^2 + r'^2 - 2rr' \cos(\phi - \phi'))^{1/2}]$. These four functions satisfy the respective hard and soft conditions (8) and all of them are required to have zero normal derivative at the interface $\phi = \alpha$ and only outgoing waves at infinity.

Then when Green's theorem is applied to $u_e^{(\pm)}(r, \phi) - [u_e^{(\pm)}(r, \phi)]_{\rho=0}$ and $G_e^{(\pm)}(r, \phi; r', \alpha)$ in the external region $\alpha < \phi < \pi$ and to $u_i^{(\pm)}(r, \phi)$ and $G_i^{(\pm)}(r, \phi; r', \alpha)$ in the interior region $0 < \phi < \alpha$, the resulting pairs of integral equations on the wedge boundary are

$$\begin{aligned} u_e^{(\pm)}(r, \alpha) - [u_e^{(\pm)}(r, \alpha)]_{\rho=0} &= \frac{1}{\pi} \int_0^\infty G_e^{(\pm)}(r, \alpha; r', \alpha) \frac{1}{r'} \frac{\partial u^{(\pm)}}{\partial \phi'}(r', \alpha) dr' \\ &= \frac{\rho}{\pi} \int_0^\infty G_e^{(\pm)}(r, \alpha; r', \alpha) f^{(\pm)}(r') dr' \quad (r > 0) \end{aligned}$$

and

$$\begin{aligned} u_i^{(\pm)}(r, \alpha) &= -\frac{1}{\pi} \int_0^\infty G_i^{(\pm)}(r, \alpha; r', \alpha) \frac{1}{r'} \frac{\partial u_i^{(\pm)}}{\partial \phi'}(r', \alpha) dr' \\ &= -\frac{1}{\pi} \int_0^\infty G_i^{(\pm)}(r, \alpha; r', \alpha) f^{(\pm)}(r') dr' \quad (r > 0), \end{aligned}$$

after use of (4), (5) and (7). Now (4) and (5) allow the unknown functions on the left hand sides to be eliminated to obtain a pair of disjoint integral equations of the first kind for the symmetric and antisymmetric source density functions $f^{(\pm)}$, namely

$$\begin{aligned} u_{inc}^{(\pm)}(r, \alpha) + [u_e^{(\pm)}(r, \alpha)]_{\rho=0} &= [u^{(\pm)}(r, \alpha)]_{\rho=0} \\ &= -\frac{1}{\pi} \int_0^\infty [G_i^{(\pm)}(r, \alpha; r', \alpha) + \rho G_e^{(\pm)}(r, \alpha; r', \alpha)] f^{(\pm)}(r') dr' \quad (r > 0). \end{aligned} \tag{9}$$

This is the equation whose reduction to a suitable integral equation of the second kind is the purpose of this work. Note that $\rho = 0$ for the hard wedge, in which limit f becomes irrelevant to the scattering problem because no transmission occurs in this case. However, the structure of the solution

procedure presented below may be regarded as a perturbation about this limit. On writing

$$f^{(\pm)} = f_0^{(\pm)} + \rho f_1^{(\pm)} + \rho^2 f_2^{(\pm)} + \dots, \quad (10)$$

the integral equation (9) reduces to the system

$$\begin{aligned} u_0^{(\pm)}(r) = [u^{(\pm)}(r, \alpha)]_{\rho=0} &= -\frac{1}{\pi} \int_0^\infty G_i^{(\pm)}(r, \alpha; r', \alpha) f_0^{(\pm)}(r') dr', \\ \frac{1}{\pi} \int_0^\infty G_e^{(\pm)}(r, \alpha; r', \alpha) f_{n-1}^{(\pm)}(r') dr' &= -\frac{1}{\pi} \int_0^\infty G_i^{(\pm)}(r, \alpha; r', \alpha) f_n^{(\pm)}(r') dr' \quad (n \geq 1) \\ & \quad (r > 0). \end{aligned} \quad (11)$$

Thus each term in the expansion (10) is determined from an interior problem while each forcing term, after the first, arises from an exterior problem. In particular, this shows that the Green's function $G_i^{(\pm)}$, with wavenumber k_i plays the dominant role in the kernel of (9).

3 The Green's Functions and Forcing Term

With the incident field defined by (3), it may be deduced from Jones (1986, section 9.19) that the forcing term in (9) is given by

$$\begin{aligned} u_0^{(\pm)}(r) = [u^{(\pm)}(r, \alpha)]_{\rho=0} &= \frac{1}{16(\pi - \alpha)} \int_{\infty - \pi i}^{\infty + \pi i} H_0^{(2)}[k_e(r^2 + R^2 - 2rR \cosh \tau)^{1/2}] \\ & \times \sinh \frac{\pi \tau}{2(\pi - \alpha)} \left[\frac{1}{\cosh \frac{\pi \tau}{2(\pi - \alpha)} - \cos \frac{\pi(\Phi - \alpha)}{2(\pi - \alpha)}} \pm \frac{1}{\cosh \frac{\pi \tau}{2(\pi - \alpha)} + \cos \frac{\pi(\Phi + \alpha)}{2(\pi - \alpha)}} \right] d\tau \\ & = \frac{1}{4} i H_0^{(2)}[k_e(r^2 + R^2 - 2rR \cos(\Phi - \alpha))^{1/2}] \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{16(\pi - \alpha)} \int_{-\infty}^{\infty} H_0^{(2)}[k_e(r^2 + R^2 + 2rR \cosh u)^{1/2}] \sinh \frac{\pi(u + \pi i)}{2(\pi - \alpha)} \\
& \times \left[\frac{1}{\cosh \frac{\pi(u + \pi i)}{2(\pi - \alpha)} - \cos \frac{\pi(\Phi - \alpha)}{2(\pi - \alpha)}} \pm \frac{1}{\cosh \frac{\pi(u + \pi i)}{2(\pi - \alpha)} + \cos \frac{\pi(\Phi + \alpha)}{2(\pi - \alpha)}} \right] du, \quad (12)
\end{aligned}$$

in which only the first term contributes to the residue at $\tau = 0$ because of the assumption $\Phi + \alpha < \pi$.

Similarly the external Green's functions are given by

$$G_e^{(+)}(r, \phi; r', \alpha) = \frac{1}{8(\pi - \alpha)} \int_{\infty - \pi i}^{\infty + \pi i} \frac{H_0^{(2)}[k_e(r^2 + r'^2 - 2rr' \cosh \tau)^{1/2}] \sinh \frac{\pi\tau}{\pi - \alpha}}{\cosh \frac{\pi\tau}{\pi - \alpha} + \cos \frac{\pi(\pi - \phi)}{\pi - \alpha}} d\tau,$$

i.e.

$$\begin{aligned}
G_e^{(+)}(r, \alpha; r', \alpha) &= \frac{1}{4} i H_0^{(2)}[k_e|r - r'|] \\
& + \frac{1}{8(\pi - \alpha)} \int_{-\infty}^{\infty} \frac{H_0^{(2)}[k_e(r^2 + r'^2 + 2rr' \cosh u)^{1/2}]}{\tanh \frac{\pi(u + \pi i)}{2(\pi - \alpha)}} du \quad (13)
\end{aligned}$$

and

$$G_e^{(-)}(r, \phi; r', \alpha) = \frac{\sin \frac{\pi(\pi - \phi)}{2(\pi - \alpha)}}{4(\pi - \alpha)} \int_{\infty - \pi i}^{\infty + \pi i} \frac{H_0^{(2)}[k_e(r^2 + r'^2 - 2rr' \cosh \tau)^{1/2}] \sinh \frac{\pi\tau}{2(\pi - \alpha)}}{\cosh \frac{\pi\tau}{\pi - \alpha} + \cos \frac{\pi(\pi - \phi)}{\pi - \alpha}} d\tau$$

i.e.

$$\begin{aligned}
G_e^{(-)}(r, \alpha; r', \alpha) &= \frac{1}{4} i H_0^{(2)}[k_e|r - r'|] \\
& + \frac{1}{8(\pi - \alpha)} \int_{-\infty}^{\infty} \frac{H_0^{(2)}[k_e(r^2 + r'^2 + 2rr' \cosh u)^{1/2}]}{\sinh \frac{\pi(u + \pi i)}{2(\pi - \alpha)}} du. \quad (14)
\end{aligned}$$

The assumption that $\alpha = \pi/2m$, for some integer $m > 1$, simplifies the corresponding expressions for the interior Green's functions which evidently can be constructed by adding image sources and sinks to the prescribed

source. Thus

$$G_i^{(\pm)}(r, \phi; r', \alpha) = \frac{1}{4}i \sum_{n=1}^{n=m} (\pm 1)^n \left\{ H_0^{(2)}[k_i(r^2 + r'^2 - 2rr' \cos[(2n-1)\alpha + \phi])^{1/2}] \right. \\ \left. \pm H_0^{(2)}[k_i(r^2 + r'^2 - 2rr' \cos[(2n-1)\alpha - \phi])^{1/2}] \right\}$$

and, in particular,

$$G_i^{(\pm)}(r, \alpha; r', \alpha) = \frac{1}{4}i \left\{ H_0^{(2)}[k_i|r - r'|] + (\pm 1)^m H_0^{(2)}[k_i(r + r')] \right. \\ \left. + 2 \sum_{n=1}^{n=m-1} (\pm 1)^n H_0^{(2)}[k_i(r^2 + r'^2 - 2rr' \cos n\pi/m)^{1/2}] \right\}. \quad (15)$$

4 Inversion of the Hankel Kernel

Consider the integral equation of the first kind

$$g(r) = \int_0^\infty \psi(r') H_0^{(2)}(k_i|r - r'|) dr' \quad (r > 0), \quad (16)$$

whose solution, which may have an integrable singularity at $x = 0$, is assumed to be bounded at infinity, and define the operator L_i by

$$(L_i \psi)(s) = -\psi'(s) + k_i \int_s^\infty \psi(r) \frac{J_1[k_i(r-s)]}{r-s} dr \\ = \left(\frac{d^2}{ds^2} + k_i^2 \right) \int_s^\infty \psi(r) J_0[k_i(r-s)] dr \quad (s \geq 0). \quad (17)$$

Porter (1983) showed that application of this operator to (16) yields the singular integral equation

$$(L_i g)(s) = -\frac{2i}{\pi} \int_0^\infty \psi(r') \frac{e^{ik_i(r'-s)}}{r' - s} dr' \quad (s \geq 0), \quad (18)$$

whose solution is given by Muskhelishvili (1953). The complementary function, $e^{-ik_i r'}/\sqrt{r'}$, satisfies the outgoing wave condition and furnishes a unique solution bounded at the origin, namely

$$\psi(r) = -\frac{i}{2\pi}\sqrt{r} \int_0^\infty \frac{(L_i g)(s)}{\sqrt{s}} \frac{e^{ik_i(s-r)}}{s-r} ds \quad (r \geq 0). \quad (19)$$

Thus, on writing (15) in the form

$$G_i^{(\pm)}(r, \alpha; r', \alpha) = \frac{1}{4}i \left[H_0^{(2)}(k_i|r-r'|) + K_i^{(\pm)}(r, r') \right], \quad (20)$$

the first of the integral equations (11) can be rearranged as

$$4\pi i u_0^{(\pm)}(r) - \int_0^\infty K_i^{(\pm)}(r, r') f_0^{(\pm)}(r') dr' = \int_0^\infty H_0^{(2)}(k_i|r-r'|) f_0^{(\pm)}(r') dr',$$

which is of the type (16) when the left hand side is regarded as known.

Hence, from (19),

$$f_0^{(\pm)}(r) = \sqrt{r} \int_0^\infty \frac{e^{ik_i(s-r)}}{\sqrt{s(s-r)}} \left[2(L_i u_0^{(\pm)})(s) + \frac{i}{2\pi} \int_0^\infty (L_i K_i^{(\pm)})(s, r') f_0^{(\pm)}(r') dr' \right] ds \quad (r \geq 0), \quad (21)$$

which is an integral equation of the second kind for each zero order density function. The two integrations introduced by the above inversion procedure can be evaluated exactly when plane wave representations are used.

The inversion of the remaining integral equations in (11) can be achieved similarly, except that the forcing terms also have a Hankel kernel, with wavenumber k_e , as given by (13) and (14) for the symmetric and antisymmetric cases. On writing these equations in the form

$$G_e^{(\pm)}(r, \alpha; r', \alpha) = \frac{1}{4}i \left[H_0^{(2)}(k_e|r-r'|) + K_e^{(\pm)}(r, r') \right], \quad (22)$$

as in (20) for the interior Green's functions, the sequence of integral equations in (11) can be arranged as

$$\begin{aligned} & - \int_0^\infty [K_e^{(\pm)}(r, r') f_{n-1}^{(\pm)}(r') + K_i^{(\pm)}(r, r') f_n^{(\pm)}(r')] dr' \\ & - \int_0^\infty [H_0^{(2)}(k_e|r-r'|) - H_0^{(2)}(k_i|r-r'|)] f_{n-1}^{(\pm)}(r') dr' \\ & = \int_0^\infty H_0^{(2)}(k_i|r-r'|) [f_n^{(\pm)}(r') + f_{n-1}^{(\pm)}(r')] dr', \end{aligned}$$

which also can be regarded as of the type (16). Hence, on defining the bounded kernel

$$K_s(r, r') = H_0^{(2)}(k_e|r-r'|) - H_0^{(2)}(k_i|r-r'|) \quad (r, r' \geq 0), \quad (23)$$

the inversion formula (19) yields

$$\begin{aligned} f_n^{(\pm)}(r) + f_{n-1}^{(\pm)}(r) &= \frac{i\sqrt{r}}{2\pi} \int_0^\infty \frac{e^{ik_i(s-r)}}{\sqrt{s(s-r)}} \int_0^\infty [(L_i K_e^{(\pm)})(s, r') f_{n-1}^{(\pm)}(r') \\ &+ (L_i K_s)(s, r') f_{n-1}^{(\pm)}(r') + (L_i K_i^{(\pm)})(s, r') f_n^{(\pm)}(r')] dr' ds \quad (n \geq 1) \quad (r \geq 0), \end{aligned} \quad (24)$$

which is an integral equation of the second kind for $f_n^{(\pm)}$ ($n \geq 1$) when $f_{n-1}^{(\pm)}$ is already determined.

The expansion (10) enables equations (21) and (24) to be combined to show that the corresponding inversion of (9) is

$$\begin{aligned} (1 + \rho) f^{(\pm)}(r) &= \sqrt{r} \int_0^\infty \frac{e^{ik_i(s-r)}}{\sqrt{s(s-r)}} \left[2(L_i u_0^{(\pm)})(s) + \frac{i}{2\pi} \int_0^\infty \{(L_i K_i^{(\pm)})(s, r') \right. \\ &\quad \left. + \rho(L_i K_e^{(\pm)})(s, r') + \rho(L_i K_s)(s, r')\} f^{(\pm)}(r') dr' \right] ds \quad (r \geq 0), \end{aligned} \quad (25)$$

which is an integral equation of the second kind for $f^{(\pm)}$.

5 Evaluation of the Inversion Integrals

The two integrations introduced by the above inversion procedure can be evaluated exactly when plane wave representations are used. Consider the inversion of $e^{-i\beta r}$ for real values of β . First, it is readily shown from the definition (17) that

$$(L_i e^{-i\beta r})(s) = e^{-i\beta s} \begin{cases} (k_i^2 - \beta^2)^{1/2} & (|\beta| < k_i) \\ \text{isgn}\beta(\beta^2 - k_i^2)^{1/2} & (|\beta| > k_i) \end{cases} \quad (26)$$

Second, the evaluation of

$$\sqrt{r} \int_0^\infty \frac{e^{ik_i(s-r)}}{\sqrt{s(s-r)}} e^{-i\beta s} ds$$

requires that of the integral I given by

$$I(\lambda, r) = \sqrt{r} \int_0^\infty \frac{e^{i\lambda(s-r)}}{\sqrt{s(s-r)}} ds \quad (r > 0), \quad (27)$$

which vanishes at $\lambda = 0$ and, by consideration of $\frac{\partial I}{\partial \lambda}$, can be expressed in terms of the Fresnel integrals C_1 and S_1 defined by

$$C_1(x) + iS_1(x) = \sqrt{\frac{2}{\pi}} \int_0^x e^{iy^2} dy. \quad (28)$$

Thus

$$\begin{aligned} I &= -\pi\sqrt{2}e^{-i\pi/4} \{C_1[(\lambda r)^{1/2}] - iS_1[(\lambda r)^{1/2}]\} \quad (\lambda > 0), \\ I &= -\pi\sqrt{2}e^{i\pi/4} \{C_1[(|\lambda|r)^{1/2}] + iS_1[(|\lambda|r)^{1/2}]\} \quad (\lambda < 0). \end{aligned} \quad (29)$$

Hence it follows from (26) and (29) that

$$\sqrt{r} \int_0^\infty \frac{e^{ik_i(s-r)}}{\sqrt{s(s-r)}} (L_i e^{-i\beta r})(s) ds = \pi\sqrt{2}e^{-i\beta r} M_i(\beta, r), \quad (30)$$

where

$$M_i(\beta, r) = \begin{cases} (\beta^2 - k_i^2)^{1/2} e^{-i\pi/4} \left\{ C_1([\beta - k_i]r^{1/2}) + iS_1([\beta - k_i]r^{1/2}) \right\} & (\beta > k_i) \\ -(k_i^2 - \beta^2)^{1/2} e^{-i\pi/4} \left\{ C_1([k_i - \beta]r^{1/2}) - iS_1([k_i - \beta]r^{1/2}) \right\} & (|\beta| < k_i) \\ (\beta^2 - k_i^2)^{1/2} e^{i\pi/4} \left\{ C_1([k_i - \beta]r^{1/2}) - iS_1([k_i - \beta]r^{1/2}) \right\} & (\beta < -k_i) \end{cases} \quad (31)$$

Now in order to use (30), it is necessary to use plane wave representations of the functions to be inverted on the right hand side of (25). The standard representations of J_0 and Y_0 (Abramovitz and Stegun, 1964) are combined in the integral representation

$$H_0^{(2)}(k|r - r'|) = \frac{1}{\pi} \int_C e^{-ik(\tau - r') \cos \tau} d\tau, \quad (32)$$

where C is the contour from $-i\infty$ to $\pi + i\infty$ drawn along the negative imaginary axis, the real axis from 0 to π and a line parallel to the imaginary axis. Equation (32), which allows (18) to be derived from (16), can be generalized to

$$H_0^{(2)}[k\{(x - x')^2 + (y - y')^2\}^{1/2}] = \frac{1}{\pi} \int_C e^{-ik[(x - x') \cos \tau + |y - y'| \sin \tau]} d\tau. \quad (33)$$

Similarly, the integral representation

$$H_0^{(2)}(k|r - r'|) = -\frac{1}{\pi i} \int_{-\infty}^{\infty} e^{-ik|r - r'| \cosh v} dv$$

can be generalized to

$$H_0^{(2)}[k\{(x - x')^2 - (y - y')^2\}^{1/2}] = -\frac{1}{\pi i} \int_{-\infty}^{\infty} e^{-ik[|x - x'| \cosh v + (y - y') \sinh v]} dv$$

$$(|x - x'| > y - y'). \quad (34)$$

Now the forcing and kernel functions in (25) can be reduced, as follows, to single or rapidly convergent double integrals involving at most the Fresnel integrals. The function $u_0^{(\pm)}(r)$, given by (12), can be written in the form

$$\begin{aligned} u_0^{(\pm)}(r) &= \frac{1}{4} i H_0^{(2)} [k_e (r^2 + R^2 - 2rR \cos(\Phi - \alpha))^{1/2}] \\ &+ \frac{1}{2} \int_{-\infty}^{\infty} H_0^{(2)} [k_e (r^2 + R^2 + 2rR \cosh u)^{1/2}] h^{(\pm)}(u) du \\ &= \frac{i}{4\pi} \int_C e^{-ik_e [r \cos \tau - R \cos(\Phi - \alpha + \tau)]} d\tau \\ &- \frac{1}{2\pi i} \int_{-\infty}^{\infty} h^{(\pm)}(u) du \int_{-\infty}^{\infty} e^{-ik_e [r \cosh v + R \cosh(u+v)]} dv, \quad (35) \end{aligned}$$

by use of (33) and (34), and hence, from (30),

$$\begin{aligned} 2\sqrt{r} \int_0^{\infty} \frac{e^{ik_i(s-r)}}{\sqrt{s(s-r)}} (L_i u_0^{(\pm)})(s) ds &= \frac{i}{\sqrt{2}} \int_C M_i(k_e \cos \tau, r) e^{-ik_e [r \cos \tau - R \cos(\Phi - \alpha + \tau)]} d\tau \\ &+ i\sqrt{2} \int_{-\infty}^{\infty} h^{(\pm)}(u) du \int_{-\infty}^{\infty} M_i(k_e \cosh v, r) e^{-ik_e [r \cosh v + R \cosh(u+v)]} dv. \quad (36) \end{aligned}$$

where M_i is defined by (31). The contribution from the interior Green's function arises, according to (15) and (20), from

$$\begin{aligned} K_i^{(\pm)}(r, r') &= (\pm 1)^m H_0^{(2)} [k_i (r+r')] + 2 \sum_{n=1}^{n=m-1} (\pm 1)^n H_0^{(2)} [k_i (r^2 + r'^2 - 2rr' \cos n\pi/m)^{1/2}] \\ &= (\pm 1)^m H_0^{(2)} [k_i (r+r')] + 2 \int_C e^{-ik_i r \cos \tau} \sum_{n=1}^{n=m-1} (\pm 1)^n e^{-ik_i r' \cos(n\pi/m + \tau)} d\tau, \end{aligned}$$

by use of (33). The first term can be inverted by comparison with (18) and hence, after further use of (30),

$$\frac{i\sqrt{r}}{2\pi} \int_0^{\infty} \frac{e^{ik_i(s-r)}}{\sqrt{s(s-r)}} (L_i K_i^{(\pm)})(s, r') ds = \frac{(\pm 1)^m}{\pi} \left(\frac{r}{r'} \right) \frac{e^{ik_i(r+r')}}{r+r'}$$

$$+ i\sqrt{2} \int_C e^{-ik_i r \cos \tau} M_i(k_i \cos \tau, r) \sum_{n=1}^{n=m-1} (\pm 1)^n e^{-ik_i r' \cos(n\pi/m+\tau)} d\tau. \quad (37)$$

The contribution from the exterior Green's function arises from K_e^\pm , given by (13), (14) and (22), and K_s , defined by (23). On using (34) and then (30), it follows that

$$\begin{aligned} \frac{i\sqrt{r}}{2\pi} \int_0^\infty \frac{e^{ik_i(s-r)}}{\sqrt{s(s-r)}} (L_i K_e^{(+)})(s, r') ds &= \frac{i}{2\sqrt{2\pi(\pi-\alpha)}} \times \\ \int_{-\infty}^\infty \frac{du}{\tanh \frac{\pi(u+\pi i)}{2(\pi-\alpha)}} \int_{-\infty}^\infty M_i(k_e \cosh v, r) e^{-ik_e[r \cosh v + r' \cosh(u+v)]} dv. & \quad (38) \end{aligned}$$

Similarly

$$\begin{aligned} \frac{i\sqrt{r}}{2\pi} \int_0^\infty \frac{e^{ik_i(s-r)}}{\sqrt{s(s-r)}} (L_i K_e^{(-)})(s, r') ds &= \frac{i}{2\sqrt{2\pi(\pi-\alpha)}} \times \\ \int_{-\infty}^\infty \frac{du}{\sinh \frac{\pi(u+\pi i)}{2(\pi-\alpha)}} \int_{-\infty}^\infty M_i(k_e \cosh v, r) e^{-ik_e[r \cosh v + r' \cosh(u+v)]} dv. & \quad (39) \end{aligned}$$

For the remaining term in (25), write

$$\begin{aligned} (L_i K_s)(s, r') &= [L_e H_0^{(2)}(k_e |r - r'|)](s, r') - [L_i H_0^{(2)}(k_i |r - r'|)](s, r') \\ &\quad - [(L_e - L_i) H_0^{(2)}(k_e |r - r'|)](s, r') \\ &= -\frac{2i}{\pi} \frac{e^{ik_e(r'-s)} - e^{ik_i(r'-s)}}{r' - s} \\ &\quad - [(L_e - L_i) H_0^{(2)}(k_e |r - r'|)](s, r') \quad (s, r' \geq 0), \quad (40) \end{aligned}$$

as in the derivation of (18). The contribution of the first term to

$$\frac{i\sqrt{r}}{2\pi} \int_0^\infty \frac{e^{ik_i(s-r)}}{\sqrt{s(s-r)}} (L_i K_s)(s, r') ds$$

is therefore

$$\begin{aligned} & \frac{\sqrt{r}}{\pi^2} e^{ik_i(r'-r)} \int_0^\infty \frac{e^{i(k_i-k_e)(s-r)} - 1}{\sqrt{s(s-r)}(r'-s)} ds \\ &= \frac{1}{\pi^2} \frac{e^{ik_i(r'-r)}}{r'-r} \left[I(k_i - k_e, r) - \left(\frac{r}{r'} \right)^{1/2} I(k_i - k_e, r') \right], \end{aligned}$$

where I is defined by (27). For the second term in (40), rewrite (32) as

$$\begin{aligned} H_0^{(2)}(k|r-r'|)(s, r') &= \frac{1}{\pi} \int_{-1}^1 e^{-ik(r-r')v} \frac{dv}{(1-v^2)^{1/2}} \\ &+ \frac{i}{\pi} \int_1^\infty [e^{-ik(r-r')v} + e^{ik(r-r')v}] \frac{dv}{(v^2-1)^{1/2}} \end{aligned} \quad (41)$$

and observe that the formula for $(L_e - L_i)$ corresponding to (26) is

$$[(L_e - L_i)e^{-ik_e v r}](s) = e^{-ik_e v s} \begin{cases} k_e(1-v^2)^{1/2} - (k_i^2 - k_e^2 v^2)^{1/2} & (|v| < 1) \\ i \operatorname{sgn} v k_e (v^2 - 1)^{1/2} - (k_i^2 - k_e^2 v^2)^{1/2} & (1 < |v| < k_i/k_e) \\ i \operatorname{sgn} v [k_e(v^2 - 1)^{1/2} - (k_e^2 v^2 - k_i^2)^{1/2}] & (|v| > k_i/k_e) \end{cases} \quad (42)$$

The factors on the right hand side of (42) effectively replace those in (26) and account must be taken of this when adapting (30) in order to use the function M_i given by (31). Hence the above results imply that

$$\begin{aligned} \frac{i\sqrt{r}}{2\pi} \int_0^\infty \frac{e^{ik_i(s-r)}}{\sqrt{s(s-r)}} (L_i K_s)(s, r') ds &= \frac{1}{\pi^2} \frac{e^{ik_i(r'-r)}}{r'-r} \left[I(k_i - k_e, r) - \left(\frac{r}{r'} \right)^{1/2} I(k_i - k_e, r') \right] \\ &+ \frac{i}{\pi\sqrt{2}} \int_{-1}^1 e^{-ik_e(r-r')v} M_i(k_e v, r) \left[\frac{k_e}{(k_i^2 - k_e^2 v^2)^{1/2}} - \frac{1}{(1-v^2)^{1/2}} \right] dv \\ &- \frac{1}{\pi\sqrt{2}} \int_1^\infty [e^{-ik_e(r-r')v} M_i(k_e v, r) \psi(v) + e^{ik_e(r-r')v} M_i(-k_e v, r) \psi(-v)] dv, \end{aligned} \quad (43)$$

where I is given by (29) and

$$\psi(v) = \begin{cases} \frac{i \operatorname{sgn} v k_e}{(k_i^2 - k_e^2 v^2)^{1/2}} - \frac{1}{(v^2 - 1)^{1/2}} & (1 < |v| < k_i/k_e) \\ \frac{k_e}{(k_e^2 v^2 - k_i^2)^{1/2}} - \frac{1}{(v^2 - 1)^{1/2}} & (|v| > k_i/k_e) \end{cases} \quad (44)$$

Evidently the expression (43) is bounded at $r = r'$, with the integral remaining convergent in this limit because the integrand is $O(v^{-2})$ as $v \rightarrow \infty$.

Thus equations (36-39) and (43) furnish simplified forms of the integrals on the right side of the integral equation (25). Additional similar terms will be introduced on relaxation of either or both of the assumptions $\alpha = \pi/2m$ and $\Phi + \alpha < \pi$. Asymptotic estimates for large r show that $f^{(\pm)}$ must be a linear combination of $e^{-ik_1 r}/\sqrt{r}$ and $e^{-ik_2 r}/\sqrt{r}$ in this limit, as might be anticipated. This information will facilitate a numerical solution for the even and odd components of the normal velocities at the interfaces of the wedge of angle 2α .

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PENETRABLE WEDGE SCATTERING VIA A PAIR OF COUPLED INTEGRAL EQUATIONS OF THE SECOND KIND

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ABSTRACT. A pair of coupled integral equations of the second kind are specialized to the even and odd symmetry components for the scattering by the penetrable wedge. The unknowns are both the scalar field and its normal derivative on the two wedge surfaces that separate the interior and exterior regions of different wave speeds and constitutive parameters (density or dielectric constant, depending on the specific physical application). A discretization and collocation procedure transforms the operator equations into coupled matrix equations, which are solved numerically. Further extensions and plans to include anticipated asymptotic behavior far from the line source and wedge apex are outlined.

I. ADAPTATION OF KLEINMAN AND MARTIN ANALYSIS

The transmission problem for the penetrable wedge of Figure 1 consists of appropriate exterior and interior waves

$$(\nabla^2 + k_e^2) u_e(r, \phi) = -\frac{1}{r} \delta(r - r_0) \delta(\phi - \phi_0) \quad (\alpha \leq \phi \leq 2\pi - \alpha) \quad (1)$$

$$(\nabla^2 + k_i^2) u_i(r, \phi) = 0 \quad (-\alpha \leq \phi \leq \alpha) \quad (2)$$

subject to the two-dimensional Sommerfeld radiation condition plus the continuity boundary conditions

$$u_e(r, \pm\alpha) = u_i(r, \pm\alpha) \quad (3)$$

$$\frac{\partial}{\partial \phi} u_e(r, \pm\alpha) = \rho \frac{\partial}{\partial \phi} u_i(r, \pm\alpha) \quad (4)$$

where $\rho = \rho_e/\rho_i$ is the ratio of exterior to interior constitutive parameters (ambient density or dielectric constant). A pair of integral equations derived by Kleinman and Martin [4] for the scalar field u and its normal derivative $\partial u/\partial n$ on the boundary of a penetrable obstacle, are adapted to the semi-infinite wedge faces of Figure 1 to effect an accurate numerical solution of the scattering problem.

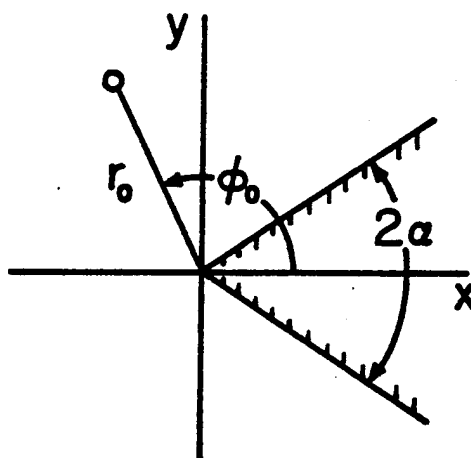


FIG.1. Penetrable Wedge and Line Source

Define the surface distributions

$$v_{\pm}(r) = u(r, \alpha) \pm u(r, -\alpha) \quad (5)$$

$$w_{\pm}(r) = \frac{\partial}{\partial n} u(r, \alpha) \pm \frac{\partial}{\partial n} u(r, -\alpha) \quad (6)$$

in order to succinctly decompose the field into its symmetric and antisymmetric components with respect to the wedge bisector (the x axis).

After careful algebra, the selected integral equations from [4] are written in a form defined on a single semi-infinite domain ($0 \leq r < \infty$) to account for the even/odd symmetry:

$$\begin{aligned} (1 + \rho)v_{\pm}(r) \pm \int_0^{\infty} dr' v_{\pm}(r') \frac{\partial}{\partial n'} [G_e(r, \alpha; r', -\alpha) - \rho G_i(r, \alpha; r', -\alpha)] \\ - \int_0^{\infty} dr' w_{\pm}(r') \{ [G_e(r, \alpha; r', \alpha) - G_i(r, \alpha; r', \alpha)] \pm [G_e(r, \alpha; r', -\alpha) - G_i(r, \alpha; r', -\alpha)] \} \\ = 2v_{\pm}^{\text{inc}}(r) \quad (7) \end{aligned}$$

$$\begin{aligned} (1 + \rho)w_{\pm}(r) \mp \int_0^{\infty} dr' w_{\pm}(r') \frac{\partial}{\partial n} [\rho G_e(r, \alpha; r', -\alpha) - G_i(r, \alpha; r', -\alpha)] + \rho \\ \cdot \int_0^{\infty} dr' v_{\pm}(r') \frac{\partial^2}{\partial n \partial n'} \{ [G_e(r, \alpha; r', \alpha) - G_i(r, \alpha; r', \alpha)] \pm [G_e(r, \alpha; r', -\alpha) - G_i(r, \alpha; r', -\alpha)] \} \\ = 2\rho w_{\pm}^{\text{inc}}(r). \quad (8) \end{aligned}$$

This arrangement ensures regular or (at worst) weakly singular kernels, which involve the differences of the scaled interior and exterior free-space Green's functions and normal derivatives thereof:

$$\frac{\partial}{\partial n'} [G_e(r, \alpha; r', -\alpha) - \rho G_i(r, \alpha; r', -\alpha)] = \frac{\sin 2\alpha}{2i} \frac{r}{R} \left[\rho k_i H_1^{(1)}(k_i R) - k_e H_1^{(1)}(k_e R) \right] \quad (9)$$

$$G_e(r, \alpha; r', \alpha) - G_i(r, \alpha; r', \alpha) = \frac{1}{2i} \left[H_0^{(1)}(k_e |r - r'|) - H_0^{(1)}(k_i |r - r'|) \right] \\ \xrightarrow{r \rightarrow r'} \frac{1}{\pi} \ln \frac{k_e}{k_i} \quad (10)$$

$$G_e(r, \alpha; r', -\alpha) - G_i(r, \alpha; r', -\alpha) = \frac{1}{2i} \left[H_0^{(1)}(k_e R) - H_0^{(1)}(k_i R) \right] \quad (11)$$

$$\frac{\partial}{\partial n} [\rho G_e(r, \alpha; r', -\alpha) - G_i(r, \alpha; r', -\alpha)] = \frac{\sin 2\alpha}{2i} \frac{r'}{R} \left[k_i H_1^{(1)}(k_i R) - \rho k_e H_1^{(1)}(k_e R) \right] \quad (12)$$

$$\frac{\partial^2}{\partial n \partial n'} [G_e(r, \alpha; r', \alpha) - G_i(r, \alpha; r', \alpha)] = \frac{\left[k_e H_1^{(1)}(k_e |r - r'|) - k_i H_1^{(1)}(k_i |r - r'|) \right]}{2i |r - r'|} \\ \xrightarrow{r \rightarrow r'} \frac{1}{2\pi} \left(k_e^2 \ln \frac{k_e |r - r'|}{2} - k_i^2 \ln \frac{k_i |r - r'|}{2} \right) \quad (13)$$

$$\frac{\partial^2}{\partial n \partial n'} [G_e(r, \alpha; r', -\alpha) - G_i(r, \alpha; r', -\alpha)] \\ = \frac{1}{2iR} \left\{ \left[\cos 2\alpha - \frac{2rr' \sin^2 2\alpha}{R^2} \right] \left[k_i H_1^{(1)}(k_i R) - k_e H_1^{(1)}(k_e R) \right] \right. \\ \left. + \frac{rr' \sin^2 2\alpha}{R} \left[k_i^2 H_0^{(1)}(k_i R) - k_e^2 H_0^{(1)}(k_e R) \right] \right\}. \quad (14)$$

The distance

$$R = (r^2 + r'^2 - 2rr' \cos 2\alpha)^{1/2} \quad (15)$$

between points on opposing wedge faces is zero only as both points coalesce at the apex ($r = r' = 0$). Excitation from the line source of unit strength at (r_0, ϕ_0) gives the forcing terms

$$v_{\pm}^{\text{inc}}(r) = \frac{i}{4} \left[H_0^{(1)} \left(k_e \sqrt{r^2 + r_0^2 - 2rr_0 \cos(\phi_0 - \alpha)} \right) \right. \\ \left. \pm H_0^{(1)} \left(k_e \sqrt{r^2 + r_0^2 - 2rr_0 \cos(\phi_0 + \alpha)} \right) \right] \quad (16)$$

$$w_{\pm}^{\text{inc}}(r) = \frac{ik_e r_0}{4} \left[\sin(\phi_0 - \alpha) \frac{H_1^{(1)} \left(k_e \sqrt{r^2 + r_0^2 - 2rr_0 \cos(\phi_0 - \alpha)} \right)}{\sqrt{r^2 + r_0^2 - 2rr_0 \cos(\phi_0 - \alpha)}} \right. \\ \left. \pm \sin(\phi_0 + \alpha) \frac{H_1^{(1)} \left(k_e \sqrt{r^2 + r_0^2 - 2rr_0 \cos(\phi_0 + \alpha)} \right)}{\sqrt{r^2 + r_0^2 - 2rr_0 \cos(\phi_0 + \alpha)}} \right]. \quad (17)$$

II. PIECEWISE CONSTANT APPROXIMATION WITH COLLOCATION

A moment-method expansion in terms of the pulse functions of width Δ in r ,

$$P_m(r) = \begin{cases} 1, & (m-1)\Delta \leq r \leq m\Delta \\ 0, & \text{otherwise} \end{cases} \quad (18)$$

proceeds formally, without regard to truncation or anticipation of asymptotic behavior, as

$$v_{\pm}(r) = \sum_{m=1}^{\infty} v_m^{\pm} P_m(r), \quad \frac{w_{\pm}(r)}{k_e} = \sum_{m=1}^{\infty} w_m^{\pm} P_m(r). \quad (19)$$

Collocation at the center points of each pulse

$$r_l = (l - 1/2)\Delta \quad (20)$$

yields the coupled matrix equations

$$(1 + \rho)[I][v^{\pm}] \pm [A][v^{\pm}] - [B^{\pm}][w^{\pm}] = [f^{\pm}] \quad (21)$$

$$(1 + \rho)[I][w^{\pm}] \mp [C][w^{\pm}] + \rho[D^{\pm}][v^{\pm}] = \rho[g^{\pm}]. \quad (22)$$

Upon discretization, the attractive feature of integral equations of the second kind is maintained in the form of diagonal dominance. The elements of the column vectors $[f^{\pm}]$ and $[g^{\pm}]$ are the right hand sides of (7) and (8), respectively, evaluated at the collocation points

$$f_l^{\pm} = 2v_{\pm}^{\text{inc}}(r_l), \quad g_l^{\pm} = \frac{2}{k_e} w_{\pm}^{\text{inc}}(r_l), \quad (23)$$

and $[I]$ is the identity matrix.

Integral forms for the dimensionless coefficients are

$$A_{lm} = \int_{(m-1)\Delta}^{m\Delta} dr' \frac{\partial}{\partial n'} [G_e(r_l, \alpha; r', -\alpha) - \rho G_i(r_l, \alpha; r', -\alpha)] \quad (24)$$

$$B_{lm}^{\pm} = k_e \int_{(m-1)\Delta}^{m\Delta} dr' \{ [G_e(r_l, \alpha; r', \alpha) - G_i(r_l, \alpha; r', \alpha)] \pm [G_e(r_l, \alpha; r', -\alpha) - G_i(r_l, \alpha; r', -\alpha)] \} \quad (25)$$

$$C_{lm} = \int_{(m-1)\Delta}^{m\Delta} dr' \frac{\partial}{\partial n} [\rho G_e(r_l, \alpha; r', -\alpha) - G_i(r_l, \alpha; r', -\alpha)] \quad (26)$$

$$D_{lm}^{\pm} = \frac{1}{k_e} \int_{(m-1)\Delta}^{m\Delta} dr' \frac{\partial^2}{\partial n \partial n'} \{ [G_e(r_l, \alpha; r', \alpha) - G_i(r_l, \alpha; r', \alpha)] \pm [G_e(r_l, \alpha; r', -\alpha) - G_i(r_l, \alpha; r', -\alpha)] \}. \quad (27)$$

III. COMPUTATIONAL DETAILS

Machine computation is facilitated by introducing the normalized parameters

$$\begin{aligned} x' &= k_e r' & x_l &= k_e r_l & \sigma &= k_i/k_e \\ \delta &= k_e \Delta & F &= k_e R = (x_l^2 + x'^2 - 2x_l x' \cos 2\alpha)^{1/2}. \end{aligned} \quad (28)$$

The required matrix elements are now obtained by quadrature, with careful attention to extract the removable and logarithmic singularities that occur in two species of the diagonal terms ($l = m$):

$$A_{lm} = \frac{x_l \sin 2\alpha}{2i} \int_{(m-1)\delta}^{m\delta} dx' \frac{1}{F} [\rho \sigma H_1^{(1)}(\sigma F) - H_1^{(1)}(F)] \quad (29)$$

$$\begin{aligned} B_{lm}^{\pm} &= B_{lm}^{(1)} \pm B_{lm}^{(2)} \\ B_{lm}^{(1)} &= \int_{(m-1)\delta}^{m\delta} dx' \underbrace{\frac{[H_0^{(1)}(|x_l - x'|) - H_0^{(1)}(\sigma|x_l - x'|)]}{2i}}_{\rightarrow -\frac{1}{\pi} \ln \sigma \text{ as } x' \rightarrow x_l} \end{aligned} \quad (30)$$

$$B_{lm}^{(2)} = \frac{1}{2i} \int_{(m-1)\delta}^{m\delta} dx' [H_0^{(1)}(F) - H_0^{(1)}(\sigma F)] \quad (31)$$

$$C_{lm} = \frac{\sin 2\alpha}{2i} \int_{(m-1)\delta}^{m\delta} dx' \frac{x'}{F} [\sigma H_1^{(1)}(\sigma F) - \rho H_1^{(1)}(F)] \quad (32)$$

$$D_{lm}^{\pm} = D_{lm}^{(1)} \pm D_{lm}^{(2)}$$

$$D_{lm}^{(1)} = \int_{(m-1)\delta}^{m\delta} dx' \frac{H_1^{(1)}(|x_l - x'|) - \sigma H_1^{(1)}(\sigma|x_l - x'|)}{2i|x_l - x'|} \quad (33)$$

$\rightarrow \frac{1}{2\pi} \left[\ln \frac{|x_l - x'|}{2} - \sigma^2 \ln \frac{\sigma|x_l - x'|}{2} \right]$ as $x' \rightarrow x_l$

$$D_{lm}^{(2)} = \frac{1}{2i} \int_{(m-1)\delta}^{m\delta} dx' \frac{1}{F} \left\{ \left[\cos 2\alpha - \frac{2x_l x' \sin^2 2\alpha}{F^2} \right] \left[\sigma H_1^{(1)}(\sigma F) - H_1^{(1)}(F) \right] \right. \\ \left. + \frac{x_l x' \sin^2 2\alpha}{F} \left[\sigma^2 H_0^{(1)}(\sigma F) - H_0^{(1)}(F) \right] \right\}. \quad (34)$$

A Fortran program that implements the above analysis is included in Appendix A. A sharp truncation of both the expansions (19) and the collocation points (20) at $l, m = N$ results in close (not large r) behavior of $v(r)$ and $w(r)$ that is surprisingly persistent. However, an accurate solution must account for the infinite domain. A physically-based approach is to include both cylindrical waves that Davis¹ extracts from an asymptotic estimate:

$$v_{\pm}(r) = \sum_{m=1}^N v_m^{\pm} P_m(r) + v_{N+1}^{\pm} \frac{e^{ik_i r}}{\sqrt{k_i r}} + v_{N+2}^{\pm} \frac{e^{ik_e r}}{\sqrt{k_e r}} \quad (35)$$

$$\frac{w_{\pm}(r)}{k_e} = \sum_{m=1}^N w_m^{\pm} P_m(r) + w_{N+1}^{\pm} \frac{e^{ik_i r}}{\sqrt{k_i r}} + w_{N+2}^{\pm} \frac{e^{ik_e r}}{\sqrt{k_e r}}. \quad (36)$$

The infinite-range, highly oscillatory integrals required by this modification are calculated with the aid of a $\pi/2$ rotation in the complex plane, resulting in exponentially decaying integrands that are more suitable for direct quadrature.

IV. EXTENSIONS AND CONCLUSIONS

Accurate and efficient implementation of the above mathematics requires careful numerical analysis. The proposed asymptotic ansatz to account for the far behavior of the unknown surface distributions is best verified first for a simpler, known problem: the soft (or hard) half-plane. Although the selected set of coupled integral equations does introduce a *pair* of unknown functions, as opposed to some methods that require only a single unknown, the attractive features are weakly singular kernels and second-kind structure. Also, having direct access to both the scalar field and its normal derivative on the boundary is a benefit to the techniques of far-field evaluation based on various Green's functions.

¹A.M.J. Davis, *Acoustical Scattering by a Penetrable Wedge*, 1994, p. 17, in this report.

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APPENDIX A. FORTRAN PROGRAM FOR COUPLED INTEGRAL EQUATIONS

C	FILE "COUPSEQ FORTRAN A" - COUPLED INTEGRAL EQUATIONS FOR	COU00010
C	DIELECTRIC WEDGE SCATTERING.	COU00020
C	15 MAY 1994 R.W.SCHARSTEIN UNIVERSITY OF ALABAMA EE DEPT.	COU00030
C		COU00040
	PARAMETER (NN=50)	COU00050
	EXTERNAL FA,FAREAL,FAIMAG,FB1,FB1REAL,FB1IMAG,FB2,FB2REAL,FB2IMAG	COU00060
	EXTERNAL FC,FCREAL,FCIMAG,FD2,FD2REAL,FD2IMAG	COU00070
	EXTERNAL FD1A,FD1AREA,FD1AIMA,FD1B,FD1BREA,FD1BIMA	COU00080
	COMMON /COM/XL,ALPHA,RHO,SIGMA	COU00090
	COMMON /SOURCE/X0,PHI0,ALPHA1	COU00100
	COMPLEX A(NN,NN),B1(NN,NN),B2(NN,NN),C(NN,NN),D1(NN,NN),D2(NN,NN)	COU00110
	COMPLEX B(NN,NN),D(NN,NN)	COU00120
	COMPLEX Q(2*NN,2*NN),Y(2*NN),VINC,WINC,V,W	COU00130
	COMPLEX CROUT(2*NN,4*NN),TEMP,DET,BIG,ERROR	COU00140
	DIMENSION LP(2*NN)	COU00150
	INTEGER P	COU00160
C		COU00170
	DATA PI/3.1415927/	COU00180
C		COU00190
C	CHOOSE EVEN (P=2) OR ODD (P=1) SYMMETRY	COU00200
	P=2	COU00210
C		COU00220
C	QUADRATURE PARAMETERS	COU00230
	EREL = 0.01	COU00240
	EABS = 0.	COU00250
C	PHYSICAL CONSTANTS	COU00260
	ALPHA = PI/6.	COU00270
	ALPHA1 = ALPHA	COU00280
	RHO = 0.1	COU00290
	SIGMA = SQRT(10.)	COU00300
C	SOURCE COORDINATES	COU00310
	X0 = 2.	COU00320
	PHI0 = PI/2.	COU00330
C	MORE INTEGRATION PARAMETERS	COU00340
	XPMAX = 25.	COU00350
	DELTA = XPMAX/FLOAT(NN)	COU00360
C		COU00370
	DO 10 L=1,NN	COU00380
	XL = (FLOAT(L)-0.5)*DELTA	COU00390
	DO 10 M=1,NN	COU00400
	X1 = FLOAT(M-1)*DELTA	COU00410
	X2 = FLOAT(M)*DELTA	COU00420
	CALL QDAG(FAREAL,X1,X2,EABS,EREL,6,AR,EREST)	COU00430
	CALL QDAG(FAIMAG,X1,X2,EABS,EREL,6,AI,EREST)	COU00440
	A(L,M) = CMLPX(AR,AI)*XL*SIN(2.*ALPHA)/CMLPX(0.,2.)	COU00450
C	WRITE(6,100) L,M,A(L,M)	COU00460
	CALL QDAG(FB1REAL,X1,X2,EABS,EREL,6,B1R,EREST)	COU00470
	CALL QDAG(FB1IMAG,X1,X2,EABS,EREL,6,B1I,EREST)	COU00480
	B1(L,M) = CMLPX(B1R,B1I)	COU00490
C	WRITE(6,100) L,M,B1(L,M)	COU00500
	CALL QDAG(FB2REAL,X1,X2,EABS,EREL,6,B2R,EREST)	COU00510
	CALL QDAG(FB2IMAG,X1,X2,EABS,EREL,6,B2I,EREST)	COU00520
	B2(L,M) = CMLPX(B2R,B2I)/CMLPX(0.,2.)	COU00530
C	WRITE(6,100) L,M,B2(L,M)	COU00540
	CALL QDAG(FCREAL,X1,X2,EABS,EREL,6,CR,EREST)	COU00550
	CALL QDAG(FCIMAG,X1,X2,EABS,EREL,6,CI,EREST)	COU00560
	C(L,M) = CMLPX(CR,CI)*SIN(2.*ALPHA)/CMLPX(0.,2.)	COU00570
C	WRITE(6,100) L,M,C(L,M)	COU00580
	IF(L.EQ.M) GO TO 20	COU00590
	CALL QDAG(FD1AREA,X1,X2,EABS,EREL,6,D1R,EREST)	COU00600

CALL QDAG(FD1AIMA,X1,X2,EABS,EREL,6,D1I,EREST)	COU00610
D1(L,M) = CMPLX(D1R,D1I)	COU00620
GO TO 30	COU00630
20 CONTINUE	COU00640
CALL QDAG(FD1BREA,X1,X2,EABS,EREL,6,D1R,EREST)	COU00650
CALL QDAG(FD1BIMA,X1,X2,EABS,EREL,6,D1I,EREST)	COU00660
D1(L,M) = CMPLX(D1R,D1I) + DELTA*((1.-SIGMA**2)*(ALOG(DELTA/4.)	COU00670
& -1.) - SIGMA**2*ALOG(SIGMA))/6.2831853	COU00680
30 CONTINUE	COU00690
C WRITE(6,100) L,M,D1(L,M)	COU00700
CALL QDAG(FD2REAL,X1,X2,EABS,EREL,6,D2R,EREST)	COU00710
CALL QDAG(FD2IMAG,X1,X2,EABS,EREL,6,D2I,EREST)	COU00720
D2(L,M) = CMPLX(D2R,D2I)/CMPLX(0.,2.)	COU00730
C WRITE(6,100) L,M,D2(L,M)	COU00740
10 CONTINUE	COU00750
C	COU00760
C BOTH EVEN (+) AND ODD (-) SYMMETRY PROBLEMS	COU00770
DO 40 L=1,NN	COU00780
DO 40 M=1,NN	COU00790
B(L,M) = B1(L,M) + (-1)**P*B2(L,M)	COU00800
40 D(L,M) = D1(L,M) + (-1)**P*D2(L,M)	COU00810
C THE BIG SUPER-MATRIX Q	COU00820
DO 42 L=1,NN	COU00830
DO 42 M=1,NN	COU00840
Q(L,M) = (-1)**P*A(L,M)	COU00850
IF(L.EQ.M) Q(L,M) = Q(L,M) + 1.+RHO	COU00860
Q(L,M+NN) = -B(L,M)	COU00870
Q(L+NN,M) = RHO*D(L,M)	COU00880
Q(L+NN,M+NN) = (-1)**(P+1)*C(L,M)	COU00890
IF(L.EQ.M) Q(L+NN,M+NN) = Q(L+NN,M+NN) + 1.+RHO	COU00900
42 CONTINUE	COU00910
C THE BIG RIGHT-HAND COLUMN VECTOR	COU00920
DO 44 L=1,NN	COU00930
XL = (FLOAT(L)-0.5)*DELTA	COU00940
Y(L) = 2.*VINC(XL,P)	COU00950
44 Y(L+NN) = 2.*RHO*WINC(XL,P)	COU00960
C INVERT THE MATRIX Q AND SOLVE THE SYSTEM OF LINEAR EQUATIONS	COU00970
C USING CROUTC (INCLUDED)	COU00980
DO 46 L=1,2*NN	COU00990
DO 46 M=1,2*NN	COU01000
46 CROUT(L,M) = Q(L,M)	COU01010
CALL CROUTC(2*NN,2*NN,0,CROUT,1.E-7,DET,IERR,LP)	COU01020
WRITE(6,700) IERR	COU01030
700 FORMAT(/,7X,'SUBROUTINE 'CROUTC' IERR = ',I3)	COU01040
C CHECK MATRIX INVERSION	COU01050
BIG = CMPLX(0.,0.)	COU01060
DO 57 K=1,2*NN	COU01070
DO 57 L=1,2*NN	COU01080
TEMP = CMPLX(0.,0.)	COU01090
DO 58 M=1,2*NN	COU01100
58 TEMP = TEMP + CROUT(K,M+2*NN)*Q(M,L)	COU01110
ERROR = TEMP	COU01120
IF(K.EQ.L) ERROR = TEMP - CMPLX(1.,0.)	COU01130
IF(CABS(ERROR).GT.CABS(BIG)) BIG = ERROR	COU01140
57 CONTINUE	COU01150
WRITE(6,710) BIG	COU01160
710 FORMAT(/,7X,'BIGGEST ERROR IN MATRIX INVERSION = ',2(E12.5,2X))	COU01170
C RESULTANT COLUMN VECTORS	COU01180
WRITE(6,200)	COU01190
200 FORMAT(/,4X,'N',14X,'V',22X,'W',/)	COU01200

210	FORMAT(2X,I3,2(2X,E12.5,2X,F7.2))	COU01210
	DO 60 L=1,NN	COU01220
	V = CMPLX(0.,0.)	COU01230
	W = CMPLX(0.,0.)	COU01240
	DO 62 M=1,2*NN	COU01250
	V = V + CROUT(L,M+2*NN)*Y(M)	COU01260
62	W = W + CROUT(L+NN,M+2*NN)*Y(M)	COU01270
	WRITE(6,210) L,CABS(V),CDEG(V),CABS(W),CDEG(W)	COU01280
60	CONTINUE	COU01290
100	FORMAT(2X,I3,2X,I3,2X,E14.7,2X,E14.7)	COU01300
	STOP	COU01310
	END	COU01320
C		COU01330
C		COU01340
C		COU01350
	FUNCTION FA(XP)	COU01360
	COMMON /COM/XL,ALPHA,RHO,SIGMA	COU01370
	COMPLEX FA,H1,H2	COU01380
	F = SQRT(XL**2+XP**2-2.*XL*XP*COS(2.*ALPHA))	COU01390
	H1 = CMPLX(BSJ1(SIGMA*F),BSY1(SIGMA*F))	COU01400
	H2 = CMPLX(BSJ1(F),BSY1(F))	COU01410
	FA = (RHO*SIGMA*H1-H2)/F	COU01420
	RETURN	COU01430
	END	COU01440
C		COU01450
C		COU01460
	FUNCTION FAREAL(XP)	COU01470
	COMPLEX FA	COU01480
	FAREAL = REAL(FA(XP))	COU01490
	RETURN	COU01500
	END	COU01510
C		COU01520
C		COU01530
	FUNCTION FAIMAG(XP)	COU01540
	COMPLEX FA	COU01550
	FAIMAG = AIMAG(FA(XP))	COU01560
	RETURN	COU01570
	END	COU01580
C		COU01590
C		COU01600
C		COU01610
	FUNCTION FB1(XP)	COU01620
	COMMON /COM/ XL,ALPHA,RHO,SIGMA	COU01630
	COMPLEX FB1,H1,H2	COU01640
	D = ABS(XL-XP)	COU01650
	IF(D.LT.0.001) GO TO 10	COU01660
	H1 = CMPLX(BSJ0(D),BSY0(D))	COU01670
	H2 = CMPLX(BSJ0(SIGMA*D),BSY0(SIGMA*D))	COU01680
	FB1 = (H1-H2)/CMPLX(0.,2.)	COU01690
	RETURN	COU01700
10	FB1 = -ALOG(SIGMA)/3.1415927	COU01710
	RETURN	COU01720
	END	COU01730
C		COU01740
C		COU01750
	FUNCTION FB1REAL(XP)	COU01760
	COMPLEX FB1	COU01770
	FB1REAL = REAL(FB1(XP))	COU01780
	RETURN	COU01790
	END	COU01800

C		COU01810
C		COU01820
	FUNCTION FB1IMAG(XP)	COU01830
	COMPLEX FB1	COU01840
	FB1IMAG = AIMAG(FB1(XP))	COU01850
	RETURN	COU01860
	END	COU01870
		COU01880
C		COU01890
C		COU01900
C		COU01910
	FUNCTION FB2(XP)	COU01920
	COMMON /COM/ XL,ALPHA,RHO,SIGMA	COU01930
	COMPLEX FB2,H1,H2	COU01940
	F = SQRT(XL**2+XP**2-2.*XL*XP*COS(2.*ALPHA))	COU01950
	H1 = CMPLX(BSJ0(F),BSY0(F))	COU01960
	H2 = CMPLX(BSJ0(SIGMA*F),BSY0(SIGMA*F))	COU01970
	FB2 = H1-H2	COU01980
	RETURN	COU01990
	END	COU02000
		COU02010
C		COU02020
C		COU02030
	FUNCTION FB2REAL(XP)	COU02040
	COMPLEX FB2	COU02050
	FB2REAL = REAL(FB2(XP))	COU02060
	RETURN	COU02070
	END	COU02080
		COU02090
C		COU02100
C		COU02110
	FUNCTION FB2IMAG(XP)	COU02120
	COMPLEX FB2	COU02130
	FB2IMAG = AIMAG(FB2(XP))	COU02140
	RETURN	COU02150
	END	COU02160
		COU02170
C		COU02180
C		COU02190
C		COU02200
	FUNCTION FC(XP)	COU02210
	COMMON /COM/ XL,ALPHA,RHO,SIGMA	COU02220
	COMPLEX FC,H1,H2	COU02230
	F = SQRT(XL**2+XP**2-2.*XL*XP*COS(2.*ALPHA))	COU02240
	H1 = CMPLX(BSJ1(SIGMA*F),BSY1(SIGMA*F))	COU02250
	H2 = CMPLX(BSJ1(F),BSY1(F))	COU02260
	FC = XP*(SIGMA*H1-RHO*H2)/F	COU02270
	RETURN	COU02280
	END	COU02290
		COU02300
C		COU02310
C		COU02320
	FUNCTION FCREAL(XP)	COU02330
	COMPLEX FC	COU02340
	FCREAL = REAL(FC(XP))	COU02350
	RETURN	COU02360
	END	COU02370
		COU02380
C		COU02390
C		COU02400
	FUNCTION FCIMAG(XP)	
	COMPLEX FC	
	FCIMAG = AIMAG(FC(XP))	
	RETURN	
	END	
C		

C		COU02410
C		COU02420
	FUNCTION FD1A(XP)	COU02430
C	USE THIS ONE FOR OFF-DIAGONAL (L NOT= M) TERMS (NO SINGULARITY!)	COU02440
	COMMON /COM/ XL,ALPHA,RHO,SIGMA	COU02450
	COMPLEX FD1A,H1,H2	COU02460
	D = ABS(XL-XP)	COU02470
	H1 = CMPLX(BSJ1(D),BSY1(D))	COU02480
	H2 = CMPLX(BSJ1(SIGMA*D),BSY1(SIGMA*D))	COU02490
	FD1A = (H1-SIGMA*H2)/CMPLX(0.,2.*D)	COU02500
	RETURN	COU02510
	END	COU02520
C		COU02530
C		COU02540
	FUNCTION FD1AREA(XP)	COU02550
	COMPLEX FD1A	COU02560
	FD1AREA = REAL(FD1A(XP))	COU02570
	RETURN	COU02580
	END	COU02590
C		COU02600
C		COU02610
	FUNCTION FD1AIMA(XP)	COU02620
	COMPLEX FD1A	COU02630
	FD1AIMA = AIMAG(FD1A(XP))	COU02640
	RETURN	COU02650
	END	COU02660
C		COU02670
C		COU02680
C		COU02690
	FUNCTION FD1B(XP)	COU02700
C	USE THIS ONE FOR DIAGONAL (L = M) TERMS (WITH SINGULARITY!)	COU02710
	COMMON /COM/ XL,ALPHA,RHO,SIGMA	COU02720
	COMPLEX FD1B,H1,H2	COU02730
	D = ABS(XL-XP)	COU02740
	IF(D.LT.0.001) GO TO 10	COU02750
	H1 = CMPLX(BSJ1(D),BSY1(D))	COU02760
	H2 = CMPLX(BSJ1(SIGMA*D),BSY1(SIGMA*D))	COU02770
	FD1B = (H1-SIGMA*H2)/CMPLX(0.,2.*D)	COU02780
	FD1B = FD1B - (ALOG(D/2.)-SIGMA**2*ALOG(SIGMA*D/2.))/6.2831853	COU02790
	RETURN	COU02800
	10 FD1B = CMPLX(0.,0.)	COU02810
	RETURN	COU02820
	END	COU02830
C		COU02840
C		COU02850
	FUNCTION FD1BREA(XP)	COU02860
	COMPLEX FD1B	COU02870
	FD1BREA = REAL(FD1B(XP))	COU02880
	RETURN	COU02890
	END	COU02900
C		COU02910
C		COU02920
	FUNCTION FD1BIMA(XP)	COU02930
	COMPLEX FD1B	COU02940
	FD1BIMA = AIMAG(FD1B(XP))	COU02950
	RETURN	COU02960
	END	COU02970
C		COU02980
C		COU02990
C		COU03000

	FUNCTION FD2(XP)	COU03010
	COMMON /COM/XL,ALPHA,RHO,SIGMA	COU03020
	COMPLEX FD2,H1,H2,H3,H4	COU03030
	F = SQRT(XL**2+XP**2-2.*XL*XP*COS(2.*ALPHA))	COU03040
	H1 = CMPLX(BSJ1(SIGMA*F),BSY1(SIGMA*F))	COU03050
	H2 = CMPLX(BSJ1(F),BSY1(F))	COU03060
	H3 = CMPLX(BSJ0(SIGMA*F),BSY0(SIGMA*F))	COU03070
	H4 = CMPLX(BSJ0(F),BSY0(F))	COU03080
	Z = XL*XP*(SIN(2.*ALPHA))**2	COU03090
	FD2 = (COS(2.*ALPHA)-2.*Z/F**2)*(SIGMA*H1-H2)+Z*(SIGMA**2*H3-H4)/F	COU03100
	FD2 = FD2/F	COU03110
	RETURN	COU03120
	END	COU03130
C		COU03140
C		COU03150
	FUNCTION FD2REAL(XP)	COU03160
	COMPLEX FD2	COU03170
	FD2REAL = REAL(FD2(XP))	COU03180
	RETURN	COU03190
	END	COU03200
C		COU03210
C		COU03220
	FUNCTION FD2IMAG(XP)	COU03230
	COMPLEX FD2	COU03240
	FD2IMAG = AIMAG(FD2(XP))	COU03250
	RETURN	COU03260
	END	COU03270
C		COU03280
C		COU03290
C		COU03300
	FUNCTION VINC(XP,P)	COU03310
	COMMON /SOURCE/ X0,PHI0,ALPHA	COU03320
	COMPLEX VINC,H1,H2	COU03330
	INTEGER P	COU03340
	G1 = SQRT(XP**2+X0**2-2.*XP*X0*COS(PHI0-ALPHA))	COU03350
	G2 = SQRT(XP**2+X0**2-2.*XP*X0*COS(PHI0+ALPHA))	COU03360
	H1 = CMPLX(BSJ0(G1),BSY0(G1))	COU03370
	H2 = CMPLX(BSJ0(G2),BSY0(G2))	COU03380
	VINC = (H1+(-1)**P*H2)*CMPLX(0.,1.)/4.	COU03390
	RETURN	COU03400
	END	COU03410
C		COU03420
C		COU03430
C		COU03440
	FUNCTION WINC(XP,P)	COU03450
	COMMON /SOURCE/ X0,PHI0,ALPHA	COU03460
	COMPLEX WINC,H1,H2	COU03470
	INTEGER P	COU03480
	G1 = SQRT(XP**2+X0**2-2.*XP*X0*COS(PHI0-ALPHA))	COU03490
	G2 = SQRT(XP**2+X0**2-2.*XP*X0*COS(PHI0+ALPHA))	COU03500
	H1 = CMPLX(BSJ1(G1),BSY1(G1))	COU03510
	H2 = CMPLX(BSJ1(G2),BSY1(G2))	COU03520
	WINC = (SIN(PHI0-ALPHA)*H1/G1+(-1)**P*SIN(PHI0+ALPHA)*H2/G2)	COU03530
	& *CMPLX(0.,X0)/4.	COU03540
	RETURN	COU03550
	END	COU03560
C		COU03570
C		COU03580
C		COU03590
	FUNCTION CDEG(Z)	COU03600

C		COU03610
C	PHASE ANGLE IN DEGREES OF A COMPLEX NUMBER Z.	COU03620
C		COU03630
	COMPLEX Z	COU03640
	ZI = AIMAG(Z)	COU03650
	ZR = REAL(Z)	COU03660
	IF((ZI.EQ.0.).AND.(ZR.EQ.0.)) GO TO 10	COU03670
	CDEG = ATAN2(ZI,ZR)*57.29578	COU03680
	RETURN	COU03690
10	CDEG = 0.	COU03700
	RETURN	COU03710
	END	COU03720
C		COU03730
C		COU03740
C		COU03750
	SUBROUTINE CROUTC(MR,NR,NCC,A,ZMCH,DT,IERR,LP)	COU03760
		COU03770
C	CROUT (1) OPERATES ON A COEFFICIENT MATRIX TO SOLVE A SYSTEM OF	COU03780
C	SIMULTANEOUS EQUATIONS OR TO COMPUTE AN INVERSE AND (2)	COU03790
C	COMPUTES A DETERMINANT.	COU03800
		COU03810
C	CROUT REDUCES THE ORIGINAL MATRIX AND RIGHT-HAND SIDES UNTIL	COU03820
C	UPON COMPLETION THE REDUCED MATRIX REPLACES THE ORIGINAL	COU03830
C	MATRIX AND THE SOLUTIONS REPLACE THE RIGHT-HAND SIDES.	COU03840
		COU03850
	COMPLEX * 8 A,DT,TEMPC	COU03860
		COU03870
	DIMENSION A(1), LP(1)	COU03880
	IF(NR.GT.MR) GO TO 210	COU03890
	MTX = MR*NR	COU03900
	MRA = MR + 1	COU03910
	MRS = MR - 1	COU03920
	MDN = MR - NR	COU03930
	MTR = MTX - MDN	COU03940
	MTRA = MTX + 1	COU03950
	DT = (1.,0.)	COU03960
	IERR = 0	COU03970
	NRS = NR - 1	COU03980
	DO 2 I = 1,NR	COU03990
	LP(I) = I	COU04000
2	CONTINUE	COU04010
	IF(NCC)210,805,1001	COU04020
805	NTC = NR + NR	COU04030
	MTT = MR*NTC - MDN	COU04040
	J = MTRA	COU04050
	DO 19 K = MTRA,MTT,MR	COU04060
	JF = K + NRS	COU04070
	DO 18 KX = K,JF	COU04080
	A(KX) = (0.,0.)	COU04090
18	CONTINUE	COU04100
	A(J) = (1.,0.)	COU04110
	J = J + MRA	COU04120
19	CONTINUE	COU04130
	GO TO 1	COU04140
1001	NTC = NR + NCC	COU04150
	MTT = MR*NTC - MDN	COU04160
1	IF(NTC.LE.NR) GO TO 210	COU04170
	DO 70 I = 1,NR	COU04180
	IS = I - 1	COU04190
	II = MR*IS + I	COU04200

	IISB = II - 1	COU04210
	IIAD = II + MR	COU04220
	ICF = MR*I - MDN	COU04230
	ICS = ICF - NRS	COU04240
	IIA = II + 1	COU04250
	TEMP = 0.	COU04260
	DO 31 J = II,MTT,MR	COU04270
	IF(I.EQ.1) GO TO 33	COU04280
	KF = J - 1	COU04290
	KS = J - I + 1	COU04300
	KX = I	COU04310
	DO 30 K = KS,KF	COU04320
	A(J) = A(J) - A(KX)* A(K)	COU04330
	KX = KX + MR	COU04340
30	CONTINUE	COU04350
33	IF(J.GT.MTR) GO TO 31	COU04360
	IF(CABS(A(J)).LE.TEMP) GO TO 31	COU04370
	TEMP = CABS(A(J))	COU04380
	NX = J/MR + 1	COU04390
31	CONTINUE	COU04400
	IF(I.EQ.NR) GO TO 35	COU04410
	IF(NX.EQ.I) GO TO 35	COU04420
	ITEMP = LP(NX)	COU04430
	LP(NX) = LP(I)	COU04440
	LP(I) = ITEMP	COU04450
	LPIS = MR*NX - MRS	COU04460
	DO 34 K = ICS,ICF	COU04470
	TEMPC = A(K)	COU04480
	A(K) = A(LPIS)	COU04490
	A(LPIS) = TEMPC	COU04500
	LPIS = LPIS + 1	COU04510
34	CONTINUE	COU04520
	DT = - DT	COU04530
35	DT = DT * A(II)	COU04540
	IF(ZMCH - CABS(A(II))) 45,45,220	COU04550
45	DO 46 J = IIAD,MTT,MR	COU04560
	A(J) = A(J)/A(II)	COU04570
46	CONTINUE	COU04580
	IF(I.EQ.1) GO TO 70	COU04590
	IF(I.EQ.NR) GO TO 78	COU04600
	DO 47 M = IIA,ICF	COU04610
	KX = M - ICS + 1	COU04620
	DO 48 KY = ICS,IISB	COU04630
	A(M) = A(M) - A(KX) * A(KY)	COU04640
	KX = KX + MR	COU04650
48	CONTINUE	COU04660
47	CONTINUE	COU04670
70	CONTINUE	COU04680
78	NRTAD = MTX + NR	COU04690
	DO 180 I = 1,NRS	COU04700
	IREV = NRTAD - I	COU04710
	KRS = IREV - MR*I	COU04720
	DO 170 IRCNT = IREV,MTT,MR	COU04730
	KCS = IRCNT + 1	COU04740
	DO 160 K = KRS,MTR,MR	COU04750
	A(IRCNT) = A(IRCNT) - A(KCS) * A(K)	COU04760
	KCS = KCS + 1	COU04770
160	CONTINUE	COU04780
170	CONTINUE	COU04790
180	CONTINUE	COU04800

	DO 6 I = 1,NRS	COU04810
9	IF(LP(I).EQ.I) GO TO 6	COU04820
	NX = LP(I)	COU04830
	LP (I) = LP(NX)	COU04840
	LP(NX) = NX	COU04850
	IXS = MTX + I	COU04860
	IY = MTX + NX	COU04870
	DO 7 IX = IXS,MTT,MR	COU04880
	TEMPC = A(IX)	COU04890
	A(IX) = A(IY)	COU04900
	A(IY) = TEMPC	COU04910
	IY = IY + MR	COU04920
7	CONTINUE	COU04930
	GO TO 9	COU04940
6	CONTINUE	COU04950
	RETURN	COU04960
210	IERR = 2	COU04970
	NTC = NR	COU04980
	DT = (999999999.,999999999.)	COU04990
	RETURN	COU5000
220	IERR = 1	COU5010
	RETURN	COU5020
	END	COU5030

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