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# IMPROVED METHOD FOR CALCULATING EXACT GEODETIC LATITUDE AND ALTITUDE


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# FOREWORD

The work described in this report was performed in the Fire Control Formulation Branch (K41), Submarine Launched Ballistic Missile (SLBM) Research and Analysis Division, Strategic and Space Systems Department, and was authorized under Strategic Systems Program Office Task Assignment 36401. This work was necessitated by the need to formulate an exact method of computing geodetic latitude and altitude whose chief attribute is the absence of singularities in the results.

This report is a refined version of NSWC TR 85–85, dated April 1985 and revised May 1986. NSWC TR 85–85 should be discarded.

This report has been reviewed and approved by Davis Owen; Johnny Boyles; Carol Rose, Head, Fire Control Formulation Branch; and Dr. David Lando, Head, SLBM Research and Analysis Division.

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## INTRODUCTION

A simple and efficient model is presented for calculating the exact geodetic latitude and altitude of an arbitrary point in space, given the coordinates of that point. The model assumes the Earth to be an oblate spheroid; i.e., an ellipsoid of revolution whose semimajor axis 'a' is the radius of the circle described by the equatorial plane, and whose semiminor axis 'b' is a line joining its center and one of its poles. The point in question can be either inside or outside the Earth ellipsoid, excluding a small open region bounded by a prolate spheroid that is concentric with and contained within the Earth ellipsoid. The cardinal merit of this model is the absence of singularities in the results.

## FORMULATION

Choose an Earth-fixed Cartesian coordinate system whose origin coincides with the center of the Earth ellipsoid. The unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ coincide with the x, y, and z axes, respectively. The +z axis points in the direction of the North Pole. The +x axis is the line of intersection of the equatorial plane with the plane of zero longitude. The +y axis completes a right-handed coordinate system.

An equation of the ellipsoid in this frame is

$$\frac{r^2}{a^2}+\frac{z^2}{b^2}=1, \text{ where } r=\sqrt{x^2+y^2}\,. \tag{1}$$

Let $P(x_0, y_0, z_0)$ be the coordinates of the given point. It is desired to find the nearest point $P'(x,y,z)$ to P on the surface of the ellipsoid.

The slope of a normal to the ellipsoid at any point on its surface is given by

$$-\frac{1}{\frac{dz}{dr}}=\frac{a^2 z}{b^2 r}, \text{ where } \frac{r^2}{a^2}+\frac{z^2}{b^2}=1\,. \tag{2}$$

Therefore, the slope of a normal from P is

$$\frac{z_0-z}{r_0-r}=\frac{a^2 z}{b^2 r}, \text{ where } r_0=\sqrt{x_0^2+y_0^2}\,;$$

i.e.,

$$(r_0-r)a^2 z=(z_0-z)b^2 r,$$

or

$$r_0 a^2 z = r\left\{\left(a^2 - b^2\right)z + b^2 z_0\right\} .$$

Squaring both sides and expressing r in terms of z,

$$a^2 b^2 r_0^2 z^2 = \left(b^2 - z^2\right)\left\{\left(a^2 - b^2\right)z + b^2 z_0\right\}^2 .$$

Writing the above equation in descending powers of z, the following quartic in z is obtained:

$$\left(a^2 - b^2\right)^2 z^4 + 2b^2\left(a^2 - b^2\right)z_0 z^3 + b^2\left\{a^2 r_0^2 + b^2 z_0^2 - \left(a^2 - b^2\right)^2\right\}z^2 - 2b^4\left(a^2 - b^2\right)z_0 z - b^6 z_0^2 = 0 .$$

Since z is either positive or negative and sign z = sign $z_0$, the above quartic can be expressed in terms of $|z|$. The result is

$$|z|^4 + 2p|z|^3 + q|z|^2 - 2pb^2|z| - p^2 b^2 = 0 , \tag{3}$$

where

$$p = \frac{b^2|z_0|}{a^2 - b^2} = \frac{|z_0|}{e'^2} , \tag{4}$$

$$q = \frac{b^2\left\{a^2 r_0^2 + b^2 z_0^2 - \left(a^2 - b^2\right)^2\right\}}{\left(a^2 - b^2\right)^2} = p^2 - b^2 + \frac{r_0^2}{e^2 e'^2} , \tag{5}$$

where

$$e^2 = 1 - \frac{b^2}{a^2} , \quad e'^2 = \frac{a^2}{b^2} - 1 .$$

Two of three powerful theorems in the Theory of Equations will now be employed to expose the nature of the roots of Equation (3). The third theorem will be needed later on. These theorems are found in the reference[1].

I. An equation f(x) = 0 cannot have more positive roots than there are changes of sign in f(x), and cannot have more negative roots than there are changes of sign in f(–x).

II. Every equation that is of an even degree and has its last term negative has at least two real roots, one positive and one negative.

III. Every equation of an odd degree has at least one real root whose sign is opposite to that of its last term.

By observing Equations (3), (4), and (5), the following conclusions are deduced:

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[1] Hall, H.S. and Knight, S.R., Higher Algebra, Fourth Edition, MacMillan and Co., London, 1927.

(i) Since the last term of Equation (3) is negative, there are at least two real roots of opposite sign (using II).

(ii) Since there is only one change of sign in Equation (3), there is at most one positive root (using I).

From (i) and (ii), it is seen immediately that Equation (3) has exactly one positive root, and this is the root that is sought.

The solution of Equation (3) is effected by a standard method known as Ferrari's method[1]. In the equation

$$|z|^4 + 2p|z|^3 + q|z|^2 - 2pb^2|z| - p^2b^2 = 0 ,$$

add to each side $(c|z|+d)^2$, the quantities c and d being determined so as to make the left-hand side a perfect square; then

$$|z|^4 + 2p|z|^3 + \left(q + c^2\right)|z|^2 + 2\left(cd - pb^2\right)|z| + d^2 - p^2b^2 = \left(c|z| + d\right)^2 .$$

Suppose that the left side of the equation is equal to $(|z|^2 + p|z| + t)^2$, then comparing the coefficients,

$$p^2 + 2t = q + c^2,\ pt = cd - pb^2,\ t^2 = d^2 - p^2b^2 . \tag{6}$$

Eliminating c and d from these equations,

$$p^2\left(t + b^2\right)^2 = \left(2t + p^2 - q\right)\left(t^2 + p^2b^2\right) ,$$

or

$$2t^3 - qt^2 - p^2b^2\left(q - p^2 + b^2\right) = 0 ,$$

or

$$2t^3 - qt^2 - \frac{p^2b^2r_0^2}{e^2e'^2} = 0, \text{ using Equation (5)}$$

or

$$2t^3 - qt^2 - \frac{a^2r_0^2z_0^2}{e'^8} = 0 . \tag{7}$$

Applying Theorems I and III to Equation (7), we see that Equation (7) has exactly one positive root. The solution of Equation (7) is accomplished by a standard method known as Cardan's Solution[1]. Eliminating the $t^2$ term in Equation (7) by making the substitution

$$t = t' + \frac{q}{6} ;$$

i.e.,

$$2\left(t'+\frac{q}{6}\right)^3 - q\left(t'+\frac{q}{6}\right)^2 - \frac{a^2 r_0^2 z_0^2}{e'^8} = 0 \ ,$$

then

$$t'^3 + Ft' + H = 0 \ , \tag{8}$$

where

$$F = -\frac{q^2}{12},\ H = -\frac{q^3}{108} - \frac{a^2 r_0^2 z_0^2}{2e'^8} \ .$$

The solution of Equation (8) is

$$t' = \left(\sqrt{\frac{H^2}{4}+\frac{F^3}{27}} - \frac{H}{2}\right)^{\frac{1}{3}} - \left(\sqrt{\frac{H^2}{4}+\frac{F^3}{27}} + \frac{H}{2}\right)^{\frac{1}{3}} \ . \tag{9}$$

Equation (9) is valid provided that

$$\frac{H^2}{4}+\frac{F^3}{27} \geq 0 \ .$$

We have

$$H = -\frac{q^3}{108} - \frac{a^2 r_0^2 z_0^2}{2e'^8} = -\frac{2}{6^3}\left(q^3 + 2P\right) ,$$

where

$$P = \frac{27 a^2 r_0^2 z_0^2}{e'^8} \ .$$

$$\frac{H^2}{4}+\frac{F^3}{27} = \frac{1}{6^6}\left(q^3+2P\right)^2 - \frac{q^6}{6^6} = \frac{4}{6^6} P\left(P+q^3\right) .$$

If

$$\frac{H^2}{4}+\frac{F^3}{27} \geq 0 \ ,$$

then

$$P + q^3 \geq 0 \ ,$$

which is satisfied when $q \geq 0$; i.e., when $a^2 r_0^2 + b^2 z_0^2 \geq \left(a^2 - b^2\right)^2$, which represents all space excluding the open region bounded by the ellipsoid

$$\frac{r_0^2}{a^2e^4}+\frac{z_0^2}{b^2e'^4}=1\,.$$

Since $ae^2 < be'^2 < 43$ km, this region is of no practical interest. Therefore, $q \geq 0$ will be the constraint. Applying Theorem I to Equation (7) with $-t$ substituted for t, we see that Equation (7) has no negative roots. Hence, Equation (7) has only one real root and it is positive.

$$\sqrt{\frac{H^2}{4}+\frac{F^3}{27}}-\frac{H}{2}=\frac{2}{6^3}\sqrt{P\left(P+q^3\right)}+\frac{1}{6^3}\left(2P+q^3\right)=\frac{1}{6^3}\left\{2P+q^3+2\sqrt{P\left(P+q^3\right)}\right\}$$

$$=\frac{1}{6^3}\left(\sqrt{P+q^3}+\sqrt{P}\right)^2\,.$$

$$\therefore\quad\left(\sqrt{\frac{H^2}{4}+\frac{F^3}{27}}-\frac{H}{2}\right)^{\frac{1}{3}}=\frac{Q}{6}\,,$$

where

$$Q=\left(\sqrt{P+q^3}+\sqrt{P}\right)^{\frac{2}{3}}\,.$$

Similarly,

$$\left(\sqrt{\frac{H^2}{4}+\frac{F^3}{27}}+\frac{H}{2}\right)^{\frac{1}{3}}=-\frac{Q'}{6}\,,$$

where

$$Q'=\left(\sqrt{P+q^3}-\sqrt{P}\right)^{\frac{2}{3}}\,.$$

(Note that $Q'=\frac{q^2}{Q}$. However, putting $Q'$ in this form is undesirable as it introduces a singularity when $Q = 0$, which occurs when either both q and $r_0$ equal zero or both q and $z_0$ equal zero.)

Substituting these values into Equation (9),

$$t'=\frac{1}{6}(Q+Q')\Rightarrow t=\frac{1}{6}(q+Q+Q')\,.$$

It will now be shown that $q \leq 2t \leq q + b^2$, where the left-hand equality occurs when either $r_0 = 0$ or $z_0 = 0$ and the right-hand equality occurs when $ar_0 = b|z_0|$.

Rewriting Equation (7) in the form

$$t^2(2t-q)=\frac{a^2r_0^2z_0^2}{e'^8}\,,$$

the lower bound of t is deduced at once. Now let

$$g = \tau^3 - q\tau^2 - \frac{4a^2 r_0^2 z_0^2}{e'^8} ,$$

where $\tau \geq q$. Then

$$\frac{dg}{d\tau} = \tau(3\tau - 2q) \geq 0 .$$

Thus, g is a monotonically increasing function of $\tau$ for all $\tau \geq q$. Since $2t \geq q$ and $g = 0$ when $\tau = 2t$, it follows that $g \geq 0$ when $\tau \geq 2t$. Putting $\tau = q + b^2$ in the expression for g,

$$g\big|_{\tau=q+b^2} = b^2(q + b^2)^2 - \frac{4a^2 r_0^2 z_0^2}{e'^8} = b^2\left(p^2 + \frac{r_0^2}{e^2 e'^2}\right)^2 - \frac{4b^2 p^2 r_0^2}{e^2 e'^2}, \text{ using Equation (5)}$$

$$= b^2\left(p^2 - \frac{r_0^2}{e^2 e'^2}\right)^2 \geq 0$$

$\Rightarrow 2t \leq q + b^2$, equality occurring when $p = \frac{r_0}{ee'}$; i.e., when $\frac{r_0}{e} = \frac{|z_0|}{e'}$, or when $ar_0 = b|z_0|$, in which case,

$$q = 2p^2 - b^2,\ t = \frac{1}{2}(q + b^2) = p^2 .$$

Solving for c and d from the system of equations (6),

$$c = \sqrt{p^2 - q + 2t},\ d = \frac{p}{c}(t + b^2) = \sqrt{t^2 + p^2 b^2} . \tag{10}$$

Since $2t \geq q$, it follows that $c \geq p$. Now, $(|z|^2 + p|z| + t)^2 = (c|z| + d)^2$ implies $|z|^2 + p|z| + t = \pm (c|z| + d)$, from which the following two quadratics in $|z|$ are obtained:

$$|z|^2 + (p - c)|z| + t - d = 0 , \tag{11}$$

and

$$|z|^2 + (p + c)|z| + t + d = 0 . \tag{12}$$

It has already been shown that Equation (3) has exactly one positive root. Furthermore, since $t^2 - d^2 = -p^2 b^2 \leq 0$ and $d \geq 0$, it follows that $t - d \leq 0 \leq t + d$. Now, $t - d$ is the last term of Equation (11) and has just been shown to be negative. Hence, by applying Theorem II to Equation (11), we see immediately that Equation (11) must contain the required positive root. Hence,

$$|z| = \frac{-(p-c)+\sqrt{(p-c)^2-4(t-d)}}{2}$$

$$= \frac{1}{2}\left(c-p+\sqrt{2p^2-q-2t-2pc+4\sqrt{t^2+p^2b^2}}\right).$$

This solution is free of singularities. It is easy to show that $\lim\limits_{z_0 \to 0}|z| = 0$ and $\lim\limits_{r_0 \to 0}|z| = b$ . We have

$$\lim_{z_0 \to 0} p = 0,\ \lim_{z_0 \to 0} P = \lim_{r_0 \to 0} P = 0 \Rightarrow \lim_{z_0 \to 0} Q = \lim_{z_0 \to 0} Q' = \lim_{r_0 \to 0} Q = \lim_{r_0 \to 0} Q' = q$$

$$\Rightarrow \lim_{z_0 \to 0} t = \lim_{r_0 \to 0} t = \frac{q}{2} \Rightarrow \lim_{z_0 \to 0} c = 0,\ \lim_{r_0 \to 0} c = p.$$

$$\therefore \lim_{z_0 \to 0}|z| = \frac{1}{2}\sqrt{-2q+2q} = 0,$$

$$\lim_{r_0 \to 0}|z| = \frac{1}{2}\sqrt{2p^2-2q-2p^2+4\sqrt{\frac{q^2}{4}+p^2b^2}}$$

$$= \frac{1}{2}\sqrt{-2\left(p^2-b^2\right)+4\sqrt{\frac{1}{4}\left(p^2-b^2\right)^2+p^2b^2}}$$

$$= \frac{1}{2}\sqrt{-2\left(p^2-b^2\right)+2\left(p^2+b^2\right)} = b\,.$$

Proceeding with the derivation,

$$r = a\sqrt{1-\frac{z^2}{b^2}},\ x = x_0\frac{r}{r_0},\ y = y_0\frac{r}{r_0},\ \text{and}\ \lambda = \tan^{-1}\frac{y_0}{x_0}\ ,$$

where $r_0 \neq 0$. (Here, $\lambda$ is the longitude at (x,y).)

To compute the geodetic latitude and altitude at $P'(x,y,z)$, it is desirable to introduce a geometric term, $N_e$, which is never zero. $N_e$ is defined to be the distance along the ellipsoidal normal from the surface of the ellipsoid to the z-axis (see Figure 1).

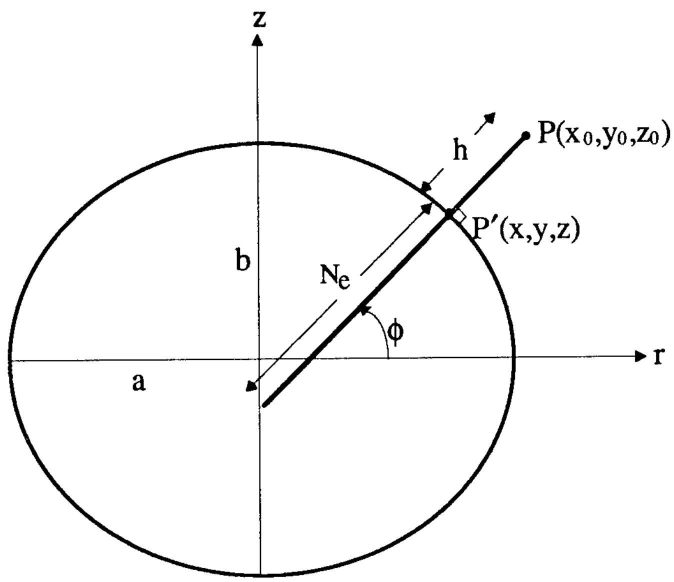


FIGURE 1. ELLIPSOIDAL NORMAL

From Figure 1,

$$\cos\phi = \frac{r}{N_e} .$$

Also,

$$\tan\phi = \frac{a^2 z}{b^2 r}, \text{ from Equation (2)}$$

$$\Rightarrow N_e = r\sec\phi = r\sqrt{1+\tan^2\phi} = r\sqrt{1+\frac{a^4 z^2}{b^4 r^2}} = \sqrt{r^2 + \frac{a^4 z^2}{b^4}}$$

$$= \sqrt{a^2\left(1-\frac{z^2}{b^2}\right)+\frac{a^4 z^2}{b^4}} = a\sqrt{1+\frac{a^2-b^2}{b^4}z^2} = a\sqrt{1+\frac{e'^2 z^2}{b^2}}, \qquad (13)$$

and

$$\sin\phi = \cos\phi\tan\phi = \frac{a^2 z}{b^2 N_e} \Rightarrow \phi = \sin^{-1}\frac{a^2 z}{b^2 N_e} ,$$

which is the desired expression for the geodetic latitude.

Also from Figure 1,

$$h\cos\phi = r_0 - r = r_0 - N_e\cos\phi,$$

$$h \sin\phi = z_0 - z = z_0 - \frac{b^2}{a^2} N_e \sin\phi .$$

Multiplying the first equation by cos $\phi$ and the second by sin $\phi$ and adding,

$$h = r_0 \cos\phi + z_0 \sin\phi - N_e\left(\cos^2\phi + \frac{b^2}{a^2}\sin^2\phi\right).$$

Now

$$\tan^2\phi = \frac{a^4 z^2}{b^4 r^2} = \frac{a^2 z^2}{b^2\left(b^2 - z^2\right)}$$

$$\Rightarrow \cos^2\phi + \frac{b^2}{a^2}\sin^2\phi = \frac{1 + \frac{b^2}{a^2}\tan^2\phi}{1 + \tan^2\phi} = \frac{1 + \frac{z^2}{b^2 - z^2}}{1 + \frac{a^2 z^2}{b^2\left(b^2 - z^2\right)}} = \frac{b^4}{b^4 + \left(a^2 - b^2\right)z^2}$$

$$= \frac{1}{1 + \frac{e'^2 z^2}{b^2}} = \frac{a^2}{N_e^2}, \text{ using Equation (13).}$$

Hence

$$h = r_0 \cos\phi + z_0 \sin\phi - \frac{a^2}{N_e} ,$$

which is the desired expression for the geodetic altitude.

To recapitulate, given a, b, $x_0$, $y_0$, and $z_0$, the algorithm is as follows:

$$e^2 = 1 - \frac{b^2}{a^2},\ e'^2 = \frac{a^2}{b^2} - 1,\ r_0 = \sqrt{x_0^2 + y_0^2},\ p = \frac{|z_0|}{e'^2},\ q = p^2 - b^2 + \frac{r_0^2}{e^2 e'^2} .$$

If $q \geq 0$, then

$$P = \frac{27 a^2 r_0^2 z_0^2}{e'^8},\ Q = \left(\sqrt{P + q^3} + \sqrt{P}\right)^{\frac{2}{3}},\ Q' = \left(\sqrt{P + q^3} - \sqrt{P}\right)^{\frac{2}{3}},\ t = \frac{1}{6}(q + Q + Q'),$$

$$c = \sqrt{p^2 - q + 2t},\ z = \frac{1}{2}\left(c - p + \sqrt{2p^2 - q - 2t - 2pc + 4\sqrt{t^2 + p^2 b^2}}\right)\text{sign } z_0 ,$$

$$N_e = a\sqrt{1 + \frac{e'^2 z^2}{b^2}},\ \phi = \sin^{-1}\frac{a^2 z}{b^2 N_e},\ h = r_0 \cos\phi + z_0 \sin\phi - \frac{a^2}{N_e} .$$

## CONCLUSION

An efficient model for transforming Earth–centered, Earth–fixed coordinates to geodetic coordinates has been presented. The model systematically derives exact expressions for the geodetic latitude and altitude, which are free of singularities. To the author's best knowledge, no such expressions exist in the literature.

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