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FORECASTING AND INVENTORY AREA MODEL CHOICE

by

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March, 1995

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Inventory control is an important element of both business and military cost control and readiness. The Taiwan Army Logistic Agency (TALA) has used a combination of mathematical inventory models, arithmetic average, three month moving average and experience to project future demand. Implicit is that the mean of monthly demand for an item remains steady over time. This assumption has proven to be incorrect during periods of force reduction, equipment is replaced or retired, or when there is a cyclical demand. Once an unusual demand pattern occurs, inventory control becomes unpredictable. Inapplicable inventory methods in the TALA have been estimated to cost as much as several million dollars. TALA has focussed on advanced forecasting methods, Exponential Weighted Moving Average (EWMA)to solve this problem. This may reduce inapplicable inventory to some extent. Residual inventory and shortage are also factors in cost control. In this research we will explore the appropriate approach to solve these problems to make the inventory control more efficient.

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FORECASTING AND INVENTORY AREA MODEL CHOICE

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ABSTRACT

Inventory control is an important element of both business and military cost control and readiness. The Taiwan Army Logistic Agency (TALA) has used a combination of mathematical inventory models, arithmetic average, three month moving average and experience to project future demand. Implicit is that the mean of monthly demand for an item remains steady over time. This assumption has proven to be incorrect during periods of force reduction, equipment is replaced or retired, or when there is a cyclical demand. Once an unusual demand pattern occurs, inventory control becomes unpredictable. Inapplicable inventory methods in the TALA have been estimated to cost as much as several million dollars. TALA has focussed on advanced forecasting methods, Exponential Weighted Moving Average (EWMA)to solve this problem. This may reduce inapplicable inventory to some extent. Residual inventory and shortage are also factors in cost control. In this research we will explore the appropriate approach to solve these problems to make the inventory control more efficient.

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I sincerely appreciate my thesis advisor David G. Brown for his endeaverous help to complete this thesis. Indeed this thesis is an frontier exploration to put the forecast methodology and the stockage (holding area) analysis both in one research, and the Crystall Ball Simulation Package is also the first attempt to simulate the complicated probability computation of the holding area.

At the beginning of this research I encountered many difficulties with the programming setting and simulation process. However, all these problem has been solved under Professor Brown's assistance. We even worked together on some weekends and evenings at campus. Eventually, we have succeeded to complete this research. We have enjoyed the accomplishment we have completed. Thanks! Professor Brown.

I also appreciate my wife Hsueh-Ping Lin for her help and encouragement when I had some trouble and frustration, and apologize to my daughter Yuan-May and my son Yuan-How. I regret not bringing them to the wonderful Monterey Bay Beach to enjoy our stay here due to lack of time on my part. What I can partly compensate them with is only the thesis I present here, and maybe the Barbie Doll and Lego is what they really want right now.

I. INTRODUCTION

A. BACKGROUND OVERVIEW

Taiwan, Republic of China is a small island country located at the western edge of the pacific. The total area of the island is only thirty-six thousand square kilometers (13,971 square miles). The population of this island is more than twenty million (the density is the second highest in the world). In addition, the island has few natural resource . Economic development is very important for Taiwan's survival and prosperity. However, the historical threat from mainland China has not decreased as predicted. They have maintained that they will "never give up resorting to forces to resolve Taiwan issue." Though we have already developed some level of business investment in mainland China, they still frequently hold military drills on isolated islands near the Taiwan Strait. So, the defense budgets still remain at a high level compared to other developing countries. This heavy burden will slow down our economic development. This is especially hard on the democratic political, environment with people eager for more comfortable life style. How to maintain a balance of investment between defense and economic development has becoming a critical issue.

B. IMPORTANCE OF INVENTORY CONTROL

There are many different approaches that can be used in making tradeoffs in the policy of balancing defense and economic development; for instance, enhance training, reorganize the facilities or upgrade weapon systems. But in this study, I will research the area of inventory control.

Inventory control is a pivotal activity of any logistics organization. Multi-item inventory systems encompass tradeoffs in balancing customer services needs with operating costs. This management task is particularly challenging in the military setting where item availability often affects mission readiness. Clearly, one objective of any inventory doctrine is success in making decisions which minimize operating costs while providing an acceptable level of service for a forecasted rate of demand. The demand forecasting process and its influence on the inventory system effectiveness, along with alternative inventory holding cost approximations are the subjects of this study.

C. THE TAIWAN LOGISTICS AGENCY (TALA)

Before the discussion of the various inventory control methodology, let's take an overview of the Taiwan Logistics Agency (TALA) organization and its functions. The organization of TALA is shown in Figure 1.

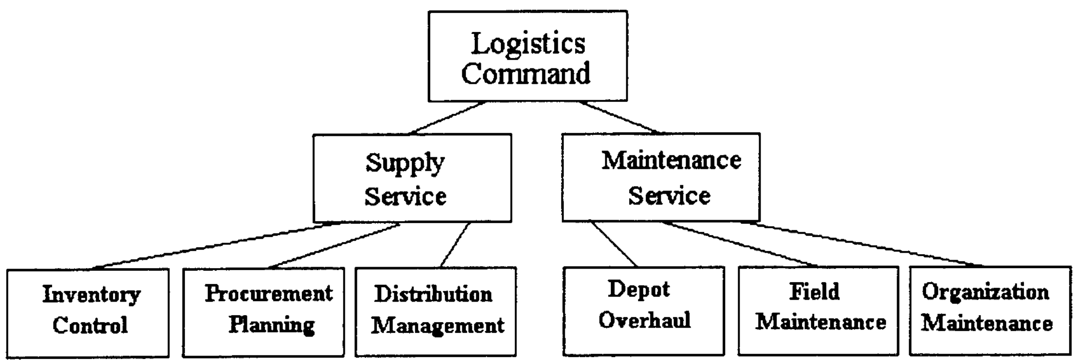

Figure 1. TALA Organization and Function

1. Functions in the TALA

a. Maintenance Service

(1) Depot Level. The Depot Maintenance level is the highest maintenance level in the TALA organization. This level of maintenance performs the sophisticated and dedicated repair work. It performs inspection, calibration, maintenance, and repair of equipment as well as the training of technicians. Equipment that has been through this level of repair can be regarded as equivalent to a new piece of equipment when it is redistributed to the units which request it. Though this level's inventory control is through BRF estimated

by skilled and experienced personnel, a deviation from real demand is unavoidable due to unexpected variation and human error. A detailed discussion will follow in the successive sections.

(2) Field Level. This level is an intermediate maintenance level. They perform the medium maintenance and repair work. Most of the work in this level is to change the main assembly parts (engine, transmission, differential gear box, wheel column and axle) instead of repairing these assembly parts (the repair of assembly parts belongs to Depot Maintenance Level). Field level also performs the function of supporting the next lower maintenance level (Organization Level) with spare parts to relieve the supply burden on the lower level. Inventory control in this level has less overhead compared to other facilities.

(3) Organization Level. This level is the lowest level of the maintenance chain. The maintenance work practiced in this level is not complicated. The main work consists of lubrication, adjustment, cleaning, and replacement of simple items, such as changing tires, replacing light bulbs, windshield blades, fan belts, spark plugs and various filters (air filter, fuel filter and oil filter).

Though this level only consists of simple maintenance work, the variation of work at this level are much greater than at other levels. Inventory control at this level is mainly done by less experienced personnel compared to other levels and the estimating method of spare parts is the simple Moving Average Approach (MAA) to take the variation into consideration. This level's facilities are widely spread in the TALA system and their contribution to the total estimated forecasting error is considerable. How to improve the MAA will be discussed in Chapter III.

b. Supply Service

Supply Services (SS) are composed of three different functional groups described as follows.

(1) Inventory Control. This group is a key functional area in the SS. They control the inventory level to meet readiness requirements and service levels, under the budget constraint. They estimate the future spare parts requirements by collecting the consumption data from all over the TALA system. The items that are controlled by this level

number more than ten thousand and with average quantities in the range of a hundred thousand. How do the people working here handle the overwhelming data? Currently, they use both computer aided algorithms (moving average approach) and experienced personnel to perform this analysis. This process works well if the average demand rate does not vary too much. However, in the real world, there are too many factors that affect the demand pattern and the accuracy of computed results. For instance, a periodic readiness inspection will make the demand increase dramatically, this will make the moving average approach less accurate. There is also a potential human factor (fatigue, sickness, temper, overburden) that will lead experienced personnel to make mistakes. How to minimize these unavoidable mistakes through the forecasting methodology is the core of this study.

(2) Procurement Group. The procurement unit working flow chart is shown in Figure 2. Basically, the procurement unit working process has little relationship to inventory control and other groups in TALA. The only case for which the procurement must communicate with the inventory unit is when vendors offers a special discount rate for a different purchasing quantity. This may affect the inventory control process to make the inventory level higher or lower according to the negotiated price and discount offered.

(3) Distribution Group. Compared with other facilities in TALA, the distribution group has a simpler working procedure. What they are concerned with is distributing spare parts at the right quantities; on schedule; to the right place, and at the same time meeting the different priorities each unit has set.

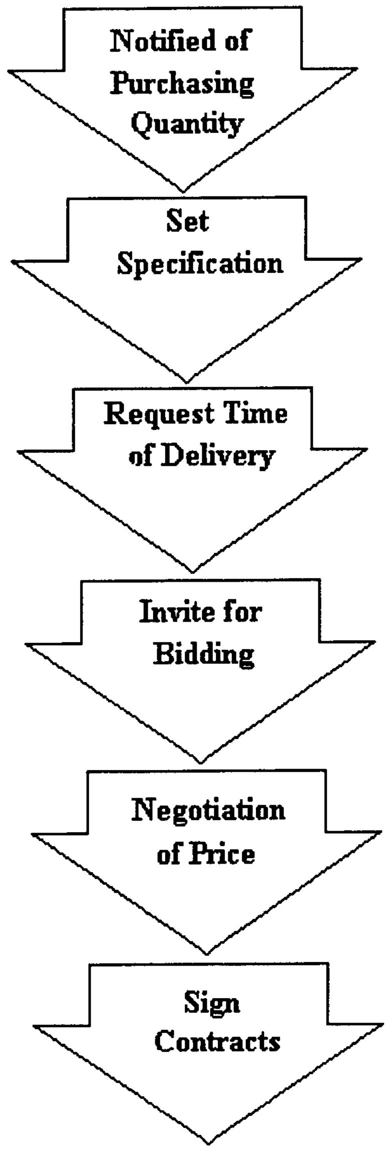

Figure 2. Procurement Unit Work Flow Chart

c. *Problems in the Current TALA Inventory Control System*

As mentioned in the previous sections, we know that out of date computer forecasting approaches and the mistakes made by the experienced personnel working in the TALA inventory system give unpredictable characteristics to inventory system performance and its efficiency. Table 1 shows the forecasting deviation made by the current forecasting procedures. Inadequate inventory methods account for five million dollars per year or almost fifteen percent of the annual inventory budget.

Year	Forecasted Demand	Real Demand	% Difference
0	14,532	11,387	+27.6%
+1	13,831	19,872	-30.4%
+2	45,372	38,754	+17.1%

Table 1. Forecasting Deviation from Real Demand

D. STOCKAGE

Stockage is the inventory level which remains at the end of each inventory cycle. There are two different kinds of stockage, residual inventory and shortage. Conceptually, residual inventory is the amount of material left over or still on hand when a new order or batch arrives. More formally, residual inventory is non-zero when the demand during leadtime is less than the reorder point, in which case it is equal to the difference between these two values. At other times, when demand during leadtime is less than or equal to the reorder point, residual inventory is defined as equal to zero. Shortage is the opposite of residual inventory. Conceptually, shortage is the amount of material back ordered because of insufficient inventory as measured when a new order arrives. It is also defined as the difference between demand during leadtime and the reorder point, but only when the demand value is larger; at other times it is zero.

Superfluous residual inventory will result in waste, as represented by inventory holding cost. While stockouts may save on holding cost, their negative impact on customer service issues is more profound. What are the factors that affect stockage status and hence the holding cost and stockouts? There are five main factors: demand rate, leadtime, average leadtime, order quantity and reorder point. Two popular formulations for approximating the holding cost area are examined is this study with respect to these factors using the Crystal Ball spreadsheet simulation package.

E. OBJECT OF THIS RESEARCH

From the discussion in the previous sections, we already know the variations of various factors will affect the inventory control. In this study, several prevailing forecasting methodologies will be explored to determine which one would minimize the total forecasting error and eventually make the best utilization of scarce resources in TALA.

Stockage is also a key area which will affect inventory control significantly and lead to higher cost for TALA. Some of the questions to be addressed in this study are: to what extent does the underlying relationships affect stockage? What are these relationships? How to obtain the best control over these underlying factors and eventually to make the inventory control more efficient.

F. LIMITATIONS OF THIS STUDY

As mentioned early, the number of spare parts processed by TALA is greater than ten thousand. The first limitation is that the data retrieved for this study is only a small part of the total inventory controlled by TALA. However, these data were retrieved randomly. According to a normal distribution characteristic, these sample data can represent the major characteristic of the whole TALA system.

Second, the minor deviation factors might not be detected by these research due to the fact that unpredictable variation factors cannot always be controlled as expected.

G. ORGANIZATIONS OF THIS STUDY

Chapter II will discuss the general forecasting theorem and discuss the specific function of each theorem. Chapter III will use the data to test each theorem discussed in Chapter II and make comparison of each forecasting approach to specify which one has the potential merits over the others. Chapter IV will discuss the stockage analysis to draw the underlying factor's relationship in the stockage. Chapter V will present the summary, conclusions, and make recommendations as a result of this research.

II. FORECAST METHODOLOGY

As stated in the introduction, forecasting methodologies have two taxonomies, the qualitative and quantitative techniques. This chapter will emphasize quantitative models for forecasting. Qualitative methods are used if adequate historical data is not available or can only be obtained at unreasonable expense. That is not the case for the data that is a subject of this thesis. Supply usage data for the TALA is readily available and the collection methods are in place and have been routinely used by TALA for several years.

This chapter is divided into two parts, the first will outline the forecasting function; the second part will discuss time series analysis.

A. FORECASTING

A broad view of forecasting functions includes specific techniques and models and the significance of the inputs and outputs to/from the forecasting model [Ref. 1]. In the design of a forecasting function the first step is to specify its intended use and the desired output. The next step is to isolate the significant inputs to be used in the model.[Ref. 1]

1. Forecasting Techniques

Some facilities replenish stock based upon historical demand. This is reactionary in nature, and does not normally reflect the timing of future demand. The replenishment system purchases for a certain demand over fixed intervals, whether or not these interval reflects actual demand over a shorter interval. This effectively supports the average demand but the greater the time interval between when the item is restocked and when it is consumed, the greater the carrying cost for the inventory. Furthermore, if the future demand is very volatile or erratic in quantity and timing, the reorder quantity will be difficult to predict.

Determining the demand quantity is an important step in ultimately determining order quantities. A common approach is to simply take an average for the preceding year. As noted above, this fails to consider the possibility of an erratic demand, seasonal factors, and trends. Where demand is computed monthly and future demand is not expected to be the same as the historical demand, some type of forecasting is required. It follows that the better the forecast of future demand, the better that inventory levels can be controlled. Improved control can translate into savings in inventory holding and ordering costs. Unless the organization is willing to maintain excessive safety stock or incur frequent stockouts, some demand forecasting is required.

There are many different forecasting models that could be used in spare parts supply inventory management control. In selecting a forecasting method, we first identify its intended use and the output results desired [Ref. 1]. The next step is to define the inputs available for the process, and then the constraints and environmental factors affecting the process. Inputs could include such items as the demand history and marketing research available. Constraints and environmental factors might include knowledge of special situations which may have affected the historical data, and the availability of expert and knowledgeable personnel. Outputs in the process include the timing of expected demand, broken down by such segments as product, customer, and region.

2. Forecasting Error

Forecasting error is used for two purposes. The first purpose is to evaluate the effectiveness of a particular forecasting method in actual use. The second method less obvious is to make a choice between forecasting methods to determine which method reflects the least error in selecting an organizational forecasting method .[Ref. 1]

There are two methods used to measure forecasting error; they include deviation statistics and bias statistics. Deviation statistics give an indication of the absolute magnitude of the average error. Bias statistics give both direction and the magnitude of forecasting errors. An example of deviation methods are the mean absolute deviation (MAD) and the mean square error (MSE). The MAD is calculated as follows:

$$MAD = \frac{\sum_{i=1}^{n} |Y_i - \hat{Y}_i|}{n}, \tag{1}$$

where:

$\hat{Y}_i$ = forecasted demand for period i,

Y_i = actual demand for period i,

n = number of observations or time periods,

$Y_i - \hat{Y}_i$ = deviation or forecast error,

$|Y_i - \hat{Y}_i|$ = absolute deviation.

The MSE is calculated as follows:

$$MSE = \frac{\sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2}{n} . \tag{2}$$

Both MAD and MSE reflect the deviation from a forecast but the difference between the two methods is that MSE penalizes a forecasting method more for larger errors then small errors.[Ref. 1]

The bias method of measuring forecasting errors is demonstrated by the mean error(ME) and the mean percent error (MPE) methods. The ME is calculated as follows:

$$ME = \frac{\sum_{i=1}^{n} (Y_i - \hat{Y}_i)}{n} . \tag{3}$$

The MPE is calculated as follows:

$$MPE = \frac{100 \sum_{i=1}^{n} (Y_i - \hat{Y}_i) \ / \ Y_i}{n} . \tag{4}$$

A positive bias indicates under forecasting while a negative result indicates that predictions were too high [Ref. 1].

Statistics of forecasting error are required for the following reasons:

- Determine the effectiveness of a particular forecasting method.
- Provide feedback for adjusting estimates of forecast model parameters.

- Set customer service level and safety stock.

3. The Forecasting Process

The development of a forecast should follow a systematic, six-step process which includes:

- Defining the forecasting problem
- Collecting and preparing the data
- Selecting and applying a forecasting method
- Reviewing and adjusting the preliminary forecasts
- Tracking the forecast's accuracy
- Updating the forecast and the forecasting system

The first step requires that the forecaster have an understanding of the problem and the purpose of the forecast. The second step involves gathering the data and ensuring that it represents the information needed in order to make an accurate projection. Step three involves selecting the forecasting technique most appropriate for the available data, the forecasting problem, and the situation or environment in which the forecast is to operate. Step four involves combining the historical data, the forecast technique, and management experience and judgment to evaluate the preliminary forecast model.

B. TIME SERIES ANALYSIS

Time series analysis is the forecasting method used in this thesis. It attempts to predict future demand based on past usage data. Time series analysis presupposes a relationship between the observations of a variable during successive and equal time periods. Time series data may contain up to five components.

- Level Component or the raw data
- Trend Component results from a long term growth or decline
- Seasonal Component results from annual movements above or below the trend line
- Cyclical Component results from long term cycles, it may or may not be periodic

- Random Component results from errors in data collection and unusual natural events

1. Techniques for Time Series Analysis

There are several common techniques for the analysis of time series data. These techniques assume that past data can be used to predict future data. Forecasters should understand that time series analysis neglects external factors which may effect results in the future but have not been experienced in the past, and that it will project unusual or unique events from the past into the future. The most common methods of time series analysis are:

- Last Period Demand
- Arithmetic Average
- Moving Average
- Regression Analysis
- Exponentially Weighted Moving Average (EWMA)

a. Last Period Demand (LPD)

This method projects that the next time period demand equals the last time period so that the forecast lags the actual demand by one period. This method responds well to trends and is very simple. Some of the disadvantages are that this method does not compensate for seasonal and that it will overreact to random influences.

Mathematically:

$$\hat{Y}_t = Y_{t-1} \qquad \textbf{(5)}$$

where:

$\hat{Y}_t$ = forecasted demand for period t,

$\hat{Y}_{t-1}$ = actual demand in the previous period.

b. Arithmetic Average

This method takes the average of all past demand and generates a projection of future demand. This method dampens random influences but does not follow trends in demand and does not follow the seasonal components.

Mathematically:

$$\hat{Y}_t = \frac{Y_1 + Y_2 + \cdots Y_n}{n} = \frac{\sum_{i=1}^{n} Y_i}{n}, \tag{6}$$

where:

$\hat{Y}_t$ = forecasted demand for period t,

Y_i = actual demand in period i,

n = number of time periods.

c. *Moving Average*

The method takes the average of a selected number of the past demand data and projects a future demand. The selection is based on the idea of selecting enough periods to reduce the effects of random influences and a small enough number of periods so that it will respond to the trend component.

The moving average method is a synthesis of the LPD and arithmetic average methods. If demand rate is smooth moving average works well and responds quickly to changes in demand but without the fluctuation of the LPD method. This method does not account for the seasonal components.

Mathematically:

$$\hat{Y}_t = \frac{Y_{t-1} + Y_{t-2} + \cdots Y_{t-n}}{n} = \frac{\sum_{i=1}^{n} Y_{t-i}}{n}, \tag{7}$$

where:

$\hat{Y}_t$ = forecasted demand for period t,

Y_{t-i} = actual demand in period $t - i$,

n = number of time periods included in moving average.

d. Regression Analysis

Regression analysis establishes a relationship between various forecast variables. The simplest linear regression relationship is used to fit a straight line to the input data. This method compensates for trend effects but does not follow seasonal effects. Because of the nature of supply data, linear regression will not perform well for those kinds of problems. Normal supply demand is usually related in time from one period to the next. In terms of linear regression this would be called autocorrelation. Autocorrelation violates the conditions required to produce a valid regression estimate. As an example, supply demand that is very high in one period may indicate that the next period demand will be very low. Regression analysis expects these kinds of fluctuations to be independent of each other.

e. Exponentially Weighted Moving Average (EWMA)

Exponentially Weighted Moving Average method is a special form of the moving average method. This method gives weight to past data as the moving average method does, but the weight varies geometrically with the increasing age of the data. Most recent data is weighted higher than older data. This method is also referred to as Exponential Smoothing. The major advantage of EWMA is that all historical data is contained in the previous projected demand and it is not necessary to calculate on large data collections. Mathematically the simplest form of EWMA is:

$$\hat{X}_t = \hat{X}_{t-1} + a(Y_{t-1} - \hat{X}_{t-1}) = a\, Y_{t-1} + (1-a)\, \hat{X}_{t-1}\,, \qquad \textbf{(8)}$$

where:

$\hat{X}_t$ = forecasted demand for period t,

Y_{t-1} = actual demand for period t-i,

n = Number of time periods included in moving average.

This is the method used in this thesis to predict supply demand. In addition, this method has been modified to incorporate provisions for trend correction. To do this, EWMA is used to project the average level and used again to project a level for the trend. For this to work it is necessary to divide historical demand data into two segments. Data in the first segment is used to initialize the trend components in the second segment of data and linear regression on the first segment of data is used to generate a slope for the trend. The second data segment is used to test various combinations of smoothing constants. Once the smoothing constants have been determined, future forecast can be made using the historical data and parameters from the last period in the second data segment. EWMA can also be used with corrections for seasonal components. For this it is necessary to generate a seasonal index that predicts demand level above and below average.

If the time series seasonal and trend components do not exist the provisions for those components should not be used. Obviously, to answer this question, it is necessary to observe some historical data to discern which components exist. This could require the storage of large amounts of data which would negate one of the advantages of this method. To avoid this, it is possible to calculate a MAD that is modified for exponential smoothing. The MAD may be recalculated after each period as:

$$MAD_t = d\,(\,Y_t - \hat{Y}_t\,) + (\,1 - d\,)\,MAD_{t-1}\,, \qquad \textbf{(9)}$$

where:

d = exponential smoothing constant.

(1) EWMA Extended for Trend Correction. To extend EWMA to account for the trend components of a time series is necessary to maintain two adjustments, average level and trend.

Average level is expanded to include previous trend:

$$\hat{X}_t = aY_{t-1} + (\,1 - a\,)\,(\,\hat{X}_{t-1} + T_{t-1}\,). \qquad \textbf{(10)}$$

The current forecast level is used to calculate the trend as the differences between consecutive forecasts and the trend component is adjusted as follows:

$$T_t = b(\hat{X}_t - \hat{X}_{t-1}) + (1 - b) T_{t-1} , \tag{11}$$

where:

b = the trend smoothing constant.

Then:

$$\hat{Y}_t = \hat{X}_t + T_t , \tag{12}$$

is used to produce the forecast for the current period. The EWMA modified with trend correction assumes level, trend and random components are contained in the time series. For this it is necessary to use historical data to determine whether these components are present and also to establish the initial trend adjustment T_{t-1} for the model. [Ref. 1]

(2) EWMA Extended for Seasonal Corrections. EWMA reacts slowly to large changes in demand. For normal services and supply demand EWMA can be adjusted for a small value of (a) and a safety stock level. For a seasonal component it is possible to modify EWMA to follow past supply demand patterns. The first step is to generate a set of indexes that represent the supply cycle over the length of a season. These indexes will represent demand above and below average. The indexes are simply generated by using the total demand for the season divided by the number of periods. This number the average demand is used to create a number which varies above and below one. This number is maintained as a normalized number (the total is equal to the number of periods). These indexes are used as an initial index and represent the previous season. This data can also come from an average of several seasons in order to reduce the effects of random components of the time series.

Actual demand data is modified to remove seasonal effects by dividing with the current seasonal index. Mathematically:

$$\hat{X}_t = \frac{aY_{t-1}}{I_{t-1}} + (1-a)\ \hat{X}_{t-1}\ , \tag{13}$$

where:

I_{t-1} = previous seasonal index,

The index is updated with:

$$I_{t+m} = \frac{cY_t}{\hat{X}_t} + (1-c)\ I_t\ , \tag{14}$$

where:

c = exponential smoothing constant,

m = number of period in a seasonal pattern.

The forecast for the current period is multiplied by the seasonal adjustment as:

$$\hat{Y}_t = \hat{X}_t\ I_t. \tag{15}$$

It is not necessary to maintain an historical data base of seasonal indexes. The seasonal index can be modified and normalized after each period of calculation as follows:

$$m\ /\ (\ m + I_t - I_{t-m})\ , \tag{16}$$

is multiplied against the index being revised and all other unrevised indexes and will maintain the seasonal index as normalized to the number of periods.[Ref. 1]

III. COMPARISON OF FORECAST METHODOLOGIES

A. INTRODUCTION

As discussed in Chapter II, there are various forecasting methodologies. Each of them have specific functions and can be used to fit the different demand patterns: flat, trend and seasonal demand pattern or the combination of the trend and seasonal demand pattern.

In recent decades, a wide variety of forecasting methods have emerged. Generally, they can be assigned to one of two taxonomies: qualitative or quantitative. Qualitative techniques are regarded as the more subjective of forecasting approaches. Usually conducted in a setting where historical data is unavailable or there is little knowledge about forecasting approaches, this method often employs expert opinion in constructing a forecasting approach. The Taiwan Army Logistics Agency (TALA) uses such a method, called Best Replaced Factor (BRF), when estimating initial stock levels for a new item of inventory.

In contrast, quantitative methods make extensive use of historical data. The data serves as an input to various types of mathematics models which compute the required forecast. Not surprisingly, advances in computer technology tend to popularize the quantitative-oriented forecasting methods.

In Chapter I, we mentioned that most of the time the TALA and its Supply Services Division adopted a combination of moving average and the direction of experienced personnel to perform analysis. This is a combination of qualitative and quantitative forecast approaches. The qualitative approach is performed by experienced personnel to judge the future demand, though it is not as accurate as we expected, it still is the applicable approach when there is no other method available. The quantitative forecast methodologies TALA uses most of the time are moving and arithmetic average. The advanced forecast methodology of exponential moving weighted average (EWMA) with trend or seasonal correction has not been used or researched by the TALA. The current approach used by TALA may be good for some specific demand patterns, for example flat demand pattern, but does not respond well to the trend or seasonal demand patterns.

In this chapter, we will explore a more advanced forecast methodology and compare it with the methodologies that TALA has adopted. We will measure what benefit and advantages we can obtain through EWMA approach. The next sections will discuss the flat demand pattern, trend demand pattern and the seasonal demand patterns. In the final section will outline the conclusions from the results of these comparisons and will also discuss mean percent error (MPE) which will be adopted to rate the accuracy of each methodology.

B. FLAT DEMAND PATTERN

Flat demand pattern is a demand with variation of less than 10% of the average demand. Table 2. shows a flat demand pattern collected from TALA's historical demand data . The forecast by TALA is using the arithmetic average approach.

Actual Demand	Forecasted Demand - TALA's - Arithmetic Average
115	130
110	123
108	118
109	116
113	114
111	114
114	114
113	114
117	114
117	114
108	114
114	114
MPE = .14 %	MAD= 3.083

Table 2. TALA Flat Demand Pattern Data Arithmetic Average

We will explore two other methodologies, simple exponential smoothing and last period forecast demand(LPD). LPD is a method that simply assume that a future period demand is the same as the last period. The forecasted result is shown in Figure 9 in Appendix A. and the difference is shown in Table 3.

We also explored the simple exponential smoothing approach in further detail by changing smoothing constant a at 0.01, 0.1, 0.2, 0.3 different values trying to receive the optimize smoothing constant value to make the forecast approach more accurate. The comparison of each different smoothing constant is shown in Appendix A. Figures 3 and 4 present a comparison of arithmetic average, LPD and EWMA.

Actual Demand	Forecasted Demand - TALA's - Last Period Forecast
115	130
110	115
108	110
109	108
113	109
111	113
114	111
113	114
117	113
117	117
108	117
114	108
MPE = .01 %	MAD= 3.25

Table 3. TALA Flat Demand Pattern Data - LPD

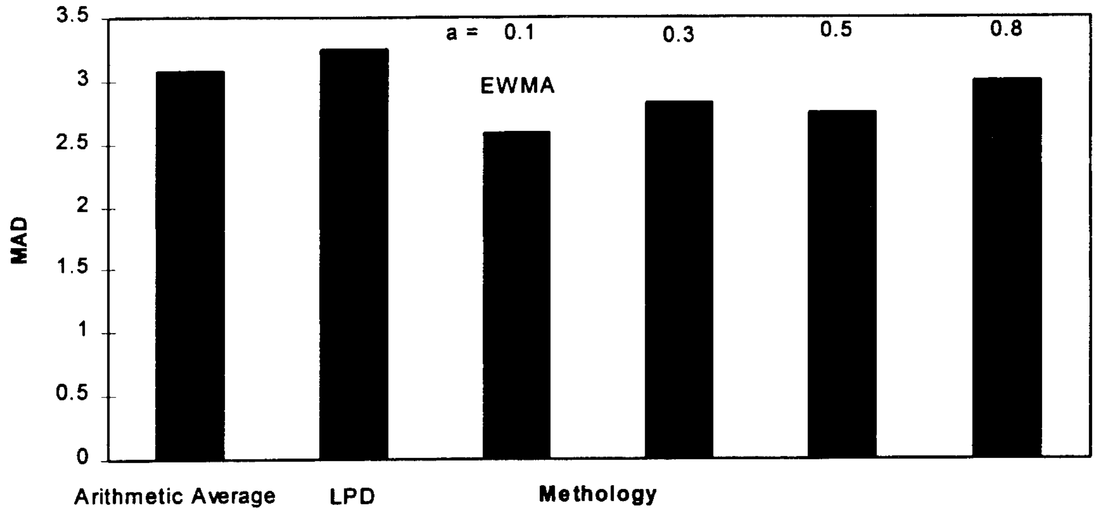

Figure 3. Flat Demand MAD

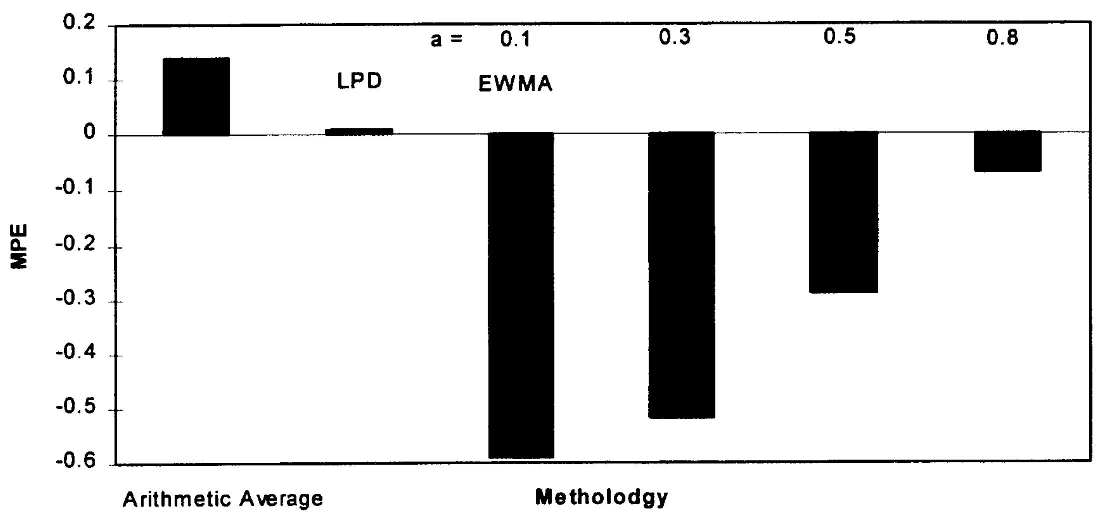

Figure 4. Flat Demand MPE

C. TREND DEMAND PATTERN

A trend demand pattern usually occurs for one or more of the following reasons:

- Downsize of the military unit results in decreased demand.
- Deteriorated or aged equipment results in increased demand.
- Subsequently activated new equipment results in a new or increase of demand.

This demand pattern can be detected by plotting the demand rate along time intervals. If the trend pattern is overlooked or being neglected, the conventional forecast methodology (arithmetic average or moving average) will be wrongly adopted by intuition, resulting in expensive forecasted deviation from the real demand. When we collected trend data and attempt to forecast demand, using moving average or arithmetic average the error from real demand can be approximately 20%.

An advanced alternative forecasting methodology is exponential smoothing average with trend correction (EWMA) is:

$$\hat{X}_t = \hat{X}_{t-1} + a(Y_{t-1} - \hat{X}_{t-1}) = a\,Y_{t-1} + (1-a)\,\hat{X}_{t-1}\,, \tag{17}$$

where:

$\hat{X}_t$ = forecasted demand for period *t*,

Y_{t-1} = actual demand for period *t-i*,

n = Number of time periods included in moving average.

With correction for trend components :

$$\hat{X}_t = aY_{t-1} + (1-a)(\hat{X}_{t-1} + T_{t-1})\,, \tag{18}$$

where:

T_{t-1} = the trend from the previous period.

The current forecast level is used to calculate the trend as the differences between consecutive forecasts and the trend component is adjusted as follows:

$$T_t = b(\hat{X}_t - \hat{X}_{t-1}) + (1-b)\,T_{t-1}\,, \tag{19}$$

where:

b = the trend smoothing constant.

Then:

$$\hat{Y}_t = \hat{X}_t + T_t \,, \qquad (20)$$

is the calculated forecast for EWMA with correction for the trend component of supply demand data.

As we explore this approach, we also use the different combinations of smoothing constant a and trend correction constant b to optimize the forecast accuracy. Appendix B present the results for some of the combinations of smoothing constants a and b. We can obtain the most accurate forecast with this approach. The deviation from real demand is reduced from 20% to 2.84%. This is a significantly improved forecast accuracy with EWMA and trend correction to forecast demand. Figures 5 and 6 provide a comparison of arithmetic average, moving average and EWMA.

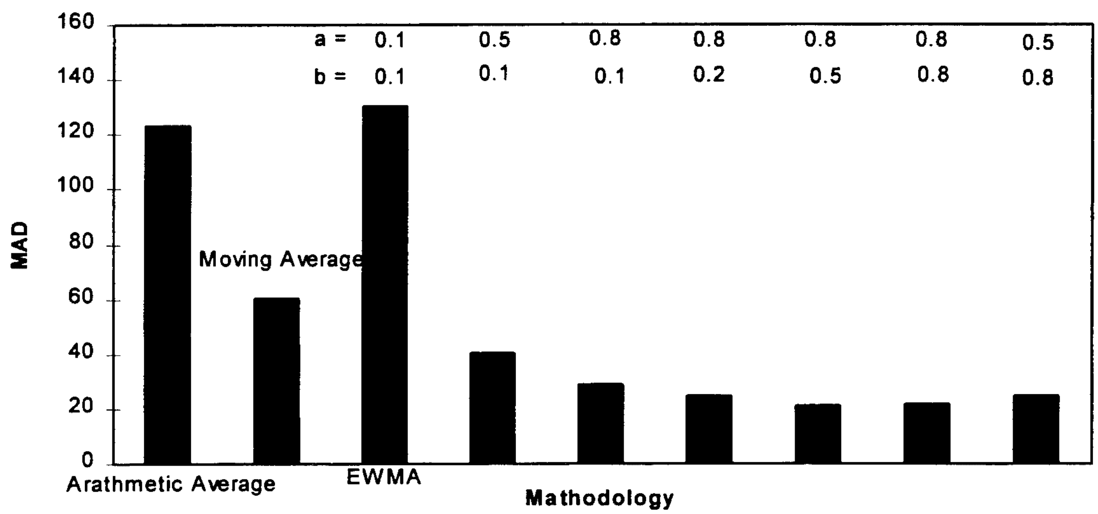

Figure 5. Trend Demand MAD

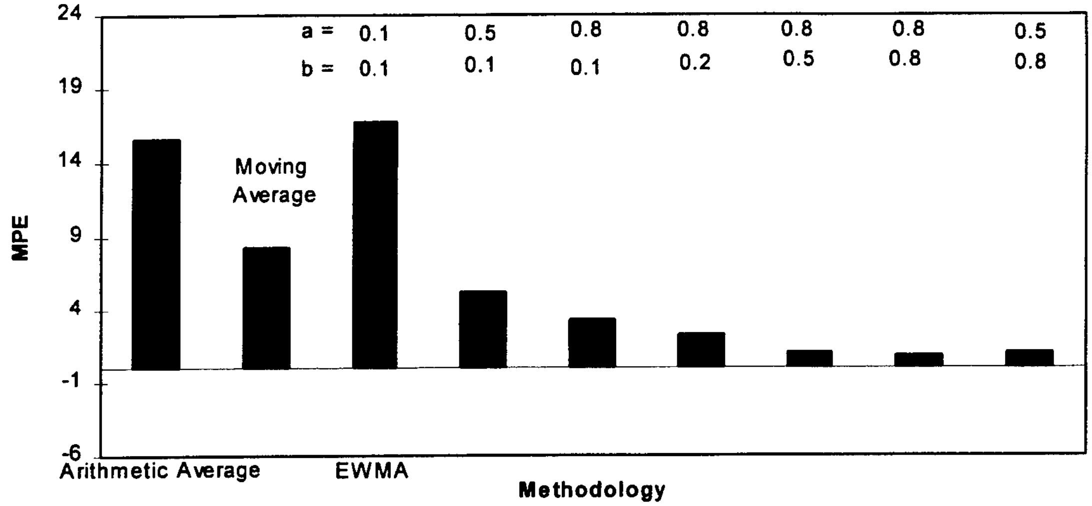

Figure 6. Trend Demand MPE

D. SEASONAL DEMAND PATTERN

As we know, seasonal demand will occur on the big event such as Christmas time in the business sector. This demand pattern will also occur in the military sector, for example during the periods before cyclic readiness inspections spare parts requisition will be higher than normal times, and seasonal equipment (anti-slip chain, anti-freeze package), grade oil (10W for winter seasons; 40W for summer season) demand will be higher or lower according to the different seasons.

How do we forecast this kind of demand pattern? Is there any forecast methodology which can forecast this kind of demand pattern? We explored the EWMA with seasonal correction approach to forecast the seasonal pattern and compared the result with the traditional ones (moving average and arithmetic average) for TALA forecast accuracy. Appendix C demonstrates EWMA adapted for seasonal components of TALA supply data. We can see that using the seasonal forecast approach to forecast seasonal demand would be better off with a MAD of 25.25 while arithmetic average and a moving average are at levels of 108.667 and 114.083. Even a MAD of 100 plus is misleading in that moving average will always lag the seasonal demand and arithmetic average will guarantee that there will be over and under stockage except perhaps two times in a seasonal cycle. Figures 7 and 8 present a comparison of arithmetic average, moving average and EWMA.

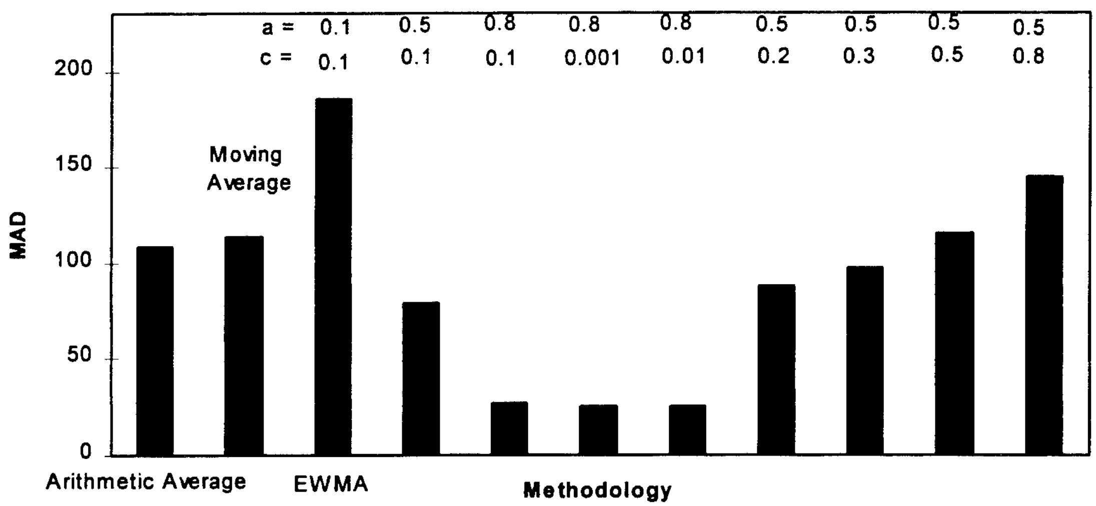

Figure 7. Seasonal Demand MAD

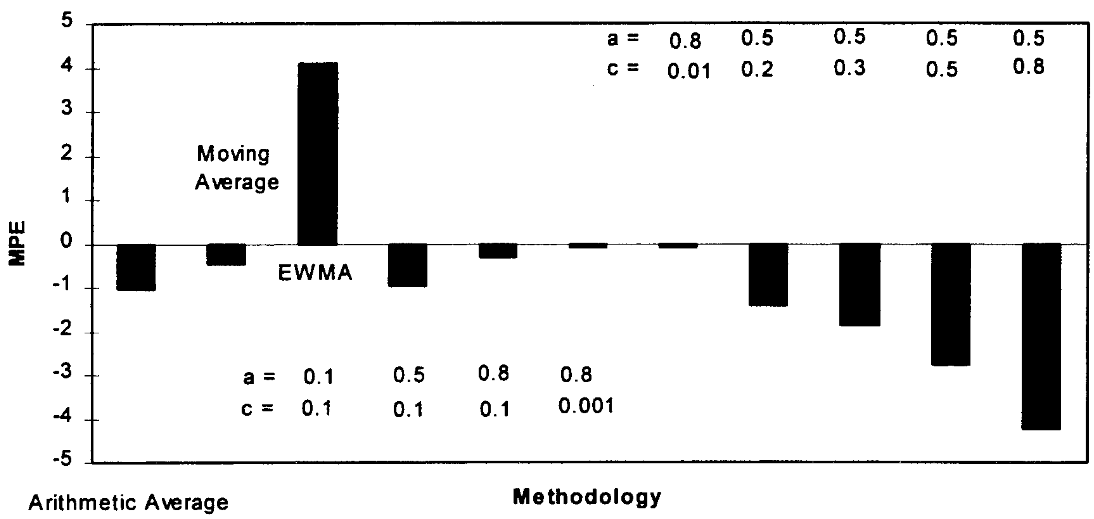

Figure 8. Seasonal Demand MPE

E. CONCLUSIONS

As we know from the previous sections, before we adopt a forecast methodology we have to know what the demand pattern is and what time series components it contains. This can be determined by plotting the historical demand rate. After deciding the demand pattern (flat , trend or seasonal demand pattern) or patterns, then an appropriate forecast approach can be adopted to create more accurate forecasts and reduce the deviation from real demand. Through this approach we would reduce the redundant and waste and reduce the defense budget or allow expenditures on other requirements.

IV HOLDING COST

It is important to remember that holding cost is per unit per unit time in this model. For any order cycle the holding cost may be calculated by multiplying the area under the inventory curve and above zero by 'h'. (See Figure 9)

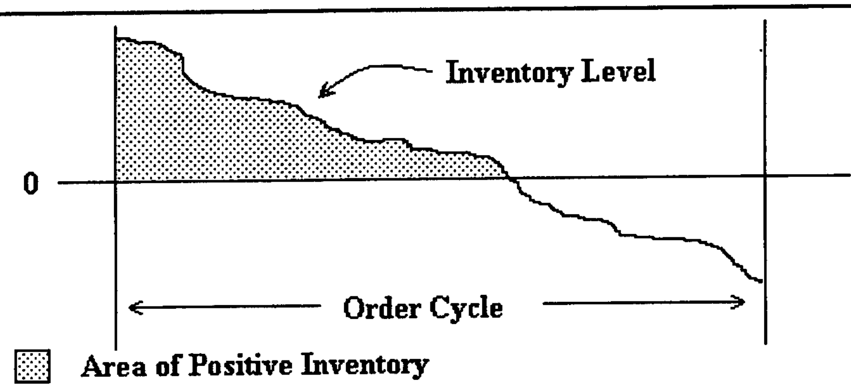

Figure 9. Order Cycle

I will present two common approximations for estimating the per unit time inventory holding area, and hence holding cost. The differences between the two are most dramatic in the case of order cycle with shortage.

In Figure 10, the exact order cycle holding area is represented by Δiqkl. The first approximation is represented by Δijl, and the second area approximation equals Δiqkl less Δkjm. Clearly the first approximation would tend to over estimate the actual value, while the second would under estimate the actual inventory holding area. In the context of inventory models such as common EOQ stochastic extensions, algebraic specifications of the exact inventory holding area either do not exist or are prohibitively cumbersome. However, both approximations can be readily specified, with the second approximation even easier than the first. The issue is then to determine which area approximation is more accurate, with the realization that this may depend on the underlying parameter values for such things as leadtime reliability.

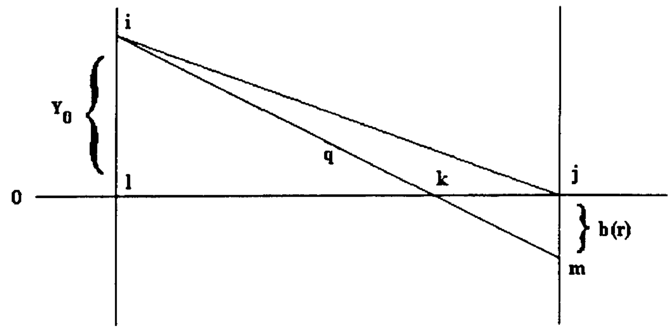

Figure 10. Stockout Approximation Geometry

In both approaches we need the expected value of Y_0, the average initial inventory of the cycle. or any given cycle, the beginning inventory is equal to the order quantity plus any residual inventory and less any stockout from the previous cycle (assuming all stockouts are back ordered). Thus the average initial cycle inventory is equal to the order quantity plus expected residual inventory and minus expected stockout:

$$Y_0 = Q + Y(r) - B(r) \tag{21}$$

where r is the reorder point, and the expected residual inventory and expected back order are respectively defined by:

$$\mathbf{Y}(r) = \int_0^r (r-x)\ \mathbf{f}_{\mathrm{DL}}(x)\ \mathbf{d}x \tag{22}$$

$$\mathbf{B}(r) = \int_r^\infty (x-r)\ \mathbf{f}_{\mathrm{DL}}(x)\ \mathbf{d}x \tag{23}$$

where $f_{DL}(x)$ is the usage during leadtime probability density function. However, note that the expected residual inventory minus the expected back order is equal to the reorder point minus the expected demand during leadtime which in turn equals the safety stock:

$$\begin{aligned}\mathbf{Y}(r) - \mathbf{B}(r) &= \int_0^\infty (r-x)\ \mathbf{f}_{\mathrm{DL}}(x)\ \mathbf{d}x = r - \int_0^\infty x\, \mathbf{f}_{\mathrm{DL}}(x)\ \mathbf{d}x \\ &= r - \mu_{DL} = SS\end{aligned} \tag{24}$$

Therefore, the expected initial cycle inventory is equal to the order quantity plus safety stock:

$$Y_0 = Q + r - \mu_{DL} = Q + SS \tag{25}$$

A. FIRST APPROXIMATION

The first approximation involves specifying the average inventory level as the simple average of the initial cycle inventory and any residual inventory:

$$\overline{Y}_1 = \frac{Y_0 + Y(r)}{2} = Y(r) + \frac{Q - B(r)}{2} \tag{26}$$

With this approach, all cycles which end in with stockout are treated as thought they end with exactly zero inventory -- no residual inventory and no back orders. The average cycle inventory area is then:

$$\frac{Q\overline{Y}_1}{\mu_D} = \frac{Q}{\mu_D}\left(Y(r) + \frac{Q - B(r)}{2} \right) \tag{27}$$

In this case the per unit time (e.g. annual) holding cost is:

$$HC_1 = h\overline{Y}_1 = h\left(Y(r) + \frac{Q - B(r)}{2} \right) \tag{28}$$

This type of holding cost approximation is adopted by Love [Ref. 2] and Brown [Ref. 3].

B. SECOND APPROXIMATION

For the second approximation, the average inventory level is simply Q/2 plus safety stock:

$$\overline{Y}_2 = \frac{Q}{2} + SS \tag{29}$$

The somewhat naive intuition behind this approach is that inventory consists of two separate parts, safety stock which is always there, and the regular inventory which is depleted through the cycle and represented by Q/2.

However, this approach ignores the fact that safety stock is not always there, and for some cycles the corresponding value is negative. Safety stock is the average ending cycle inventory -- averaged over both residual inventories for some cycles and stockouts for other cycles. To be more precise, safety stock is the expected difference between the reorder point and the actual demand during leadtime:

$$SS = \int_0^{\infty} (r-x)\ f_{DL}(x)\ dx \tag{30}$$

Normally, for most cycles this difference is positive, resulting in residual inventories. However, there will always be some cycles for which the difference is negative, producing stockouts. The second approximation treats these back orders as negative inventory for the purpose of calculating holding cost. That is, they are treated as a negative cost, which they surely are not.

For cycles with stockout, the average cycle inventory with this approximation is an average of a positive number (Y_0) and a negative number, the conditional expected stockout:

$$\overline{Y_2}\big|_{x>r} = \frac{Y_0 - \dfrac{B(r)}{\int_r^{\infty} f_{DL}(x)\ dx}}{2} \tag{31}$$

Note that the negative number in this case may be relatively large compared to B(r) because the integral, representing the probability of stocking out, may be quite small.

With the second approach, the average cycle inventory area is:

$$\frac{Q\,\overline{Y_2}}{\mu_D} = \frac{Q}{\mu_D}\left(\frac{Q}{2} + SS\right) \tag{32}$$

and the per unit time (e.g. annual) holding cost is:

$$HC_2 = h\overline{Y}_2 = h\left(\frac{Q}{2} + SS\right) \qquad \textbf{(33)}$$

This type of holding cost approximation is adopted by several authors including by Ballou [Ref. 4]. The difference between the two average inventory approximations is half of the expected stockout:

$$\overline{Y}_1 - \overline{Y}_2 = \left(Y(r) + \frac{Q - B(r)}{2} \right) - \left(\frac{Q}{2} + Y(r) - B(r) \right) = \frac{B(r)}{2} \qquad \textbf{(34)}$$

C. EXACT INVENTORY HOLDING AREA

The exact inventory area may be determined by first dividing the inventory cycle into two periods based on when the order is placed. Thus the first period is the time following the arrival of an order up until inventory gets down to the reorder point, and the second period occurs over the length of the leadtime. The inventory area geometry of the second period will then depend on whether the cycle ends with residual inventory or stockout.

The inventory areas for these periods are discussed below. An exact average inventory area specification is developed for the first period. However for the second period, the specifications are not averages, but are rather based on actual leadtime and demand values. These second period specifications are used through simulation to determine average values.

An implicit assumption of this whole development is that the demand in the second period is statistically independent of the demand in the first. This is an logical extension of the independence assumption for demand during any two inventory cycles.

1. First Period Area

The first period area A is the trapezoidal area which is the area between the initial inventory level and reorder level, with the base t_r and slope μ_D.

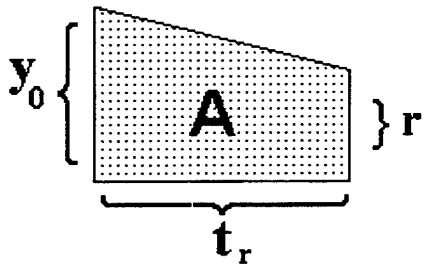

Figure 11. First Period

To calculate the trapezoidal area we can use the variables specified above and substitute them into the trapezoidal area equation (35).

$$\begin{aligned} \mu_D &= \frac{Y_0 - r}{t_r} \\ t_r &= \frac{Y_0 - r}{\mu_D} \\ A &= \frac{(Y_0 + r)\, t_r}{2} = \frac{(Y_0 + r)}{2}\left(\frac{Y_0 - r}{\mu_D}\right) \\ &= \frac{Y_0^2 - r^2}{2\mu_D} \end{aligned} \tag{35}$$

So, we can transform equation 35 into 36:

$$A = \frac{(Q + SS)^2 - r^2}{2\mu_D}. \tag{36}$$

2. Second Period with Nonzero Residual Inventory

The second period with nonzero residual inventory is the area demonstrated in Figure 12. This figure represents the area from the reorder point to the end of the cycle with a base of the leadtime (LT), and with the right hand side equal to residual inventory, i.e., the reorder point minus demand during leadtime (DL):

$$
\begin{aligned}
B &= \frac{r + (r - DL)}{2} \times LT \\
&= LT\left(r - \frac{D \cdot LT}{2} \right)
\end{aligned}
\tag{37}
$$

where D is the actual demand rate, LT is the actual leadtime, and DL is the consequent actual demand during leadtime (DL=D·LT).

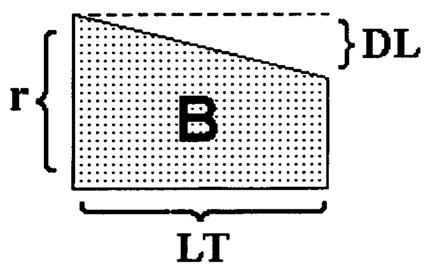

Figure 12. Second Period

3. Second Period with Nonzero Shortage

The second period area with nonzero shortage is the area shown in Figure 13 which is after the reorder point and before the next order quantity has arrived. The inventory in this area will be depleted at some point before the end of the cycle which results in a nonzero shortage during this period. We can express this area as:

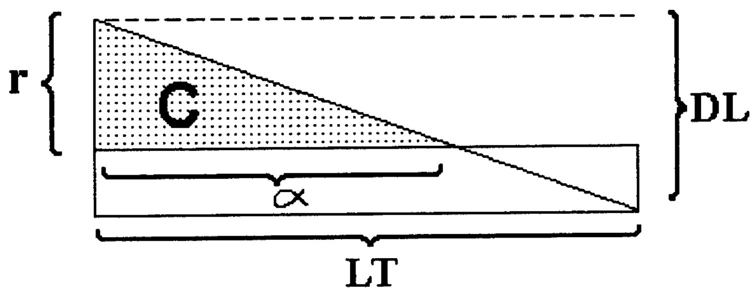

Figure 13. Second Period, Non-zero Shortage

$$C = \frac{r\,\alpha}{2} \tag{38}$$

But by similar triangles:

$$\begin{aligned} \frac{\alpha}{r} &= \frac{LT}{DL} = \frac{LT}{D \cdot LT} = \frac{1}{D} \\ \alpha &= \frac{r}{D} \end{aligned} \tag{39}$$

And hence:

$$C = \frac{r^2}{2D} \tag{40}$$

D. SUMMARY

The accuracy of both approximations will depend on several parameters:

- Average Demand Rate
- Demand Variability (Demand Coefficient of Variation)
- Average Leadtime
- Leadtime Variability (Leadtime Coefficient of Variation)

- Order Quantity
- Reorder Point

To analyze each parameter's effect on the inventory holding area we utilized the crystal ball spreadsheet simulation package. By changing each parameter value one at a time, and keeping other parameters unchanged, we can observe each parameter's effect on the holding area for both the first and second approximations, as well as the exact formulation. For all simulations, both leadtime and demand were assumed to have log-normal distributions. Each simulation included 1,200 trials.

Initially all parameter values were specified as in Table 4. Average demand was always kept at this value (1,000 units per unit time). However, all of the other parameters were varied to determine if the relative accuracy of the two approximations depended on these parameter values. First, demand variability, leadtime variability and average leadtime were independently varied as indicated in Table 5. Then, order quantity and reorder point were varied together as presented in Table 6. For example, the bottom row indicates that a reorder point value of 5,000 units was examined in conjunction with two order quantity values, 5,000 and 10,000.

Figure 14 presents the results of this analysis for the case where demand variability was varied. These results are typical in the following respects:

- As expected, the simulated exact average area was always less than the first (Love/Brown) approximation.
- Contrary to expectations, the simulated exact average area was sometimes less than and sometimes greater than the second (Ballou) approximation
- In absolute terms (ignoring signs), the second (Ballou) approximation was always superior to the first (Love/Brown) approximation, usually by an order of magnitude or more. Occasionally they provided approximately the same accuracy.

The results for the other variations (2nd and 3rd columns of Table 5, and Table 6) are presented in Appendix D.

These results, as illustrated by Figure 14, merit further investigation. In particular, the unexpected relationship between the second (Ballou) approximation and the simulated exact values needs to be explained. The next logical step would involve using the real analysis (calculus) associated with probability theory to investigate these questions.

Average Demand	1000
Demand Coeffience of Variation	0.01
Average Leadtime	1
Leadtime Coeffience of Variation	0.01
Reorder Point (ROP)	1000
Qrder Quantity (Q)	5000

Table 4. Primary Parameter Values

Demand Coefficient of Variation	Leadtime Coefficient of Variation	Average Leadtime
0.01	0.01	0.001
0.05	0.05	0.01
0.1	0.1	0.1
0.2	0.2	0.2
0.3	0.3	0.5
0.5	0.5	0.75
0.8	0.8	1.0
1.0	1.0	1.1
1.2	1.2	
1.5	1.5	
2.0	2.0	
5.0	5.0	
10.0	10.0	

Table 5. Leadtime and Demand Parameter Values

Reorder Point (ROP)	Order Quantity(Q)					
1	1	10	100	1000	5000	10000
100			100	1000	5000	10000
1000				1000	5000	10000
5000					5000	10000

Table 6. Combination of ROP & Q

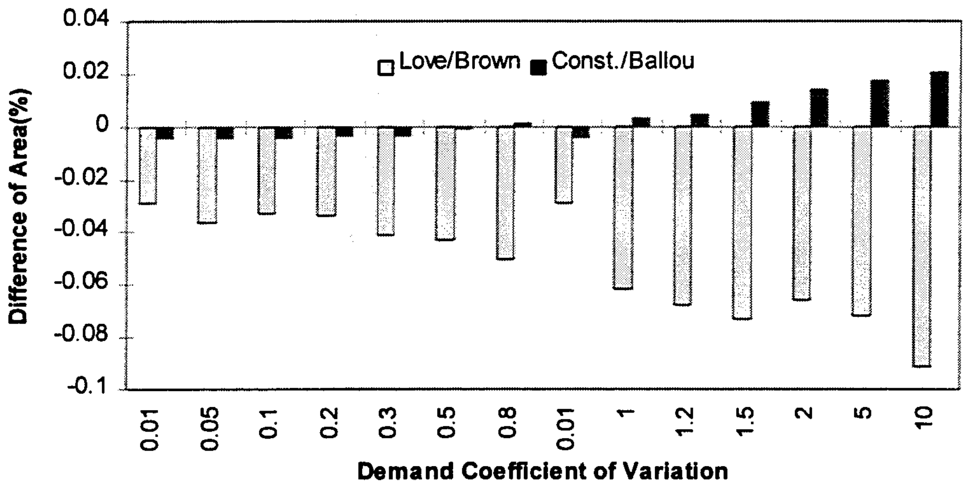

Figure 14. Demand Coefficient of Variation V.S. Inventory Area

V. SUMMARY, CONCLUSIONS AND RECOMMENDATION

A. SUMMARY

The TALA's inventory management system has been historically ineffective in preventing the accumulation of inapplicable inventories when equipment is retired from service and has failed to anticipate shortage due to increasing demand when new equipment has entered service. Both of these shortcomings are due in large part to inappropriate or nonprofessional forecasting approach. The moving average or arithmetic average forecasting methodology will only perform well for some special demand patterns (in most cases the flat demand pattern). The flat demand pattern rarely occurs and these forecasting approaches cannot provide reliable demand prediction. There are several advanced forecasting methodologies designed for tracking complicated demand patterns (trend demand pattern, seasonal demand pattern and the combination of trend and seasonal demand pattern). These Exponential Weighted Moving Average (EWMA) methodologies [Ref. 1. pp. 53-63] can be applied efficiently if the demand pattern for the item has been researched by plotting the items historical demand rate along a time line. Once the demand pattern has been established, we can use the relevant forecast methodology (simple EWMA, EWMA with trend correction, EWMA with seasonal correction) to project the future demand.

In this research we collected some TALA's historical forecast demand data, analyzed those items' demand patterns, and reforecasted them at a projected demand based on EWMA. The results shows there was an improved forecast accuracy using EWMA. If TALA would adopt this approach, they may save millions of dollars per year and improve the readiness and efficiency of their inventory system.

As mentioned, there are two different kinds of stockage, residual inventory and shortage. Superfluous residual inventory will result in waste, as represented by inventory holding cost, and while stockout may save on holding cost, there is a negative impact on customer service and readiness. There are complicated factors that interact and effect the stockage level and inventory control. The most influential are average leadtime, leadtime variation, average demand rate, demand variation, reorder point and order quantity. Two popular formulations which take these factors into consideration to compute the holding cost area were examined. The first approximation involves specifying the average inventory level as the simple average of the initial cycle inventory and any residual inventory. The second approximation, the average inventory level is simply $\frac{Q}{2}$ plus the safety stock. The assumption behind this approach is that inventory consists of two separate parts, safety stock which is always there, and the regular inventory which is depleted through the cycle and represented by $\frac{Q}{2}$.

To compare the two approximation's accuracy, we have used the crystal ball spreadsheet simulation package to simplify the solvability of this sophisticated probability integral problem.

B. CONCLUSIONS

As is true with most logistical problems, Inventory Officers in TALA are faced with complicated demand forecasting. Before advanced forecasting methodologies were proven to be effective they have preferred the simple and traditional forecasting approaches to approaches that are puzzling or confusing. Through this research, we can make the statement that there is no one specific forecasting methodology that can be applied universally. The best way to make demand forecasting accurate and efficient is to adopt a suitable approach that fits the demand pattern. In this research we have collected empirical data to explore the various forecasting methodology. The results show that arithmetic average and moving average are adequate for the flat demand pattern; however, more sophisticated forecasting methods (EWMA) are required when demand data contains seasonal or trend components. This result would seem to require that data must be kept on each item in the system and the method chosen for forecasting must fit the demand pattern for that item.

In comparing the two holding cost area approximations with the Crystal Ball simulation package, the second approximation demonstrated clear superiority and is recommended for use in modeling with appropriate inventory policies.

C. RECOMMENDATIONS

That TALA implement advanced demand forecasting methodologies. The advanced forecasting methodologies discussed in Chapter II and III have demonstrated an improvement in accuracy over the methods currently used by TALA. Since real demand pattern changes seasonally and due to the effect of trends and random factors, the current methods used by TALA are inadequate and significant costs could be saved if TALA were to change their forecasting approach. Additional accuracy could be realized if holding costs and Ballou approximations of holding area calculations were performed by TALA.

Future research by TALA should be performed since the demand pattern varies over time and it is necessary to continually update the smoothing constants used in the EWMA demand forecasting method. At this time updating smoothing constants can only be done by trial and error. It may be possible for TALA to develop computer software to calculate optimal smoothing constants based on their supply system demand data. This would be more efficient and more accurate than hand calculations and is likely to be more timely.

APPENDIX A. FLAT DEMAND PATTERN

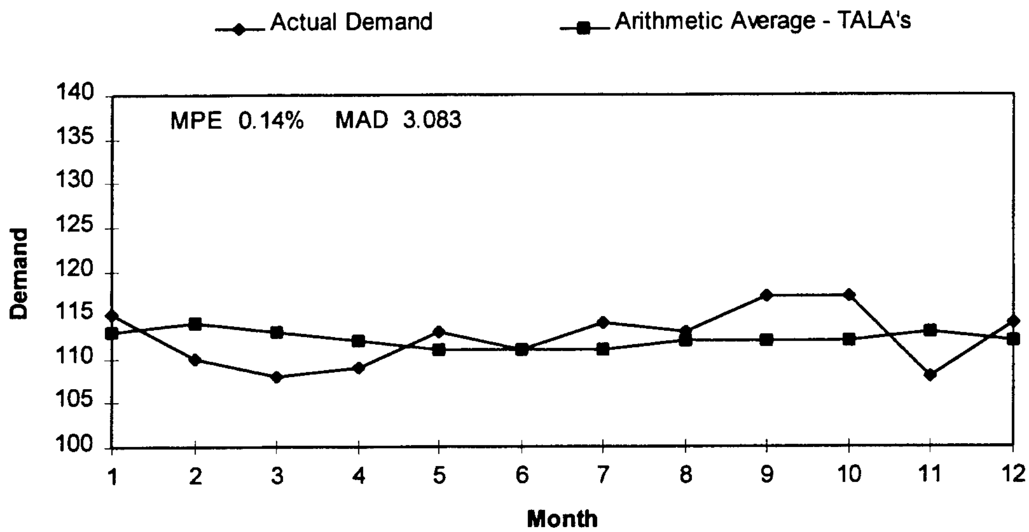

Figure 15. Flat Demand Pattern - Arithmetic Average

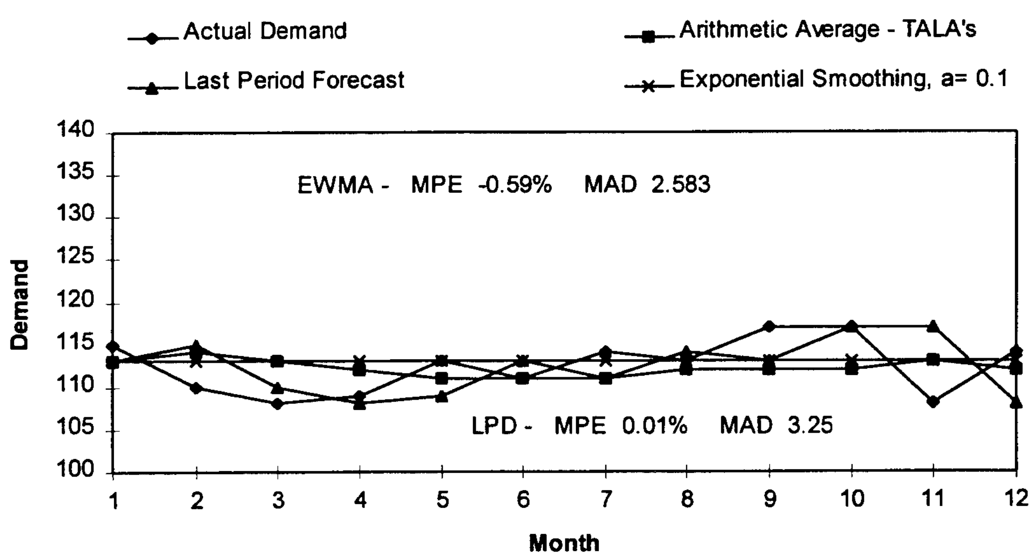

Figure 16. Flat Demand Pattern - Exponential Smoothing a=0.1

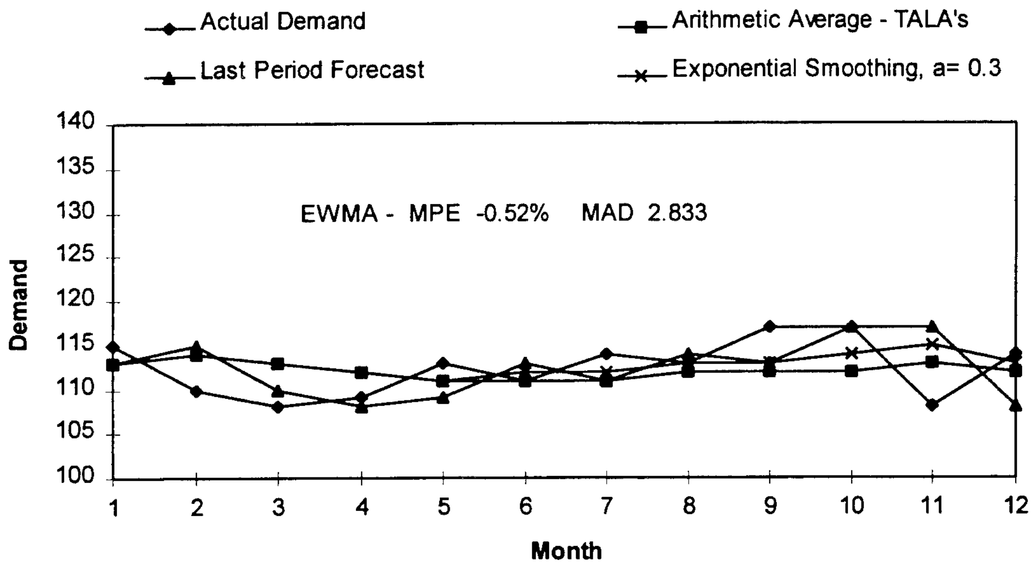

Figure 17. Flat Demand Pattern - Exponential Smoothing a=0.3

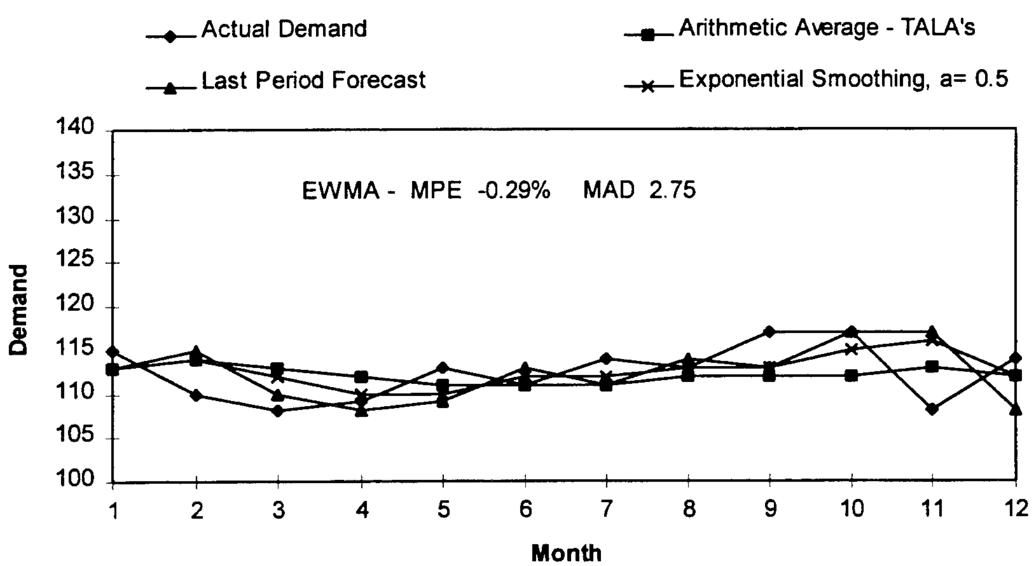

Figure 18. Flat Demand Pattern - Exponential Smoothing a=0.5

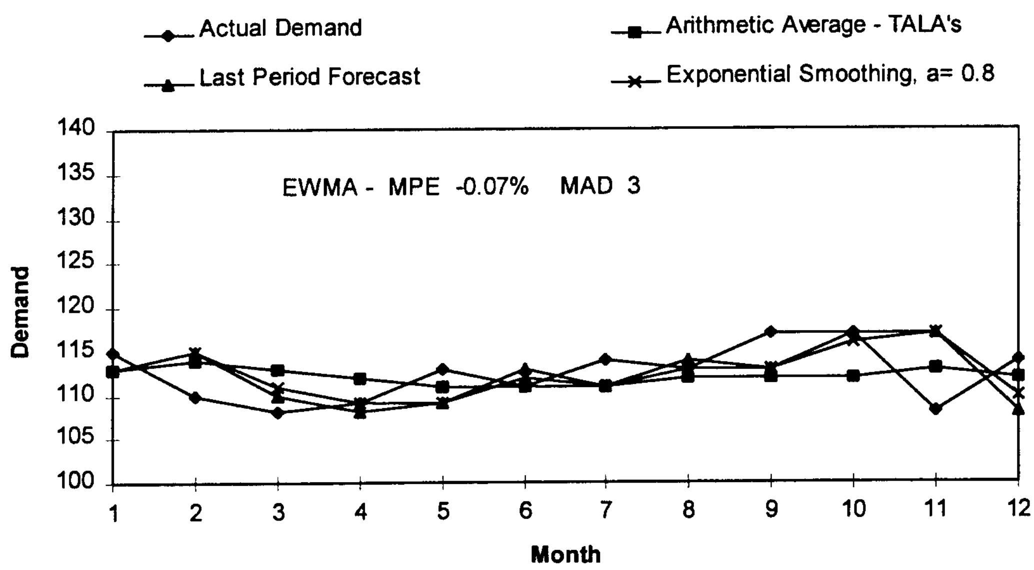

Figure 19. Flat Demand Pattern - Exponential Smoothing a=0.8

APPENDIX B. INCREASING DEMAND PATTERN

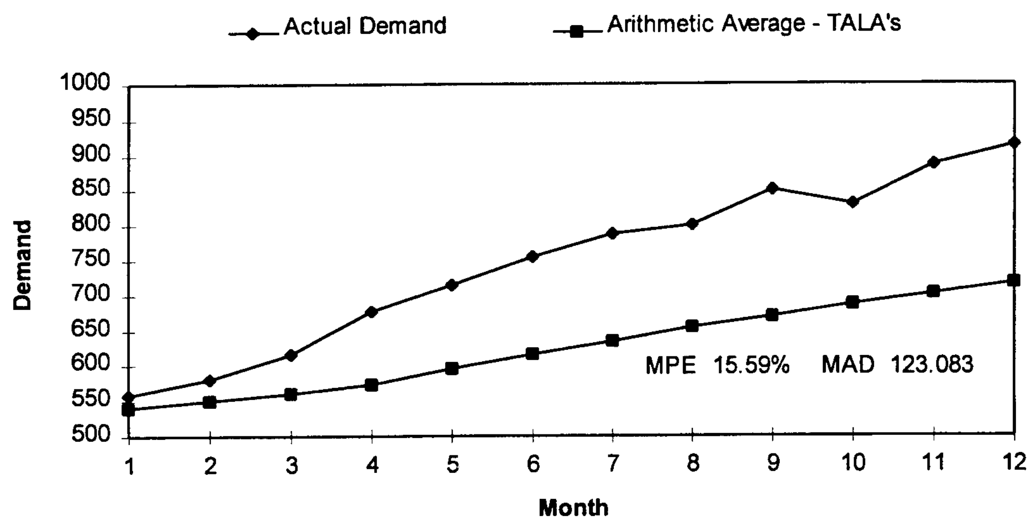

Figure 20. Trend Demand Pattern - Arithmetic Average

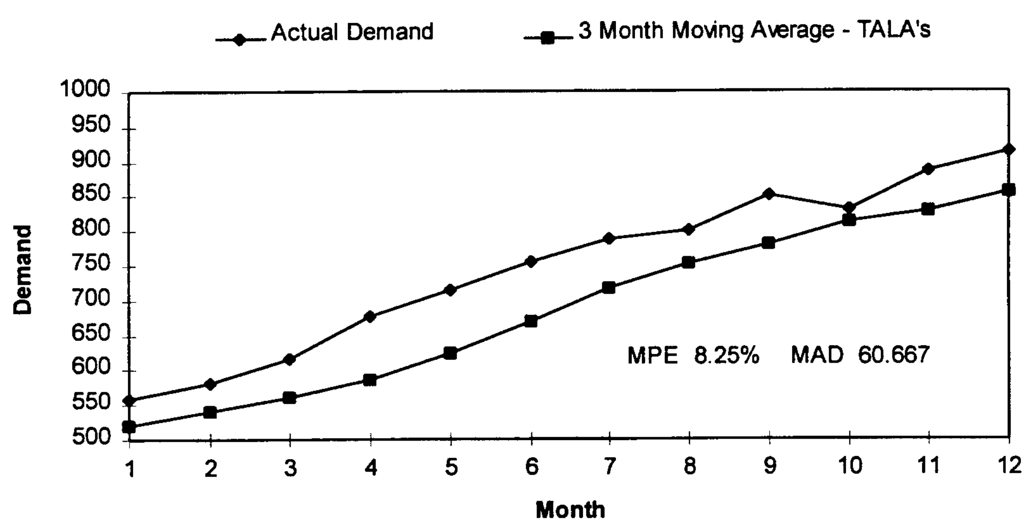

Figure 21. Trend Demand Pattern - Three Month Moving Average

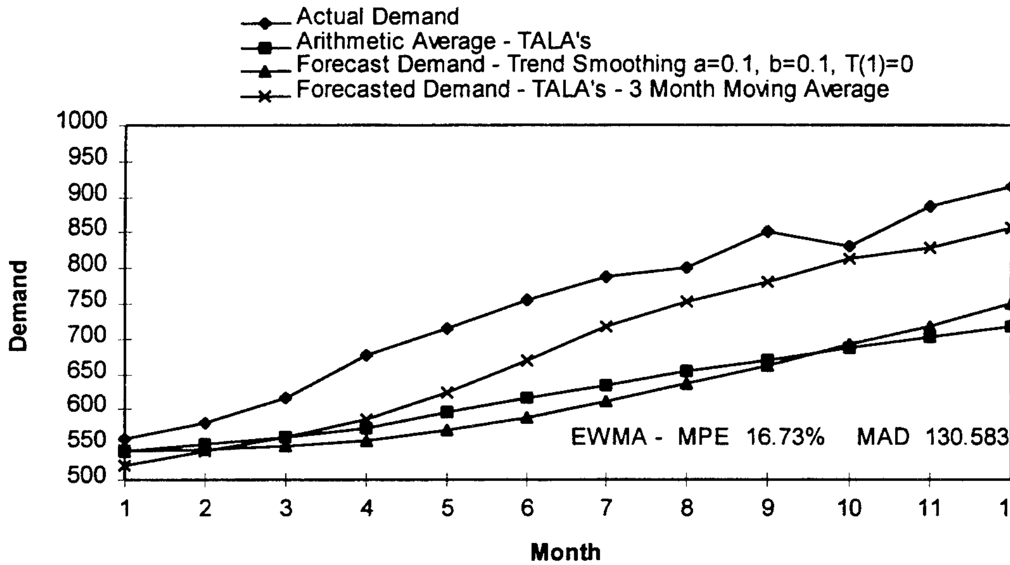

Figure 22. Trend Demand Pattern - Trend Smoothing a=0.1, b=0.1, t(1)=0

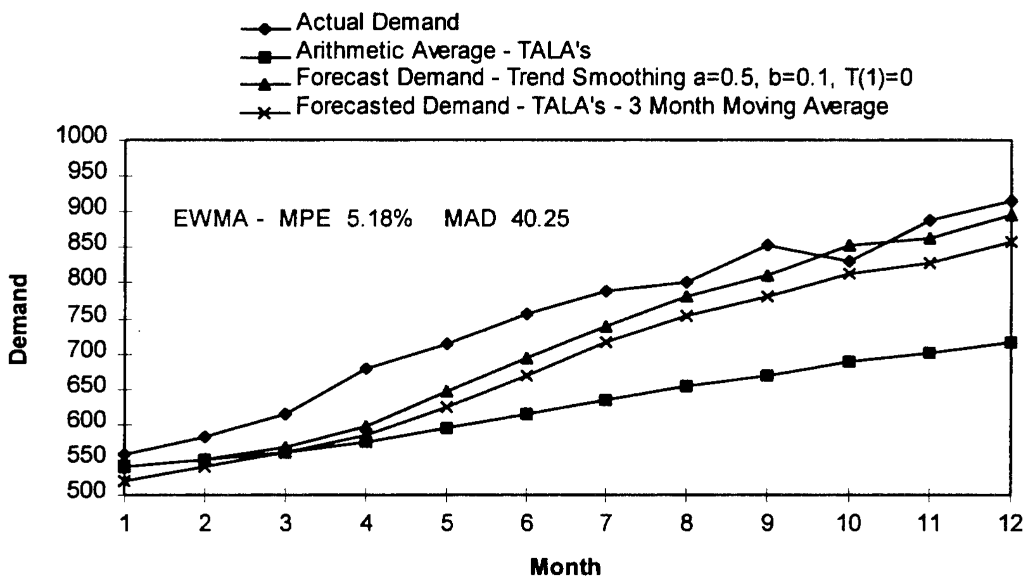

Figure 23. Trend Demand Pattern - Trend Smoothing a=0.5, b=0.1, t(1)=0

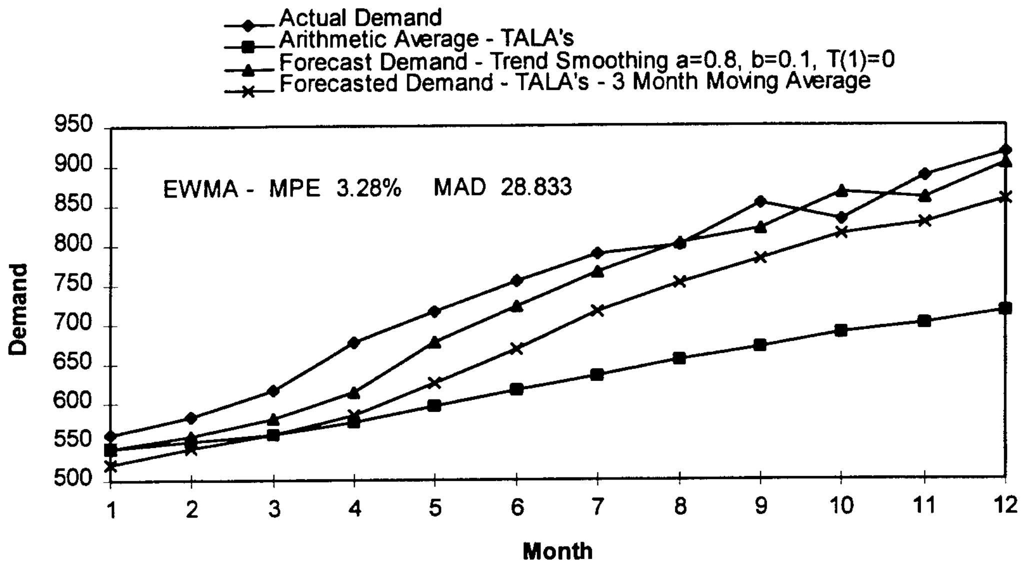

Figure 24. Trend Demand Pattern - Trend Smoothing a=0.8, b=0.1, t(1)=0

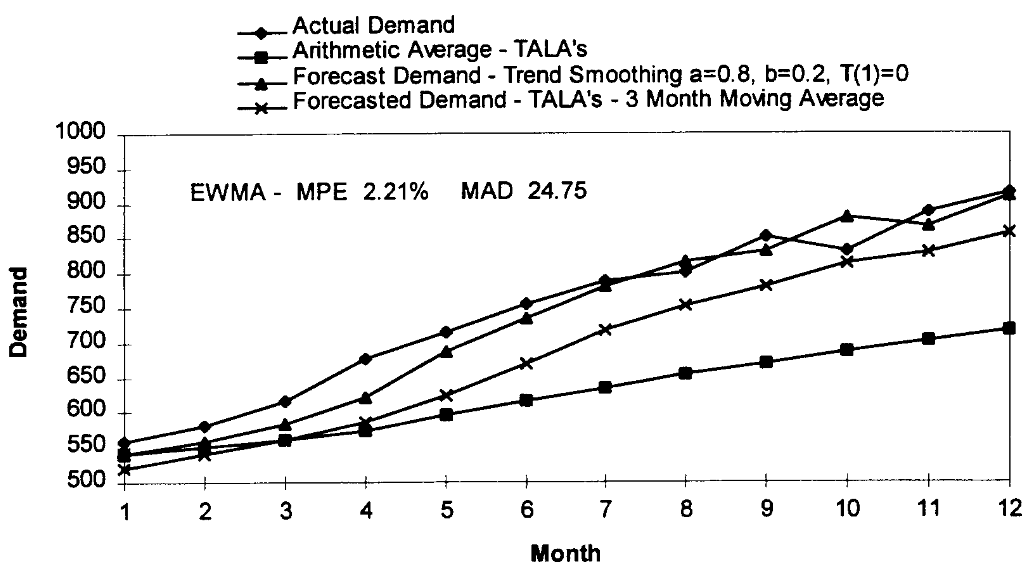

Figure 25. Trend Demand Pattern - Trend Smoothing a=0.8, b=0.2, t(1)=0

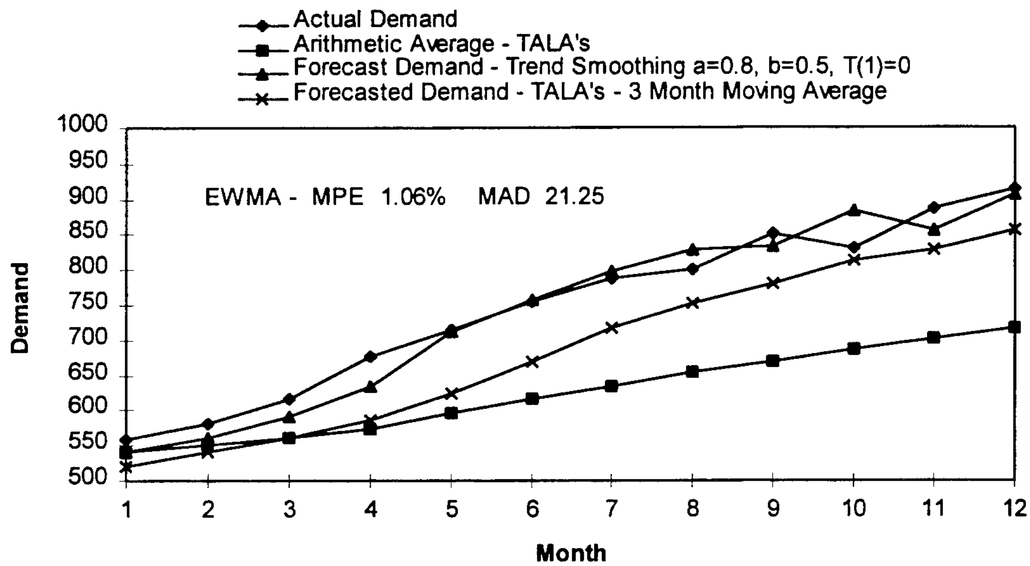

Figure 26. Trend Demand Pattern - Trend Smoothing a=0.8, b=0.5, t(1)=0

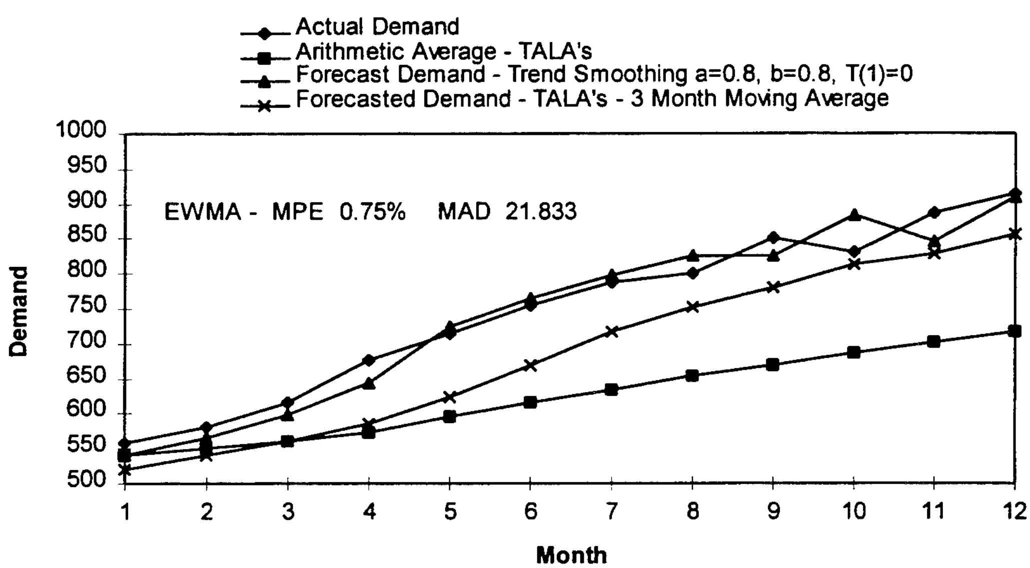

Figure 27. Trend Demand Pattern - Trend Smoothing a=0.8, b=0.8, t(1)=0

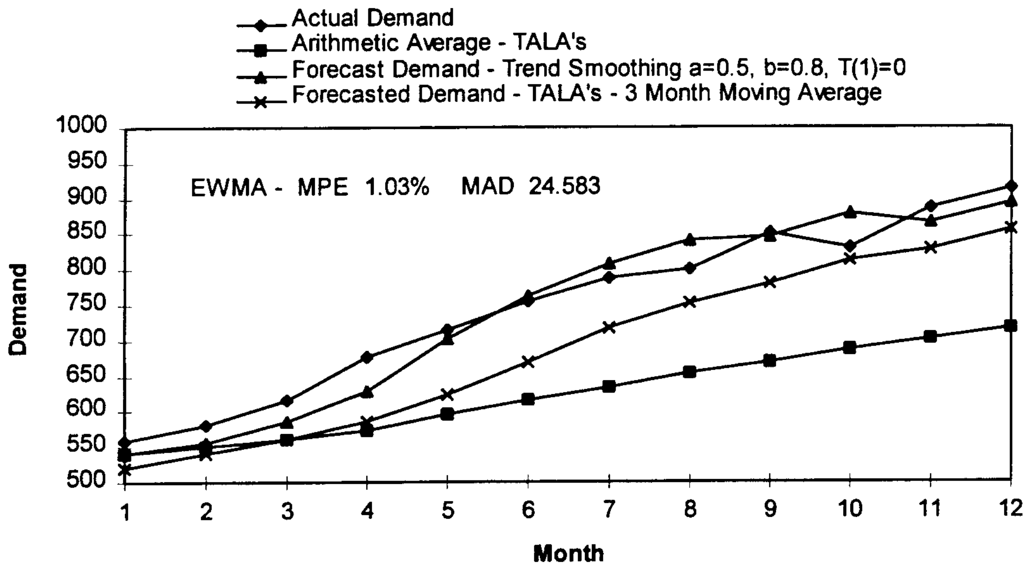

Figure 28. Trend Demand Pattern - Trend Smoothing a=0.5, b=0.8, t(1)=0

APPENDIX C. SEASONAL DEMAND PATTERN

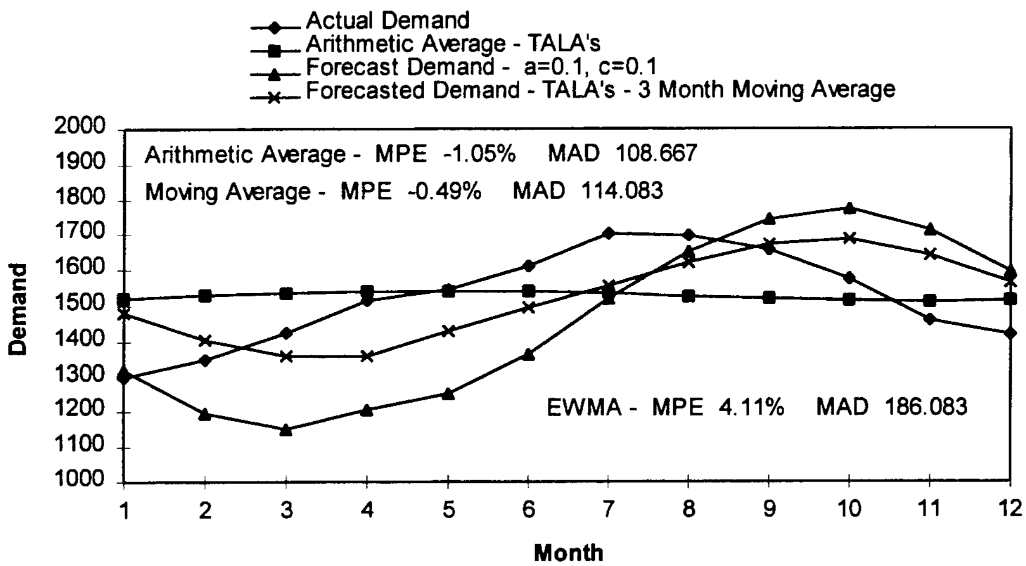

Figure 29. Seasonal Demand Pattern a=0.1, c=0.1

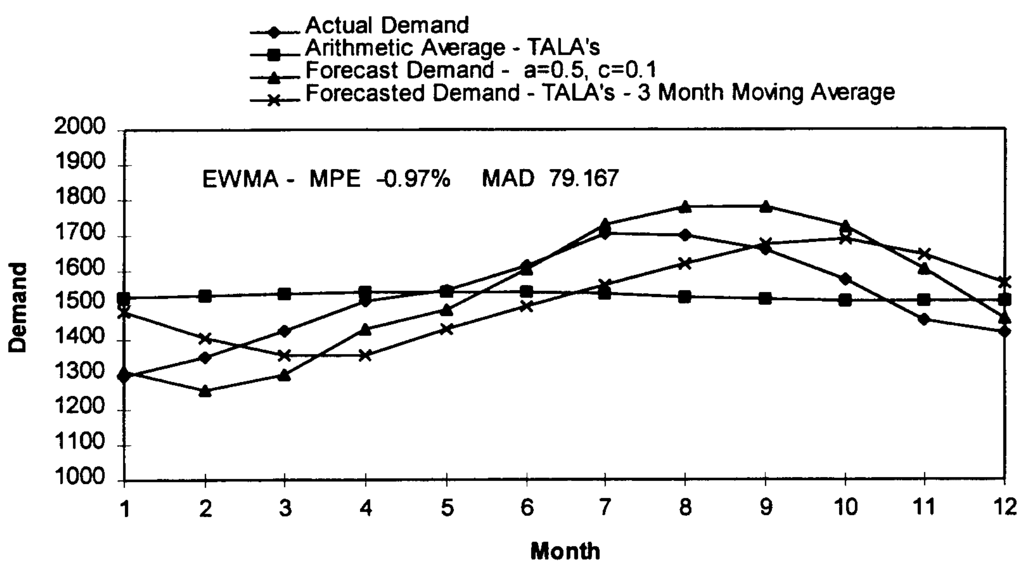

Figure 30. Seasonal Demand Pattern a=0.5, c=0.1

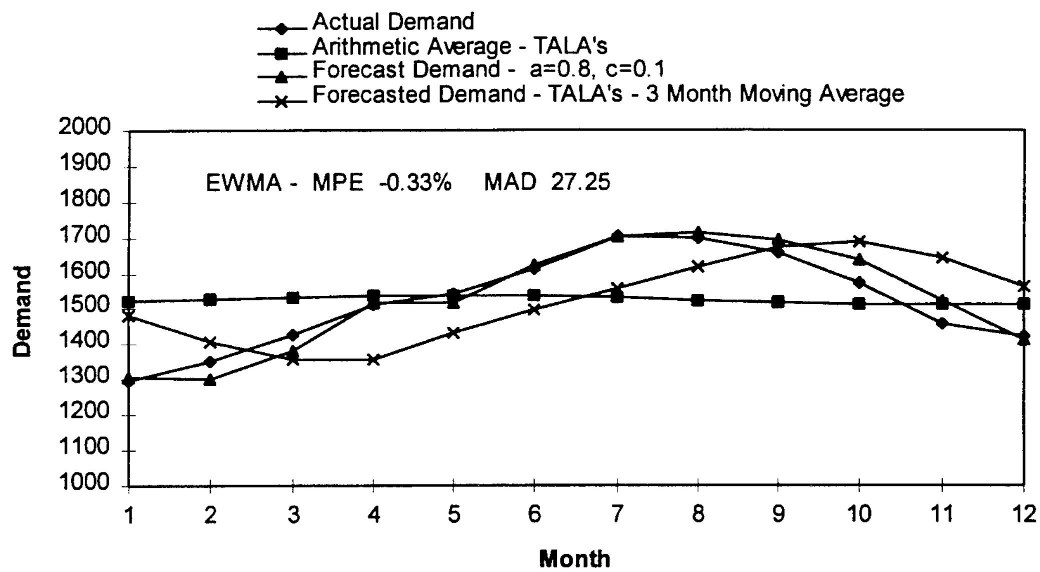

Figure 31. Seasonal Demand Pattern a=0.8, c=0.1

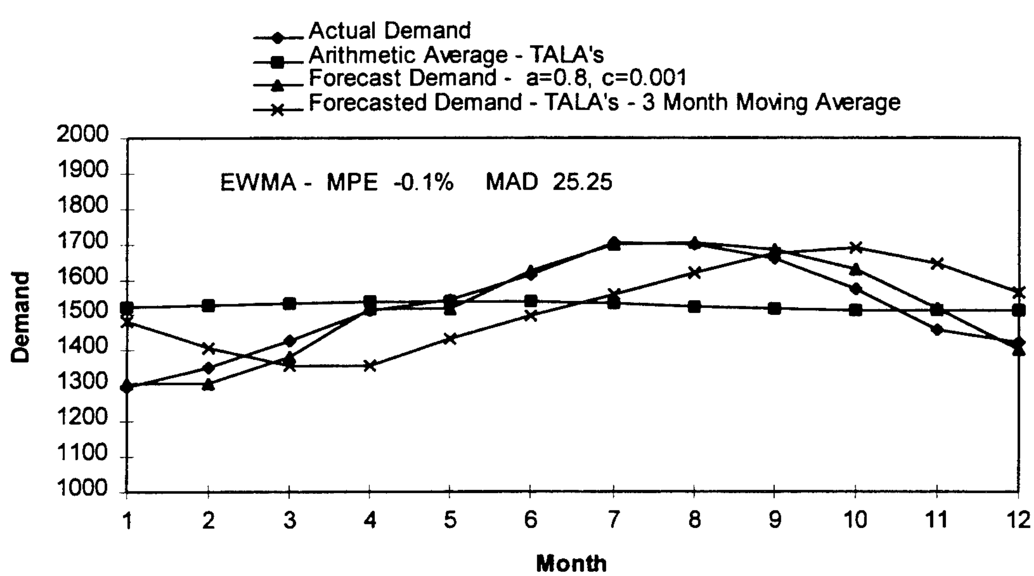

Figure 32. Seasonal Demand Pattern a=0.8, c=0.001

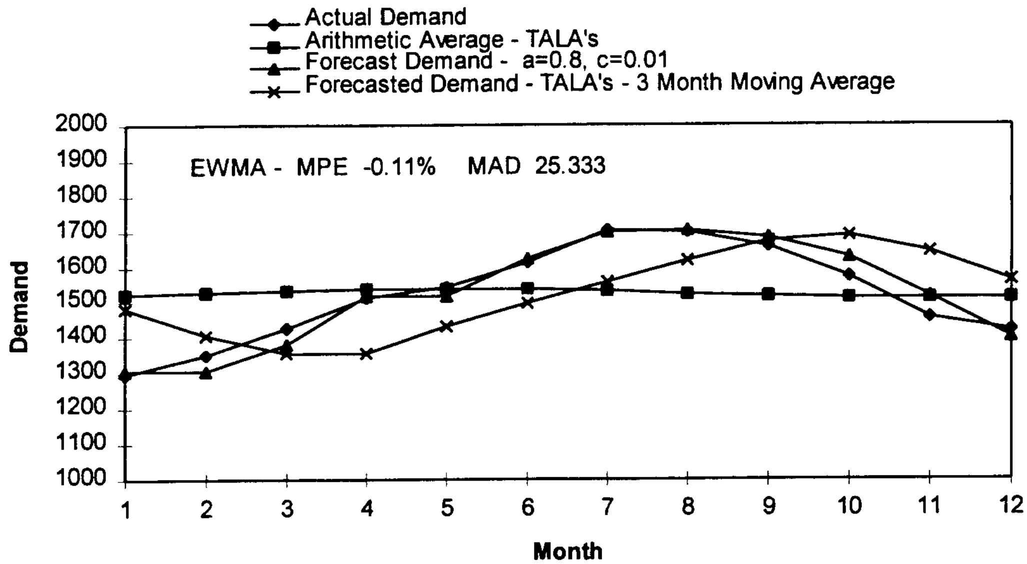

Figure 33. Seasonal Demand Pattern a=0.8, c=0.01

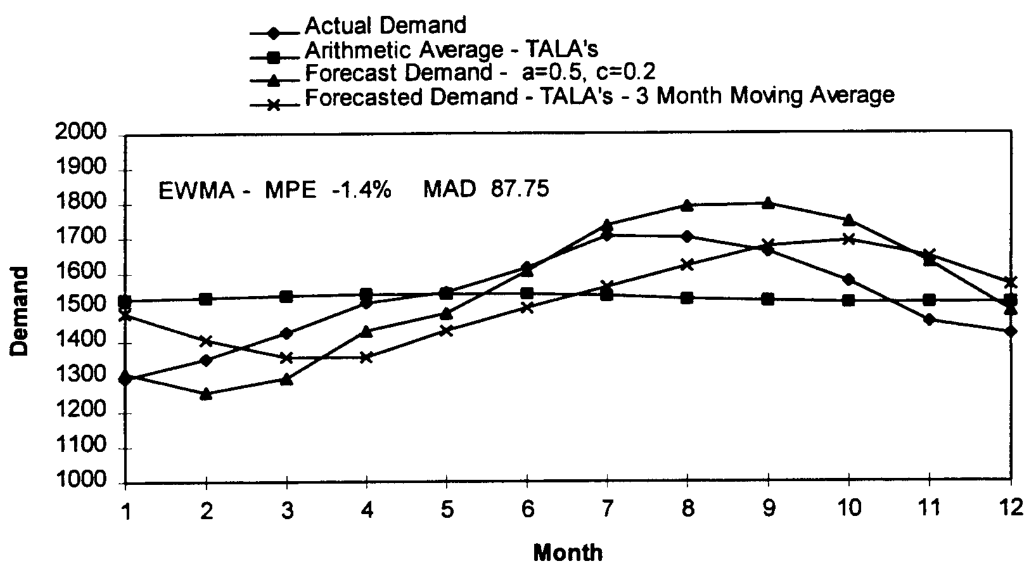

Figure 34. Seasonal Demand Pattern a=0.5, c=0.2

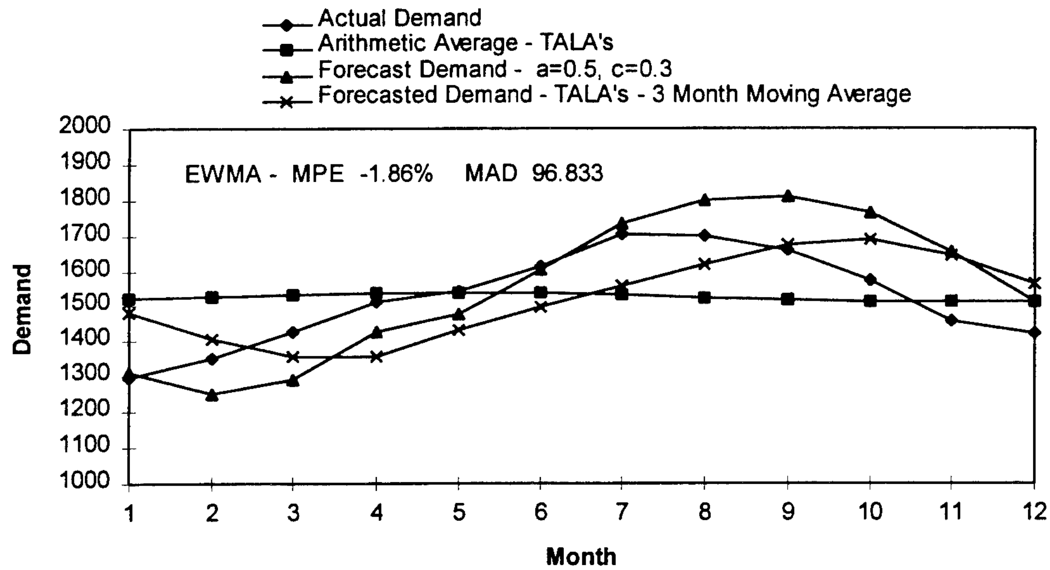

Figure 35. Seasonal Demand Pattern a=0.5, c=0.3

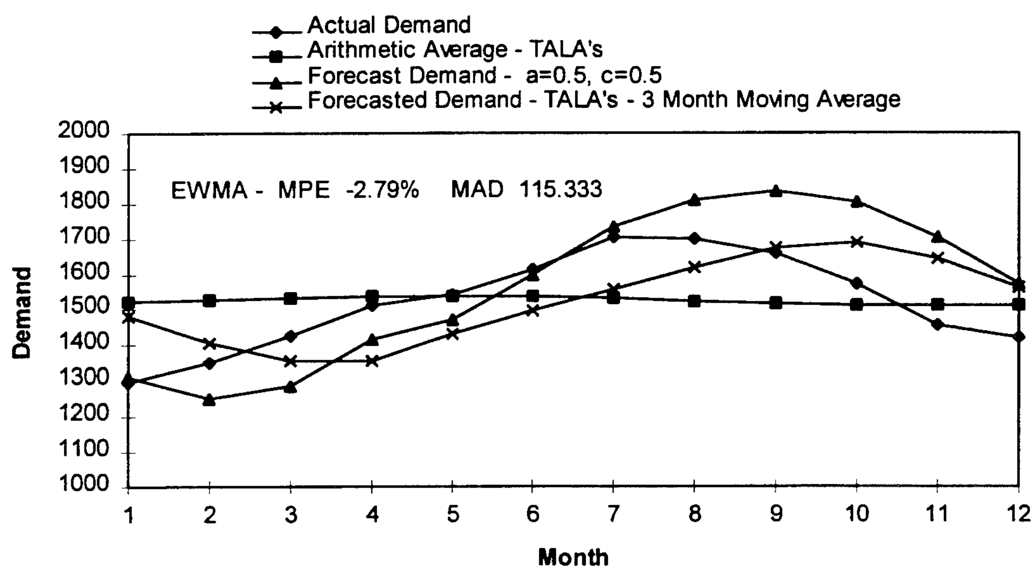

Figure 36. Seasonal Demand Pattern a=0.5, c=0.5

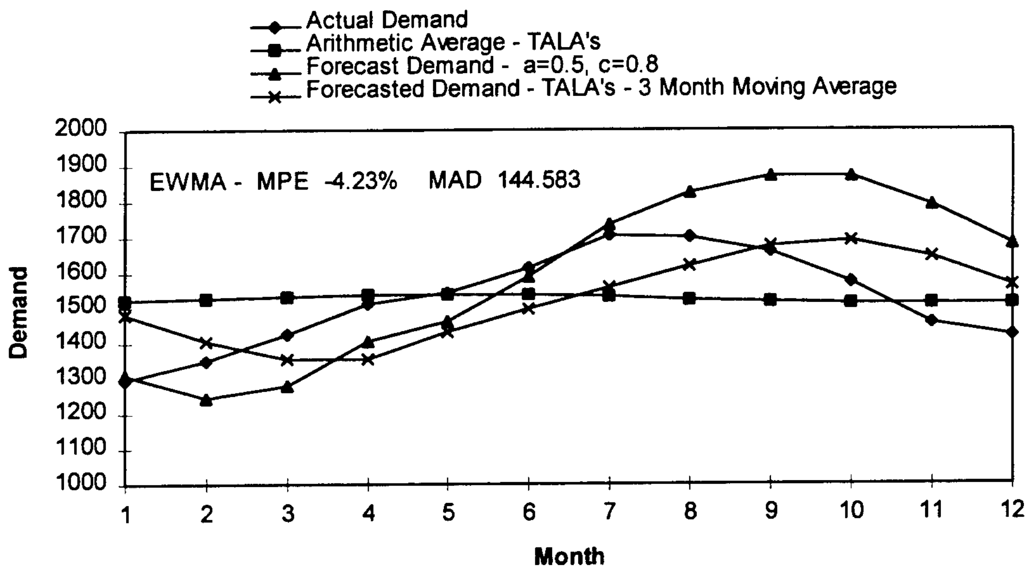

Figure 37. Seasonal Demand Pattern a=0.5, c=0.8

APPENDIX D. HOLDING COST

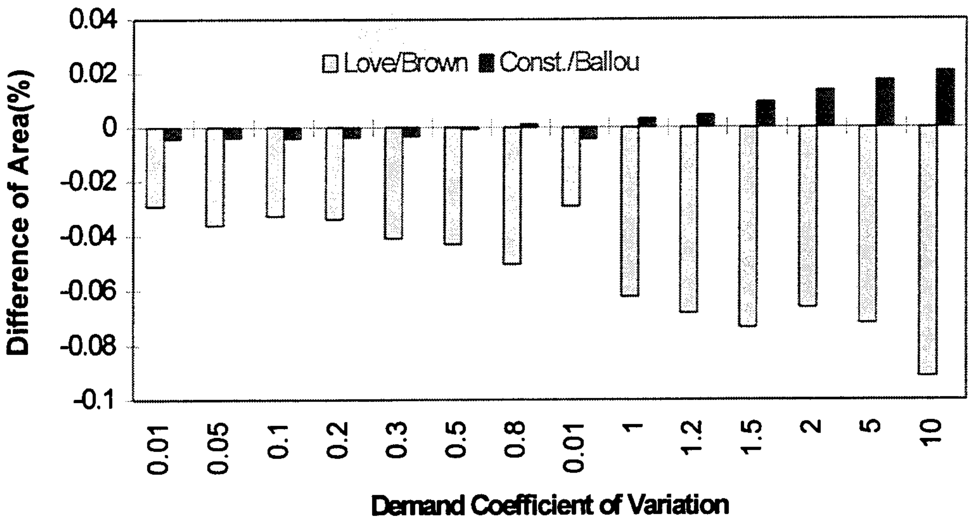

Figure 38. Demand Coefficient of Variation vs Inventory Area

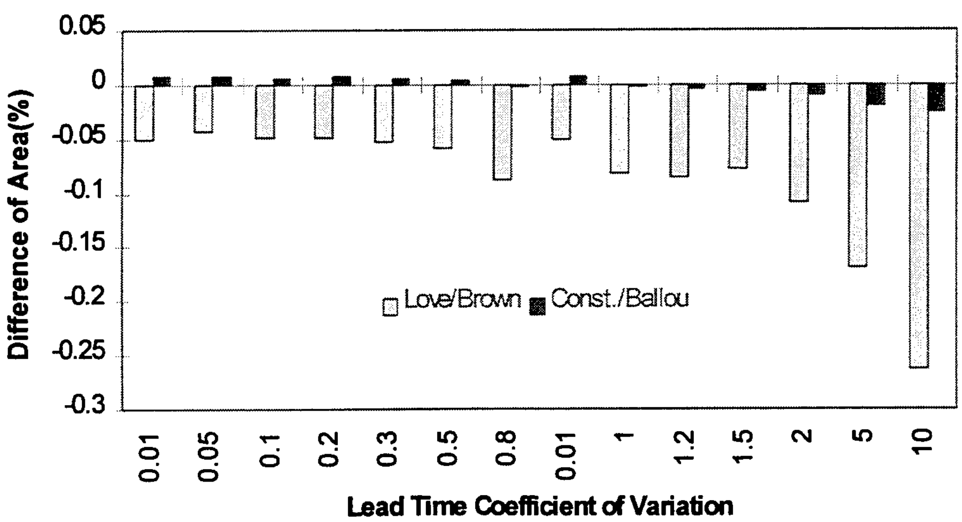

Figure 39. Leadtime Coefficient of Variation vs Inventory Area

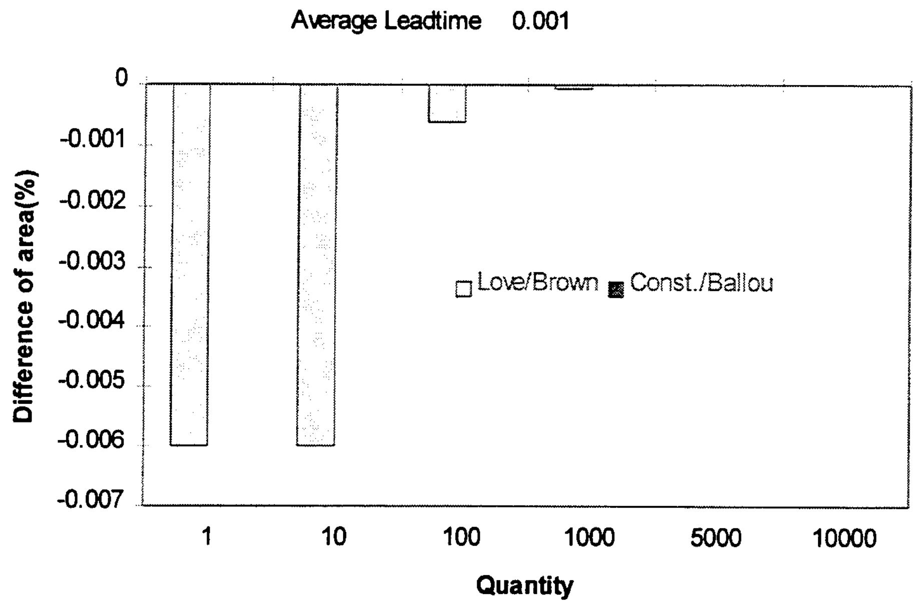

Figure 40. Combination of ROP & Q (ROP=1, AVELT=0.001)

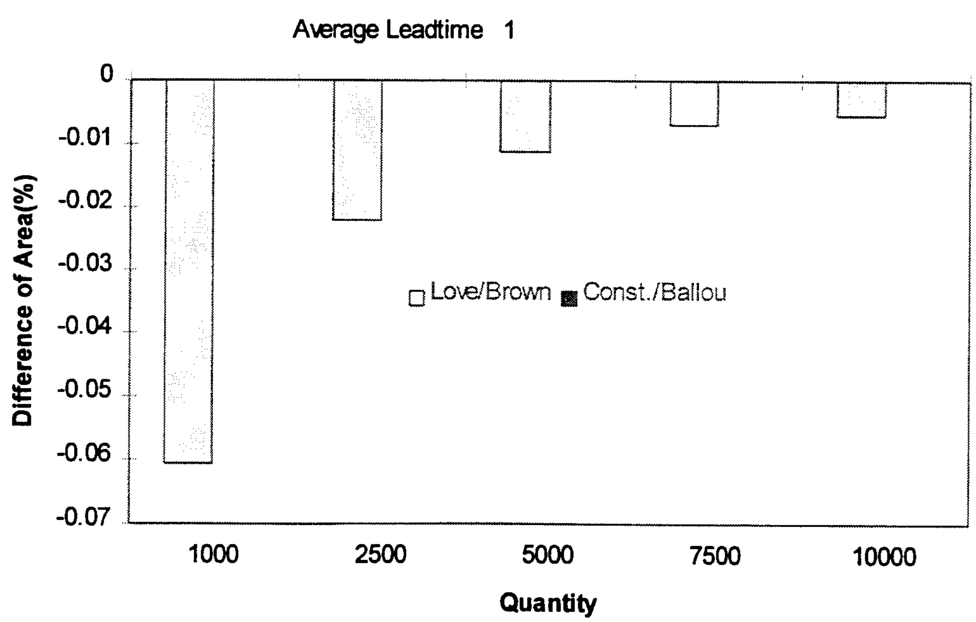

Figure 41. Combination of ROP and Q (ROP=1000, AVELT=1)

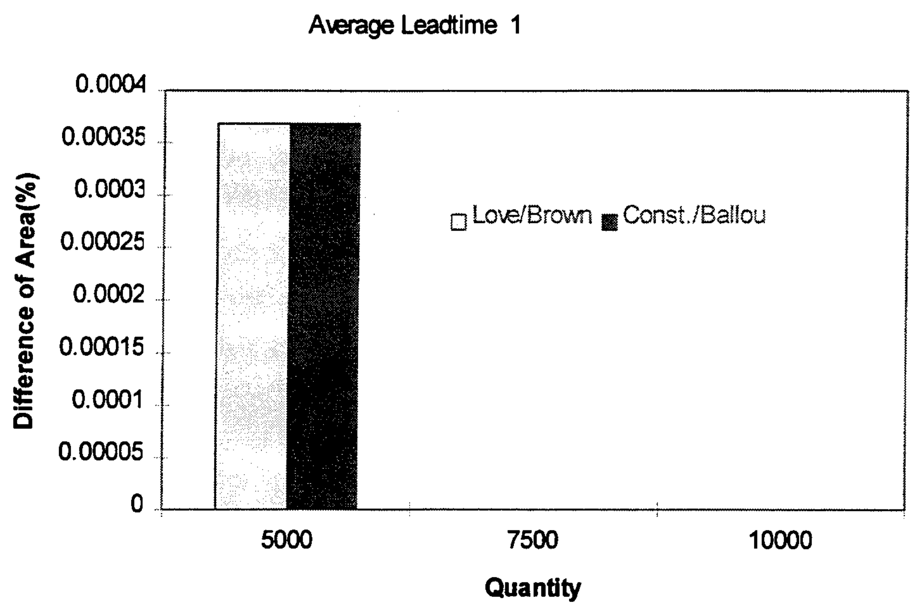

Figure 42. Combination of ROP and Q (ROP=5000, AVELT=1)

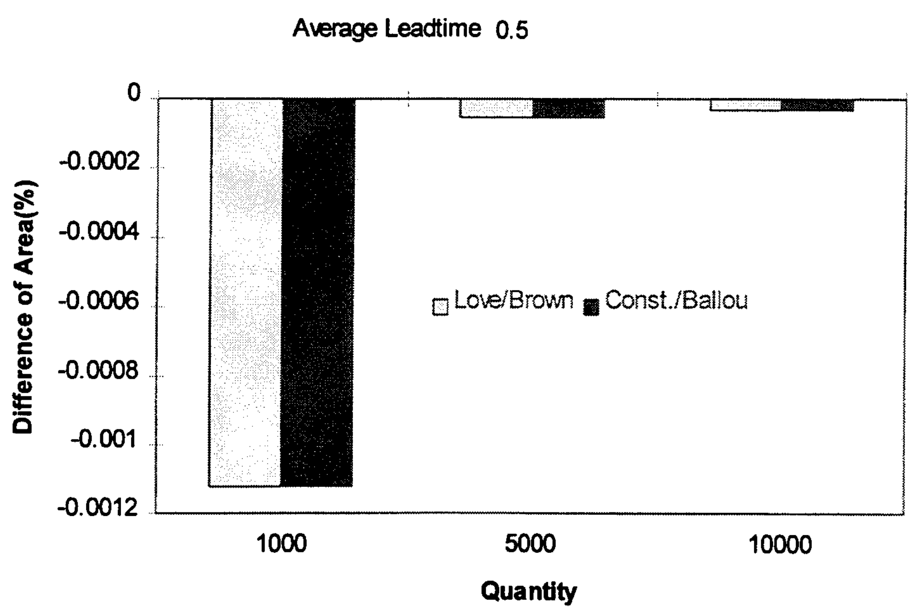

Figure 43. Combination of ROP and Q (ROP=1000, AVELT=0.5)

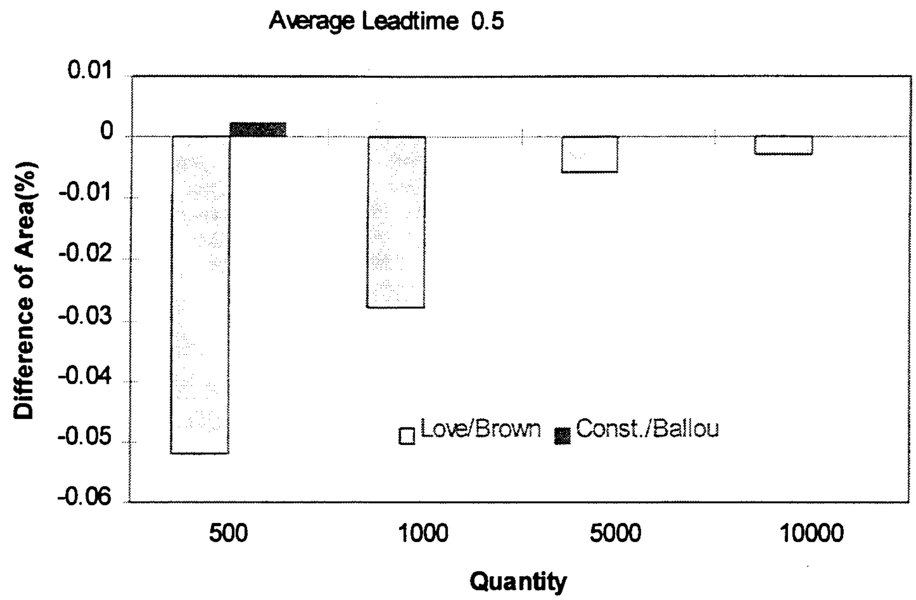

Figure 44. Combination of ROP and Q (ROP=500, AVELT=0.5)

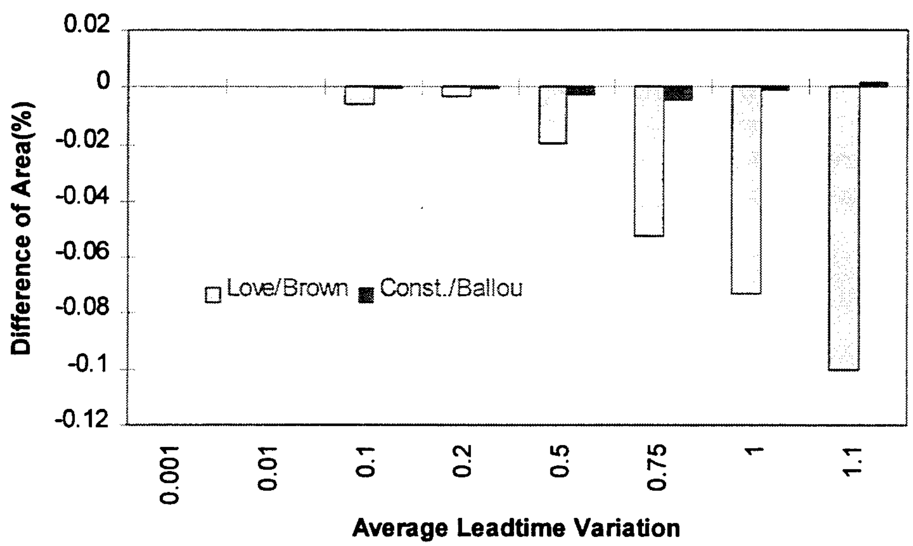

Figure 45. Average Leadtime Coefficient of Variation vs Inventor Area

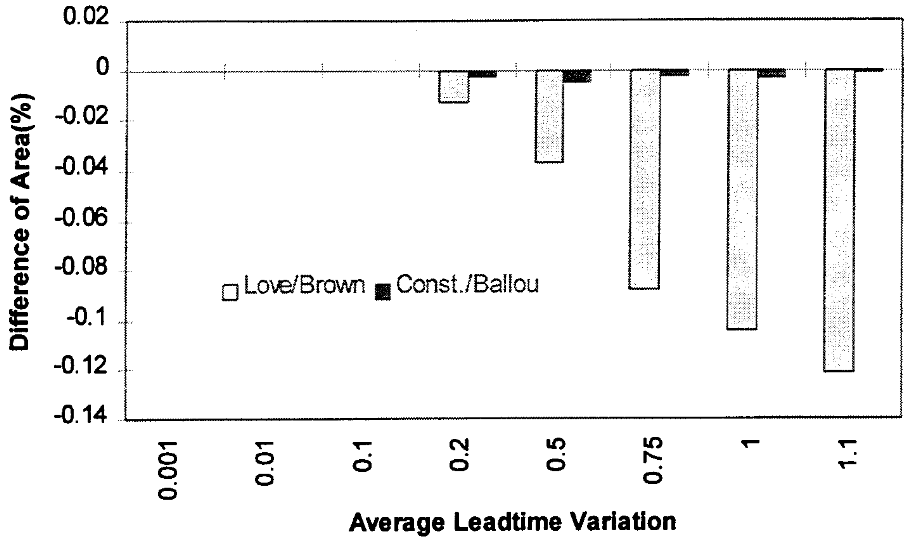

Figure 46. Average Leadtime Coefficient of Variation vs Inventory Area

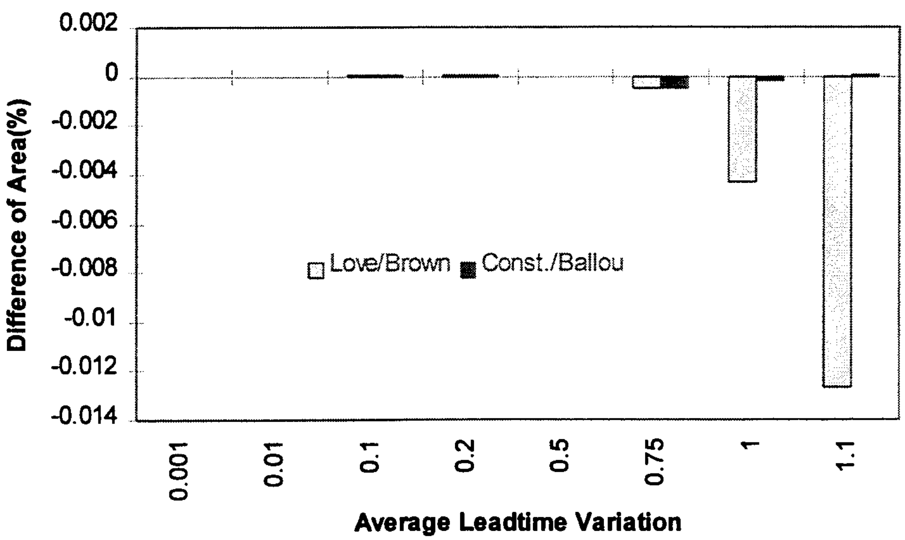

Figure 47. Average Leadtime Coefficient of Variation vs Inventory Area

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