ARMY RESEARCH LABORATORY

On Finite Strain Plasticity Constitutive Modeling in Computational Mechanics

Norris J. Huffington, Jr.

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13. ABSTRACT ***(Maximum 200 words)***

The utility of employing discrete point input constitutive data (effective stress vs. effective plastic strain) for finite elastoplastic response problems is examined. A difficulty that occurs when the response effective plastic strain passes points of slope discontinuity in the input data is noted, and an algorithm that circumvents this difficulty is introduced. This algorithm has been successfully incorporated in the DYNA3D hydrocode and the NIKE3D structural mechanics code.

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1. INTRODUCTION

In classical elastoplasticity (Hill 1950; Malvern 1969), it is customary to assume the existence of a plastic potential function in stress space, which is usually taken to be the yield function. Consistent with this assumption is the associated flow rule, which specifies that each component of plastic natural strain increment is proportional to the partial derivative of the potential function with respect to the corresponding stress component. For materials that work harden, the effect of inelastic straining on the "size" of the yield function must be considered. One method of accomplishing this is to introduce the concept of a universal plastic stress-strain curve that relates two scalar quantities: an "effective" stress and an accumulation of "effective" plastic strain increments (defined in sequel). This concept gives good agreement with physical experiments for proportional loading, but less satisfactory predictions for combined stresses deviating significantly from proportional loading. Nevertheless, this concept for modeling work hardening is widely used, both in analysis and in computational methodology.

An example of a universal stress-strain curve is the popular Johnson-Cook (1983) strength model, particularized to the rate-independent, isothermal case.* While the parameters for this model were derived by fitting experimental data, it will be shown that such a fit can only be satisfactory for a limited range of finite strains. An alternate approach is to provide the universal stress-strain curve as a discrete set of data points over as wide a range of effective plastic strains as desired and to use an interpolation procedure for intermediate values. This report will elaborate on the latter alternative and certain refinements.

2. DISCRETE PAIRS CONSTITUTIVE INPUT

Computer programs developed at Lawrence Livermore National Laboratory have provided options to input constitutive data as coordinate pairs of points on an effective stress vs. effective plastic strain curve. Specifically, these options apply to Material Types 10 and 24 of DYNA3D (Hallquist 1988) and Material Types 3 and 24 of NIKE3D (Hallquist 1984). An illustration of this form of input is shown in Figure 1, where experimentally determined values (Weerasooriya and Swanson 1991) for annealed OFHC copper are displayed as asterisks. The computer programs then calculate values of effective stress for intermediate values of the accumulated effective plastic strain by linear interpolation.

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* The author believes that any rate-/temperature-dependant constitutive model should provide agreement with quasi-static experimental data when so particularized.

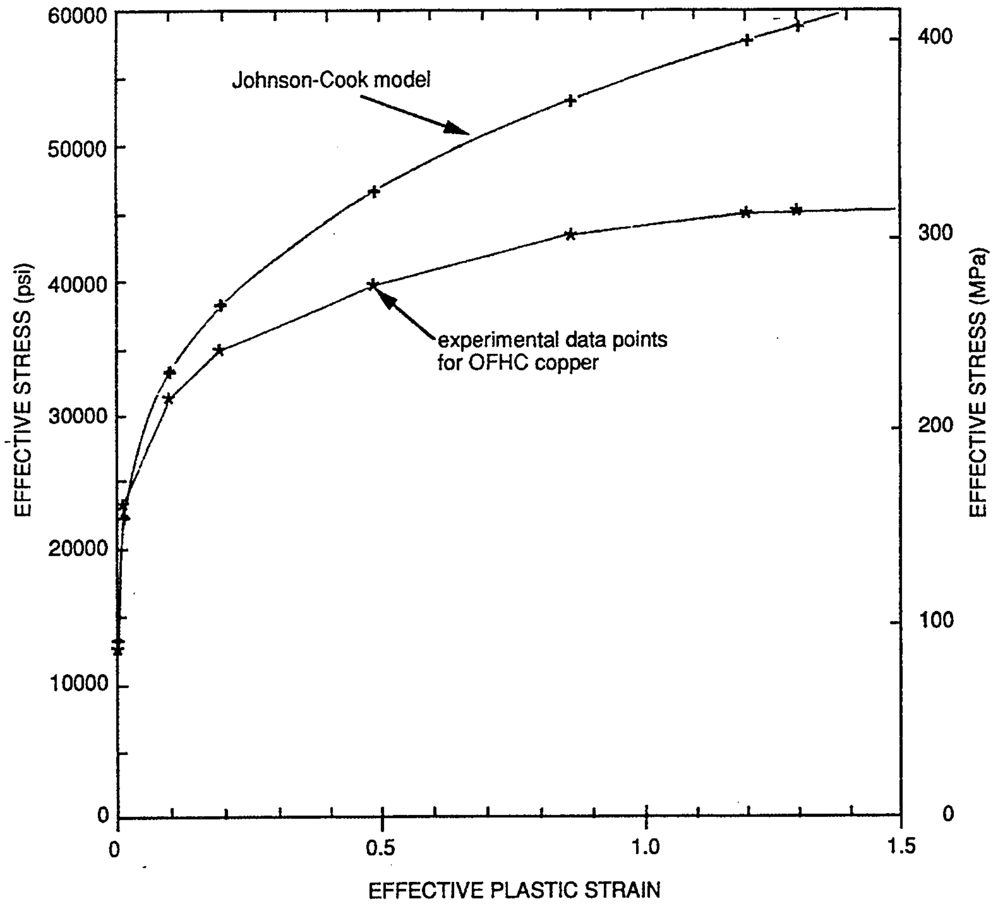

Figure 1. Effective stress vs. effective plastic strain curves.

Figure 1 also shows a corresponding curve for the Johnson-Cook model, based on their published parameters for OFHC copper. The differences between the two curves for small to moderate effective plastic strains are not regarded as significant, since there may have been differences in heat treatments between the two sets of experiments. However, the Johnson-Cook model, which uses a single power law term, cannot represent the stress saturation that is observed physically. The same limitation also applies to the Zerilli-Armstrong (1987) model.

The discrete point representation of constitutive data can also give rise to undesired effects. When faced with a limited number of input points (16 for DYNA3D, 8 for NIKE3D), one would pick points from available experimental data, not necessarily at uniform increments of effective plastic strain, which appear to include the essential features of these data. If the experimental data are known to represent a smoothly continuous phenomenon, the first (and possibly the second) differences of the selected input points should be checked for proper smoothness.

Another difficulty can arise, at least with the computational procedure for stress incrementation employed by the DYNA/NIKE codes. This is illustrated in Figure 2, which was obtained from an NIKE3D computation for the linearly increasing twist of a tube, using the copper data from Figure 1. The large "spike" that is observed at 0.1 s occurred when the computed effective plastic strain exceeded one of the input values (where there is a slope discontinuity). The problem is that the constitutive subroutine uses the previous slope of the yield function to evaluate the new yield stress and is not aware of the slope change until the next time step. In the case of NIKE3D, large time increments can be employed, which accentuates the obvious discrepancy.

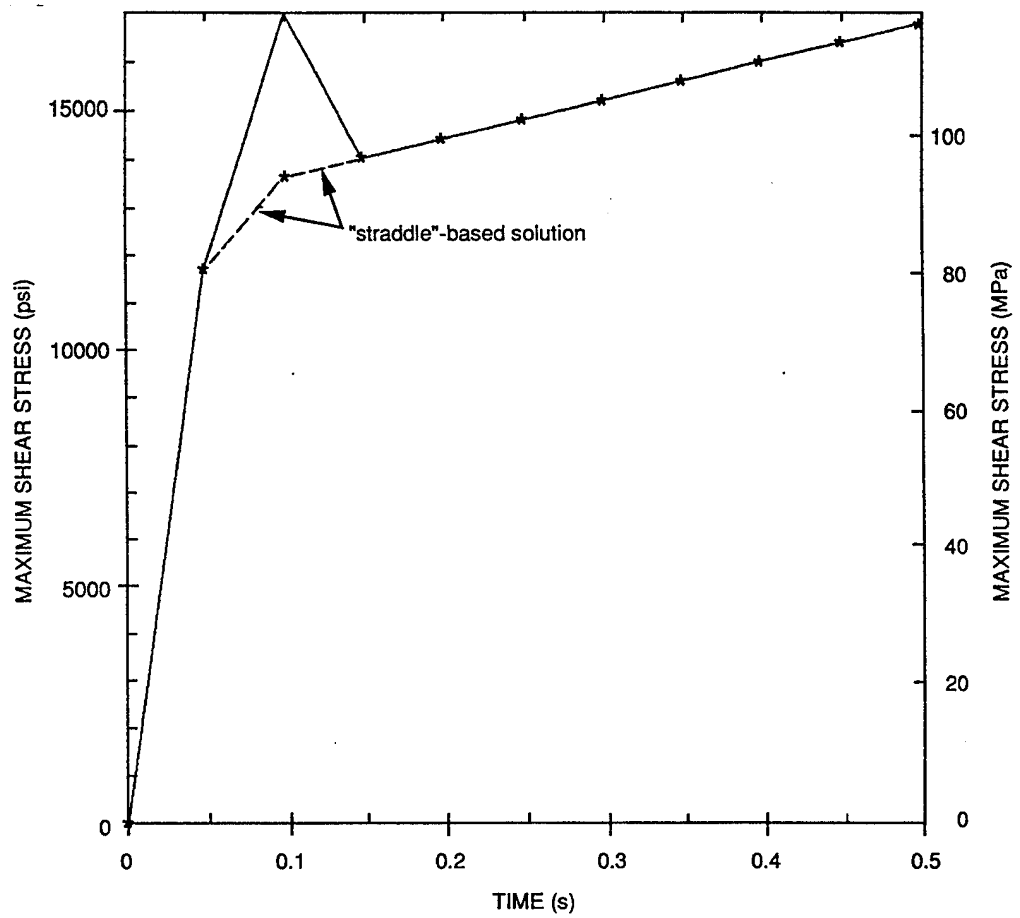

Figure 2. "Before and After" stress-time solution.

For the DYNA3D code, where the time increment is constrained to be small for numerical stability, the "spike" also appears (perhaps with a smaller amplitude), but is followed by an oscillatory response that does not rapidly die out. This behavior has been shown to be associated with the occurrence of spurious negative plastic work.

The foregoing phenomenon can be avoided by use of a special algorithm developed by the author that is invoked whenever the computed effective plastic strain reaches one of the input data values.

3. THE "STRADDLE" ALGORITHM

The discussion of this algorithm refers to the manner in which it was implemented in the DYNA3D code for the solid hexahedron element; however, for other applications, the procedure is essentially the same. The notation employed is defined in the Nomenclature section of this report.

Subroutine F3DM10 takes the stress components from the previous cycle and a new set of strain increments and calculates a set of trial deviatoric stress components on the basis of an elastic stress change. It recalls the previous effective plastic strain ε_j^p and calls the interpolation subroutine YIELDS to reconstruct the current yield stress σ_j^Y (see Figure 3). The trial stresses are used to calculate the von Mises effective (trial) stress σ^T. If $\sigma^T \leq \sigma_j^Y$, the stress change was elastic and the trial deviatoric stresses are accepted as valid. Otherwise, the stress change is plastic and the program calculates an incremental effective plastic strain (scalar) using

$$(\Delta\varepsilon^p)^* = \frac{\sigma^T - \sigma_j^Y}{3G + H_k}. \tag{1}$$

In the original version, of F3DM10 a new yield stress is determined using

$$(\sigma_{j+1}^Y)^* = \sigma_j^Y + H_k(\Delta\varepsilon^p)^* \tag{2}$$

It is this procedure that results in a stress overshoot when ε_j^p and $\varepsilon_j^p + \Delta\varepsilon^p$ straddle a point such as $\bar{\varepsilon}_{k+1}$ on the yield curve.

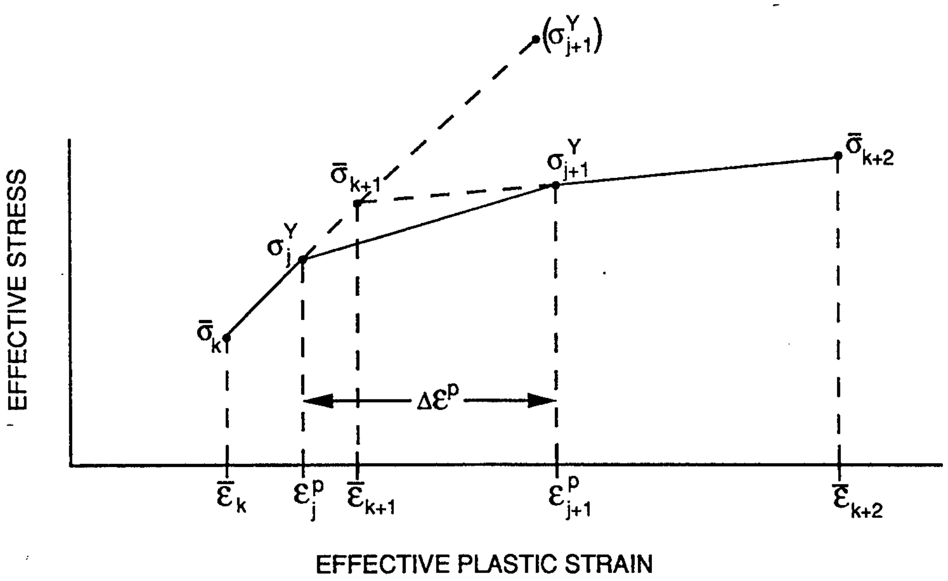

Figure 3. Geometry of the straddle analysis.

In the revised version of F3DM10, the new trial effective plastic strain

$$(\varepsilon^p_{j+1})^T = \varepsilon^p_j + (\Delta\varepsilon^p)^* \tag{3}$$

is calculated and compared to $\bar{\varepsilon}_{k+1}$. If $(\varepsilon^p_{j+1})^T > \bar{\varepsilon}_{k+1}$, the analysis branches to the straddle algorithm to deal with this special case. It is desired to determine a $\Delta\varepsilon^p$ and an intermediate slope H_e that will result in a value of σ^Y_{j+1} which is consistent with the input data. The following conditions must be satisfied:

$$\varepsilon^p_{j+1} = \varepsilon^p_j + \Delta\varepsilon^p \tag{4}$$

$$\Delta\varepsilon^p = \frac{\sigma^T - \sigma^Y_j}{3G + H_e} \tag{5}$$

$$H_e = \frac{\sigma^Y_{j+1} - \sigma^Y_j}{\varepsilon^p_{j+1} - \varepsilon^p_j} \tag{6}$$

$$\sigma^Y_{j+1} = \sigma^Y_{k+1} + H_{k+1}\left(\varepsilon^p_{j+1} - \bar{\varepsilon}_{k+1}\right) \tag{7}$$

These simultaneous equations may be solved algebraically to yield

$$H_e = \frac{3GC_1 + H_{k+1}\left(\sigma^T - \sigma_j^Y\right)}{\sigma^T - \sigma_j^Y - C_1} \tag{8}$$

and

$$\Delta\varepsilon^p = \frac{\sigma^T - \sigma_j^Y - C_1}{3G + H_{k+1}} \tag{9}$$

where

$$C_1 = \overline{\sigma}_{k+1} - \sigma_j^Y - H_{k+1}\left(\overline{\varepsilon}_{k+1} - \varepsilon_j^p\right) \tag{10}$$

Values of ε_{j+1}^p and σ_{j+1}^Y may then be obtained by use of equations (4) and (7). Certain tests to verify that ε_{j+1}^p falls within the range $\overline{\varepsilon}_{k+1} < \varepsilon_{j+1}^p < \overline{\varepsilon}_{k+2}$ are required.

Following these calculations, the normal flow of this subroutine is rejoined and the stress components are scaled back (radial return method [Krieg and Key 1976]) so that the new total stresses will lie on the enlarged yield surface.

4. IMPLEMENTATION OF THE ALGORITHM IN COMPUTER CODES

Although the straddle algorithm was formulated for use with the solid element in DYNA3D, it is also directly applicable for the NIKE3D solid element, as well as for the elements in the 2-D versions of these codes, wherever the yield function is introduced as discrete points. The 3-D programs also feature beam and shell elements for which, for certain options, discrete yield data are accepted.

In DYNA3D, both the Hughes-Liu (1981) and Belytschko-Tsay (1981, 1984) shells have this feature and there is a choice between iterative and noniterative plane stress plasticity (Hallquist and Whirley 1989). In NIKE3D, the Hughes-Liu beam and shell formulations both provide for discrete yield data and both require iterative stress evaluation to satisfy the requirement that the stress component normal to the lamina system be zero. This iteration process complicates the introduction of the straddle algorithm, but it has been successfully incorporated for the Hughes-Liu shell in subroutine s3mns. In fact, the dashed line solution in Figure 2 resulted from a NIKE3D calculation after this subroutine was modified.

To date, the straddle algorithm has only been incorporated in those portions of the DYNA and NIKE codes where a need has arisen. As an example, the inclusion of this algorithm in DYNA3D Material Model 10 is listed in the appendix.

5. CONCLUDING REMARKS

The straddle algorithm was developed to cope with nonphysical "spikes" and oscillations observed in solutions obtained using the Lagrangian hydrocodes NIKE3D and DYNA3D, in conjunction with discrete yield function input data. In many applications, these perturbations may be only a minor nuisance in the graphical output. However, when tracking material response to transient loading in conjunction with a material failure criterion, the decision as to whether survivability or failure will result may be altered by the presence of spurious spikes.

Consequently, it is argued that incorporation of the algorithm is an inexpensive means of avoiding unpleasant surprises (since the number of branches to the algorithm at each integration point cannot exceed the number of input data points minus one regardless of the number of cycles computed).

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APPENDIX:

STRADDLE ALGORITHM IN DYNA3D MATERIAL MODEL 10

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The changes made to DYNA3D to incorporate the straddle algorithm in the following two subroutines are indicated in bold lower case letters.

```
      SUBROUTINE F3DM10(CM)
      COMMON/BK02/IBURN,DT1,DT2,ISDO
      COMMON/AUX2/D1(128),D2(128),D3(128),D4(128),D5(128),D6(128),
     1 WZZDT(128),WYYDT(128),WXXDT(128),EINC(128)
      COMMON/AUX11/PO(128)
      COMMON/AUX14/
     &SIGN1(128),SIGN2(128),SIGN3(128),SIGN4(128),
     &SIGN5(128),SIGN6(128),
     &EPX1(128),EPX2(128),EPX3(128),EPX4(128),EPX5(128),AUX(128,5)
      COMMON/AUX18/DD(128),DF(128)
      COMMON/AUX19/SP,BFAC(128),DR1V(128),DR2V(128),W1
      COMMON/AUX20/
     & AJ2(128),SJ2(128),SCALE(128),FJL(128),CC(128),P(128),
     & POLD(128),DAVG(128),FJK(128),SPECEN(128),AK(128),AKT(128),
     & YWH(128)
      COMMON/AUX33/IX1(128),IX2(128),IX3(128),IX4(128),IX5(128),
     1              IX6(128),IX7(128),IX8(128),MXT(128),NMEL
      COMMON/AUX35/RHOA(128),CXXA(128)
      COMMON/AUX36/LFT,LLT
      COMMON/EOSD/PC(128),SHRM(128)
      logical plastic(128)
      common/state/plastic
      DIMENSION  CM(1),il(128)
      DATA THIRD/.3333333333333/
      MX=48*(MXT(LFT)-1)
      SP=0.0
      G=CM(MX+1)
      QH=CM(MX+3)
      QS=CM(MX+2)
      A1=CM(MX+5)
      A2=CM(MX+6)
      ISPALL=CM(MX+7)-1.
      NPT=CM(MX+16)
      G2=2.*DT1*G
      DO 10 I=LFT,LLT
```

```
   10 DAVG(I)=-THIRD*(D1(I)+D2(I)+D3(I))
      DO 20 I=LFT,LLT
      SHRM(I)=G
      AKT(I)=QH
      PC(I)=EPX5(I)
   20 AK(I)=QS+QH*EPX1(I)+(A1+A2*PO(I))*MAX(0.,PO(I))
      DO 30 I=LFT,LLT
      SIGN1(I)=SIGN1(I)+PO(I)+G2*(D1(I)+DAVG(I))
      SIGN2(I)=SIGN2(I)+PO(I)+G2*(D2(I)+DAVG(I))
      SIGN3(I)=SIGN3(I)+PO(I)+G2*(D3(I)+DAVG(I))
      SIGN4(I)=SIGN4(I)+.50*G2*D4(I)
      SIGN5(I)=SIGN5(I)+.50*G2*D5(I)
   30 SIGN6(I)=SIGN6(I)+.50*G2*D6(I)
      IF (ISPALL.NE.2) GO TO 80
      DO 40 I=LFT,LLT
   40 DAVG(I)=PO(I)-EPX5(I)
      DO 50 I=LFT,LLT
   50 SCALE(I)=.50*(1.+SIGN(1.,DAVG(I)))
      DO 60 I=LFT,LLT
      PC(I)=SCALE(I)*EPX5(I)
      SIGN1(I)=SCALE(I)*SIGN1(I)
      SIGN2(I)=SCALE(I)*SIGN2(I)
      SIGN3(I)=SCALE(I)*SIGN3(I)
      SIGN4(I)=SCALE(I)*SIGN4(I)
      SIGN5(I)=SCALE(I)*SIGN5(I)
   60 SIGN6(I)=SCALE(I)*SIGN6(I)
      DO 70 I=LFT,LLT
   70 EPX5(I)=PC(I)
   80 IF (NPT.EQ.0) GO TO 100
      CALL YIELDS(NPT,CM(MX+17),CM(MX+33),EPX1,AK,AKT,LFT,LLT,il)
  100 DO 110 I=LFT,LLT
      AJ2(I)=.5*(SIGN1(I)**2+SIGN2(I)**2+SIGN3(I)**2)+SIGN4(I)**2+SIGN5
     1 (I)**2+SIGN6(I)**2
      SJ2(I)=SQRT(3.*AJ2(I))
  110 CONTINUE
      do 115 i = lft,llt
      plastic(i)= .false.
      if(ak(i).le.sj2(i)) plastic(i) = .true.
  115 continue
```

```
C.... CALCULATE INCREMENT IN EFFECTIVE PLASTIC STRAIN & NEW YIELD STRESS
      THRG=THIRD/G
      DO 120 I=LFT,LLT
      if(plastic(i)) then
        CC(I)=DIM(SJ2(I),AK(I))/(1.E-30+DIM(1.,-THRG*AKT(I)))
        CC(I)=THRG*AMIN1(SJ2(I),CC(I))
        epxt=epx1(i)+cc(i)
        m=mx+il(i)+16
        if(epxt.gt.cm(32)) go to 120
        if(epxt.gt.cm(m) .and. epxt.le.cm(m+1)) then
          hb=(cm(m+17)-cm(m+16))/(cm(m+1)-cm(m))
          c1=cm(m+16)-ak(i)-hb*(cm(m)-epx1(i))
          akt(i)=(3.*g*c1+hb*(sj2(i)-ak(i)))/(sj2(i)-ak(i)-c1)
          cc(i)=(sj2(i)-ak(i)-c1)/(3.*g+hb)
          ak(i)=max(cm(m+16)+hb*(epx1(i)+cc(i)-cm(m)),0.)
        elseif(epxt.gt.cm(m+1)) then
          write(13,1) i
          call adios(2)
        else
          AK(I)=MAX(AK(I)+AKT(I)*CC(I),0.)
        endif
      else
        cc(i)= 0.
      endif
  120 CONTINUE

C.... CALCULATE FACTOR TO SCALE STRESSES BACK TO YIELD SURFACE
      SEPS=1.E-30*G
      DO 130 I=LFT,LLT
      SCALE(I)=(AK(I)+SEPS)/(MAX(AK(I),SJ2(I))+SEPS)
  130 CONTINUE
C.... SCALE BACK STRESS AND INCREMENT PLASTIC STRAIN
      DO 140 I=LFT,LLT
      SIGN1(I)=SCALE(I)*SIGN1(I)
      SIGN2(I)=SCALE(I)*SIGN2(I)
      SIGN3(I)=SCALE(I)*SIGN3(I)
      SIGN4(I)=SCALE(I)*SIGN4(I)
      SIGN5(I)=SCALE(I)*SIGN5(I)
      SIGN6(I)=SCALE(I)*SIGN6(I)
```

```
      EPX1(I)=EPX1(I)+CC(I)
  140 CONTINUE

      IF(ISPALL.NE.1) RETURN
      DO 150 I=LFT,LLT
      AJ2(I)=.5*(SIGN1(I)**2+SIGN2(I)**2+SIGN3(I)**2)+SIGN4(I)**2+SIGN5
     1 (I)**2+SIGN6(I)**2+1.E-12
  150 SJ2(I)=SIGN1(I)*SIGN5(I)**2+SIGN2(I)*SIGN6(I)**2+SIGN3(I)*SIGN4(I)
     1     **2-SIGN1(I)*SIGN2(I)*SIGN3(I)-2.*SIGN4(I)*SIGN5(I)*SIGN6(I)
      DO 160 I=LFT,LLT
      AKT(I)=-SQRT(27./AJ2(I))*SJ2(I)*0.5/AJ2(I)
      AKT(I)=SIGN(AMIN1(ABS(AKT(I)),1.),AKT(I))
  160 YWH(I)=ACOS(AKT(I))*THIRD
      DO 170 I=LFT,LLT
  170 SJ2(I)=2.*SQRT(AJ2(I)*THIRD)*COS(YWH(I))
      DO 180 I=LFT,LLT
  180 DAVG(I)=PO(I)-SJ2(I)-EPX5(I)
      DO 190 I=LFT,LLT
  190 SCALE(I)=.50*(1.+SIGN(1.,DAVG(I)))
      DO 200 I=LFT,LLT
      PC(I)    =SCALE(I)*EPX5(I)
      SIGN1(I)=SCALE(I)*SIGN1(I)
      SIGN2(I)=SCALE(I)*SIGN2(I)
      SIGN3(I)=SCALE(I)*SIGN3(I)
      SIGN4(I)=SCALE(I)*SIGN4(I)
      SIGN5(I)=SCALE(I)*SIGN5(I)
  200 SIGN6(I)=SCALE(I)*SIGN6(I)
      DO 210 I=LFT,LLT
  210 EPX5(I)=PC(I)
      RETURN
    1 format(' EXCESSIVE PLASTIC STRAIN IN ELEMENT',i10,/)
      END

      SUBROUTINE YIELDS(NPT,EPS,SIGE,EPX,AK,QH,LFT,LLT,il)

C     Rewritten by Glenn Randers-Pehrson, BRL, 6 April 1992, to
C     interpolate correctly within the beginning and end segments,
C     to extrapolate correctly, and to run faster.
C
```

```
      INTEGER   NPT,LFT,LLT
      REAL      EPS(16),SIGE(16),EPX(LLT),AK(LLT),QH(LLT)
C
      INTEGER   I,L,IL(128)
      REAL      SLOPE(16)

      DO 10 I=LFT,LLT
        IL(I)=0
        IF(EPX(I).LT.EPS(2)) IL(I)=2
        IF(EPX(I).GT.EPS(NPT-1)) IL(I)=NPT
   10 CONTINUE

      DO 30 L=2,NPT
        DO 20 I=LFT,LLT
          IF (IL(I).EQ.0.AND.(EPX(I).GE.EPS(L-1).AND.EPX(I).LE.EPS(L)))
     +          IL(I)=L
   20   CONTINUE
        SLOPE(L)=(SIGE(L)-SIGE(L-1)) / (EPS(L)-EPS(L-1))
   30 CONTINUE

      DO 40  I=LFT,LLT
        QH(I)=SLOPE(IL(I))
        AK(I)=SIGE(IL(I)-1)+QH(I) * (EPX(I)-EPS(IL(I)-1))
   40 CONTINUE

      RETURN
      END
```

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NOMENCLATURE

C_1	$= \bar{\sigma}_{k+1} - \sigma_j^Y - H_{k+1}\left(\bar{\varepsilon}_{k+1} - \varepsilon_j^p\right)$
G	shear modulus
H_e	slope determined by the straddle algorithm
H_k	$= \left(\bar{\sigma}_{k+1} - \bar{\sigma}_k\right)/\left(\bar{\varepsilon}_{k+1} - \bar{\varepsilon}_k\right)$; slope of yield curve for interval $\bar{\varepsilon}_k$ to $\bar{\varepsilon}_{k+1}$
H_{k+1}	$= \left(\bar{\sigma}_{k+2} - \bar{\sigma}_{k+1}\right)/\left(\bar{\varepsilon}_{k+2} - \bar{\varepsilon}_{k+1}\right)$
N	number of input $\left(\bar{\varepsilon}_m, \bar{\sigma}_m\right)$ pairs
$\Delta\varepsilon^p$	incremental change in the effective plastic strain
$\bar{\varepsilon}_m$	input values of effective plastic strain, m = 1, 2, ..., N
ε_j^p	$= \sum \sqrt{\frac{2}{3}\Delta\varepsilon_{ij}^p \Delta\varepsilon_{ij}^p}$; current value of effective plastic strain
σ_j^Y	current value of yield stress
$\bar{\sigma}_m$	input values of effective stress, m = 1, 2, ···, N
σ^T	$= \sqrt{\frac{3}{2}\tau_{ij}^T \tau_{ij}^T}$; trial value of von Mises effective stress
τ_{ij}	deviatoric components of Cauchy stress tensor
$(\ )^T$	trial value
$(\ )^*$	quantity calculated using original program

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