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Massachusetts Institute of Technology
Department of Earth, Atmospheric, and Planetary Sciences

Final report: ONR # N00014–90–J–1839
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Theoretical Study of Gulf Stream Instabilities, Eddy Production and Interactions

Goals

This project examined the dynamics of nonlinear, thin, baroclinic jets, with applications to Gulf Stream meandering and eddy formation. We also studied nonlinear eddies and their interactions in order to understand thebetter physics of mesoscale eddies and rings.

Students/ Theses

Two graduate students, Luanne Thompson and Bingjian Ni, were supported by this grant. Luanne examined unsteady flows over a cylindrical obstacle in the homogeneous fluid model and in a two layer model (with the obstacle filling the lower layer). She also solved for steady state solutions on the beta plane and in the presence of a frictional Taylor column (the latter in the barotropic case only). Her thesis, *Flow over finite isolated topography* was completed in 1990. The abstract reads:

> One and two layer models are used to study flow over axisymmetric isolated topography. Inviscid or nearly inviscid flow in which non-linear effects have order one importance is considered, and both effects of B and finite topography are included.
>
> A one-layer quasi-geostrophic model is used to find the shape of Taylor columns on both the f-plane and the B-plane in the inviscid limit of the frictional problem. In this limit, the boundary of the Taylor column is a streamline, and the velocity in both directions vanishes on the boundary. The fluid within the Taylor column is stagnant, corresponding to the solution that Ingersoll (1969) found for flow over a right circular cylinder on the f-plane. In this case, the Taylor column is circular. An iterative boundary integral technique is used to find the solutions for flow over a cone on the f-plane. In this case the Taylor column has a tear drop shape. Solutions are also found for flow on the B-plane over a cylinder, and the Taylor column is approximately elliptical for westward flow with the major axis in the x direction, while it is slightly elongated in the y direction for the eastward flow. The stagnation point of the Taylor column is located on the edge of the topography for all the solutions found. It was not possible to find solutions for smooth topographic shapes.

Steady solutions for flow over a right circular cylinder of finite height are studied when the quasi-geostrophic approximation no longer applies. The solution consists of two parts, one which is similar to the quasi-geostrophic solution and is driven by the potential vorticity anomaly over the topography and the other which is similar to the solution of potential flow around an cylinder and is driven by the matching conditions on the edge of the topography. When the effect of B is large, the transport over the topography is enhanced as the streamlines follow lines of constant background potential vorticity. For eastward flow, the Rossby wave drag can be much larger than predicted form quasi-geostrophic theory.

A two-layer model over finite topography using the quasi-geostrophic approximation is developed. The topography is a right circular cylinder which goes all of the way through the lower layer and an order Rossby number amount into the upper layer, so that the quasi-geostrophic approximation can be applied consistently. This geometry allows description of flow in which an isopycnal intersects the topography. The model is valid for a different regime than existing models of steady flow over finite topography in a continuously stratified fluid in which the bottom boundary is an isopycnal surface. The solutions contain the two components that are found in the barotropic model of flow over finite topography. The model breaks down when the interface goes above the topography which occurs more easily when the stratification is weak. Closed streamlines occur more readily over the topography when the stratification is strong. Near the topography, the interface is depressed to the right and raised to the left (looking downstream).

A hierarchy of time-dependent models is used to examine the initial value problem of flow initiation over topography on the f-plane. A modified contour dynamics method is developed that extends the range of problems to which contour dynamics can be applied. The method allows boundary and matching conditions to be applied on a circular boundary. A one-layer quasi-geostrophic model is used to show that more fluid that originates over the topography remains there when the flow is turned on slowly than when it is turned on quickly. Flow over finite topography in a one-layer model shows a variety of different behaviors depending on the topographic height. When the topography has moderate height, two cyclonic eddies are created; when the topography fills up most of the water column, the fluid oscillates on and off the topography as it moves around the topography in a clockwise direction, and none of the fluid is shed downstream. Two quasi-geostrophic stratified models are considered, one in which the topography is small, and the other in which it is finite. In the small topography model, an eddy is shed which is cyclonic, warm-core, and bottom-trapped. In contrast, the shed eddy is cyclonic, cold-core, and surface-intensified in the finite depth model using the geometry described above.

Luanne is currently on the faculty at the University of Washington; she has published three papers from her thesis.

Bingjing Ni began his thesis work under this project, investigating the stability of flows in shallow water primitive equation models. He has used the hydrographic section taken by Hall and Fofonoff during SYNOP to define the jet flow field. He finds the dispersion relation is quite similar to that predicted by quasigeostrophic theory. However, the surface temperature gradients (equivalent by potential vorticity inversion to near surface PV gradients) also play an important role in the stability. His thesis will be completed this spring.

Linear and nonlinear waves on jets

We published a study on finite amplitude waves on barotropic shear layers and jets (*Geophys. Astrophys. Fluid Dyn.*):

> We give a detailed derivation of the amplitude equations governing weak, slowly modulated, varicose wavetrains on a barotropic, triangular jet on a B-plane. As one might expect, at wavenumbers that are neutral by linear theory, well-behaved wavetrain solutions can be found. Such solutions are not unique. The properties of the wavetrain (in particular, the form of the nonlinear coefficient in the amplitude equation) dependent upon assumptions that one makes about the structure of the wave field in the region far from the jet. Such assumptions turn out to be equivalent to assumptions made about the initial conditions of the problem. This link is established by a wave packet analysis that retains separate, nondimensional parameters for the amplitude of the wavetrain and the length scale of the modulation envelope. We briefly state similar results for sinuous waves on a triangular jet and for waves on a shear layer composed of a strip of uniform potential vorticity.

We have explored the dispersion relationship for waves on a jet in some detail. We have demonstrated that the "n$\frac{1}{2}$" layer model gives a different long wave dispersion relationship than a model which includes the barotropic mode. The former predicts long waves are stable and the phase speed is proportional to k^2, as in the work of Pratt and Stern, while the latter has instability at the longest wave lengths with the imaginary part of the phase speed proportional to $k^{1/2}$, as in the study of Flierl and Robinson. In addition, we have shown that these conclusions also hold for a primitive equation model as well as the quasigeostrophic model. We have looked at when the long wave behavior of a two layer model is adequately represented by a $1\frac{1}{2}$ layer model. The long wave dispersive properties resemble each other only when the lower layer is very deep, $H_2/H_1 >> (kR_d)^{-3}$. We have also explored dispersion properties for waves on a front in a continuously stratified ocean. We are comparing dispersion relationships with the data from the SYNOP study; however, it is necessary to include aspects of spatial or wave–packet instabilities to do this properly.

For QG jets, we have developed a new technique for linear and nonlinear analyses of jet instability based on multi–layer contour dynamics. We express the potential vorticity q in each layer by a stair–step profile:

$$q_i = q_{i,0} + \sum \Delta_{i,j}\mathcal{H}(y - y_i - \eta_i)$$

with y_i the mean position of the potential vorticity contour and $\eta_i(x,t)$ the deviation (Figure 1). The zonal and meridional flow are expressed as an integral operator on the potential vorticities. Thus the streamfunction is just

$$\psi_i = \int\int dx'dy' G_{ij}(x,y|x,y')q_j(x',y')$$

and the interfaces between the regions with different q values behave as material surfaces:

$$\frac{\partial}{\partial t}\eta_i = \frac{d}{dx}\psi(x, y_i + \eta_i(x,t), t)$$

We have shown that these equations can be linearized in a very simple way, essentially because the Taylor expansion of the step function $\mathcal{H}$ is a delta function. The resulting stability equation is then a standard matrix eigenvalue problem, and we can set up and solve the dispersion relation in less than a page of MATLAB code. Figure 2 compares the results of a six–front jet (three in the upper layer and three in the lower layer) to one derived directly from Hall and Fofonoff's CTD section across the stream. The full model exhibits weak growth for short waves and also has two growing modes; however, the overall growth rates and phase speeds are similar.

The contour dynamical model offers additional benefits: by casting the theory in Hamiltonian form (Morrison, Flierl, and Bell, Meacham and Flierl), it is possible to explore the weakly nonlinear behavior even in the case with multiple fronts. Essentially, one can express the Hamiltonian H for the dynamics as a functional of the displacements of the q contours and define a suitable differential operator, called a Poisson bracket, such that

$$\frac{d}{dt}\eta_i = \{\eta_i, H\}$$

Nonlinear evolution equations can then be derived systematically by Taylor expanding the Hamiltonian and substituting the results in the equation above. Symbolically, then

$$\frac{d}{dt}\eta_i = L_{ij}\eta_j + N_{ijk}\eta_j\eta_k + \ldots$$

and this can be used to explore whether the nonlinearity increases the growth rate or decreases it.

To summarize the results (Flierl and Meacham, 1992, *Synoptician*), we find that, as shown in Figure 3, there are several instabilities. For long waves, we can see the unstable meandering mode $c \sim ik^{0.5}$. If the jet width is short enough and the lower layer centerline velocity is positive, there is a second long wave instability with $c \sim i const$. In the case of a barotropic jet, this mode represents an instability to a baroclinic disturbance; as the wavenumber $k \to 0$, the effective wavenumber $\sqrt{k^2 + \gamma^2}$ remains within the strongly unstable band. The peak growth rate occurs at a scale on the order of the jet width/ radius of deformation. There may be secondary maxima, depending on the jet structure.

In collaboration with Nathan Paldor, we have demonstrated that the contour dynamics approach with maultiple contours in the lower layer compares very well with a different analytical approach for the case when the deep fluid is at rest. These linear results are being put together into a paper now (Flierl, Meacham and Paldor).

Nonlinearity has been examined near the critical wavenumber. We find that narrow jets (width from northern $u = 0$ point to southern $u = 0$ point less than 4.6 deformation radii) have growth rates which increase with amplitude. For wider jets, however, the nonlinearity is stabilizing.

We are also examining fully nonlinear steadily propagating solutions using an iteration technique. This allows us, in the case of the $1\frac{1}{2}$ layer model, to bridge the gap between the finite amplitude long wave theory of Pratt and our weakly nonlinear results. The wave profiles resemble those obtained by Pratt for choices of deformation radii that are comparable to or shorter than the wavelength of the disturbance; however, for large deformation radii, and in the barotropic case, there seems to be a limiting state in which the wave resembles a sawtooth pattern. We have extended this study to two-layer fronts and narrow, or barotropic, jets. Preliminary results suggest that the supercritical transition associated with baroclinic or wide jets — the fact that nonlinearity is stabilizing — may not hold very far from the transition point or for very large initial amplitudes. Finally, we are examining fully nonlinear time–dependent calculations (c.f. Meacham, 1991). The nonlinear results are being compiled into a second paper (Meacham and Flierl).

The Rossby number of the Gulf Stream is large enough that non–quasigeostrophic effects should be significant. In collaboration with Richard Williams, (Williams, 1991, *GFD Summer School Notes*), we looked at the corrections to the dispersion relationship for waves on a front separating two regions of constant potential vorticity $q_{PE} = (\zeta + f)/h$ in a $1\frac{1}{2}$ layer model. The linear wave problem can be written

$$(\overline{u} - c)u + (\overline{u}_y - f)v + g'h = 0 \tag{1}$$

$$fu - k^2(\overline{u} - c)v + g'\frac{\partial}{\partial y}h = 0 \tag{2}$$

$$\overline{h}u + \frac{\partial}{\partial y}(\overline{h}v) + (\overline{u} - c)h = 0 \tag{3}$$

or

$$-u_y - k^2 v - \overline{q}h = 0 \tag{4}$$

The last equation only holds in a region of constant potential vorticity; however, we can see that it offers the same advantage as the constant q *ansatz* in QG flow: the critical layer is no longer a problem. If we were to use (2) and (3) to advance v and h in steps in y, we would have to solve (1) for u and that involves dividing by $\overline{u} - c$. In contrast, we can use (2) and (4) to advance the fields h and u and then solve (1) for v, thereby avoiding the singularity. The equations are solved numerically in each region where $\overline{q}$ is constant and matched at the edges. (The matching does involve a $\overline{u} - c$ factor, but this is not a problem since it need be done only at a finite set of y points.) The numerical solutions have been compared with asymptotic solutions carried out to order Rossby number squared, to long wave limits, and to the QG limit. Although the dispersion relationship is rather similar to the QG case, the structure appears to be different: the semigeostrophic or PE model has asymmetries in the basic jet profile and we believe these will extend to asymmetries in the meanders.

Ni has also applied a difference formulation for the stability problem of more complex, two–layer jets in the PE model and finds the differences with the QG model are small, except, perhaps, for the influence of topography.

We have begun a comparative study of the contour dynamics model, conventional layered QG models, the SPEM model, and the Bleck–Boudra isopycnal coordinate model. The work, in collaboration with Meacham, Malanotte–Rizzoli, and Chaussignet, involves:

1) Analysis of the Hall and Fofonoff CTD section at 68° W. We calculated at each station the deformation radius and the parameter measuring the self–nonlinearity of the first baroclinic mode

$$H_{upper}/H_{lower} = \frac{1}{4}(\sqrt{\xi^2+4}-\xi)^2$$

where $\xi = \int F^3 / \int F^2$ (Figure 4). Then, we calculated the projection of the baroclinic geostrophic velocity unto the first mode. The resulting profile was fit to a $\mathrm{sech}^2(y)$ jet. There was a weak asymmetry with the shear on the north being somewhat larger than that on the south. These analyses provide the jet structure and, for two layer models, the appropriate H_1 and g' values.

2) From this data analysis, we initialize each model with the best approximation to the jet and add a perturbation in the axis path. The models are integrated forward and various fields compared (Figure 5).

Eddy dynamics and interactions

We have developed "Hamiltonian contour dynamics," in conjunction with Phil Morrison (U. Texas) and George Bell (Boulder), a collaboration begun at the GFD school. This theory gives a much simpler and more direct approach to finite amplitude instability theory as described above. The advantage of Hamiltonian approximations is that they conserve potential vorticity, energy, and angular momentum. The manuscript describing this work is in preparation.

We have also examined the evolution of vortices in a stratified quasigeostrophic fluid. V. Zhmur and K. Pankratov have developed an analytical approximation for the critical separation of a pair of spheroidal vortex lenses that distinguishes instances of merger from non-merger. We have explored this criterion numerically, using a stratified contour dynamics model on the NRL Cray to simulate the interaction between a pair of vortex lenses in a continuously stratified fluid, and have obtained examples of merger that show it to be a rather complicated process.

We also studied intense vortex motion on the beta plane and the development of the beta gyres (*J. Atmos. Sci.*):

> An analytical theory is presented for the self-induced translation of an intense vortex relative to a uniform background flow on the B plane. The equivalent barotropic approximation is used to formulate the initial value problem within a polar coordinate frame translating with the vortex center. A contour dynamical model of the vortex is melded with the regular beta-plane model of the residual flow. Evolution of vortex asymmetries for azimuthal mode number one, the so-called beta gyres, which are responsible for the relative vortex motion, is considered for a period of time while the Rossby wave radiation is not important.
>
> It is shown for an initially axisymmetric vortex that the beta gyres and corresponding vortex translational velocity consist of two parts. The first one is generated by advection of the background potential vorticity gradient and rotates differently because of the symmetric vortex circulation. The second part arises due to distortion in the vortex shape represented by displacements of the piecewise constant potential vorticity contours relative to the vortex center. The distortion of the vortex shape is described by the sum of normal modes generated by the first part. Explicit solutions for both parts are obtained, and approximate expressions for different stages of the vortex motion are presented.
>
> For a vortex with a uniform potential vorticity core (single contour), the beta gyres are found to consist only of the first part so that the vortex translation depends on the ratio of the core size to the radius of deformation. A small core corresponds to the geostrophic point vortex limit with initially predominantly meridional motion. Asymptotically, after a large number of fluid revolutions at a radical distance on the order of the radius of deformation, the westward translation dominates: the meridional velocity and the deviation of zonal velocity form the maximum linear Rossby wave speed decay linearly with time. This tendency is explained to be a result of effective symmetrization of the potential vorticity due to differential rotation of fluid around the vortex. The period of initial predominantly meridional motion is negligible when the core size is on the order of the deformation radius.
>
> For the vortex with two steps in the potential vorticity, the normal mode rotates faster than the fluid if the potential vorticities in the core and at the periphery have different signs. The effect of the distortion in the vortex shape on the vortex translation increases with increasing deformation radius relative to the vortex size. In a stationary beta gyre, for a finite vortex, the relative contour shift contributes to the westward translation just up to the long Rossby wave speed.

In the nondivergent limit a universal approximate trajectory has been found for large outer contour radius. The center of a finite vortex moves northwestward with permanent meridional acceleration due to degeneracy of a zero-frequency normal mode. The zonal translational velocity approaches a limit proportional to the vortex area. The effect of the distortion in the vortex shape in this nondivergent limit results in decreasing the westward translation and increasing the meridional one.

Applications of the theory to hurricanes in the atmosphere and rings in the ocean are discussed.

In a review article in *Chaos*, we examined the application of coherent feature models to oceanic rings:

Solitary wave or isolated eddy models are often invoked to explain the longevity of strong nonlinear features in oceans and atmospheres. But when we look at the physics in detail, we find that models of isolated eddies often hinge either on an oversimplification of the dynamics or on constraints which are not appropriate for the observed eddies. In a more complete model, as this study demonstrates, rings (and probably other nonlinear goephysical eddies as well) will interact with their surroundings via Rossby wave radiation, primarily in the barotropic mode. Such wave generation leads to a slow decay of the eddy as energy leaks into the wave field.

Biological–physical dynamics

We have been collaborating with the BIOSYNOP program, developing models of the influence of meandering and oceanic eddies upon the biological fields. Our first study was published in *J. Marine Res.*:

A modeling study was conducted to examine the effects of time-dependent mesoscale meandering of a jet on nutrient-phytoplankton-zooplankton (NPZ) dynamics. The jet was represented as a quasi-geostrophic flow using the method of contour dynamics. Two cases for biology were examined: 1) plankton in a mixed layer of fixed depth and 2) plankton at the base of a mixed layer (i.e., pycnocline) of variable depth. When the mixed layer depth is fixed, nutrient upwelling and dilution of the phytoplankton and zooplankton populations occur along the northward branch of the meander. The additional nutrients and reduced grazing pressure leads to significant enhancement (10-20production and biomass, while the zooplankton biomass decreases similarly. For plankton on a material surface of variable depth, phytoplankton growth in the pycnocline is increased by the higher light levels encountered during along-isopycnal upwelling. The nutrients decrease and the zooplankton mass in the pycnocline increases by a small amount downstream of the phytoplankton peak. Although the biological enhancements found are not large, the results suggest that vertical motions resulting from mesoscale oceanographic features such as jet meanders and mid-ocean eddies can be an important source of new nutrients for oceanic plankton production.

Subsequent studies have been made with spectral QG models and PE models.

Publications

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Meacham, S.P. and G.R. Flierl (1991) Finite amplitude waves on barotropic shear layers and jets. *Geophys. Astrophys. Fluid Dyn.*, **56**, 3–57.

Meacham, S.P. (1991). Meander evolution on quasigeostrophic jets. *J. Phys. Oceanogr.*, **21**, 1139–1170.

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Thompson, L. and G.R. Flierl (1993) Bbarotropic flow over finite isolated topography: steady solutions on the beta–plane and the initial value problem. *J. Fluid Mech.*, **250**, 553–586.

Flierl, G.R. and C.S.Davis (1993) Biological Effects of Gulf Stream Meandering. *J. Mar. Res.*, **51**, 529–560.

Sutyrin, G.G. and G.R. Flierl (1993) Intense vortex motion on the beta–plane. Part 1. Development of the beta–gyres. *J. Atmos. Sci.*, **51**, 773–790.

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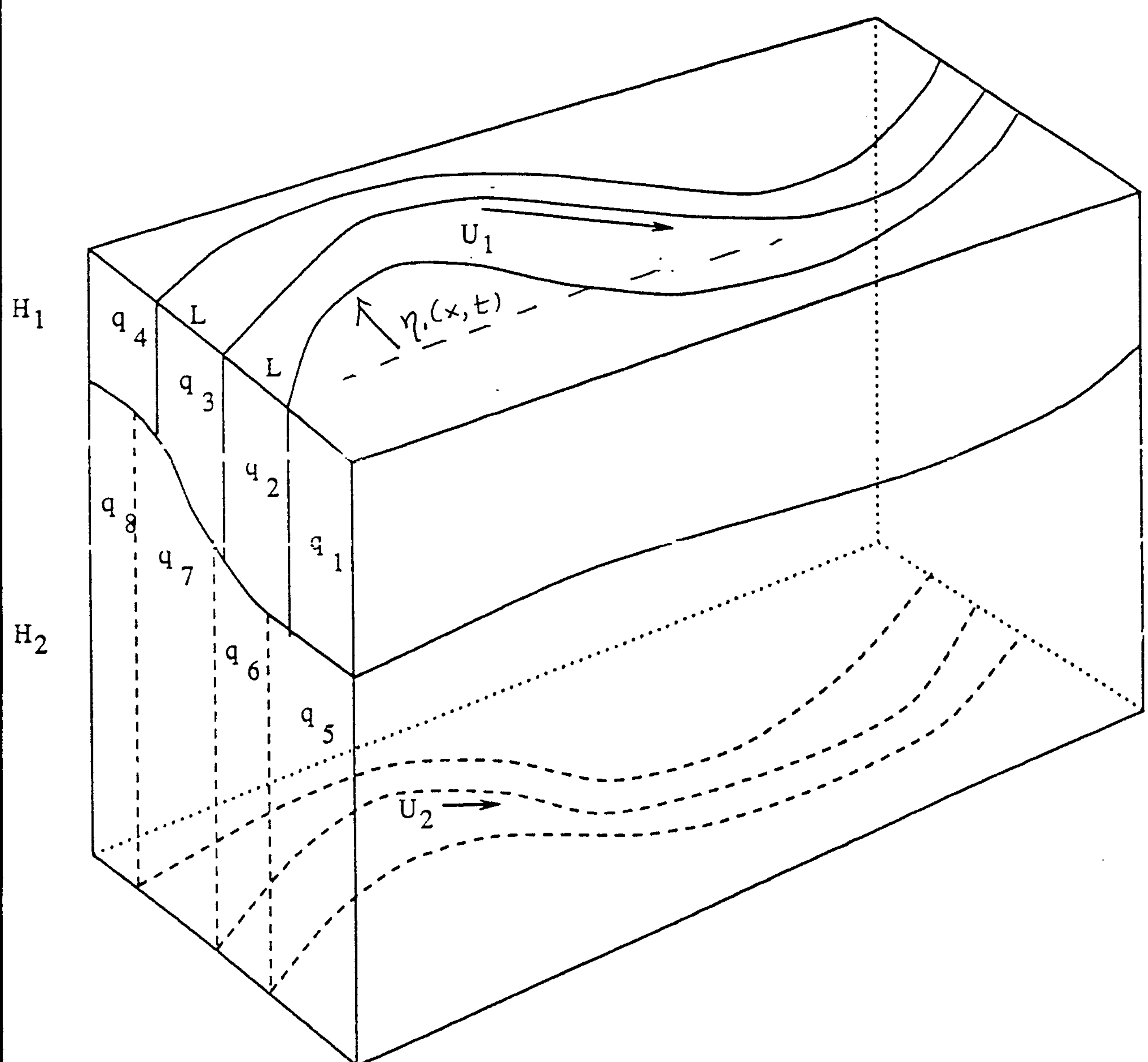
Flierl, G. R., and K. Haines. The decay of modons due to Rossby wave radiation. *Pageophys.*,

Flierl, G., S. Meacham, and N. Paldor. Instabilities and Waves on Thin Jets.

Meacham, S.P. and G.R. Flierl. Nonlinear Waves on Thin Jets.

Morrison, P., G.R. Flierl, and G. Bell. Hamiltonian Contour Dynamics.

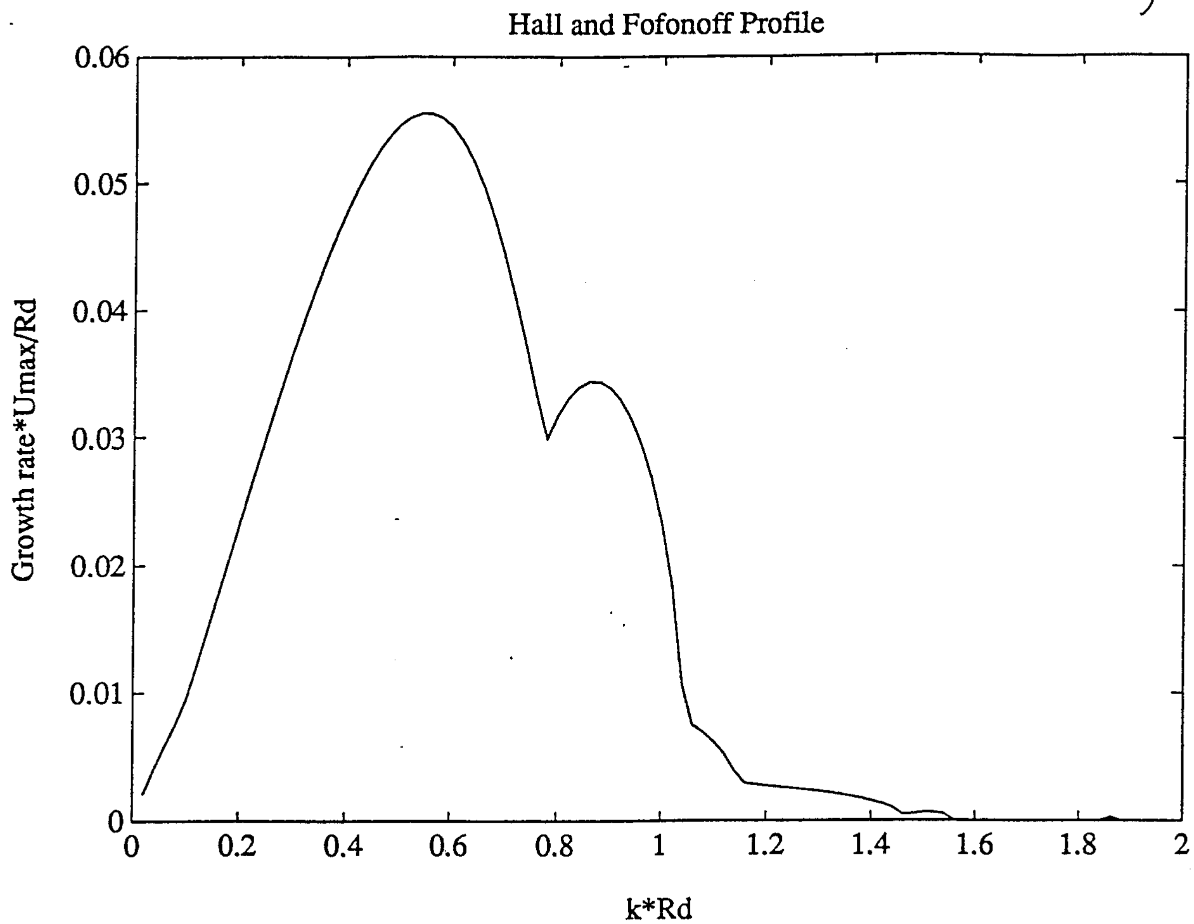
Figure 1



Six front model of a jet

The jet is modelled by three fronts in the upper layer, separating regions in which the potential vorticity is uniform, and three lower layer fronts. The basic state profile has zonal velocity equal to zero on the four outer fronts.

Figure 2



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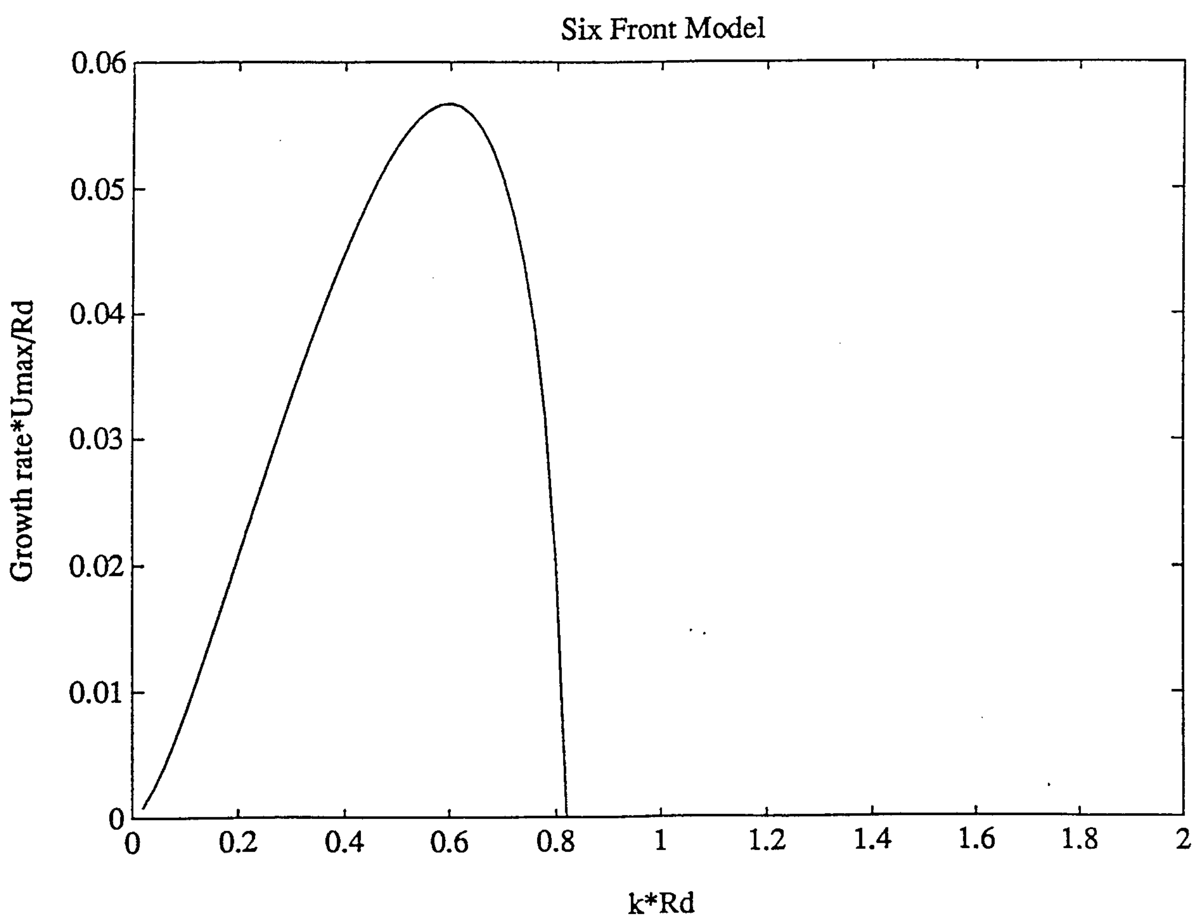
k*Rd



Six Front Model
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0.05
0.04
0.03
0.02
0.01
0
Growth rate*Umax/Rd
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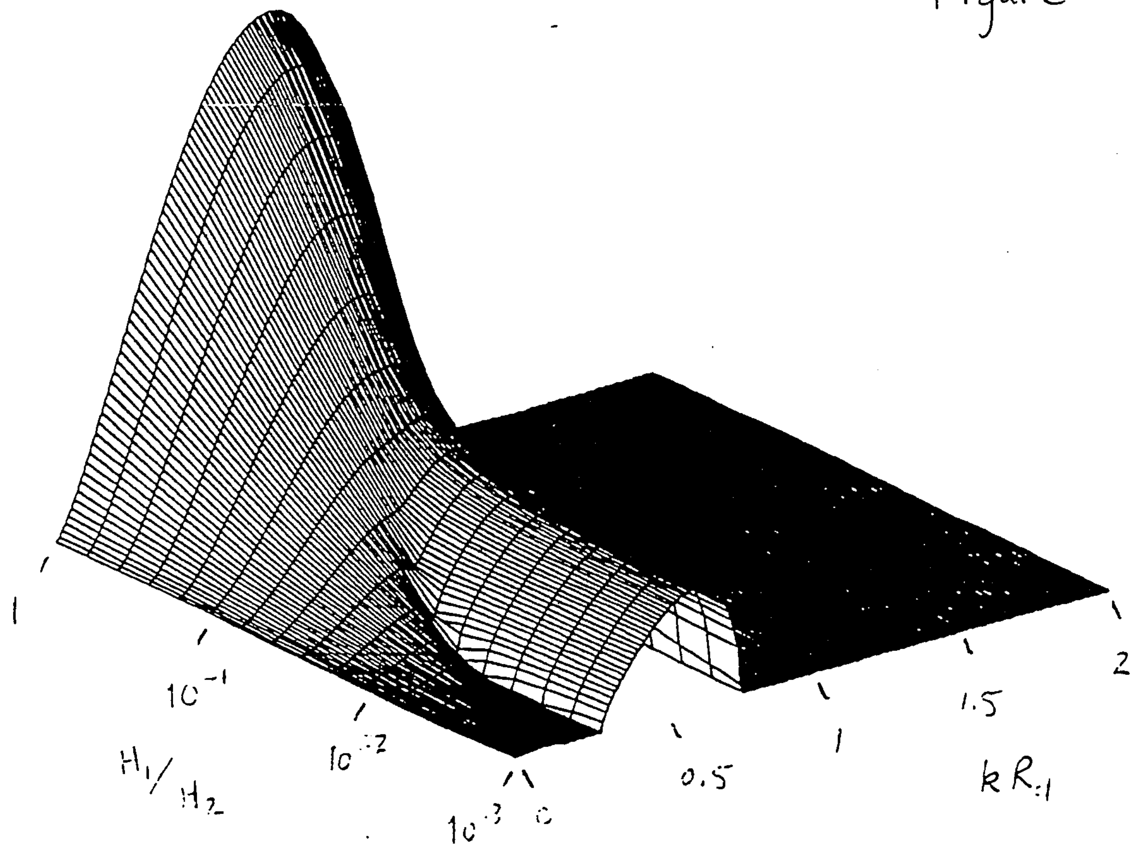
k*Rd

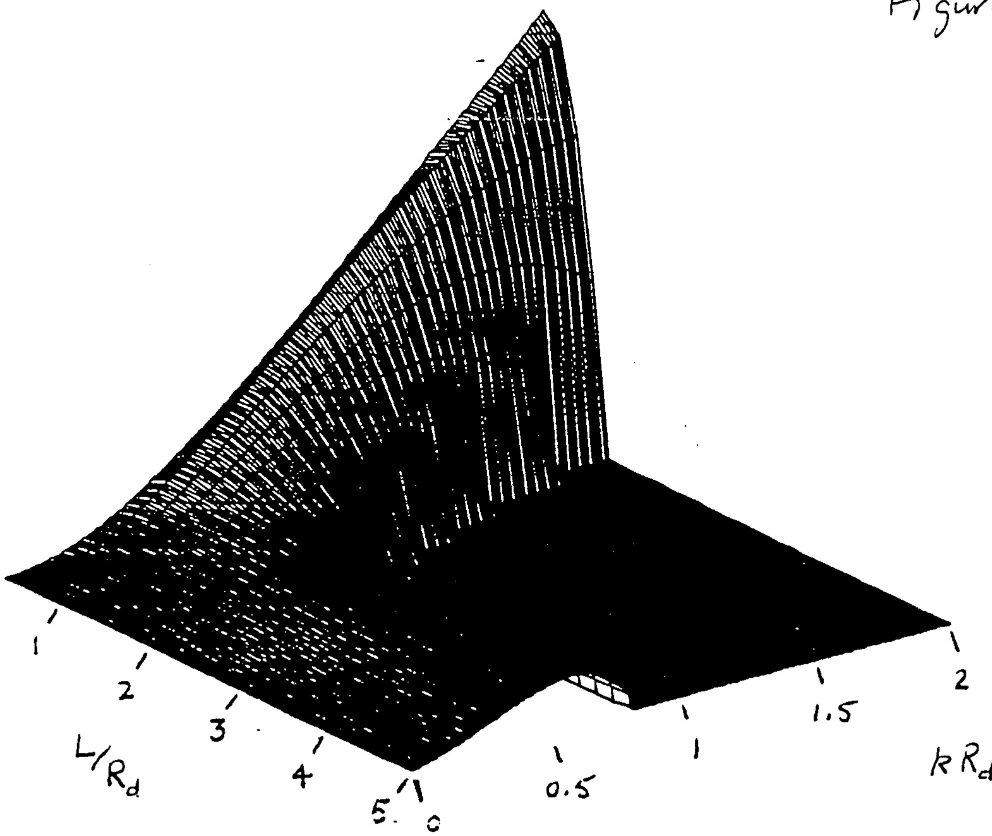
Figure 3a



Growth rate versus wavenumber and upper layer/ lower layer depth

Growth rate for instabilities of the six front model as a function of kR_d and H_1/H_2. There is a barotopic instability occuring at intermediate wavenumbers even when the lower layer depth is infinite. The baroclinic instability becomes dominant when $H_1/H_2 \sim 0.1$.

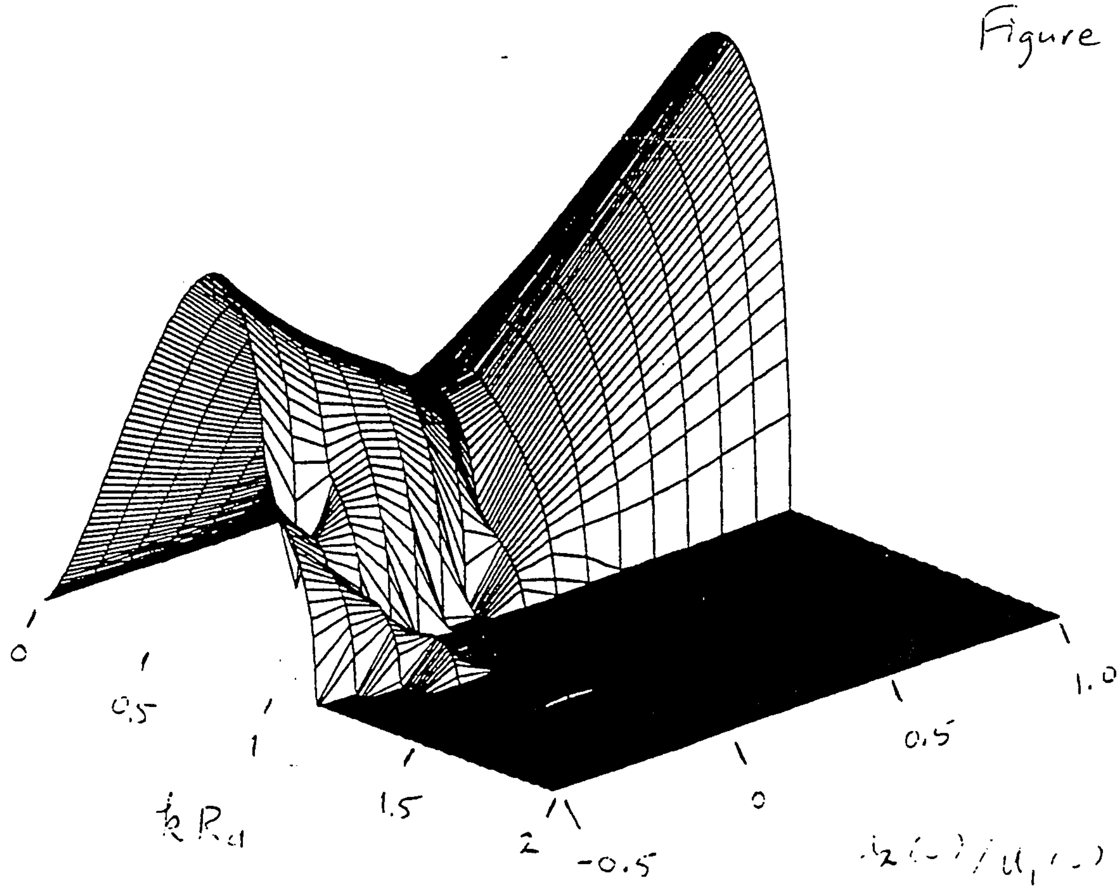
Figure 3b



Growth rate versus jet width

Growth rate for instabilities of the six front model as a function of kR_d and L. For very wide jets, the instability is essentially a baroclinic instability; as L decreases, the barotropic instability becomes important at higher wavenumbers.

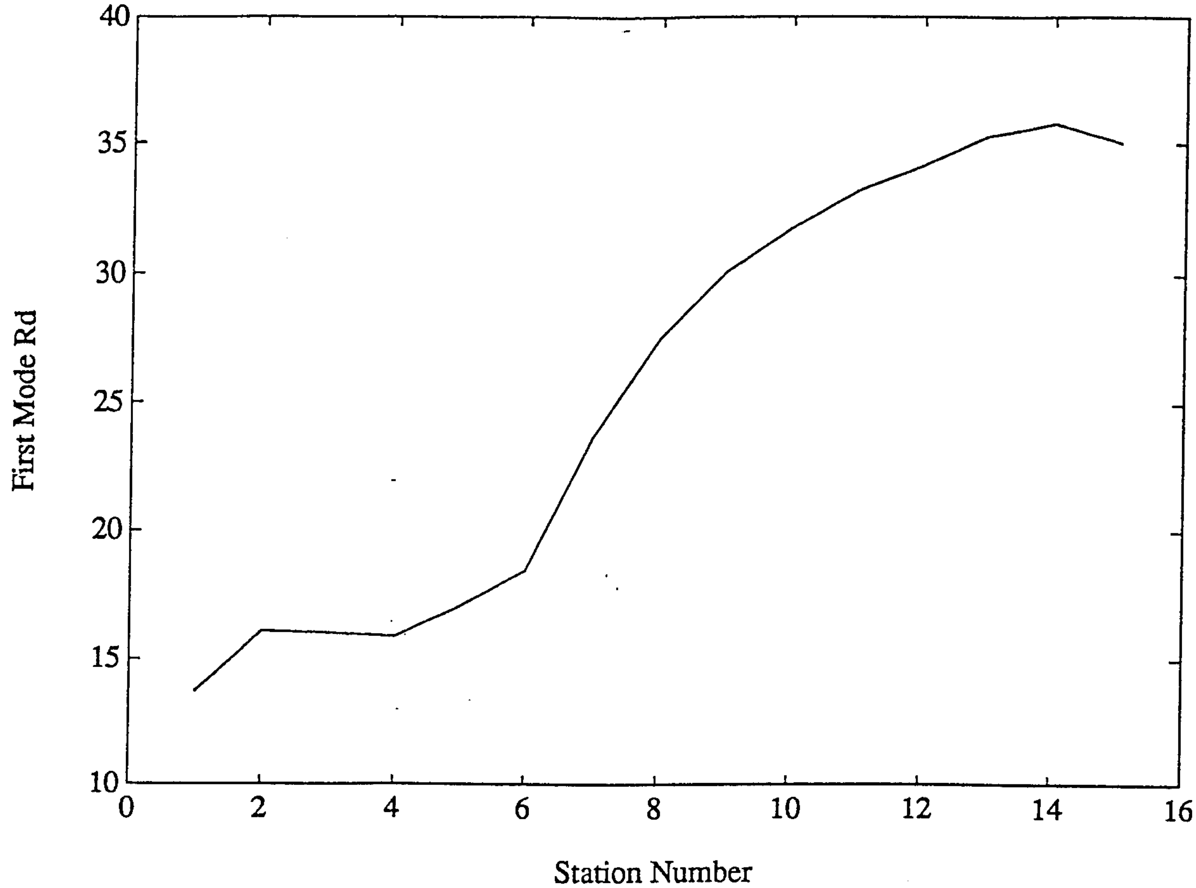
Figure 3c



Growth rate versus wavenumber and lower layer velocity

Growth rate for instabilities of the six front model as a function of kR_d and $u_2(0)$. For negative lower layer centerline velocities, there are several unstable modes near the high wavenumber cutoff.

Figure 4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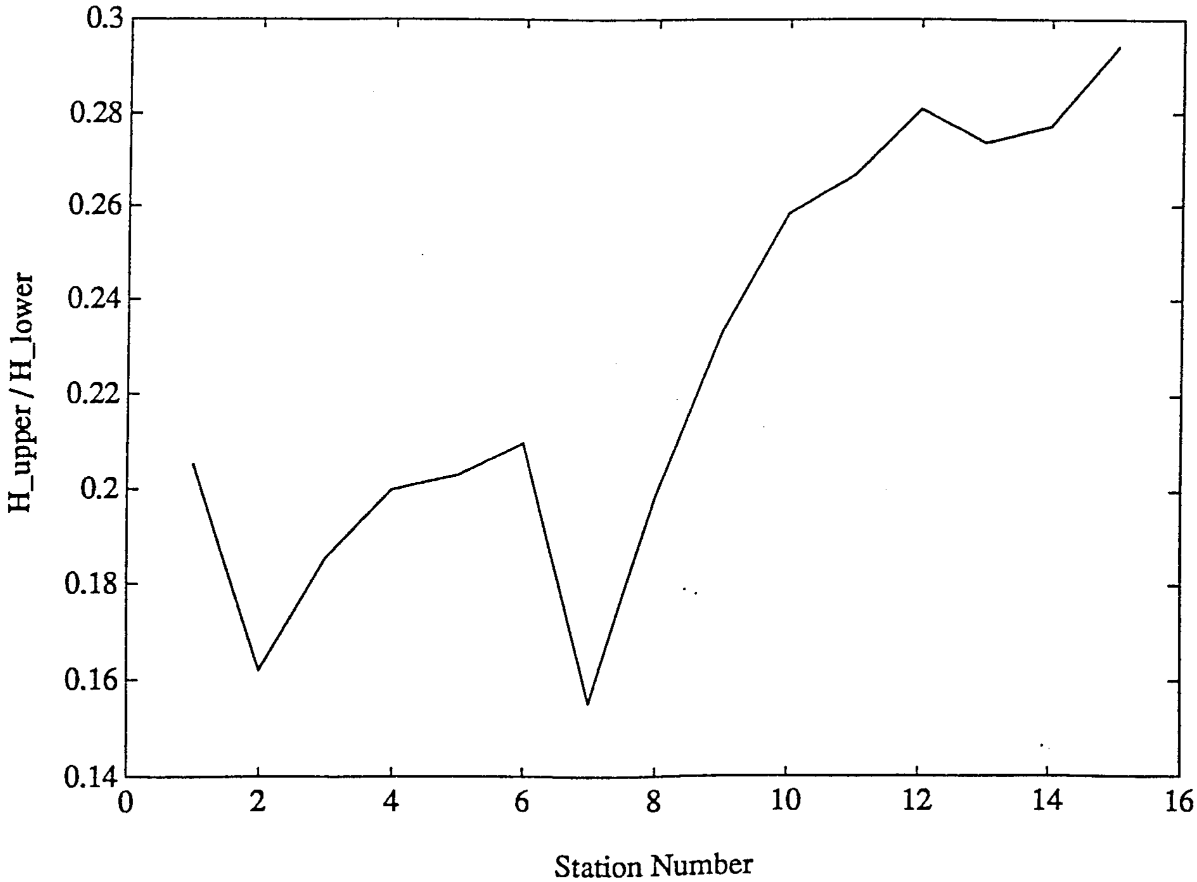
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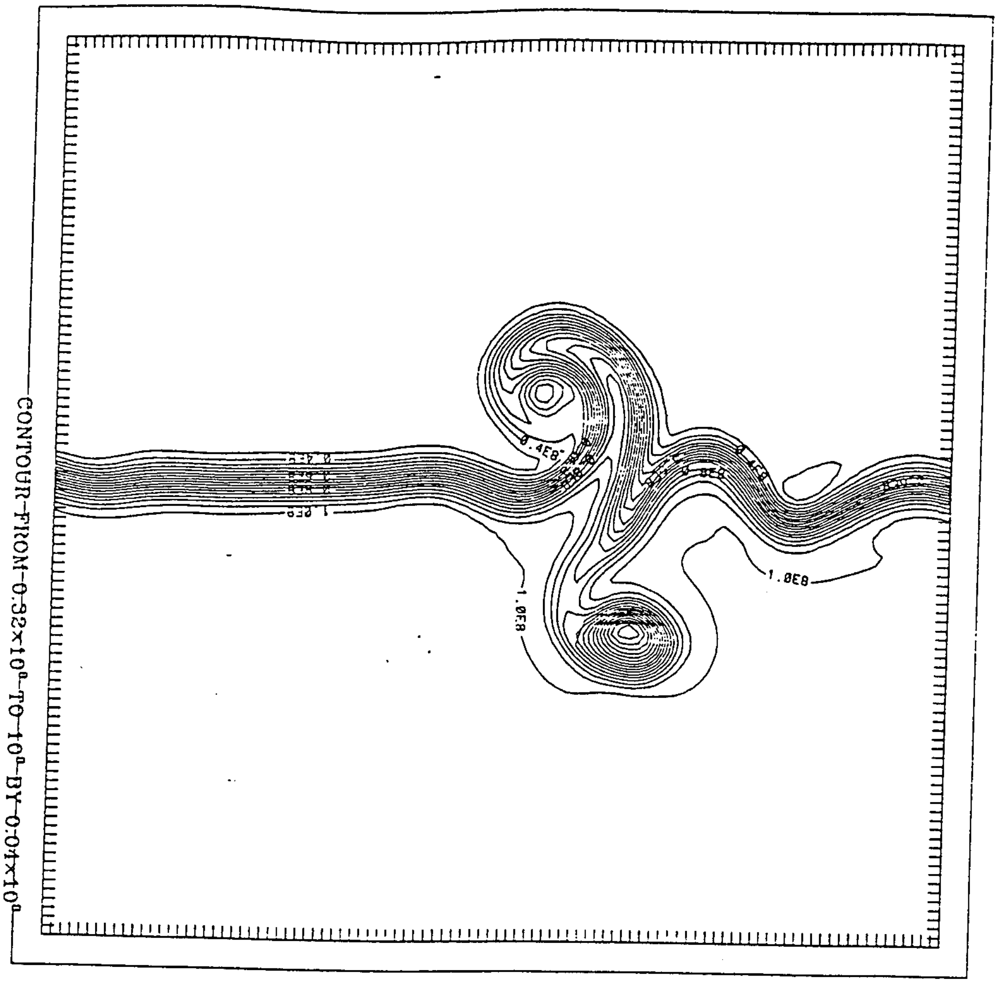
Figure 5

h_1

PE



Day 20

QG