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WAVELET TRANSFORM SIGNAL PROCESSING APPLIED TO ULTRASONICS

A. ABBATE

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DEVELOPMENT AND ENGINEERING CENTER
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13. ABSTRACT *(Maximum 200 words)*
The wavelet transform is applied to the analysis of ultrasonic waves for improved signal detection and analysis of the signals. In instances where the mother wavelet is well defined, the wavelet transform has relative insensitivity to noise and does not need windowing. Peak detection of ultrasonic pulses using the wavelet transform is described and results show good detection even when large white noise was added. The use of the wavelet transform to extract the frequency dispersion relation of the Lamb wave velocity is also described. The two-dimensional wavelet transform allows for both time and frequency analysis, thus making it particularly suitable for dispersion studies. Experimental and numerical results show the superior performance of the wavelet transform signal processor.

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INTRODUCTION

The science of ultrasonics has many applications in various fields of physics, chemistry, technology, and medicine. The study of the propagation of ultrasonic waves in different media has been the basis of evaluation for a wide variety of physical properties of gases, liquids, and solids. Conventionally, an ultrasound wave is generated on the surface or in the bulk of the material under study and its time evolution is utilized to characterize the travelling medium. Since the travel time τ_d required for the wave to travel between two positions in the medium is inversely proportional to the ultrasound velocity, in some instances accurate determinations of absolute velocities have been hindered by unsatisfactory time measurements due to the presence of background noise. In ultrasonic nondestructive evaluation of materials such as steel, aluminum, and copper, the detection of small flaws is often limited as a result of the masking effect caused by scattering of ultrasound due to grain boundaries.

The problem of detecting a transient signal of unknown amplitude and arrival time buried in noise is not restricted to ultrasonics, but it is general to many practical situations. In many instances the measurement of the amplitude and velocity of transient signals propagating in dispersive media cannot be done using conventional pulse-echo overlap or peak identification methods (ref 1). The use of the phase and magnitude of the spectrum to measure the velocity and amplitude of the signals may also not be possible since the multiple modes often overlap and the relative power of the modes may vary.

The Wavelet Transform (WT) is the latest technique to emerge for processing signals with time varying spectra (refs 2,3). There has been an explosion of papers and interest in wavelets and their potential applications; some have called them the most significant mathematical event of the past decade. Applications range in many fields such as geophysics, mathematics, theoretical physics, and communications (ref 4). Multiresolution signal decomposition using wavelet transform has been extensively studied, especially in the development of Perfect Reconstruction-Quadrature Mirror Filter (PR-QMF) (ref 5).

The WT is defined in terms of basis functions obtained by compression/dilation and shifting of a 'mother wavelet'. It does not suffer from the time-bandwidth resolution tradeoff like the short-time Fourier transform. It is also a linear transform, and hence avoids many of the problems associated with the Wigner-Ville distribution (ref 6). All these make the WT an attractive analysis technique as compared to existing time-frequency analysis procedures. This new technique--with the ability to use different windows at different frequencies--tends to model the signal in an efficient way.

In the following, the wavelet transform is briefly introduced and its application in ultrasonics is presented. In particular we have found the processing of the ultrasonic signals using wavelet transform extremely useful for noise suppression and peak detection and for the joint time-frequency representation of ultrasonic signals.

ULTRASONIC SIGNAL PROCESSING USING WAVELET TRANSFORM

By definition, the WT is the correlation between the signal and a set of basic wavelets (ref 7). An appropriate square integrable mother wavelet $h(t)$ is chosen to analyze a specific transient signal of finite energy. Then a complete orthogonal set of daughter wavelets $h_{a,b}(t)$ is generated from the mother wavelet $h(t)$ by dilation (a) and shift (b) operations. The WT expansion coefficients $W_s(a,b)$ of the signal $s(t)$ are given by:

$$W_s\,(a,b) = \int_{-\infty}^{\infty} s(t) \cdot h^{*}_{a,b}(t)\; dt = s(t) \otimes \frac{1}{\sqrt{a}}\, h^{*}\!\left(\frac{t}{a}\right) \tag{1}$$

where the function $h_{a,b}(t)$ is given by:

$$h_{a,b}(t) = a^{-1/2} \cdot h\!\left(\frac{t-b}{a}\right) \tag{2}$$

A difference from the Fourier transform is that the mother wavelet can be any function that satisfies certain admissibility conditions. Also, as discussed in detail in the following section, the choice of the mother wavelet for a particular problem improves the signal processing capability of the system. Tailoring of the wavelet is possible and should be done. In the frequency domain, Eq. (2) becomes:

$$H_{a,b}(f) = \sqrt{a} \cdot H(af) \cdot e^{j2\pi fb}$$

which represents the equation of a band-pass filter. This equation shows the important concept that a dilation t/a in the time domain is equivalent to a frequency change of af. The WT transform can thus be seen as a bank of filters constructed by dilation/compression of a single function $h(t)$. The filter constructed by the dilated version of the mother wavelet processes the low frequency information of the signal $s(t)$, and the one related to the compressed version of $h(t)$ analyzes the high frequency information. The ability of adapting the window size of the signal processed makes the WT a natural candidate for the analysis of transient waveforms with a wide spectral range. In many practical applications the signal is sampled and thus also the dilation factor a and the translation b are discretized. Discretization of the time-scale parameters (ref 8) leads to setting $a = a_o^{\,m}$, where $a_0 > 1$ and $b = n\, b_o\, a_o^{\,m}$. The Discrete Wavelet Transform is a very powerful technique that is intimately related to the subband and filter bank techniques. The correlation between the scale and shift parameters implies that the signal sampling frequency is reduced from scale-to-scale in accordance with Nyquist's rule to avoid redundancy. If $a_o=2$, the frequency is represented using a 'dyadic' scale that corresponds to a frequency stepping in an octave-by-octave fashion.

PULSE DETECTION AND NOISE SUPPRESSION

Pulse detection consists of determining the presence or absence of a pulse and estimating its amplitude and arrival time. In some cases, as in many instances in ultrasonic testing, the pulse shape is known in advance. From signal processing theory, it is well known that the optimum receiving filter for detecting pulses buried in white Gaussian noise is a matched filter (ref 9), whose impulse response has the same shape as the pulse to be detected, except that it is reversed in time. Convolution with a reversed pulse is the same as correlation with an unreversed pulse, hence the optimum way to detect a pulse in the presence of white noise is to correlate the pulse with itself. The WT can be an effective noise reduction tool if the mother wavelet can be chosen such that it can be considered an efficient representation of the signal itself. This is because the WT can be represented as a optimal filter and thus noise can be reduced (ref 10).

Computer simulation was utilized to verify the improvements on the signal detection for an ultrasonic wave embodied in white noise. The ultrasonic signal $s(t)$ was generated using a 5-MHz commercial longitudinal transducer on a steel block of known thickness. The echo train of the pulse-echo experiment, given in Figure 1a, was digitized using a Hewlett Packard digital oscilloscope. The sampling rate is 1 GHz. Computer-generated white Gaussian noise $n(t)$ of variable amplitude was added to the waveform, and the resulting signal $y(t)=s(t)+n(t)$ was processed using the wavelet transform. If the shape of the signal $s(t)$ to be detected is known a priori, a replica of $s(t)$ can be utilized as the mother wavelet function $h(t)$. The WT is thus performed by a bank of matched filters whose frequency response is given by $H(af)$. When the input signal $s(t)$ is detected by one of the matched filters, there is a correlation peak whose coordinates indicate the dilation factor a and the position b of the signal. When the input is unknown a priori, the WT may be performed by a bank of filters that does the local analysis of the signal. In this case, the mother wavelet $h(t)$ can be chosen from a set of theoretical signals. The mother wavelet labelled in the figures as 'base' represents the filtered response of the transducer utilized in the test and closely matches the shape of the first echo in Figure 1a. More conventional mother wavelets such as the real Gaussian signal modulated at the frequency of the ultrasonic pulse, the mexican hat, the exponential decay, and the complex Morlet wavelet, were also used. A complex wavelet can be used to eliminate the ripple effects due to the ringing of the transducer. The noisy signal $y(t)$ for the case of a signal-to-noise ratio (SNR) of -11 db is shown in Figure 1b, and its WT obtained using the 'base' mother wavelet with $a = 1$ is shown in Figure 1c. It is obvious that the second echo cannot be detected using $y(t)$, whereas the WT shows the two echoes clearly.

In order to quantify the improvements in the signal detected using WT, the output SNR and the estimated time delay between two successive pulses were evaluated as a function of SNR. The output SNR_{out}, calculated as the ratio of the output signal power computed in the time window where the ultrasonic signal is present and the output signal power computed in a time window for which the original echo train is null, is given in Figure 2 as a function of the input SNR_{in}. The change in measured time delay between the two pulses is plotted in Figure 3 as a function of SNR_{in}. In this case, the time delay measured using the original signal buried in noise and time averaging is also reported for comparison purposes.

It is clear from these graphs that matched filtering using WT facilitates the pulse detection process by making the echoes more clearly defined. To verify this result experimentally, a thinner steel block sample was used. The experimental ultrasonic echo train is shown in Figure 4 as the lower curve, while the top curve represents its WT. The base mother wavelet was utilized with a dilation value of $a=1$. In the original signal, only five echoes can be clearly determined, while nine echoes can be determined using the output of the WT. Since the medium in which the ultrasonic wave travelled was non-dispersive, the nine echoes detected were equally spaced. Echoes found beyond the ninth were not evenly spaced and are probably erroneous. However, it should be noted that there was very little detectable signal beyond the ninth echo.

The detection of flaws is often limited by the masking effects of noise and grain boundary scattering. In order to increase the detection of defects by ultrasonics, several signal processing techniques have been developed (refs 11,12). Since the WT seems to be an efficient procedure to improve the signal strength and reduce the noise, a WT-based technique can be extremely useful for flaw detection. In order to simulate flaws in a material and quantify the performance of the technique proposed, a series of holes with variable size were machined in a block of steel approximately 5 cm thick. The depth of the holes was 1.2 cm from the back surface, and their size varied from 5.2 mm (0.2031 in.) to 0.6 mm (0.0234 in.). The holes were not flat bottomed in order to reduce the area of interaction of the bottom of the hole with the wavefront of the ultrasonic wave. A 5-MHz unfocussed longitudinal transducer with an active area of 90 mm^2 was utilized to generate and detect bulk waves. It was chosen in this manner to show the enhancement of the proposed technique. The mother wavelet was chosen as the first back echo obtained from an A-scan of the sample in a region where no defects were present. The results are shown in Figure 5. The left column represents the square of the amplitude of the original ultrasonic signals, and the related WT is plotted on the right. The number in percent, shown between the two plots, represents the ratio between the area of the hole and the active area of the transducer. For flaw sizes of about 10 percent of the size of the acoustic beam, the detection is still possible using the original signal, but the WT gives optimal results even with the smallest of the holes. From Figure 5 it is obvious that the size of the hole can be reduced even further. Thus, it can be concluded that signal processing of the ultrasonic signals using wavelet transform can be extremely useful in reducing the noise and improving the detection of small flaws in materials.

TIME-SCALE REPRESENTATION OF ULTRASONIC SIGNALS USING WT

Time-frequency representation of ultrasonic signals is a useful tool to describe travelling ultrasound waves in dispersive materials. In a dispersive medium, an arbitrary waveform evolves in time and space due to the dependence of the phase velocity on the frequency (ref 13). Since each progressive wave component propagates with a different phase velocity, the initial shape of the transient waveform is distorted in time. The majority of absolute ultrasonic velocity data has been obtained using time-of-flight methods (ref 14) such as pulse-echo-overlap or double-pulse superposition (refs 15,16). The time-scale representations obtained using the wavelet transform can be used to analyze the dispersive nature of the material in which the ultrasonic wave is propagating. The variable width of the window size, naturally built in the WT, can be extremely useful in analyzing transient phenomena and in particular in identifying the distinct spectral characteristics of a transient waveform.

Computer simulation of an ultrasonic signal travelling in a dispersive medium was performed. The initial pulse is shown in Figure 6 as the pulse occurring at the time unit *800*. The spectrum of the waveform was modified by introducing a delay τ_d and a linear phase shift in frequency to simulate dispersion. The dispersed pulse is also given in Figure 6 as the signal occurring in time $t > 1000$. In this case dispersion can be clearly seen as the higher frequency components of the signal have propagated with lower velocity. When the components making up an original transient waveform are spread over a wide spectral range, the resulting evolution of the spectrum becomes a natural candidate for wavelet transform analysis. The reason is that the self-adjusting window structure of the WT results in a time-scale representation that displays the growth of the spectral components with varying resolutions. In the Fourier transform, the signal is decomposed into a series of harmonic waves of single frequency. In the wavelet transform, this is not true, as the signal is decomposed in a series of 'wavelet components' that are not single frequency. For example, in the case of Morlet mother wavelet, such 'components' have a Gaussian bandwidth about a center frequency. Thus, each 'wavelet component' is characterized by an associated group of waves that has a particular propagation velocity. The center frequency of a wavelet component determines the group velocity and thus the arrival time. The detected point for the group delay time τ_g is the peak point of the envelope curve of the received signal waveform (ref 17).

The Fourier transform of the undispersed signal is shown in Figure 7a as the larger bell-shaped curve. The narrower one represents the spectral distribution of the Morlet mother wavelet used in the time-scale processing of the signals. In Figure 7b the spectral contents of several daughter wavelets are shown. It is clear that each filter related to a particular value of the dilation a_j processes a particular frequency band of the signal, and only information of this range is contained in the resulting $WT(a_j,b)$ signal. If the spectral width of the filter bank determined by the wavelet is very narrow (high $Q = f / \Delta f$), the wavelet will retain its Gaussian envelope with the peak amplitude traveling at the group velocity, even though the pulse width may spread (ref 18). Due to the linearity of the WT, comparison between the time-scale representations of the two signals in Figure 6 yields the group delay as a function of the central frequency of the mother wavelet. The time-scale representation of the two signals is given as a density plot in Figures 8 and 9, respectively. The x- and y-axes in the plot represent the dilation coefficient m and the time t, respectively. Please note that as previously discussed, a negative dilation coefficient m represents compression of the mother wavelet and thus higher frequencies. The first pulse retains its Gaussian shape showing a constant delay with frequency, on the other hand, the time-scale representation of the second signal clearly shows a different time delay with frequency. The delay is longer for higher frequency wavelet components and linearly decreases as the frequency decreases. This result matches the calculated values for the two signals, as shown in Figure 10.

The time-scale representation of ultrasonic signals was applied to the analysis of Lamb waves generated by a laser on a steel sample 0.9 mm thick. The laser pulse was generated using a Nd:YAG laser with a pulse energy of 10 mJ and a pulse width of 5 nsec. The transient ultrasonic signals were detected using conventional wedge transducers at two locations (P_1 and P_2) on the sample. To avoid interference between the two transducers, the ultrasonic wave was generated in a location between the two transducers and in particular in a spot extremely close to

P_1. The two detected signals are given in Figure 11. The signal detected at P_1 has travelled very little in the material and thus can be considered as the impulse response of the generation/detection system. On the other hand, the signal at P_2 has travelled a distance of 28 mm, therefore it represents the impulse response of the generation/detection system as well as the signature of the travelling medium. The time-scale representation of the P_2 signal is given in Figure 12. The ultrasonic wave detected at P_1 shows a pulse at 6 μsec with a scale component of approximately -0.5 (equivalent to 4.24 MHz) and a lower frequency trail ($a=1$ thus at 3 MHz) that shows a modest dispersion characteristic. The time-scale representation of P_2 shows three wave components of different frequency, two of which travel with similar velocity. For the three components, the group delay, and thus the group velocity, was calculated. The results are given in Table 1. The three velocities represent the three modes generated in the plate. The big advantage of using the WT technique is evident here, since we are trying to analyze wave modes that have different frequency but similar wave velocities and thus arrive at the transducer at the same time.

TABLE 1
Summary of Results

$k_t \cdot d$	Frequency-thickness (MHz mm)	Group velocity (m/sec)
7.01	3.8	3417
6.44	2.7	2634
3.57	1.9	3360

CONCLUSIONS

The preceding results and discussion demonstrated the usefulness and effectiveness of wavelet transform as a signal processing technique for the analysis of ultrasonic waveforms. The WT seen as a bank of matched filters can be extremely efficient in eliminating noise in ultrasonic signals and thus enhance the detection of flaw signatures that in many cases are buried in noise. A computer simulation was performed and a practical application for the detection of flaws was presented. Also, due to the constant Q property of the wavelet transform, this technique can be used to analyze transient waves propagating in a dispersive medium. The time-scale representations resulting from the WT demonstrate a very efficient means of obtaining the velocity dispersion in an ultrasonic medium. The self-adjusting window structure provides an enhanced resolution compared to the short-time Fourier transform. The technique was successful in estimating the group velocities of the first three modes of a plate wave induced by laser generation.

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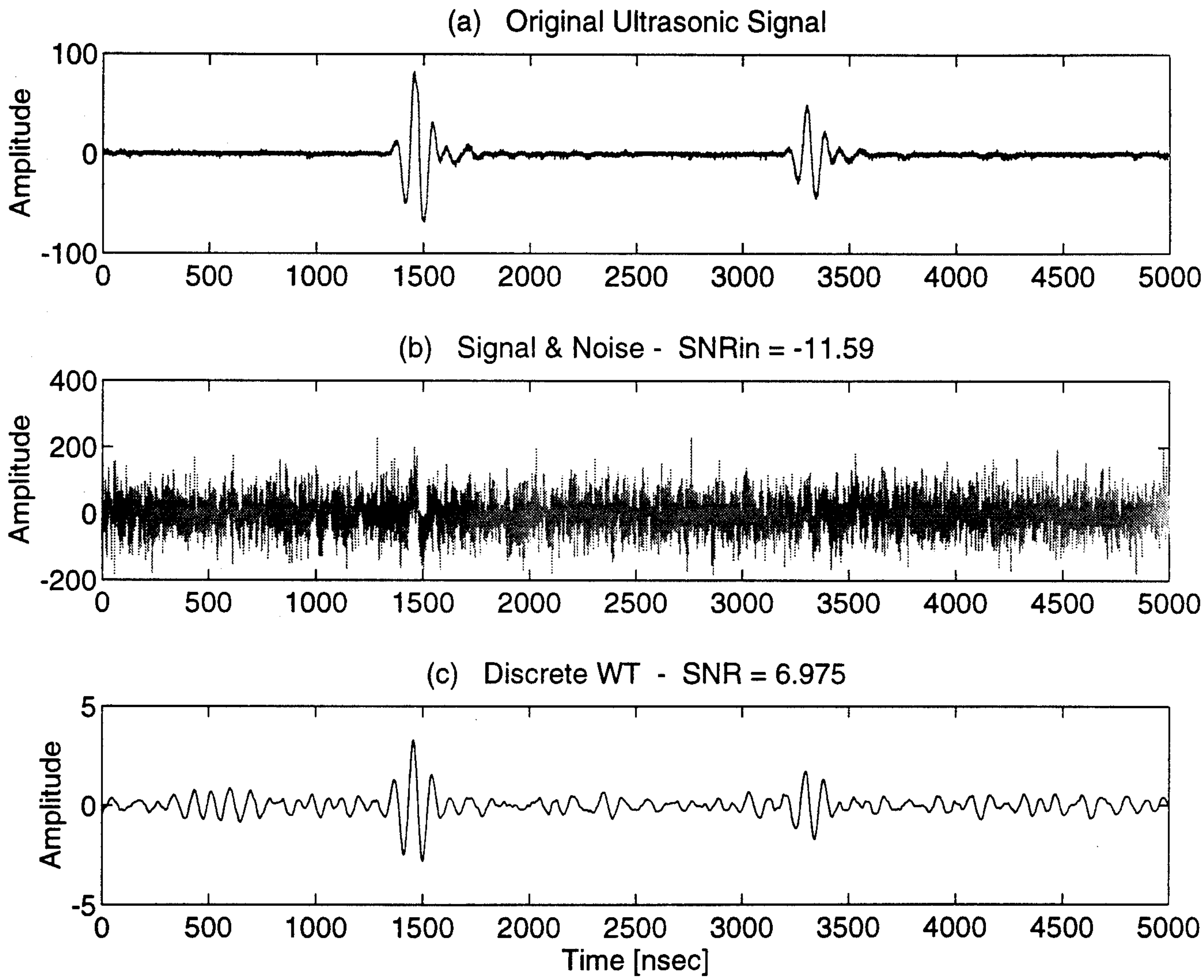

Figure 1. (a) Original ultrasonic signal and (b) with a white noise. The output of the WT using a "base" mother wavelet is shown in (c) with $a = 1$.

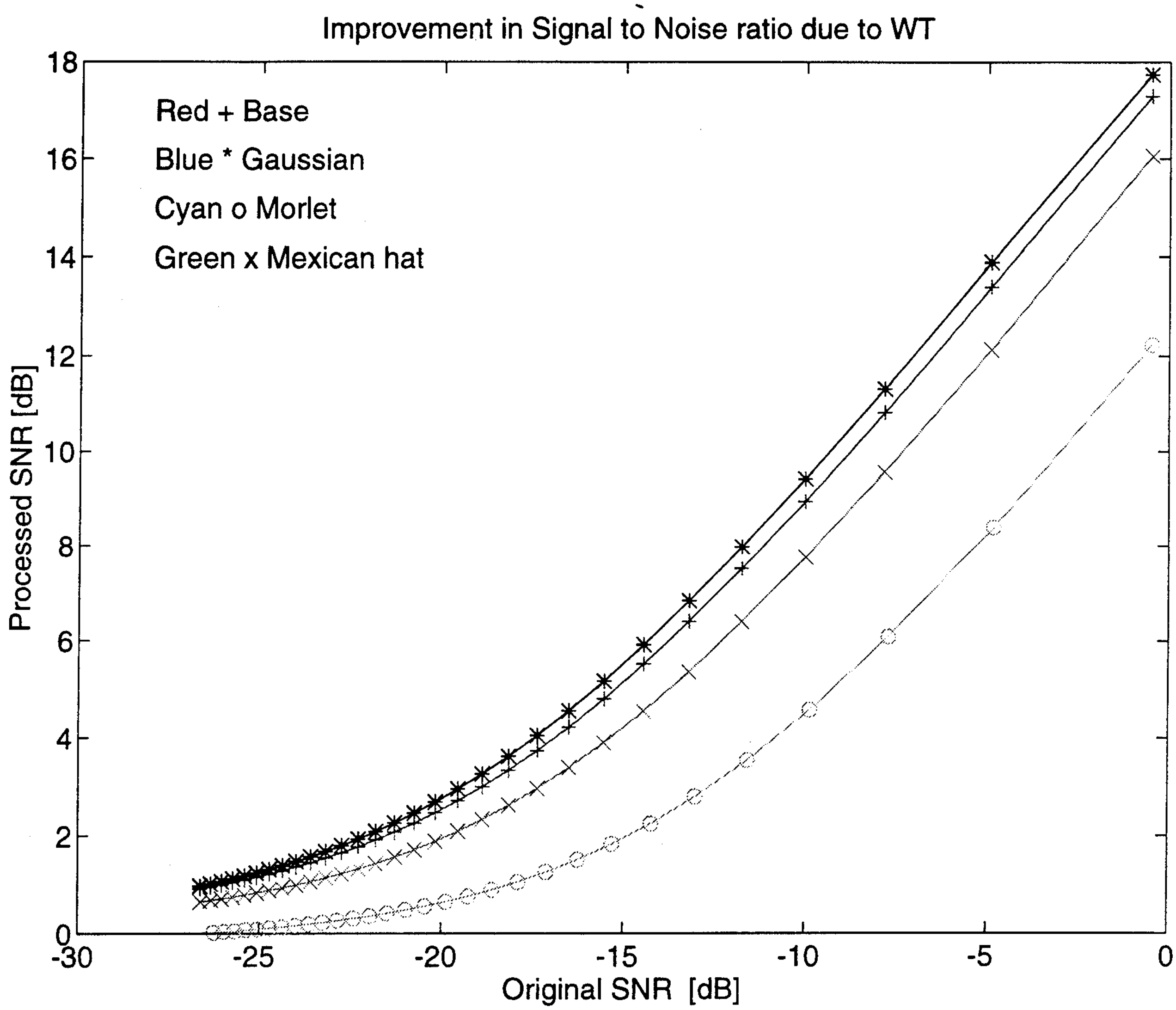

Figure 2. Output SNR of the WT signal processor plotted as a function of the input SNR. Results for four different mother wavelets are shown.

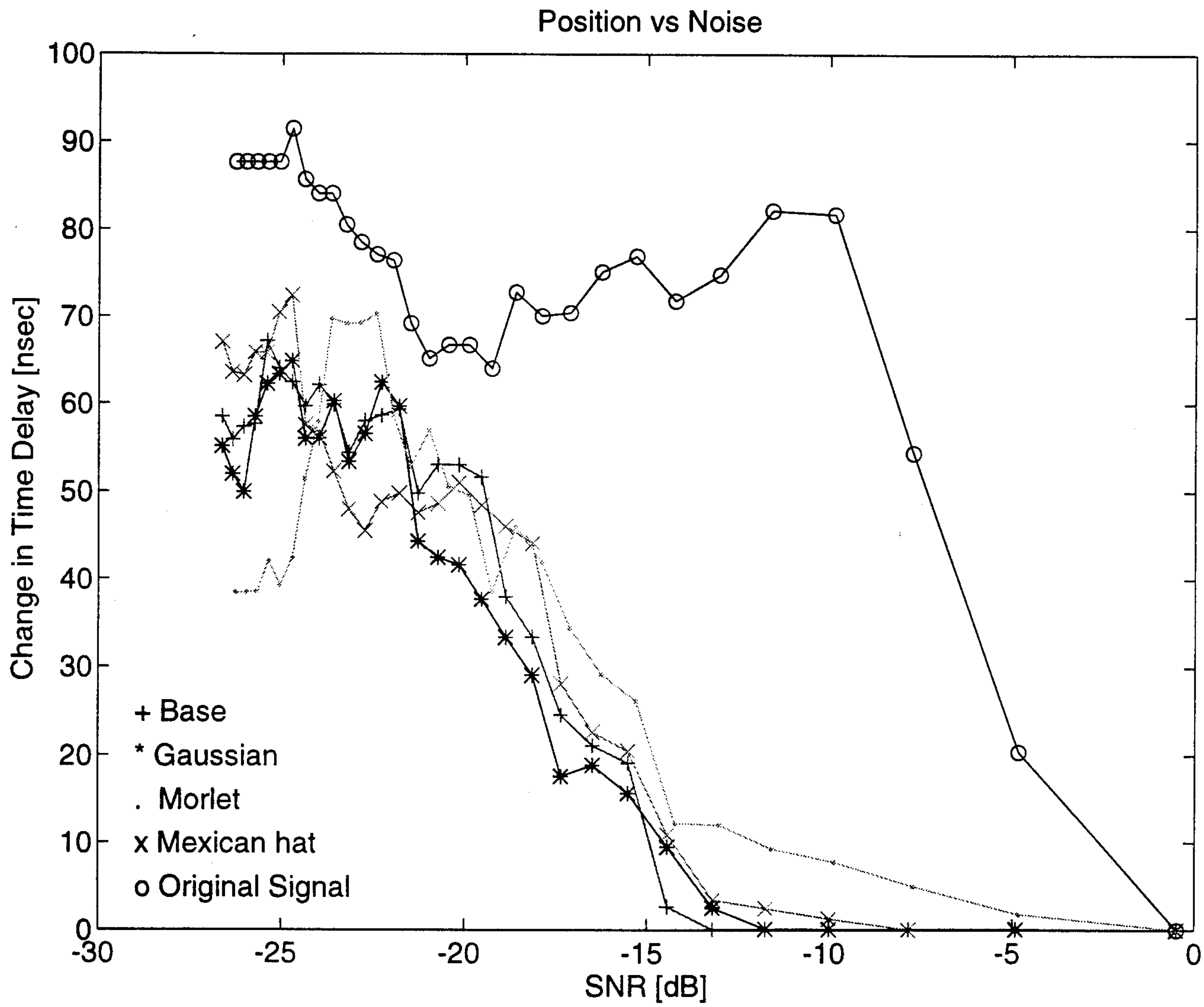

Figure 3. Change in delay time between the two echoes as a function of the SNR.

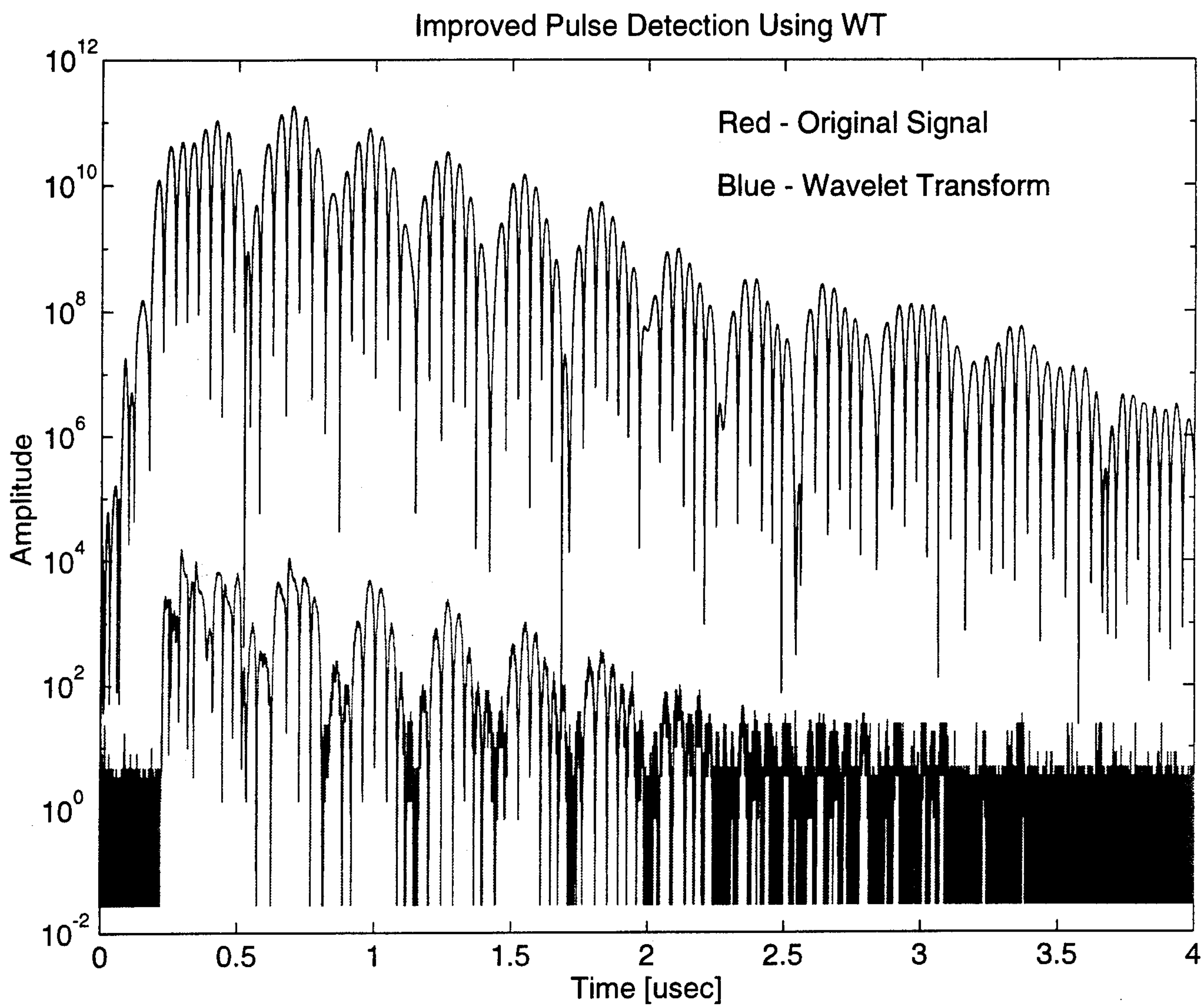

Figure 4. Logarithmic scale representation of the amplitude of the ultrasonic signal and its wavelet transform (a = 1). The top curve represents the WT and the bottom represents the original signal.

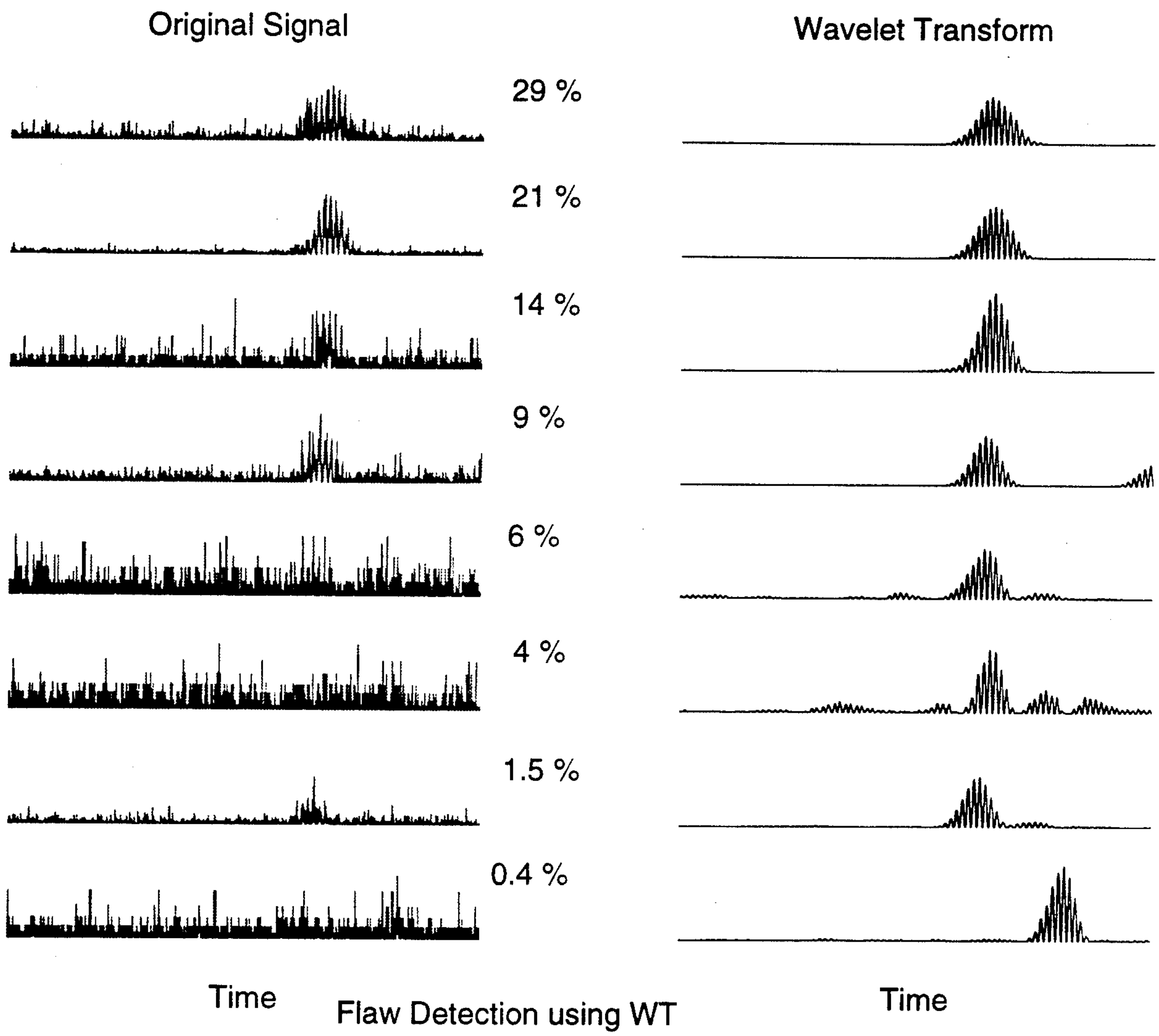

Figure 5. Plot of the original and related WT signals of ultrasonic signals used to probe a steel test block with holes of different sizes. The ratio of the area of the hole and the area of the transducer is given in percent.

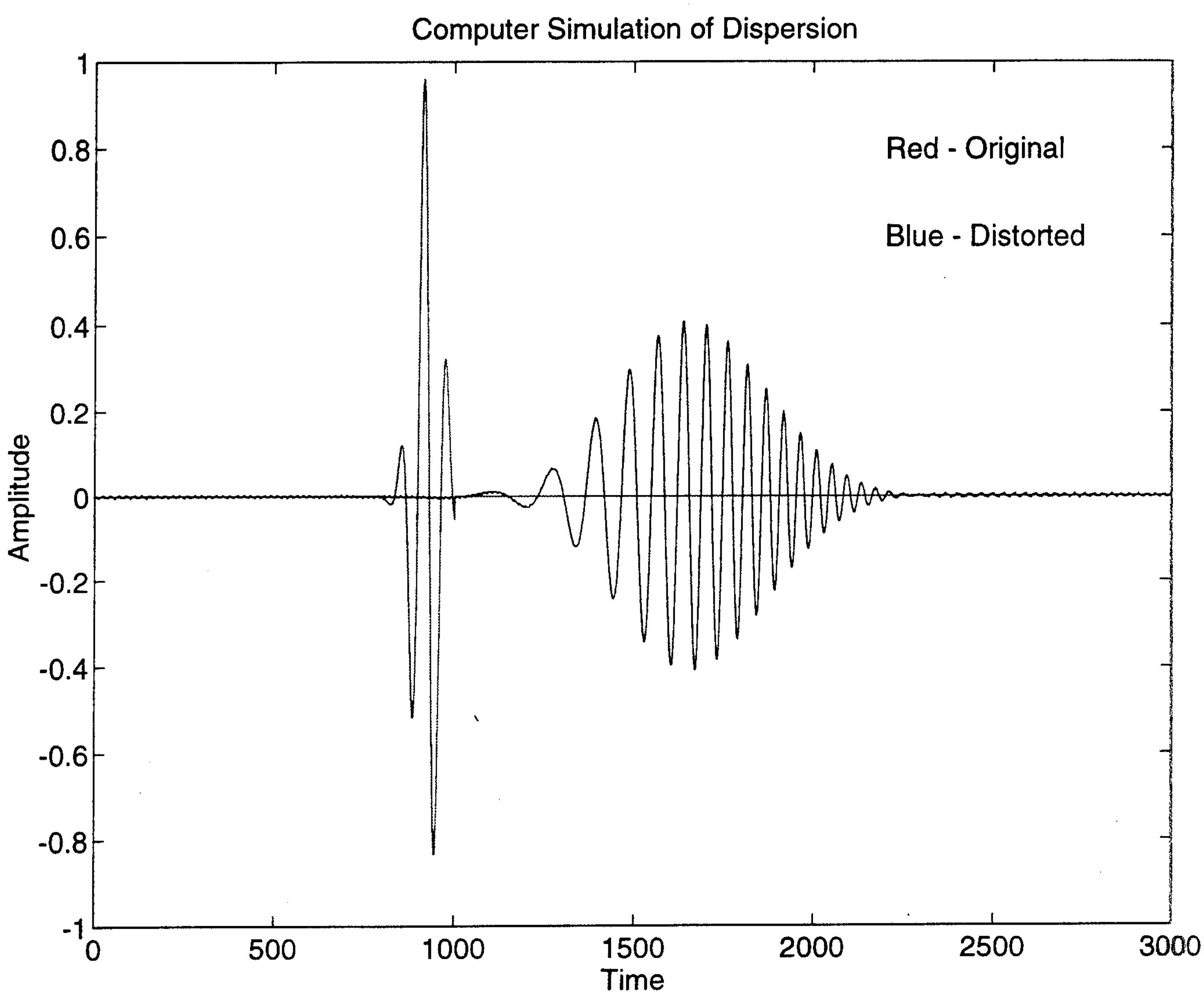

Figure 6. Calculated signals used in computer simulation of ultrasonic propagation in a dispersive medium.

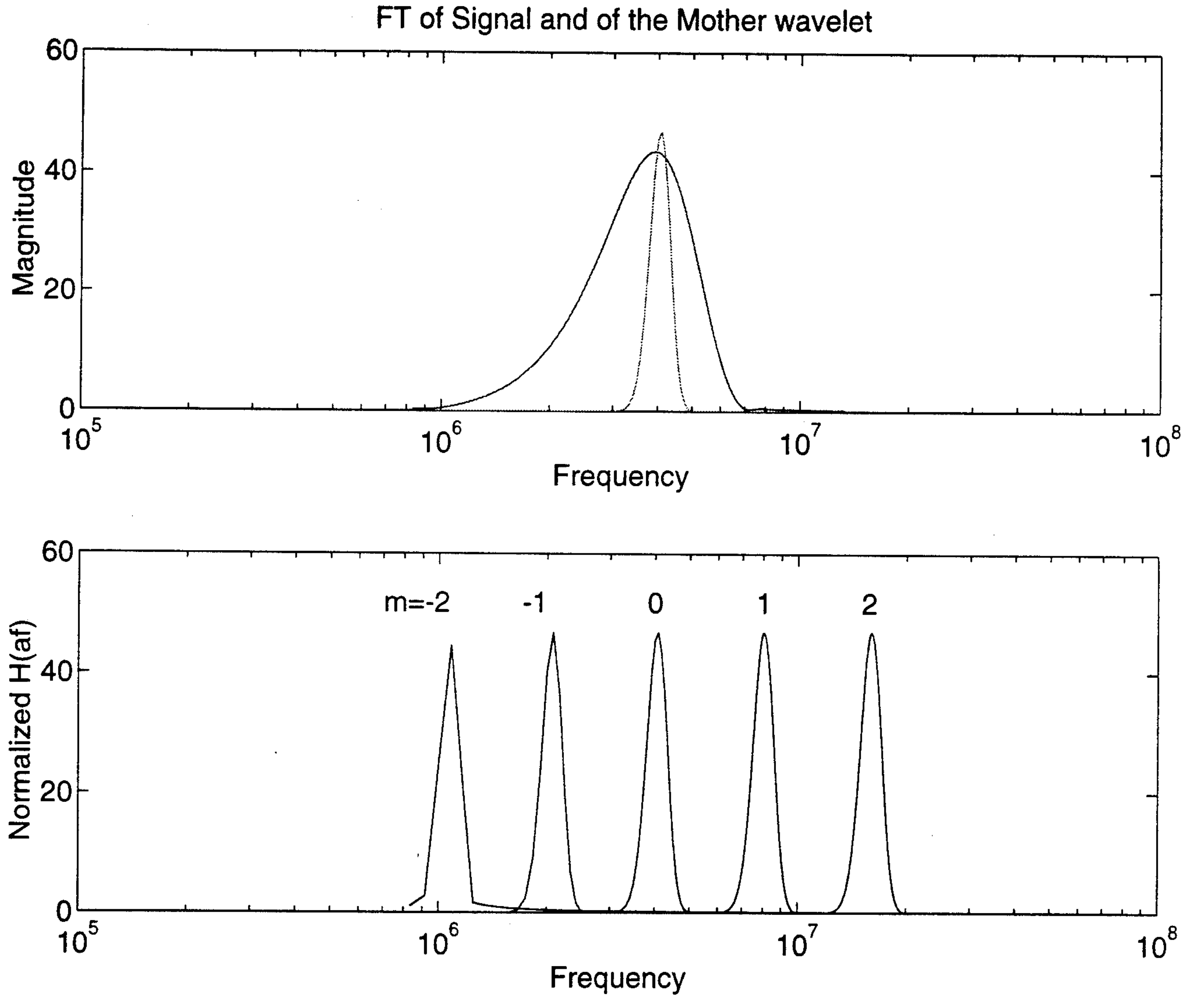

Figure 7. (a) Fourier transform of the signals in Figure 6 and of the Morlet mother wavelet used. (b) In the bottom plot, the spectral contents of several daughter wavelets are shown.

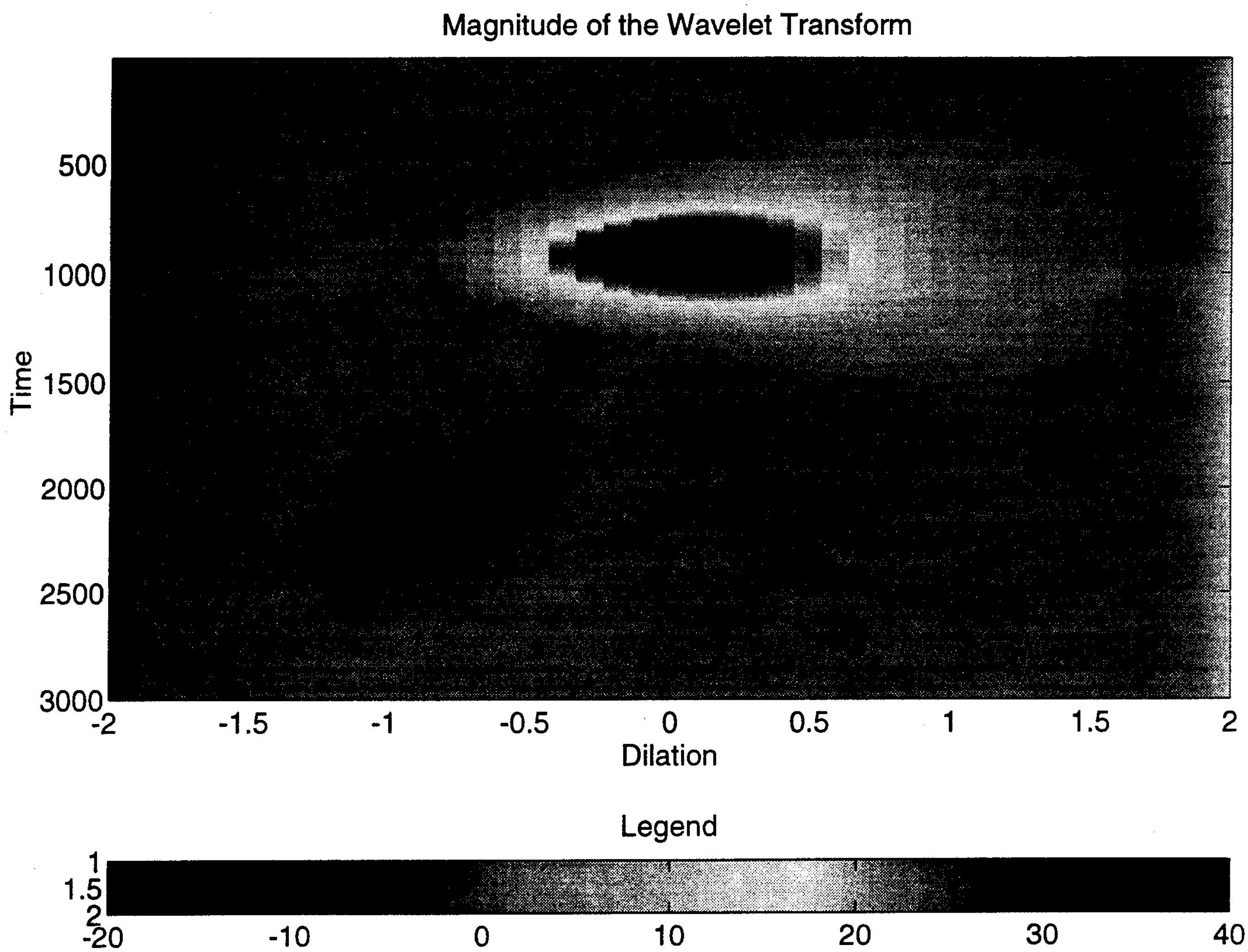

Figure 8. Magnitude plot of the WT of the undistorted signal in Figure 6. The x- and y-axes represent the scale coefficient m ($a = 2^m$) and the time t, respectively.

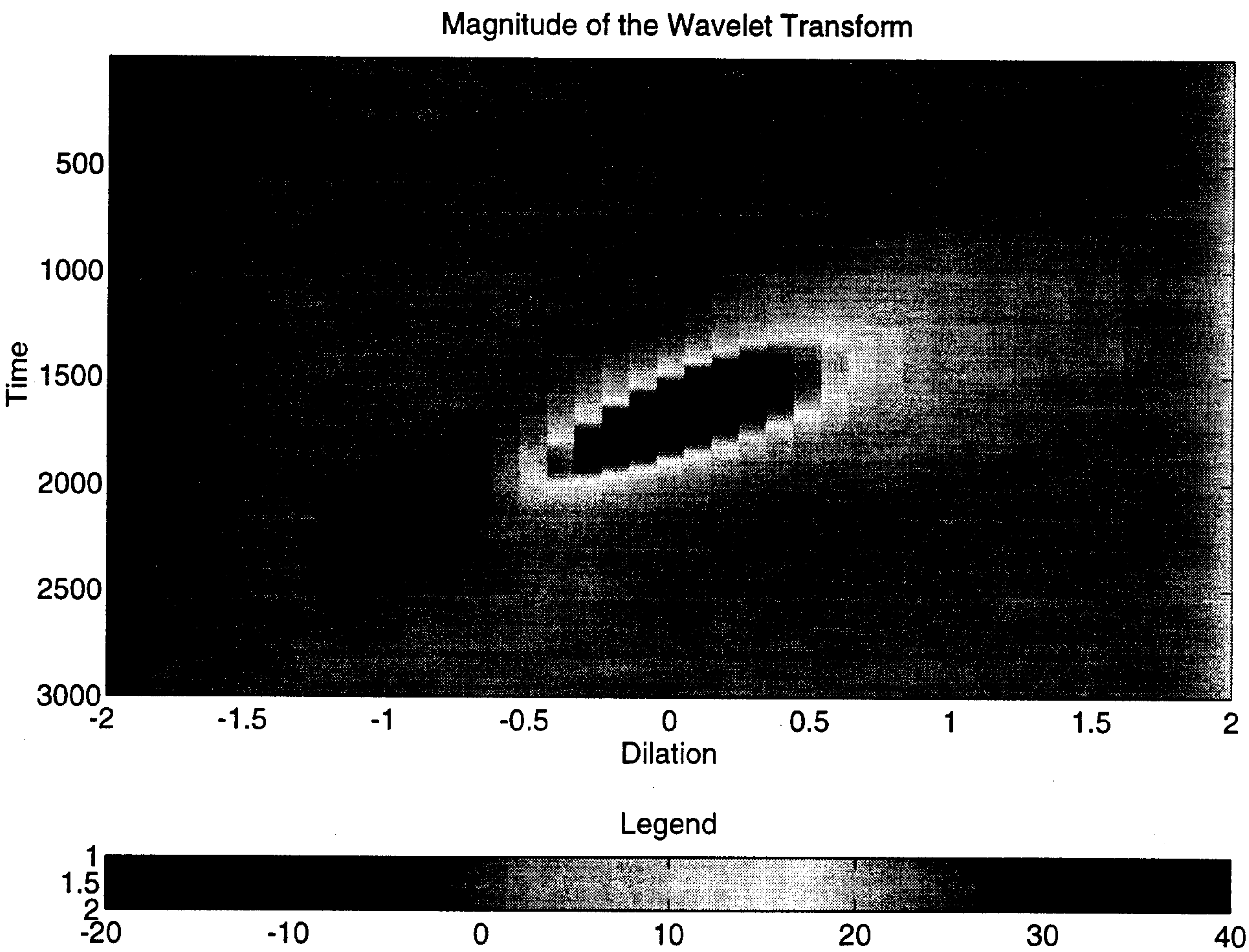

Figure 9. Magnitude plot of the WT of the distorted signal in Figure 6. The linear group delay can be clearly seen.

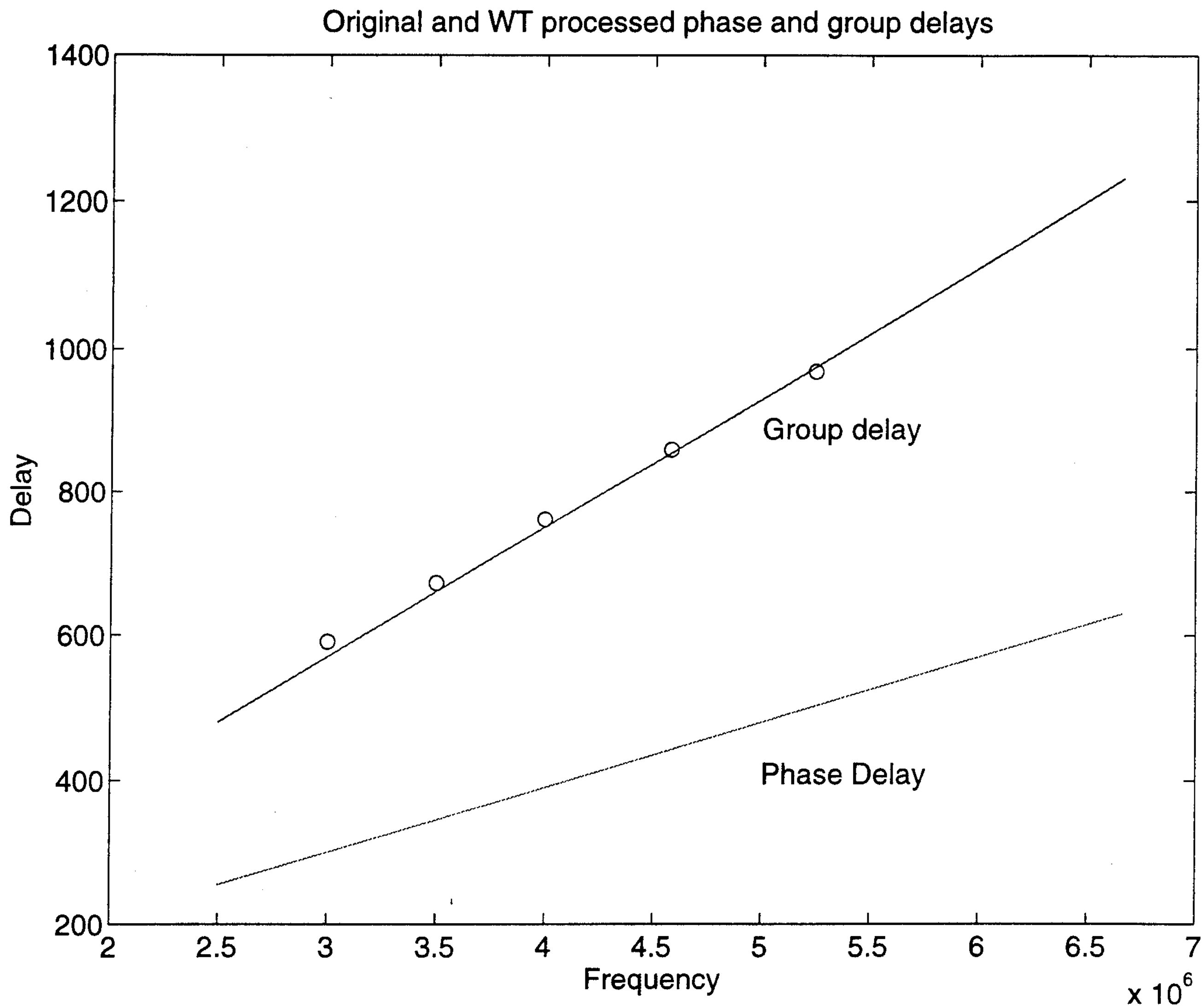

Figure 10. Plot of the group and phase delay as a function of frequency. The lines represent the calculated values and the circles represent values obtained using the wavelet transform time-scale representation.

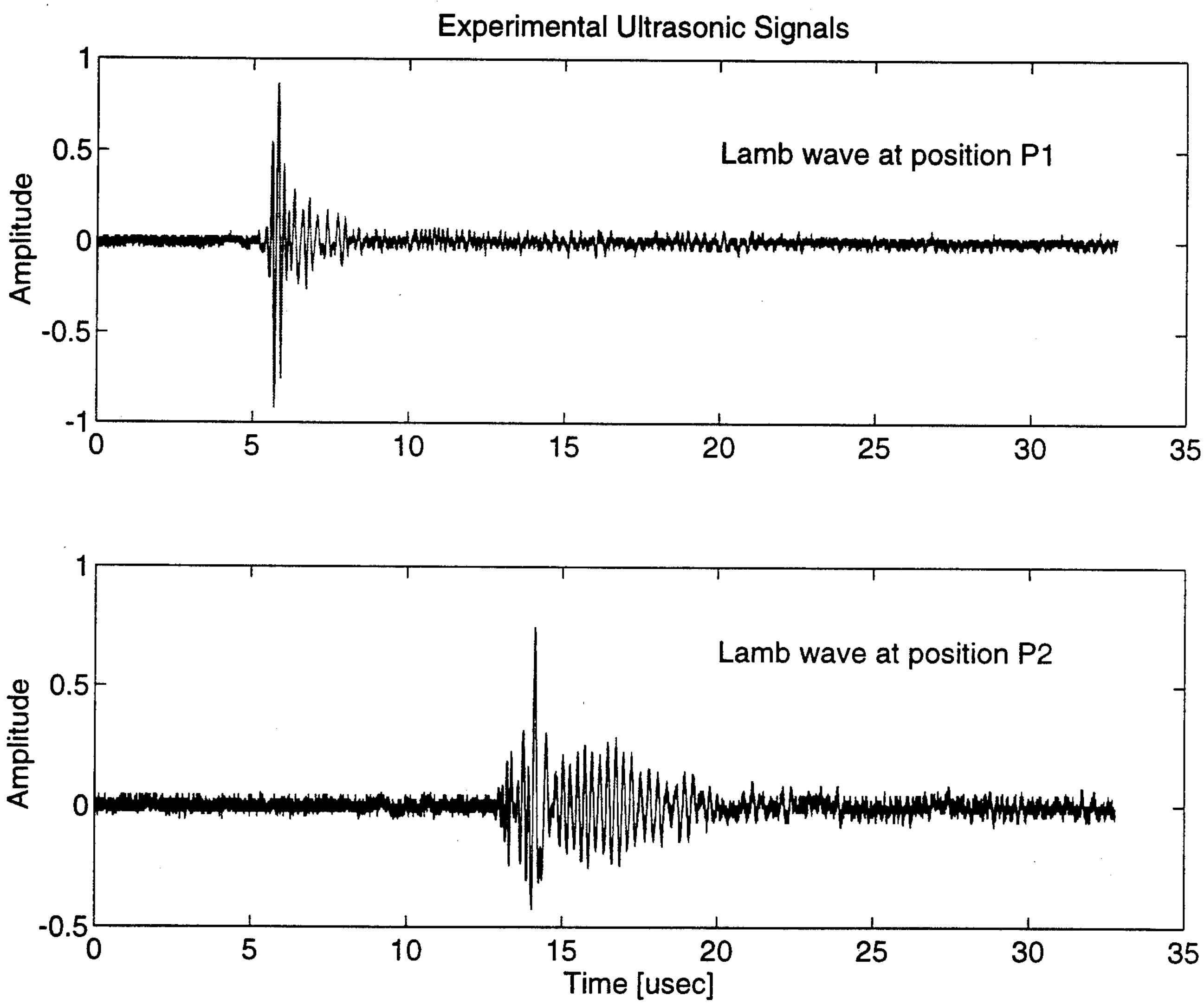

Figure 11. Experimental ultrasonic signals detected in two different locations on a thin steel plate. Dispersion is evident in the signal in P_2.

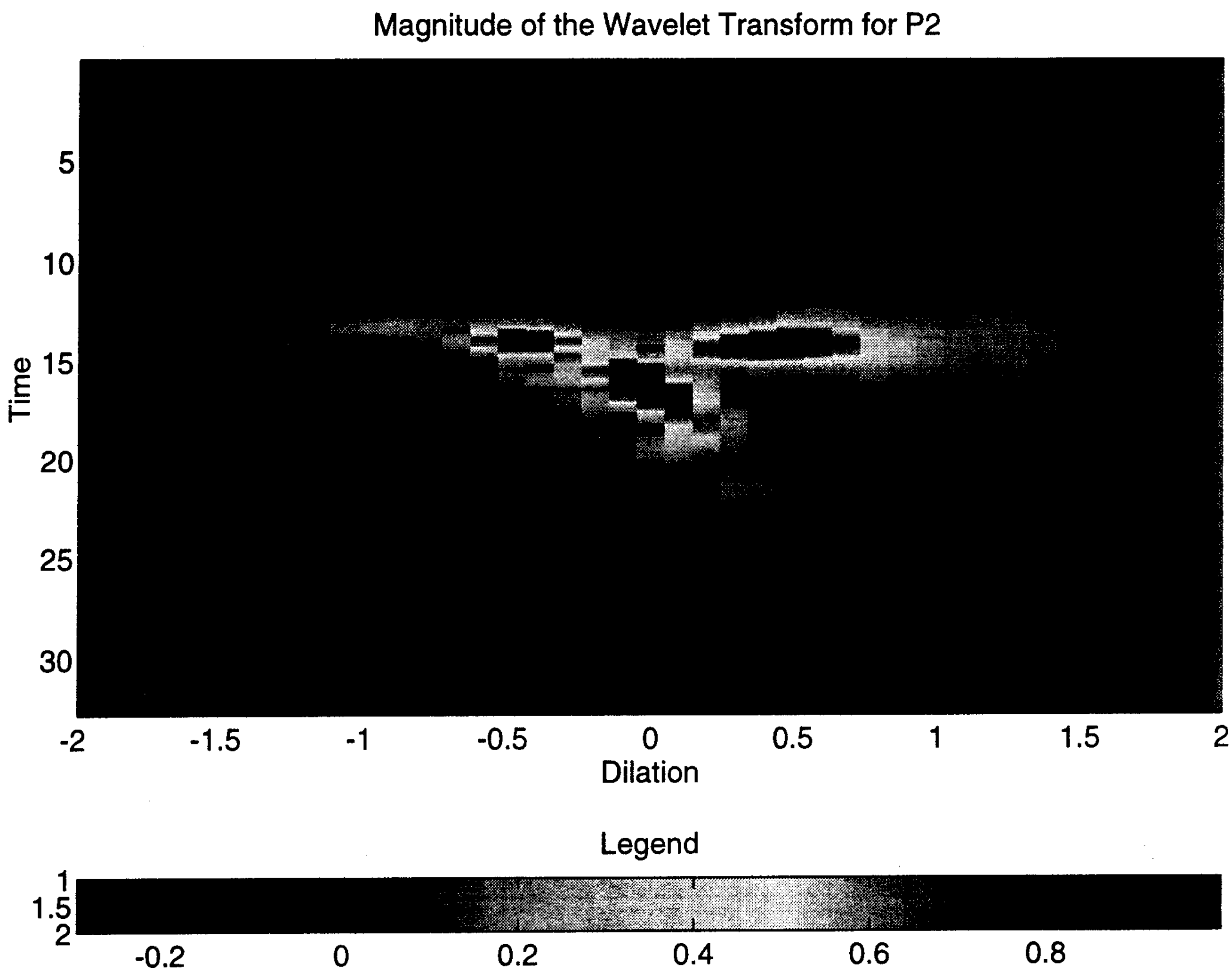

Figure 12. Magnitude plot of the WT of the signal detected in P_2 (Figure 11). Three different modes, of which two have similar group delay, are observed.

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