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NASA CR-165433

AN ANALYSIS OF CRACKS EMANATING FROM A CIRCULAR HOLE IN UNIDIRECTIONAL FIBER REINFORCED COMPOSITES

Final Report - Part II

by

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prepared for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

NASA Lewis Research Center
Grant NSG 3044

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA CR 165433		

4. Title and Subtitle	5. Report Date
An Analysis of Cracks Emanating From a Circular Hole in Unidirectional Fiber Reinforced Composites	February 1981
	6. Performing Organization Code
7. Author(s)	8. Performing Organization Report No.
S. S. Wang and J. F. Yau	
9. Performing Organization Name and Address	10. Work Unit No.
University of Illinois Urbana, IL 61801	11. Contract or Grant No. NSG 3044
	13. Type of Report and Period Covered Final Report - Part II
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington DC 20546	14. Sponsoring Agency Code

15. Supplementary Notes

Project Manager: C. C. Chamis, Structures & Mechanical Technologies Division
NASA Lewis Research Center, Mail Stop 49-6
21000 Brookpark Road
Cleveland, OH 44135

16. Abstract

An analytical method is developed for cracks emanating from a circular hole in an off-axis unidirectional fiber-reinforced composite. The method is formulated by using conservation laws of elasticity and fundamental relationships in anisotropic fracture mechanics. The method provides a very convenient and accurate means to examine the complicated crack behavior, when used in conjunction with a suitable numerical scheme such as the finite element method. The formulation is eventually reduced to a system of linear algebraic equations of mixed-mode stress intensity factors. Fracture parameters, describing crack-tip of deformation and fracture in the composite, are obtained explicitly. Effects of material anisotropy and crack/hole geometry are examined also. Of particular interest are the energy release rates associated with crack extension; their values are evaluated for various cases. Results show that mixed-mode stress intensity factors and energy release rates associated with the cracks emanating from a hole change very appreciably with fiber orientation in the composite. K_I and G increase monotonically with increasing θ; but K_{II} reaches its maximum at $\theta = 45^o$, and then decreases gradually as θ increases further.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
stress intensity, energy release, holes-with-cracks, stress analysis, fiber composites, anisotropy, J-integral	Unclassified, Unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified		

* For sale by the National Technical Information Service, Springfield, Virginia 22161

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FOREWORD

This report describes a portion of the results obtained on NASA Grant NSG 3044. This work was done under subcontract to the University of Illinois, Urbana, with Prof. S.S. Wang as the Principal Investigator. The prime grantee was the Massachusetts Institute of Technology, with Prof. F.J. McGarry as the Principal Investigator and Dr. J.F. Mandell as a major participant. The NASA - LeRC Project Manager was Dr. C.C. Chamis.

Efforts in this project are primarily directed towards the development for finite element analyses for the study of flaw growth and fracture of fiber composites. This report presents a convenient method of analysis for cracks emanating from holes in anisotropic materials.

ABSTRACT

A method of analysis has been developed for cracks emanating from a circular hole in an off-axis unidirectional fiber-reinforced composite. This analysis method is formulated on the basis of conservation laws of elasticity and of fundamental relationships in anisotropic fracture mechanics. The problem is eventually reduced to a system of linear algebraic equations in mixed-mode stress intensity factors. Superiority of this current method to other approaches in investigating the problem with a very complicated crack geometry and material anisotropy is demonstrated when used in conjunction with any numerical method such as a finite element analysis. This method is found to be convenient and accurate. Mixed-mode stress intensity factors and the associated energy release rates in the crack problem are determined for the composites with various fiber orientations. Solutions for both single and double cracks emanating from the edge of a hole in the composites are presented also to illustrate the fundamental nature of the problem.

1. INTRODUCTION

Fracture of fiber-reinforced composite materials containing stress concentrations such as cutouts has been of great concern for years in the design and analysis of composite structures. Composites with circular cutouts of various sizes exhibit the so-called hole-size effects, which have recently received considerable attention from researchers because of their significant variations of material strength reduction [1]. Thus, the problem of crack extension from a hole in composites has been subjected to extensive investigation both analytically and experimentally. While the importance of the problem has been well recognized, understanding of the fundamental nature of the problem is still very limited due to its inherent complexities. Various studies have been conducted to examine the stress concentration around cutouts in infinite and finite-dimensional anisotropic composites and have achieved significant success [2-5]. However, research progress in studying crack initiation and growth from a hole in anisotropic composite has been relatively slow. The difficulties involve not only the complex crack geometry but also the strong material anisotropy. In addition, the crack-tip singular behavior and its interaction with the hole and the finite-dimensional boundaries of the composites are inherently associated with the problem. These complications make it extremely difficult to be solved mathematically, even for a purely elastic case.

Early work by Bowie [6] has provided solutions for the problem of a crack (or cracks) emanating from a circular hole in an isotropic infinite plate subjected to uniaxial and biaxial stresses. The stress intensity factor coefficients, expressible as a function of the ratio of crack length to hole radius, are given explicitly in [7] in a tabulated form. Problems of crack extension from a hole in a finite-dimensional isotropic strip have also been investigated by many researchers using various numerical and experimental techniques [8-10]. However, relatively few studies of the problem for anisotropic composites have been reported. Waddoups et al [1] were among the first to examine the failure of finite-dimensional composites with circular cutouts using a fracture mechanics approach. Although Bowie's solution for an isotropic infinite solid was assumed to be held for the finite-dimensional anisotropic composites in the study, results seem to predict the proper trend of the fracture behavior of the composites. Recently, a number of alternative approaches have been proposed [11-13]. Because of the complex nature of the problem and the aforementioned difficulties involved, most of the solutions are approximate in nature. While some of the approximate studies provide reasonable correlation between solutions and experimental data, no vigorous analysis, to the authors' knowledge, of the current problem has been reported.

This paper describes an analytical method for crack extension from a hole in a unidirectional fiber-reinforced composite strip. The unidirectional fiber composite, which has the simplest laminate configuration, is deliberately chosen in this study to provide basic understanding of the problem and to serve as a reference for further investigation of more complex multilayered composites. General concepts underlying the basic formulation of the current approach are discussed in the next section. Solution procedure for numerical computation is presented in section 3. The mixed-mode stress intensity factors characterizing the crack-tip behavior are determined explicitly in section 4. Effects of fiber orientation on deformation and crack extension are examined for both single and double crack cases. Of particular interest are the energy release rates during crack extension; their values are evaluated for several cases to illustrate the fundamental nature of the crack problem. Crack initiation and growth from a circular cutout in a multilayered fiber composite give an extremely complicated three-dimensional crack geometry and have to be studied by a fully three-dimensional fracture analysis. Some progress has been made by the authors and will be reported elsewhere [14].

2. FORMULATION

The current approach, using conservation laws of anisotropic elasticity and fundamental relationships of anisotropic fracture mechanics, can provide rigorous solutions for the mixed-mode fracture parameters without the need of solving the corresponding complex boundary value problem. The method has been demonstrated to be superior to other approaches in its mathematical rigorousness and operational simplicity [15], since the conservation integral can be evaluated conveniently along a path removed from the crack-tip region. Details of the formulation for the mixed-mode crack problem in anisotropic composites have been reported elsewhere [16]; only a brief description of the fundamentals of the analysis is given in this section.

Consider a unidirectional composite plate with rectilinearly anisotropic elastic properties. Deformation and stress fields in the solid are governed by the generalized Hooke's law,

$$\varepsilon_i = \sum_j a_{ij}\,\sigma_j, \qquad i,j = 1,2,6 \tag{1}$$

where a_{ij} are compliance coefficients with $a_{ij} = a_{ji}$. Compatibility and equilibrium conditions of the problem lead to a fourth order governing partial differential equation with a characteristic equation of the form

$$a_{11}\,\mu^4 - 2a_{16}\,\mu^3 + (2a_{12} + a_{66})\mu^2 - 2a_{26}\,\mu + a_{22} = 0 \tag{2}$$

Lekhnitskii showed [17] that the roots of Eq. 2 are always complex or purely imaginary and occur in conjugate pairs as μ_1, $\bar{\mu}_1$ and μ_2, $\bar{\mu}_2$. For infinitesimal deformations of anisotropic elastic bodies, a form of general conservation laws for the plane elasticity problem may be written as

$$J_i = \int_S \left[W\, n_i - \sigma_{jk}\, n_k \frac{\partial u_j}{\partial x_i} \right] ds = 0 \tag{3}$$

in which S is an arbitrary closed boundary, enclosing a portion of the continuum with unit outward normals n_i, and W is the strain energy density. This conservation law can be reduced [18] to the well-known path-independent J-integral for an arbitrary path Γ, which begins on one crack surface, encloses the crack tip, and terminates on the opposite face, as shown in Fig. 1,

$$J = J_1 = \int_\Gamma \left(W dy - T_i \frac{\partial u_i}{\partial x} ds \right) \tag{4a}$$

where

$$T_i = \sigma_{ij}\, n_j \tag{4b}$$

The crack-tip stresses have been proved to possess a classical fracture mechanics $r^{-\frac{1}{2}}$ singularity in a linear elastic anisotropic body [19] and can be characterized by stress intensity factors K_I and K_{II} (opening and in-plane shearing modes, respectively). For a mixed-mode fracture problem, the corresponding energy release rates, G_I and G_{II}, may be related to K_I and K_{II} by

$$G_I = -\frac{K_I}{2} a_{22} \, \mathrm{Im} \left[\frac{K_I(\mu_1 + \mu_2) + K_{II}}{\mu_1 \mu_2} \right] \tag{5}$$

$$G_{II} = \frac{K_{II}}{2} a_{11} \, \mathrm{Im} \left[K_{II}(\mu_1 + \mu_2) + K_I \mu_1 \mu_2 \right] \tag{6}$$

and

$$G_{Total} = G_I + G_{II} \tag{7}$$

Using Eqs. 5-7, J can be shown to be related to the stress intensity factors of a cracked, anisotropic composite plate subjected to in-plane loading by

$$J = \alpha_{11} K_I^2 + \alpha_{12} K_I K_{II} + \alpha_{22} K_{II}^2 \quad (8)$$

where

$$\alpha_{11} = -\frac{a_{22}}{2} \mathrm{Im}\left(\frac{\mu_1 + \mu_2}{\mu_1 \mu_2}\right) \quad (9)$$

$$\alpha_{22} = \frac{a_{11}}{2} \mathrm{Im}(\mu_1 + \mu_2) \quad (10)$$

and

$$\alpha_{12} = -\frac{a_{22}}{2} \mathrm{Im}\left(\frac{1}{\mu_1 \mu_2}\right) + \frac{a_{11}}{2} \mathrm{Im}(\mu_1 \mu_2) \quad (11)$$

It should be noted that Eq. 8 alone does not provide adequate information to determine K_I and K_{II} separately for the present mixed-mode crack problem.

If, however, the conservation law for two independent equilibrium states [18] is introduced, additional information may be obtained to remove this deficiency. Denoting the two equilibrium states, 1° and 2°, by the superscripts of field variables, the J-integral for the superimposed state 0° may be expressed by

$$J^{(0)} = J^{(1)} + J^{(2)} + M^{(1,2)} \quad (12)$$

where

$$M^{(1,2)} = \int_\Gamma \left[W^{(1,2)}\, dy - \left(T_i^{(1)} \frac{\partial u_i^{(2)}}{\partial x} + T_i^{(2)} \frac{\partial u_i^{(1)}}{\partial x} \right) ds \right] \quad (13)$$

$$J^{(k)} = \int_{\Gamma} \left[W^{(k)} \, dy - T_i^{(k)} \frac{\partial u_i^{(k)}}{\partial x} \, ds \right], \qquad k = 0,1,2 \text{ (no summation on k)} \tag{14}$$

$$u_i^{(0)} = u_i^{(1)} + u_i^{(2)} \tag{15}$$

and $W^{(1,2)}$ is the mutual potential energy density, denoted by

$$W^{(1,2)} = C_{ijk\ell} \, u_{i,j}^{(1)} \, u_{k,\ell}^{(2)} = C_{k\ell ij} \, u_{i,j}^{(2)} \, u_{k,\ell}^{(1)} \tag{16}$$

Recalling the J-K relationship in Eq. 8 and the orthogonality of the stress intensity factors, $M^{(1,2)}$ can be shown [16] to have the form

$$M^{(1,2)} = 2\alpha_{11} \, K_I^{(1)} \, K_I^{(2)} + \alpha_{12}\left(K_I^{(1)} \, K_{II}^{(2)} + K_I^{(2)} \, K_{II}^{(1)}\right) + 2\alpha_{22} \, K_{II}^{(1)} \, K_{II}^{(2)} \tag{17}$$

The M-integral in Eqs. 13 and 17 deals with interaction terms only and can be used directly to determine the mixed-mode stress intensity solutions of the present problem.

3. SOLUTION PROCEDURE

Equation 13 together with Eq. 17 provides, in fact, sufficient information for determining $K_I^{(1)}$ and $K_{II}^{(1)}$ for the current mixed-mode anisotropic fracture problem if proper auxiliary solutions are provided. Introducing the first auxiliary solution, denoted by the superscript 2a, for the cracked body subjected to mode I deformation only so that

$$K_I^{(2a)} = 1 \qquad \text{and} \qquad K_{II}^{(2a)} = 0 \tag{18}$$

Eq.17 becomes

$$M^{(1,2a)} = 2\alpha_{11} K_I^{(1)} + \alpha_{12} K_{II}^{(1)} \tag{19}$$

where the integral $M^{(1,2a)}$ has the form

$$M^{(1,2a)} = \int_\Gamma \left(C_{ijk\ell}\, u_{i,j}^{(1)}\, u_{k,\ell}^{(2a)}\, dy - \left[T_i^{(1)} \frac{\partial u_i^{(2a)}}{\partial x} + T_i^{(2a)} \frac{\partial u_i^{(1)}}{\partial x} \right] ds \right. \tag{20}$$

$T_i^{(1)}$ and $u_i^{(1)}$ in Eq. 20 can be determined easily along a properly chosen integration path Γ. Any convenient method such as the commonly used finite-element analysis, which can provide accurate far field solutions, may be used. $u_i^{(2a)}$ and $T_i^{(2a)}$ can be calculated from the following relationships:

$$u_i^{(2a)} = \sqrt{\frac{r}{\pi}}\, f_i^{(I)}(\mu_1, \mu_2, \theta) \tag{21}$$

and

$$T_i^{(2a)} = \sigma_{ij}^{(2a)}\, n_j \tag{22}$$

where

$$\sigma_{ij}^{(2a)} = \frac{1}{\sqrt{2\pi r}}\, g_{ij}^{(I)}(\mu_1, \mu_2, \theta) \tag{23}$$

and $f_i^{(I)}$ and $g_{ij}^{(I)}$ are given explicitly in Ref. [19].

Similarly, a second auxiliary solution, denoted by the superscript 2b, for pure mode II deformation of the cracked body is introduced also so that

$$K_I^{(2b)} = 0 \qquad \text{and} \qquad K_{II}^{(2b)} = 1 \tag{24}$$

This gives

$$M^{(1,2b)} = \alpha_{12} K_I^{(1)} + 2\alpha_{22} K_{II}^{(1)} \tag{25}$$

where

$$M^{(1,2b)} = \int_\Gamma \left\{ C_{ijk\ell}\, u_{i,j}^{(1)}\, u_{k,\ell}^{(2b)}\, dy - \left[T_i^{(1)} \frac{\partial u_i^{(2b)}}{\partial x} + T_i^{(2b)} \frac{\partial u_i^{(1)}}{\partial x} \right] ds \right\} \tag{26}$$

The quantities $T_i^{(2b)}$ and $u_i^{(2b)}$ in Eq. 26 may be determined by

$$T_i^{(2b)} = \sigma_{ij}^{(2b)}\, n_j \tag{27}$$

where

$$\sigma_{ij}^{(2b)} = \frac{1}{\sqrt{2\pi r}}\, g_{ij}^{(II)}(\mu_1,\mu_2,\theta) \tag{28}$$

and

$$u_i^{(2b)} = \sqrt{\frac{r}{\pi}}\, f_i^{(II)}(\mu_1,\mu_2,\theta) \tag{29}$$

Thus, Eqs. 19 and 25 provide a system of linear algebraic equations, and $K_I^{(1)}$ and $K_{II}^{(1)}$ can be determined easily. It should be noted that in the process of solving for $K_I^{(1)}$ and $K_{II}^{(1)}$, the integrals, $M^{(1,2a)}$ and $M^{(1,2b)}$, need to be evaluated accurately and explicitly. For a given crack geometry and loading condition, this can be achieved by integrating the functionals in Eqs. 20 and 26 numerically along a convenient path in the far field so as to avoid the crack-tip complications. In this paper a finite element analysis is used in conjunction with a second-order Gaussian quadrature for evaluating the M-integral in each element (Fig. 2) by

$$M^{(1,2)} = \sum_{n=1}^{N} \sum_{m=1}^{3} \left\{ H_m F_n(x_m, y_m) \right\} \Delta s_n \tag{30a}$$

where

$$F_n(x_m, y_m) = \left\{ W^{(1,2)} n_1 - \left[T_i^{(1)} \frac{\partial u_i^{(2)}}{\partial x} + T_i^{(2)} \frac{\partial u_i^{(1)}}{\partial x} \right] \right\}_{(x_m, y_m)} \tag{30b}$$

and H_m is a weight coefficient in the Gaussian quadrature; (x_m, y_m), the Cartesian coordinates of Gaussian stations; N, the total number of elements associated with the integration path Γ, and Δs_n is a segment of Γ in the n-th element.

4. RESULTS AND DISCUSSION

The study of mixed-mode crack extension from a circular hole in an anisotropic fiber composite strip has been conducted successfully using the method of analysis mentioned in the previous section. The traction and displacement fields, $T_i^{(1)}$ and $u_i^{(1)}$, involved in Eqs. 20 and 26 for determining $M^{(1,2a)}$ and $M^{(1,2b)}$ may be evaluated along a properly selected path removed from the crack tip by a finite-element method, which employs eight-node isoparametric elements of the displacement model. Accuracy of current results is illustrated first by examining a crack-hole problem whose solution is available in the open literature. Mixed-mode crack-tip stress intensity solutions and the associated energy release rates during crack extension are obtained then for various cases of interest.

4.1 Accuracy of Solutions

Accuracy of current solutions for the mixed-mode fracture mechanics problem has been studied by examining the problem of a crack emanating from a circular hole in an isotropic finite-dimensional plate subjected to a uniaxial in-plane stress. Figure 3 shows the geometry and individual dimensions of the problem. A crack of length a is assumed to emanate radically at a position (R,ϕ) on the hole boundary, where R is the radius of the hole and ϕ is the direction along which the crack extends. For comparative purposes, the problem with $\phi = 0°$ was selected for evaluating the accuracy of the solution for the crack-hole problem. The analysis, employing only 56 elements in the numerical computation, gives the result shown in Table 1, which is in excellent agreement with the existing solution reported by Stalk and Orringer [20]. A detailed study of the convergence and accuracy of the present method of analysis for mixed-mode fracture problems has been reported elsewhere [15,16]. The results indicate that, in general, solution accuracy within one percent deviation from the reference ones obtainable in literature can be achieved easily using only one-third of elements in other highly sophisticated, singular finite-element analyses [20,21]. The accuracy of the current solutions is further reported to be insensitive to the degree of discretization in the numerical calculation. This situation is expected, since the present approach bypasses the highly sensitive and complicated crack-tip region and evaluates the conservation integral along a selected path in the far field, where field variables are generally smooth.

4.2 Cracks Emanating from a Circular Hole in Composites with Various Fiber Orientations

More complex fracture problems of a crack (or cracks) emanating from a circular hole in off-axis unidirectional fiber composite strips are examined in this section. Since the response of the problem is antisymmetric in nature, two cracks of equal length, a, are assumed to be initiated from the boundary of the hole in the composite, where high stress concentration occurs. The corresponding single crack case is also studied for comparative purposes. Figure 3 shows the geometry of the cracks and the circular hole located at the center of a finite-dimensional 2b x 2L anisotropic panel subjected to uniaxial nominal stress, σ_∞, along the y-direction. The material axes, denoted by 1 and 2, of the plate are oriented with respect to the structural axes x and y by an angle θ, as indicated in the figure. The local stress field around the hole (without a crack) in the finite-dimensional composite strip is solved first to determine accurately local stress concentrations by an anisotropic finite-element analysis developed in the present study. The Tsai-Hill stress failure criterion [22] is introduced then into the analysis to determine the locations, (R,ϕ) and $(R,\pi+\phi)$, of crack initiation from the edge of the hole by

$$F(\sigma_{ij}) = \left(\frac{\sigma_1}{X}\right)^2 - \frac{\sigma_1 \sigma_2}{X^2} + \left(\frac{\sigma_2}{Y}\right)^2 + \frac{\sigma_{12}^2}{S^2} = 1 \qquad (31)$$

where subscripts 1 and 2 refer to the stresses in the material coordinates, and X, Y, and S are the longitudinal, transverse, and shear strength of the unidirectional composite, respectively. While other criteria can be implemented easily in the current analysis, no attempt has been made to evaluate the accuracy of different failure theories. For an E-glass/epoxy composite strip panel, X, Y, and S have the following values [23]:

$$X = 150 \times 10^3 \text{ psi}$$

$$Y = 4 \times 10^3 \text{ psi}$$

and

$$S = 6 \times 10^3 \text{ psi}$$

Values of ϕ for various degrees of material anisotropy (i.e. fiber orientation) of the composite are obtained by Eq. 31 in the present analysis. Figure 4 gives the location of crack emanation ϕ as a function of θ for the E-glass/epoxy composite indicating that ϕ decreases appreciably with increasing θ.

It is a common practice in studying fracture of the unidirectional composite to assume that cracks extend, after initiation, along a direction parallel to the major material axis, i.e. the fiber direction, since the trans-fiber strength of the composite is generally much lower than that along the fiber direction. Now consider the problem of two cracks of equal length, $a/R = 0.4$, emanating at (R,ϕ) and $(R,\pi+\phi)$ from the edge of the hole in the glass/epoxy composite, where ϕ depends upon the fiber orientation as shown in Fig. 4. Fracture parameters such as mixed-mode stress intensity factors, K_I and K_{II}, which describe the crack-tip deformation and fracture of the composite, are calculated explicitly for cases of various fiber orientations. For comparative purposes, corresponding single-crack problems are also analyzed. Results, expressed in terms of $K_I/(\sigma_\infty\sqrt{\pi a})$ and $K_{II}/(\sigma_\infty\sqrt{\pi a})$ for both single and double crack cases, are reported in Fig. 5. The associated energy release rates G are determined also in Fig. 6. The results indicate that K_I increases continuously with the increase of fiber orientation and that K_{II} rises to a maximum as θ approaches 45^o and then gradually decreases to zero. The opening mode stress intensity factors determined for most cases are generally larger than that of the shearing mode. For the particular crack length studied, differences in K_I, K_{II}, and G between the single and double crack cases do not seem to be appreciable. Nonetheless, a significant discrepancy may be expected if the a/R ratio becomes larger, which will be discussed in the next section. One extreme situation is the case of $\theta = 90^o$ in which the crack estension is shown to occur along the direction normal to the loading, i.e. $\phi = 0^o$, as expected. This gives a purely opening mode fracture, i.e. $K_{II} = 0.00$, due to the symmetry of loading and geometric conditions.

4.4 Crack Extension in Off-Axis Unidirectional Composites

Problems concerning crack extension from the edge of a hole in the composites are also examined in detail in this study. The case of an off-axis unidirectional glass/epoxy with fibers oriented along $\theta = 45^o$ is presented here for an illustrative purpose. The geometry and loading conditions are taken the same as previous examples (Fig. 3). Both single and double crack cases are considered again. Solutions for the problem have been obtained by an incremental scheme using the method of analysis described in section 2. Results given in Figs. 7 and 8 illustrate the fundamental behavior of crack extension from a hole in the off-axis composites. Effects of the crack length on stress intensity solutions and energy release rates are shown clearly in the figures. Values of $K_I/(\sqrt{\pi}\ \sigma_\infty)$ remain relatively constant for a wide range of the a/R ratio studied and change significantly for both smaller and larger cracks in the single crack case. For the double crack case, $K_I/(\sqrt{\pi}\ \sigma_\infty)$ increases monotonically during crack extension. It is interesting to note that, for both single and double crack cases, $K_{II}/(\sqrt{\pi}\ \sigma_\infty)$ varies approximately linearly with the crack length. Slight deviations from linearity in $K_{II}/(\sqrt{\pi}\ \sigma_\infty)$ are observed as the crack (or cracks) extends beyond $a/R = 1$. In general, the results indicate that all K_I, K_{II}, and G have larger values in a double crack case than those in the corresponding single crack problem, presumably due to the larger "effective" crack length. Moreover, the larger the crack length, the greater the difference between the two cases. Composites with other fiber orientations may exhibit different values of K's and G's during crack extension, but fundamental features of the crack-tip deformation and fracture behavior are essentially the same.

Extension of this method to more complicated fracture problems of composites, such as viscoelastic anisotropic crack analysis, can be easily achieved. These further developments have been very successful and are reported elsewhere [24].

5. SUMMARY AND CONCLUSION

An analytical investigation of cracks emanating from a circular hole in an off-axis unidirectional fiber-reinforced composite has been conducted. The method of analysis employed in the study is formulated by using conservation laws of elasticity and fundamental relationships in anisotropic fracture mechanics. The current approach provides a very convenient and accurate means to examine the complicated crack behavior, when used in conjunction with a suitable numerical scheme such as the finite element method. The problem is eventually reduced to a system of linear algebraic equations in mixed-mode stress intensity factors. Fracture parameters, describing crack-tip deformation and fracture in the composite, are obtained explicitly. Effects of material anisotropy and crack/hole geometry are examined also. Of particular interest are the energy release rates associated with crack extension; their values are evlauated for various cases.

The results obtained in this study lead to the following conclusions:

(1) The current method of analysis is demonstrated to be superior to other approaches in its operational simplicity and solution accuracy for investigating mixed-mode crack problems with extremely complex crack hole geometry and material anisotropy.

(2) Mixed-mode stress intensity factors and energy release rates associated with the cracks emanating from a hole change very appreciably with fiber orientation in the composite. K_I and G increase monotonically with increasing θ; but K_{II} reaches its maximum at $\theta = 45^o$, and then decreases gradually as θ increases further.

(3) During crack extension from a circular hole in the composite with a given fiber orientation, the opening mode stress intensity factors are found to be relatively constant for a certain range of the crack length (i.e., $a/R < 1.0$) in the single crack case, but increase rather rapidly in the double crack case. K_{II} increases linearly with the crack length, but deviates gradually from linearity as the length of the crack exceeds the hole radius, i.e $a/R > 1$, for both cases.

(4) The energy release rates and mixed-mode stress intensity solutions for single and double crack cases do not have much difference for problems with small crack-length-to-hole-radius ratio i.e. $a/R < 0.5$. However, the difference increases and becomes very appreciable as a/R increases during crack extension.

6. ACKNOWLEDGMENT

The authors wish to express their deep gratitude to Professor H. T. Corten, Department of Theoretical and Applied Mechanics, University of Illinois, for his fruitful discussion and encouragement during the course of this study.

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TABLE 1

STRESS INTENSITY FACTORS FOR A CRACK EMANATING FROM A CIRCULAR HOLE IN AN ISOTROPIC FINITE-DIMENSIONAL PANEL*

	K_I	K_{II}
Stalk and Orringer [20]	2.302	0.000
Present Solution	2.333	0.000

*$a/R = 0.4$, $\phi = 0°$, 56 eight-node isoparametric elements

LIST OF FIGURE CAPTIONS

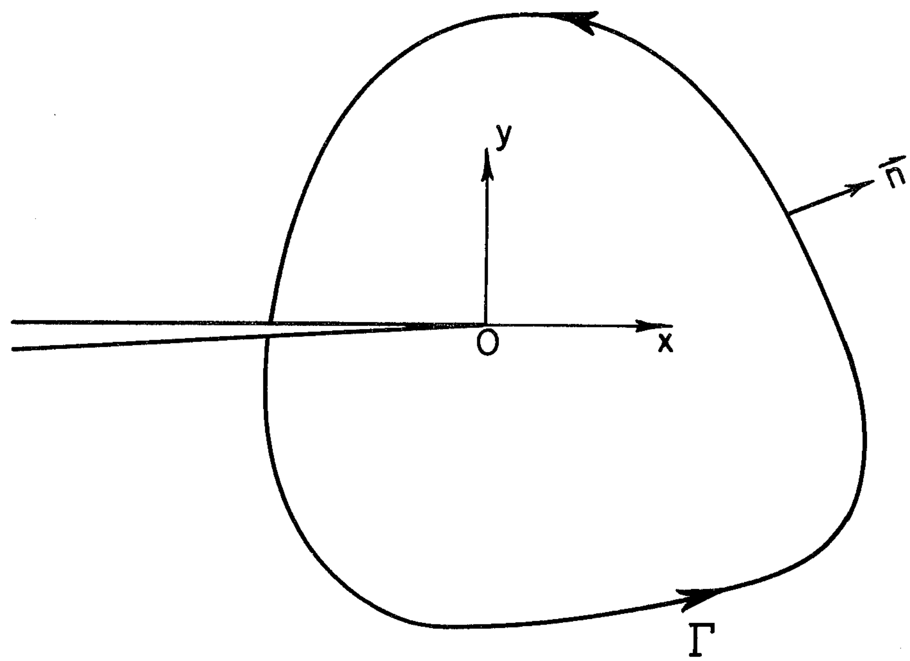

FIG. 1 COORDINATES OF THE CRACK AND PATH FOR J INTEGRAL.

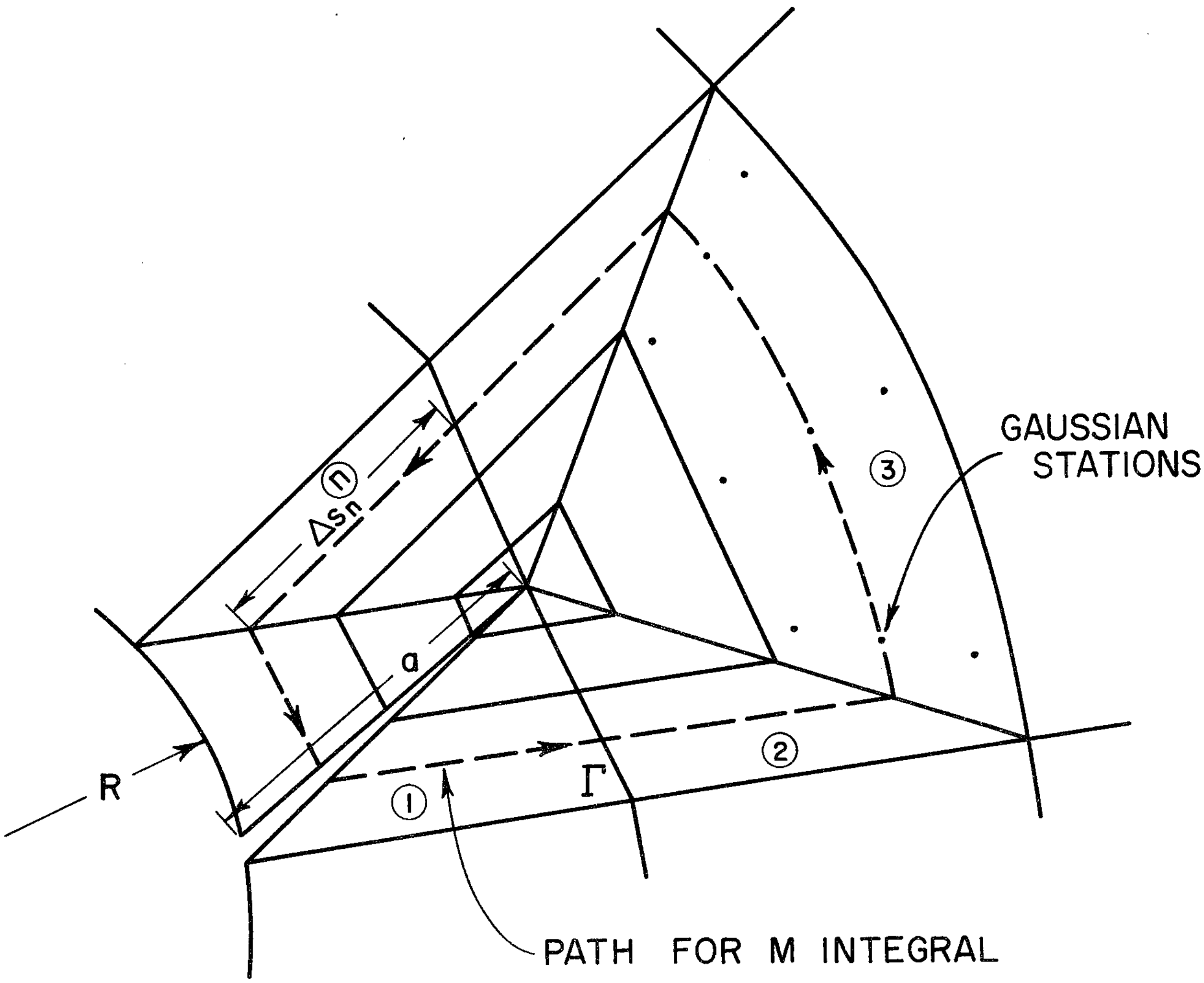

FIG. 2 GAUSSIAN STATIONS, FINITE ELEMENT MESH AND PATH FOR M INTEGRAL AROUND CRACK TIP.

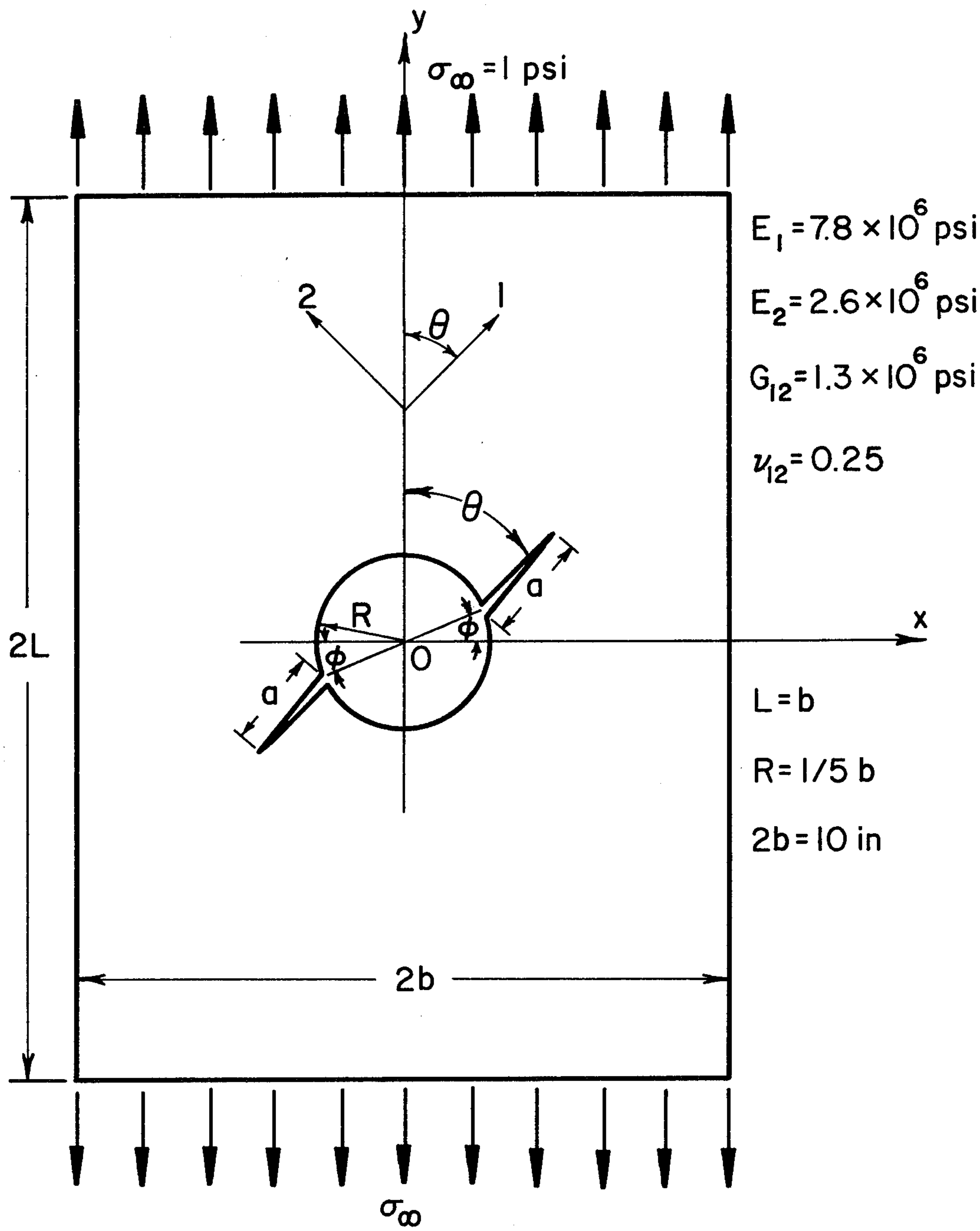

FIG. 3 CRACKS EMANATING FROM A CIRCULAR HOLE IN OFF-AXIS UNIDIRECTIONAL FIBER COMPOSITE WITH MATERIAL AXES ORIENTED AT 1-2 DIRECTIONS.

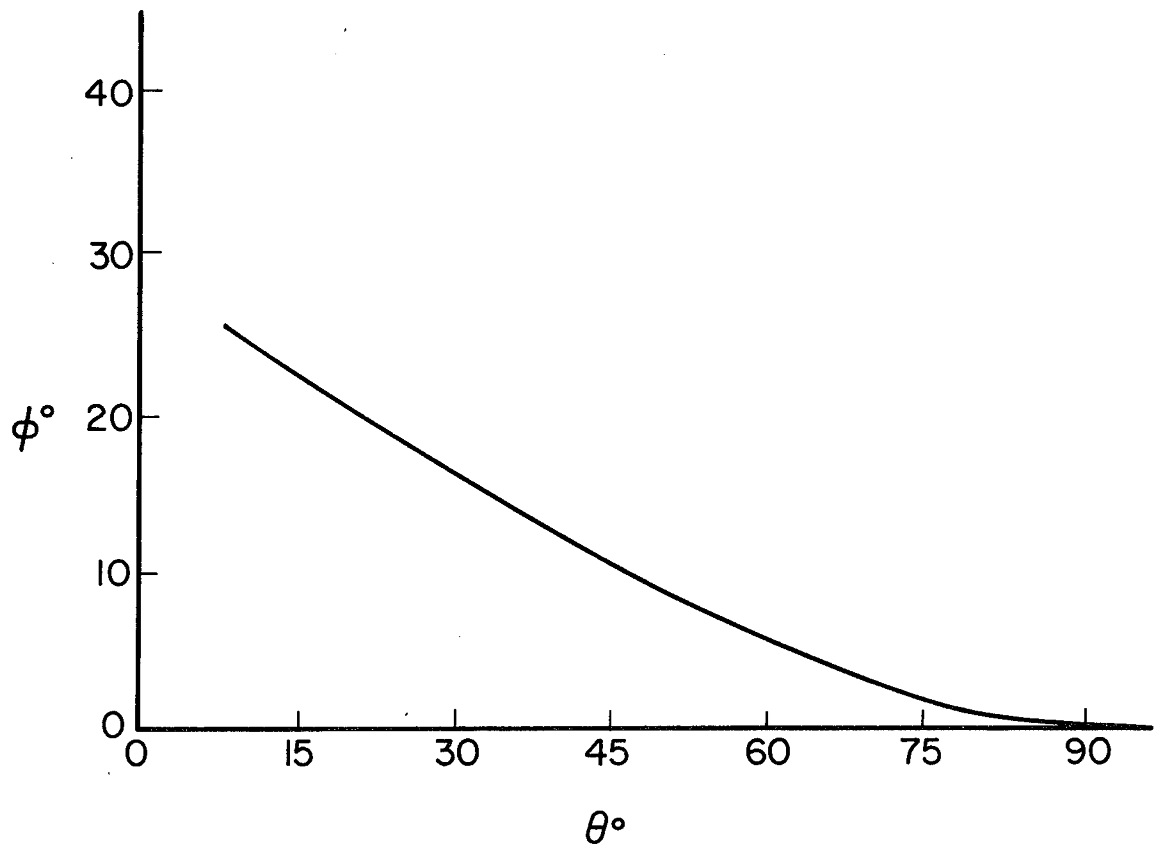

FIG. 4 LOCATION OF CRACK EMANATION FROM A CIRCULAR HOLE IN GLASS/EPOXY COMPOSITES WITH VARIOUS FIBER ORIENTATIONS, a/R = 0.4.

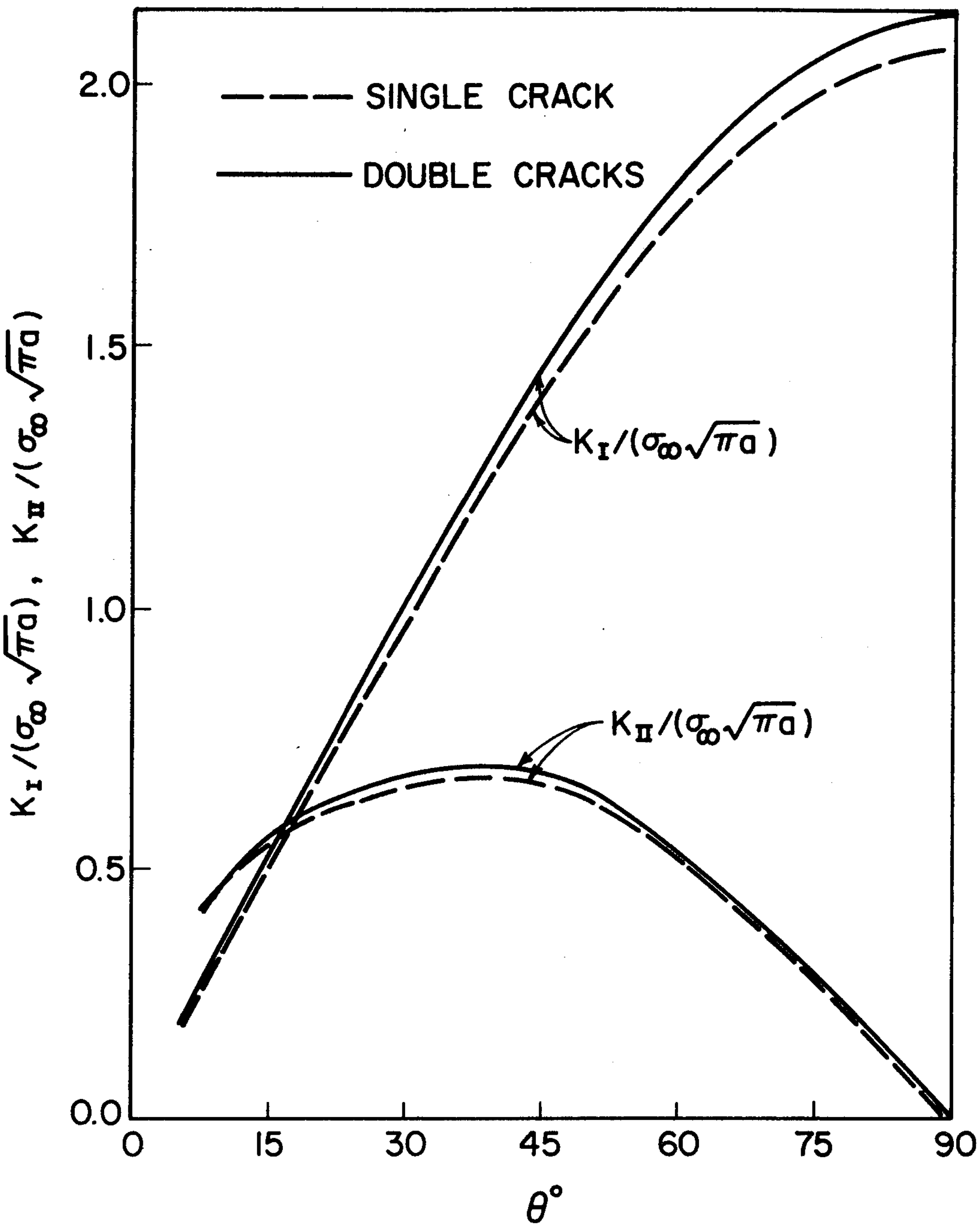

FIG. 5 MIXED-MODE STRESS-INTENSITY FACTORS OF CRACK(S) EMANATING FROM A CIRCULAR HOLE IN UNIDIRECTIONAL GLASS/EPOXY COMPOSITES WITH VARIOUS FIBER ORIENTATIONS, a/R = 0.4.

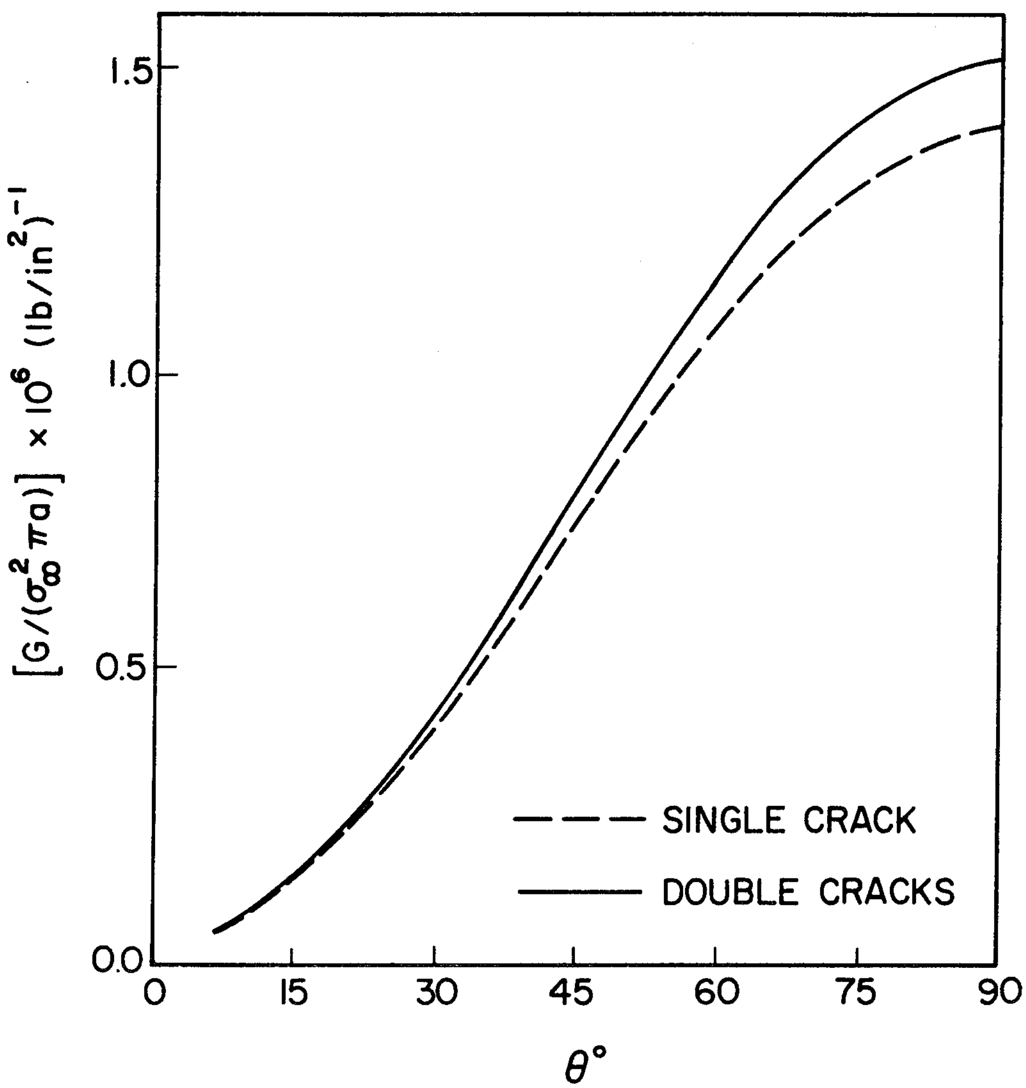

FIG. 6 ENERGY RELEASE RATES OF CRACK(S) EMANATING FROM A CIRCULAR HOLE IN UNIDIRECTIONAL GLASS/EPOXY COMPOSITES WITH VARIOUS FIBER ORIENTATION, a/R = 0.4.

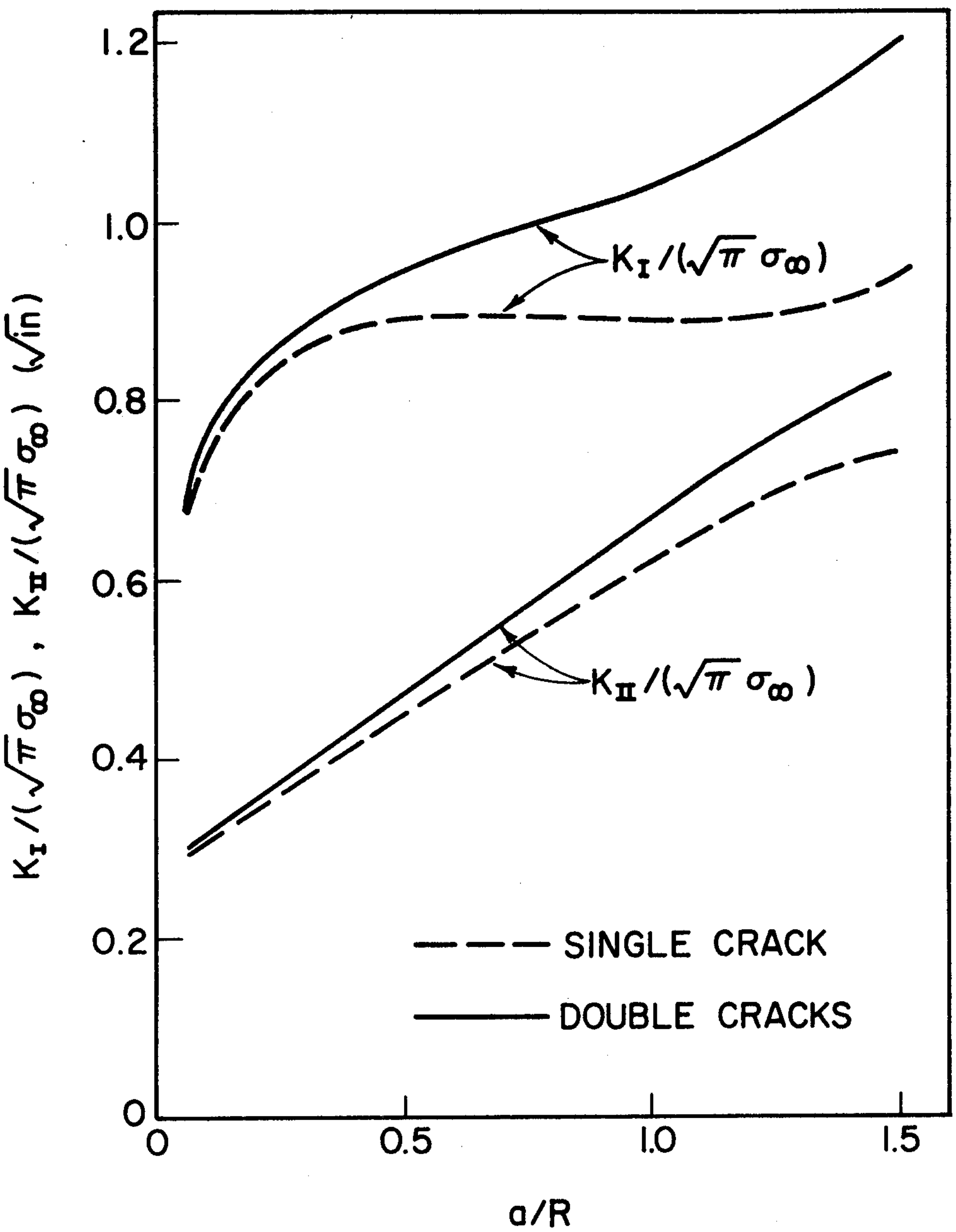

FIG. 7 MIXED-MODE STRESS INTENSITY FACTORS VS. CRACK LENGTH IN UNIDIRECTIONAL GLASS/EPOXY COMPOSITE WITH θ = 45°.

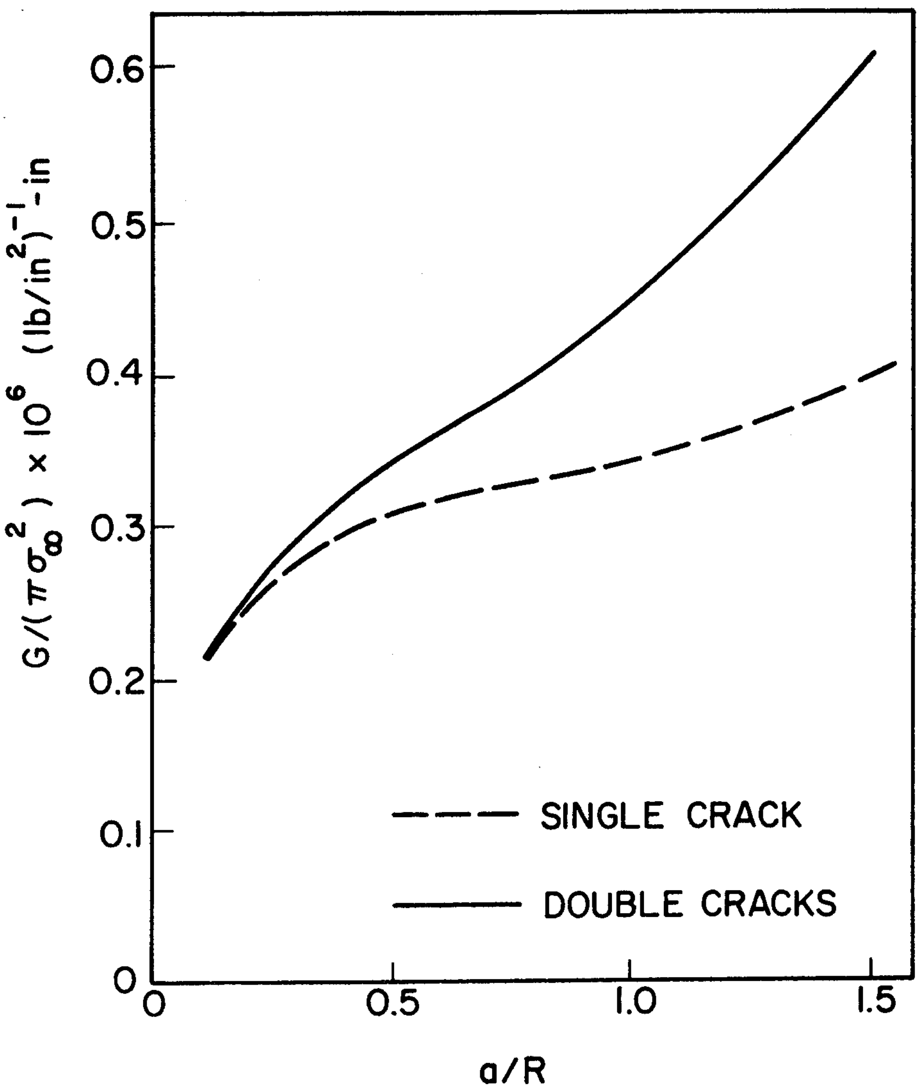

FIG. 8 ENERGY RELEASE RATES VS. CRACK LENGTH IN UNIDIRECTIONAL GLASS/EPOXY COMPOSITE WITH $\theta = 45°$.

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