# FINAL REPORT

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Title:

**Mismatch, signal-gain degradation and stabilized source localization in matched field acoustic array signal processing**

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Abstract of the effort:

The use of high resolution methods of processing acoustic array data for source localization in shallow-water waveguide environments has been the focus of much research in recent years. It has been noted that, whereas these methods rely on sophisticated models of the acoustic environment, they are subject to degraded performance in the presence of mismatch, such as might occur when environmental parameters are incorrectly estimated. Estimators that have been analyzed in this way include Capon's "maximum likelihood method" (MLM), also called the "minimum variance distortionless response", and the "sector-focusing" method (SFM) of Byrne.

Because of the nonlinear nature of the estimators of interest it is difficult to quantify the sensitivity to mismatch using usual measures of signal-gain degradation. While no single measure of mismatch will be applicable to all estimators under all circumstances, associated with each estimator and noise environment there is a single quantity that decreases as mismatch increases and upon which the success of the source localization depends. Measuring the dependence of this quantity on changes in the various environmental parameters involved will help to isolate those parameters for which accurate estimation is most important.

Source detection and localization in complex acoustic environments has been studied by Navy lab scientists and others in the community for more than a decade now. There seems to be concensus that so-called "matched field" processing can be used effectively, provided the stability problem can be controlled. Several methods have been offered to achieve this goal and the next step is to quantify both their performance and the mismatch to which they are subjected. We have isolated quantitative measures of mismatch with which to gauge the effects of perturbation of the various parameters on the stability of the related matched field techniques. This will tell us which parameters must be estimated most accurately. To help estimate these parameters we propose using high resolution methods on data from known sources.

## Background:

The degraded performance of high resolution methods [7,16,19] for array processing in the presence of modeling errors or other forms of perturbation has been an issue since shortly after these methods were introduced in the late 1960's [8,9,20]. It was noticed, in particular, that these methods became more sensitive to perturbations if the background noise was nonwhite; the cause of this was later discovered in the varying behavior of certain eigenfunctions associated with the cross-spectral matrix and stable adaptive methods were developed to improve the performance [3,4]. High resolution methods and stabilized versions thereof were initially designed to be used with planewave propagation models, but by the middle of the 1980's were being applied to more complex acoustical environments using matched field processing (MFP) [1,2,13,14,15]. Since more prior information about environmental parameters is required in MFP there is an increased risk of model error, and therefore of degraded performance [10,11,17,18,21]. In addition, the ambient noise propagating through the waveguide appears nonwhite to the array, leading to greater instability. We are led to ask the following questions: How do we design estimators to reduce sensitivity to perturbations while retaining increased resolution? and Do we need to know all the parameters of the model equally well or are some more important than others?

In answer to the first question, there have been several methods suggested for stabilizing high resolution estimation in the presence of perturbations, including dimensionality reduction and sector-focused stability [5,6,12]. What is needed are quantitative measures of the improvements achieved by these methods and additional improvements based on these quantitative measures. In this proposed research we shall consider various measures of array-gain-degradation in attempt to quantify the effect on each estimator of mismatch caused by particular forms of perturbation. Having quantified the effect we can study the dependence of this quantity on changes in each of the parameters in turn.

High resolution methods for localizing unknown sources in essentially known environments can also be used to estimate the environmental parameters, given a known source; some work along these lines has already been done [22]. We shall apply the estimators we develop to solve such inverse problems.

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MISMATCH AND SIGNAL GAIN DEGRADATION (SGD)

by Charles Byrne, Dept. of Mathematics, UMass-Lowell

## 0. Introduction

When we plot an ambiquity surface we use a certain finite collection of replica vectors. These replica vectors are constructed from a model of the environment (range-independent normal mode model, e.g.), with specific choices of the parameters the model requires (e.g. sound speed profile, channel depth, sensor locations, etc.). The actual plotting routine may use only a selected subset of the replicas of interest, the grid chosen depending on the amount of calculation one wishes to do and on the desired resolution. The hope is that the model is sufficiently accurate so that the actual source vector is (nearly) contained within the set of replica vectors used; mismatch refers to the variety of ways in which this can fail to be the case.

The purpose of this report is to discuss the problem of mismatch and its effects on estimators. As we shall see, there is no single measure of mismatch that is the relevant quantity to consider in all cases and with all estimators. Also, the signal-gain-degradation (SGD), measured here as the ratio of peak-to-background values for the mismatched vs the no-mismatch cases, is a somewhat complicated expression involving both a measure of the mismatch itself and the output signal-to-noise ratio. The greater this output signal-to-noise ratio the greater the degradation.

We begin by describing the kinds of mismatch that are possible and selecting the one that interests us here. We then go on to consider the normalized Bartlett, or conventional estimator (NCONV), the normalized maximum likelihood estimator (NML) and the normalized sector-focused estimator (NSF), for both white and correlated noise fields.

## I. Types of mismatch

(1) coarse mesh: the subset of replica vectors actually used in the plotting can correspond to a grid that is too coarse to approximate well the actual source vector. This is a problem only in a high SNR situation, and mainly in simulations.

(2) modeling errors: the model chosen may not be appropriate, even as an approximation of the true environment. For example, a planewave model will not do in a shallow-water situation in which bottom interactions are important; a range-independent normal mode model will not adequately describe a highly range-dependent situation. This kind of mismatch is usually avoidable (e.g. by not using planewave models in shallow water); all models will involve some degree of mismatch, however. This is not the sort of mismatch we are concerned with in this report.

(3) parameter errors: Even when the model chosen is appropriate it is necessary to determine the values of the parameters the model requires. Errors in choosing these parameters can lead to erroneous estimator output; this is the sort of mismatch we focus on in these notes. From now on we use the term "mismatch" to mean "parameter errors". Our chief concern will be the growing inability of estimators to extract signal from noise, as the degree of mismatch increases; the generic term for this is "signal gain degradation" (SGD).

## II. Quantifying mismatch

When the parameters chosen are incorrect then one of two things can happen: either the true signal vector is now associated with a replica vector with incorrect values of the ambiquity surface coordinates (r=range, z=depth, e.g.); or the true signal no longer corresponds to any of the replica vectors. It is the second situation that concerns us here; the degree to which the true signal vector fails to correspond to any of the replicas is a measure of the mismatch present.

Let the set of replica vectors be $R$ ={p=p(r,z) | (r,z) in the grid used to plot the ambiquity surface}, and let q be the signal vector. For each p in $R$ the dot product $p \cdot q = p^{+}q$ can be used to determine the angle between p and q: we have $|p \cdot q| = ||p||\ ||q|| \cos(t)$, where t is the angle between p and q and $||\ ||$ denotes the usual Euclidean length of a vector. Written differently, we have

$$\cos^2(t) = p^+qq^+p/(p^+p)(q^+q) \quad . \qquad (1)$$

If we maximize the quantity on the right side over all p in *R*, then we have a measure of how close q comes to being a member of *R* : let

$$m = \max \ p^+qq^+p/(p^+p)(q^+q) \ \text{, over p in } R \ . \qquad (2)$$

Then m is a measure of mismatch.

To calculate m in the manner above we need to know q. In practice we would not know q, but might have a loud source of opportunity, from which we could calculate a cross spectral matrix R. If the acoustic field consists of the single source in white noise then the matrix R will be approximately

$$R = \alpha^2\, qq^+ + \sigma^2\, I\ , \qquad (3)$$

where I is the identity matrix. Then we can calculate m using the eigenvalues of R and the conventional (Bartlett) estimator: the largest eigenvalue of R is $\lambda_1 = \alpha^2\, q^+q + \sigma^2$ and the smallest is $\lambda_N = \sigma^2$ . Then the conventional estimator gives

$$p^+Rp = \alpha^2\, p^+qq^+p + \sigma^2\, p^+p\ , \qquad (4)$$

so that

$$\cos^2(t) = [p^+Rp/p^+p - \lambda_N]/(\lambda_1 - \lambda_N) \quad . \qquad (5)$$

Then m is calculated by taking the maximum of this quantity, as p ranges over the members of $R$ .

For the conventional estimator (CONV) generally, and for Capon's maximum likelihood estimator (ML) in white noise, we can relate the behavior of the estimator directly to changes in the value of m. For ML in correlated noise and for other estimators, such as the sector-focusing (SM) method, the quantity m is not the relevant measure of mismatch; in both cases signal and replica vectors undergo a transformation before they are matched and the quantity of mismatch after transformation is what affects the behavior of the estimator. We begin by looking at the effects on CONV and ML of changes in m.

## III. Signal gain degradation for NCONV and NML

We define the normalized conventional estimator to be

$$NCONV(r,z)= p(r,z)^{+}Rp(r,z)/\ p(r,z)^{+}p(r,z)\quad , \tag{6}$$

and the normalized maximum likelihood estimator to be

$$NML(r,z)=\ p(r,z)^{+}p(r,z)/\ p(r,z)^{+}R^{-1}p(r,z)\quad . \tag{7}$$

For each estimator we evaluate its behavior in the presence of mismatch by calculating the percentage drop in the quantity "peak height minus background". For both cases we assume that $R = \alpha^2 qq^+ + \sigma^2 Q$, where Q denotes the noise-only cross spectral matrix, normalized to have trace equal to the number of phones. For NCONV and m=1 (no mismatch) we have a peak height of $\alpha^2 q^+q + \sigma^2 q^+Qq/q^+q$ and a background of $\sigma^2 q^+Qq/q^+q$. For general m we have a peak height of $\alpha^2 q^+q + \sigma^2 q^+Qq/q^+q$ and a background of $\sigma^2 q^+Qq/q^+q$. Therefore, the quantity "peak height minus background" goes from $\alpha^2 q^+q$ to $m \alpha^2 q^+q$ ; therefore the ratio is m. The NCONV estimator suffers SGD in direct proportion to m.

For the NML case we use the Woodbury identity to simplify the calculations:

$$R^{-1} = \sigma^{-2} Q^{-1} - ( \alpha^2 \sigma^{-4}/1+ \sigma^{-2} \alpha^2 q^+Q^{-1}q)Q^{-1}qq^+Q^{-1} . \qquad (8)$$

Then we see that when Q=I (white noise) and m=1 (no mismatch) the peak height is $\alpha^2 q^+q + \sigma^2$ , while for general m the peak height of NML is $(\alpha^2 q^+q+\sigma^2)/ (\alpha^2\sigma^{-2} q^+q(1-m) +1)$. The background in both cases is approximately $\sigma^2$ so that the ratio of the quantities "peak height minus background" in the two cases becomes

$$\text{ratio}= m/(1+(1-m)SNR) , \qquad (9)$$

where the output signal-to-noise ratio is $SNR= \alpha^2q^+q/\sigma^2$. From this we see that, when the SNR is low, NML and NCONV suffer roughly the same degradation as m decreases while, for higher SNR, NML degrades more quickly as m drops below 1. Defining the function $f(m)= m/(1+(1-m)SNR)$, we find that the derivative of f at m=1 is $f'(1)= 1 + SNR$, meaning that if we use f(m) as our measure of SGD, then NML is more sensitive to a drop in m when SNR is high than when it is low. When the noise is correlated, that is $Q \neq I$, then, after some algebra, we find that the ratio of "peak minus background" for the mismatched case versus the no-mismatch case becomes

$$\text{ratio}= m' (p^{+}p/p^{+}Q^{-1}p)(q^{+}Q^{-1}q/q^{+}q)[1/1+ (\alpha^{2}q^{+}Q^{-1}q/\sigma^{2})(1-m')], \qquad (10)$$

where $m'=|p^{+}Q^{-1}q|^{2}/(p^{+}Q^{-1}p)(q^{+}Q^{-1}q)$ is now the relevant measure of mismatch. Note that the quantity $(\alpha^{2}q^{+}Q^{-1}q/\sigma^{2})$ is the output signal-to-noise ratio from the NML estimator now.

It has been noted elsewhere that the ML estimator is particularly sensitive to mismatch in the presence of certain types of correlated noise, such as spherical isotropic noise for an oversampled line array and planewave processing, or for so-called modal noise in MFP. The difficulty can be traced to the three appearances of the term $p^{+}Q^{-1}p$ in (10). In both noise situations just described the noise field appears to the array as a collection of sources, all similar to the potential signals being sought. The Q matrix has several dominant eigenvalues and some very small ones; the large eigenvectors nearly span the space of sources and q has essentially no projection onto the eigenvectors associated with the very small eigenvalues; therefore $q^{+}Q^{-1}q$ is not large. But, when there is mismatch, replica vectors do not look so much like true source vectors and they begin to have sizable projections onto the small eigenvectors of Q. The result is that $p^{+}Q^{-1}p$ becomes large. This term appears in the denominator of m', making m' smaller, and in the denominator of (10) itself. The overall effect is to make the ratio smaller, hence increasing the SGD.

The message from this is that SGD depends not only on the estimator, but on the context; f(m) as a measure of SGD involves not only m but SNR as well. When the noise is not white m need not be a good measure of the mismatch; we need to consider m' in the case of NML and $Q \neq I$. Comparisons of various estimators as to SGD must take into account this dependence on context and be made with some reasonable level of white noise background; if the SGD of NML is compared to that of other estimators at very high SNR it looks terrible.

Other estimators also cannot be analyzed directly in terms of the quantity m; the SGD of estimators such as the sector focusing (SF) method does not depend directly on m. We turn to the SF estimator now.

## IV. SIGNAL GAIN DEGRADATION FOR THE SF ESTIMATOR

The SF estimator is the following: first a sector S in (r,d)-space is selected, an N by K vector V (with $V^+V=I$) is constructed to describe the sector S, and then, with (r,z) fixed within the sector, the normalized SF estimator is

$$NSF(r,z) = p(r,z)^+VV^+p(r,z) / \; p(r,z)^+V(V^+RV)^{-1}V^+p(r,z) \quad . \qquad (11)$$

Using the Woodbury identity again, and with $G=V^+QV$, $g=V^+q$, $h= h(r,z) = V^+p(r,z)$ and $R=\alpha^2qq^+ + \sigma^2Q$, we have

$$1/NSF = [\sigma^{-2}h^+G^{-1}h \; - (\alpha^2\sigma^{-4}/(1+ \alpha^2\sigma^{-2}g^+G^{-1}g))|h^+G^{-1}g|^2 \; ]/h^+h \; . \qquad (12)$$

The peak height of NSF, with no mismatch (g=h), is $\alpha^2g^+g + \sigma^2(g^+g/g^+G^{-1}g)$; the background is roughly $\sigma^2 (g^+g/g^+G^{-1}g)$, so the "peak minus background" is $\alpha^2g^+g$. When there is mismatch the background is still approximately $\sigma^2(g^+g/g^+G^{-1}g)$; the peak height will occur at some h and "peak minus background" will have the value

$$(h^+h/h^+G^{-1}h)(\alpha^2g^+G^{-1}g \; m^*)/(1+\alpha^2\sigma^{-2}g^+G^{-1}g \; (1-m^*)) \; , \qquad (13)$$

where $m^* = |h^+G^{-1}g|^2/(h^+G^{-1}h)(g^+G^{-1}g)$ is the relevant measure of mismatch. Again, note that the quantity $\alpha^2\sigma^{-2}g^+G^{-1}g$ is the output signal-to-noise ratio from the NSF estimator. The ratio measuring SGD then becomes

$$\text{ratio}= m^*(h^+h/h^+G^{-1}h)(g^+G^{-1}g/g^+g)/(1+\alpha^2\sigma^{-2}g^+G^{-1}g\ (1-m^*))\ . \qquad (14)$$

The NSF method is more robust than NML, particularly in the presence of correlated noises of the sort described earlier, because the quantity $h^+G^{-1}h$ that appears in (14) and in $m^*$ is smaller than $p^+Q^{-1}p$. View $1/p^+Q^{-1}p$ and $1/h^+G^{-1}h$ as ML estimators of the noise-only fields corresponding to Q and G respectively. When Q is a signal-like noise (as discussed earlier), but replica p involves erroneous choices of parameters, then in "direction" p there will not be much noise, hence $1/p^+Q^{-1}p$ will be small and $p^+Q^{-1}p$ large. Since $G= V^+QV$ and $h=V^+p$ both involve projection into the column space of $V^+$, even though p involves erroneous choices of parameters, h cannot be that different from the collection of signal-like vectors that make up the G noise field. Therefore $1/h^+G^{-1}h$ is not as small as $1/p^+Q^{-1}p$ and $h^+G^{-1}h$ not as large as $p^+Q^{-1}p$.

## V. CONCLUSIONS

We have measured SGD in terms of the ratio of peak-to-background values for the mismatched and unmismatched cases. We found that for NCONV and for NML in white noise the SGD is a function of the quantity m, which measures the maximum cosine squared of the angles between the actual signal and the replica family. For these cases, then, we can examine the effects of errors in the choice of various parameter values solely in terms of what happens to m. For NML in correlated noise and for the NSF estimator generally, m is not the relevant measure of mismatch.

For NML in correlated noise the SGD depends on the quantity m'. Effects of errors in the choice of parameters can be assessed in terms of what happens to m'. For the NSF estimator the relevant measure of mismatch is $m^*$. Now the importance of accuracy in estimating the various parameters can be determined by measuring their effects on $m^*$.

It would be nice if mismatch could be quantified using a single measure (m, say) and if the SGD of each estimator always depended on m in some clear way. Although this is not the case, it is still convenient that in the various cases considered here there is a single quantity, which does depend on the estimator and the context, however, that can be used to measure the effects of parameter error and to help us determine which parameters need to be determined with the greatest accuracy.

We have seen that the effect of mismatch is more complicated for nonlinear estimators than for the linear NCONV estimator. For both of the nonlinear methods studied we saw that both the mismatch measure (m' or $m^*$) and the output signal-to-noise ratio contributed to SGD, with greater degradation (relatively speaking) when the output signal-to-noise ratio is higher. Clearly, then, when estimator SGD is being compared it is important to study the estimators in typical contexts, not in artificially high signal-to-noise situations.

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The use of high resolution methods of processing acoustic array data for source localization in shallow-water waveguide environments has been the focus of much research in recent years. It has been noted that, whereas these methods rely on sophisticated models of the acoustic environment, they are subject to degraded performance in the presence of mismatch, such as might occur when environmental parameters are incorrectly estimated. Estimators that have been analyzed in this way include Capon's "maximum likelihood method" (MLM), also called the "minimum variance distortionless response," and the "sector-focusing" method (SFM) of Byrne.

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