

# REPORT DOCUMENTATION PAGE

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13. ABSTRACT (Maximum 200 words)  Mark Johnson's research during the past two years has addressed low-dimensional behavior in dynamical systems and dissipative partial differential equations - specifically 1) Computation of 2-D Invariant Manifolds and 2) Low-Dimensional Dynamics in PDEs. His emphasis has been on scientific computing and the successful approximation of global dynamical phenomena. He has obtained preliminary results in several directions, including numerical and interactive representations of invariant manifolds and global bifurcation result on discretization of PDEs. This work provides Johnson with several alternatives to be pursued toward his thesis.				
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TECHNICAL REPORT  
THIRD AND FINAL YEAR

*David W. McLaughlin*

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Mark Johnson has been supported on the AASERT Grant for the past two years, which constitute his first two years as a graduate student. During this time, he has made significant progress on beginning research projects related to chaotic behavior in PDEs, such as the nonlinear Schroedinger equation of nonlinear optics. These studies will extend to research for his Ph.D. dissertation. We expect he will be able to complete requirements for the Ph.D. in the next two or three years. The projects he has worked on are described below.

Computation of 2-D Invariant Manifolds

Mr. Johnson has combined scientific computation and interactive graphics to numerically approximate two-dimensional invariant manifolds of fixed and periodic saddle-type solutions in  $n$ -dimensional space as well as their dependence on parameters. While one-dimensional invariant manifolds have been successfully studied by many numerical researchers in the past, the evolution, parametrization and post-processing of two-dimensional surfaces is much more complicated, and no definitive algorithms exist. Mr. Johnson has implemented the capabilities of initializing, parametrizing, triangulating and evolving several such two-dimensional surfaces within a general purpose scientific computing environment. He has used this to study global bifurcations in low-dimensional dynamical systems (from the Lorenz equations and the Minea "model" of the Navier-Stokes to low-dimensional truncations of the driven damped nonlinear Schroedinger equations). Current research issues regard the parametrization of the surfaces in the presence of intense rotation, as well as post-processing to determine intersections of such surfaces. The possibility of using texture and light to study the interaction of such 2-D surfaces in 4-D space is also a subject of current study. He has made several presentations of aspects of this work (at SIAM annual meetings as well as at the 1994 Dynamics Days) and won the award for best video-poster presentation at the 1994 SIAM annual Meeting. This work is expected to also give rise to usable software that will be made available to other researchers, and we are currently actively discussing the subject with researchers at the NSF Geometry Center at the University of Minnesota.

Low-Dimensional Dynamics in PDEs

The second aspect of Mr. Johnson's research is exploiting scientific computing to study the dynamics of low-dimensional approximations and truncations of PDEs, like the nonlinear Schroedinger equation. For the latter equation, Mr. Johnson works with Mr. Mark Winograd in studying global bifurcations and homoclinic orbit families (pulses for the PDEs). Mr. Johnson has also made considerable progress in the study of global bifurcations in approximate inertial forms of the Kuramoto-Sivashinsky amplitude equation, involving the interactions of two-dimensional stable and unstable manifolds of more than one fixed points, as well as of invariant manifolds of saddle-type steady states with those of limit cycle saddle-type solutions. These are solutions of the full PDE and they could not be discovered and outlined without his scientific computing and visualization skills. Finally, we have started an effort towards studying low-dimensional dynamics and

pattern formation in PDEs on *non uniform* media, in which case the usual "traveling frame" transformation to ODEs does not apply and the full PDE has to be considered. A paper is in preparation from his study of global bifurcations in the Kuramoto-Shivashinsky equation that should be submitted for publication this semester. Overall, Mr. Johnson has shown both skill and initiative in his work, often taking new paths (like his initiative in studying the effects of knotting between limit cycles as a topological constraint on the global bifurcations that involve their invariant manifolds).