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13. ABSTRACT (Maximum 200 words) This project concerned numerical methods for solving linear systems of equations of the type arising from discretization of boundary value problems of elliptic and mixed type. The problems considered were of fundamental use in mathematical models used in structural analysis and fluid dynamics. The emphasis was on preconditioning techniques in which properties of the problem are used to construct approximations that are easy to compute with and that lead to rapid convergence of iterative methods. The approach included both analytic studies producing bounds on convergence rates, and computational experiments that confirm and supplement the analysis. <div style="text-align: right; font-weight: bold;">DTIC QUALITY INSPECTED 4</div>			
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Numerical Solution of Discrete Boundary Value Problems
Final Report
Howard C. Elman

December 7, 1995

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A. Problem Statement

This project concerned numerical methods for solving linear systems of equations of the type arising from discretization of boundary value problems of elliptic and mixed type. The problems considered were of fundamental use in mathematical models used in structural analysis and fluid dynamics. The emphasis was on preconditioning techniques in which properties of the problem are used to construct approximations that are easy to compute with and that lead to rapid convergence of iterative methods. The approach included both analytic studies producing bounds on convergence rates, and computational experiments that confirm and supplement the analysis.

B. Summary of Major Results

1. **Iterative methods for the discrete convection-diffusion equation.** An analysis has been conducted of the effect of ordering of unknowns on performance of relaxation methods for solving linear systems arising from discretization of the (steady-state) convection-diffusion equation. Main results consist of analytic bounds on norms of relaxation operators associated with both one-dimensional and two-dimensional problems. The results rigorously establish that relaxation that "follows the flow" associated with the problem displays fast convergence, but that there are latencies associated with relaxation based on red-black ordering and sweeping in a direction opposite to the direction of flow.

2. **Parallel algorithms for discrete elliptic problems.** A study of parallel implementation of the *hp*-version of the finite element method on a shared-memory computer shows that performance can be improved by grouping computations into blocks to avoid memory conflicts. This enhances performance on shared memory machines and, more importantly, is needed to make efficient use of hierarchical memory on arbitrary architectures.

3. **Solution algorithms for the Stokes and Navier-Stokes equations.** The most recent emphasis has been on numerical solution techniques for problems arising in models of incompressible fluid flow. The typical linear systems produced are nonsymmetric and indefinite, which present challenges for solution algorithms, but the structure of the problems can be used to develop effective algorithms.

- a. New preconditioning techniques for the linearized discrete Navier-Stokes equations have been developed that produce preconditioned linear systems with eigenvalues that are bounded independent of the discretization mesh size. Experimental results indicate that Krylov subspace iterative methods combined with these preconditioners display rates of convergence that are independent of the mesh parameter. In addition, they show that performance can be improved through the use of inner iterations for solving subproblems consisting of a set of discrete convection-diffusion equations.
- b. An analysis and empirical study of a variant of the Uzawa method for self-adjoint saddle point problems shows that the most costly part of the algorithm, solution of a symmetric positive-definite subsystem at every iteration, can be replaced by an approximate solution obtained by an inner iteration. The convergence behavior of the resulting "inexact Uzawa" algorithm is relatively insensitive to the accuracy of the approximate solution. Analysis

and experiments with the Stokes equations confirm and supplement these results.

- c. An empirical comparison of four solvers for the discrete Stokes equations (variants of the Uzawa, preconditioned conjugate gradient, preconditioned conjugate residual, and multigrid methods) indicates that multigrid with smoothing based on incomplete factorization is more efficient than the other methods, but typically by no more than a factor of two. The conjugate residual method has the advantages of being both independent of iteration parameters and widely applicable.

4. Other results for elliptic problems. A comparison of the performance of low order and high order finite elements for solving the discrete Poisson equation has been made, with results showing the efficiencies achieved by high order *hp*-elements.

C. List of Publications and Technical Reports

Refereed Journal Publications

1. H. C. Elman and M. P. Chernesky, "Ordering effects on relaxation methods applied to the discrete one-dimensional convection-diffusion equation," *SIAM Journal on Numerical Analysis* 30:1268-1290, 1993.
2. H. C. Elman and G. H. Golub, "Inexact and preconditioned Uzawa algorithms for saddle point problems," *SIAM Journal on Numerical Analysis* 30:1645-1661, 1994.
3. H. C. Elman and X. Zhang, "Algebraic analysis of the hierarchical basis preconditioner," *SIAM Journal on Matrix Analysis and Applications* 16:192-206, 1995.
4. H. C. Elman and D. K.-Y. Lee, "Use of linear algebra kernels to build an efficient finite element solver," *Parallel Computing* 21:161-173, 1995.
5. Howard Elman and David Silvester, "Fast Nonsymmetric Iterations and Preconditioning for Navier-Stokes Equations," UMIACS-TR-94-66, University of Maryland, 1994. To appear in *SIAM Journal on Scientific Computing*.
6. Howard C. Elman, "Multigrid and Krylov Subspace Methods for the Discrete Stokes Equations," UMIACS-TR-94-76, University of Maryland, 1994. To appear in *International Journal for Numerical Methods in Fluids*.

Reports Submitted for Publication

1. Aaron E. Naiman, Ivo M. Babuška and Howard C. Elman, "A Note on Conjugate Gradient Convergence," UMIACS-TR-95-86, University of Maryland, 1995. Submitted to *Numerische Mathematik*.
2. Howard C. Elman, Perturbation of Eigenvalues of Preconditioned Navier-Stokes Operators, UMIACS-TR-95-110, University of Maryland, 1995. Submitted to *SIAM Journal on Matrix Analysis and Applications*.

Conference Proceedings and Book Chapters

1. Howard C. Elman, "Iterative methods for linear systems," in J. Gilbert and D. Kershaw, Editors, *Large-Scale Matrix Problems and the Numerical Solution of Partial Differential Equations*, Oxford University Press, New York, 1994, pp. 69-118.

2. Howard C. Elman and Michael P. Chernesky, "Ordering effects on relaxation methods applied to the discrete convection-diffusion equation," In G. H. Golub, A. Greenbaum and M. Luskin, Editors, *Recent Advances in Iterative Methods*, Springer-Verlag, New York, 1994, pp. 45-57.

3. Howard C. Elman, "Multigrid and Krylov subspace methods for the discrete Stokes equations", in M. Natori and T. Nodera, *Advances in Numerical Methods for Large Sparse Sets of Linear Equations*, Proceedings of the PCG94 Symposium, Keio University, Yokohama, Japan, 1994, pp. 151-164.

Theses

"Multilevel Preconditioners: Analysis, Performance Enhancements and Parallel Algorithms," by Xian-Zhong Guo, Ph.D. Thesis, Applied Mathematics, 1992.

D. Scientific Personnel

Principal Investigator: Howard C. Elman

Graduate Research Assistants: Xian-Zhong Guo (Ph.D., Applied Mathematics, May 1992)
Santiago Arteaga (Ph.D., Applied Mathematics, expected May 1996)

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