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STATISTICAL CHARACTERIZATION OF THE FATIGUE  
BEHAVIOR OF COMPOSITE LAMINA

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STATISTICAL CHARACTERIZATION OF THE FATIGUE BEHAVIOR OF COMPOSITE  
LAMINA

Final Technical Report

NASA Langley Research Center

Grant NSG 1415

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## ABSTRACT

This report summarizes the research that was performed under NASA Grant 1415, Statistical Characterization of the Fatigue Behavior of Composite Lamina, during the life of the grant, May 1, 1977, to January 31, 1979. The technical monitors of this grant were Drs. G. L. Roderick and T. Kevin O'Brien of the NASA Langley Research Center, Hampton, Virginia 23665.

Essentially all of the work that was included in the proposals has been completed. A theoretical model was developed to predict statistically the effects of constant and variable amplitude fatigue loadings on the fatigue life and residual strength of composite lamina. The model parameters were evaluated by one set of static and one set of fatigue tests and a number of verification test series were performed. Good agreement was found between predicted and actual test results. The results of this research have been or will be presented at three conferences and published in conference proceedings or in appropriate journals.

## INTRODUCTION

Under the sponsorship of this grant, a theoretical model was developed to predict statistically the effects of constant and variable amplitude fatigue loadings on the residual strength and fatigue life of composite lamina. The parameters in the model were established from the results of a series of static tensile tests (15-25 tests) and a fatigue scan (30-35 tests) and a number of verification tests were performed. It was found that predictions of the fatigue behavior based on the tensile and fatigue tests agreed very well with the verification test series, including predictions of the effect of load sequence effects. A total of approximately 200 tests were performed on specimens made of 5208/T300 graphite/epoxy  $[\pm 45^\circ]_{2s}$  laminates. In addition some initial tests were performed on some graphite/epoxy  $[\pm 35^\circ]_{2s}$  laminates as a further verification of the model. These tests will be completed under a new grant which will extend the range of applications of this theory.

The results of this work have been well received as indicated by the conference presentations and publications produced under this grant. The results of the initial work of establishing and verifying the theoretical model were presented at the ASTM 5th Conference on Composite Materials; Testing and Design and published in the Journal of Composite Materials. A second paper on the effect of load sequence on the statistical fatigue of composites has been accepted for presentation at the AIAA/ASME/ASCE/AHS 20th Structures, Structural Dynamics, and Materials

Conference in St. Louis, April 4-6 and will be published in the conference proceedings. A third paper, also on load sequence effects, has been accepted for presentation at an ASTM symposium on Fatigue of Fibrous Composite Materials, May 22-23. Abstracts of these papers are included with this report and, since the papers discuss essentially all of the works performed under the grant, this report will present only a brief summary of the work done.

## PROGRESS

### (1) DEVELOPMENT OF THE THEORETICAL MODEL

A three-parameter fatigue and residual strength degradation model for unnotched composite laminates under cyclic loading has been derived in the form

$$R^c(n_1) = R^c(n_0) - \beta^c K S^b (n_1 - n_0). \quad (1)$$

In eq. (1)  $R(n_1)$  and  $R(n_0)$  are the residual strengths at  $n_1$  and  $n_0$  cycles ( $n_1 > n_0$ ) respectively,  $\beta$  is the scale parameter of the ultimate strength, and  $b, c, K$  are three parameters to be determined from the test data. The stress range  $S$  is defined as

$$S = \sigma_{\max} - \sigma_{\min} = (1-R)\sigma_{\max}, \quad (2)$$

where  $\sigma_{\max}$  and  $\sigma_{\min}$  represent the maximum and minimum cyclic stresses, respectively, and  $R$  is the stress ratio.

For  $n_1 = n$  and  $n_0 = 0$ , eq. (1) reduces to

$$R^c(n) = R^c(0) - \beta^c K S^b n \quad (3)$$

where the ultimate strength  $R(0)$  is a statistical variable assumed to follow a two-parameter Weibull distribution

$$F_{R(0)}(x) = P[R(0) \leq x] = 1 - \exp[-(x/\beta)^\alpha]. \quad (4)$$

In eq. (4)  $F_{R(0)}(x)$  represents the probability that the ultimate strength is smaller than  $x$  and  $\alpha$  is the shape parameter.

Constant Amplitude Fatigue Loadings. Under the special case of constant amplitude fatigue loadings, it was shown that the distribution function of the fatigue life could be written as

$$F_N(n) = \begin{cases} 0 & , n \leq 0 \\ 1 - \exp \left\{ - \left[ \frac{n + (\sigma_{\max}^c / \beta^c KS^b)}{1/KS^b} \right] \right\} & , n > 0 \end{cases} \quad (5)$$

The statistical distribution of the residual strength  $F_R(n)$  was also obtained as

$$F_R(n)(\sigma_{\max}) = P[R(n) \leq \sigma_{\max}] = 1 - \exp \left\{ - \left[ \frac{\sigma_{\max}^c + \beta^c KS^b n}{\beta^c} \right]^{\alpha/c} \right\} \quad (6)$$

which represents the probability that the specimen will not survive  $n$  cycles of  $\sigma_{\max}$ .

A Sequence of Two Fatigue Loadings. It was also shown that this theory could be applied to predict the effect of variable amplitude fatigue loadings, including dual amplitude and spectrum loadings, on the fatigue life and residual strength of composite lamina. The dual amplitude case was examined in detail for high-low and low-high load sequences. It was shown that the distribution function of the residual strength can be written as

$$F_{R(n_1+n_2)}(x) = 1 - \exp \left\{ - \left[ \left( \frac{x}{\beta} \right)^c + \frac{n_1}{N_1^*} + \frac{n_2}{N_2^*} \right]^{\alpha/c} \right\} ; \quad x \geq \sigma_{2\max} \quad (7)$$

where  $N_1^* = 1/KS_1^b$  and  $N_2^* = 1/KS_2^b$  are the characteristic lives and the subscripts 1 and 2 refer to the first and second series of load applications. The distribution function of the fatigue life under the second fatigue loading,  $N_{12}$ , was obtained as

$$F_{N_{12}}(n) = 1 - \exp \left\{ - \left[ \frac{n_1}{N_1^*} + \frac{n}{N_2^*} + \left( \frac{\sigma_{2\max}}{\beta} \right)^c \right]^{\alpha/c} \right\} \quad (8)$$

After these various formulas were established, five material

parameters,  $\alpha, \beta, b, c$  and  $K$  were evaluated from one static and one fatigue test series. Then the predictions made from these equations were verified with additional test series.

## (2) EXPERIMENTAL TEST PROGRAM

A test program using coupon specimens of 5208/T300 graphite/epoxy  $[+45^\circ]_{2S}$  laminates was initiated for the purpose of generating statistically significant test data to evaluate the validity of the theoretical model. The specimens were nominally 1.5 inches wide, 10 inches long and 0.04 inches thick. Fifteen static tests were performed at a rate that would approximate failure during the first load excursion of the fatigue test program (0.05 seconds to failure). These tests were used to evaluate  $\alpha$  and  $\beta$  in the two-parameter Weibull distribution for the ultimate strength.

A set of thirty-three fatigue tests were performed under various applied loads. The test frequency was 10 Hz and the stress ratio was 0.1. From these two sets of tests the constants  $b, c$ , and  $K$  were established. After those five constants were obtained, predictions of the fatigue and residual strength distributions were made and compared with additional test results. It was seen that the predictions agreed quite well with the test results.

The same values of the test parameters were used to predict the fatigue lives under low-high and high-low fatigue test series using eq. (8). These predictions were also found to agree

well with the test results. Altogether, approximately 200 specimens were tested in the test program described above. In addition some initial tests were performed on some graphite/epoxy  $[\pm 35^\circ]_{2s}$  laminates. The purpose of these additional tests will be to establish the test parameters for this laminate and examine further the load sequence effects.

In summary, a considerable amount of work was performed under this grant and the results were encouraging. The principal investigators would like to again express their appreciation to Drs. J. R. Davidson, G. L. Roderick and T. Kevin O'Brien for their support of and interest in this work.

ABSTRACTS

# STATISTICAL FATIGUE OF GRAPHITE/EPOXY ANGLE-PLY LAMINATES IN SHEAR

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## ABSTRACT

A three-parameter fatigue and residual strength degradation model has been proposed to predict statistically the fatigue behavior of composite laminae under axial shear loadings. The fatigue behavior includes the fatigue life and the fatigue damage expressed in terms of the residual strength degradation. An experimental test program using graphite/epoxy  $[+45^\circ]_{2S}$  laminates has been conducted to generate statistically meaningful data in order to examine the validity of the theoretical model. It is shown that the correlation between the theoretical predictions and the test results on the statistical distributions of the fatigue life and the residual strength is excellent. Test results on the shear modulus degradation are also presented and discussed in detail in order to provide insight for the establishment of a shear modulus degradation model.

THE EFFECT OF LOAD SEQUENCE ON THE STATISTICAL FATIGUE OF  
COMPOSITES

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ABSTRACT

A theoretical model to predict the effect of loading sequence on the statistical distributions of the fatigue life and the residual strength under  $n$ -stress levels of cyclic loading has been derived based on a three-parameter model. In particular, the dual stress fatigue cumulative damage is studied in detail and the Miner's sum (the cumulative damage sum at fatigue failure) is shown to be a statistical variable. It is proved theoretically that the Miner's sum is always greater than or equal to unity for the high-low load sequence, while it is always smaller than or equal to unity for the low-high load sequence, with the deviation from unity increasing as the difference between the high and the low stress levels increases. An experimental test program using graphite/epoxy  $[\pm 45^\circ]_{2s}$  angle-ply laminates has been carried out to generate statistically meaningful data for verifying the proposed model. It is shown that the correlation between the test results and the theoretical predictions of the fatigue life distribution for graphite/epoxy  $[\pm 45^\circ]_{2s}$  angle-ply laminates is very reasonable.

LOAD SEQUENCE EFFECTS ON THE FATIGUE BEHAVIOR OF COMPOSITE  
MATERIALS

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ABSTRACT

In recent years a considerable amount of research has been performed for the purpose of characterizing statistically the strength degradation and the failure of composite materials under cyclic loading conditions. However, with only a few exceptions these research efforts have been restricted to constant amplitude cyclic loading and are thus incapable of dealing with load sequence effects. The importance of load sequence effects is apparent since two or more loads will normally represent a better model of actual loads applied to engineering structures than can be obtained through constant amplitude fatigue testing. In addition, a proper treatment of load sequence effects will serve as a basis for the analysis of spectrum or random loadings, which are normally applied to engineering structures. Therefore, the purpose of this paper will be to examine more fully an initial treatment of load sequence effects recently developed by the authors for unnotched composite laminates.

The three-parameter fatigue and residual strength degradation model that has been proposed recently to predict the statistical distribution of the fatigue life under a multiple loading sequence will be examined in greater detail for two load levels and correlated with additional experimental results. The three parameters in the model are established by a statistical analysis of one set of static tensile tests and one fatigue scan.

The model then can be exercised to predict the fatigue and residual strength distribution under a variety of cyclic loading programs. Initial predictions of the effect of low-high and high-low load sequences on the fatigue lives of unnotched graphite/epoxy laminates have been in good agreement with the experimental results. However, since different materials cannot be expected to respond identically to load sequence effects, the existing theoretical model will be generalized and correlated with the test results from additional composite laminates.

This paper will present comparisons between the predicted and actual fatigue lives for several test series on graphite/epoxy  $[\pm 45]_{2S}$ ,  $[\pm 35]_{2S}$ , and fiberglass/epoxy  $[0,90]_{4S}$  laminates. All of the specimens are unnotched, with the graphite/epoxy specimens having nominal dimensions of 1.5x0.04x10 inches, and all fatigue tests are being performed at 10 Hz. The static tests are performed at a loading rate which would correspond to a failure during one-half of a fatigue cycle. Several additional test series have been planned, in addition to those already completed, so that these results will provide considerable additional information about the validity and accuracy of this model.