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A FINITE-DIFFERENCE PROGRAM FOR STRESSES IN ANISOTROPIC, LAYERED PLATES IN BENDING

Nicholas J. Salamon

*George C. Marshall Space Flight Center
Marshall Space Flight Center, Ala. 35812*

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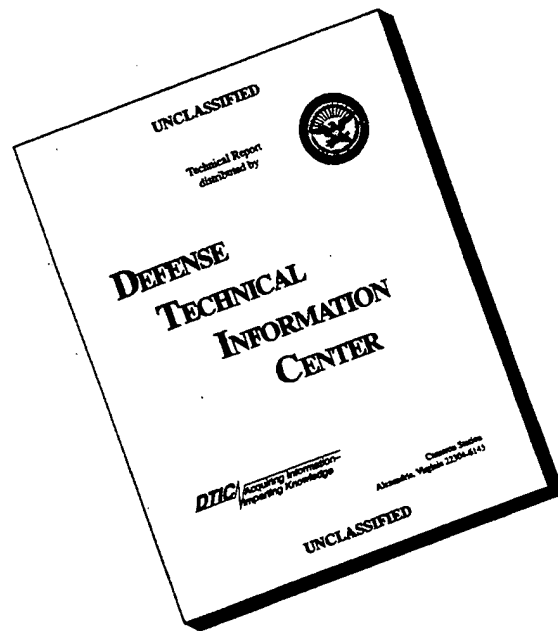


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16. ABSTRACT Results from the initial phase of a study of the interlaminar stresses induced in a layered laminate that is bent into a cylindrical surface are given. The laminate is modeled as a continuum, and the resulting elasticity equations are solved using the finite-difference method. The report sets forth the mathematical framework, presents some preliminary results, and provides a listing and explanation of the computer program. Significant among the results are apparent symmetry relationships that will reduce the numerical size of certain problems and an interlaminar stress behavior having a sharp rise at the free edges.			
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LIST OF SYMBOLS

<u>Symbol</u>	<u>Definition</u>
A	laminate configuration; coefficient matrix [equation (22)]
B	laminate configuration; load vector [equation (22)]
B'_{ij}	constitutive matrix (Appendix A)
B_u, B_v	laminate load constants [equation (7)]
C_i	laminate load constants [equation (5)]
c'_{ij}	elastic coefficients with respect to x', y', z'
c_{ij}	elastic coefficients with respect to x, y, z [equation (1)]
C, D	load values [equation (33)]
D_v	laminate load constant [equation (7)]
D'_{ij}	constitutive matrix (Appendix A)
E_{ii}	Young's moduli
G_{ij}	shear moduli
h_i	node spacing (Fig. 2)
I, J	nodal coordinates (Figs. 2 and 3)
M, M_i	applied moments [equation (4a)]
m	layer number (Fig. 1)
U, V, W	displacement functions [equation (6)]
u, v, w	displacements with respect to x, y, z [equations (3) and (8)]
x, y, z	laminate coordinate axes (Fig. 1)
x', y', z'	lamina orthotropic axes (Fig. 1)

LIST OF SYMBOLS (Concluded)

<u>Symbol</u>	<u>Definition</u>
X	unknown vector [equation (22)]
γ_{ij}	shear strains [equation (2)]
ϵ_i	normal strains [equation (2)]
θ	lamina orientation angle (Fig. 1)
σ_i	normal stress [equation (1)]
τ_{ij}	shear stress [equation (1)]
ν_{ij}	Poisson's ratio

Symbols appearing in the computer program are defined in the subsection entitled "The Mesh Simulation."

A FINITE-DIFFERENCE PROGRAM FOR STRESSES IN ANISOTROPIC, LAYERED PLATES IN BENDING

INTRODUCTION

One critical feature associated with structural composites of laminated construction, using materials or geometrical arrangements that exhibit different elastic properties from layer to layer, is the possibility that the glued layers will separate or delaminate. This was undoubtedly realized from the outset of their use, and a brief historical sketch of the American scene is presented by Pipes [1]. However, the earliest serious investigation into the cause of delamination-type failure, namely the interlaminar stress problem, was apparently done in Japan by Hayashi [2,3], who reported that the maximum interlaminar shearing stresses occurred at the free edge of a laminate under tension. Hayashi used a plane stress model for the layers and approximated the interlaminar shears by a strain-averaging technique. Using a similar model, Puppo and Evensen [4] likewise discovered a sharp rise in the interlaminar stresses near a free edge. Notably, the use of the above models ignored the interlaminar normal stress. In two publications, Pipes and Pagano [5,6] developed a finite-difference program to solve the exact elasticity equations for a long laminate in uniaxial extension. In their development, the stresses are assumed independent of the axial coordinate and include all six components. The results of this investigation show that a sharp rise in both the interlaminar shear stresses and the normal stress occurs near the free edge. Thereafter, Oplinger [7] did an analysis of angle ply laminates in tension using a model similar to that of References 2 through 4. His approach allows a large number of layers to be considered. Indeed it was discovered that a singularity in the interlaminar shear occurs at the free edge of a laminate of one particular type of construction. An alternative solution to that employed in the above references is used by Rybicki [8] who applied a three-dimensional finite element formulation. His results agree with References 5 and 6.

The present report marks the initial phase of a study of the interlaminar stresses induced in a layered laminate by bending. Following the approach used by Pipes [5], the laminate is modeled as a continuum and the resulting elasticity equations are solved using the finite-difference method. This solution technique is made possible by assuming that the laminate is bent into a cylindrical surface such that the stresses are independent of the axial coordinate. The objective of this report is to set forth the mathematical framework, present some preliminary results, and to avail the computer program to others. The results reveal a simplifying symmetry relationship in the displacements that will allow significant reduction in the size of certain numerical problems. In addition, trends in the interlaminar stress distribution are somewhat similar to those found for stretching problems, in that a sharp rise occurs at the free edge.

PROBLEM FORMULATION

Laminate Description

The laminated composites considered in this report consist of rectangular laminae symmetrically stacked with respect to a midplane and bonded together to form a flat laminate. The bonding is assumed to provide perfect adhesion between the laminae, which nullifies the possibility of slip between adjacent laminae thus establishing the conditions of continuous displacements and tractions at each interface. Each individual lamina is considered to be elastic, homogeneous, and orthotropic (i.e., each lamina possesses three planes of elastic symmetry). The assumption of homogeneity eliminates micromechanical effects such as those involving fibers or matrix. The geometry of a typical lamina and laminate is illustrated in Figure 1. One may note that the orthotropic coordinate axes (x', y', z) of a lamina are referred through a clockwise rotation about z to the fixed coordinate axes (x, y, z) of the laminate. The laminae are stacked along z to form a laminate whose sides are normal to x, y , and z . Each lamina is given a layer number m .

Limiting the analysis to linear elastic materials, the constitutive relation for each lamina referred to the x, y, z coordinate system is

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & c_{16} \\ & c_{22} & c_{23} & 0 & 0 & c_{26} \\ & & c_{33} & 0 & 0 & c_{36} \\ \text{(symmetric)} & & & c_{44} & c_{45} & 0 \\ & & & & c_{55} & 0 \\ & & & & & c_{66} \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{bmatrix}, \quad (1)$$

where the elastic constants c_{ij} are related to the nine orthotropic constants c'_{ij} through the well known transformation equations of References 9 and 10.¹ By associating the displacements u, v , and w with x, y , and z , respectively, the strains for each lamina are defined as

1. In using the transformation equations in References 9 and 10 substitute $-\theta$ for $+\theta$ since here the constants are referred to the unprimed coordinate axes of the laminate.

$$\begin{aligned} \epsilon_x^m &= u_{,x}^m & \epsilon_y^m &= v_{,y}^m & \epsilon_z^m &= w_{,z}^m \\ \gamma_{yz}^m &= w_{,y}^m + v_{,z}^m & \gamma_{xz}^m &= w_{,x}^m + u_{,z}^m & \gamma_{xy}^m &= v_{,x}^m + u_{,y}^m \end{aligned} \quad , \quad (2)$$

where the comma denotes partial differentiation.

Loading and Field Quantities

Consider a laminate loaded by bending about y at the ends $x = \text{constant}$. Assuming that the laminate is long enough in the x -direction and that Saint-Venant's principle holds for a laminate, the resulting stress distribution will be independent of x in regions sufficiently removed from the areas of loading. Using this assumption and following Lekhnitskii [11], the elastic strain-stress relations can be integrated to yield displacements for each lamina of the form

$$\begin{aligned} u^m &= (C_1 y + C_2 z + C_3) x + U^m(y, z) \\ v^m &= -\frac{1}{2} C_1 x^2 + C_4 x z + V^m(y, z) \\ w^m &= -\frac{1}{2} C_2 x^2 - C_4 x y + W^m(y, z) \end{aligned} \quad , \quad (3)$$

where U^m , V^m , and W^m are unknown functions of y , z . The layer number, m , is left off the constants C_i because it results that each C_i must be the same for every lamina in order to satisfy the displacement continuity conditions at the interfaces. Thus, the C_i are found to be properties of the entire laminate. The displacement equations (3) represent the full three-dimensional elasticity solution that holds for all points in the laminate.

To evaluate the C_i , the scheme is as follows. Since equations (3) hold for all points in the laminate, they must converge to the plane stress solution, which is an exact solution, in the interior region of the laminate. Integrating the relation [10,12]

$$e_i = B'_{ij} M_j + z D'_{ij} M_j \quad ; \quad i, j = 1, 2, 6 \quad (4a)$$

for the case where $M_1 = -M$ and $M_2 = M_6 = 0$, the plane stress displacements are found to be

$$\begin{aligned}
 u_{ps} &= (-D'_{11}Mz - B'_{11}M)x - B'_{61}My - \frac{1}{2}D'_{16}Myz + f(z) \\
 v_{ps} &= -\frac{1}{2}D'_{16}Mxz - (B'_{21}M + D'_{12}Mz)y + g(z) \\
 w_{ps} &= \frac{1}{2}D'_{11}Mx^2 + \frac{1}{2}D'_{16}Mxy + \frac{1}{2}D'_{12}My^2 + f^*(x) + g^*(y) \quad , \quad (4b)
 \end{aligned}$$

where B'_{ij} and D'_{ij} are laminate properties defined in Appendix A, and M is the applied moment. Comparing equations (3) and (4b) leads to the results:

$$\begin{aligned}
 C_1 &= 0 & C_2 &= -D'_{11}M \\
 C_3 &= -B'_{11}M & C_4 &= -\frac{1}{2}D'_{16}M
 \end{aligned} \quad (5)$$

and

$$\begin{aligned}
 U^m(y, z) &\rightarrow B_u y + C_4 yz + U^m(y, z) \\
 V^m(y, z) &\rightarrow B_v y + D_v yz + V^m(y, z) \\
 W^m(y, z) &\rightarrow -\frac{1}{2}D_v y^2 + W^m(y, z) \quad , \quad (6)
 \end{aligned}$$

where²

$$B_u = -B'_{61}M \quad , \quad B_v = -B'_{21}M \quad , \quad \text{and} \quad D_v = -D'_{12}M \quad . \quad (7)$$

2. The extended forms (6) for U^m , V^m , and W^m are not necessary to the solution.

Substituting the results (6) into equations (3) yields displacements of the following functional form for each layer

$$\begin{aligned}
 u^m &= (C_2 z + C_3)x + (B_u + C_4 z)y + U^m(y, z) \\
 v^m &= C_4 xz + (B_v + D_v z)y + V^m(y, z) \\
 w^m &= -\frac{1}{2} C_2 x^2 - C_4 xy - \frac{1}{2} D_v y^2 + W^m(y, z) \quad , \quad (8)
 \end{aligned}$$

where C_i , B_i , and D_v are defined by equations (5) and (7). The strains are found by substituting the displacements (8) into the strain relations (2). The stresses then follow directly using the constitutive relation (1).

It is of interest to examine the strain ϵ_x^m which is

$$\epsilon_x^m = C_2 z + C_3 \quad . \quad (9)$$

Should the laminate be a balanced composite, i.e., the laminae are symmetrically stacked, according to composition and orientation with respect to the midplane $z = 0$, then $B'_{ij} = 0$ and from equations (5) $C_3 = 0$, which results in a case of pure bending. For the opposite case, an unbalanced composite exhibits an extensional strain, C_3 , in bending. Such coupling effects are common to laminated composites.

Field Equations and Boundary Conditions

In regions sufficiently removed from the load planes, the nonboundary points must satisfy the reduced equilibrium equations

$$\begin{aligned}
 \tau_{xy,y}^m + \tau_{xz,z}^m &= 0 \\
 \sigma_{y,y}^m + \tau_{yz,z}^m &= 0 \\
 \tau_{yz,y}^m + \sigma_{z,z}^m &= 0 \quad , \quad (10)
 \end{aligned}$$

where the stresses exhibit no x-dependence, which conforms to an earlier assumption. Substituting for the stresses in terms of displacements yields the field equations for each lamina

$$\begin{aligned}
c_{66}^m U_{,yy}^m + c_{55}^m U_{,zz}^m + c_{26}^m V_{,yy}^m + c_{45}^m V_{,zz}^m + (c_{36}^m + c_{45}^m) W_{,yz}^m &= 0 \\
c_{26}^m U_{,yy}^m + c_{45}^m U_{,zz}^m + c_{22}^m V_{,yy}^m + c_{44}^m V_{,zz}^m + (c_{23}^m + c_{44}^m) W_{,yz}^m &= 0 \\
(c_{36}^m + c_{45}^m) U_{,yz}^m + (c_{23}^m + c_{44}^m) V_{,yz}^m + c_{44}^m W_{,yy}^m + c_{33}^m W_{,zz}^m \\
&= -(c_{13}^m C_2 + c_{23}^m D_V + 2c_{36}^m C_4) \quad . \quad (11)
\end{aligned}$$

The boundary conditions on the free surfaces normal to y are

$$\sigma_y^m = \tau_{xy}^m = \tau_{yz}^m = 0 \quad (12)$$

and on the free surfaces normal to z are

$$\sigma_z^m = \tau_{xz}^m = \tau_{yz}^m = 0 \quad . \quad (13)$$

For continuity at the interfaces, the boundary conditions are:

$$(u^m, v^m, w^m) = (u^{m+1}, v^{m+1}, w^{m+1})$$

and (14)

$$(\sigma_z^m, \tau_{xz}^m, \tau_{yz}^m) = (\sigma_z^{m+1}, \tau_{xz}^{m+1}, \tau_{yz}^{m+1}) \quad ,$$

respectively.

It is noted that the corner conditions are ambiguous in that there are five possible conditions out of which only three can be employed at any one time. The remaining two may or may not be satisfied by the solution. Thus, combinations may be tried until some satisfying results are achieved.

FINITE-DIFFERENCE SIMULATION

Function Representation

The mathematical basis for the finite-difference method is Taylor's Series. Referring to Figure 2, the Taylor Series expansion for a function f at some point y, z about the point (or node) I, J is

$$\begin{aligned}
 f(y, z) = & f(I, J) + yf_{,y}(I, J) + zf_{,z}(I, J) \\
 & + \frac{1}{2} y^2 f_{,yy}(I, J) + \frac{1}{2} z^2 f_{,zz}(I, J) + yzf_{,yz}(I, J) + \dots \quad (15)
 \end{aligned}$$

Thus, for the specific node $I-1, J$, the expansion is

$$f(I-1, J) = f(I, J) - h_1 f_{,y} + \frac{1}{2} h_1^2 f_{,yy} - \dots \quad (16)$$

Writing similar expansions for the remaining seven points neighboring the node I, J and simultaneously solving expansions for the first and second derivatives yields the finite-difference approximations for these derivatives. All but the last of these expressions, given below, are taken from Forsythe and Wasow [13]. They are

$$\begin{aligned}
 f_{,y}(I, J) &= \frac{1}{h_1 + h_2} \left[\frac{h_1}{h_2} f(I+1, J) - \frac{h_2}{h_1} f(I-1, J) \right] + \frac{h_2 - h_1}{h_1 h_2} f(I, J) + O(h^2) \\
 f_{,z}(I, J) &= \frac{1}{2h_3} \left[f(I, J+1) - f(I, J-1) \right] + O(h^2) \\
 f_{,yy}(I, J) &= \frac{2}{h_1 + h_2} \left[\frac{1}{h_2} f(I+1, J) + \frac{1}{h_1} f(I-1, J) \right] - \frac{2}{h_1 h_2} f(I, J) + O(h^2) \\
 f_{,zz}(I, J) &= \frac{1}{h_3^2} \left[f(I, J+1) + f(I, J-1) - 2f(I, J) \right] + O(h^2) \\
 f_{,yz}(I, J) &= \frac{1}{2h_3(h_1 + h_2)} \left[f(I+1, J+1) - f(I-1, J+1) - f(I+1, J-1) \right. \\
 &\quad \left. + f(I-1, J-1) \right] + O(h^2) \quad , \quad (17)
 \end{aligned}$$

where h is an order of magnitude equal to h_1 , h_2 , or h_3 . The difference equations (17) are "central" differences.

At boundaries and interfaces it is convenient to use "forward" and "backward" differences. Such difference equations are one-sided in that they express a boundary point in terms of neighboring points interior to the boundary. For the present problem, only first derivatives are of concern.

To derive such difference equations, expand two points, both lying on one side of the reference point I, J , by using equation (15) in conjunction with Figure 2. For example, a forward expansion yields

$$\begin{aligned}
 f(I + 1, J) &= f(I, J) + h_2 f_{,y}(I, J) + \frac{1}{2} h_2^2 f_{,yy}(I, J) + O(h_2^3) \\
 f(I + 2, J) &= f(I, J) + 2h_2 f_{,y}(I, J) + \frac{1}{2} (4h_2^2) f_{,yy}(I, J) + O(h_2^3) \quad . \quad (18)
 \end{aligned}$$

Subtracting one expression from the other to eliminate the second derivative leads to the difference equation for the first derivative. Thus, the forward differences are

$$\begin{aligned}
 f_{,y}(I, J) &= \frac{1}{2h_2} \left[4f(I + 1, J) - 3f(I, J) - f(I + 2, J) \right] - O(h_2^2) \\
 f_{,z}(I, J) &= \frac{1}{2h_3} \left[4f(I, J + 1) - 3f(I, J) - f(I, J + 2) \right] - O(h_3^2) \quad . \quad (19)
 \end{aligned}$$

Similarly, the backward differences are

$$\begin{aligned}
 f_{,y}(I, J) &= \frac{1}{2h_1} \left[3f(I, J) + f(I - 2, J) - 4f(I - 1, J) \right] + O(h_1^2) \\
 f_{,z}(I, J) &= \frac{1}{2h_3} \left[3f(I, J) + f(I, J - 2) - 4f(I, J - 1) \right] + O(h_3^2) \quad . \quad (20)
 \end{aligned}$$

It should be pointed out that more simplified, but less accurate, forward and backward expressions can be written; however, the present application requires all the accuracy that it is possible to attain near the free boundaries. Thus, the higher order difference was chosen. In addition, this choice yields a magnitude of error equal to that found in equations (17).

Using the representations just obtained, equations (11) through (14) can be transformed into difference equations characterizing the problem. For example, the last equation in (11) becomes

$$\begin{aligned}
& \frac{h_1 h_2}{2h_3(h_1 + h_2)} \left\{ (c_{36}^m + c_{45}^m) [U(I + 1, J + 1) - U(I - 1, J + 1) - U(I + 1, J - 1) \right. \\
& \quad + U(I - 1, J - 1)] + (c_{23}^m + c_{44}^m) [V(I + 1, J + 1) \\
& \quad \left. - V(I - 1, J + 1) - V(I + 1, J - 1) + V(I - 1, J - 1)] \right\} \\
& + \frac{2h_1}{h_1 + h_2} c_{44}^m \left[W(I + 1, J) + \frac{h_2}{h_1} W(I - 1, J) \right] \\
& + \frac{h_1 h_2}{h_3^2} c_{33}^m [W(I, J + 1) + W(I, J - 1)] \\
& - 2(c_{44}^m + \frac{h_1 h_2}{h_3^2} c_{33}^m) W(I, J) = -h_1 h_2 [c_{13}^m C_2 + c_{23}^m D_V \\
& + 2c_{36}^m C_4] \quad , \quad (21)
\end{aligned}$$

where the layer number, m , is left off U , V , and W since their location is determined by the node I, J .

Developing the Matrix Equation

In this section, the difference equations, like (21), are transformed into a linear matrix equation of the form

$$[A] [X] = [B] \quad , \quad (22)$$

where A is an $N \times N$ coefficient matrix (N being the number of unknowns or equations), X is the solution vector, and B is the load or input vector. To accomplish this, the three unknowns (U , V , and W) must be uniquely collapsed into the single unknown X so that at each node three unique equations in X will be created. For instance, let

$$\begin{array}{l}
 U \rightarrow X(1) \\
 V \rightarrow X(2) \\
 W \rightarrow X(3)
 \end{array}
 \left. \vphantom{\begin{array}{l} U \\ V \\ W \end{array}} \right\} \text{ at Node 1}
 \qquad
 \begin{array}{l}
 U \rightarrow X(4) \\
 V \rightarrow X(5) \\
 W \rightarrow X(6)
 \end{array}
 \left. \vphantom{\begin{array}{l} U \\ V \\ W \end{array}} \right\} \text{ at Node 2} \quad . \quad (23)$$

It remains to generalize such a transformation for all nodes.

It is convenient to follow Pipes [1] and his notation is adopted. If LAT is the number of nodes in one column along the vertical axis (LAminate Thickness direction), then the nodes, unknowns, and equations can be identified by a unique number in terms of the nodal position (I, J). If

$$JJ1 = 3[LAT(I - 1) + J] - 2 \quad , \quad (24)$$

then

$$\begin{aligned}
 NODE &= LAT(I - 1) + J \\
 U(I, J) &= X(JJ1) \\
 V(I, J) &= X(JJ1 + 1) \\
 W(I, J) &= X(JJ1 + 2)
 \end{aligned} \quad (25)$$

and

$$\begin{aligned}
 \text{Number the 1st equation: } &JJ1 \\
 \text{Number the 2nd equation: } &JJ1 + 1 \\
 \text{Number the 3rd equation: } &JJ1 + 2 \quad . \quad (26)
 \end{aligned}$$

Letting I = 1 and J = 1, 2 consecutively generates the results in (23).

Since the finite-difference equations involve unknowns at nodes neighboring the JJ1 node, it is necessary to develop transformation relations like (24) in order to number unknowns at these nodes as well. For example, using I, J as the reference node, a

transformation relation for an unknown at the node $I - 1, J + 1$ is found by letting $I \rightarrow I - 1$ and $J \rightarrow J + 1$ in (24) and giving the result a unique name, for example JJ7. Thus,

$$JJ7 = 3[LAT(I - 2) + J] + 1 \quad . \quad (27)$$

Using Table 1, which identifies all the unknowns at nodes neighboring I, J, and following the above procedure yields the transformation relations that uniquely number each unknown. In summary, all of these transformations are

$$\begin{aligned} JJ1 &= 3*(LAT*I1 + J) - 2 \\ JJ2 &= 3*(LAT*I2 + J) - 2 \\ JJ3 &= 3*(LAT*I2 + J) - 5 \\ JJ4 &= 3*(LAT*I + J) - 2 \\ JJ5 &= 3*(LAT*I + J) + 1 \\ JJ6 &= 3*(LAT*I1 + J) + 1 \\ JJ7 &= 3*(LAT*I2 + J) + 1 \\ JJ8 &= 3*(LAT*I1 + J) - 5 \\ JJ9 &= 3*(LAT*I + J) - 5 \\ JJ10 &= 3*(LAT*I1 + J) - 8 \\ JJ11 &= 3*(LAT*(I + 1) + J) - 2 \\ JJ12 &= 3*(LAT*I1 + J) + 4 \\ JJ13 &= 3*(LAT*(I - 3) + J) - 2 \quad , \quad (28) \end{aligned}$$

where

$$\begin{aligned} I1 &= I - 1 \\ I2 &= I - 2 \end{aligned} \quad (29)$$

TABLE 1. NODE IDENTIFICATION

Node	U	V	W
I, J	X(JJ1)	X(JJ1 + 1)	X(JJ1 + 2)
I - 1, J	X(JJ2)	X(JJ2 + 1)	X(JJ2 + 2)
I - 1, J - 1	X(JJ3)	X(JJ3 + 1)	X(JJ3 + 2)
I + 1, J	X(JJ4)	X(JJ4 + 1)	X(JJ4 + 2)
I + 1, J + 1	X(JJ5)	X(JJ5 + 1)	X(JJ5 + 2)
I, J + 1	X(JJ6)	X(JJ6 + 1)	X(JJ6 + 2)
I - 1, J + 1	X(JJ7)	X(JJ7 + 1)	X(JJ7 + 2)
I, J - 1	X(JJ8)	X(JJ8 + 1)	X(JJ8 + 2)
I + 1, J - 1	X(JJ9)	X(JJ9 + 1)	X(JJ9 + 2)
I, J - 2	X(JJ10)	X(JJ10 + 1)	X(JJ10 + 2)
I + 2, J	X(JJ11)	X(JJ11 + 1)	X(JJ11 + 2)
I, J + 2	X(JJ12)	X(JJ12 + 1)	X(JJ12 + 2)
I - 2, J	X(JJ13)	X(JJ13 + 1)	X(JJ13 + 2)

Generation of the matrix equation (22) now remains. To do this, straightforward substitution for U, V, and W, using Table 1, into equations (11) through (14) yields the desired results in equation form. For example, equation (21) becomes

$$\begin{aligned}
& \frac{h_1 h_2}{2h_3(h_1 + h_2)} \left\{ (c_{36}^m + c_{45}^m) [X(JJ5) - X(JJ7) - X(JJ9) + X(JJ3)] \right. \\
& \quad + (c_{23}^m + c_{44}^m) [X(JJ5 + 1) - X(JJ7 + 1) - X(JJ9 + 1) \\
& \quad \left. + X(JJ3 + 1)] \right\} + \frac{2h_1}{h_1 + h_2} c_{44}^m [X(JJ4 + 2) + \frac{h_2}{h_1} X(JJ2 + 2)] \\
& \quad + \frac{h_1 h_2}{h_3^2} c_{33}^m [X(JJ6 + 2) + X(JJ8 + 2)] \\
& \quad - 2(c_{44}^m + \frac{h_1 h_2}{h_3^2} c_{33}^m) X(JJ1 + 2) \\
& = -h_1 h_2 [c_{13}^m C_2 + c_{23}^m D_V + 2c_{36}^m C_4] \quad . \quad (30)
\end{aligned}$$

To assure non-zero diagonal terms in the A-matrix, an appropriate equation number for (30) is JQ2 (in this case there is only one possibility) where

$$JQ2 = JJ1 + 2 \quad . \quad (31)$$

Now, from equation (30), the only nonzero elements for the JQ2 row in the A-matrix are

$$\begin{aligned}
A(JQ2, JJ5) &= A(JQ2, JJ3) = C \\
A(JQ2, JJ7) &= A(JQ2, JJ9) = -C \\
A(JQ2, JJ5 + 1) &= A(JQ2, JJ3 + 1) = D \\
A(JQ2, JJ7 + 1) &= A(JQ2, JJ9 + 1) = -D \\
A(JQ2, JJ4 + 2) &= 2h_1 c_{44}^m / (h_1 + h_2) \\
A(JQ2, JJ2 + 2) &= (h_2/h_1) \cdot 2h_1 c_{44}^m / (h_1 + h_2) \\
A(JQ2, JJ6 + 2) &= A(JQ2, JJ8 + 2) = h_1 h_2 c_{33}^m / h_3^2 \\
A(JQ2, JJ1 + 2) &= -2(c_{44}^m + h_1 h_2 c_{33}^m / h_3^2) \quad , \quad (32)
\end{aligned}$$

where

$$C = h_1 h_2 (c_{36}^m + c_{45}^m) / 2h_3 (h_1 + h_2)$$
$$D = h_1 h_2 (c_{23}^m + c_{44}^m) / 2h_3 (h_1 + h_2) \quad . \quad (33)$$

Note that the material constants c_{44}^m and c_{33}^m are non-zero ensuring a non-zero diagonal element $A(JQ2, JJ1 + 2)$. In addition to this, the load vector is

$$B(JQ2) = -h_1 h_2 [c_{13}^m C_2 + c_{23}^m D_V + 2c_{36}^m C_4] \quad . \quad (34)$$

Of course, these results only apply to node numbers where the third equilibrium equation in (11) holds. The computer program logically connects appropriate equations with each node. The matrix elements for the remaining equations (11) through (14) are generated in a similar fashion.

The Mesh Simulation

The continuum is to be simulated by a number of nodal points that form a finite-difference mesh. The mesh is distributed over a cross section of the laminate as shown in Figure 3. The mesh is defined by the following parameters:

- NLAY: the number of laminae
- LAT: the number of nodes along one column in the LAMinate Thickness direction
- LAW: the number of nodes along one row in the LAMinate Width direction
- FSW1: the first change in nodal spacing termed Fine Simulation Width
- K: magnification factor of the fine simulation width
- H: the fine simulation width

Given these parameters, the following parameters can be determined:

INF(M): values of J at the upper INterFace of the mth layer

FSW2: the second change in nodal spacing

KH: the coarse simulation width (K = 1, 2, 3, ...)

JQMAX = 3*LAT*LAW: the number of unknowns or equations

IBW = 2*(3*LAT + 1): the half bandwidth

NBAND = 2*IBW + 1: the full band

The bandwidth of the coefficient matrix is found by considering that the maximum number of nodes involved in the difference equations is three, as can be seen from expressions (19) and (20), and calculating the maximum number of consecutive elements on both sides of the diagonal to and including the last off-diagonal non-zero element.

Selecting equations representing the conditions to be imposed at each node remains to be accomplished. Because of the arbitrariness of the corner conditions, a number of choices are possible. Those selected for this program are illustrated in Figure 4.

A user's guide and a more detailed description of the computer program are presented in Appendix C. A program listing is provided also in Appendix C.

RESULTS

The results given below were obtained using a square mesh, magnification factor $K = 1$, of size (LAW, LAT) = (13, 9). A complete mesh description, taken from the program output, is displayed in Table 2. It is seen that these dimensions represent a beam rather than a plate. The program was run on an IBM 370 computer utilizing virtual storage.

A single material having properties typical of a high modulus graphite-epoxy was chosen for the above mesh. Using standard notation,

$$E_{11} = 20.0 \times 10^6 \text{ psi} \quad , \quad \nu_{12} = \nu_{13} = \nu_{23} = 0.21$$

$$E_{22} = E_{33} = 2.1 \times 10^6 \text{ psi}$$

$$G_{12} = G_{13} = G_{23} = 0.85 \times 10^6 \text{ psi} \quad ,$$

TABLE 2. MESH DESCRIPTION TAKEN FROM PROGRAM OUTPUT

```

*** UNIFORM BENDING OF A LAMINATED PLATE ***
*** INPUT DATA ***
NUMBER OF LAYERS IN CROSS SECTION, NLAY = 4
NUMBER OF NODES ON VERTICAL AXIS, LAT = 13
NUMBER OF NODES ON HORIZONTAL AXIS, LAH = 9
CHANGE IN MESH WIDTH (FSW1) AT I = 3
CHANGE IN MESH WIDTH (FSW2) AT I = 7
MESH WIDTH MAGNIFICATION FACTOR, K = 1
LAYER NO. 1 INTERFACE AT J = 4
LAYER NO. 2 INTERFACE AT J = 7
LAYER NO. 3 INTERFACE AT J = 10
LAYER NO. 4 INTERFACE AT J = 13
FINE SIMULATION WIDTH, H = 0.00167

```

where the subscript "1" refers to the fiber direction. The two laminate configurations which are considered are

$$A = [\theta, 0, 0, \theta]$$

and

$$B = [0, \theta, \theta, 0]$$

with θ as in Figure 1 such that $0 \text{ degree} \leq \theta \leq 90 \text{ degrees}$. Typical laminate data and load constants are displayed in Table 3.³ Here the additional constant MT is the resulting moment required to produce a specified maximum strain which, for the present analysis, is $\epsilon_x = 1.0 \times 10^{-3}$ inch/inch (see Appendix B).

A sample of the results for the displacement functions U, V, and W is presented in Table 4. Examination of their variation with respect to z reveals the apparent symmetry relations,

U, V antisymmetric in z

W symmetric in z

within an accuracy of two digits.

Symmetries with respect to y are evident for the strains within three-digit accuracy. Samples of these results are plotted in Figures 5 and 6. Coupling these apparent symmetries with the strain relations (2) in an expanded form yields

U, V antisymmetric in y

W symmetric in y

The displacement results verify this precisely for U (to four places), but show some deviation in V and W.⁴

To illustrate the effect of bending on the stress distribution, Figures 7 through 19 are presented. Although convergence to the exact values has yet to be demonstrated, the results do have qualitative merit. The following cases result from a bending strain of $\epsilon_x = 1.0 \times 10^{-3}$ inch/inch prescribed at the bottom surface.

Of principal interest are the interlaminar stresses illustrated in Figures 7 through 12. We note that laminates composed of 30 degree or 45 degree layers produce the greatest stress rise in σ_z at the free edge with a more pronounced effect occurring if the angle plies are on the outside, i.e., system A = $[\theta, 0, 0, \theta]$. A similar effect is seen in the shear stress τ_{yz} , although the rise in stress is sharply blunted by the requirement of zero

3. The thermal problem is neglected in this preliminary analysis even though expansion coefficients appear in the program.

4. It is interesting to note that the y-symmetries for V and W are verified precisely using the coarser mesh (LAW, LAT) = (8, 9) which decreases the relative size of the bandwidth.

TABLE 3. (Concluded)

```

6.745E+35 5.745E+06 5.210E+05 0.0 0.0 4.506E+06 2.024E+07 5.648E+05 5.648E+05 0.0 0.0 0.0
6.745E+06 5.210E+05 0.0 0.0 4.506E+06 2.213E+06 4.771E+05 0.0 0.0 0.0 0.0
2.213E+06 0.0 0.0 4.506E+06 2.213E+06 0.0 0.0 0.0 0.0 0.0 0.0
8.500E+05 0.0 0.0 0.0 0.0 0.0 8.500E+05 0.0 0.0 0.0 0.0
3.599E+35 7.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
5.330E+06 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

```

*** COEFFICIENTS OF THERMAL EXPANSION ***

LAYER	THETA	AL1	AL2	AL3	AL5	AL1P	AL2P	AL3P
1	45.0	0.600E-05	0.670E-05	0.120E-04	-0.120E-04	0.0	0.120E-04	0.120E-04
2	0.0	0.0	0.120E-04	0.120E-04	0.0	0.0	0.120E-04	0.120E-04
3	0.0	0.0	0.120E-04	0.120E-04	0.0	0.0	0.120E-04	0.120E-04
4	45.0	0.600E-05	0.670E-05	0.120E-04	-0.120E-04	0.0	0.120E-04	0.120E-04

*** THE LIMITATE LOAD CONSTANTS ***

```

C1 = 0.0 C2 = 0.0 C3 = 0.0 C4 = 1.311E-01 3U = 0.0 3V = 0.0 3W = -2.908E-01 MT = -5.537E-01
ERROR CONDITION OF SOLVER ROUTINE IS 0.0 PANK IS 351.0 DETERMINANT = 1.00

```

NOTE: MT IS THE RESULTING MOMENT REQUIRED TO PRODUCE THE SPECIFIED MAXIMUM STRAIN.

TABLE 4. DISPLACEMENT FUNCTION RESULTS TAKEN FROM
PROGRAM OUTPUT FOR LAMINATE DESCRIBED IN
TABLES 2 AND 3

*** GRID POINT DISPLACEMENT FUNCTIONS ***

NO DE	U-DISPLACEMENT	V-DISPLACEMENT	W-DISPLACEMENT
1	0.161561D-04	0.264686D-04	-0.909039D-05
2	0.149580D-04	0.219831D-04	-0.936804D-05
3	0.125381D-04	0.173176D-04	-0.951939D-05
4	0.953594D-05	0.127014D-04	-0.953590D-05
5	0.611696D-05	0.818977D-05	-0.940002D-05
6	0.304395D-05	0.403364D-05	-0.924686D-05
7	0.487189D-09	0.250769D-08	-0.916589D-05
8	-0.304291D-05	-0.403918D-05	-0.925084D-05
9	-0.611575D-05	-0.819432D-05	-0.937698D-05
10	-0.926689D-05	-0.126308D-04	-0.952892D-05
11	-0.125354D-04	-0.173211D-04	-0.954418D-05
12	-0.149550D-04	-0.219880D-04	-0.937161D-05
13	-0.161503D-04	-0.264803D-04	-0.909891D-05

stress at the free edge, and here the stress in system B = [0, θ , θ , 0] is slightly more pronounced than that in A. The largest stress rise, an order of magnitude greater than σ_z and τ_{yz} , is created in the A-system in τ_{xz} . Again it is the 30 degree laminate incurring the sharpest stress rise, but here the 15 degree laminate overshadows the 45 degree laminate. In summary, the laminates containing 15 degree through 45 degree layers located adjacent to 0 degree layers have the largest interlaminar stresses for the cases considered; i.e., 0 degree $\leq \theta \leq$ 90 degrees taken in 15 degree intervals.

Some results peculiar to the numerical method of solution should be pointed out. Referring to Figure 9, we note a sharp rise in the stress σ_z at the midpoint node (I, J) = (5, 7). This is a result of fixing the displacements at I = 5 and 6, J = 7 in the program in order to zero-out rigid body motion and drift in the solution routine. However averaging the values for σ_z just above and just below the interface (at J = 7, m = 2 and m = 3) yields a more plausible result. Since the tractions must be continuous at the interface anyway, this averaging technique was also applied at the free edges where the free surface conditions were adopted in lieu of the continuity conditions. This technique had varying success as illustrated by the 75 degree and 90 degree configurations in Figures 10 and 11.

The in-plane stresses are illustrated in Figures 13 through 19. In Figure 13, we find that σ_x in the 0 degree layers is independent of the orientation of the adjacent layer when the maximum strain is specified.⁵ This facilitates the presentation of both systems A and B in one figure. It is interesting to note in Figure 14 that σ_x rises at the free edge if the 0 degree layers are outside the laminate and drops if these layers are inside the laminate.

Observation of Figures 15 and 17 for the distribution of σ_y and τ_{xy} with respect to z reveals that the off-axis layers, particularly again for 15 degrees through 45 degrees, serve as stress raisers with the effect considerably more pronounced if the 0 degree layers are inside.

Typical distributions of σ_y and τ_{xy} with respect to y are shown in Figures 18 and 19. The disturbing feature of these plots is that the stresses just above an interface do not approach zero at the free surface. One cause of this problem is the placement of nodes directly on the interface, which requires their occupation by both layers. Then at the corners, as stated previously, the multitude of boundary conditions cannot be satisfied.⁶ However this problem is confined to the free surface nodes and one line of

5. In agreement with the beam theory approximation.

6. Placing the interface between two nodal lines may alleviate this problem.

interior nodes. To see this, one may examine the curves for the A-system at $J = 4-$ and $J = 10+$ and note that they are reflections of each other within the range $3 \leq I \leq 7$. Since, from above, σ_y and τ_{xy} appear, in general, to be antisymmetric in z , the correct values at $J = 10+$ are recovered within this range if we accept the values at $J = 4-$.

CONCLUSIONS

Although only two types of laminate systems were considered, namely $A = [\theta, 0, 0, \theta]$ and $B = [0, \theta, \theta, 0]$, it is reasonable to assume from these results and from physical considerations that the following symmetry relations hold for balanced ($B_{ij} = 0$) composites:

U, V antisymmetric in y and z

W symmetric in y and z ,

where U, V, and W are displacement functions of y and z . Based on the stress results, laminates containing layers oriented within the range $15 \text{ degrees} \leq \theta \leq 45 \text{ degrees}$ produce the largest interlaminar stresses out of the cases studied, $0 \text{ degrees} \leq \theta \leq 90 \text{ degrees}$ taken in 15 degree intervals. In fact this same group of laminates produces high values in the in-plane stresses as well, with the effect considerably more pronounced for the A-system. Although some deviations in stress occur in the numerical solution, they are localized to a double line of nodes at the boundary. This is a disconcerting feature of the solution in that the boundary region stresses appear to be critically involved in delamination-type failure, which makes their accurate determination desirable.

This study provides a base for future work in this area. Using the present program coupled with an out-of-core equation solver routine, unbalanced laminates may be studied. Using the symmetry relations discussed above, the present computer program may be modified to more efficiently handle balanced laminates ($B_{ij} = 0$).

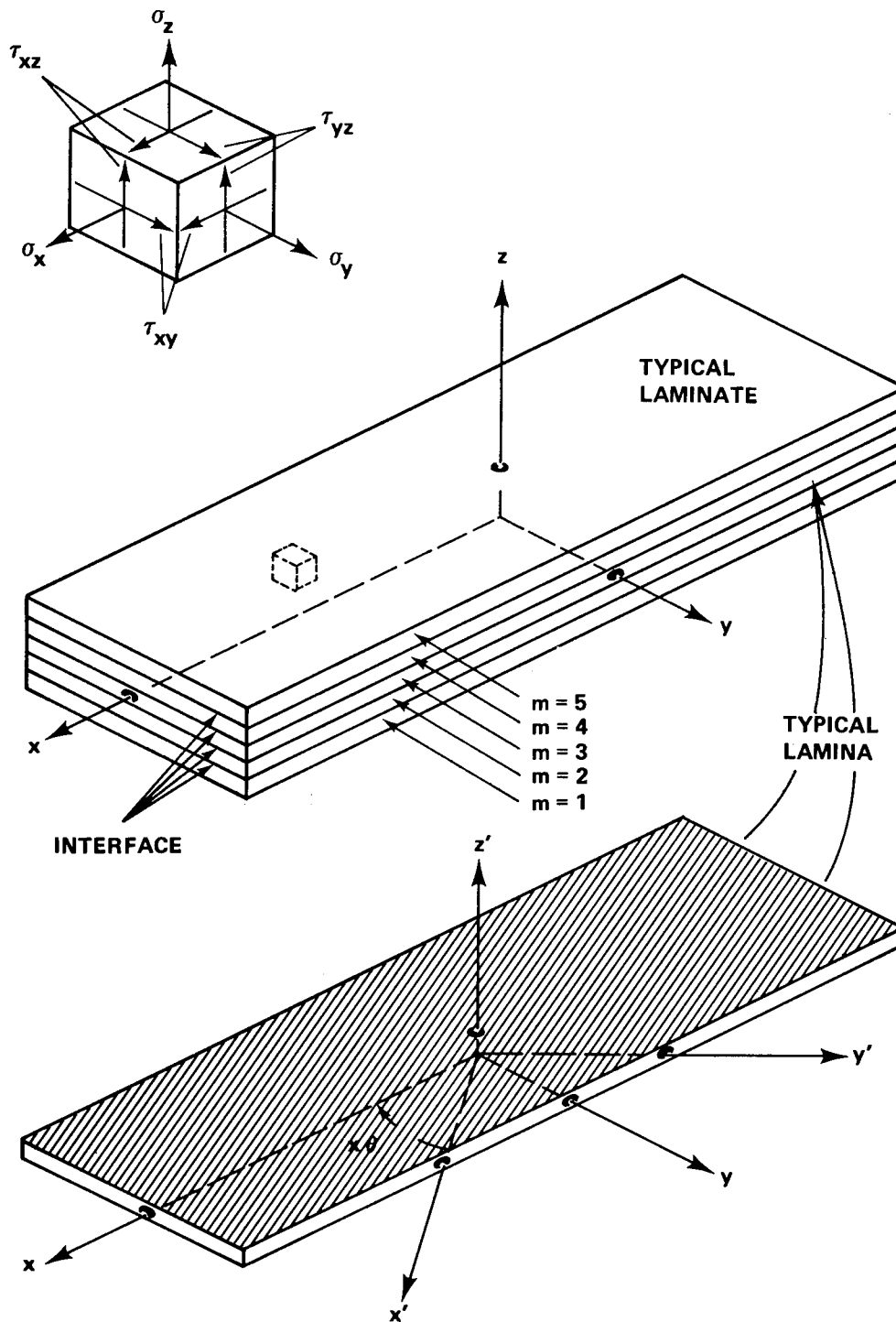


Figure 1. Laminate geometry.

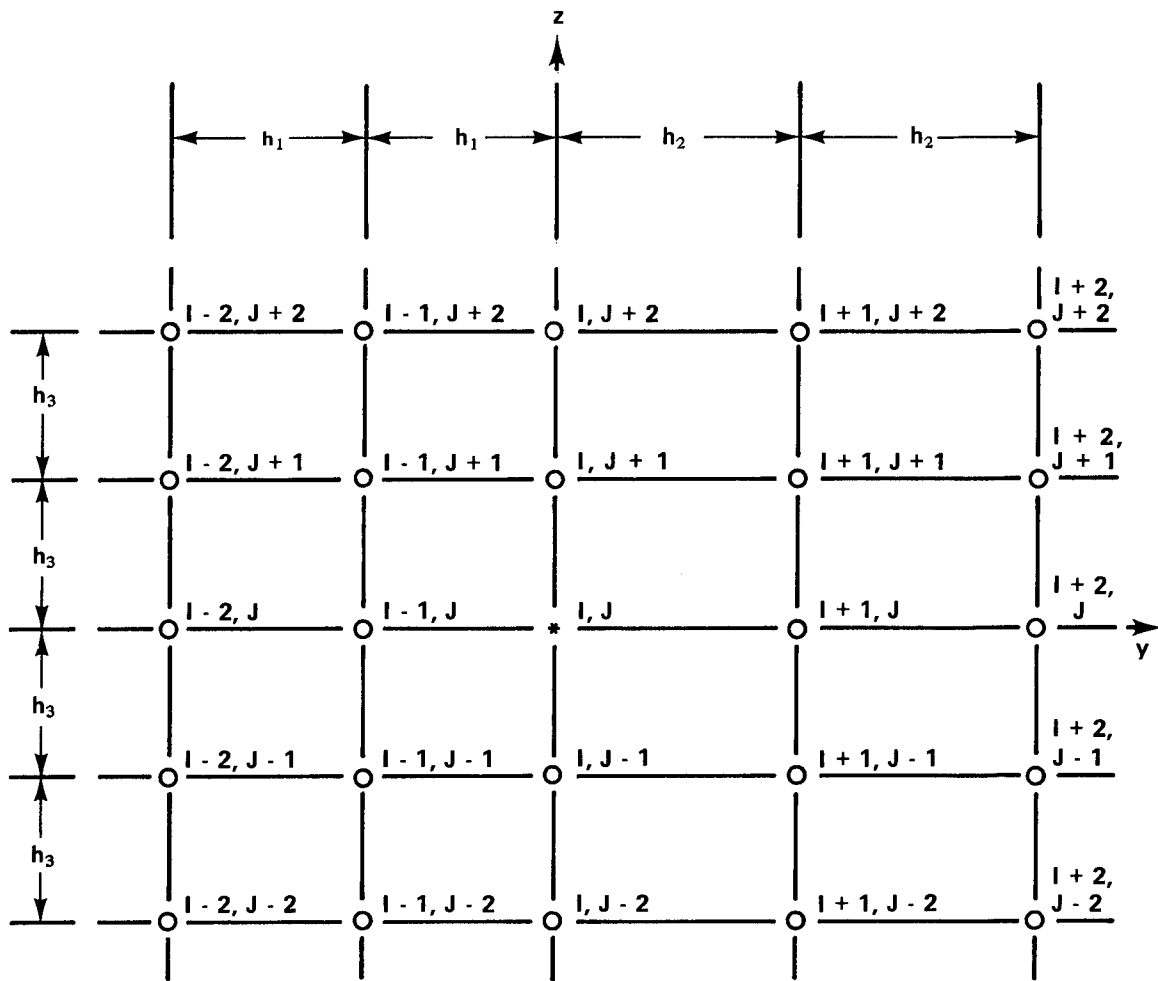


Figure 2. Finite-difference mesh.

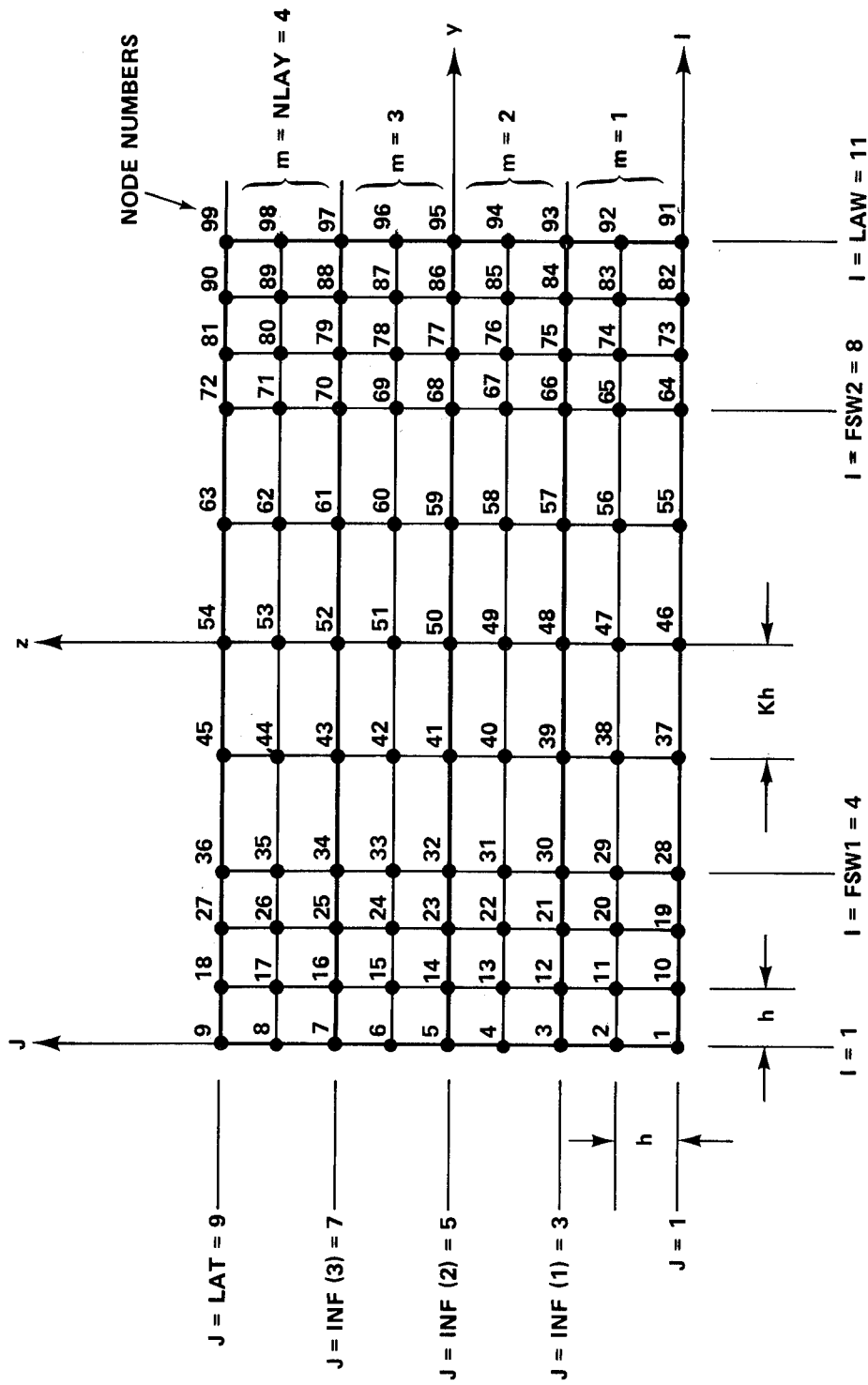
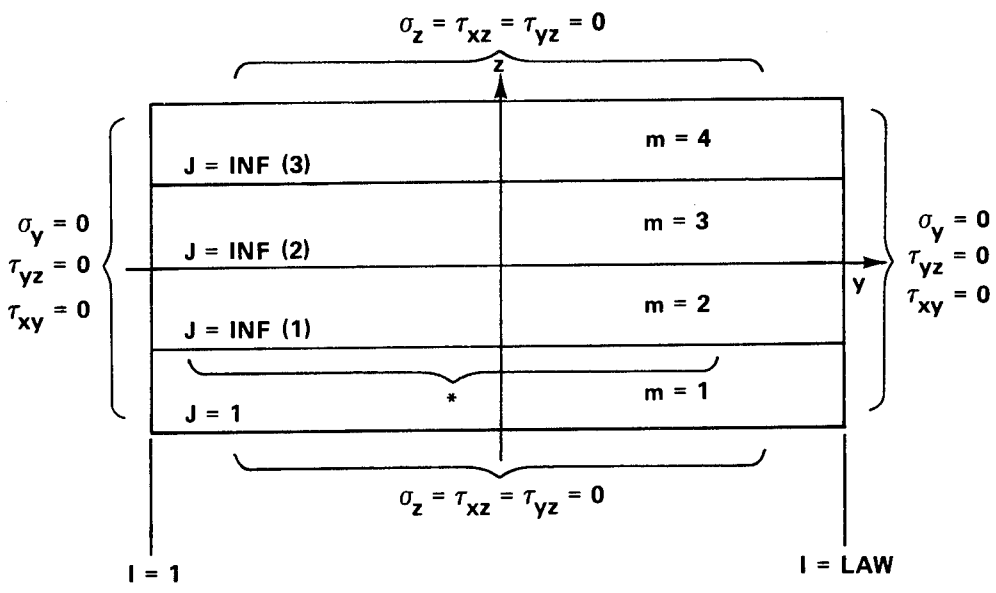
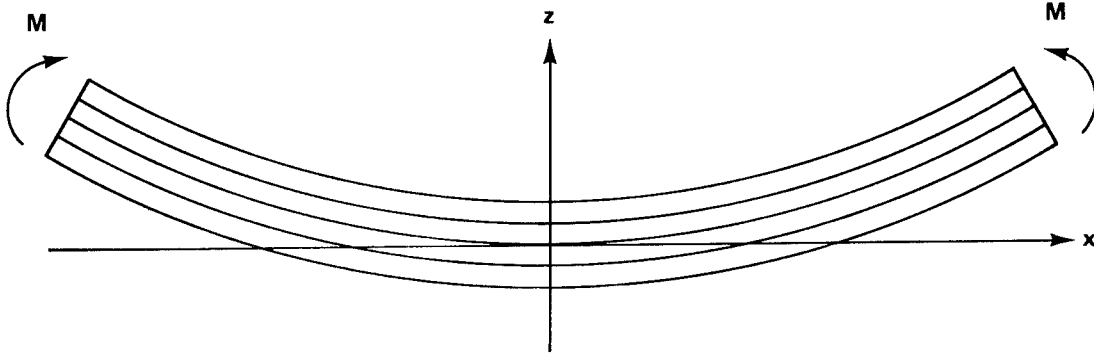


Figure 3. A typical laminate mesh.



*AT INF(m) WHERE $1 < I < LAW$ AND $1 \leq m < NLAY$:

$$[u^m, v^m, w^m] = [u^{m+1}, v^{m+1}, w^{m+1}]$$

$$[\sigma_z^m, \tau_{yz}^m, \tau_{xz}^m] = [\sigma_z^{m+1}, \tau_{yz}^{m+1}, \tau_{xz}^{m+1}]$$

● STATIC EQUILIBRIUM IS IMPOSED AT ALL INTERIOR POINTS

Figure 4. Equations selected for each node.

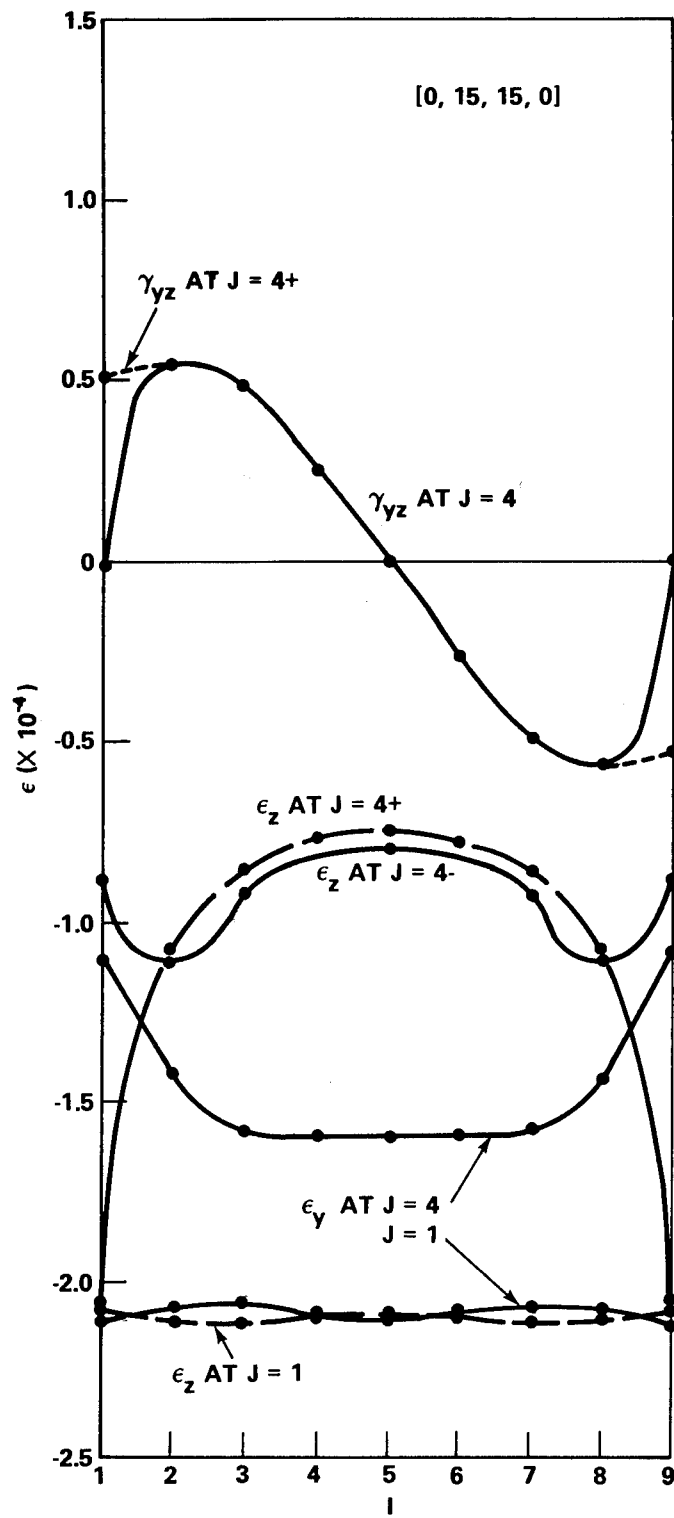


Figure 5. Variation of strain with y .

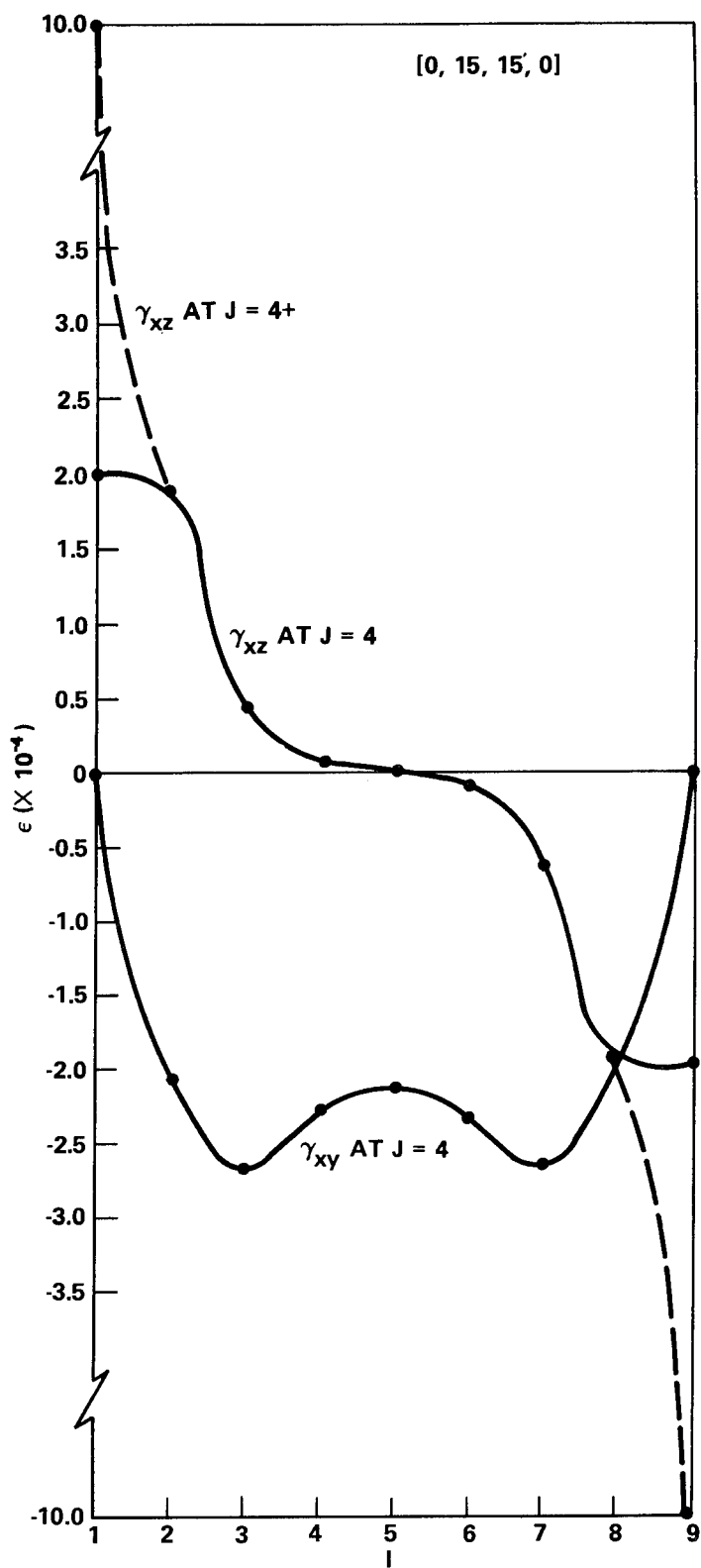


Figure 6. Variation of shear strain with y.

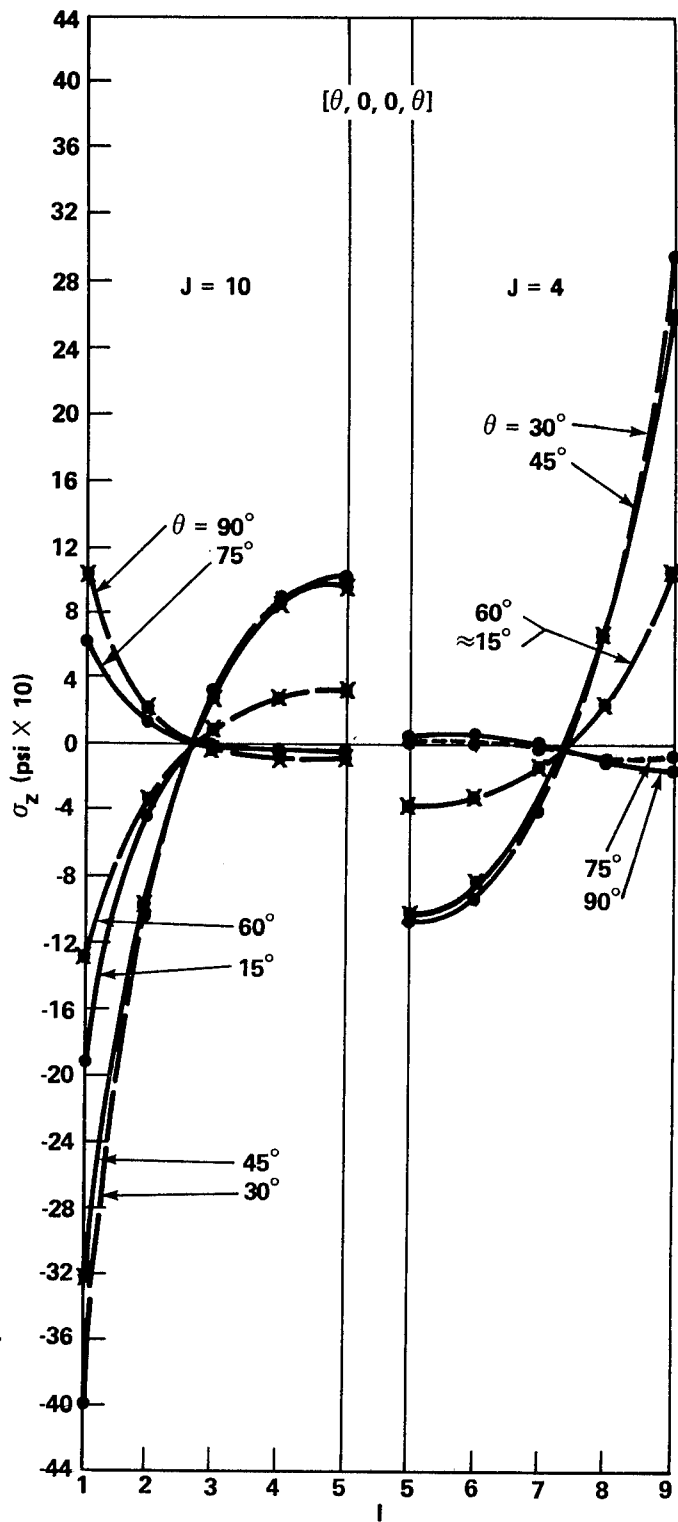


Figure 7. Variation of the normal stress σ_z (symmetric in y) with y for a $[\theta, 0, 0, \theta]$ laminate.

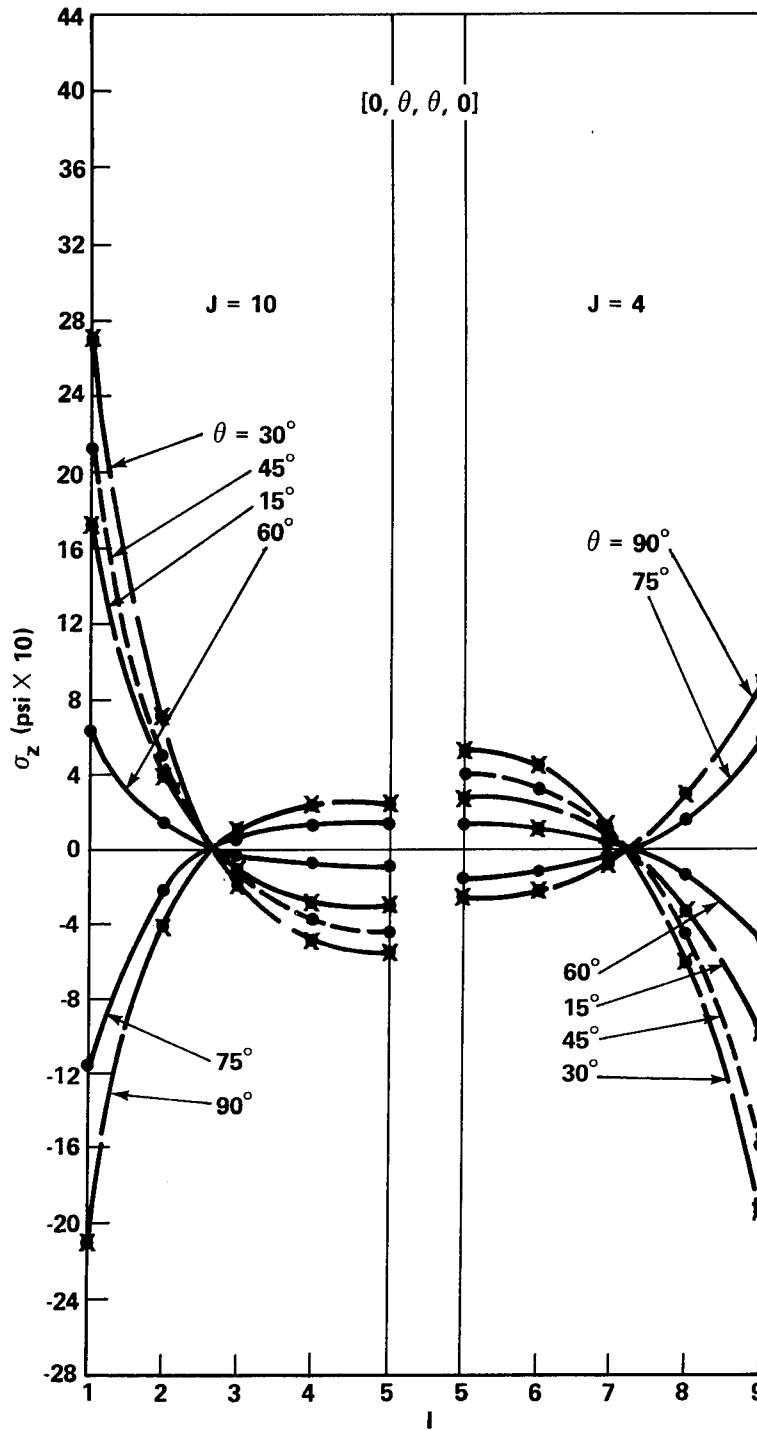


Figure 8. Variation of the normal stress σ_z (symmetric in y) with y for a $[0, \theta, \theta, 0]$ laminate.

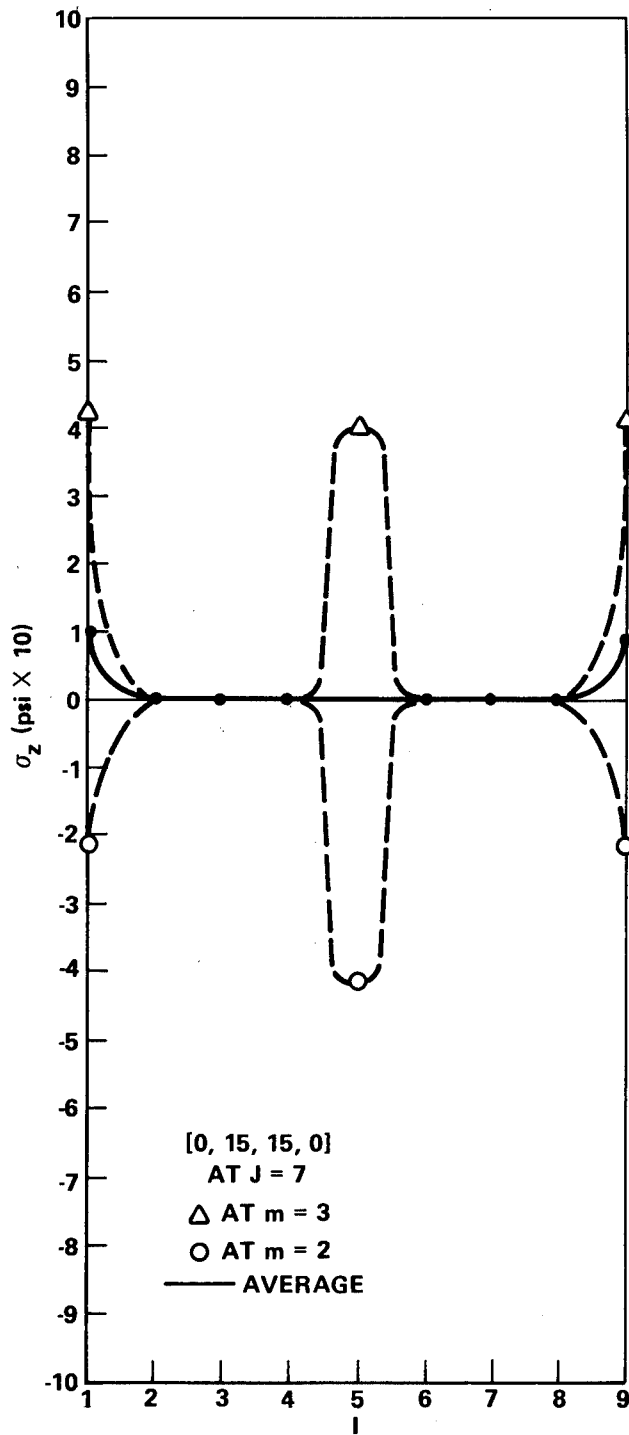


Figure 9. Numerical peculiarities in the normal stress σ_z .

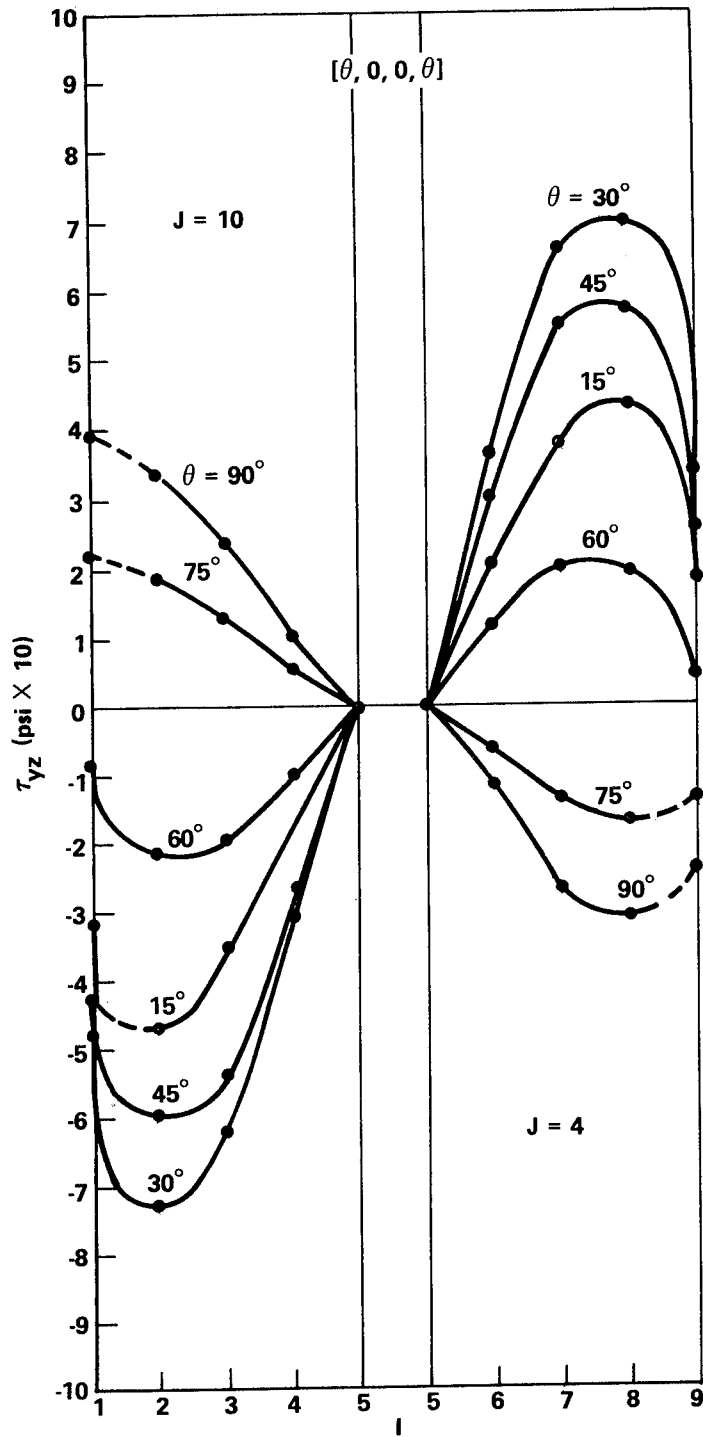


Figure 10. Variation of the shear stress τ_{yz} (antisymmetric in y) with y for a $[\theta, 0, 0, \theta]$ laminate.

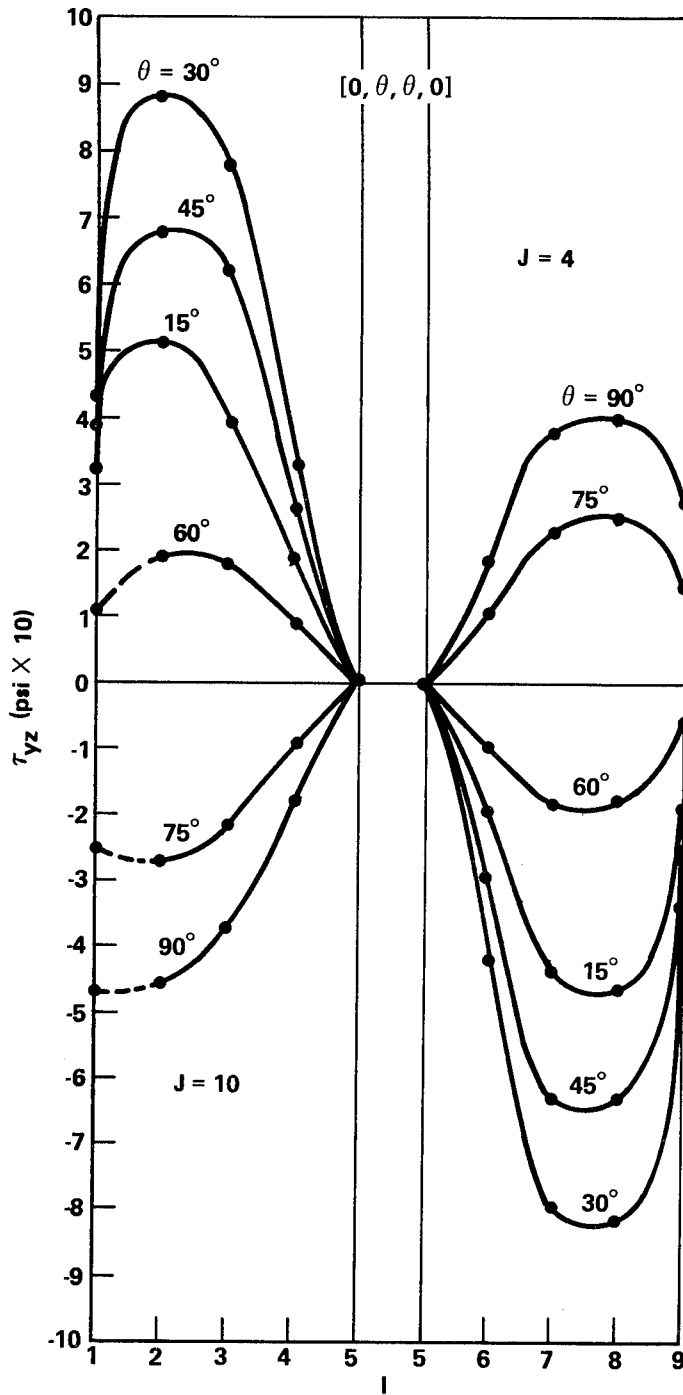


Figure 11. Variation of the shear stress τ_{yz} (antisymmetric in y) with y for a $[0, \theta, \theta, 0]$ laminate.

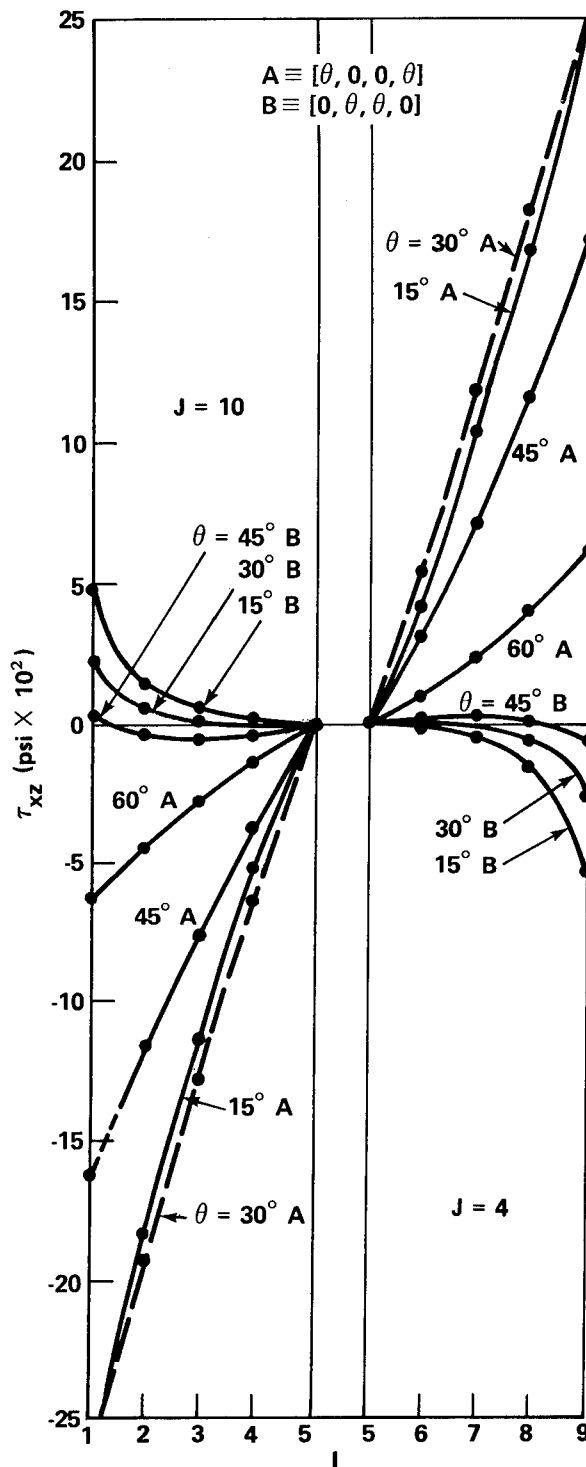


Figure 12. Variation of the shear stress τ_{xz} (antisymmetric in y) with y .

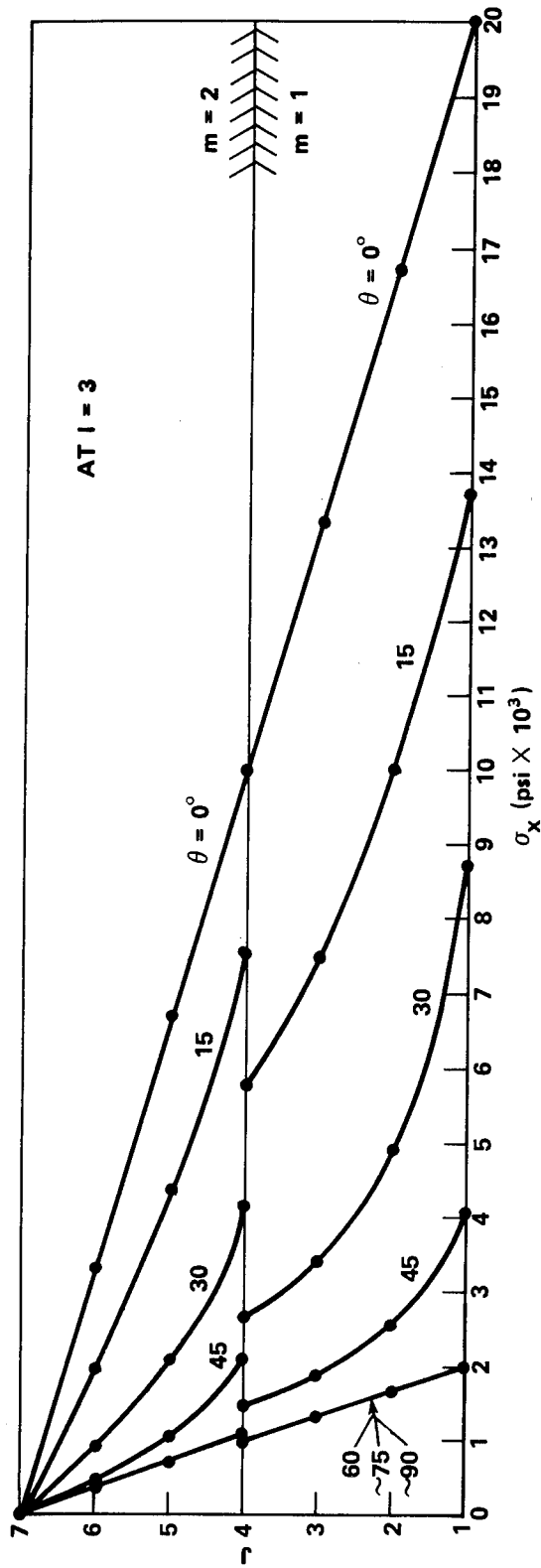


Figure 13. Variation of the normal stress σ_x (antisymmetric in z) with z for each layer with respect to position where the adjacent layer is oriented at $\theta = 0$ degree.

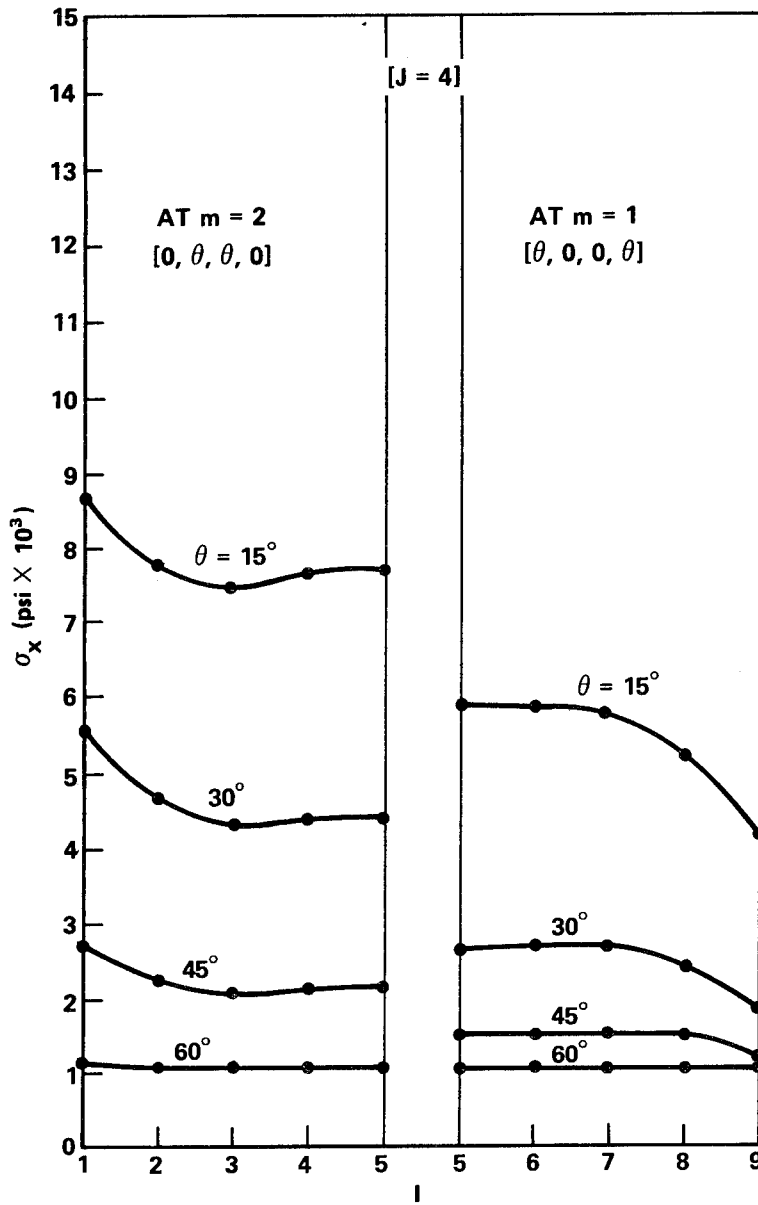


Figure 14. Variation of the normal stress σ_x (symmetric in y) with y .

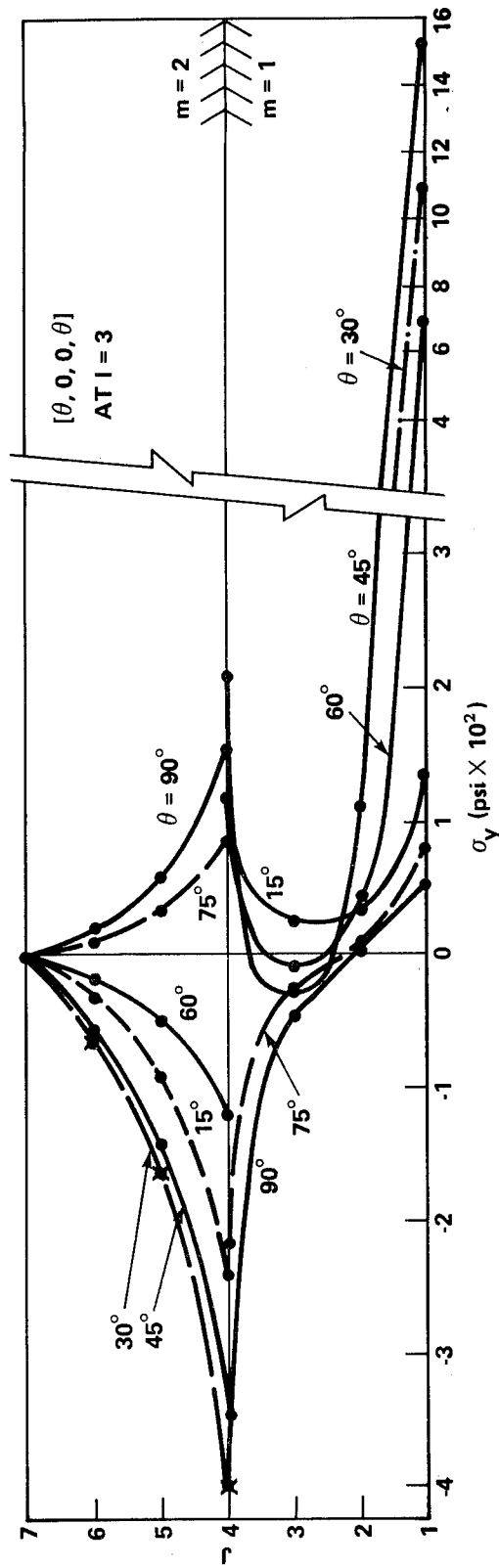


Figure 15. Variation of the normal stress σ_y (antisymmetric in z) with z for a $[\theta, 0, 0, \theta]$ laminate.

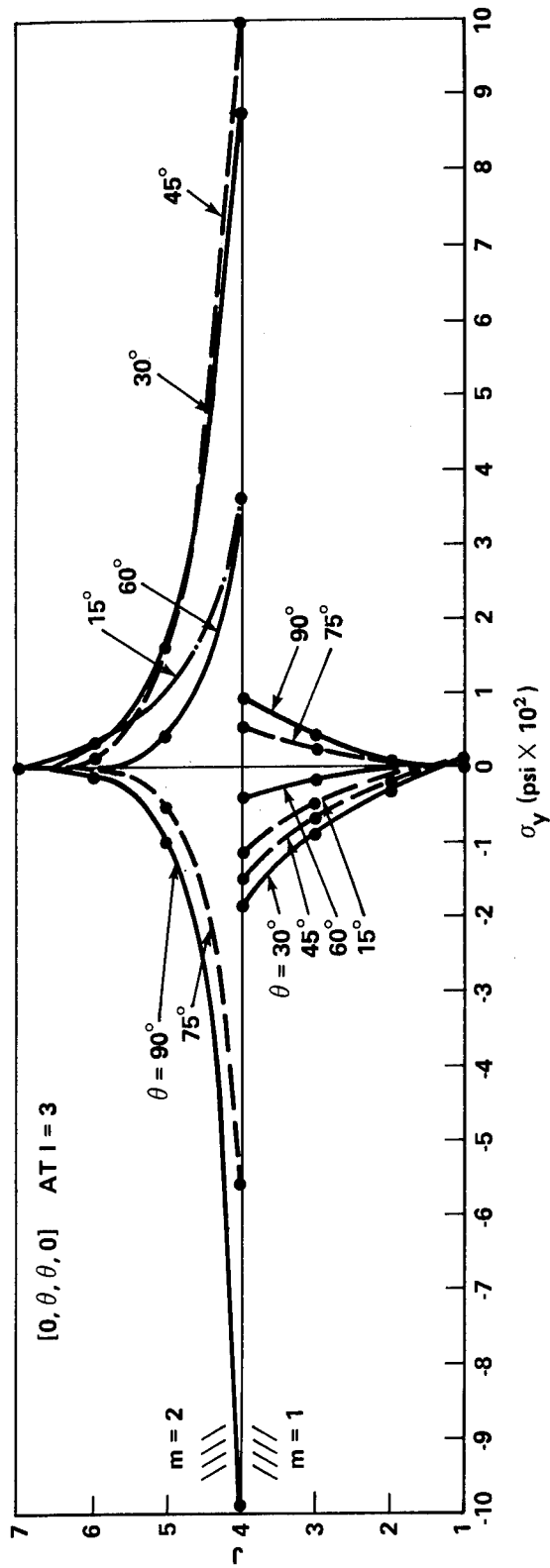


Figure 16. Variation of the normal stress σ_y (antisymmetric in z) with z for a $[0, \theta, \theta, 0]$ laminate.

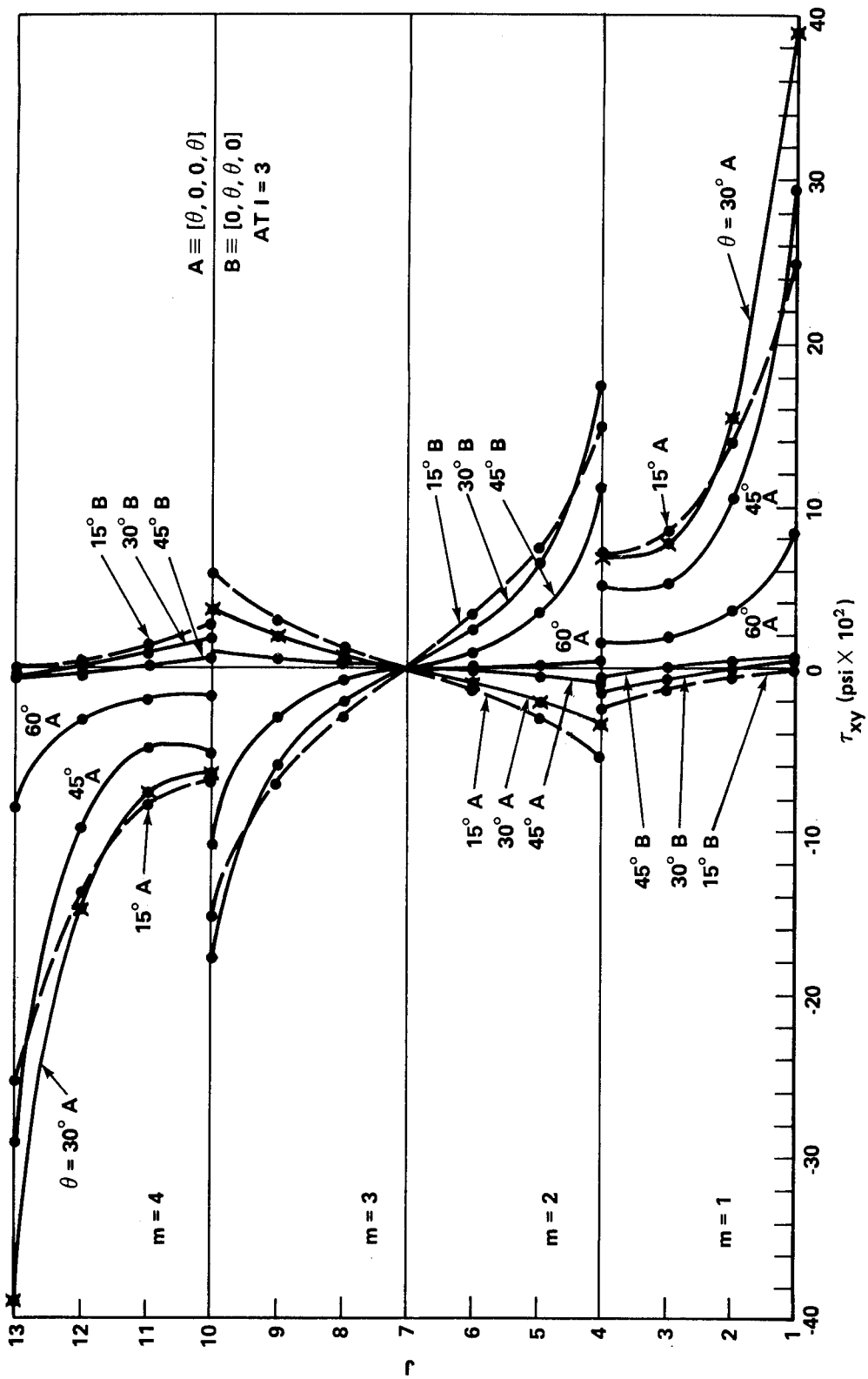


Figure 17. Variation of the shear stress τ_{xy} with z .

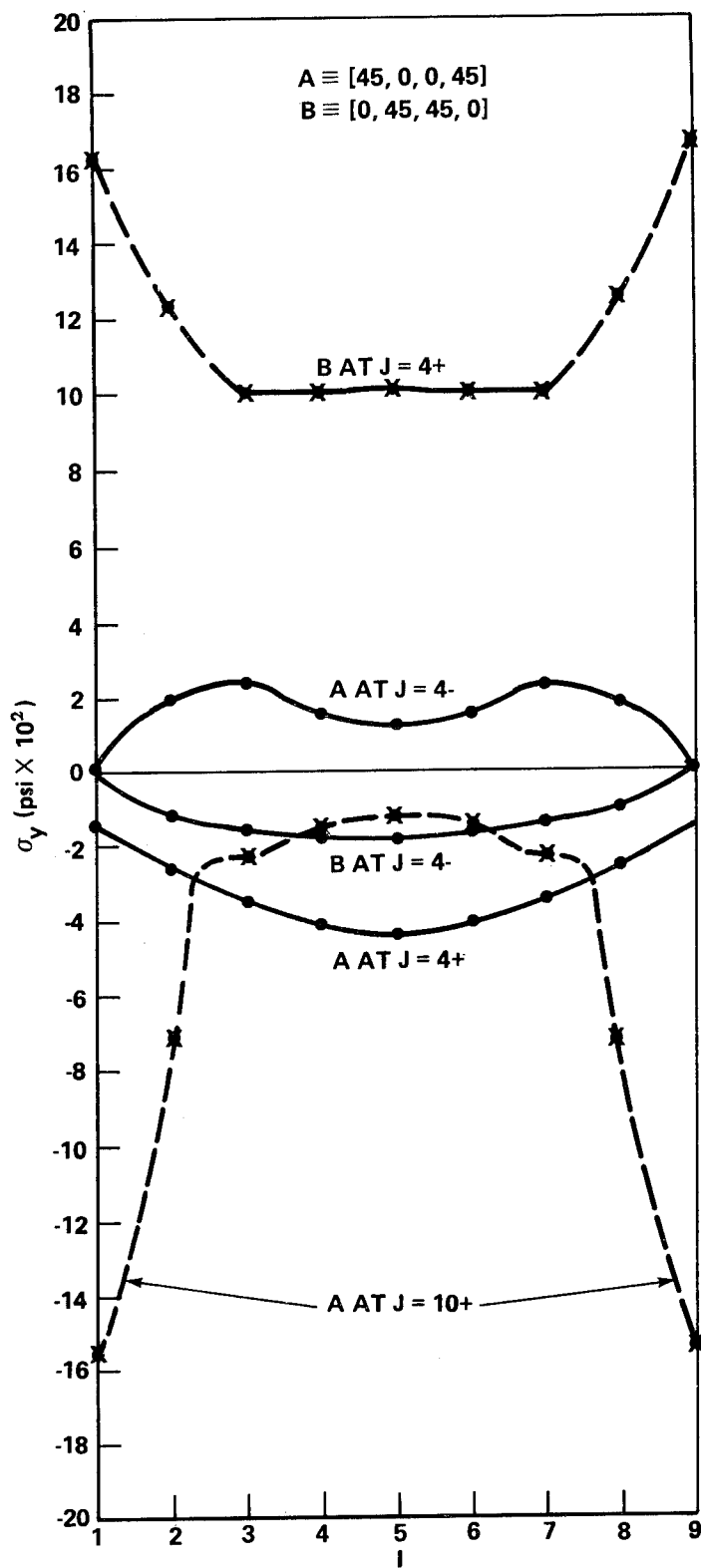


Figure 18. Variation of the normal stress σ_y with y .

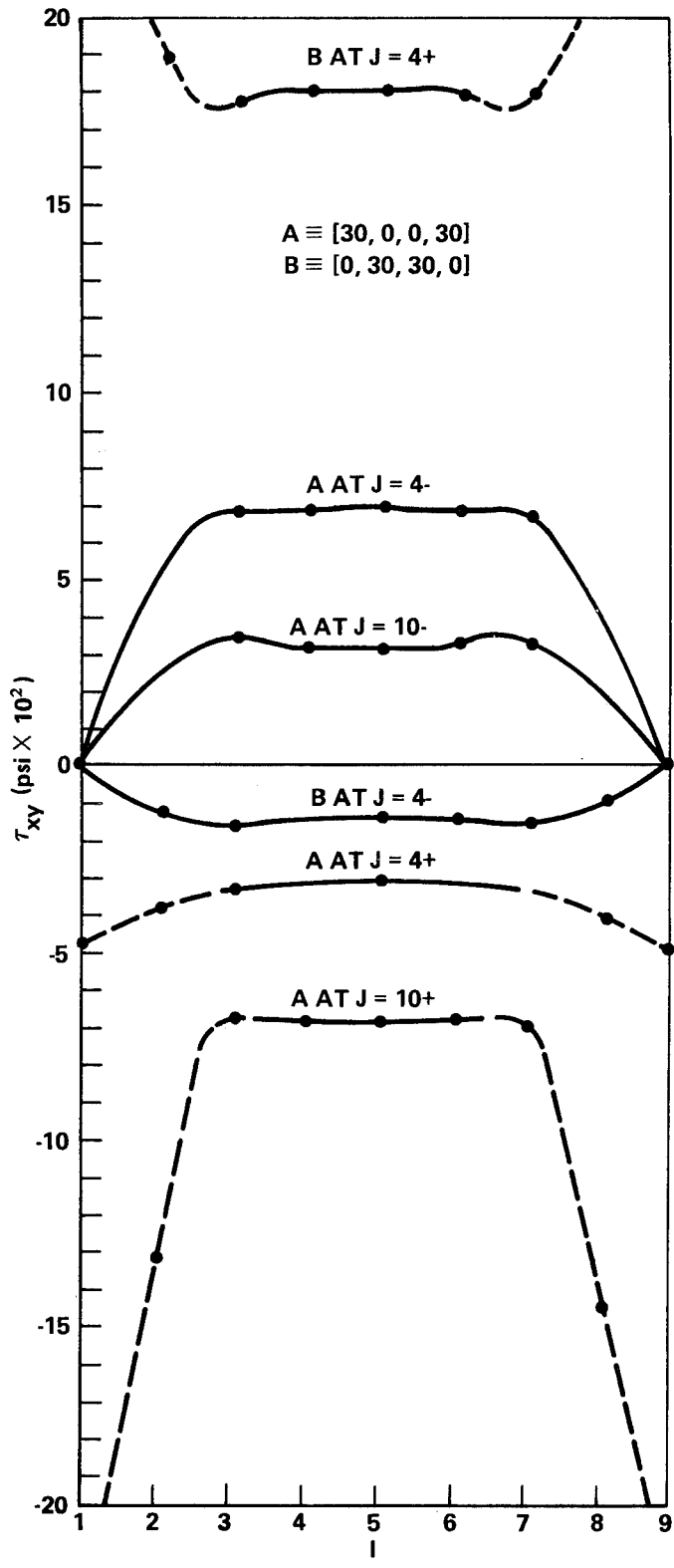


Figure 19. Variation of the shear stress τ_{xy} with y.

APPENDIX A
LAMINATE CONSTANTS

Following Reference 9 or 10, define

$$Q_{ij}^m = c_{ij}^m - \frac{c_{i3}^m c_{j3}^m}{c_{33}^m} ; \quad i, j = 1, 2, 6$$

and let t be the half-thickness of the laminate, h_0 the thickness of a lamina, and N the total number of layers; then

$$\begin{aligned} A_{ij} &= h_0 \sum_{m=1}^N Q_{ij}^m \\ B_{ij} &= \frac{h_0^2}{2} \left\{ \sum_{m=1}^N Q_{ij}^m (2m - 1) - N \sum_{m=1}^N Q_{ij}^m \right\} \\ D_{ij} &= \frac{h_0^3}{3} \left\{ \sum_{m=1}^N Q_{ij}^m (3m^2 - 3m + 1) \right. \\ &\quad \left. - \frac{3}{2} N \sum_{m=1}^N Q_{ij}^m (2m - 1) \right. \\ &\quad \left. + \frac{3}{4} N^2 \sum_{m=1}^N Q_{ij}^m \right\} \end{aligned}$$

with $i, j = 1, 2, 6$. Finally, let

$$A^* = A^{-1} , \quad B^* = -A^{-1}B , \quad \text{and} \quad D^* = D - BA^{-1}B$$

where the letters symbolize 3×3 matrices. Then,

$$B' = B^*(D^*)^{-1}$$

and

$$D' = (D^*)^{-1}$$

Considerable simplification is attained if the laminate is balanced, which implies $B_{ij} = B'_{ij} = 0$.

APPENDIX B

STRAIN SPECIFICATION

Rather than prescribe the laminate loading as end moments, the maximum strain, ϵ_x^{\max} , at the top and bottom surfaces, $z = \pm z^{\max}$, will be prescribed. From equation (9), we have

$$\epsilon_x^{\max} = C_2 z^{\max} + C_3 \quad .$$

Now from equations (5),

$$C_3 = -B'_{11} M = \frac{B'_{11}}{D'_{11}} C_2 \quad ,$$

so that

$$\epsilon_x^{\max} = C_2 \left(z^{\max} + \frac{B'_{11}}{D'_{11}} \right)$$

and, thus,

$$C_2 = \frac{D'_{11} \epsilon_x^{\max}}{B'_{11} + D'_{11} z^{\max}} \quad .$$

In the computer program, we set $\epsilon_x^{\max} = -1.0 \times 10^{-3}$ inch/inch at the top surface $z = +z^{\max}$ to evaluate the constant C_2 which represents the inverse bending radius.

APPENDIX C

THE COMPUTER PROGRAM

Program Description

The computer program is an in-core program and is not overlaid. It is felt that a flow chart of the program would be no less complicated than the presentation of a listing with an accompanying explanation, so the latter choice will be followed. Certain statements in the program are extraneous to the problem in this report because the program is in steady transition to handle more general problems. A part-by-part description follows.

Part I. Part I contains a brief definition of terms and an explanation of the order and format of the data cards. The dimensions of the data are: H is in inches, E is material constants in psi (the shear moduli G_{12} , etc., are read into the E12, etc., arrays), ALPHA is the coefficient of expansion in inches/inch/ $^{\circ}$ F, and THETA is the lamina orientation in angular degrees. Precision and dimension statements are then established, data are read in, and mesh parameters are calculated. The letter M refers to the layer number. In the loop, D0 9000, IRAN counts each laminate layup from one to IRUN (only changes in lamina orientation are allowed for within this loop).

Part II. Part II calculates the anisotropic stiffness matrix. BETA is in radians. CP11, etc., are the orthotropic elastic constants in the primed coordinate system. C11(M), etc., are the anisotropic elastic constants for the Mth lamina in the x, y, z coordinate system. AL1P(M), etc., are the coefficients of thermal expansion in the primed coordinate system and AL1(M) are those coefficients in the x, y, z coordinate system, both the the Mth lamina. Finally, the subroutine MATCON, which calculates the laminate MATERIAL CONSTANTS, is called.

Part III. Part III calculates the coefficient matrix for the difference equations. The loops D0 100 and D0 101 count through the mesh node-by-node. D0 3000 zeroes out the A-matrix.

The logic that associates the various field conditions with each node and correctly fills out the A-matrix is contained in D0 102. First the node I, J is tested to determine the proper layer number, M. Then the node is checked to see if it lies on a boundary, along J equals a constant, or lies at some select position (in this case, IMID or JMID). If it does, the program is routed to the statement number that contains the non-zero matrix elements satisfying the conditions imposed at this node. Should the node not lie at any of these preselected locations, the program passes through the IF statements on J to statement number 193, which initiates a series of checks to see if the node lines on selected values of I. These values include the boundaries $I = 1$ or $I = LAW$, the changes in

nodal spacing $I = \text{FSW1}$ or $I = \text{FSW2}$, and all points in the region between FSW1 and FSW2 . Should the node not lie at any of these locations, the program passes through the IF statements on I and evaluates the non-zero coefficients for the only remaining possibility, the equilibrium terms for a square mesh.

When a node does lie on some select location, say J equals LAT , then the logic in that statement series, say the series starting from statement number 202, guides the program through the checks on selected values of I in a fashion similar to that above. The logic is easily understood by reading directly from the listing.

Upon reaching statement number 102, the A-matrix ($3 \times \text{JQMAX}$) is full. The elements of the A-matrix lying within the bandwidth are then stored in the banded matrix AX. The loops D0 100 or D0 101 then continue for the next node, if any. The previous A-matrix is destroyed and regenerated for the new node until the loop D0 100 is satisfied.

At rewind 9, the matrix AX and the load vector X are stored for later use. The loop D0 107 stores the load vector $X(I)$ in $\text{AX}(I, \text{NBD})$. Then a series of WRITE statements (listed as comments) will output the coefficient matrix AX and load vector X should they be desired. Finally, the solver routine, TRMSTR, is called.⁷

Part IV. Part IV outputs the functional displacements and provides an accuracy check. Just below statement number 4006, the STOP 1 statement will terminate the program if the coefficient matrix AX is singular. (Such an occurrence probably indicates an error.) The loop D0 108 stores the solution vector $\text{AX}(I, 1)$ in $X(I)$. Then the original values of the matrix AX and load vector X are read back into the AX array and R vector, respectively.

The loops D0 11 and D0 12 output the values for the functions $U(y, z)$, $V(y, z)$, and $W(y, z)$ which occur in the displacements u , v , and w , respectively.

The series of statements from the one above 9950 to 9990 outputs the accuracy results. These results provide the difference between the original load vector, now stored in the R-array, with the calculated load vector, which is found by substituting the appropriate solution vectors, $X(I)$, into each matrix equation. In addition to giving the accuracy of each equation, an average accumulated accuracy is provided.

Part V. Part V outputs the strains and stresses. The logic is similar to that in Part III. Knowing that the finite-difference relations for the strains differ for various mesh locations, the strains are split into terms dependent upon the value of I and terms dependent upon the value of J . The strain SX , which represents ϵ_{xx} , depends upon neither the value of I nor the value of J and is determined prior to any logical branching.

7. Actually the AX-matrix stores a transposed A-matrix; i.e., instead of storing row elements crosswise or in a row, they are stored in the AX-matrix vertically or in a column. The result is a drastic reduction in "wall-time" on the IBM 370. This necessitated a slight revision in the solver routine, TRIMSS, as written by Billy Gibbs, U.S. Army, Redstone Arsenal [14]. So here it is called TRMSTR or TRIMSS transposed.

First, the node is checked to determine its location with respect to I, and I-dependent strains (or the partial strain, SYZI) are calculated. Then the loop DO 392 establishes the correct layer number, M, in order to check if J lies on the interface, INF(M). Upon determining the correct location of the node with respect to J, the J-dependent strains (or the partial strain, SYZJ) are calculated. Statement number 391 totals the partial strains to obtain SYZ. The stresses are then calculated in a straight forward manner using equation (1). Note that the stresses are calculated twice at interface nodes, once for the material below the interface and again for the material above.

Part VI. The subroutine MATCON calculates the MATERIAL CONstants C_j , BU, BV, and DV as defined earlier in the text.

Part VII. The subroutine MAMULT is a MATRIX MULTIplier and is easily understood from the listing.

Part VIII. The subroutine MATIN4 is a MATRIX INversion routine which is described in Reference 14.

Part IX. The subroutine TRMSTR is the equation solver which is described in the listing.

Part X. The subroutine RITE is used to WRITE out a matrix or vector.

Program Listing

The complete listing of the program is contained in the following pages.

```

C JQMAX IS THE NUMBER OF UNKNOWN OR EQUATIONS TO BE SOLVED.          00000010
C A IS A FULL MATRIX (3 X JQMAX) REPRESENTING EACH NODE.              00000020
C AX IS THE BANDED MATRIX (NBAND+1 X JQMAX).                          00000030
C X IS THE LOAD VECTOR. AFTER TRIMSS X BECOMES THE SOLUTION VECTOR.    00000040
C                                                                        00000050
C IF THE NUMBER OF LAYERS EXCEED 6, THE COMMON /MC/ AND DIMENSION (E11,00000060
C E22, ETC.) STATEMENTS MUST BE REDIMENSIONED TO AGREE WITH LAT.      00000070
C REMEMBER TO PLACE A COMMON /MC/ STATEMENT IN SUBROUTINE MATCON.     00000080
C                                                                        00000090
C USE THE FOLLOWING ORDER FOR DATA CARDS                               00000100
C                                                                        00000110
C DATA CARD NO.              DATA              FORMAT 00000120
C 1          NLAY, LAT, LAW, FSW1, K            5I10  00000130
C 2          H                                G12.5 00000140
C 3          E11, E22, E33, E12, E13, E23      8G12.5 00000150
C 4          NU12, NU13, NU23                  8G12.5 00000160
C 5          ALPHA 1 PRIME, ALPHA 2 PRIME, ALPHA 3 PRIME 8G12.5 00000170
C NOTE, REPEAT CARDS OF THE TYPE 3, 4, 5 FOR EACH ADDITIONAL LAYER 00000180
C 6          SXMAX, C3E                        10G10.3 00000190
C 7          IRUN                              5I10  00000200
C 8          THETA(1), THETA(2), THETA(3), ETC. 10G10.3 00000210
C NOTE, REPEAT CARD 8 FOR EACH ADDITIONAL LAYUP.                      00000220
C                                                                        00000230
C                                                                        00000240
0001          INTEGER P, FSW1, FSW2          00000250
0002          DOUBLE PRECISION TEST, R, ERR, AVE, DT                    00000260
0003          DOUBLE PRECISION AX, X                                       00000270
0004          DOUBLE PRECISION THETA, BETA                                  00000280
0005          DOUBLE PRECISION CM, CN, CM4, CN4, CM3N, CN3M, CM2, CN2, GNU21, 00000290
1          GNU31, GNU32, DET, CP11, CP22, CP33, CP12, CP13, 00000300
2          CP23, CP44, CP55, CP66                                          00000310
C                                                                        00000320
0006          DIMENSION AX(162,351),A(3,351), X(351), R(351)            00000330
C                                                                        00000340
0007          COMMON /MC/ C11(6),C12(6),C16(6),C22(6),C26(6),C66(6),C13(6), 00000350
1          C23(6),C36(6),C44(6),C45(6),C55(6),C33(6),AL1(6),AL2(6), 00000360
2          AL3(6),AL6(6),C2,C3,C3E,C4,BU,DU,BV,DV,H,SXMAX,NLAY,INF(6) 00000370
C                                                                        00000380
0008          DIMENSION E11(6),E22(6),E33(6),E12(6),E13(6),E23(6),GNU12(6), 00000390
1          GNU13(6),GNU23(6),THETA(6), AL1P(6), AL2P(6), AL3P(6) 00000400
C                                                                        00000410
0009          TEMP = 0.0                                                  00000420
C                                                                        00000430
0010          WRITE(6,600)                                                00000440
0011          READ(5,601)NLAY,LAT,LAW,FSW1,K                              00000450
C                                                                        00000460
0012          FSW2=LAW-FSW1+1                                              00000470
0013          JQMAX = 3*LAW*LAT                                           00000480
0014          IBW = 2*(3*LAT+1)                                           00000490
0015          IBW1 = IBW+1                                                00000500
0016          NBAND = 2*IBW+1                                             00000510
C                                                                        00000520
0017          WRITE(6,602)NLAY,LAT,LAW,FSW1,FSW2,K                      00000530
0018          LAT1=LAT-1                                                  00000540
0019          IMID = (LAW+1)/2                                           00000550
0020          JMID = (LAT+1)/2                                           00000560
C                                                                        00000570
0021          DO 501 M=1, NLAY                                           00000580

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0022          INF(M)=1+M*LAT1/NLAY          00000590
0023          WRITE(6,608)M,INF(M)        00000600
0024          501 CONTINUE                 00000610
C          C                               00000620
C          NOTE THAT INF(NLAY) EQUALS LAT AND IS NOT AN ACTUAL INTERFACE. 00000630
C          C                               00000640
0025          READ(5,603) H                00000650
0026          WRITE(6,607) H              00000660
0027          HSQ = H**2                  00000670
C          C                               00000680
0028          WRITE(6,604)                00000690
C          C                               00000700
0029          DO 500 M=1,NLAY              00000710
0030          READ(5,603)E11(M),E22(M),E33(M),E12(M),E13(M),E23(M) 00000720
0031          READ(5,603)GNU12(M),GNU13(M),GNU23(M) 00000730
0032          WRITE(6,605) M, E11(M), E22(M), E33(M), E12(M), E13(M), E23(M), 00000740
          1 GNU12(M), GNU13(M), GNU23(M) 00000750
0033          READ(5,603)AL1P(M), AL2P(M), AL3P(M) 00000760
0034          500 CONTINUE                 00000770
C          C                               00000780
0035          READ(5,606) SXMAX, C3E       00000790
0036          READ(5,601) IRUN            00000800
C          C                               00000810
0037          DO 9000 IRAN = 1, IRUN       00000820
0038          READ(5,606) (THETA(M),M=1,NLAY) 00000830
C          C                               00000840
C          *****00000850
C          C                               00000860
C          CALCULATION OF ANISOTROPIC STIFFNESS MATRIX TERMS REFERRED TO X,Y,Z 00000870
C          C                               00000880
C          *****00000890
C          C                               00000900
0039          WRITE(6,613)                 00000910
0040          XX = 0.0                     00000920
0041          DO 3001 M=1,NLAY              00000930
0042          BETA = .017453292519943300*THETA(M) 00000940
0043          CM=DCOS(BETA)                 00000950
0044          CN=DSIN(BETA)                 00000960
0045          IF(DABS(CM).LT.1.E-08) CM = 0. 00000970
0046          IF(DABS(CN).LT.1.E-08) CN = 0. 00000980
0047          CM4=CM**4                     00000990
0048          CN4=CN**4                     00001000
0049          CM3N=CM**3*CN                 00001010
0050          CN3M=CN**3*CM                 00001020
0051          CM2=CM**2                     00001030
0052          CN2=CN**2                     00001040
0053          GNU21=GNU12(M)*E22(M)/E11(M) 00001050
0054          GNU31=GNU13(M)*E33(M)/E11(M) 00001060
0055          GNU32=GNU23(M)*E33(M)/E22(M) 00001070
0056          DET=1.-GNU12(M)*GNU21-GNU23(M)*GNU32-GNU13(M)*GNU31 00001080
          1-2.*GNU12(M)*GNU23(M)*GNU31 00001090
0057          CP11=E11(M)*(1.-GNU23(M)*GNU32)/DET 00001100
0058          CP22=E22(M)*(1.-GNU13(M)*GNU31)/DET 00001110
0059          CP33=E33(M)*(1.-GNU12(M)*GNU21)/DET 00001120
0060          CP12=E11(M)*(GNU21+GNU23(M)*GNU31)/DET 00001130
0061          CP13=E11(M)*(GNU31+GNU21*GNU32)/DET 00001140
0062          CP23=E22(M)*(GNU32+GNU12(M)*GNU31)/DET 00001150
0063          CP44=E23(M)                   00001160

```

```

0064          CP55=E13(M)                                00001170
0065          CP66=E12(M)                                00001180
0066          C11(M)=CM4*CP11+2.*CM2*CN2*CP12+CN4*CP22+4.*CM2*CN2*CP66 00001190
0067          C12(M)=CM2*CN2*CP11+(CM4+CN4)*CP12+CM2*CN2*CP22-CM2*CN2*4.*CP66 00001200
0068          C16(M)=CM3N*CP11-(CM3N-CN3M)*CP12-CN3M*CP22-2.*(CM3N-CN3M)*CP66 00001210
0069          C22(M)=CN4*CP11+2.*CM2*CN2*CP12+CM4*CP22+4.*CM2*CN2*CP66 00001220
0070          C26(M)=CN3M*CP11-(CN3M-CN3N)*CP12-CN3N*CP22-2.*(CN3M-CN3N)*CP66 00001230
0071          C66(M)=CM2*CN2*CP11-2.*CM2*CN2*CP12+CM2*CN2*CP22+(CM2-CN2)**2*CP66 00001240
0072          C13(M)=CM2*CP13+CN2*CP23                    00001250
0073          C23(M)=CN2*CP13+CM2*CP23                    00001260
0074          C36(M)=CM*CN*(CP13-CP23)                   00001270
0075          C44(M)=CM2*CP44+CN2*CP55                   00001280
0076          C45(M)=CM*CN*(CP55-CP44)                   00001290
0077          C55(M)=CN2*CP44+CM2*CP55                   00001300
0078          C33(M)=CP33                                  00001310
C                                                     00001320
C                                                     00001330
C*****00001340
C                                                     00001350
C CALCULATION OF THE COEF. OF THERMAL EXPANSION REFERRED TO X,Y,Z 00001360
C                                                     00001370
C*****00001380
C                                                     00001390
0079          AL1(M)=CM2*AL1P(M)+CN2*AL2P(M)             00001400
0080          AL2(M)=CN2*AL1P(M)+CM2*AL2P(M)             00001410
0081          AL3(M)=AL3P(M)                              00001420
0082          AL6(M)=2.*CM*CN*(AL1P(M)-AL2P(M))          00001430
C                                                     00001440
0083          WRITE(6,620) M, C11(M), C12(M), C13(M), XX, XX, C16(M), CP11, 00001450
1              CP12, CP13, XX, XX, XX, C22(M), C23(M), XX, XX, 00001460
2              C26(M), CP22, CP23, XX, XX, XX, C33(M), XX, XX, 00001470
3              C36(M), CP33, XX, XX, XX, THETA(M), C44(M), C45(M), 00001480
4              XX, CP44, XX, XX, C55(M), XX, CP55, XX, C66(M), CP66 00001490
C                                                     00001500
0084          3001 CONTINUE                               00001510
C                                                     00001520
0085          WRITE(6,611)                                00001530
C                                                     00001540
0086          DO 503 M=1,NLAY                             00001550
0087          WRITE(6,614) M, THETA(M), AL1(M), AL2(M), AL3(M), AL6(M), 00001560
1              AL1P(M), AL2P(M), AL3P(M)                 00001570
0088          503 CONTINUE                               00001580
C                                                     00001590
0089          CALL MATCON                                  00001600
C                                                     00001610
C                                                     00001620
C*****00001630
C                                                     00001640
C CALCULATION OF THE COEFFICIENT MATRIX FOR THE DIFFERENCE EQUATIONS 00001650
C                                                     00001660
C*****00001670
C                                                     00001680
C                                                     00001690
0090          KJ1 = 1                                     00001700
0091          KQ1 = KJ1 + 1                               00001710
0092          KQ2 = KJ1 + 2                               00001720
C                                                     00001730
0093          DO 100 I=1,LAW                              00001740
0094          DO 101 J=1,LAT

```

```

C
0095      DO 3000 IM = KJ1, KQ2
0096      DO 3000 JM = 1, JQMAX
0097      A(IM,JM) = 0.
0098      3000 CONTINUE
C
0099      I1=I-1
0100      I2=I-2
0101      Z = (FLOAT(J)-(FLOAT(LAT)+1.)/2.)*H
0102      NODE = LAT*I1+J
0103      JJ1 = 3*(LAT*I1+J)-2
0104      JJ2 = 3*(LAT*I2+J)-2
0105      JJ3 = 3*(LAT*I2+J)-5
0106      JJ4 = 3*(LAT*I+J)-2
0107      JJ5 = 3*(LAT*I+J)+1
0108      JJ6 = 3*(LAT*I1+J)+1
0109      JJ7 = 3*(LAT*I2+J)+1
0110      JJ8 = 3*(LAT*I1+J)-5
0111      JJ9 = 3*(LAT*I+J)-5
0112      JJ10 = 3*(LAT*I1+J)-8
0113      JJ11 = 3*(LAT*(I+1)+J)-2
0114      JJ12 = 3*(LAT*I1+J)+4
0115      JJ13 = 3*(LAT*(I-3)+J)-2
C
0116      JQ1 = JJ1+1
0117      JQ2 = JJ1+2
C
0118      DO 102 M=1, NLAY
0119      IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 102
0120      IF(M.EQ.1) GO TO 192
0121      IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 102
0122      192 IF(J.EQ.1) GO TO 200
0123      IF(I.EQ.IMID.AND.J.EQ.JMID) GO TO 203
0124      IF(I.EQ.IMID+1.AND.J.EQ.JMID) GO TO 203
0125      IF(J.EQ.LAT) GO TO 202
0126      IF(J.EQ.INF(M)) GO TO 201
C
C SHOULD J EQUAL NONE OF THE ABOVE, CONTINUE ON BELOW TO STATEMENT 193
C
0127      193 IF(I.EQ.1) GO TO 194
0128      IF(I.EQ.FSW1.OR.I.EQ.FSW2) GO TO 195
0129      IF(I.LT.FSW2.AND.I.GT.FSW1) GO TO 197
0130      IF(I.EQ.LAW) GO TO 198
C
C EQUILIBRIUM MATRIX TERMS FOR A SQUARE MESH, H1=H2=H3=H
C
0131      A(KJ1,JJ1) = -8.*(C66(M)+C55(M))
0132      A(KJ1,JJ2) = 4.*C66(M)
0133      A(KJ1,JJ4) = 4.*C66(M)
0134      A(KJ1,JJ6) = 4.*C55(M)
0135      A(KJ1,JJ8) = 4.*C55(M)
0136      A(KJ1,JJ1+1) = -8.*(C26(M)+C45(M))
0137      A(KJ1,JJ2+1) = 4.*C26(M)
0138      A(KJ1,JJ4+1) = 4.*C26(M)
0139      A(KJ1,JJ6+1) = 4.*C45(M)
0140      A(KJ1,JJ8+1) = 4.*C45(M)
C
0141      C = C36(M)+C45(M)

```

```

00001750
00001760
00001770
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00002240
00002250
00002260
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00002290
00002300
00002310
00002320

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0142      C      A(KJ1, JJ3+2) = C      00002330
0143      A(KJ1, JJ5+2) = C      00002340
0144      A(KJ1, JJ7+2) = -C     00002350
0145      A(KJ1, JJ9+2) = -C     00002360
                                00002370
0146      C      X(JJ1) = 0.      00002380
                                00002390
                                00002400
0147      C      A(KQ1, JJ1) = -8.*(C26(M)+C45(M)) 00002410
0148      A(KQ1, JJ2) = 4.*C26(M) 00002420
0149      A(KQ1, JJ4) = 4.*C26(M) 00002430
0150      A(KQ1, JJ6) = 4.*C45(M) 00002440
0151      A(KQ1, JJ8) = 4.*C45(M) 00002450
0152      A(KQ1, JJ1+1) = -8.*(C22(M)+C44(M)) 00002460
0153      A(KQ1, JJ2+1) = 4.*C22(M) 00002470
0154      A(KQ1, JJ4+1) = 4.*C22(M) 00002480
0155      A(KQ1, JJ6+1) = 4.*C44(M) 00002490
0156      A(KQ1, JJ8+1) = 4.*C44(M) 00002500
                                00002510
0157      C      D = C23(M)+C44(M) 00002520
                                00002530
0158      C      A(KQ1, JJ3+2) = D 00002540
0159      A(KQ1, JJ5+2) = D 00002550
0160      A(KQ1, JJ7+2) = -D     00002560
0161      A(KQ1, JJ9+2) = -D     00002570
                                00002580
0162      C      X(JQ1) = 0.      00002590
                                00002600
0163      C      A(KQ2, JJ3) = C 00002610
0164      A(KQ2, JJ5) = C 00002620
0165      A(KQ2, JJ7) = -C      00002630
0166      A(KQ2, JJ9) = -C      00002640
0167      A(KQ2, JJ3+1) = D 00002650
0168      A(KQ2, JJ5+1) = D 00002660
0169      A(KQ2, JJ7+1) = -D     00002670
0170      A(KQ2, JJ9+1) = -D     00002680
0171      A(KQ2, JJ1+2) = -8.*(C44(M)+C33(M)) 00002690
0172      A(KQ2, JJ2+2) = 4.*C44(M) 00002700
0173      A(KQ2, JJ4+2) = 4.*C44(M) 00002710
0174      A(KQ2, JJ6+2) = 4.*C33(M) 00002720
0175      A(KQ2, JJ8+2) = 4.*C33(M) 00002730
                                00002740
0176      C      X(JQ2) = -4.*(C13(M)*C2 + C23(M)*DV + 2.*C36(M)*C4)*HSQ 00002750
0177      GO TO 102              00002760
                                00002770
0178      C      FREE SURFACE MATRIX TERMS FOR I=1 AND J NOT EQUAL TO 1, INF OR LAT 00002780
0179      C      194 A(KJ1, JJ1) = -3.*C66(M) 00002790
0180      A(KJ1, JJ4) = 4.*C66(M) 00002800
0181      A(KJ1, JJ11) = -C66(M) 00002810
0182      A(KJ1, JJ1+1) = -3.*C26(M) 00002820
0183      A(KJ1, JJ4+1) = 4.*C26(M) 00002830
0184      A(KJ1, JJ11+1) = -C26(M) 00002840
0185      A(KJ1, JJ6+2) = C36(M) 00002850
                                00002860
                                00002870
0186      C      A(KQ1, JJ1) = -3.*C26(M) 00002880
0187      A(KQ1, JJ4) = 4.*C26(M) 00002890
                                00002900

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0188      A(KQ1,JJ11) = -C26(M)          00002910
0189      A(KQ1,JJ1+1) = -3.*C22(M)     00002920
0190      A(KQ1,JJ4+1) = 4.*C22(M)      00002930
01  1     A(KQ1,JJ11+1) = -C22(M)       00002940
01       A(KQ1,JJ6+2) = C23(M)          00002950
0193     A(KQ1,JJ8+2) = -C23(M)         00002960
        C                                00002970
0194     A(KQ2,JJ6) = C45(M)             00002980
01  35    A(KQ2,JJ8) = -C45(M)           00002990
0196     A(KQ2,JJ6+1) = C44(M)          00003000
0197     A(KQ2,JJ8+1) = -C44(M)         00003010
0198     A(KQ2,JJ1+2) = -3.*C44(M)      00003020
0199     A(KQ2,JJ4+2) = 4.*C44(M)        00003030
0200     A(KQ2,JJ11+2) = -C44(M)        00003040
        C                                00003050
0201     CY1 = C12(M)*C3 + C22(M)*BV + C26(M)*BU 00003060
0202     CY2 = C12(M)*C2 + C22(M)*DV + 2.*C26(M)*C4 00003070
0203     CXY1 = C16(M)*C3 + C26(M)*BV + C66(M)*BU 00003080
0204     CXY2 = C16(M)*C2 + C26(M)*DV + 2.*C66(M)*C4 00003090
        C                                00003100
0205     X(JJ1) = -2.*H*(CXY1 + CXY2*Z) 00003110
0206     X(JQ1) = -2.*H*(CY1 + CY2*Z)    00003120
0207     X(JQ2) = 0.                     00003130
0208     GO TO 102                        00003140
        C                                00003150
0209     195 H1 = H                       00003160
0210     H2 = FLOAT(K)*H                  00003170
0211     H3 = H                           00003180
        C                                00003190
0212     IF(I,NE,FSW2) GO TO 196          00003200
0213     H1 = FLOAT(K)*H                  00003210
0214     H2 = H                           00003220
        C                                00003230
0215     196 CONTINUE                     00003240
0216     HH = H2/H1                       00003250
0217     HR = HH/(1.+HH)                  00003260
0218     HH1 = H1/H3                      00003270
0219     HH2 = H2/H3                      00003280
0220     HH3 = H1*H2                      00003290
0221     HMU = HH1*HH2                    00003300
0222     GO TO 199                        00003310
        C                                00003320
0223     197 H1 = FLOAT(K)*H              00003330
0224     H2 = H1                          00003340
0225     H3 = H                            00003350
0226     GO TO 196                        00003360
        C                                00003370
        C FREE SURFACE MATRIX TERMS FOR I=LAW AND J NOT EQUAL TO 1, INF OR LAT 00003380
        C                                00003390
0227     198 A(KJ1,JJ1) = 3.*C66(M)       00003400
0228     A(KJ1,JJ2) = -4.*C66(M)         00003410
0229     A(KJ1,JJ13) = C66(M)            00003420
0230     A(KJ1,JJ1+1) = 3.*C26(M)        00003430
0231     A(KJ1,JJ2+1) = -4.*C26(M)       00003440
0232     A(KJ1,JJ13+1) = C26(M)          00003450
0233     A(KJ1,JJ6+2) = C36(M)           00003460
0234     A(KJ1,JJ8+2) = -C36(M)          00003470
        C                                00003480

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0235          A(KQ1, JJ1) = 3.*C26(M)          00003490
0236          A(KQ1, JJ2) = -4.*C26(M)        00003500
0237          A(KQ1, JJ13) = C26(M)           00003510
0238          A(KQ1, JJ1+1) = 3.*C22(M)       00003520
0239          A(KQ1, JJ2+1) = -4.*C22(M)     00003530
0240          A(KQ1, JJ13+1) = C22(M)         00003540
0241          A(KQ1, JJ6+2) = C23(M)         00003550
0242          A(KQ1, JJ8+2) = -C23(M)         00003560
          C
0243          A(KQ2, JJ6) = C45(M)            00003570
0244          A(KQ2, JJ8) = -C45(M)          00003580
0245          A(KQ2, JJ6+1) = C44(M)         00003590
0246          A(KQ2, JJ8+1) = -C44(M)        00003600
0247          A(KQ2, JJ1+2) = 3.*C44(M)      00003610
0248          A(KQ2, JJ2+2) = -4.*C44(M)     00003620
0249          A(KQ2, JJ13+2) = C44(M)        00003640
          C
0250          CY1 = C12(M)*C3 + C22(M)*BV + C26(M)*BU 00003650
0251          CY2 = C12(M)*C2 + C22(M)*DV + 2.*C26(M)*C4 00003660
0252          CXY1 = C16(M)*C3 + C26(M)*BV + C66(M)*BU 00003670
0253          CXY2 = C16(M)*C2 + C26(M)*DV + 2.*C66(M)*C4 00003680
          C
0254          X(JJ1) = -2.*H*(CXY1 + CXY2*Z) 00003700
0255          X(JQ1) = -2.*H*(CY1 + CY2*Z)   00003710
0256          X(JQ2) = 0.                    00003720
0257          GO TO 102                      00003730
          C
          C EQUILIBRIUM MATRIX TERMS FOR A VARIABLE MESH, H1, H2 , H3 INDEPENDENT 00003740
          C
0258          199 A(KJ1, JJ1) = -2.*(C66(M)+HMU*C55(M)) 00003750
0259          A(KJ1, JJ2) = 2.*HR*C66(M)      00003760
0260          A(KJ1, JJ4) = 2.*C66(M)/(1.+HH) 00003770
0261          A(KJ1, JJ6) = HMU*C55(M)        00003780
0262          A(KJ1, JJ8) = HMU*C55(M)        00003790
0263          A(KJ1, JJ1+1) = -2.*(C26(M)+HMU*C45(M)) 00003800
0264          A(KJ1, JJ2+1) = 2.*HR*C26(M)    00003810
0265          A(KJ1, JJ4+1) = 2.*C26(M)/(1.+HH) 00003820
0266          A(KJ1, JJ6+1) = HMU*C45(M)      00003830
0267          A(KJ1, JJ8+1) = HMU*C45(M)      00003840
          C
0268          C = HH1*HR*(C36(M)+C45(M))/2.   00003850
          C
0269          A(KJ1, JJ3+2) = C                00003860
0270          A(KJ1, JJ5+2) = C                00003870
0271          A(KJ1, JJ7+2) = -C              00003880
0272          A(KJ1, JJ9+2) = -C              00003890
          C
0273          A(KQ1, JJ1) = -2.*(C26(M)+HMU*C45(M)) 00003900
0274          A(KQ1, JJ2) = 2.*HR*C26(M)      00003910
0275          A(KQ1, JJ4) = 2.*C26(M)/(1.+HH) 00003920
0276          A(KQ1, JJ6) = HMU*C45(M)        00003930
0277          A(KQ1, JJ8) = HMU*C45(M)        00003940
0278          A(KQ1, JJ1+1) = -2.*(C22(M)+HMU*C44(M)) 00003950
0279          A(KQ1, JJ2+1) = 2.*HR*C22(M)    00003960
0280          A(KQ1, JJ4+1) = 2.*C22(M)/(1.+HH) 00003970
0281          A(KQ1, JJ6+1) = HMU*C44(M)      00003980
0282          A(KQ1, JJ8+1) = HMU*C44(M)      00003990
          C

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0283          D = HH1*HR*(C23(M)+C44(M))/2.          00004070
C                                                    00004080
0284          A(KQ1,JJ3+2) = D                      00004090
0285          A(KQ1,JJ5+2) = D                      00004100
0286          A(KQ1,JJ7+2) = -D                     00004110
0287          A(KQ1,JJ9+2) = -D                     00004120
C                                                    00004130
0288          A(KQ2,JJ3) = C                         00004140
0289          A(KQ2,JJ5) = C                         00004150
0290          A(KQ2,JJ7) = -C                        00004160
0291          A(KQ2,JJ9) = -C                        00004170
0292          A(KQ2,JJ3+1) = D                       00004180
0293          A(KQ2,JJ5+1) = D                       00004190
0294          A(KQ2,JJ7+1) = -D                      00004200
0295          A(KQ2,JJ9+1) = -D                      00004210
0296          A(KQ2,JJ1+2) = -2.*(C44(M)+HMU*C33(M)) 00004220
0297          A(KQ2,JJ2+2) = 2.*HR*C44(M)           00004230
0298          A(KQ2,JJ4+2) = 2.*C44(M)/(1.+HH)      00004240
0299          A(KQ2,JJ6+2) = HMU*C33(M)              00004250
0300          A(KQ2,JJ8+2) = HMU*C33(M)              00004260
C                                                    00004270
0301          X(JJ1) = 0.                            00004280
0302          X(JQ1) = 0.                            00004290
0303          X(JQ2) = -HH3*(C13(M)*C2 + C23(M)*DV + 2.*C36(M)*C4) 00004300
0304          GO TO 102                               00004310
C                                                    00004320
0305          200 IF(I.EQ.1) GO TO 210                00004330
0306          IF(I.EQ.LAW) GO TO 211                  00004340
C                                                    00004350
C FREE SURFACE MATRIX TERMS FOR I BETWEEN 1 AND LAW AND J=1. 00004360
C                                                    00004370
0307          A(KJ1,JJ1) = -3.*C55(M)                00004380
0308          A(KJ1,JJ6) = 4.*C55(M)                 00004390
0309          A(KJ1,JJ12) = -C55(M)                  00004400
C                                                    00004410
0310          A(KJ1,JJ1+1) = -3.*C45(M)              00004420
0311          A(KJ1,JJ6+1) = 4.*C45(M)              00004430
0312          A(KJ1,JJ12+1) = -C45(M)                00004440
C                                                    00004450
0313          A(KQ1,JJ1) = -3.*C45(M)                00004460
0314          A(KQ1,JJ6) = 4.*C45(M)                00004470
0315          A(KQ1,JJ12) = -C45(M)                  00004480
C                                                    00004490
0316          A(KQ1,JJ1+1) = -3.*C44(M)              00004500
0317          A(KQ1,JJ6+1) = 4.*C44(M)              00004510
0318          A(KQ1,JJ12+1) = -C44(M)                00004520
C                                                    00004530
0319          A(KQ2,JJ1+2) = -3.*C33(M)              00004540
0320          A(KQ2,JJ6+2) = 4.*C33(M)              00004550
0321          A(KQ2,JJ12+2) = -C33(M)                00004560
C                                                    00004570
0322          CZ1 = C13(M)*C3 + C23(M)*BV + C36(M)*BU 00004580
0323          CZ2 = C13(M)*C2 + C23(M)*DV + 2.*C36(M)*C4 00004590
C                                                    00004600
0324          X(JJ1) = 0.                            00004610
0325          X(JQ1) = 0.                            00004620
0326          X(JQ2) = -2.*H*(CZ1 + CZ2*Z)           00004630
C                                                    00004640

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0327		IF(I.EQ.FSW1) GO TO 206	00004650
0328		IF(I.EQ.FSW2) GO TO 206	00004660
0329		IF(I.GT.FSW1.AND.I.LT.FSW2) GO TO 209	00004670
	C		00004680
	C	IF I IS BETWEEN 1 AND FSW1 OR BETWEEN FSW2 AND LAW, CONTINUE BELOW	00004690
	C		00004700
0330		A(KJ1, JJ2+2) = -C45(M)	00004710
0331		A(KJ1, JJ4+2) = C45(M)	00004720
	C		00004730
0332		A(KQ1, JJ2+2) = -C44(M)	00004740
0333		A(KQ1, JJ4+2) = C44(M)	00004750
	C		00004760
0334		A(KQ2, JJ2) = -C36(M)	00004770
0335		A(KQ2, JJ4) = C36(M)	00004780
0336		A(KQ2, JJ2+1) = -C23(M)	00004790
0337		A(KQ2, JJ4+1) = C23(M)	00004800
0338		GO TO 102	00004810
	C		00004820
	C	CASE WHERE I=FSW1 OR FSW2 AND J=1	00004830
	C		00004840
0339		206 XK = FLOAT(K)	00004850
0340		D1 = 2.*(XK-1.)/XK	00004860
0341		D2 = 2.*XK/(XK+1.)	00004870
0342		D3 = 2./((XK+1.)*XK)	00004880
	C		00004890
0343		IF(I.EQ.FSW2) GO TO 207	00004900
	C		00004910
0344		A(KJ1, JJ1+2) = D1*C45(M)	00004920
0345		A(KJ1, JJ2+2) = -D2*C45(M)	00004930
0346		A(KJ1, JJ4+2) = D3*C45(M)	00004940
	C		00004950
0347		A(KQ1, JJ1+2) = D1*C44(M)	00004960
0348		A(KQ1, JJ2+2) = -D2*C44(M)	00004970
0349		A(KQ1, JJ4+2) = D3*C44(M)	00004980
	C		00004990
0350		A(KQ2, JJ1) = D1*C36(M)	00005000
0351		A(KQ2, JJ2) = -D2*C36(M)	00005010
0352		A(KQ2, JJ4) = D3*C36(M)	00005020
	C		00005030
0353		A(KQ2, JJ1+1) = D1*C23(M)	00005040
0354		A(KQ2, JJ2+1) = -D2*C23(M)	00005050
0355		A(KQ2, JJ4+1) = D3*C23(M)	00005060
0356		GO TO 102	00005070
	C		00005080
0357		207 A(KJ1, JJ1+2) = -D1*C45(M)	00005090
0358		A(KJ1, JJ2+2) = -D3*C45(M)	00005100
0359		A(KJ1, JJ4+2) = D2*C45(M)	00005110
	C		00005120
0360		A(KQ1, JJ1+2) = -D1*C44(M)	00005130
0361		A(KQ1, JJ2+2) = -D3*C44(M)	00005140
0362		A(KQ1, JJ4+2) = D2*C44(M)	00005150
	C		00005160
0363		A(KQ2, JJ1) = -D1*C36(M)	00005170
0364		A(KQ2, JJ2) = -D3*C36(M)	00005180
0365		A(KQ2, JJ4) = D2*C36(M)	00005190
	C		00005200
0366		A(KQ2, JJ1+1) = -D1*C23(M)	00005210
0367		A(KQ2, JJ2+1) = -D3*C23(M)	00005220

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0368          A(KQ2,JJ4+1) = D2*C23(M)          00005230
0369          GO TO 102                          00005240
C                                                  00005250
C CASE WHERE I IS BETWEEN FSW1 AND FSW2 AND J=1 00005260
C                                                  00005270
0370          209 XK = FLOAT(K)                  00005280
0371          A(KJ1,JJ2+2) = -C45(M)/XK          00005290
0372          A(KJ1,JJ4+2) = C45(M)/XK          00005300
C                                                  00005310
0373          A(KQ1,JJ2+2) = -C44(M)/XK          00005320
0374          A(KQ1,JJ4+2) = C44(M)/XK          00005330
C                                                  00005340
0375          A(KQ2,JJ2) = -C36(M)/XK           00005350
0376          A(KQ2,JJ4) = C36(M)/XK           00005360
C                                                  00005370
0377          A(KQ2,JJ2+1) = -C23(M)/XK         00005380
0378          A(KQ2,JJ4+1) = C23(M)/XK         00005390
0379          GO TO 102                          00005400
C                                                  00005410
C FREE SURFACE MATRIX TERMS FOR I=J=1          00005420
C                                                  00005430
C                                                  00005440
0380          210 A(KJ1,JJ1) = -3.*C66(M)        00005450
0381          A(KJ1,JJ4) = 4.*C66(M)           00005460
0382          A(KJ1,JJ11) = -C66(M)            00005470
0383          A(KJ1,JJ1+1) = -3.*C26(M)         00005480
0384          A(KJ1,JJ4+1) = 4.*C26(M)         00005490
0385          A(KJ1,JJ11+1) = -C26(M)          00005500
0386          A(KJ1,JJ1+2) = -3.*C36(M)        00005510
0387          A(KJ1,JJ6+2) = 4.*C36(M)         00005520
0388          A(KJ1,JJ12+2) = -C36(M)          00005530
C                                                  00005540
0389          A(KQ1,JJ1) = -3.*C26(M)           00005550
0390          A(KQ1,JJ4) = 4.*C26(M)           00005560
0391          A(KQ1,JJ11) = -C26(M)            00005570
0392          A(KQ1,JJ1+1) = -3.*C22(M)        00005580
0393          A(KQ1,JJ4+1) = 4.*C22(M)         00005590
0394          A(KQ1,JJ11+1) = -C22(M)          00005600
0395          A(KQ1,JJ1+2) = -3.*C23(M)        00005610
0396          A(KQ1,JJ6+2) = 4.*C23(M)         00005620
0397          A(KQ1,JJ12+2) = -C23(M)          00005630
C                                                  00005640
0398          A(KQ2,JJ1) = -3.*C45(M)           00005650
0399          A(KQ2,JJ6) = 4.*C45(M)           00005660
0400          A(KQ2,JJ12) = -C45(M)            00005670
0401          A(KQ2,JJ1+1) = -3.*C44(M)        00005680
0402          A(KQ2,JJ6+1) = 4.*C44(M)         00005690
0403          A(KQ2,JJ12+1) = -C44(M)          00005700
0404          A(KQ2,JJ1+2) = -3.*C44(M)        00005710
0405          A(KQ2,JJ4+2) = 4.*C44(M)         00005720
0406          A(KQ2,JJ11+2) = -C44(M)          00005730
C                                                  00005740
0407          CY1 = C12(M)*C3 + C22(M)*BV + C26(M)*BU 00005750
0408          CY2 = C12(M)*C2 + C22(M)*DV + 2.*C26(M)*C4 00005760
0409          CXY1 = C16(M)*C3 + C26(M)*BV + C66(M)*BU 00005770
0410          CXY2 = C16(M)*C2 + C26(M)*DV + 2.*C66(M)*C4 00005780
C                                                  00005790
0411          X(JJ1) = -2.*H*(CXY1 + CXY2*Z)    00005800
0412          X(JQ1) = -2.*H*(CY1 + CY2*Z)

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0413           X(JQ2) = 0.                00005810
0414           GO TO 102                  00005820
C                                                     00005830
C FREE SURFACE MATRIX TERMS FOR J=1 AND I=LAW      00005840
C                                                     00005850
0415     211 A(KJ1, JJ1) = 3.*C66(M)      00005860
0416           A(KJ1, JJ2) = -4.*C66(M)    00005870
0417           A(KJ1, JJ13) = C66(M)       00005880
0418           A(KJ1, JJ1+1) = 3.*C26(M)   00005890
0419           A(KJ1, JJ2+1) = -4.*C26(M)  00005900
0420           A(KJ1, JJ13+1) = C26(M)     00005910
0421           A(KJ1, JJ1+2) = -3.*C36(M)  00005920
0422           A(KJ1, JJ6+2) = 4.*C36(M)   00005930
0423           A(KJ1, JJ12+2) = -C36(M)    00005940
C                                                     00005950
0424           A(KQ1, JJ1) = 3.*C26(M)     00005960
0425           A(KQ1, JJ2) = -4.*C26(M)    00005970
0426           A(KQ1, JJ13) = C26(M)       00005980
0427           A(KQ1, JJ1+1) = 3.*C22(M)   00005990
0428           A(KQ1, JJ2+1) = -4.*C22(M)  00006000
0429           A(KQ1, JJ13+1) = C22(M)     00006010
0430           A(KQ1, JJ1+2) = -3.*C23(M)  00006020
0431           A(KQ1, JJ6+2) = 4.*C23(M)   00006030
0432           A(KQ1, JJ12+2) = -C23(M)    00006040
C                                                     00006050
0433           A(KQ2, JJ1) = -3.*C45(M)    00006060
0434           A(KQ2, JJ6) = 4.*C45(M)     00006070
0435           A(KQ2, JJ12) = -C45(M)      00006080
0436           A(KQ2, JJ1+1) = -3.*C44(M)  00006090
0437           A(KQ2, JJ6+1) = 4.*C44(M)   00006100
0438           A(KQ2, JJ12+1) = -C44(M)    00006110
0439           A(KQ2, JJ1+2) = 3.*C44(M)   00006120
0440           A(KQ2, JJ2+2) = -4.*C44(M)  00006130
0441           A(KQ2, JJ13+2) = C44(M)     00006140
C                                                     00006150
0442           CY1 = C12(M)*C3 + C22(M)*BV + C26(M)*BU 00006160
0443           CY2 = C12(M)*C2 + C22(M)*DV + 2.*C26(M)*C4 00006170
0444           CXY1 = C16(M)*C3 + C26(M)*BV + C66(M)*BU 00006180
0445           CXY2 = C16(M)*C2 + C26(M)*DV + 2.*C66(M)*C4 00006190
C                                                     00006200
0446           X(JJ1) = -2.*H*(CXY1 + CXY2*Z) 00006210
0447           X(JQ1) = -2.*H*(CY1 + CY2*Z) 00006220
0448           X(JQ2) = 0.                00006230
0449           GO TO 102                  00006240
C                                                     00006250
0450     201 P = M+1                    00006260
0451           IF(I.EQ.1) GO TO 220       00006270
0452           IF(I.EQ.FSW1) GO TO 221    00006280
0453           IF(I.LT.FSW2.AND.I.GT.FSW1) GO TO 222 00006290
0454           IF(I.EQ.FSW2) GO TO 221    00006300
0455           IF(I.EQ.LAW) GO TO 223     00006310
C                                                     00006320
C MATRIX TERMS AT INTERFACE FOR I BETWEEN 1 AND FSW1 OR FSW2 AND LAW 00006330
C                                                     00006340
0456           A(KJ1, JJ1) = 3.*(C55(M)+C55(P)) 00006350
0457           A(KJ1, JJ6) = -4.*C55(P)    00006360
0458           A(KJ1, JJ8) = -4.*C55(M)    00006370
0459           A(KJ1, JJ10) = C55(M)      00006380

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0460          A(KJ1,JJ12) = C55(P)                                00006390
C                                                    00006400
0461          A(KJ1,JJ1+1) = 3.*(C45(M)+C45(P))                00006410
0462          A(KJ1,JJ6+1) = -4.*C45(P)                        00006420
0463          A(KJ1,JJ8+1) = -4.*C45(M)                        00006430
0464          A(KJ1,JJ10+1) = C45(M)                            00006440
0465          A(KJ1,JJ12+1) = C45(P)                            00006450
C                                                    00006460
0466          A(KJ1,JJ2+2) = C45(P)-C45(M)                      00006470
0467          A(KJ1,JJ4+2) = C45(M)-C45(P)                      00006480
C                                                    00006490
0468          A(KQ1,JJ1) = 3.*(C45(M)+C45(P))                    00006500
0469          A(KQ1,JJ6) = -4.*C45(P)                            00006510
0470          A(KQ1,JJ8) = -4.*C45(M)                            00006520
0471          A(KQ1,JJ10) = C45(M)                               00006530
0472          A(KQ1,JJ12) = C45(P)                               00006540
C                                                    00006550
0473          A(KQ1,JJ1+1) = 3.*(C44(M)+C44(P))                 00006560
0474          A(KQ1,JJ6+1) = -4.*C44(P)                          00006570
0475          A(KQ1,JJ8+1) = -4.*C44(M)                          00006580
0476          A(KQ1,JJ10+1) = C44(M)                             00006590
0477          A(KQ1,JJ12+1) = C44(P)                             00006600
C                                                    00006610
0478          A(KQ1,JJ2+2) = C44(P)-C44(M)                      00006620
0479          A(KQ1,JJ4+2) = C44(M)-C44(P)                      00006630
C                                                    00006640
0480          A(KQ2,JJ2) = C36(P)-C36(M)                         00006650
0481          A(KQ2,JJ4) = C36(M)-C36(P)                         00006660
C                                                    00006670
0482          A(KQ2,JJ2+1) = C23(P)-C23(M)                      00006680
0483          A(KQ2,JJ4+1) = C23(M)-C23(P)                      00006690
C                                                    00006700
0484          A(KQ2,JJ1+2) = 3.*(C33(M)+C33(P))                 00006710
0485          A(KQ2,JJ6+2) = -4.*C33(P)                          00006720
0486          A(KQ2,JJ8+2) = -4.*C33(M)                          00006730
0487          A(KQ2,JJ10+2) = C33(M)                             00006740
0488          A(KQ2,JJ12+2) = C33(P)                             00006750
C                                                    00006760
0489          CZ1 = (C13(P)-C13(M))*C3 + (C23(P)-C23(M))*BV + (C36(P)-C36(M))*BU 00006770
0490          CZ2 = (C13(P)-C13(M))*C2+(C23(P)-C23(M))*DV+2.*(C36(P)-C36(M))*C4 00006780
C                                                    00006790
0491          X(JJ1) = 0.                                         00006800
0492          X(JQ1) = 0.                                         00006810
0493          X(JQ2) = 2.*H*(CZ1 + CZ2*Z)                         00006820
0494          GO TO 102                                           00006830
C                                                    00006840
C FREE SURFACE MATRIX TERMS AT ANY INTERFACE WHERE I=1 AND J=INF OR AT 00006850
C THE FREE SURFACE POINT I=1, J=LAT                             00006860
C                                                    00006870
0495          220 A(KJ1,JJ1) = -3.*C66(M)                         00006880
0496          A(KJ1,JJ4) = 4.*C66(M)                             00006890
0497          A(KJ1,JJ11) = -C66(M)                               00006900
C                                                    00006910
0498          A(KJ1,JJ1+1) = -3.*C26(M)                          00006920
0499          A(KJ1,JJ4+1) = 4.*C26(M)                           00006930
0500          A(KJ1,JJ11+1) = -C26(M)                            00006940
C                                                    00006950
0501          A(KJ1,JJ1+2) = 3.*C36(M)                            00006960

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0502          A(KJ1, JJ8+2) = -4.*C36(M)          00006970
0503          A(KJ1, JJ10+2) = C36(M)             00006980
C                                                    00006990
0504          A(KQ1, JJ1) = -3.*C26(M)           00007000
0505          A(KQ1, JJ4) = 4.*C26(M)            00007010
0506          A(KQ1, JJ11) = -C26(M)             00007020
C                                                    00007030
0507          A(KQ1, JJ1+1) = -3.*C22(M)         00007040
0508          A(KQ1, JJ4+1) = 4.*C22(M)         00007050
0509          A(KQ1, JJ11+1) = -C22(M)          00007060
C                                                    00007070
0510          A(KQ1, JJ1+2) = 3.*C23(M)         00007080
0511          A(KQ1, JJ8+2) = -4.*C23(M)        00007090
0512          A(KQ1, JJ10+2) = C23(M)           00007100
C                                                    00007110
0513          A(KQ2, JJ1) = 3.*C45(M)           00007120
0514          A(KQ2, JJ8) = -4.*C45(M)          00007130
0515          A(KQ2, JJ10) = C45(M)             00007140
C                                                    00007150
0516          A(KQ2, JJ1+1) = 3.*C44(M)         00007160
0517          A(KQ2, JJ8+1) = -4.*C44(M)        00007170
0518          A(KQ2, JJ10+1) = C44(M)           00007180
C                                                    00007190
0519          A(KQ2, JJ1+2) = -3.*C44(M)        00007200
0520          A(KQ2, JJ4+2) = 4.*C44(M)        00007210
0521          A(KQ2, JJ11+2) = -C44(M)         00007220
C                                                    00007230
0522          CY1 = C12(M)*C3 + C22(M)*BV + C26(M)*BU 00007240
0523          CY2 = C12(M)*C2 + C22(M)*DV + 2.*C26(M)*C4 00007250
0524          CX1 = C16(M)*C3 + C26(M)*BV + C66(M)*BU 00007260
0525          CX2 = C16(M)*C2 + C26(M)*DV + 2.*C66(M)*C4 00007270
C                                                    00007280
0526          X(JJ1) = -2.*H*(CX1 + CX2*Z)       00007290
0527          X(JQ1) = -2.*H*(CY1 + CY2*Z)       00007300
0528          X(JQ2) = 0.                        00007310
0529          GO TO 102                          00007320
C                                                    00007330
C MATRIX TERMS AT THE INTERFACE FOR J=INF AND I=FSW1 OR I=FSW2 00007340
C                                                    00007350
221          XK = FLOAT(K)                      00007360
0531          D1 = (XK-1.)/XK                    00007370
0532          D2 = XK/(XK+1.)                   00007380
0533          D3 = 1./((XK+1.)*XK)              00007390
C                                                    00007400
0534          A(KJ1, JJ1) = 3.*(C55(M)+C55(P))   00007410
0535          A(KJ1, JJ6) = -4.*C55(P)          00007420
0536          A(KJ1, JJ8) = -4.*C55(M)          00007430
0537          A(KJ1, JJ10) = C55(M)             00007440
0538          A(KJ1, JJ12) = C55(P)             00007450
C                                                    00007460
0539          A(KJ1, JJ1+1) = 3.*(C45(M)+C45(P)) 00007470
0540          A(KJ1, JJ6+1) = -4.*C45(P)        00007480
0541          A(KJ1, JJ8+1) = -4.*C45(M)        00007490
0542          A(KJ1, JJ10+1) = C45(M)           00007500
0543          A(KJ1, JJ12+1) = C45(P)           00007510
C                                                    00007520
0544          A(KQ1, JJ1) = 3.*(C45(M) + C45(P)) 00007530
0545          A(KQ1, JJ6) = -4.*C45(P)          00007540

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0546      A(KQ1,JJ8) = -4.*C45(M)          00007550
0547      A(KQ1,JJ10) = C45(M)            00007560
0548      A(KQ1,JJ12) = C45(P)            00007570
          C                                00007580
0549      A(KQ1,JJ1+1) = 3.*(C44(M)+C44(P)) 00007590
0550      A(KQ1,JJ6+1) = -4.*C44(P)       00007600
0551      A(KQ1,JJ8+1) = -4.*C44(M)       00007610
0552      A(KQ1,JJ10+1) = C44(M)          00007620
0553      A(KQ1,JJ12+1) = C44(P)          00007630
          C                                00007640
0554      A(KQ2,JJ1+2) = 3.*(C33(M)+C33(P)) 00007650
0555      A(KQ2,JJ6+2) = -4.*C33(P)       00007660
0556      A(KQ2,JJ8+2) = -4.*C33(M)       00007670
0557      A(KQ2,JJ10+2) = C33(M)          00007680
0558      A(KQ2,JJ12+2) = C33(P)          00007690
          C                                00007700
0559      CZ1 = (C13(P)-C13(M))*C3 + (C23(P)-C23(M))*BV + (C36(P)-C36(M))*BU 00007710
0560      CZ2 = (C13(P)-C13(M))*C2+(C23(P)-C23(M))*DV+2.*(C36(P)-C36(M))*C4 00007720
          C                                00007730
0561      X(JJ1) = 0.                      00007740
0562      X(JQ1) = 0.                      00007750
0563      X(JQ2) = 2.*H*(CZ1 + CZ2*Z)      00007760
          C                                00007770
0564      C=C45(M)-C45(P)                  00007780
0565      D=C44(M)-C44(P)                  00007790
0566      E=C23(M)-C23(P)                  00007800
0567      CC=C36(M)-C36(P)                 00007810
          C                                00007820
0568      IF(I.EQ.FSW2) GO TO 227          00007830
          C                                00007840
0569      A(KJ1,JJ1+2) = 2.*D1*C           00007850
0570      A(KJ1,JJ2+2) = -2.*D2*C         00007860
0571      A(KJ1,JJ4+2) = 2.*D3*C          00007870
          C                                00007880
0572      A(KQ1,JJ1+2) = 2.*D1*D           00007890
0573      A(KQ1,JJ2+2) = -2.*D2*D         00007900
0574      A(KQ1,JJ4+2) = 2.*D3*D          00007910
          C                                00007920
0575      A(KQ2,JJ1) = 2.*D1*CC            00007930
0576      A(KQ2,JJ2) = -2.*D2*CC          00007940
0577      A(KQ2,JJ4) = 2.*D3*CC           00007950
          C                                00007960
0578      A(KQ2,JJ1+1) = 2.*D1*E           00007970
0579      A(KQ2,JJ2+1) = -2.*D2*E         00007980
0580      A(KQ2,JJ4+1) = 2.*D3*E          00007990
0581      GO TO 102                        00008000
          C                                00008010
0582      227 A(KJ1,JJ1+2) = -2.*D1*C       00008020
0583      A(KJ1,JJ2+2) = -2.*D3*C         00008030
0584      A(KJ1,JJ4+2) = 2.*D2*C          00008040
          C                                00008050
0585      A(KQ1,JJ1+2) = -2.*D1*D           00008060
0586      A(KQ1,JJ2+2) = -2.*D3*D         00008070
0587      A(KQ1,JJ4+2) = 2.*D2*D          00008080
          C                                00008090
0588      A(KQ2,JJ1) = -2.*D1*CC            00008100
0589      A(KQ2,JJ2) = -2.*D3*CC          00008110
0590      A(KQ2,JJ4) = 2.*D2*CC           00008120

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C
0591      A(KQ2, JJ1+1) = -2.*D1*E      00008130
0592      A(KQ2, JJ2+1) = -2.*D3*E      00008140
0593      A(KQ2, JJ4+1) = 2.*D2*E      00008150
0594      GO TO 102                      00008160
                                           00008170
C
C MATRIX TERMS AT AN INTERFACE FOR J=INF AND I BETWEEN FSW1 AND FSW2
C
0595      222 XK = FLOAT(K)              00008180
0596      A(KJ1, JJ1) = 3.*(C55(M)+C55(P)) 00008190
0597      A(KJ1, JJ6) = -4.*C55(P)      00008200
0598      A(KJ1, JJ8) = -4.*C55(M)      00008210
0599      A(KJ1, JJ10) = C55(M)         00008220
0600      A(KJ1, JJ12) = C55(P)         00008230
                                           00008240
C
0601      A(KJ1, JJ1+1) = 3.*(C45(M)+C45(P)) 00008250
0602      A(KJ1, JJ6+1) = -4.*C45(P)    00008260
0603      A(KJ1, JJ8+1) = -4.*C45(M)    00008270
0604      A(KJ1, JJ10+1) = C45(M)       00008280
0605      A(KJ1, JJ12+1) = C45(P)       00008290
                                           00008300
C
0606      A(KJ1, JJ2+2) = (C45(P)-C45(M))/XK 00008310
0607      A(KJ1, JJ4+2) = (C45(M)-C45(P))/XK 00008320
                                           00008330
C
0608      A(KQ1, JJ1) = 3.*(C45(M)+C45(P)) 00008340
0609      A(KQ1, JJ6) = -4.*C45(P)      00008350
0610      A(KQ1, JJ8) = -4.*C45(M)      00008360
0611      A(KQ1, JJ10) = C45(M)         00008370
0612      A(KQ1, JJ12) = C45(P)         00008380
                                           00008390
C
0613      A(KQ1, JJ1+1) = 3.*(C44(M)+C44(P)) 00008400
0614      A(KQ1, JJ6+1) = -4.*C44(P)    00008410
0615      A(KQ1, JJ8+1) = -4.*C44(M)    00008420
0616      A(KQ1, JJ10+1) = C44(M)       00008430
0617      A(KQ1, JJ12+1) = C44(P)       00008440
                                           00008450
C
0618      A(KQ1, JJ2+2) = (C44(P)-C44(M))/XK 00008460
0619      A(KQ1, JJ4+2) = (C44(M)-C44(P))/XK 00008470
                                           00008480
C
0620      A(KQ2, JJ2) = (C36(P)-C36(M))/XK 00008490
0621      A(KQ2, JJ4) = (C36(M)-C36(P))/XK 00008500
                                           00008510
C
0622      A(KQ2, JJ2+1) = (C23(P)-C23(M))/XK 00008520
0623      A(KQ2, JJ4+1) = (C23(M)-C23(P))/XK 00008530
                                           00008540
C
0624      A(KQ2, JJ1+2) = 3.*(C33(M)+C33(P)) 00008550
0625      A(KQ2, JJ6+2) = -4.*C33(P)    00008560
0626      A(KQ2, JJ8+2) = -4.*C33(M)    00008570
0627      A(KQ2, JJ10+2) = C33(M)       00008580
0628      A(KQ2, JJ12+2) = C33(P)       00008590
0629      X(JQ1) = 0.                   00008600
                                           00008610
C
0630      CZ1 = (C13(P)-C13(M))*C3 + (C23(P)-C23(M))*BV + (C36(P)-C36(M))*BU 00008620
0631      CZ2 = (C13(P)-C13(M))*C2 + (C23(P)-C23(M))*DV + 2.*(C36(P)-C36(M))*C4 00008630
                                           00008640
C
0632      X(JJ1) = 0.                   00008650
0633      X(JQ2) = 2.*H*(CZ1 + CZ2*Z)   00008660
0634      GO TO 102                      00008670
                                           00008680
                                           00008690
                                           00008700

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C
C FREE SURFACE MATRIX TERMS AT ANY INTERFACE WHERE I=LAW, J=INF OR AT
C THE FREE SURFACE POINT I=LAW, J=LAT
C
0635      223 A(KJ1,JJ1) = 3.*C66(M)
0636      A(KJ1,JJ2) = -4.*C66(M)
0637      A(KJ1,JJ13) = C66(M)
C
0638      A(KJ1,JJ1+1) = 3.*C26(M)
C
0644      A(KQ1,JJ1) = 3.*C26(M)
0645      A(KQ1,JJ2) = -4.*C26(M)
0646      A(KQ1,JJ13) = C26(M)
C
0647      A(KQ1,JJ1+1) = 3.*C22(M)
0648      A(KQ1,JJ2+1) = -4.*C22(M)
0649      A(KQ1,JJ13+1) = C22(M)
C
0650      A(KQ1,JJ1+2) = 3.*C23(M)
0651      A(KQ1,JJ8+2) = -4.*C23(M)
0652      A(KQ1,JJ10+2) = C23(M)
C
0653      A(KQ2,JJ1) = 3.*C45(M)
0654      A(KQ2,JJ8) = -4.*C45(M)
0655      A(KQ2,JJ10) = C45(M)
C
0656      A(KQ2,JJ1+1) = 3.*C44(M)
0657      A(KQ2,JJ8+1) = -4.*C44(M)
0658      A(KQ2,JJ10+1) = C44(M)
C
0659      A(KQ2,JJ1+2) = 3.*C44(M)
0660      A(KQ2,JJ2+2) = -4.*C44(M)
0661      A(KQ2,JJ13+2) = C44(M)
C
0662      CY1 = C12(M)*C3 + C22(M)*BV + C26(M)*BU
0663      CY2 = C12(M)*C2 + C22(M)*DV + 2.*C26(M)*C4
0664      CXY1 = C16(M)*C3 + C26(M)*BV + C66(M)*BU
0665      CXY2 = C16(M)*C2 + C26(M)*DV + 2.*C66(M)*C4
C
0666      X(JJ1) = -2.*H*(CXY1 + CXY2*Z)
0667      X(JQ1) = -2.*H*(CY1 + CY2*Z)
0668      X(JQ2) = 0.
0669      GO TO 102
C
C MATRIX TERMS TO FIX THE RIGID TRANSLATIONS
C
0670      203 A(KJ1,JJ1) = 1.0
0671      A(KQ1,JJ1+1) = 1.0
0672      A(KQ2,JJ1+2) = 1.0
C
0673      X(JJ1) = 0.
0674      X(JQ1) = 0.

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0675          X(JQ2) = 0.                00009290
0676          GO TO 102                  00009300
C                                                    00009310
C 202 IF(I.EQ.1) GO TO 220              00009320
0677          IF(I.EQ.LAW) GO TO 223    00009330
0678          C                           00009340
C FREE SURFACE MATRIX TERMS FOR I BETWEEN 1 AND LAW AND J=LAT 00009350
C                                                    00009360
0679          A(KJ1, JJ1) = 3.*C55(M)   00009370
0680          A(KJ1, JJ8) = -4.*C55(M)  00009380
0681          A(KJ1, JJ10) = C55(M)     00009390
C                                                    00009400
0682          A(KJ1, JJ1+1) = 3.*C45(M) 00009410
0683          A(KJ1, JJ8+1) = -4.*C45(M) 00009420
0684          A(KJ1, JJ10+1) = C45(M)    00009430
C                                                    00009440
0685          A(KQ1, JJ1) = 3.*C45(M)   00009450
0686          A(KQ1, JJ8) = -4.*C45(M)  00009460
0687          A(KQ1, JJ10) = C45(M)     00009470
C                                                    00009480
0688          A(KQ1, JJ1+1) = 3.*C44(M) 00009490
0689          A(KQ1, JJ8+1) = -4.*C44(M) 00009500
0690          A(KQ1, JJ10+1) = C44(M)    00009510
C                                                    00009520
0691          A(KQ2, JJ1+2) = 3.*C33(M)  00009530
0692          A(KQ2, JJ8+2) = -4.*C33(M) 00009540
0693          A(KQ2, JJ10+2) = C33(M)    00009550
C                                                    00009560
0694          CZ1 = C13(M)*C3 + C23(M)*BV + C36(M)*BU 00009570
0695          CZ2 = C13(M)*C2 + C23(M)*DV + 2.*C36(M)*C4 00009580
C                                                    00009590
0696          X(JJ1) = 0.                00009600
0697          X(JQ1) = 0.                00009610
0698          X(JQ2) = -2.*H*(CZ1 + CZ2*Z) 00009620
C                                                    00009630
0699          IF(I.EQ.FSW1) GO TO 231    00009640
0700          IF(I.EQ.FSW2) GO TO 231    00009650
0701          IF(I.GT.FSW1.AND.I.LT.FSW2) GO TO 234 00009660
C                                                    00009670
C IF I IS BETWEEN 1 AND FSW1 OR BETWEEN FSW2 AND LAW, CONTINUE BELOW 00009680
C                                                    00009690
0702          A(KJ1, JJ2+2) = -C45(M)    00009700
0703          A(KJ1, JJ4+2) = C45(M)     00009710
C                                                    00009720
0704          A(KQ1, JJ2+2) = -C44(M)    00009730
0705          A(KQ1, JJ4+2) = C44(M)     00009740
C                                                    00009750
0706          A(KQ2, JJ2) = -C36(M)      00009760
0707          A(KQ2, JJ4) = C36(M)       00009770
C                                                    00009780
0708          A(KQ2, JJ2+1) = -C23(M)    00009790
0709          A(KQ2, JJ4+1) = C23(M)     00009800
0710          GO TO 102                  00009810
C                                                    00009820
C CASE WHERE I=FSW1 OR FSW2 AND J=LAT 00009830
C                                                    00009840
0711          231 XK = FLOAT(K)          00009850
0712          D1 = 2.*(XK-1.)/XK         00009860

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0713          D2 = 2.*XK/(XK+1.)          00009870
0714          D3 = 2./((XK+1.)*XK)       00009880
          C                               00009890
0715          IF(I.EQ.FSW2) GO TO 232    00009900
          C                               00009910
0716          A(KJ1, JJ1+2) = D1*C45(M)   00009920
0717          A(KJ1, JJ2+2) = -D2*C45(M)  00009930
0718          A(KJ1, JJ4+2) = D3*C45(M)   00009940
          C                               00009950
0719          A(KQ1, JJ1+2) = D1*C44(M)   00009960
0720          A(KQ1, JJ2+2) = -D2*C44(M)  00009970
0721          A(KQ1, JJ4+2) = D3*C44(M)   00009980
          C                               00009990
0722          A(KQ2, JJ1) = D1*C36(M)     00010000
0723          A(KQ2, JJ2) = -D2*C36(M)    00010010
0724          A(KQ2, JJ4) = D3*C36(M)     00010020
          C                               00010030
0725          A(KQ2, JJ1+1) = D1*C23(M)   00010040
0726          A(KQ2, JJ2+1) = -D2*C23(M)  00010050
0727          A(KQ2, JJ4+1) = D3*C23(M)   00010060
0728          GO TO 102                   00010070
          C                               00010080
0729          232 A(KJ1, JJ1+2) = -D1*C45(M) 00010090
0730          A(KJ1, JJ2+2) = -D3*C45(M)  00010100
0731          A(KJ1, JJ4+2) = D2*C45(M)   00010110
          C                               00010120
0732          A(KQ1, JJ1+2) = -D1*C44(M)   00010130
0733          A(KQ1, JJ2+2) = -D3*C44(M)  00010140
0734          A(KQ1, JJ4+2) = D2*C44(M)   00010150
          C                               00010160
0735          A(KQ2, JJ1) = -D1*C36(M)     00010170
0736          A(KQ2, JJ2) = -D3*C36(M)    00010180
0737          A(KQ2, JJ4) = D2*C36(M)     00010190
          C                               00010200
0738          A(KQ2, JJ1+1) = -D1*C23(M)   00010210
0739          A(KQ2, JJ2+1) = -D3*C23(M)  00010220
0740          A(KQ2, JJ4+1) = D2*C23(M)   00010230
0741          GO TO 102                   00010240
          C                               00010250
          C CASE WHERE I IS BETWEEN FSW1 AND FSW2 AND J=LAT 00010260
          C                               00010270
0742          234 XK = FLOAT(K)           00010280
0743          A(KJ1, JJ2+2) = -C45(M)/XK   00010290
0744          A(KJ1, JJ4+2) = C45(M)/XK    00010300
          C                               00010310
0745          A(KQ1, JJ2+2) = -C44(M)/XK   00010320
0746          A(KQ1, JJ4+2) = C44(M)/XK    00010330
          C                               00010340
0747          A(KQ2, JJ2) = -C36(M)/XK     00010350
0748          A(KQ2, JJ4) = C36(M)/XK     00010360
          C                               00010370
0749          A(KQ2, JJ2+1) = -C23(M)/XK  00010380
0750          A(KQ2, JJ4+1) = C23(M)/XK   00010390
          C                               00010400
0751          102 CONTINUE                00010410
          C                               00010420
          C FORM THE NONSYMETRIC BANDED MATRIX AX 00010430
          C                               00010440

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0752          IL = KJ1+3*(NODE-1)          00010450
0753          IN = IL+2                    00010460
C                                                    00010470
0754          DO 103 IK=IL, IN             00010480
0755          II = IK-IL+1                 00010490
C                                                    00010500
0756          DO 104 JK=1,NBAND            00010510
0757          JJ = IK+JK-IBW-1             00010520
0758          IF(IK.LE.IBW1) JJ = JK       00010530
0759          IF(JJ.GT.JQMAX) GO TO 105    00010540
0760          AX(JK,IK) = A(II,JJ)        00010550
0761          GO TO 104                    00010560
0762          105 AX(JK,IK) = 0.0          00010570
0763          104 CONTINUE                 00010580
0764          103 CONTINUE                 00010590
0765          101 CONTINUE                 00010600
0766          100 CONTINUE                 00010610
C                                                    00010620
0767          REWIND 9                      00010630
0768          WRITE(9) ((AX(J,I),J=1,NBAND),I=1,JQMAX) 00010640
0769          WRITE(9) (X(I),I=1,JQMAX)     00010650
0770          END FILE 9                    00010660
0771          REWIND 9                      00010670
C                                                    00010680
0772          NBD = NBAND+1                00010690
0773          DO 107 I=1, JQMAX             00010700
0774          AX(NBD,I) = X(I)              00010710
0775          107 CONTINUE                  00010720
C                                                    00010730
C          WRITE(6,4000)                   00010740
C4000 FORMAT(1H1, ' EQUATION', 35X, ' THE BANDED MATRIX TERMS AX(I,J)' //) 00010750
C          CALL RITE(1, JQMAX, NBD, JQMAX, NBD, AX) 00010760
C          WRITE(6,4003)                   00010770
C4003 FORMAT(1H1, 45X, ' *** THE LOAD VECTOR X(I) ***' // ) 00010780
C          WRITE(6,4004) (X(I), I=1, JQMAX) 00010790
C4004 FORMAT(28(2X, 10D12.3 / ))          00010800
C                                                    00010810
0776          CALL TRMSTR(AX, JQMAX, NBD , IBW, IBW, NBAND, DT, RT, ET) 00010820
C                                                    00010830
C          WRITE(6,4006) ET, RT, DT        00010840
0777          4006 FORMAT(/// ' ERROR CONDITION OF SOLVER ROUTINE IS ', F4.1, 5X, 00010850
0778          1 'RANK IS ', F6.1, 5X, 'DETERMINANT = ', G10.3) 00010860
0779          IF(ET.EQ.1.) STOP 1          00010870
C                                                    00010880
0780          DO 108 I=1,JQMAX              00010890
0781          X(I) = AX(1,I)                00010900
0782          108 CONTINUE                 00010910
C                                                    00010920
0783          READ(9) ((AX(J,I),J=1,NBAND),I=1,JQMAX) 00010930
0784          READ(9) (R(I),I=1,JQMAX)      00010940
C                                                    00010950
C          *****                        00010960
C          *****                        00010970
C          *****                        00010980
C          OUTPUT OF THE NODAL DISPLACEMENTS, U, V, W 00010990
C          *****                        00011000
C          *****                        00011010
C          *****                        00011020

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0785          WRITE(6,650)                                00011030
0786          J = 1                                        00011040
0787          DO 12 IK = 1, LAW                            00011050
0788          DO 11 JK = 1, LAT                            00011060
0789          WRITE(6,651) J, X(3*J-2), X(3*J-1), X(3*J) 00011070
0790          J = J+1                                       00011080
0791          11 CONTINUE                                    00011090
0792          WRITE(6,653)                                    00011100
0793          12 CONTINUE                                    00011110
C                                                     00011120
0794          WRITE(6,9950)                                    00011130
0795          9950 FORMAT(1H1, 5X, 'EQUATION', 5X, '*** THE ACCURACY TEST, TEST-R(I) 00011140
1 ***', 10X, '*** THE AVERAGE ABSOLUTE ERROR ***' ///) 00011150
0796          ERR = 0.00                                       00011160
0797          DO 9990 I=1,JQMAX                               00011170
0798          TEST = 0.00                                       00011180
0799          DO 9960 J=1,NBAND                               00011190
0800          IC = I+J-IBW-1                                    00011200
0801          IF(I.LE.IBW1) IC = J                               00011210
0802          IF(IC.GT.JQMAX) GO TO 9970                       00011220
0803          TEST = TEST+AX(J,I)*X(IC)                       00011230
0804          9960 CONTINUE                                    00011240
0805          9970 TEST = TEST-R(I)                            00011250
0806          ERR = ERR+DABS(TEST)                             00011260
0807          AVE = ERR/I                                       00011270
0808          WRITE(6,9980) I, TEST, AVE                       00011280
0809          9980 FORMAT(5X, 14,10X, G15.8, 32X, G15.8)     00011290
0810          9990 CONTINUE                                    00011300
C                                                     00011310
C *****00011320
C                                                     00011330
C          CALCULATION OF THE STRAIN (S) AND STRESS (T)     00011340
C                                                     00011350
C *****00011360
C                                                     00011370
0811          SXM = SXMAX * 1.E06                             00011380
0812          SXE = C3E * 1.E06                             00011390
0813          WRITE(6,670) SXM, SXE                         00011400
0814          WRITE(6,671)                                    00011410
0815          HR = 1./(2.*H)                                  00011420
0816          XK = FLOAT(K)                                   00011430
C                                                     00011440
0817          DO 399 I=1, LAW                                 00011450
0818          DO 398 J=1, LAT                                 00011460
C                                                     00011470
0819          I1=I-1                                         00011480
0820          I2=I-2                                         00011490
0821          NODE = LAT*I1+J                                00011500
0822          JJ1 = 3*(LAT*I1+J)-2                          00011510
0823          JJ2 = 3*(LAT*I2+J)-2                          00011520
0824          JJ3 = 3*(LAT*I2+J)-5                          00011530
0825          JJ4 = 3*(LAT*I+J)-2                           00011540
0826          JJ5 = 3*(LAT*I+J)+1                           00011550
0827          JJ6 = 3*(LAT*I1+J)+1                          00011560
0828          JJ7 = 3*(LAT*I2+J)+1                          00011570
0829          JJ8 = 3*(LAT*I1+J)-5                          00011580
0830          JJ9 = 3*(LAT*I+J)-5                           00011590
0831          JJ10 =3*(LAT*I1+J)-8                          00011600

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0832          JJ11 =3*(LAT*(I+1)+J)-2          00011610
0833          JJ12 =3*(LAT*I+J)+4             00011620
0834          JJ13 =3*(LAT*(I-3)+J)-2         00011630
C                                                    00011640
0835          Z = (FLODAT(J)-(FLOAT(LAT)+1.)/2.)*H 00011650
0836          SX = C2*Z + C3                   00011660
C                                                    00011670
0837          IF(I.EQ.1) GO TO 385             00011680
0838          IF(I.EQ.LAW) GO TO 386           00011690
0839          IF(I.GT.FSW1.AND.I.LT.FSW2) GO TO 382 00011700
0840          IF(I.EQ.FSW1.OR.I.EQ.FSW2) GO TO 383 00011710
C                                                    00011720
0841          H1 = H                           00011730
0842          H2 = H1                           00011740
0843          GO TO 384                         00011750
C                                                    00011760
0844          382 H1 = XK*H                     00011770
0845          H2 = H1                           00011780
0846          GO TO 384                         00011790
C                                                    00011800
0847          383 H1 = H                       00011810
0848          H2 = XK*H                         00011820
0849          IF(I.EQ.FSW1) GO TO 384          00011830
0850          H1 = XK*H                         00011840
0851          H2 = H                           00011850
C                                                    00011860
0852          384 H12 = H1/H2                  00011870
0853          H21 = H2/H1                      00011880
0854          HRD = (H2-H1)/(H1*H2)           00011890
0855          HRS = 1./(H1+H2)                 00011900
C                                                    00011910
0856          SY = HRS*(H12*X(JJ4+1)-H21*X(JJ2+1))+HRD*X(JJ1+1) + DV*Z + BV 00011920
0857          SXY = HRS*(H12*X(JJ4)-H21*X(JJ2)) + HRD*X(JJ1) + 2.*C4*Z + BU 00011930
0858          SYZI = HRS*(H12*X(JJ4+2)-H21*X(JJ2+2))+HRD*X(JJ1+2) 00011940
0859          GO TO 387                          00011950
C                                                    00011960
0860          385 SY = HR*(4.*X(JJ4+1)-3.*X(JJ1+1)-X(JJ11+1)) + DV*Z + BV 00011970
0861          SXY = HR*(4.*X(JJ4)-3.*X(JJ1)-X(JJ11)) + 2.*C4*Z + BU 00011980
0862          SYZI = HR*(4.*X(JJ4+2)-3.*X(JJ1+2)-X(JJ11+2)) 00011990
0863          GO TO 387                          00012000
C                                                    00012010
0864          386 SY = HR*(3.*X(JJ1+1)+X(JJ13+1)-4.*X(JJ2+1)) + DV*Z + BV 00012020
0865          SXY = HR*(3.*X(JJ1)+X(JJ13)-4.*X(JJ2)) + 2.*C4*Z + BU 00012030
0866          SYZI = HR*(3.*X(JJ1+2)+X(JJ13+2)-4.*X(JJ2+2)) 00012040
C                                                    00012050
0867          387 DO 392 M=1, NLAY              00012060
0868          IF(M.EQ.1.AND.J.GT.INF(1)) GO TO 392 00012070
0869          IF(M.EQ.1) GO TO 388              00012080
0870          IF(J.LE.INF(M-1).OR.J.GT.INF(M)) GO TO 392 00012090
0871          388 IF(J.EQ.1) GO TO 389          00012100
0872          IF(J.EQ.INF(M).OR.J.EQ.LAT) GO TO 390 00012110
C                                                    00012120
0873          SZ = HR*(X(JJ6+2)-X(JJ8+2))      00012130
0874          SYZJ = HR*(X(JJ6+1)-X(JJ8+1))    00012140
0875          SXZ = HR*(X(JJ6)-X(JJ8))         00012150
0876          GO TO 391                          00012160
C                                                    00012170
0877          389 SZ = HR*(4.*X(JJ6+2)-3.*X(JJ1+2)-X(JJ12+2)) 00012180

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0878          SYZJ = HR*(4.*X(JJ6+1)-3.*X(JJ1+1)-X(JJ12+1))      00012190
0879          SXZ = HR*(4.*X(JJ6)-3.*X(JJ1)-X(JJ12))              00012200
0880          GO TO 391                                           00012210
C                                                     00012220
0881          390 SZ = HR*(3.*X(JJ1+2)+X(JJ10+2)-4.*X(JJ8+2))    00012230
0882          SYZJ = HR*(3.*X(JJ1+1)+X(JJ10+1)-4.*X(JJ8+1))      00012240
0883          SXZ = HR*(3.*X(JJ1)+X(JJ10)-4.*X(JJ8))              00012250
C                                                     00012260
0884          391 SYZ = SYZI + SYZJ                                00012270
C                                                     00012280
C          CALCULATION OF THE STRESS (T)                        00012290
C                                                     00012300
0885          TX = C11(M)*SX + C12(M)*SY + C13(M)*SZ + C16(M)*SXY 00012310
0886          TY = C12(M)*SX + C22(M)*SY + C23(M)*SZ + C26(M)*SXY 00012320
0887          TZ = C13(M)*SX + C23(M)*SY + C33(M)*SZ + C36(M)*SXY 00012330
C                                                     00012340
0888          TYZ = C44(M)*SYZ + C45(M)*SXZ                      00012350
0889          TXZ = C45(M)*SYZ + C55(M)*SXZ                      00012360
0890          TXY = C16(M)*SX + C26(M)*SY + C36(M)*SZ + C66(M)*SXY 00012370
C                                                     00012380
0891          WRITE(6,672) NODE, TX, TY, TZ, TYZ, TXZ, TXY, SY, SZ, SYZ, SXZ,SXY 00012390
0892          WRITE(6,397) SX                                      00012400
C                                                     00012410
C          STRESS AND STRAINS JUST ABOVE AN INTERFACE           00012420
C                                                     00012430
0893          IF(J.NE.INF(M).OR.J.EQ.LAT) GO TO 392                00012440
0894          P = M+1                                              00012450
0895          SZ = HR*(4.*X(JJ6+2)-3.*X(JJ1+2)-X(JJ12+2))        00012460
0896          SYZJ = HR*(4.*X(JJ6+1)-3.*X(JJ1+1)-X(JJ12+1))      00012470
0897          SXZ = HR*(4.*X(JJ6)-3.*X(JJ1)-X(JJ12))              00012480
0898          SYZ = SYZI + SYZJ                                    00012490
C                                                     00012500
0899          TX = C11(P)*SX + C12(P)*SY + C13(P)*SZ + C16(P)*SXY 00012510
0900          TY = C12(P)*SX + C22(P)*SY + C23(P)*SZ + C26(P)*SXY 00012520
0901          TZ = C13(P)*SX + C23(P)*SY + C33(P)*SZ + C36(P)*SXY 00012530
C                                                     00012540
0902          TYZ = C44(P)*SYZ + C45(P)*SXZ                      00012550
0903          TXZ = C45(P)*SYZ + C55(P)*SXZ                      00012560
0904          TXY = C16(P)*SX + C26(P)*SY + C36(P)*SZ + C66(P)*SXY 00012570
C                                                     00012580
0905          WRITE(6,672) NODE, TX, TY, TZ, TYZ, TXZ, TXY, SY, SZ, SYZ, SXZ,SXY 00012590
C                                                     00012600
0906          392 CONTINUE                                         00012610
0907          398 CONTINUE                                         00012620
0908          WRITE(6,652)                                         00012630
0909          399 CONTINUE                                         00012640
0910          9000 CONTINUE                                        00012650
C                                                     00012660
C          *****00012670
C                                                     00012680
C          FORMATS                                               00012690
C                                                     00012700
C          *****00012710
C                                                     00012720
0911          397 FORMAT(14X,1P1E11.3/)                            00012730
0912          600 FORMAT(1H1, 44X, 44H*** UNIFORM BENDING OF A LAMINATED PLATE ***) 00012740
0913          601 FORMAT(5I10)                                      00012750
C                                                     00012760

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0914      602 FORMAT(//// 5X, 18H*** INPUT DATA *** ///          00012770
          1      18X, 'NUMBER OF LAYERS IN CROSS SECTION, NLAY =', I4 // 00012780
          2      18X, 'NUMBER OF NODES ON VERTICAL AXIS, LAT =', I4 // 00012790
          3      18X, 'NUMBER OF NODES ON HORIZONTAL AXIS, LAW =', I4 /// 00012800
          4      18X, 37HCHANGE IN MESH WIDTH (FSW1) AT I = , I4 // 00012810
          5      18X, 37HCHANGE IN MESH WIDTH (FSW2) AT I = , I4 // 00012820
          6      18X, 37HMESH WIDTH MAGNIFICATION FACTOR, K = , I4 /) 00012830
          C      00012840
0915      603 FORMAT(8G12.5)          00012850
          C      00012860
0916      604 FORMAT(1H1, 55X, 21H*** MATERIAL DATA *** //// 2X, 5HLAYER, 7X, 00012870
          1      3HE11, 9X, 3HE22, 9X, 3HE33, 9X, 3HE12, 9X, 3HE13, 9X, 00012880
          2      3HE23, 8X, 4HNU12, 4X, 4HNU13, 4X, 4HNU23 // ) 00012890
          C      00012900
0917      605 FORMAT(3X, I2, 6X, 2PE10.3, 2(2X, 1PE10.3), 3(2X, OPE10.3), 00012910
          1      3(3X, F5.2) / ) 00012920
          C      00012930
0918      606 FORMAT(10G10.3)          00012940
          C      00012950
0919      607 FORMAT(/// 18X, 26HFINE SIMULATION WIDTH, H = ,F8.5) 00012960
          C      00012970
0920      608 FORMAT(// 18X, 9HLAYER NO., 2X, I3, 5X, 17HINTERFACE AT J = ,I3) 00012980
          C      00012990
0921      611 FORMAT(// 45X, 41H*** COEFFICIENTS OF THERMAL EXPANSION ***, /// 00013000
          1      1X, 5HLAYER, 8X, 5HTHETA, 12X, 3HAL1, 12X, 3HAL2, 12X, 00013010
          2      3HAL3, 12X, 3HAL6, 12X, 4HAL1P, 11X, 4HAL2P, 11X, 4HAL3P 00013020
          3      /// ) 00013030
          C      00013040
0922      613 FORMAT(// 53X, 26H*** STIFFNESS MATRICES *** /// 1X, 00013050
          1      11HLAYER/THETA, 21X, 12HX-Y-Z MATRIX, 44X, 00013060
          2      18HX-Y-Z PRIME MATRIX /// ) 00013070
          C      00013080
0923      614 FORMAT(2X, I2, 9X, F5.1, 5X, 7(5X, E10.3)) 00013090
          C      00013100
0924      620 FORMAT(2X, I2, 5X, 1P12E10.3 // 19X, 5E10.3, 10X, 5E10.3 // 29X, 00013110
          1      4E10.3, 20X, 4E10.3 // 1X,OPF5.1, 33X, 1P3E10.3, 30X, 00013120
          2      3E10.3 // 49X, 2E10.3, 40X, 2E10.3 // 59X, E10.3, 50X, 00013130
          3      E10.3 /// ) 00013140
          C      00013150
0925      650 FORMAT(1H1 // 10X, '*** GRID POINT DISPLACEMENT FUNCTIONS ***' ///00013160
          1      16X, 5H NODE, 5X, 14HU-DISPLACEMENT, 6X, 14HV-DISPLACEMENT, 00013170
          2      6X, 14HW-DISPLACEMENT /// ) 00013180
          C      00013190
0926      651 FORMAT(10X, I10, 3E20.6 // ) 00013200
0927      652 FORMAT(// 12H ***** // ) 00013210
0928      653 FORMAT(// 10X, 12H ***** // ) 00013220
          C      00013230
0929      670 FORMAT(1H1, 10X, 77H*** OUTPUT STRESSES AND STRAINS FOR A MAXIMUM 00013240
          1LONGITUDINAL BENDING STRAIN OF , F6.0, 22H MICRO-INCHES/INCH AND /00013250
          2 48X, 40H AN APPLIED AXIAL EXTENSIONAL STRAIN OF , F6.0, 00013260
          3 19H MICRO-INCHES/INCH. // 10X, 'NOTE: INTERFACE NODES ARE REPEAT00013270
          4ED WITH VALUES GIVEN BELOW AND ABOVE THE INTERFACE RESPECTIVELY.' 00013280
          5 //// ) 00013290
          C      00013300
0930      671 FORMAT(1X,5HNODE , 5X, 5HSIG-X, 6X, 5HSIG-Y, 6X, 5HSIG-Z, 6X, 00013310
          1      6HTAU-YZ, 5X, 6HTAU-XZ, 5X, 6HTAU-XY, 5X, 5HEPS-Y, 6X, 00013320
          2      5HEPS-Z, 6X, 6HEPS-YZ, 5X, 6HEPS-XZ, 5X, 6HEPS-XY / 17X, 00013330
          3      5HEPS-X /// ) 00013340
          C      00013350
0931      672 FORMAT(1X, I3, 4X, 1P11E11.3 /) 00013360
          C      00013370
0932      STOP 00013380
0933      END 00013390

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0001          SUBROUTINE MATCON                                00013400
C                                                     00013410
C *****00013420
C                                                     00013430
C   CALCULATION OF LAMINATE LOAD CONSTANTS FOR A FULL CROSS SECTION 00013440
C                                                     00013450
C *****00013460
C                                                     00013470
C   THIS SUBROUTINE IS GOOD FOR BENDING OF AN ARBITRARILY LAID UP 00013471
C   LAMINATE WHICH IS SYMMETRIC OR NONSYMMETRIC ABOUT THE MIDPLANE. 00013472
C                                                     00013480
C   THE CONSTANTS ARE  C2 = INVERSE BENDING RADIUS          00013490
C                     C3E = APPLIED UNIFORM EXTENSIONAL STRAIN 00013500
C                     C3 = EXTENSIONAL COUPLING DUE TO BENDING PLUS C3E 00013510
C                     C4 = IN-PLANE SHEAR COUPLING          00013520
C                                                     00013530
C                     BU OCCURS IN THE FCTN. U(Y,Z)         00013540
C                     BV AND DV OCCUR IN THE FCTN. V(Y,Z)   00013550
C                                                     00013560
C   SXMAX (EFFECTIVELY THE LOAD INPUT) IS A MAXIMUM STRAIN    00013570
C                                                     00013580
0002          INTEGER ORDER                                00013590
C                                                     00013600
0003          COMMON /MC/ C11(6),C12(6),C16(6),C22(6),C26(6),C66(6),C13(6),
1             C23(6),C36(6),C44(6),C45(6),C55(6),C33(6),AL1(6),AL2(6),
2             AL3(6),AL6(6),C2,C3,C3E,C4,BU,DU,BV,DV,H,SXMAX,NLAY,INF(6) 00013610
C                                                     00013620
C                                                     00013630
0004          DIMENSION A(3,3), B(3,3), D(3,3), QM(3,3)    00013640
C                                                     00013650
0005          DOUBLE PRECISION A, B, D                    00013660
C                                                     00013670
0006          ORDER = 3                                    00013680
C                                                     00013690
0007          LAY = INF(1)-1                                00013700
0008          HL = H*FLOAT(LAY)                             00013710
0009          HL2 = HL**2/2.                                00013720
0010          HL3 = HL**3/3.                                00013730
0011          RN = FLOAT(NLAY)                              00013740
0012          RN2 = RN**2                                   00013750
C                                                     00013760
C                                                     00013770
0013          DO 20 I=1,3                                   00013780
0014             DO 20 J=1,3                                 00013790
0015             A(I,J) = 0.00                               00013800
0016             B(I,J) = 0.00                               00013810
0017             D(I,J) = 0.00                               00013820
0018          20 CONTINUE                                  00013830
C                                                     00013840
0019          DO 30 I=1,3                                   00013850
0020             DO 30 J=1,3                                 00013860
0021             DO 30 M=1,NLAY                              00013870
0022             QM(1,1) = C11(M)-C13(M)*C13(M)/C33(M)      00013880
0023             QM(1,2) = C12(M)-C13(M)*C23(M)/C33(M)      00013890
0024             QM(1,3) = C16(M)-C13(M)*C36(M)/C33(M)      00013900
0025             QM(2,1) = QM(1,2)                          00013910
0026             QM(2,2) = C22(M)-C23(M)*C23(M)/C33(M)     00013920
0027             QM(2,3) = C26(M)-C23(M)*C36(M)/C33(M)     00013930
0028             QM(3,1) = QM(1,3)                          00013940
0029             QM(3,2) = QM(2,3)                          00013950

```

```

0030      QM(3,3) = C66(M)-C36(M)*C36(M)/C33(M)          00013960
C
C NOTE THAT THE SUBSCRIPT 3 IN QM REPLACES A 6 IN STANDARD NOTATION. 00013970
C THE SAME IS TRUE BELOW IN A(I,J), B(I,J), D(I,J), ETC. 00013980
C
C
C
0031      M1 = 2*M-1          00014000
0032      M2 = 3*M*(M-1)+1  00014010
C
C
0033      A(I,J) = A(I,J) + HL*QM(I,J)          00014020
0034      B(I,J) = B(I,J) + HL2*QM(I,J)*(M1-RN) 00014030
0035      D(I,J) = D(I,J) + HL3*QM(I,J)*(M2-1.5*RN*M1+.75*RN2) 00014040
0036      30. CONTINUE          00014050
C
C
C INVERT (A). STORE IN (A).          00014060
C
C
C CALL MATIN4 (A,ORDER)          00014070
0037
C
C MULTIPLY (A)INVERSE * (B). STORE IN A. 00014080
C
C
C CALL MAMULT (A,B,ORDER,A)          00014090
0038
C
C MULTIPLY (B) * (A)INVERSE * (B). STORE IN B. 00014100
C
C
C CALL MAMULT (B,A,ORDER,B)          00014110
0039
C
C DO 40 I=1,3          00014120
0040      DO 40 J=1,3          00014130
0041      A(I,J) = -1.*A(I,J)          00014140
0042      D(I,J) = D(I,J) - B(I,J)          00014150
0043      40 CONTINUE          00014160
0044
C
C INVERT NEW MATRIX (D). THE RESULT IS D-PRIME. STORE IN D. 00014170
C
C
C CALL MATIN4 (D,ORDER)          00014180
0045
C
C MULTIPLY -(A)INVERSE * B * D-PRIME WHICH YIELDS B-PRIME. STORE IN B. 00014190
C
C
C CALL MAMULT (A,D,ORDER,B)          00014200
0046
C
C DETERMINE THE LOAD CONSTANTS. MINUS C2 IMPLIES A SMILING PLATE. 00014210
C
C
C ZMAX = RN*HL/2.          00014220
0047      C2 = -D(1,1)*SXMAX/(B(1,1) +D(1,1)*ZMAX)          00014230
0048      RATIO = C2/D(1,1)          00014240
0049
C
C C3 = B(1,1)*RATIO + C3E          00014250
0050      C4 = .5*D(1,3)*RATIO          00014260
0051      BU = B(3,1)*RATIO          00014270
0052      BV = B(2,1)*RATIO          00014280
0053      DV = D(1,2)*RATIO          00014290
0054
C
C RATIO = -RATIO          00014300
0055      WRITE(6,50)          00014310
0056      50 FORMAT(///// 48X, 35H*** THE LAMINATE LOAD CONSTANTS *** /// ) 00014320
0057      WRITE(6,60) C2, C3, C4, BU, BV, DV, RATIO          00014330
0058      60 FORMAT(' C2 = ', 1PE10.3, 4X, ' C3 = ', E10.3, 4X, ' C4 = ', E10.3, 00014340
1          4X, ' BU = ', E10.3, 4X, ' BV = ', E10.3, 4X, ' DV = ', E10.3, 00014350
2          4X, ' MT = ', E10.3 )          00014360
0059      RETURN          00014370
0060      END          00014380

```

```
*OPTIONS IN EFFECT* NOTERM,NOID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP,NOTEST
*OPTIONS IN EFFECT* NAME = MATCON , LINECNT = 60
*STATISTICS* SOURCE STATEMENTS = 60,PROGRAM SIZE = 2060
*STATISTICS* NO DIAGNOSTICS GENERATED
```

```
0001          SUBROUTINE MAMULT(B,C,N,A)                                00014550
C                                                     00014551
C MAMULT POSTMULTIPLIES MATRIX (B) BY MATRIX (C) AND STORES THE 00014552
C RESULT IN MATRIX (A) WHERE N IS THE ORDER OF THE MATRICES. 00014553
C                                                     00014554
0002          DOUBLE PRECISION A,B,C,SUM                            00014560
0003          DIMENSION A(N,N), B(N,N), C(N,N)                    00014570
0004          DO 1 I=1,N                                           00014580
0005          DO 1 J=1,N                                           00014590
0006          SUM = 0.                                             00014600
0007          DO 2 K=1,N                                           00014610
0008          SUM = SUM + B(I,K)*C(K,J)                            00014620
0009          2 CONTINUE                                           00014630
0010          A(I,J) = SUM                                         00014640
0011          1 CONTINUE                                           00014650
0012          RETURN                                              00014660
0013          END                                                  00014670
```

```
*OPTIONS IN EFFECT* NOTERM,NOID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP,NOTEST
*OPTIONS IN EFFECT* NAME = MAMULT , LINECNT = 60
*STATISTICS* SOURCE STATEMENTS = 13,PROGRAM SIZE = 702
*STATISTICS* NO DIAGNOSTICS GENERATED
```

```
0001          SUBROUTINE MATIN4(ARRAY,N)                            00014680
C                                                     00014681
C MATIN4 INVERTS THE MATRIX (ARRAY) WHICH IS OF ORDER N. 00014682
C                                                     00014683
0002          DIMENSION ARRAY(N,N)                                00014690
0003          DOUBLE PRECISION ARRAY                              00014700
0004          DO 604 I=1,N                                         00014710
0005          STORE = ARRAY(I,I)                                   00014720
0006          ARRAY(I,I) = 1.                                       00014730
0007          DO 601 J=1,N                                         00014740
0008          601 ARRAY(I,J) = ARRAY(I,J)/STORE                    00014750
0009          DO 604 K=1,N                                         00014760
0010          IF(K-1)602,604,602                                    00014770
0011          602 STORE = ARRAY(K,I)                                00014780
0012          ARRAY(K,I) = 0.                                       00014790
0013          DO 603 J=1,N                                         00014800
0014          603 ARRAY(K,J) = ARRAY(K,J) - STORE*ARRAY(I,J)    00014810
0015          604 CONTINUE                                           00014820
0016          RETURN                                              00014830
0017          END                                                  00014840
```

```

0001          SUBROUTINE TRMSTR(A,N,ND,NLD,NRD,NED,D,R,E)          00014850
C                                                     00014860
C TRMSTR IS THE SUBROUTINE TRIMSS WITH MATRIX A TRANSPOSED.    00014870
C THE SIMULTANEOUS SOLUTIONS IS GAUSSIAN ELIMINATION,          00014880
C MODIFIED TO TAKE ADVANTAGE OF THE REDUCED MATRIX. THE        00014890
C ROUTINE ALSO USES PARTIAL PIVOTING TO REDUCE ROUND OFF ERROR. 00014900
C INPUT                                                         00014910
C   1 A FIRST LOCATION OF COEFFICIENT MATRIX, I.E. A(1,1).     00014920
C   THE BAND ELEMENTS IN EACH ROW MUST BE LEFT                 00014930
C   JUSTIFIED AND EXTEND TO THE RIGHT M PLACES                 00014940
C   (M=MIN(N,NLD+NRD+1)). IF IN ANY PARTICULAR ROW             00014950
C   THERE ARE ONLY K BAND ELEMENTS AND K IS LESS                00014960
C   THAN M, THEN THE M-K RIGHT MOST ELEMENTS OF THAT          00014970
C   ROW WILL BE SET TO ZERO. THE ROW WHOSE LEFT                00014980
C   MOST COLUMN IN THE FULL BLOWN MATRIX CONTAINS              00014990
C   A NON-ZERO ELEMENT MUST BE THE FIRST ROW OF THE            00015000
C   REDUCED MATRIX AND ETC. THE COLUMN TO THE                  00015010
C   IMMEDIATE RIGHT OF THE REDUCED MATRIX (FORMED AS           00015020
C   ABOVE) MUST CONTAIN THE RIGHT HAND SIDE OF THE              00015030
C   EQUATION SET IN QUESTION. IT SHOULD NOW BE                 00015040
C   OBVIOUS THAT AN N X N+1 FULL BLOWN SYSTEM WOULD           00015050
C   BE REDUCED BY THE ABOVE METHOD TO AN N X M+1                00015060
C   SYSTEM.                                                       00015070
C   2 N NUMBER OF SIMULTANEOUS EQUATIONS TO BE SOLVED.         00015080
C   3 ND VARIABLE DIMENSION INTEGER. MUST BE EQUAL TO          00015090
C   ROW DIMENSION OF A IN CALLING PROGRAM.                     00015100
C   4 NLD MAXIMUM NUMBER OF BAND ELEMENTS TO THE LEFT           00015110
C   OF PRINCIPAL DIAGONAL IN ANY ROW OF SYSTEM TO              00015120
C   BE DETERMINED.                                              00015130
C   5 NRD MAXIMUM NUMBER OF BAND ELEMENTS TO THE RIGHT          00015140
C   OF PRINCIPAL DIAGONAL IN ANY ROW OF SYSTEM TO              00015150
C   BE DETERMINED.                                              00015160
C   6 NED NED=MIN(N,NLD+NRD+1)                                  00015170
C OUTPUT                                                         00015180
C   1 A THE FIRST COLUMN OF A CONTAINS THE SOLUTION             00015190
C   VECTOR.                                                       00015200
C   2 D CONTAINS DETERMINANT OF A.                               00015210
C   3 R CONTAINS RANK OF A.                                       00015220
C   4 E E=0., SOLUTION O.K. E=1., A SINGULAR.                   00015230
C   E=2., SOLUTION ATTEMPTED, BUT A ILL CONDITIONED            00015240
C   OR SINGULAR. IN THIS CASE SOLUTIONS SHOULD BE              00015250
C   CHECKED TO ASSURE VALIDITY.                                   00015260
C   00015270
C   00015280
SUBROUTINE TRMSTR(A,N,ND,NLD,NRD,NED,D,R,E)          00015280
DIMENSION A(ND,1)          00015290
DOUBLE PRECISION A,D,Y,W,S 00015300
X1 = 1.                    00015310
L1 = 1                      00015320
E=0.                        00015330
R = 0.                      00015340
D=1.                        00015350
ND1=NED+1                   00015360
M=NLD                       00015370
NM1=N-1                     00015380
DO 1 I=1,NM1                 00015390
IF(I.GT.(N-NLD))M=M-1       00015400
NN=I+M-1                    00015410
DO 2 II=I,NN                 00015420
0002
0003
0004
0005
0006
0007
0008
0009
0010
0011
0012
0013
0014
0015

```

```

0016          IF(DABS(A(1,I)).GE.DABS(A(1,II+1))) GO TO 2          00015430
0017          D=-D          00015440
0018          DO 3 J=1,ND1          00015450
0019          Y=A(J,I)          00015460
0020          A(J,I)=A(J,II+1)          00015470
0021          3 A(J,II+1)=Y          00015480
0022          2 CONTINUE          00015490
C          D=D*A(1,I)          00015500
0023          IF(A(1,I) .EQ. 0.) GO TO 10          00015510
0024          GO TO (5,13),L1          00015520
0025          13 IF(DABS(DABS((X1-A(1,I))/X1)-1.).LT.1.E-07) E=2.          00015530
0026          X1 = A(1,I)          00015540
0027          5 R = R + 1.          00015550
0028          L1 = 2          00015560
0029          DO 4 J=2,ND1          00015570
0030          4 A(J,I)=A(J,I )/ A(1,I)          00015580
0031          K=I+1          00015590
0032          NN=I+M          00015600
0033          DO 1 II=K,NN          00015610
0034          W=A(1,II)          00015620
0035          DO 6 J=1,NED          00015630
0036          6 A(J,II)=A(J+1,II)-A(J+1,I)*W          00015640
0037          A(ND1,II)=A(NED,II)          00015650
0038          1 A(NED,II)=0.          00015660
0039          IF(A(1,N).EQ.0.)GO TO 10          00015670
0040          IF(DABS(DABS((X1-A(1,N))/X1)-1.).LT.1.E-07) E=2.          00015680
0041          9 R = R + 1.          00015690
0042          A(1,N)=A(ND1,N)/A(1,N)          00015700
0043          K=NM1          00015710
0044          NN=2          00015720
0045          8 IF(NN.GT.NED)NN=NED          00015730
0046          J=K+1          00015740
0047          S=0.          00015750
0048          DO 7 I=2,NN          00015760
0049          S=S+A(1,J)*A(I,K)          00015770
0050          7 J=J+1          00015780
0051          A(1,K)=A(ND1,K)-S          00015790
0052          NN=NN+1          00015800
0053          K=K-1          00015810
0054          IF(K.NE.0)GO TO 8          00015820
0055          RETURN          00015830
0056          10 E=1.          00015840
0057          RETURN          00015850
0058          END          00015860

```

```

*OPTIONS IN EFFECT* NOTERM,NOID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP,NOTEST
*OPTIONS IN EFFECT* NAME = TRMSTR , LINECNT = 60
*STATISTICS* SOURCE STATEMENTS = 58,PROGRAM SIZE = .2294
*STATISTICS* NO DIAGNOSTICS GENERATED

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```

0001      SUBROUTINE RITE(IDUM,NR,NC,MR,MC,A)      00015870
0002      DOUBLE PRECISION A                      00015880
0003      DIMENSION A(MR,MC)                      00015890
0004      IPRINT= 12                               00015900
0005      IF(IDUM.NE.1) IPRINT= 30                00015910
0006      IPR= IPRINT-1                            00015920
0007      DO 35 K=1,NC,IPRINT                     00015930
0008      MAX= K+IPR                              00015940
0009      IF(MAX.GT.NC) MAX=NC                    00015950
0010      IF(K.NE.1) WRITE(6,103)                00015960
0011      45 WRITE(6,102) (I,I=K,MAX)            00015970
0012      DO 40 J=1,NR                            00015980
0013      40 WRITE(6,105) J,(A(J,I),I=K,MAX)     00015990
0014      35 CONTINUE                             00016000
0015      RETURN                                  00016010
0016      101 FORMAT(6X,30I4)                    00016020
0017      102 FORMAT(6X,12I10)                   00016030
0018      103 FORMAT('1')                        00016040
0019      104 FORMAT(' ',15,30I4)                00016050
0020      105 FORMAT(' ',15,12G10.3)             00016060
0021      END                                    00016070

```

```

*OPTIONS IN EFFECT*  NOTERM,NOID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP,NOTEST
*OPTIONS IN EFFECT*  NAME = RITE      , LINECNT =      60
*STATISTICS*        SOURCE STATEMENTS =      21,PROGRAM SIZE =      864
*STATISTICS*        NO DIAGNOSTICS GENERATED
*STATISTICS*        NO DIAGNOSTICS THIS STEP

```

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