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WAVE PROPAGATION IN GRAPHITE/EPOXY  
LAMINATES DUE TO IMPACT

by

T.M. Tan and C.T. Sun

December, 1982

**COMPOSITE  
MATERIALS  
LABORATORY**

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16. Abstract The low velocity impact response of graphite/epoxy laminates is investigated theoretically and experimentally. A 9-node isoparametric finite element in conjunction with an empirical contact law was used for the theoretical investigation. Flat laminates subjected to pendulum impact were used for the experimental investigation. Theoretical results are in good agreement with strain gage experimental data. The collective results of the investigation indicate that the theoretical procedure describes the impact response of the laminate up to about 150 in/sec. impact velocity.					
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## LIST OF SYMBOLS

$A$	Cross-sectional area of the projectile
$A_{ij}, B_{ij}, D_{ij}$	Laminate stiffnesses
$E_s$	Young's modulus of the steel indenter
$E_1$	Young's modulus of lamina in the fiber direction
$E_2$	Young's modulus of lamina in the transverse direction
$F$	Contact force
$F_m$	Maximum contact force
$G$	Shear modulus
$[K_p], [K_r]$	Stiffness matrices
$[M_p], [M_r]$	Mass matrices
$M$	Stress couples of laminate
$N$	Stress resultants of laminate
$\{P_p\}, \{P_r\}$	Assembled global load vectors
$Q$	Transverse shear force of laminate
$Q_{ij}$	Reduced stiffnesses
$\bar{Q}_{ij}$	Transformed reduced stiffnesses
$R_s$	Radius of steel indenter
$S_i$	Shape functions of plate element
$V_F$	Output voltage of the force transducer
$V_a$	Output voltage of the accelerometer

a	Acceleration
c	Phase velocity
$c_a$	Sensitivity of the accelerometer
$c_F$	Sensitivity of the impact-force transducer
$c_n$	Normal velocity of wave front
$f_i$	Shape functions of rod element
[f]	Discontinuity of f across wave front surface
h	Laminate thickness
k	Wave number
k	Contact coefficient
$k_1$	Reloading rigidity
$[k_p], [k_r]$	Element stiffness matrices
$[m_p], [m_r]$	Element mass matrices
n	Power index of loading law
$n_i$	Unit normal on the wave front
p	Power index of reloading law
$p_i$	Slowness vector
$\{p_p\}_e, \{p_r\}_e$	Element load vectors
q	Power index of unloading law
$\{q_p\}, \{q_r\}$	Assembled global displacement vectors
$\{q_p\}_e, \{q_r\}_e$	Element displacement vectors
s	Unloading rigidity
t	Time
$t^*$	Non-dimensional time
u, v, w	Displacement components of laminate
$u^0, v^0, w^0$	Midplane displacement components

$x, y, z$	Laminate coordinate system
$x_1, x_2, x_3$	Laminar coordinate system
$\Omega$	Wave front surface
$\alpha$	Indentation depth
$\alpha_0$	Permanent indentation
$\alpha_m$	Maximum indentation
$\alpha_{cr}, \alpha_p$	Critical indentations
$\gamma$	Shearing strain
$\epsilon$	Normal strain
$\kappa_x, \kappa_y, \kappa_{xy}$	Rotation gradients
$\lambda$	Wave length
$\nu$	Poisson's ratio
$\nu_s$	Poisson's ratio of the steel indenter
$\xi, \eta$	Normalized local coordinates of plate element
$\rho$	Mass density of laminate
$\sigma$	Normal stress
$\tau$	Shearing stress
$\phi_x, \phi_y$	Rotations of cross-sections of laminate
$\omega$	Frequency

## CHAPTER 1

### INTRODUCTION

Advanced fiber-reinforced composite materials such as boron/epoxy and graphite/epoxy have been successfully employed as structural materials in aircrafts, missiles and space vehicles in recent years, and the performance of these composites has shown their superiority over metals in applications requiring high strength, high stiffness as well as low weight. The advantages of these composites, however, are overshadowed by their relatively poor resistance to the impact loadings, which has prevented the application of these materials to turbine fan bladings. Many other reports dealing with the responses of advanced composites to various types of impact have further increased the need for a better understanding of the problem so that the survivability of these composites can be improved.

It is obvious that impact produces damage and consequently reduces the strength of composite materials. The damage modes usually include local permanent deformations, breakage of fibers, delaminations, etc.. While the cause of these damages are still unknown and may not be simple in nature, in general, the impact of a soft object could give a longer contact duration, and the dynamic

response of the whole structure is of importance. The hard object impact usually gives a short contact time and results in the initial transmission of impact energy into a local region of the structure. This initial energy will propagate into the rest of the structure in the form of stress waves. Far field damage away from the impact area could result from the reflection of stress waves. It is generally agreed that the cause of the sudden failure must be examined from the point of transient wave propagation phenomena.

Flexural waves induced by dynamic loads in laminated composites are more complicated than those in homogeneous and isotropic plates due to the anisotropic and nonhomogeneous properties in the laminate. Moreover, because of the low transverse shear modulus in fiber composites, the effect of transverse shear deformation becomes significant and should be considered in the formulation. In Chapter 2, the laminate theory which includes the transverse shear deformation effect is reviewed, and harmonic waves in a graphite/epoxy laminated plate are studied. The propagation of wave front which, for a given time after impact, bound the stressed region surrounding the impact point, is also investigated.

A survey of wave propagation and impact in composite materials has been given by Moon [1]. Many analytical [2-5], numerical [6-7] and experimental [8-10] methods have been employed to study the transient impact problems. The

response of a laminated plate can be analyzed using these methods provided the applied load history is prescribed. However if the dynamic load results from an impact of an object on the laminated plate, then the resulting contact force must be determined first. An accurate account of the contact behavior becomes the most important step in analyzing the impact response problems.

A classical contact law between two elastic spheres was derived by Hertz [11]. When letting the radius of one of the spheres go to infinity, one obtains the contact law between an elastic sphere and an elastic half-space. Many authors have used the Hertzian contact law for the study of impact on metals and composites [12-13]. Recently, Yang and Sun [14] performed statical indentation tests on graphite/epoxy composite laminates using spherical steel indenters of different sizes and found that the Hertzian law of contact was not adequate. In particular, they found that significant permanent indentations existed and that the unloading paths were very different from the loading path. Noting that energy dissipation takes place during the process of impact, Yang and Sun [14] suggested that this energy is responsible for the local damage of the target materials. The unloading curves and permanent indentations obtained from the statical indentation tests may provide a useful information in estimating the amount of damage due to impact since this energy is simply the area enclosed by the

loading-unloading curves. In this study, similar statical indentation tests were conducted and the results are presented in Chapter 3.

Wang [15] has performed a number of impact tests on graphite/epoxy laminated beams and plates. It was shown that the strain responses calculated using finite element method and the statically determined contact laws from [14] agreed with the experimental measurements quite well. This indicates that the statical indentation law is reasonably adequate in the dynamical impact analysis. It was also suggested that the contact force should be measured experimentally to provide an additional basis for comparison with the finite element solution which could allow further evaluation the applicability of the contact laws in impact analysis. Chapter 4 describes an impact experiment on graphite/epoxy laminated plate using an impact-force transducer with a spherical steel cap as the impactor. The contact force history and strain responses at various points on the plate were measured by means of the transducer and surface strain gages, respectively, and were compared with the predictions of finite element analysis using the statically determined contact law.

Chapter 5 summarizes the results obtained in Chapter 2, 3 and 4.

## CHAPTER 2

### STRESS WAVE IN A LAMINATED PLATE

A laminated plate theory which includes the effects of transverse shear deformation and rotatory inertia was developed by Yang, Norris and Stavsky [16] in a way suggested by Mindlin [17] for homogeneous isotropic plates. It was shown that the frequency curves for the propagation of harmonic waves in an infinite two-layer isotropic plate in plane strain agreed with the predictions of the exact solution obtained from theory of elasticity very well. A similar laminated plate theory was developed by Whitney and Pagano [18] and was employed in the study of static bending and vibration for antisymmetric angle-ply composite plates with particular layer properties. It was found that the effect of shear deformation can be quite significant for composite plates with span-to-depth ratio as high as 20. Good agreement was also observed in numerical results for plate bending as comparing with exact solutions of elasticity. In this study, the laminate theory developed by Whitney and Pagano was used for its simplicity yet quite satisfactory in describing the harmonic wave propagation [19].

## 2.1 Laminate Theory with Transverse Shear Effects

### 2.1.1 Lamina Constitutive Equations

A laminated plate of constant thickness  $h$  consists of a number of thin laminas of unidirectionally fiber-reinforced composite perfectly bonded together. Each lamina, whose fiber may orient in any arbitrary direction, can be regarded as a homogeneous orthotropic solid. Consider a typical  $k$ -th lamina. A coordinate system  $(x_1, x_2, x_3)$  is chosen in such a way that the  $x_1$ - $x_2$  plane coincides with the midplane of lamina, and  $x_1$  and  $x_2$  axes are parallel and perpendicular to the fiber direction, respectively. If a state of plane stress parallel to the  $x_1$ - $x_2$  plane is assumed, then the in-plane stress-strain relations are given by

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \end{Bmatrix}^k = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \gamma_{12} \end{Bmatrix}^k \quad (2-1)$$

The transverse shear stress-strain relations are given by

$$\begin{Bmatrix} \tau_{23} \\ \tau_{13} \end{Bmatrix}^k = \begin{bmatrix} Q_{44} & 0 \\ 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{23} \\ \gamma_{13} \end{Bmatrix}^k \quad (2-2)$$

in which

$$\begin{aligned}
Q_{11} &= E_1 / (1 - \nu_{12}\nu_{21}) \\
Q_{22} &= E_2 / (1 - \nu_{12}\nu_{21}) \\
Q_{12} &= \nu_{12}E_2 / (1 - \nu_{12}\nu_{21}) = \nu_{21}E_1 / (1 - \nu_{12}\nu_{21}) \\
Q_{66} &= G_{12} \\
Q_{44} &= G_{23} \\
Q_{55} &= G_{13}
\end{aligned} \tag{2-3}$$

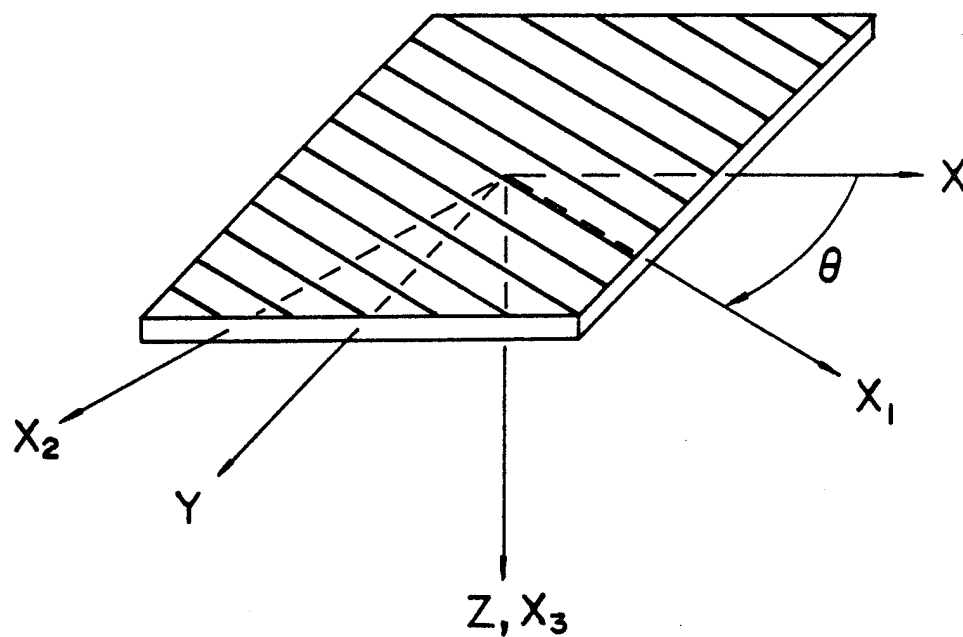
are the so-called reduced stiffnesses, where  $E$ ,  $G$  and  $\nu$  are Young's modulus, shear modulus and Poisson's ratio, respectively, and subscripts 1 and 2 denote the directions parallel to  $x_1$  and  $x_2$  axes, respectively.

The coordinate system for an arbitrarily oriented lamina does not, in general, coincide with the reference axes  $(x, y, z)$  of laminated plate (see Figure 2.1). Using the coordinate transformation laws for stress and strain, we obtain the stress-strain relations in laminate reference system as

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix}^k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} \\ 0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55} \end{bmatrix} \begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}^k \tag{2-4}$$

in which  $\bar{Q}_{i,j}$  are given by

$$\bar{Q}_{11} = Q_{11}m^4 + 2(Q_{12} + 2Q_{66})m^2n^2 + Q_{22}n^4$$



$(X_1, X_2, X_3)$  — Lamina Reference Axes

$(X, Y, Z)$  — Laminate Reference Axes

Figure 2.1 Lamina reference axes and laminate reference axes

$$\begin{aligned}
\bar{Q}_{22} &= Q_{11}n^4 + 2(Q_{12} + 2Q_{66})m^2n^2 + Q_{22}m^4 \\
\bar{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})m^2n^2 + Q_{12}(m^4 + n^4) \\
\bar{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66})m^3n + (Q_{12} - Q_{22} + 2Q_{66})mn^3 \\
\bar{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})mn^3 + (Q_{12} - Q_{22} + 2Q_{66})m^3n \\
\bar{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})m^2n^2 + Q_{66}(m^4 + n^4) \\
\bar{Q}_{44} &= Q_{44}m^2 + Q_{55}n^2 \\
\bar{Q}_{45} &= (Q_{44} - Q_{55})mn \\
\bar{Q}_{55} &= Q_{44}n^2 + Q_{55}m^2
\end{aligned} \tag{2-5}$$

where

$$m = \cos\theta \quad n = \sin\theta$$

and  $\theta$  is the angle between  $x$ -axis and  $x_1$ -axis measured from  $x$  to  $x_1$  counterclockwise as shown in Figure 2.1.

### 2.1.2 Plate Strain-Displacement Relations

The displacement components of the laminated plate are assumed to be of the form [16]

$$\begin{aligned}
u(x,y,z) &= u^0(x,y) + z\phi_x(x,y) \\
v(x,y,z) &= v^0(x,y) + z\phi_y(x,y) \\
w(x,y,z) &= w^0(x,y) = w(x,y)
\end{aligned} \tag{2-6}$$

where  $u^0$ ,  $v^0$  and  $w^0$  are the midplane displacement components in the  $x$ -,  $y$ - and  $z$ -directions, respectively, and  $\phi_x$  and  $\phi_y$  are rotations of cross-sections perpendicular to  $x$ - and  $y$ -axis, respectively (see Figure 2.2). In Equation (2.6) we

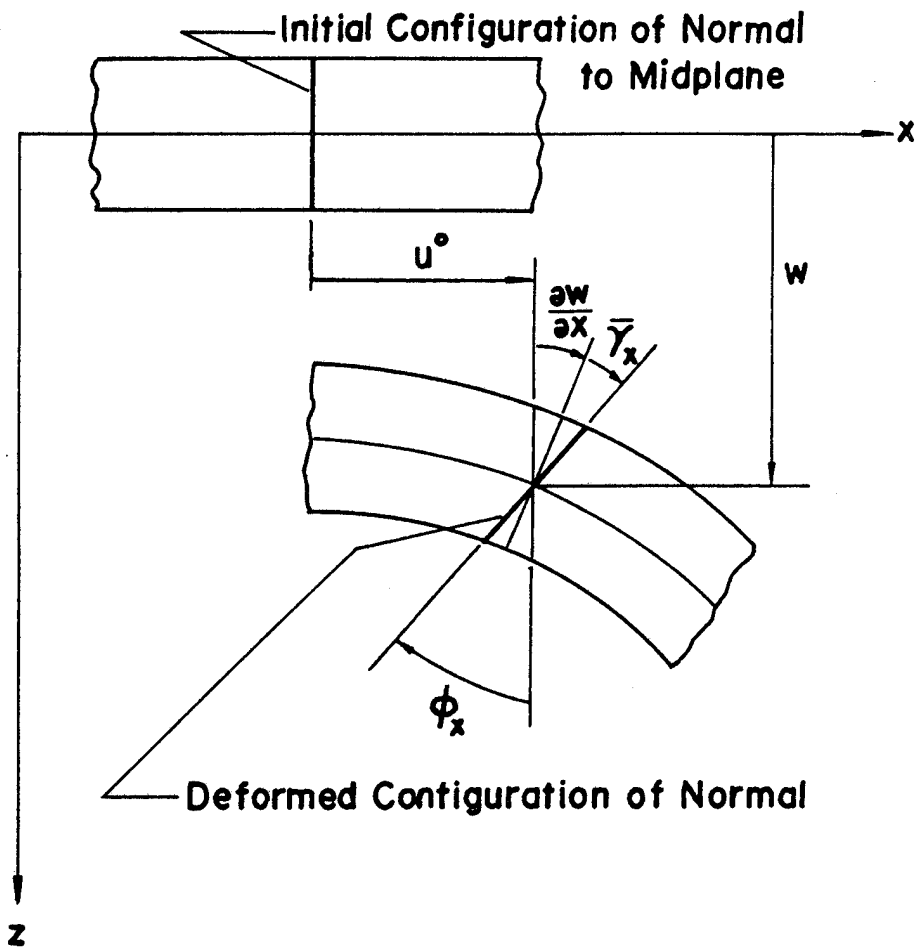


Figure 2.2 Laminate displacement components for a cross-section perpendicular to the y-axis

have assumed that  $u$  and  $v$  vary linearly in the thickness direction, while  $w$  is constant through the thickness.

The strain components for a point in  $k$ -th lamina of the laminated plate with a distance  $z$  from the midplane can be computed as

$$\begin{aligned}
 \epsilon_{xx}^k &= \epsilon_x^0 + z\kappa_x \\
 \epsilon_{yy}^k &= \epsilon_y^0 + z\kappa_y \\
 \gamma_{xy}^k &= \gamma_{xy}^0 + z\kappa_{xy} \\
 \gamma_{yz}^k &= \partial w/\partial y + \partial v/\partial z = \partial w/\partial y + \phi_y = \gamma_{yz}^0 \\
 \gamma_{xz}^k &= \partial w/\partial x + \partial u/\partial z = \partial w/\partial x + \phi_x = \gamma_{xz}^0
 \end{aligned} \tag{2-7}$$

where

$$\begin{aligned}
 \gamma_x^0 &= \partial u^0/\partial x \\
 \gamma_y^0 &= \partial v^0/\partial y \\
 \gamma_{xy}^0 &= \partial u^0/\partial y + \partial v^0/\partial x
 \end{aligned} \tag{2-8}$$

are the in-plane strain components of midplane, and

$$\begin{aligned}
 \kappa_x &= \partial \phi_x/\partial x \\
 \kappa_y &= \partial \phi_y/\partial x \\
 \kappa_{xy} &= \partial \phi_x/\partial y + \partial \phi_y/\partial x
 \end{aligned} \tag{2-9}$$

are the rotation gradients.

In Equation (2-7), since  $w$ ,  $\phi_x$  and  $\phi_y$  are independent of  $z$ , it follows that the transverse shear strains are constant through the thickness of the plate.

Equation (2-7) can be written in concise matrix form as

$$\begin{Bmatrix} \epsilon \\ \gamma \end{Bmatrix}^k = \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}^k = \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \\ \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} \epsilon \\ \gamma \end{Bmatrix}^0 + z \begin{Bmatrix} \kappa \\ 0 \end{Bmatrix} \quad (2-10)$$

Thus, the strain components at any point in the plate can be determined from the extensional strain components of the midplane, the rotation gradients of the plate and the distance  $z$  from the midplane.

### 2.1.3 Stress-Resultants and Laminate Constitutive Equations

Substitution of Equation (2-10) in Equation (2-4) gives the stress components for a point in the  $k$ -th lamina as:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix}^k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} \\ 0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55} \end{bmatrix} \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \\ \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \\ 0 \\ 0 \end{Bmatrix} \quad (2-11)$$

The stress-resultants acting on a laminate can be obtained by integration of the stresses in each lamina through the laminate thickness. Specifically, the in-plane

stress-resultants are given by

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \gamma_{xy} \end{Bmatrix} dz = \sum_{k=1}^N \int_{h_{k-1}}^{h_k} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix}^k dz \quad (2-12)$$

the stress couples are given by

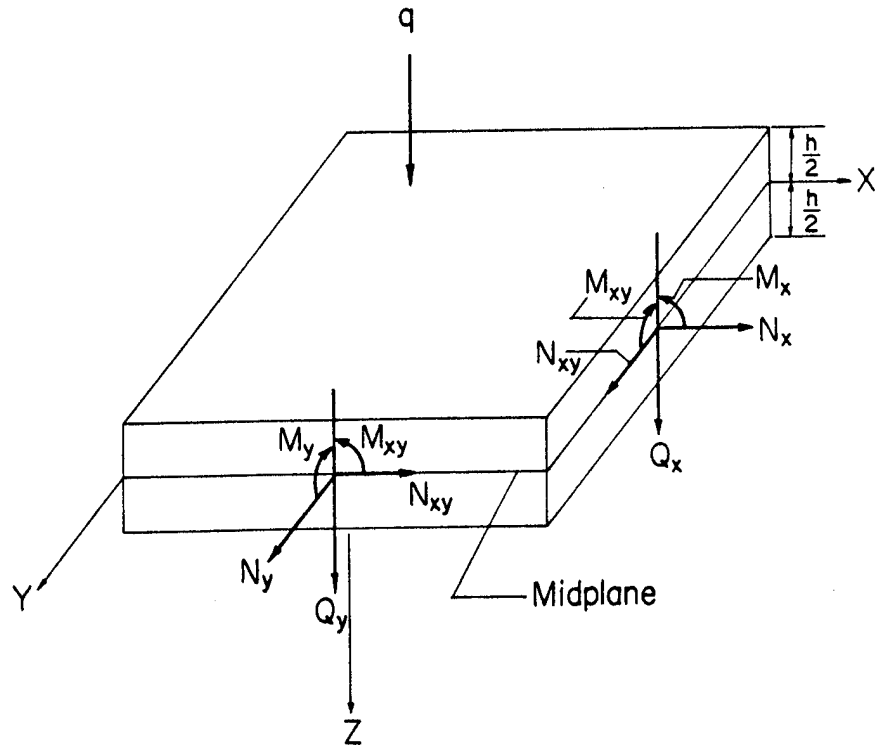
$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \gamma_{xy} \end{Bmatrix} z dz = \sum_{k=1}^N \int_{h_{k-1}}^{h_k} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix}^k z dz \quad (2-13)$$

and the transverse shear forces are given by

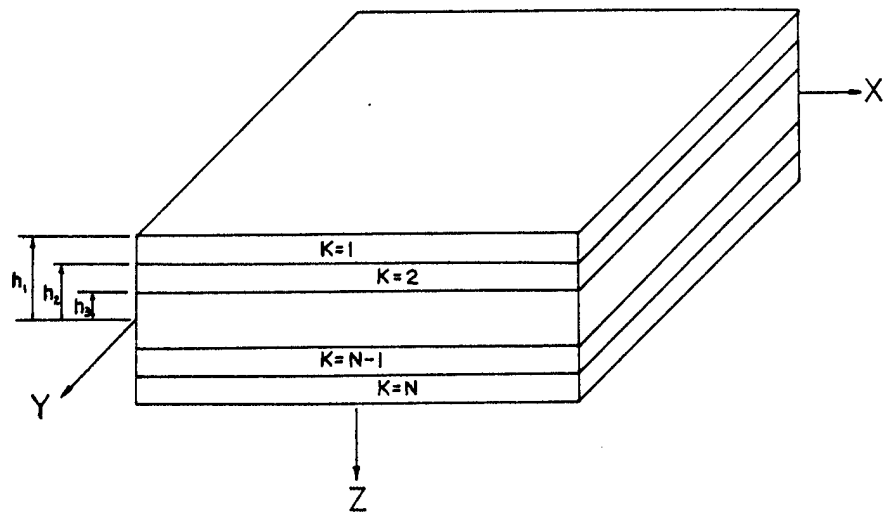
$$\begin{Bmatrix} Q_y \\ Q_x \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix} dz = \sum_{k=1}^N \int_{h_{k-1}}^{h_k} \begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix}^k dz \quad (2-14)$$

The sign convention for these stress-resultants along with the geometry of a typical N-layer laminated plate are shown in Figure 2.3.

Substituting Equation (2-11) into the right hand sides of the above three equations and performing the integrations, we obtain



(a) STRESS RESULTANTS OF A LAMINATE

(b) GEOMETRY OF AN  $N$ -LAYER LAMINATEFigure 2.3 Stress-resultants and geometry of a typical  $N$ -layer laminate

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \quad (2-15)$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \quad (2-16)$$

$$\begin{Bmatrix} Q_y \\ Q_x \end{Bmatrix} = \begin{bmatrix} A^*_{44} & A^*_{45} \\ A^*_{45} & A^*_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \quad (2-17)$$

where

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-h/2}^{h/2} \bar{Q}_{ij}(1, z, z^2) dz \quad i, j = 1, 2, 6 \quad (2-18)$$

and

$$A^*_{ij} = \int_{-h/2}^{h/2} \bar{Q}_{ij} dz \quad i, j = 4, 5 \quad (2-19)$$

Equations (2-15) through (2-17) are usually written symbolically as

$$\begin{Bmatrix} N \\ M \\ Q \end{Bmatrix} = \begin{bmatrix} A & B & 0 \\ B & D & 0 \\ 0 & 0 & A^* \end{bmatrix} \begin{Bmatrix} \epsilon^0 \\ \kappa \\ \gamma \end{Bmatrix} \quad (2-20)$$

which is the laminate constitutive equation with transverse shear effect included.

### 2.1.4 Plate Equations of Motion

The stress-equations of motion for the  $k$ -th lamina are given by

$$\begin{aligned}\sigma_{xx,x} + \tau_{xy,y} + \tau_{xz,z} &= \rho\ddot{u} \\ \tau_{xy,x} + \sigma_{yy,y} + \tau_{yz,z} &= \rho\ddot{v} \\ \tau_{xz,x} + \tau_{yz,y} + \sigma_{zz,z} &= \rho\ddot{w}\end{aligned}\quad (2-21)$$

where  $\rho$  is the mass density. Integrating Equation (2-21) through the thickness of laminate and then substituting Equation (2-12), (2-14) and (2-6) in, we obtain

$$\begin{aligned}N_{x,x} + N_{xy,y} &= P\ddot{u}^0 + R\ddot{\phi}_x \\ N_{xy,x} + N_{y,y} &= P\ddot{v}^0 + R\ddot{\phi}_y \\ Q_{x,x} + Q_{y,y} + q &= P\ddot{w}\end{aligned}\quad (2-22)$$

where  $q$  is the normal traction on the plate. Multiplying the first two equations of Equation (2-21), integrating through the thickness of laminate and then substituting Equations (2-13), (2-14) and (2-5) in, we obtain

$$\begin{aligned}M_{x,x} + M_{xy,y} - Q_x &= R\ddot{u}^0 + I\ddot{\phi}_x \\ M_{xy,x} + M_{y,y} - Q_y &= R\ddot{v}^0 + I\ddot{\phi}_y\end{aligned}\quad (2-23)$$

in which  $P$ ,  $R$  and  $I$  are defined as

$$(P, R, I) = \int_{-h/2}^{h/2} \rho(1, z, z^2) dz \quad (2-24)$$

Equations (2-22) and (2-23) are the plate equations of

motion. Substitution of Equation (2-20) and then the strain-displacement relations in these two equations yield the equations of motion in terms of midplane displacements and rotations of the plate.

A graphite/epoxy laminated plate provided by NASA Lewis Research Center was used throughout this study. This laminate is a  $[0^0/45^0/0^0/-45^0/0^0]_{2s}$  graphite/epoxy composite with 0.0053 inch ply thickness and the following ply properties [15]:

$$\begin{aligned}
 E_1 &= 17.5 \times 10^6 \text{ psi.} \\
 E_2 &= 1.15 \times 10^6 \text{ psi.} \\
 G_{12} = G_{13} = G_{23} &= 0.8 \times 10^6 \text{ psi.} \\
 \nu_{12} &= 0.30 \\
 \rho &= 0.000148 \text{ lb-sec}^2/\text{in}^4
 \end{aligned}
 \tag{2-25}$$

For symmetrically laminated composite plate,  $B_{ij} = 0$  and  $R = 0$ . In addition, by choosing the x-axis of the laminate reference system to coincide with the  $0^0$  fiber direction, we obtain  $A_{16} = A_{26} = 0$  and  $D_{16} = D_{26}$ . Further, in this study, we assume  $G_{13} = G_{23} = G_{12}$ , and consequently,  $A^*_{45} = 0$  and  $A^*_{44} = A^*_{55}$ . For this particular laminate, the displacement-equations of motion are given by

$$\begin{aligned}
 A_{11}\partial^2 u^0/\partial x^2 + A_{66}\partial^2 u^0/\partial y^2 + (A_{12} + A_{66})\partial^2 v^0/\partial x\partial y &= P\ddot{u}^0 \\
 (A_{12} + A_{66})\partial^2 u^0/\partial x\partial y + A_{66}\partial^2 v^0/\partial x^2 + A_{22}\partial^2 v^0/\partial y^2 &= P\ddot{v}^0
 \end{aligned}$$

$$\begin{aligned}
& D_{11}\partial^2\phi_x/\partial x^2 + 2D_{16}\partial^2\phi_x/\partial x\partial y + D_{66}\partial^2\phi_x/\partial y^2 \\
& + D_{16}(\partial^2\phi_y/\partial x^2 + \partial^2\phi_y/\partial y^2) + (D_{12} + D_{66})\partial^2\phi_y/\partial x\partial y \\
& -A^*_{44}(\partial w/\partial x + \phi_x) = I\ddot{\phi}_x
\end{aligned} \tag{2-26}$$

$$\begin{aligned}
& D_{16}(\partial^2\phi_x/\partial x^2 + \partial\phi_x/\partial y^2) + (D_{12} + D_{66})\partial^2\phi_x/\partial x\partial y \\
& + D_{66}\partial^2\phi_y/\partial x^2 + 2D_{16}\partial^2\phi_y/\partial x\partial y + D_{22}\partial^2\phi_y/\partial y^2 \\
& -A^*_{44}(\partial w/\partial y + \phi_y) = I\ddot{\phi}_y
\end{aligned}$$

$$A^*_{44}(\partial^2 w/\partial x^2 + \partial^2 w/\partial y^2 + \partial\phi_x/\partial x + \partial\phi_y/\partial y) + q = P\ddot{w}$$

In Equation (2-26), the first two equations govern the in-plane motion while the last three equations govern the flexural motion.

## 2.2 Propagation of Harmonic Waves

Consider a infinitely large laminated plate governed by the equations of motion (2-26). We assume plane harmonic waves in the form

$$\begin{aligned}
u^0 &= U \exp[ik(\eta - ct)] \\
v^0 &= V \exp[ik(\eta - ct)] \\
w &= W \exp[ik(\eta - ct)] \\
\phi_x &= \Phi_x \exp[ik(\eta - ct)] \\
\phi_y &= \Phi_y \exp[ik(\eta - ct)]
\end{aligned} \tag{2-27}$$

propagating over the plate, where  $U$ ,  $V$ ,  $W$ ,  $\Phi_x$  and  $\Phi_y$  are constant amplitudes,  $k$  is the wave number,  $c$  is the phase

velocity and  $\eta$  is given by

$$\eta = x \cos\alpha + y \sin\alpha \quad (2-28)$$

In which  $\alpha$  is the angle between the direction of wave propagation and x-axis.

Substitution of Equation (2-27) into Equation (2-26) with  $q = 0$  yields a system of five homogeneous equations for the five constant amplitudes. In order to have a nontrivial solution, the determinant of the coefficient matrix is set equal to zero. Since the equations are uncoupled into two groups, the determinantal equation can be separated into two equations as

$$|a_{ij}| = 0 \quad (2-29)$$

for the in-plane extensional and in-plane shear waves, and

$$|b_{ij}| = 0 \quad (2-30)$$

for the flexural waves. In Equations (2-29) and (2-30) the coefficients  $a_{ij}$  and  $b_{ij}$  are given by

$$\begin{aligned} a_{11} &= A_{11}\cos^2\alpha + A_{66}\sin^2\alpha - Pc^2 \\ a_{12} &= a_{21} = (A_{12} + A_{66})\sin\alpha\cos\alpha \\ a_{22} &= A_{66}\cos^2\alpha + A_{22}\sin^2\alpha - Pc^2 \end{aligned} \quad (2-31)$$

and

$$\begin{aligned} b_{11} &= D_{11}k^2\cos^2\alpha + 2D_{16}k^2\sin\alpha\cos\alpha + D_{66}k^2\sin^2\alpha \\ &\quad + A_{44}^* - Ik^2c^2 \end{aligned}$$

$$\begin{aligned}
b_{12} &= b_{21} = D_{16}k^2\cos^2\alpha + (D_{12} + D_{66})k^2\sin\alpha\cos\alpha \\
&\quad + D_{16}k^2\sin^2\alpha \\
b_{13} &= b_{31} = iA_{44}^*k\cos\alpha \\
b_{22} &= D_{66}k^2\cos^2\alpha + 2D_{16}k^2\sin\alpha\cos\alpha + D_{22}k^2\sin^2\alpha \\
&\quad + A_{44}^* - Ik^2c^2 \\
b_{23} &= b_{32} = iA_{44}^*k\sin\alpha \\
b_{33} &= -A_{44}^*k^2 + Pk^2c^2
\end{aligned} \tag{2-32}$$

Expanding Equation (2-29) we obtain a quadratic equation in  $c^2$  as

$$c^4 - d_1c^2 + d_2 = 0 \tag{2-33}$$

where

$$d_1 = (A_{11}\cos^2\alpha + A_{22}\sin^2\alpha + A_{66})/P \tag{2-34}$$

$$d_2 = \begin{vmatrix} A_{11}\cos^2\alpha + A_{66}\sin^2\alpha & (A_{12} + A_{66})\sin\alpha\cos\alpha \\ (A_{12} + A_{66})\sin\alpha\cos\alpha & A_{66}\cos^2\alpha + A_{22}\sin^2\alpha \end{vmatrix}$$

It is noted that the phase velocity  $c$  does not depend on the wave number  $k$ , thus these waves are nondispersive. In studying of transverse impact problem where in-plane deformation is negligible, this nondispersive property has no significant effect. Should in-plane deformation become important, higher order approximation of displacement

components  $u$  and  $v$  must be assumed and the dispersive property of these in-plane waves could be included.

From Equation (2-34) it is evident that there exist two phase velocities corresponding to two modes of wave. Although these two waves involve both in-plane extensional deformation as well as in-plane shear, from the eigenvectors we are able to tell which one is dominant. Thus we label the two waves as in-plane extensional wave and in-plane shear wave accordingly.

The determinantal equation given by Equation (2-30) yields three positive roots in  $c^2$  indicating that three flexural waves exist. These phase velocities are functions of the wave number  $k$ , thus they are dispersive. Among these three modes of wave, only the lowest one corresponding to the transverse shear wave has a finite velocity as  $k \rightarrow 0$  or as the wave length becomes infinite. The dispersion curves for the waves of the lowest mode propagating in the directions of  $0^\circ$ ,  $45^\circ$  and  $90^\circ$  respectively are plotted in Figure 2.4 with the non-dimensional phase velocity vs. the non-dimensional wavelength  $\lambda/h$ . It can be seen that they all approach the value of  $\sqrt{G_{13}/\rho}$  as the wavelength becomes shorter. The phase velocities for the two higher modes, however, approach different values in different propagation directions when  $\lambda \rightarrow 0$ . For laminated composite which are anisotropic in general, the phase velocity varies from one direction to another. As a result the wave surface will

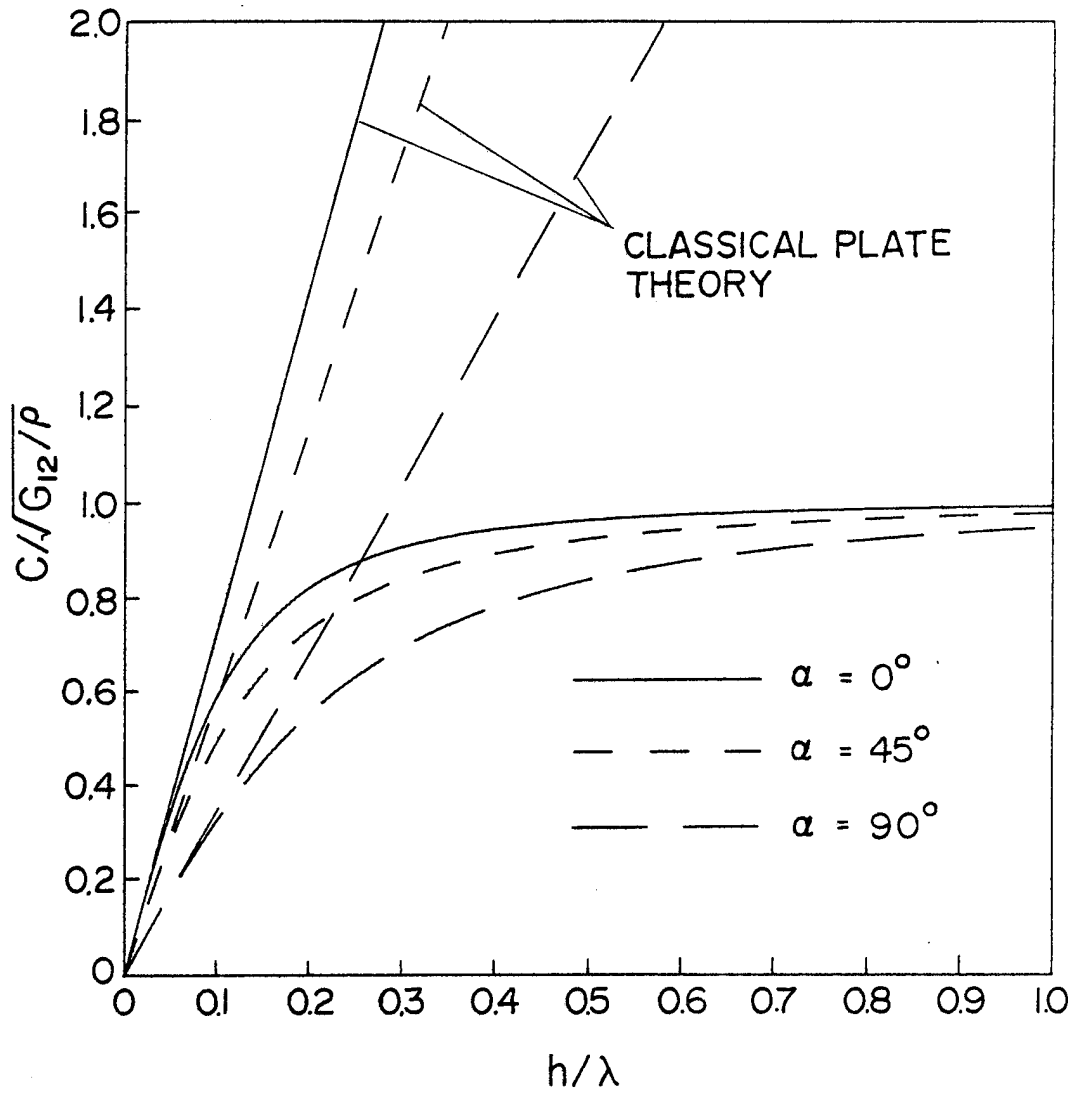


Figure 2.4 Dispersion curves for plane harmonic waves propagating in the  $0^\circ$ -  $45^\circ$ - and  $90^\circ$ - directions

become a rather complicated shape as it propagates. This will be discussed in the next section.

Substitution of  $\omega = kc$  in Equation (2-32) yields a set of frequency equations for flexural waves. Figure 2.5 shows the frequency curves of these waves for  $\alpha = 0^\circ$ ,  $45^\circ$  and  $90^\circ$ , respectively, with the non-dimensional frequency vs. the non-dimensional wavelength. The cutoff frequencies for the two higher modes have a value of  $\sqrt{12G_{13}/\rho}/h$ . Comparing with the exact cutoff frequency  $(\pi/h)\sqrt{G_{13}/\rho}$ , it can be seen that if the shear correction factor  $\pi^2/12$  is introduced, this theory will predict the correct cutoff frequency.

### 2.3 Propagation of Wave Front

Impact of foreign objects on a laminated plate with a very short duration could generate weak shock waves which will propagate into the rest of the structure with finite velocities, and the positions of the wave fronts define the regions being disturbed at any instant after impact. Damages to the laminated plate may possibly occur as the first wave front hits the weakest part. It is hence important to investigate the propagation of these shocks in the plate. There have been works dealing with the propagation of wave front in anisotropic elastic media [20-22]. Moon [23] presented an analysis of wave surfaces in a laminate by treating it as an equivalent homogeneous

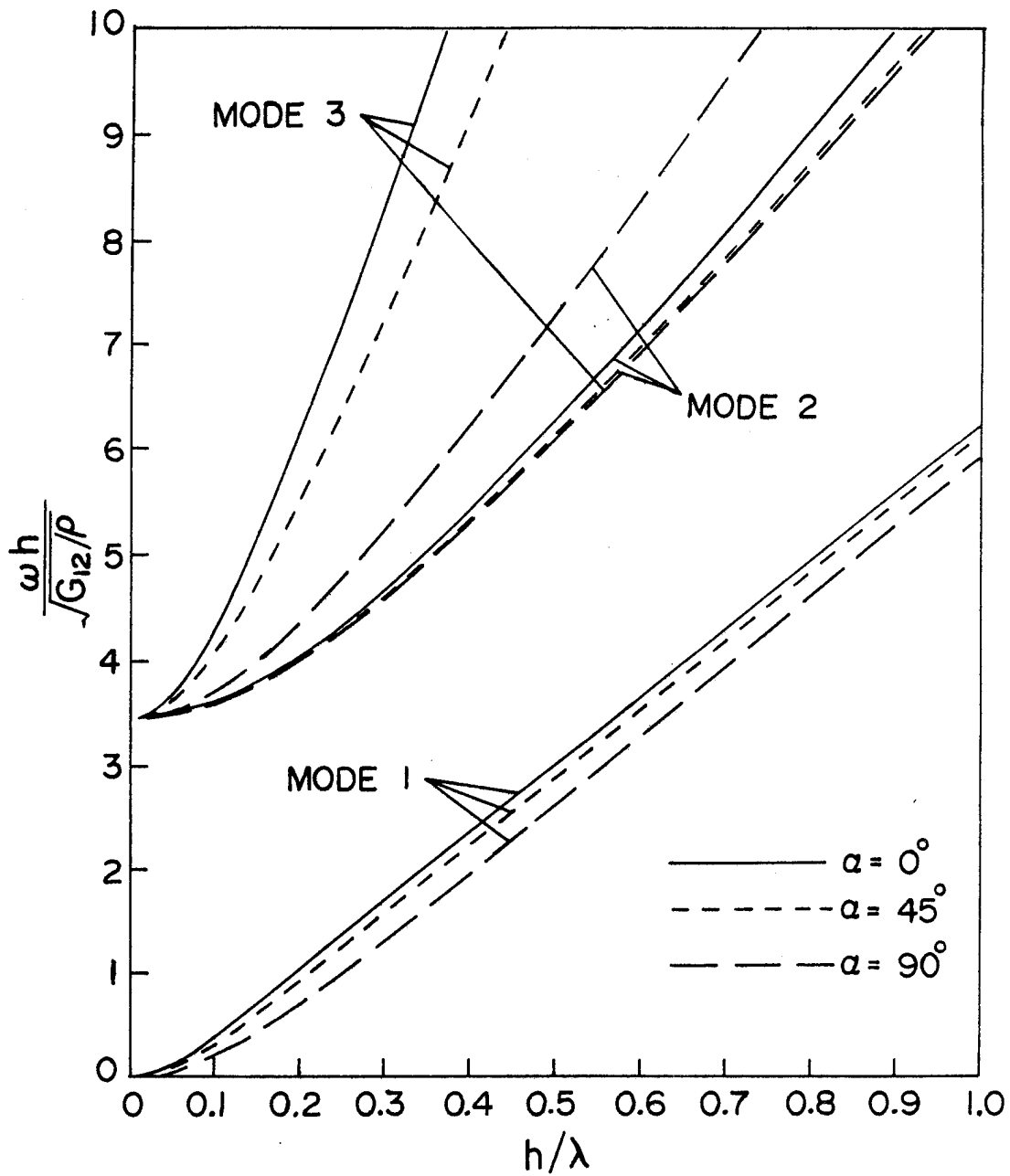


Figure 2.5 Frequency curves for flexural waves propagating in the  $0^\circ$ -  $45^\circ$ - and  $90^\circ$ - directions

orthotropic plate. The acceleration waves and their wave fronts were investigated. The propagation of shock waves in more general laminates which exhibit the bending-extensional coupling were studied by Sun [2]. The ray theory was employed to construct the wave front surface. The growth and decay of the shock strength were also discussed. In this section, the analytical procedures developed by Sun [2] were applied on a  $[0^\circ/45^\circ/0^\circ/-45^\circ/0^\circ]_{2s}$  graphite/epoxy laminated plate.

### 2.3.1. Kinematic Conditions of Compatibility on the Wave Front

A wave front, which will be denoted by  $\Omega$ , is defined as a surface travelling over the plate as time varies continuously, and across which there may exist a discontinuity in the stress, particle velocity and their derivatives.

Consider a discontinuous surface  $\Omega$  passing some observation point in a medium at a certain time  $t$ . Let  $f^-$  be the value of a field function  $f(x_i, t)$  (e.g. stress, particle velocity, etc.) behind the surface  $\Omega$ , and  $f^+$  be the value of  $f$  in front of it, then the discontinuity of function  $f$  can be expressed as

$$[f] = f^+ - f^- \quad (2-35)$$

In which the right hand side is to be evaluated at the time and location on  $\Omega$  passing the observation point, and the jump across the wave front is denoted by square bracket.

Surface  $\Omega$  may be expressed mathematically by an equation of the form

$$\psi(x_i, t) = 0 \quad (2-36)$$

or, by making  $t$  explicit, as

$$\psi(x_i, t) = F(x_i) - t = 0 \quad (2-37)$$

which represents a family of surfaces in  $x_i$ -space with  $t$  as a parameter. By evaluating  $f^+$  and  $f^-$  at  $t = F(x_i)$ , the jump of  $f$  across the wave front becomes

$$[f(x_i)] = f^+(x_i, F(x_i)) - f^-(x_i, F(x_i)) \quad (2-38)$$

The rate of change of  $[f]$  for an observer moving with  $\Omega$  is given by

$$\begin{aligned} d[f]/dt &= (\partial f^+/\partial x_i - \partial f^-/\partial x_i) dx_i/dt + (\partial f^+/\partial t - \partial f^-/\partial t) \\ &= c_i [\partial f/\partial x_i] + [\partial f/\partial t] \end{aligned} \quad (2-39)$$

where  $t = F(x_i)$  is substituted, and  $c_i = dx_i/dt$  are velocity components of wave front relative to the material.

If the laminate theory introduced in previous section is used, then the plate displacement components are  $u^0$ ,  $v^0$ ,  $w$ ,  $\phi_x$  and  $\phi_y$ , while the spatial variables are  $x_1 = x$  and  $x_2 = y$ . It is assumed that the integrity of the material is not

affected by the propagation of the stress wave front, then these displacement components will remain continuous. Consequently, we have

$$[u^0] = [v^0] = [w] = [\phi_x] = [\phi_y] = 0 \quad (2-40)$$

across the wave front. Applying the general condition of Equation (2-39) on  $u^0$ , together with Equation (2-40), we obtain

$$[\partial u^0 / \partial x_j] c_j + [\dot{u}^0] = 0 \quad j = 1, 2 \quad (2-41)$$

Let  $c_n$  and  $n_j$  be the normal velocity and the unit normal on the wave front, respectively, it follows that

$$n_j c_j = c_n \quad (2-42)$$

and Equation (2-41) becomes

$$[\partial u^0 / \partial x_j] = -[\dot{u}^0] n_j / c_n \quad j = 1, 2 \quad (2-43)$$

Similar relations can be derived for the other displacement components  $v^0$ ,  $w$ ,  $\phi_x$  and  $\phi_y$ . Together they specify the kinematic conditions of compatibility on the wave front.

### 2.3.2 Dynamical Conditions on the Wave Front

Consider a finite volume  $V$  of a material medium and denoted by  $S$  the boundary or surface of  $V$ . For a continuous and differentiable function  $f(x_i, t)$  in  $V$ , it can be shown

[23] that

$$\frac{d}{dt} \int_V f(x_i, t) dV = \int_V f_{,t} dV + \int_S G f dS \quad (2-44)$$

under deformation of the medium, where  $G$  is the normal velocity of the surface  $S$ . In the case where the deformation of the volume  $V$  is determined solely by the motion of material particles, we have

$$G = \dot{u}_i n_i = \dot{u}_n \quad (2-45)$$

where  $u_i$  is the displacement components,  $n_i$  is the outward normal on  $S$ , and  $\dot{u}_n$  is the normal velocity of material particle on  $S$ . If there exists a discontinuity surface (or wave front) travelling with velocity  $c_i$  in the medium, by choosing this surface as the boundary of  $V$ , we have

$$G = c_i n_i = c_n \quad (2-46)$$

where  $c_n$  is the normal velocity of wave front.

Suppose that a volume  $V$  whose motion is determined by the deformation of the material medium, is divided by a travelling surface  $\Omega$  into two volumes  $V^-$  and  $V^+$  as shown in Figure 2.6. The surface  $S$  is also divided into two portions  $S^-$  and  $S^+$  which form parts of the boundaries of  $V^-$  and  $V^+$ , respectively. The remaining part of the boundary is formed by  $\Omega_0$  which is a segment of  $\Omega$ . In Figure 2.6,  $n_i$  denotes the unit normal of  $\Omega$  in the direction of travelling, and  $n_i^*$  denotes the unit outward normal of  $S$ .

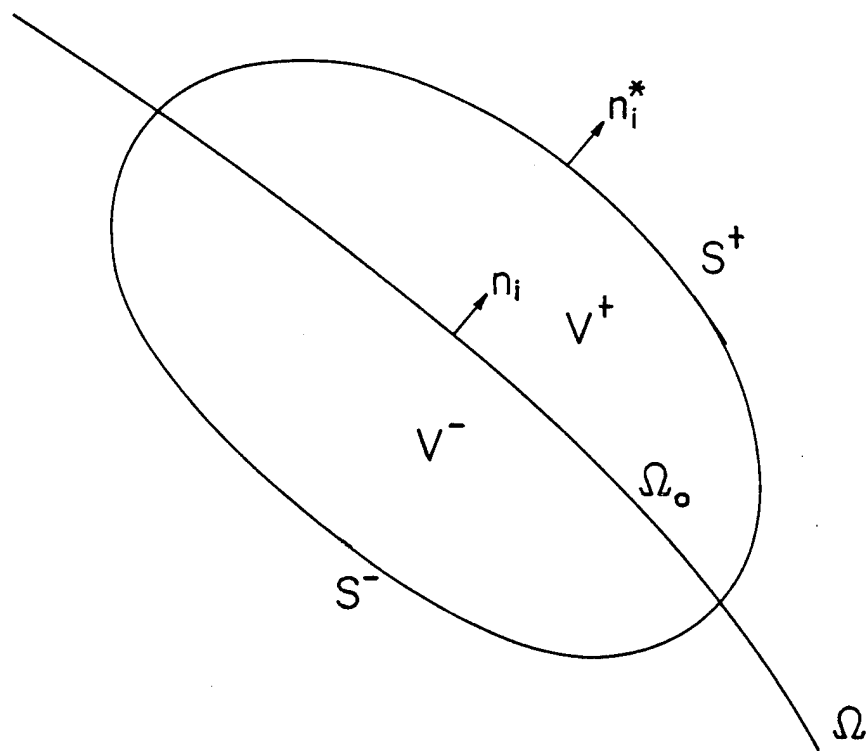


Figure 2.6 A deformed volume  $V$  divided by a travelling surface  $\Omega$

Taking  $f = \rho \dot{u}_i$  in Equation (2-44) and using equation (2-45) and (2-46), we obtain

$$\frac{d}{dt} \int_{V^-} \rho \dot{u}_i^- dV = \int_{V^-} (\rho \dot{u}_i^-)_{,t} dV + \int_{S^-} \dot{u}_n^- \rho \dot{u}_i^- dS + \int_{\Omega_0} c_n \rho \dot{u}_i^- d\Omega \quad (2-47)$$

$$\frac{d}{dt} \int_{V^+} \rho \dot{u}_i^+ dV = \int_{V^+} (\rho \dot{u}_i^+)_{,t} dV + \int_{S^+} \dot{u}_n^+ \rho \dot{u}_i^+ dS - \int_{\Omega_0} c_n \rho \dot{u}_i^+ d\Omega \quad (2-48)$$

where  $\dot{u}_i^-$  and  $\dot{u}_i^+$  are the velocity components of material particle in  $V^-$  and  $V^+$ , respectively. Combining the above two equations gives

$$\begin{aligned} \frac{d}{dt} \int_V \rho \dot{u}_i dV &= \int_V (\rho \dot{u}_i)_{,t} dV + \int_{S^-} \dot{u}_n^- \rho \dot{u}_i^+ dS + \int_{S^+} \dot{u}_n^+ \rho \dot{u}_i^+ dS \\ &\quad + \int_{\Omega_0} c_n \rho (\dot{u}_i^- - \dot{u}_i^+) d\Omega \end{aligned} \quad (2-49)$$

From theory of elasticity we have

$$\frac{d}{dt} \int_V \rho \dot{u}_i dV = \int_S \sigma_{ij} n_j dS \quad (2-50)$$

If we let the volume  $V$  approach zero at a fixed time in such a way that it will pass into  $\Omega_0$ , then the volume integral in Equation (2-49) will evidently approach zero; however

$$\int_{S^+} \dot{u}_n^+ \rho \dot{u}_i^+ dS \rightarrow - \int_{\Omega_0} \dot{u}_n^+ \rho \dot{u}_i^+ d\Omega \quad (2-51)$$

$$\int_{s^-} \dot{u}_n^- \rho \dot{u}_i^- dS \rightarrow \int_{\Omega_0} \dot{u}_n^- \rho \dot{u}_i^- d\Omega \quad (2-52)$$

$$\int_s \sigma_{ij} n_j dS \rightarrow \int_{\Omega_0} (\sigma_{ij}^+ - \sigma_{ij}^-) n_j d\Omega \quad (2-53)$$

where  $\sigma_{ij}^-$  and  $\sigma_{ij}^+$  are the stress components on the sides of  $\Omega_0$ , respectively.

Substituting Equations (2-50) through (2-53) into Equation (2-49) gives

$$\int_{\Omega_0} (\sigma_{ij}^+ - \sigma_{ij}^-) n_j d\Omega = \int_{\Omega_0} \rho \dot{u}_i^- (c_n - \dot{u}_n^-) d\Omega - \int_{\Omega_0} \rho \dot{u}_i^+ (c_n - \dot{u}_n^+) d\Omega \quad (2-54)$$

Using  $[\sigma_{ij}]$  and  $[\dot{u}_i]$  to represent the jumps of stress and particle velocity across the wave front, and utilizing the fact that  $c_n \gg \dot{u}_n$ , we obtain

$$\int_{\Omega_0} [\sigma_{ij}] n_j d\Omega = - \int_{\Omega_0} \rho c_n [\dot{u}_i] d\Omega \quad (2-55)$$

Since this condition is independent of the extent of the surface integration  $\Omega_0$ , it follows that

$$[\sigma_{ij}] n_j = - \rho c_n [\dot{u}_i] \quad (2-56)$$

In the case of laminated plate,  $i = x, y, z$  and  $j = x, y$ .

Substitution of Equation (2-6) into Equation (2-56) yields

$$\begin{aligned}
[\sigma_{1j}]n_j &= -\rho c_n \{ [\dot{u}^0] + z[\dot{\phi}_x] \} \\
[\sigma_{2j}]n_j &= -\rho c_n \{ [\dot{v}^0] + z[\dot{\phi}_y] \} \\
[\sigma_{3j}]n_j &= -\rho c_n [\dot{w}]
\end{aligned}
\tag{2-57}$$

Integrating Equation (2-57) over the thickness of plate gives

$$\begin{aligned}
[N_x]n_x + [N_{xy}]n_y &= -Pc_n [\dot{u}^0] - Rc_n [\dot{\phi}_x] \\
[N_{xy}]n_x + [N_y]n_y &= -Pc_n [\dot{v}^0] - Rc_n [\dot{\phi}_y] \\
[Q_x]n_x + [Q_y]n_y &= -Pc_n [\dot{w}]
\end{aligned}
\tag{2-58}$$

Multiplying the first two equations of Equation (2-57) by  $z$  and then integrating over the thickness, we obtain two more equations

$$\begin{aligned}
[M_x]n_x + [M_{xy}]n_y &= -Rc_n [\dot{u}^0] - Ic_n [\dot{\phi}_x] \\
[M_{xy}]n_x + [M_y]n_y &= -Rc_n [\dot{v}^0] - Ic_n [\dot{\phi}_y]
\end{aligned}
\tag{2-59}$$

where  $P$ ,  $R$  and  $I$  have been defined in Equation (2-24)

The five equations given by Equations (2-58) and (2-59) are the dynamical conditions on the wave front for the laminated plate.

### 2.3.3 Propagation Velocity of the Wave Front

Across the wave front, the laminate constitutive relations given by Equation (2-20) can be written as

$$\begin{Bmatrix} [N] \\ [M] \\ [Q] \end{Bmatrix} = \begin{bmatrix} A & B & 0 \\ B & D & 0 \\ 0 & 0 & A^* \end{bmatrix} \begin{Bmatrix} [\epsilon] \\ [\kappa] \\ [\gamma] \end{Bmatrix} \quad (2-60)$$

where

$$\begin{aligned} \{[N]\}^T &= \{[N_x], [N_y], [N_{xy}]\} \\ \{[M]\}^T &= \{[M_x], [M_y], [M_{xy}]\} \\ \{[Q]\}^T &= \{[Q_x], [Q_y]\} \end{aligned} \quad (2-61)$$

are the jumps of the stress resultants, and

$$\begin{aligned} \{[\epsilon]\}^T &= \{[\partial u^0/\partial x], [\partial v^0/\partial y], [\partial u^0/\partial y] + [\partial v^0/\partial x]\} \\ \{[\kappa]\}^T &= \{[\partial \phi_x/\partial x], [\partial \phi_y/\partial y], [\partial \phi_x/\partial y] + [\partial \phi_y/\partial x]\} \\ \{[\gamma]\}^T &= \{[\partial w/\partial y], [\partial w/\partial x]\} \end{aligned} \quad (2-62)$$

are the jumps of the strain components. In Equation (2-62), the conditions  $[\phi_x] = [\phi_y] = 0$  are substituted.

Substituting of Equation (2-43) and the similar relations for other kinematic variables in Equation (2-60), we can express the jumps of the stress resultants in terms of the jumps of the time derivatives of the displacement components  $u^0$ ,  $v^0$ ,  $w$ ,  $\phi_x$  and  $\phi_y$ . These relations are then substituted in Equations (2-58) and (2-59), which results in five homogeneous equations. For  $[0^0/45^0/0^0/-45^0/0^0]_{2s}$  graphite/epoxy laminated plate which is symmetrical and balanced (i.e.  $B_{ij} = 0$ ,  $A_{16} = A_{26} = 0$ ,  $R = 0$  and  $D_{16} = D_{26}$ ), these five equations are uncoupled into three groups as

$$[a_{ij}] \begin{Bmatrix} [\dot{u}^0] \\ [\dot{v}^0] \end{Bmatrix} = 0 \quad (2-63)$$

$$[b_{ij}] \begin{Bmatrix} [\dot{\phi}_x] \\ [\dot{\phi}_y] \end{Bmatrix} = 0 \quad (2-64)$$

$$(A^*_{44} - Pc_n^2) [\dot{w}] = 0 \quad (2-65)$$

In which  $[a_{ij}]$  and  $[b_{ij}]$  are both two by two symmetric matrices, and their entries are given by

$$\begin{aligned} a_{11} &= n_x^2 A_{11} + n_y^2 A_{66} - Pc_n^2 \\ a_{12} &= a_{21} = n_x n_y (A_{12} + A_{66}) \\ a_{22} &= n_x^2 A_{66} + n_y^2 A_{22} - Pc_n^2 \end{aligned} \quad (2-66)$$

$$\begin{aligned} b_{11} &= n_x^2 D_{11} + 2n_x n_y D_{16} + n_y^2 D_{66} - Ic_n^2 \\ b_{12} &= b_{21} = D_{16} + n_x n_y (D_{12} + D_{66}) \\ b_{22} &= n_x^2 D_{66} + 2n_x n_y D_{16} + n_y^2 D_{22} - Ic_n^2 \end{aligned} \quad (2-67)$$

It can be seen that Equation (2-63) describes the in-plane extensional and the in-plane shear wave fronts, Equation (2-64) describes the bending moment and the twisting moment wave fronts and Equation (2-65) describes the transverse shear wave front.

From Equation (2-65), we obtain the normal velocity with which the transverse shear wave front propagates as

$$c_n^2 = A^*_{44}/P \quad (2-68)$$

It is noted that this velocity is independent of the direction of propagation, and is called directionally

Isotropic wave front.

Equations (2-63) and (2-64) yield non-trivial solutions only if the determinant of the coefficients matrices vanish, i.e.

$$|a_{ij}| = 0 \quad (2-69)$$

$$|b_{ij}| = 0 \quad (2-70)$$

Each of the above equations can be expanded into a quadratic equation of  $c_n^2$ . For  $[0^\circ/45^\circ/0^\circ/-45^\circ/0^\circ]_{2s}$  graphite/epoxy laminated plate, the normal velocities of wave fronts corresponding to the in-plane modes and flexural modes are plotted in Figure 2.7 and 2.8, respectively. It is noted that the normal velocities of the in-plane extensional and in-plane shear modes are symmetrical about x-axis and y-axis, while there is no such symmetry for the bending moment and twisting moment modes.

#### 2.3.4 Wave Surface and Ray

From Figure 2.7 and 2.8, it can be seen that for laminated composites which are anisotropic in general, the in-plane and flexural wave fronts travel with different normal velocities in different directions. In other words, the initial shape of a wave surface will be distorted after it propagates. However, Equations (2-66) and (2-67) show

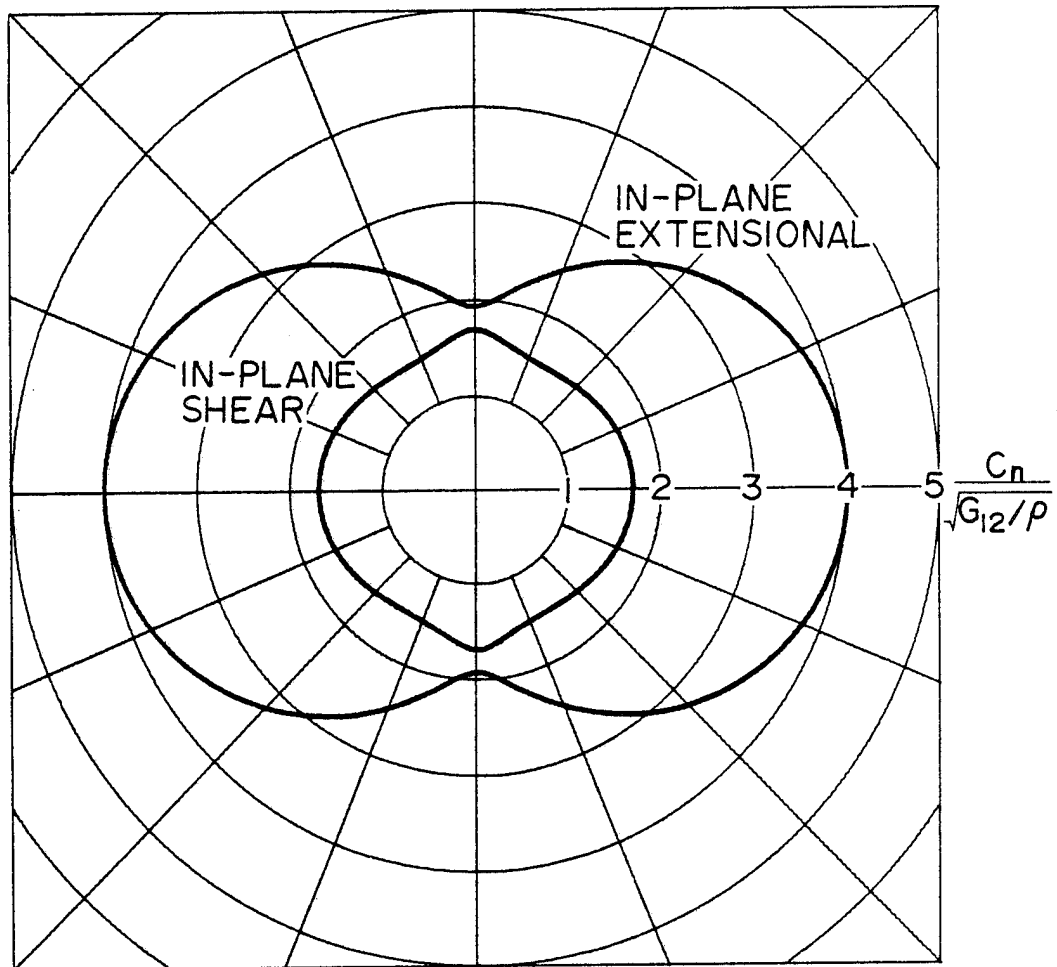


Figure 2.7 Normal velocities of in-plane wave fronts

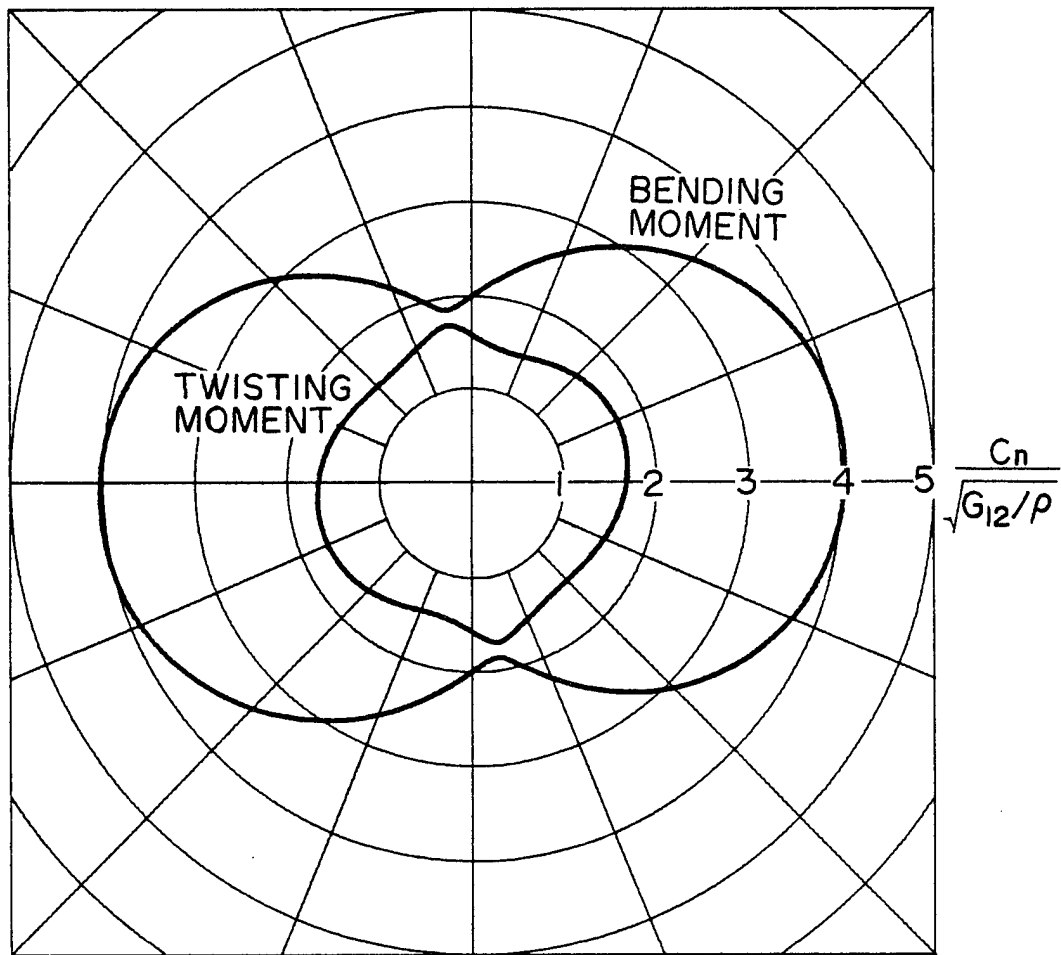


Figure 2.8 Normal velocities of flexural wave fronts

that for any fixed normal direction  $n_i$ ,  $c_n$  is a constant. Connecting the points having the same unit normals to the travelling wave front surface, we obtain a family of lines which are called rays. Thus, along a ray, the normal velocity of wave front remains unchanged. By using the ray theory which has been well established in the field of geometrical optics, we are able to construct the wave front surface.

Recall Equation (2.37)

$$F(x_i) - t = 0 \quad i = 1, 2 \quad (2-37)$$

which represents a family of wave fronts propagating over the plate with  $t$  as a parameter. It follows that

$$dF/dt = (\partial F/\partial x_i)(dx_i/dt) = (\partial F/\partial x_i)c_i = 1 \quad (2-71)$$

By putting

$$p_i = \partial F/\partial x_i = \nabla F \quad (2-72)$$

Equation (2-71) becomes

$$p_i c_i = 1 \quad (2-73)$$

Since  $p_i$  is normal to the surface  $F$ , it can be written as

$$p_i = |p_i| n_i \quad (2-74)$$

where  $|p_i|$  denotes the length of  $p_i$ . Combining (2-73) and (2-74), we obtain

$$|p_i|n_i c_i = |p_i|c_n = 1 \quad (2-75)$$

from which we obtain

$$p_i = n_i/c_n \quad (2-76)$$

In Equation (2-76),  $p_i$  is called the slowness vector which has the direction normal to the wave front with the magnitude being equal to the inverse of normal velocity  $c_n$ .

Substituting Equation (2-76) in Equation (2-69) and (2-70), we obtain two equations in terms of  $p_i$

$$\begin{vmatrix} p_x^2 A_{11} + p_y^2 A_{66} - P & p_x p_y (A_{12} + A_{66}) \\ p_x p_y (A_{12} + A_{66}) & p_x^2 A_{66} + p_y^2 A_{22} - P \end{vmatrix} = 0$$

$$\begin{vmatrix} p_x^2 D_{11} + 2p_x p_y D_{16} + p_y^2 D_{66} - I & D_{16} + p_x p_y (D_{12} + D_{66}) \\ D_{16} + p_x p_y (D_{12} + D_{66}) & p_x^2 D_{66} + 2p_x p_y D_{16} + p_y^2 D_{22} - I \end{vmatrix} = 0$$

which can be written in a general form as

$$g(p_i) = 0 \quad i = 1, 2 \quad (2-77)$$

In view of Equation (2-72), we recognize that Equation (2-77) may be regarded as a set of first-order partial differential equation for  $F$ . A standard method of solving first-order partial differential equation is by means of characteristics [24], which reduces the equation to a system of first-order ordinary differential equations. In our case, Equation (2-77) then is equivalent to the following

$$dx/ds = \partial g/\partial p_x \quad dy/ds = \partial g/\partial p_y \quad (2-78)$$

$$dp_x/ds = -\partial g/\partial x \quad dp_y/ds = -\partial g/\partial y \quad (2-79)$$

where  $s$  is a parameter. These equations, together with Equation (2-77) describe the ray geometry and the normal direction of the wave front propagating along the ray.

From Equation (2-78), we have

$$dy/dx = (\partial g/\partial p_y)/(\partial g/\partial p_x) \quad (2-80)$$

Since the normal direction of wave front along a ray is constant, it can be seen from Equation (2-76) that  $p_i$  is also constant along a ray. For laminated composite which is assumed to have homogeneous material properties, Equation (2-77) shows that  $g(p_i)$  does not depend on  $x_i$ , consequently,  $\partial g/\partial p_x$  and  $\partial g/\partial p_y$  are all constants along a ray. Thus, the solution of Equation (2-80) is then given by

$$y = \zeta(x - x_0) + y_0 \quad (2-81)$$

where  $x_0$  and  $y_0$  are the initial values of  $x$  and  $y$  at  $t = 0$ , and  $\zeta = (\partial g/\partial p_y)/(\partial g/\partial p_x)$ . This equation shows that the rays in a homogeneous solid are straight lines.

From Equations (2-73) and (2-77), we have

$$c_i dp_i = 0 \quad (2-82)$$

$$dg = (\partial g/\partial p_i) dp_i = 0 \quad (2-83)$$

Eliminating  $dp_i$  from these equations yields

$$dx_i/dt = c_i = (\partial g/\partial p_i)/(p_j \partial g/\partial p_j) \quad (2-84)$$

where summation over  $j$  is understood.

Equation (2-84) can be solved to obtain the position of wave front at time  $t$ . Again, since  $\partial g/\partial p_i$  and  $p_i$  are all constant along a ray, we obtain the solution of Equation (2-84) as

$$x = (\partial g/\partial p_x)t/(p_j \partial g/\partial p_j) + x_0 \quad (2-85)$$

$$y = (\partial g/\partial p_y)t/(p_j \partial g/\partial p_j) + y_0 \quad (2-86)$$

where  $x_0$  and  $y_0$  denote the initial wave position at  $t = 0$ .

Consider at  $t = 0$ , a wave front forms a circle given by

$$\begin{aligned} x_0 &= h \cos \alpha \\ y_0 &= h \sin \alpha \end{aligned} \quad (2-87)$$

At this instant, the normal directions to the wave front coincide with the radial directions. Due to the different velocities of propagation in directions, this initial shape would be distorted. By using Equations (2-85) and (2-86), the subsequent positions of the wave front can be determined. Figures 2.9-2.12 show the wave front positions at two consecutive instants after  $t = 0$  for the in-plane extensional, in-plane shear, bending moment and twisting moment modes, respectively, for the  $[0^\circ/45^\circ/0^\circ/-45^\circ/0^\circ]_{2s}$

graphite/epoxy laminated plate. It is noted that for symmetrical laminates, the in-plane modes are uncoupled from the bending modes. The rays along which the normal directions to the wave front are  $0^\circ$ ,  $45^\circ$  and  $90^\circ$ , respectively, are also shown in the figures. It is seen that the wave fronts of the in-plane extensional and the in-plane shear modes possess symmetry with respect to x-axis and y-axis. The wave fronts of the bending and twisting moments, however, lose their original symmetry with respect to x-axis and y-axis. This is an indication that in performing analysis of flexural deformation of this laminate, one can not take a quadrant for analysis, a practice followed by many authors dealing with homogeneous and isotropic plates.

From Figures 2.9-2.12, it is also interesting to note that ray geometries for these two groups of wave fronts are quite different. For the in-plane extensional and in-plane shear wave fronts, the rays coincide with the normal directions when  $\alpha = 0^\circ$  and  $90^\circ$ . Along other directions, the direction of the ray deviates from the normal direction of the wave front. It was discussed in [2] that the degree of spreading of rays is proportional to the decay of the stress amplitude at the wave front. Thus, from Figures 2.9 and 2.11, one can conclude that the strength of the in-plane extensional and bending moment wave fronts decay more rapidly in the y-direction than in the x-direction.

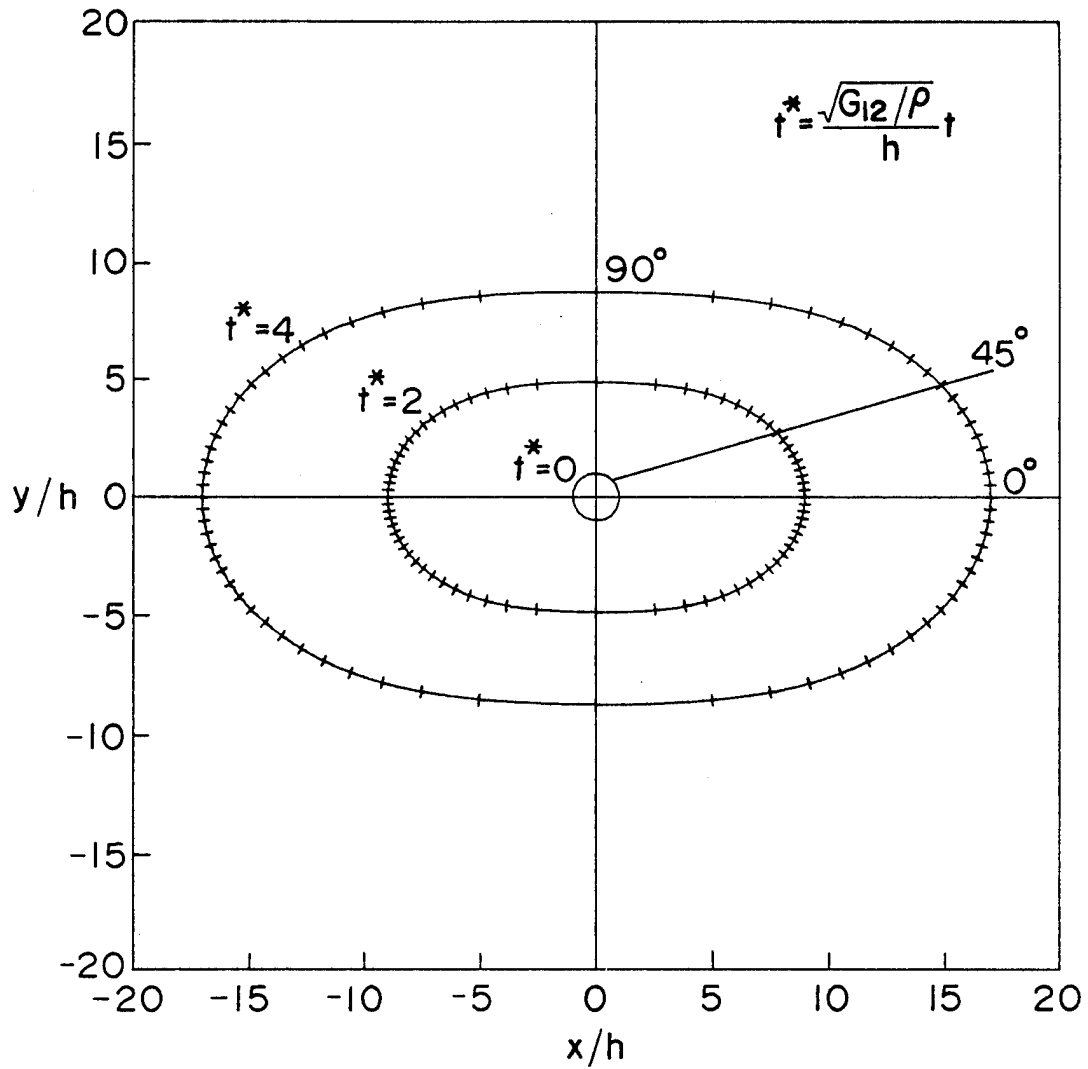


Figure 2.9 Wave front positions at different times and rays for in-plane extensional mode

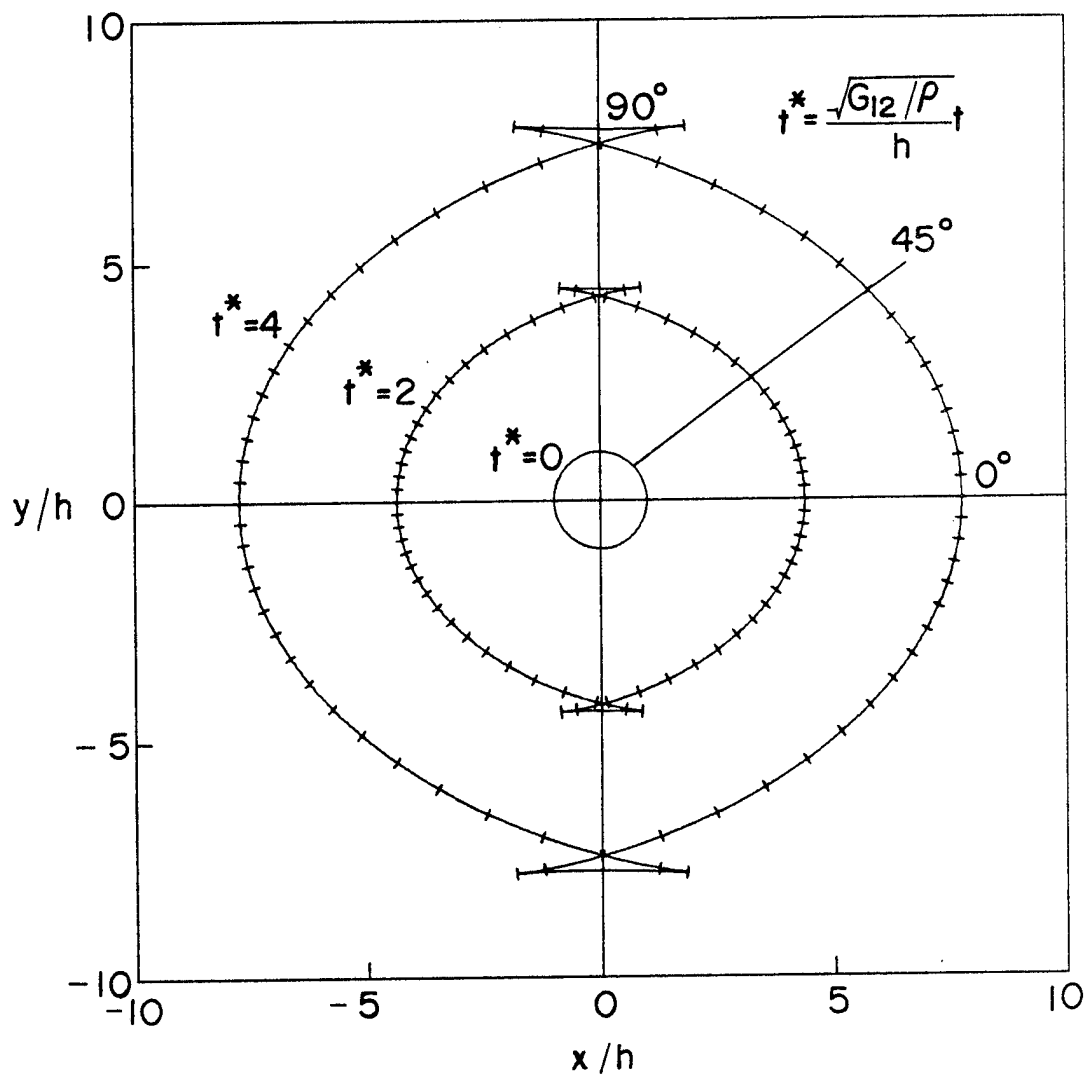


Figure 2.10 Wave front positions at different times and rays for in-plane shear mode

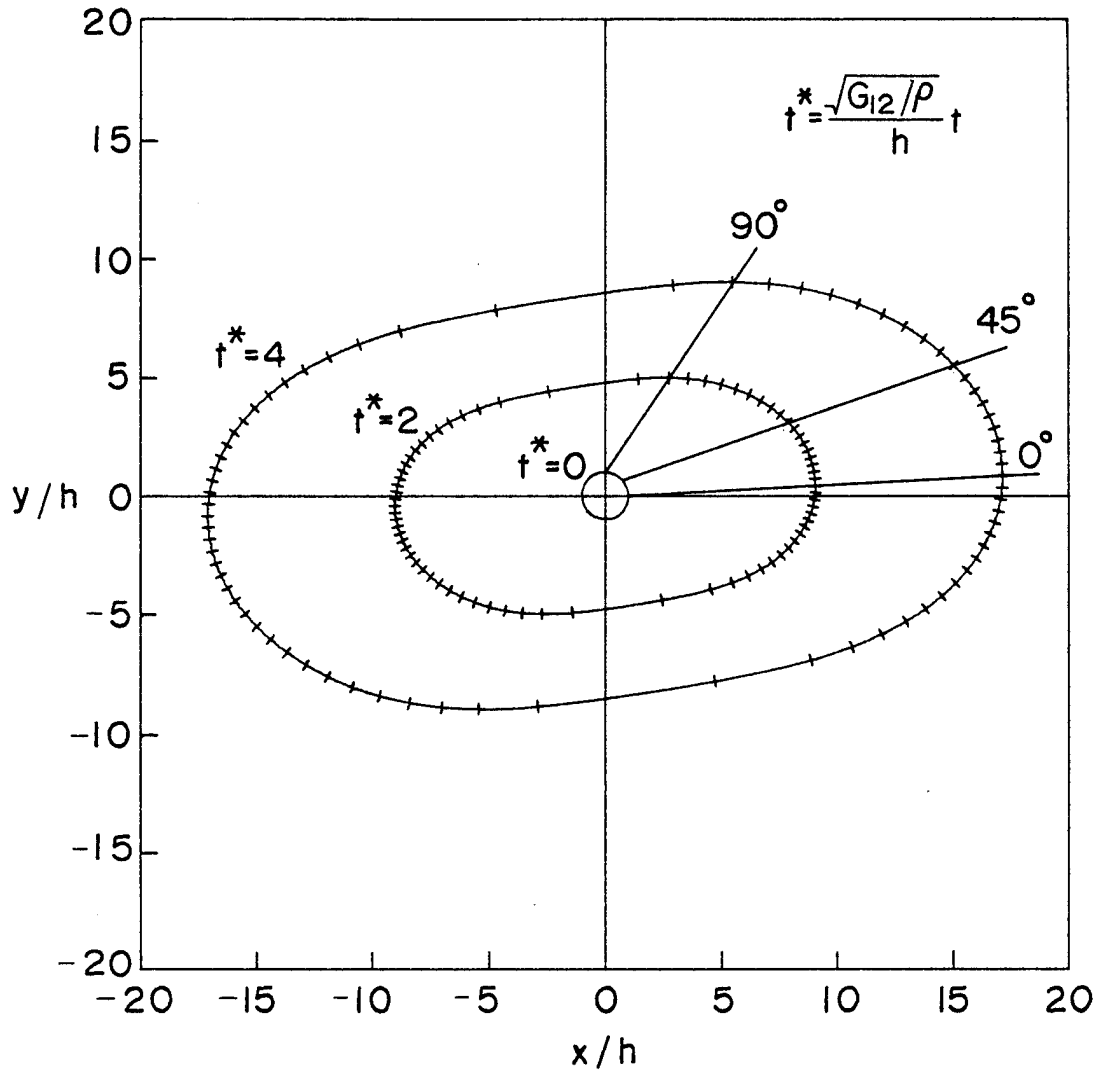


Figure 2.11 Wave front positions at different times and rays for bending mode

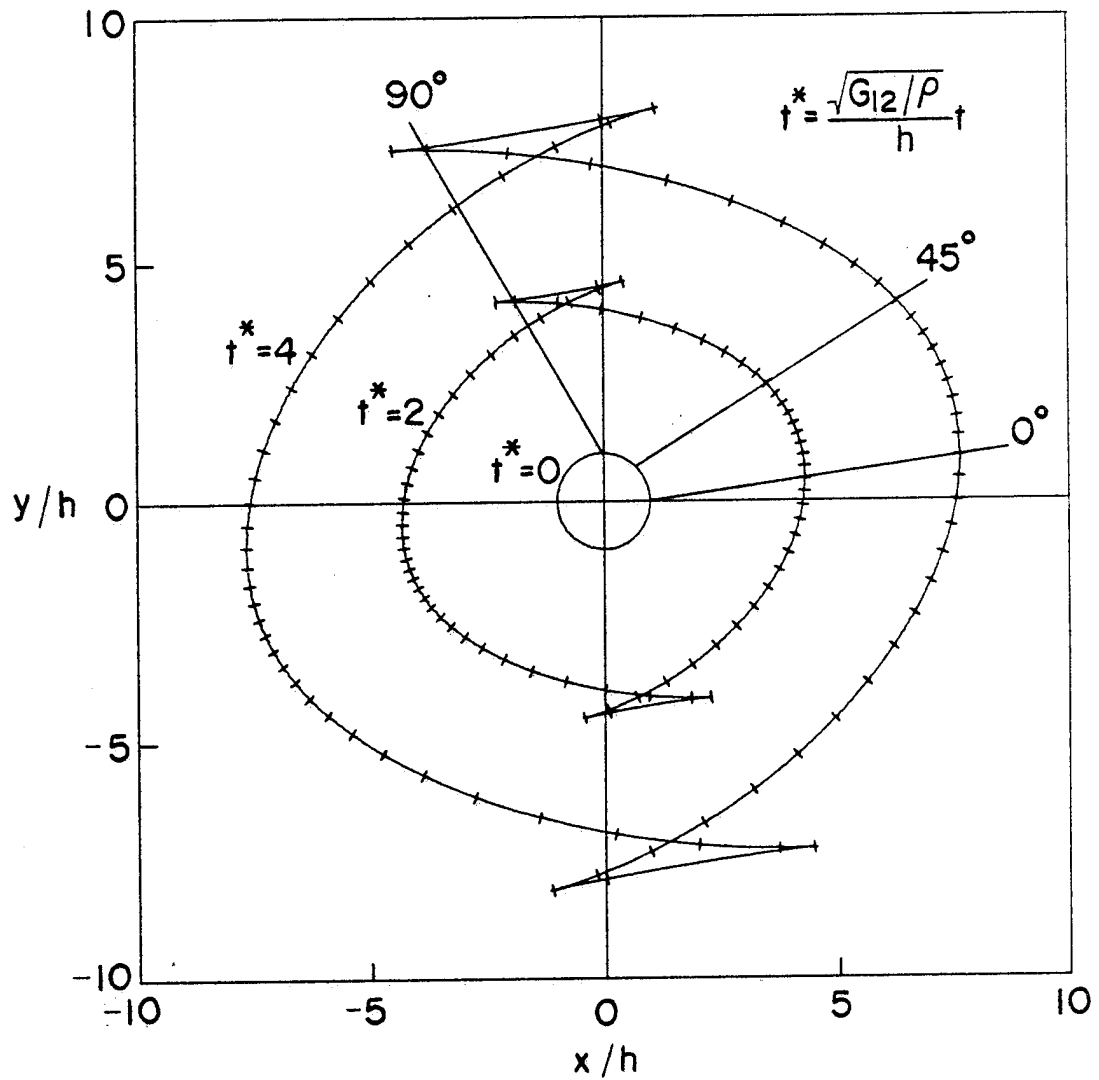


Figure 2.12 Wave front positions at different times and rays for twisting mode

A photoelastic study of anisotropic waves in a fiber reinforced composite has been done by Dally et al. [9]. The waves was produced by a explosive charge in a small hole on the plate. The result showed clearly an elliptic-like stress wave front pattern. This indicates that stress waves in anisotropic materials propagate with different velocities in different directions.

### CHAPTER 3

#### STATICAL INDENTATION LAWS

A brief introduction of the historical development on impact problem involving homogeneous isotropic materials was given by Goldsmith [12]. Hertz [11] was the first to obtain a satisfactory solution on contact law for two isotropic elastic spherical bodies. When letting the radius of one of the spheres go to infinity, this law then describes the contact behavior between a sphere and an elastic half-space. The Hertzian law, in spite of being static and elastic in nature, has been widely applied to impact analyses where permanent deformations were produced. The use of this law beyond the elastic limit has been justified on the basis that it appears to predict accurately most of the impact parameters that can be experimentally verified.

In studying impact responses of laminated composites, the problem becomes extremely complicated. One may easily realize that the Hertzian contact law which was derived based on homogeneous isotropic materials may not be adequate in describing the contact behavior of laminated composites due to their anisotropic and nonhomogeneous properties. Moreover, most of the laminated composites have finite thickness which can not be represented by a half-space. In

many existing analytical works [25], loadings to the laminates were assumed known, and the responses of the laminates were assumed elastic.

Willis [26] obtained explicit formulas for Hertzian contact law for transversely isotropic half-space pressed by a rigid sphere, and extended it to the application of impact problems. It was shown that

$$F = k\alpha^n \quad (3-1)$$

with  $n = 3/2$  is valid for the contact force  $F$  and the indentation  $\alpha$ , where  $k$  is a contact coefficient whose value depends on the material properties of the target and the sphere, and the radius of sphere.

A modified contact law with

$$k = (4/3) \frac{R_s^{1/2}}{\frac{1 - \nu_s^2}{E_s} + \frac{1}{E_t}} \quad (3-2)$$

was used [13] in an analytical study on impact of laminated composites. In Equation (3-2),  $R_s$ ,  $\nu_s$  and  $E_s$  are the radius, Poisson's ratio and Young's modulus of the sphere, respectively, and  $E_t$  is the Young's modulus of the laminates in thickness direction. It was also suggested by Sun et al. [27] that the value of  $k$  can be experimentally determined.

Recently Yang and Sun [14] have conducted static indentation tests on the  $[0^\circ/45^\circ/0^\circ/-45^\circ/0^\circ]_{2s}$  graphite/epoxy laminates using spherical steel indenters of 0.25 in. and 0.5 in. diameters. The results were fitted into Equation (3-1) and were found that the  $3/2$  power is valid. In addition, it was also observed that even for small amounts of load there were significant permanent indentations. This implies that the unloading curve has to be different from the loading curves. In order to account for the permanent deformation, the equation

$$F = F_m \left( \frac{\alpha - \alpha_0}{\alpha_m - \alpha_0} \right)^q \quad (3-3)$$

proposed by Crook [28] was used to model the unloading path where  $F_m$  is the contact force at which unloading begins,  $\alpha_m$  is the indentation corresponding to  $F_m$ , and  $\alpha_0$  denotes the permanent indentation in an unloading cycle. Equation (3-3) can be rewritten as

$$F = s(\alpha - \alpha_0)^q \quad (3-4)$$

in which

$$s = F_m / (\alpha_m - \alpha_0)^q \quad (3-5)$$

is called unloading rigidity. In order to simplify the modeling of the unloading law, it was assumed [14] that the value of  $s$  for all the unloading curves remains the same.

Consequently, a constant  $\alpha_{cr}$  given by

$$\alpha_{cr} = k/s \quad (3-6)$$

was introduced. It was also shown that  $q=5/2$  fitted the unloading path very well, and the permanent indentation  $\alpha_0$  was then related to  $\alpha_m$  by

$$\begin{aligned} \alpha_0/\alpha_m &= 1 - (\alpha_{cr}/\alpha_m)^{2/5} & \text{as } \alpha_m > \alpha_{cr} \\ \alpha_0 &= 0 & \text{as } \alpha_m \leq \alpha_{cr} \end{aligned} \quad (3-7)$$

The value of  $\alpha_{cr}$  was found to be independent of the size of the indenter and hence can be regarded as a material constant.

It was also mentioned in [14] and [29] that there were some practical difficulties in performing the tests. Since the indentation was measured step by step using a dial gage and readings on the gage were taken about 10 to 20 seconds after the load was increased by one step, the creep effect may cause an appreciable error to the results. Another important problem was that it was almost impossible to measure the permanent indentation accurately using the dial gage. In order to overcome these problems, a Linear Variable Differential Transformer (LVDT) was used in this study to measure the indentation.

The LVDT is an electromechanical transducer that produces an electrical output proportional to the displacement.

Connecting this output and the one from the strain indicator which is used to measure the applied loading to a X-Y plotter, one can obtain a continuous loading-unloading curve. By changing the loading rate which can be applied as fast as 50 lb./sec., it is possible to examine the significance of creep effect on the contact law. The starting point and final point of a loading-unloading cycle, which represent respectively the instants of contact and separation of the indenter and the specimen, can be easily determined from the curve. Thus, the measurements of permanent indentations are much more accurate than those using the dial gage.

### 3.1 Specimens and Experimental Procedure

Two groups of test specimens were prepared from a  $[0^\circ/45^\circ/0^\circ/-45^\circ/0^\circ]_{2s}$  graphite/epoxy laminate. They were cut in the way such that the longitudinal axis of the beam specimen of the first group was parallel to the  $0^\circ$  fiber direction while the second one was perpendicular to it. The latter then becomes  $[90^\circ/45^\circ/90^\circ/-45^\circ/90^\circ]_{2s}$  laminated beams. The thickness of the beam was 0.106 in. and the width was approximately 1.25 in.. In all tests, the specimens were clamped at both ends. It was shown in [14] that the span of the specimen in the range of 2 in. to 6 in. has little effect on the contact law. Hence, only one span, i.e. 2 in., was used in the test.

The experimental set-up is shown schematically in Figure 3.1. LVDT was mounted on a 'C' bracket fixed to the loading piston so that only the relative movement between the indenter and the specimen was recorded. The load was applied pneumatically by a plunger and it was measured using a load cell and a strain indicator. Outputs from LVDT and strain indicator were fed into an X-Y plotter so that a continuous force-indentation curve can be obtained. Two spherical steel indenters of diameters 0.5 in. and 0.75 in. were used.

### 3.2 Experimental Results

#### 3.2.1 Loading Curves

The experimental curves were first digitized into some discrete data points and then fitted into Equation (3-1) using least-squares method. Figures 3.2 and 3.3 show the test data and the fitted curves for 0.5 in. diameter indenter. It can be seen from these figures that the  $3/2$  power index gives very good results. However, the contact coefficient  $k$  of  $[0^\circ/45^\circ/0^\circ/-45^\circ/0^\circ]_{2s}$  specimen is less than the one of  $[90^\circ/45^\circ/90^\circ/-45^\circ/90^\circ]_{2s}$  specimen by about 7%. During the test, larger deflections were observed for the second group of specimen due to their lower flexural rigidity. This means that the contact area is also larger and the indentation under same amount of loading should be

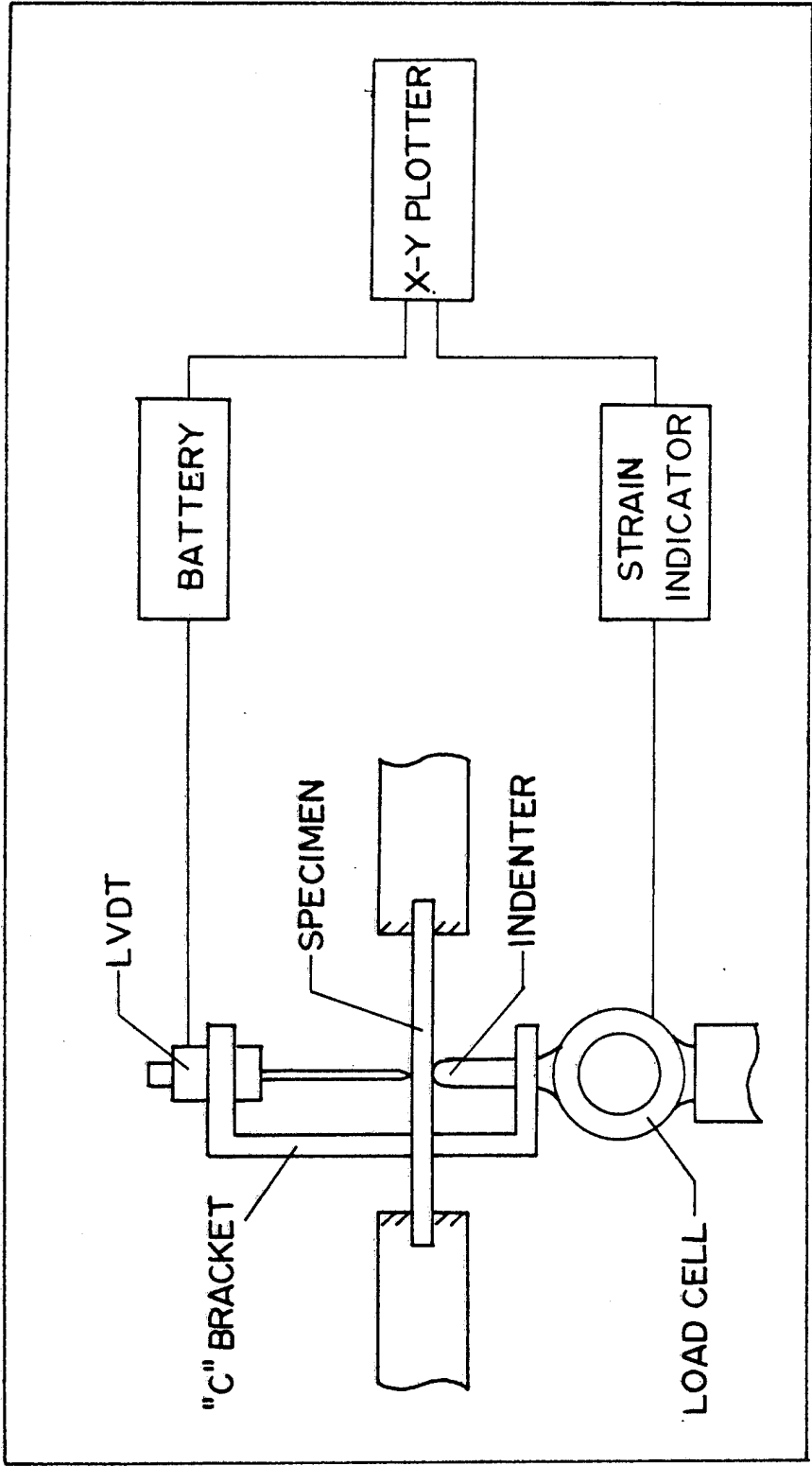


Figure 3.1 Schematic diagram for the indentation test set-up

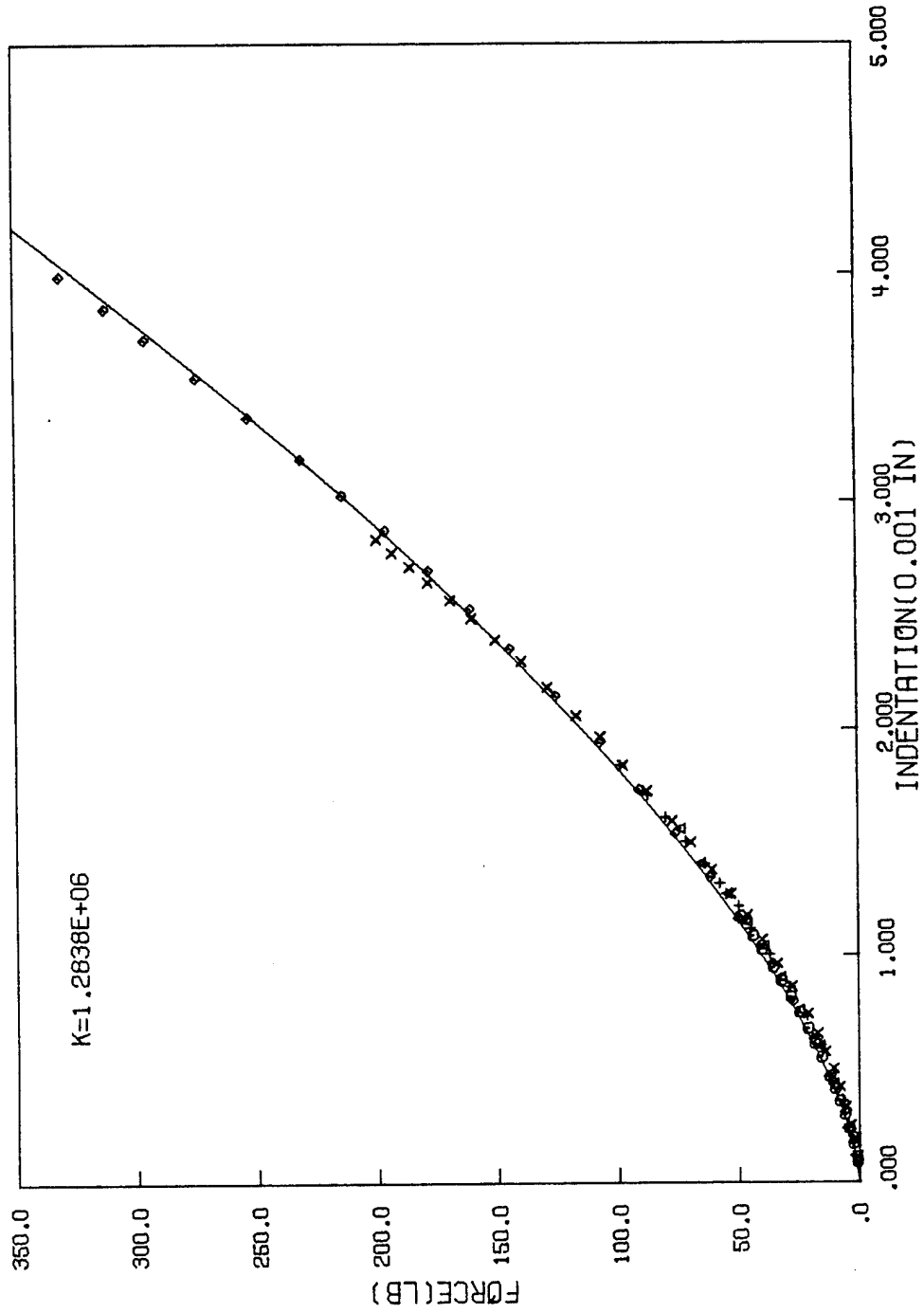


Figure 3.2 Loading curve of [0°/45°/0°/-45°/0°]<sub>2s</sub> specimens with 0.5 inch indenter (n=3/2)

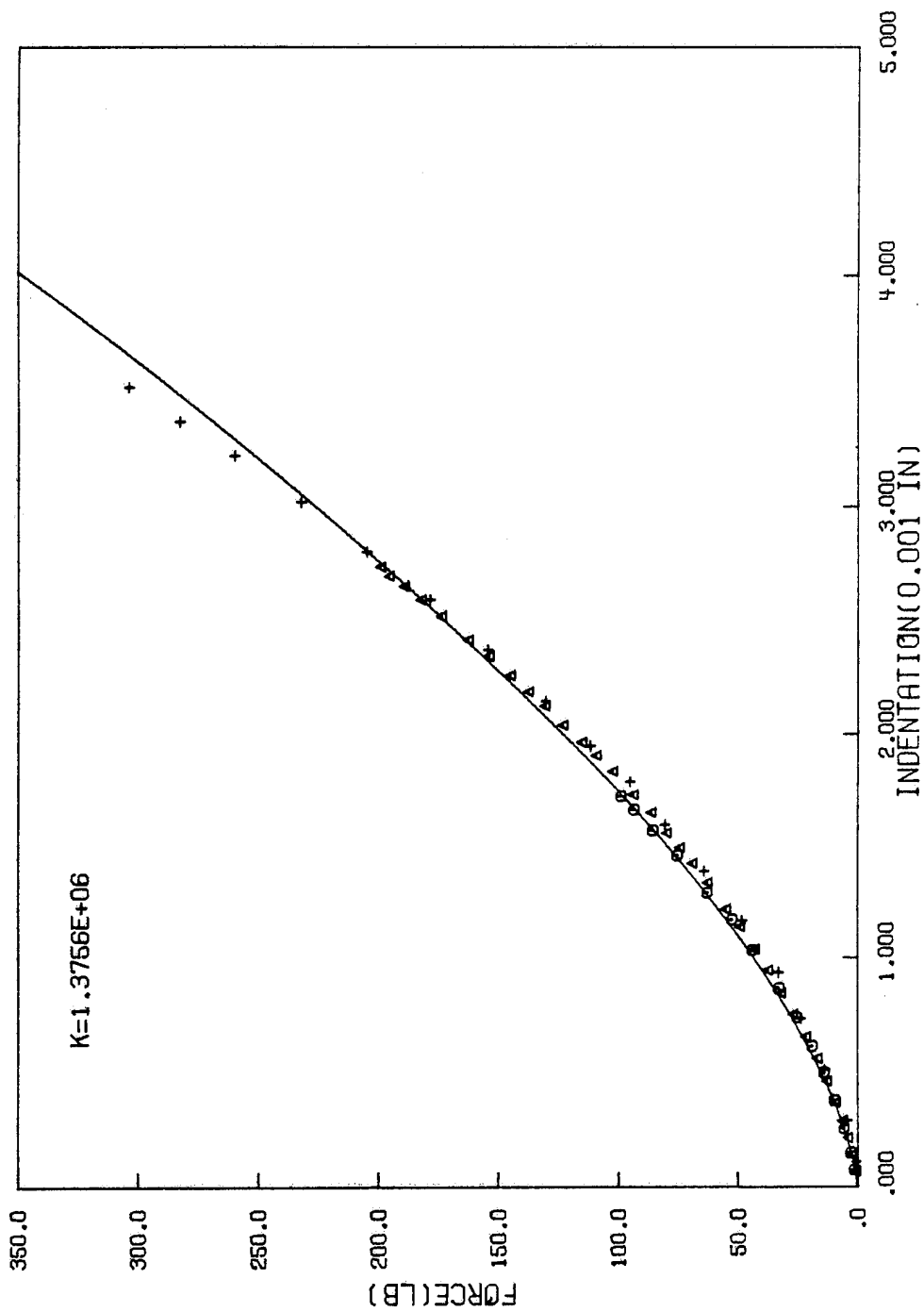


Figure 3.3 Loading curve of  $[90^\circ/45^\circ/90^\circ/-45^\circ/90^\circ]_{2s}$  specimens with 0.5 inch indenter ( $n=3/2$ )

smaller comparing with the first group of specimens. Consequently, the higher value of  $k$  for the  $[90^\circ/45^\circ/90^\circ/-45^\circ/90^\circ]_{2s}$  specimens is reasonable.

The results for 0.75 in. diameter indenter are presented in Figures 3.4 and 3.5. Again, good agreement between the experimental data and fitted curves indicates that the  $3/2$  power index for loading law is valid. The values of  $k$  for both indenters are summarized in Table 3.1. It should be noted that the average value of  $k$  obtained from the two groups of specimens was used later in a finite element analysis of impact responses.

### 3.2.2 Unloading Curves

By choosing a suitable value for  $q$ , it can be seen from Equation (3-5) that once the relation between  $\alpha_0$  and  $\alpha_m$  is established, the unloading rigidity  $s$  is then determined. Test results show that the permanent indentations  $\alpha_0$  and the corresponding maximum indentations  $\alpha_m$  exhibit a rather linear relationship. The equation given by

$$\alpha_0 = s_p (\alpha_m - \alpha_p) \quad (3-8)$$

is obtained from the test data for both 0.5 in. and 0.75 in. indenters using least-squares fitting method, and are plotted in Figure 3.6. In Equation (3-8),  $\alpha_p$  can be considered as a critical value of indentation. Once the amount of indentation exceeds  $\alpha_p$ , permanent deformation will occur.

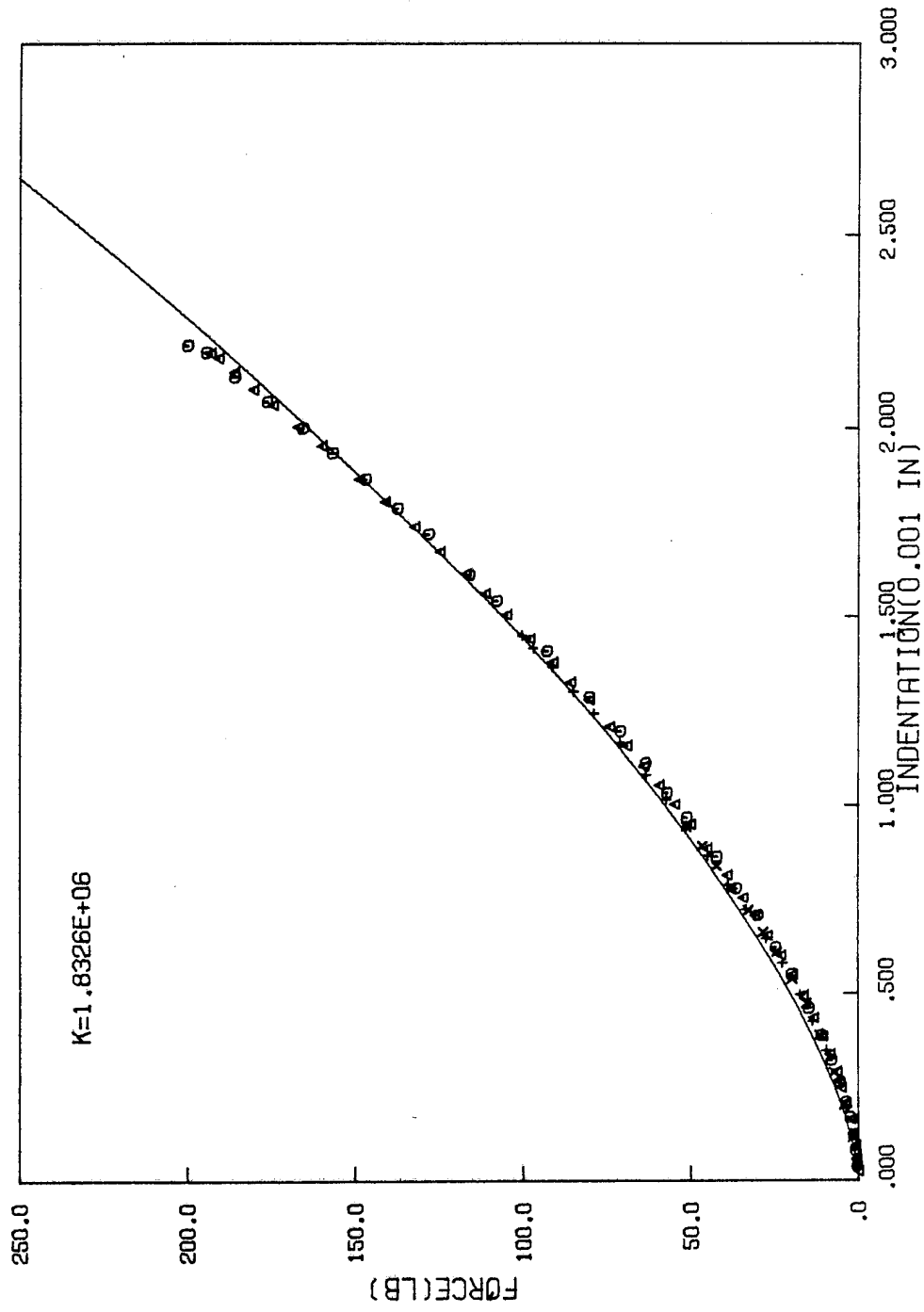


Figure 3.4 Loading curve of  $[0^\circ/45^\circ/0^\circ/-45^\circ/0^\circ]_{2s}$  specimens with 0.75 inch indenter ( $n=3/2$ )

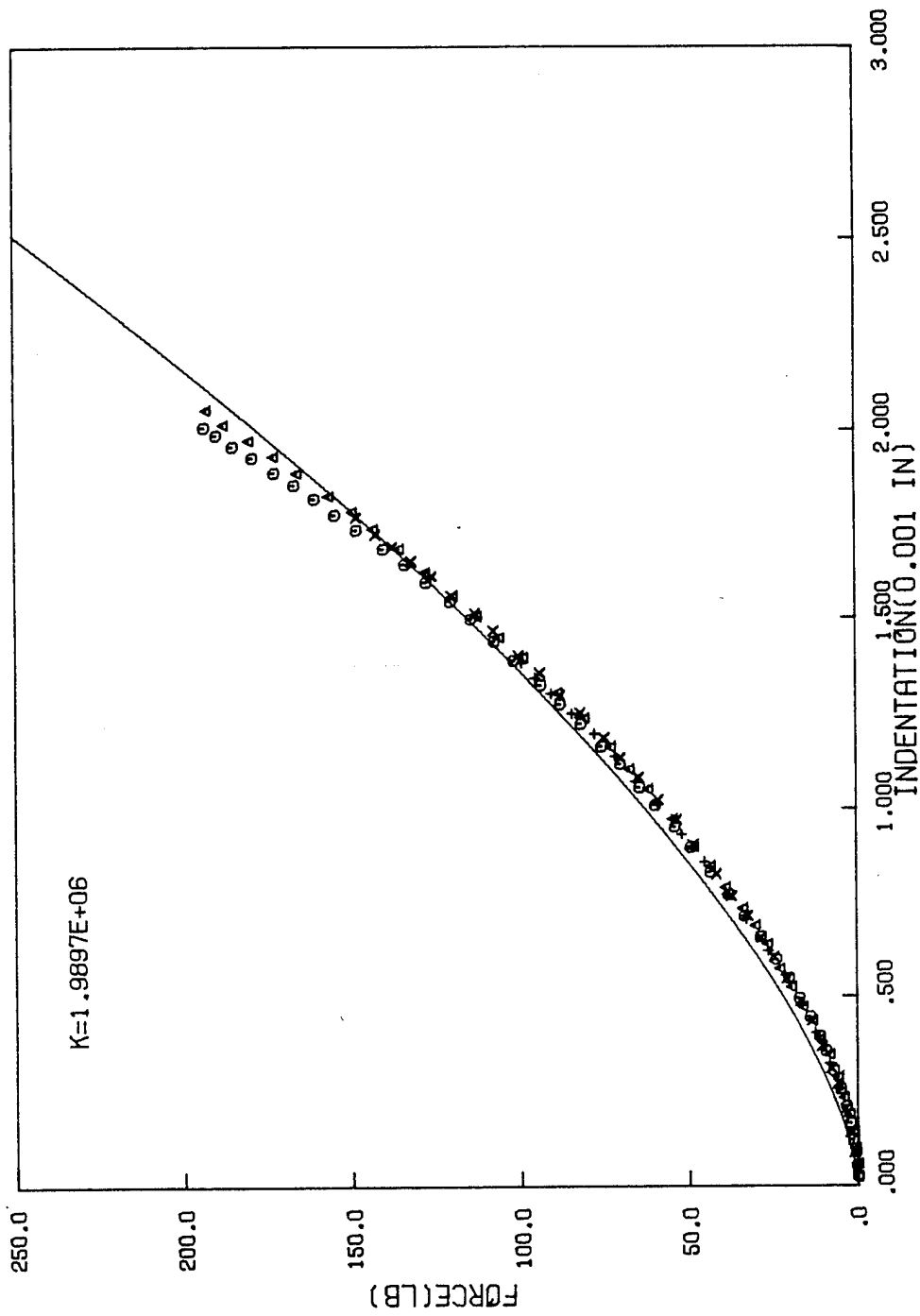


Figure 3.5 Loading curve of  $[90^\circ/45^\circ/90^\circ/-45^\circ/90^\circ]_{2s}$  specimens with 0.75 inch indenter ( $n=3/2$ )

Table 3.1  
 Contact coefficient  $k$  of loading law  $F = k\alpha^{1.5}$

Size of Indenter(in)	0.5		0.75	
Specimen	Group 1 <sup>+</sup>	Group 2 <sup>‡</sup>	Group 1 <sup>+</sup>	Group 2 <sup>‡</sup>
$k(\text{lb}/\text{in}^{1.5})$	$1.284 \times 10^6$	$1.376 \times 10^6$	$1.833 \times 10^6$	$1.990 \times 10^6$
Average $k$	$1.330 \times 10^6$		$1.912 \times 10^6$	
Ref. [14]	$9.694 \times 10^5$			

<sup>+</sup>  $[0^\circ/45^\circ/0^\circ/-45^\circ/0^\circ]_{2S}$  specimens

<sup>‡</sup>  $[90^\circ/45^\circ/90^\circ/-45^\circ/90^\circ]_{2S}$  specimens

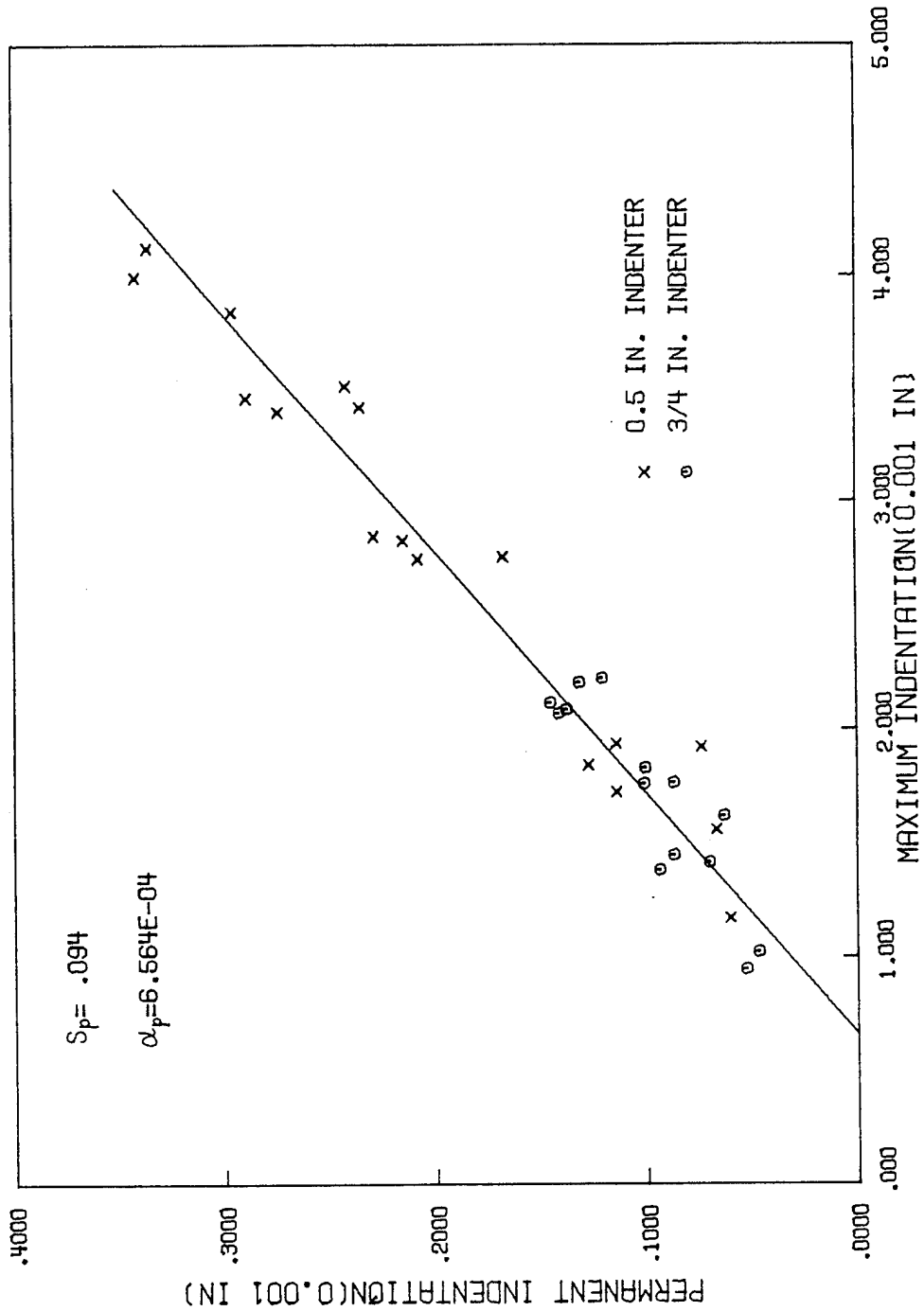


Figure 3.6 Relation between permanent indentation and maximum indentation

Substitution of Equation (3-8) and (3-1) into Equation (3-5) yields

$$s = \frac{k\alpha_m^{3/2}}{[(1 - s_p)\alpha_m + s_p\alpha_p]^q} \quad \text{if } \alpha_m \geq \alpha_p \quad (3-9)$$

$$s = \frac{k\alpha_m^{3/2}}{\alpha_m^q} \quad \text{if } \alpha_m < \alpha_p \quad (3-10)$$

These two equations along with Equation (3-4) are then used to fit the experimental unloading curves in finding the value of  $q$ .

Yang [14] has shown that  $q = 2.5$  fits the test results for both 0.25 in. and 0.5 in. indenters quite well. In this study, however, the values of 2.2 and 1.8 were found to give the best fitting for 0.5 in. and 0.75 in. indenters, respectively using the aforementioned method (Figures 3.7-3.10). For convenience,  $q = 2.5$  was used for 0.5 in. indenter while  $q = 2.0$  was chosen for 3/4 in. indenter. The results of the curve-fitting are presented in Figures 3.11-3.14. Further discussions on the unloading law will be given in Section 3.3.

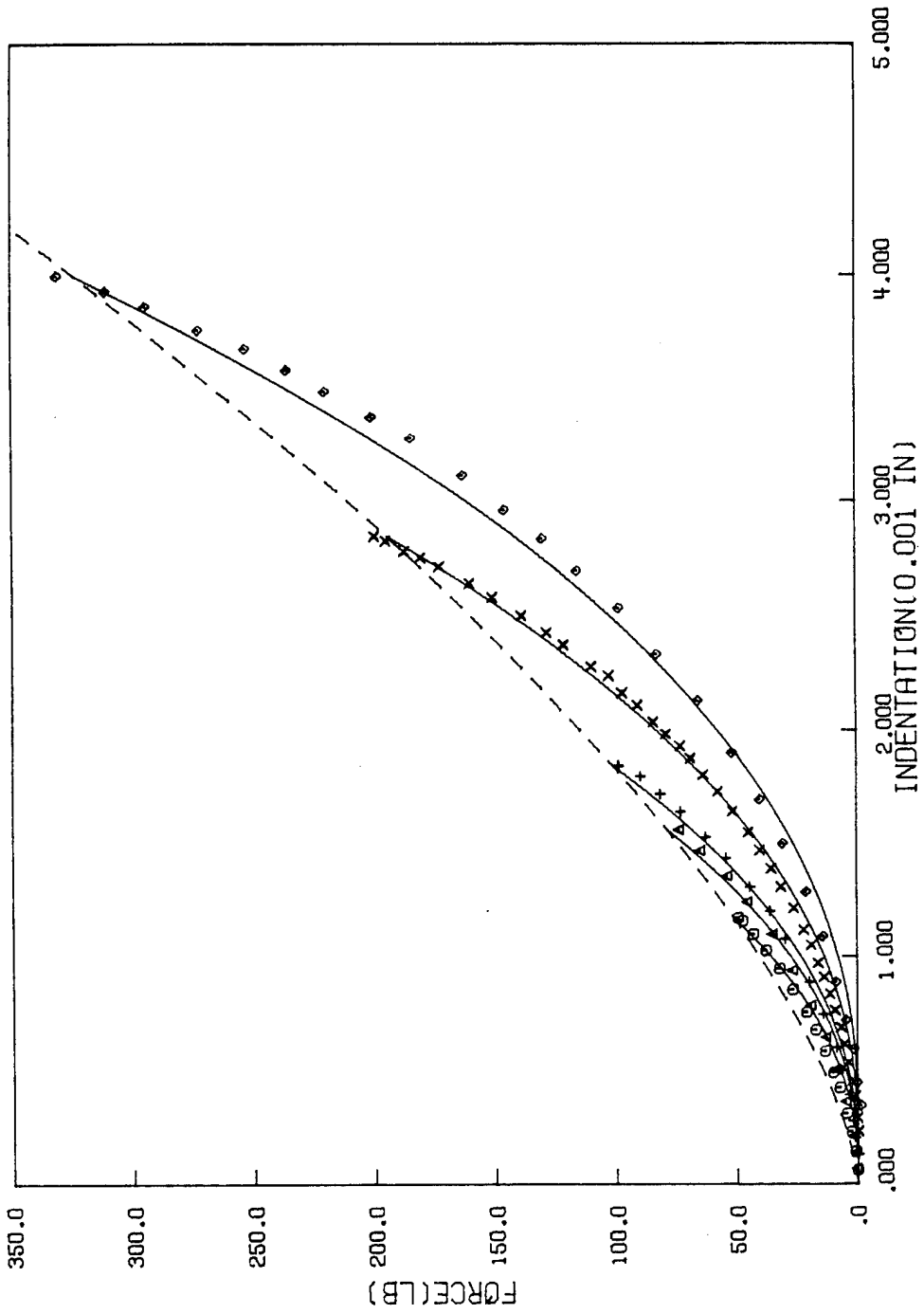


Figure 3.7 Unloading curves of  $[0^\circ/45^\circ/0^\circ/-45^\circ/0^\circ]_{2s}$  specimens with 0.5 Inch Indenter ( $q=2.2$ )

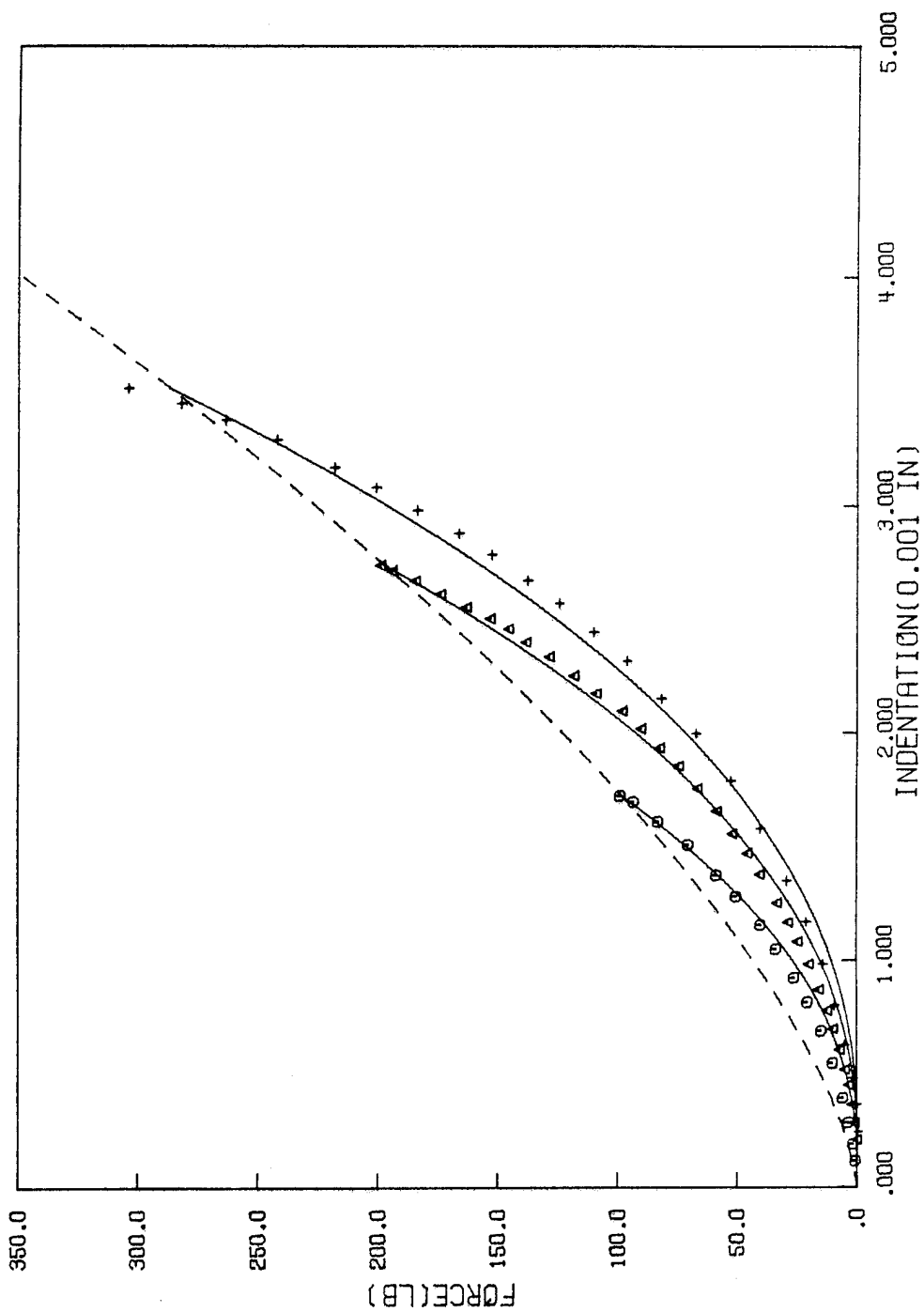


Figure 3.8 Unloading curves of  $[90^\circ/45^\circ/90^\circ/-45^\circ/90^\circ]_{2s}$  specimens with 0.5 inch indenter ( $q=2.2$ )

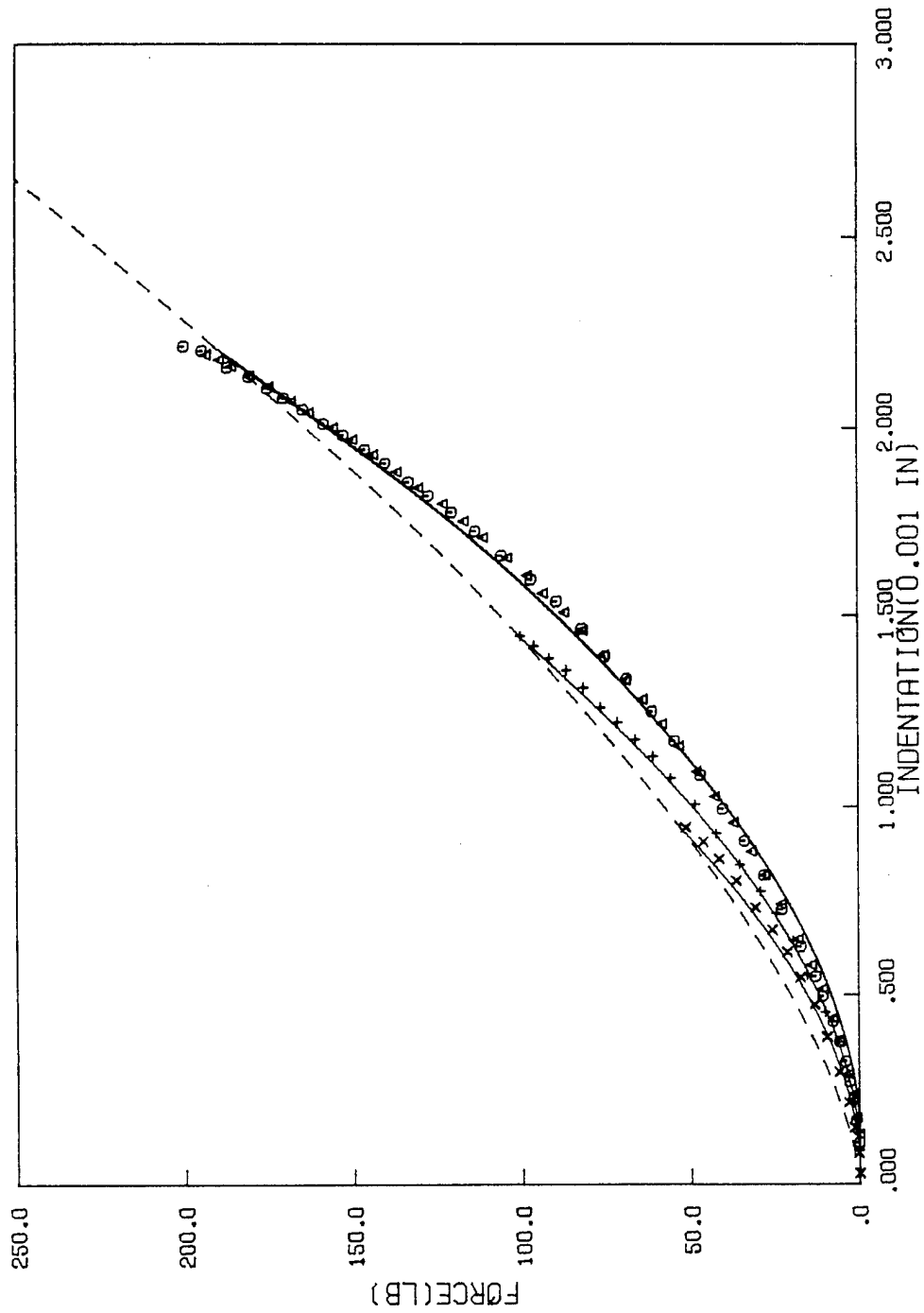


Figure 3.9 Unloading curves of  $[0^{\circ}/45^{\circ}/0^{\circ}/-45^{\circ}/0^{\circ}]_{2S}$  specimens with 0.75 inch indenter ( $q=1.8$ )

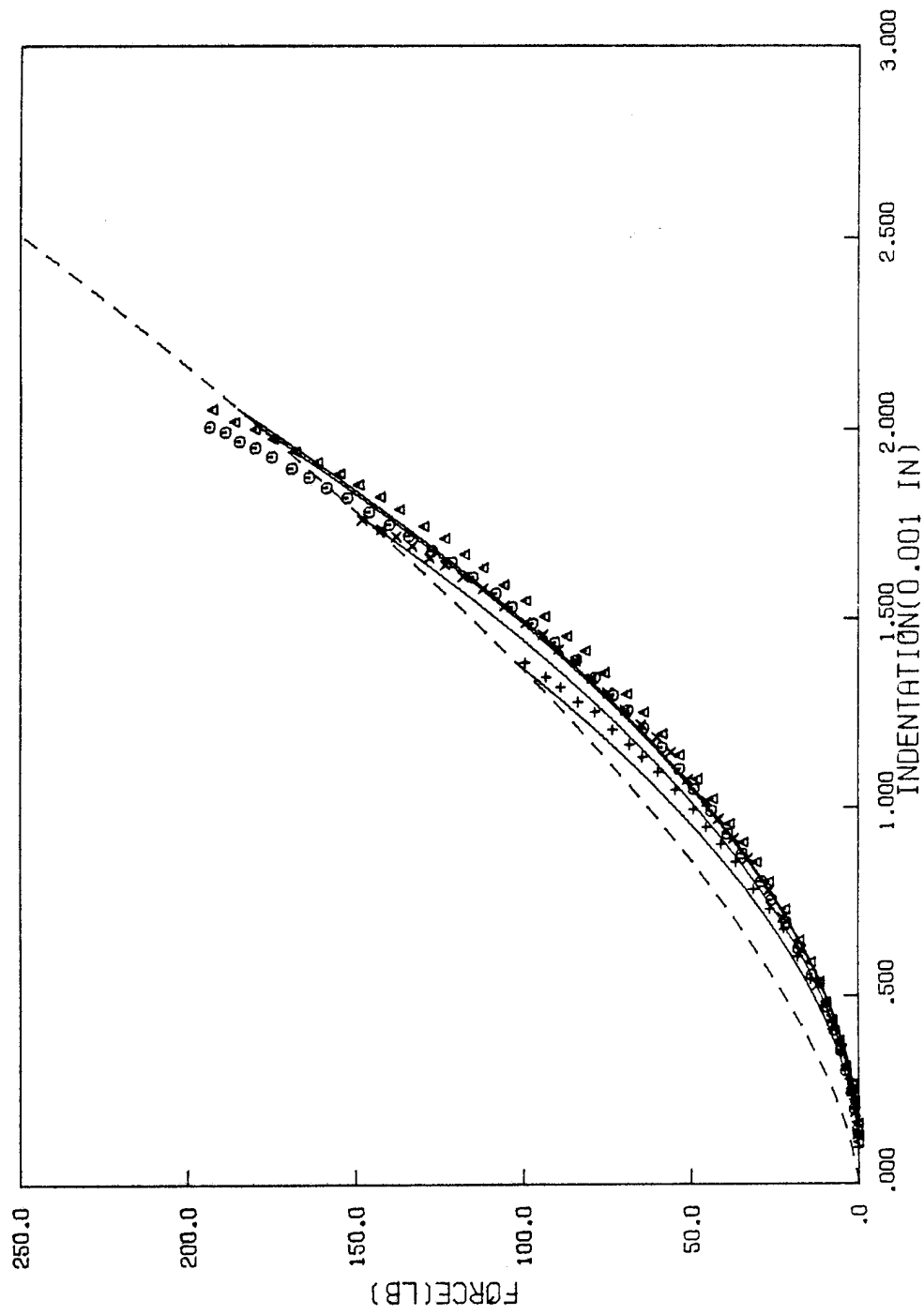


Figure 3.10 Unloading curves of  $[90^\circ/45^\circ/90^\circ/-45^\circ/90^\circ]_2s$  specimens with 0.75 inch indenter ( $q=1.8$ )

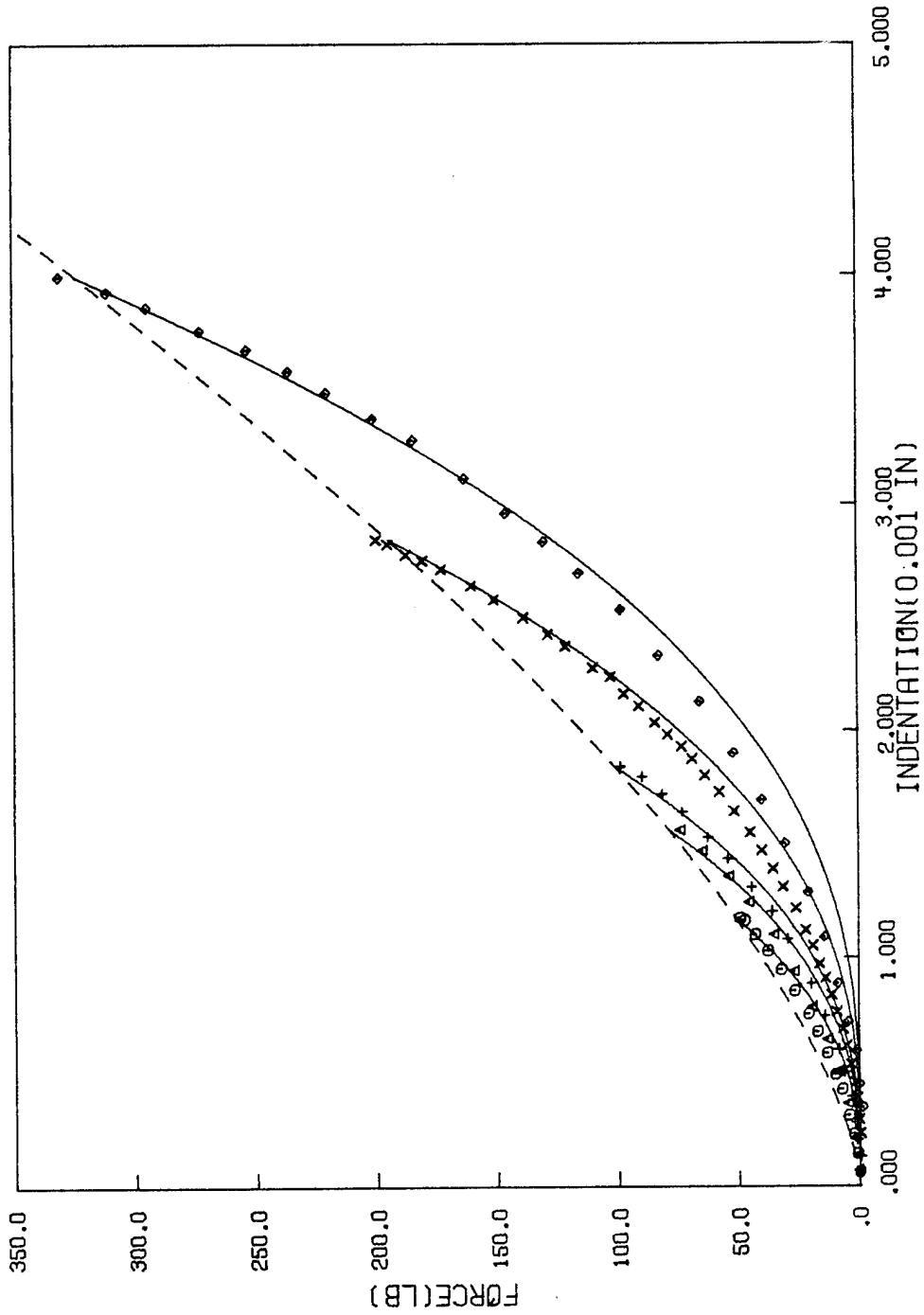


Figure 3.11 Unloading curves of  $[0^\circ/45^\circ/0^\circ/-45^\circ/0^\circ]_{2s}$  specimens with 0.5 inch indenter ( $q=2.5$ )

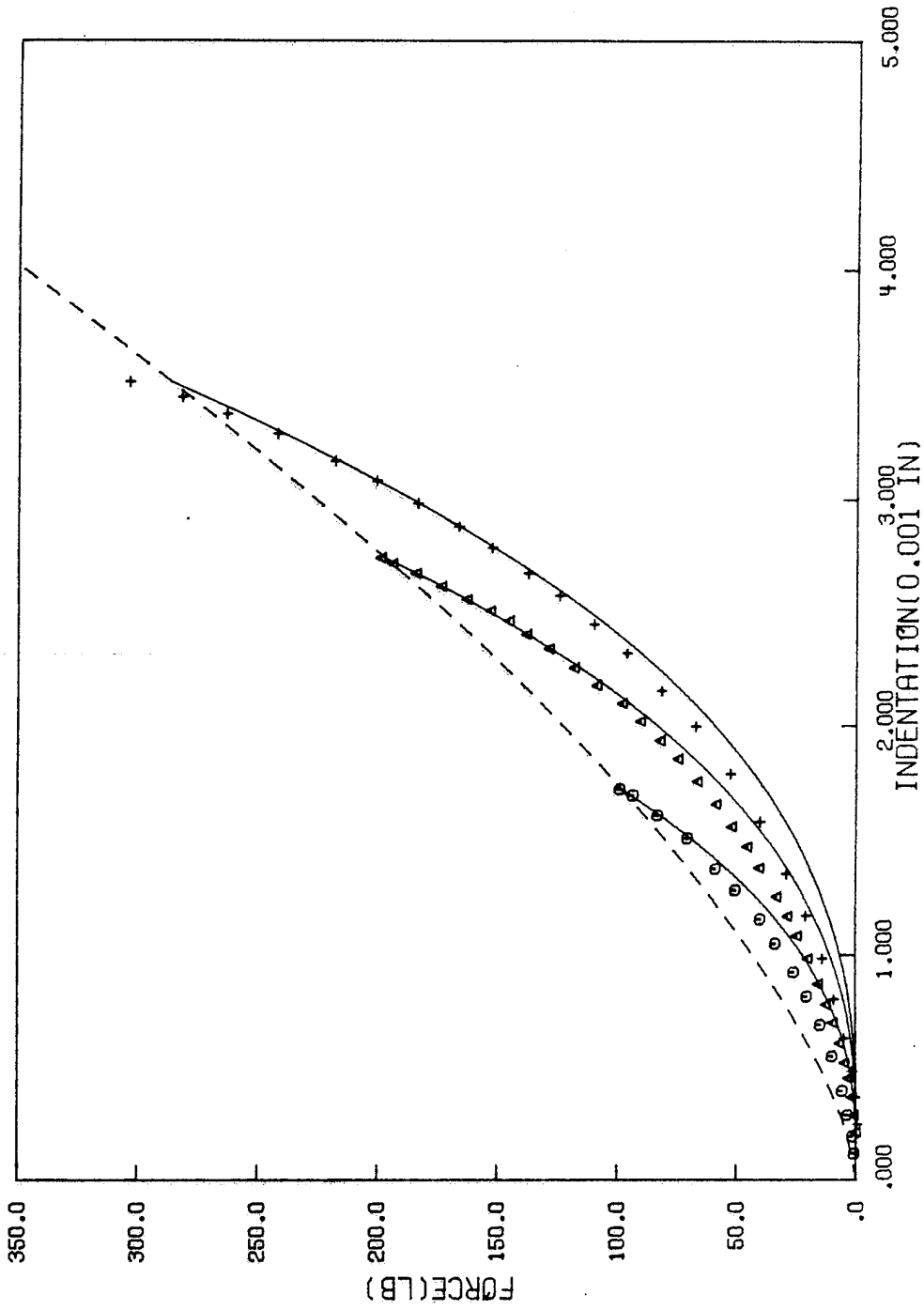


Figure 3.12 Unloading curves of  $[90^\circ/45^\circ/90^\circ/-45^\circ/90^\circ]_{2s}$  specimens with 0.5 inch indenter ( $q=2.5$ )

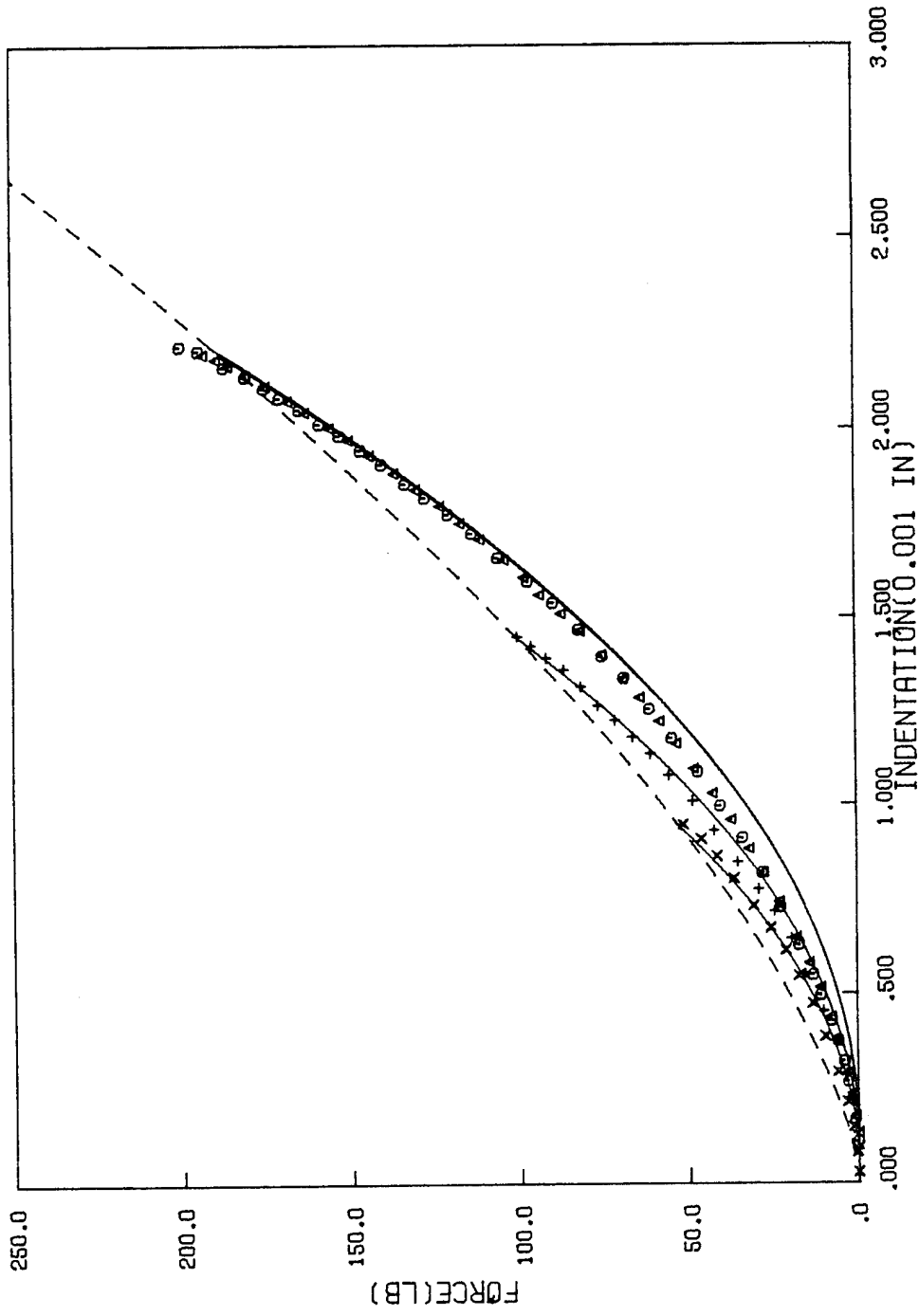


Figure 3.13 Unloading curves of  $[0^\circ/45^\circ/0^\circ/-45^\circ/0^\circ]_{2s}$  specimens with 0.75 Inch indenter ( $q=2.0$ )

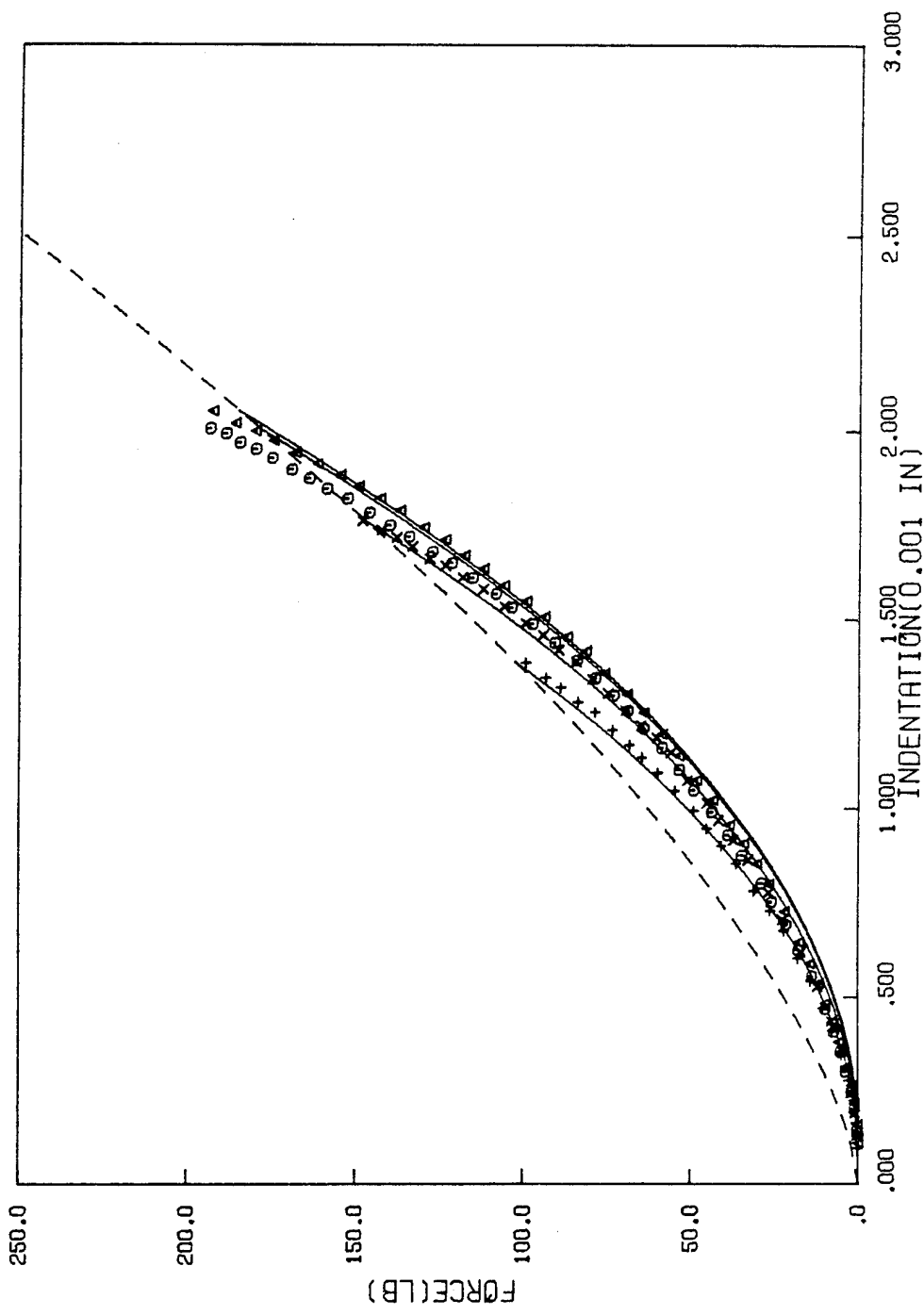


Figure 3.14 Unloading curves of  $[90^\circ/45^\circ/90^\circ/-45^\circ/90^\circ]_{2s}$  specimens with 0.75 inch indenter ( $q=2.0$ )

### 3.2.3 Reloading Curves

The equation

$$F = k_1 (\alpha - \alpha_0)^p \quad (3-11)$$

suggested by Yang [14] was used to model the reloading curve, where  $k_1$  is called reloading rigidity and  $p = 3/2$  was found to fit the experimental data quite well. It was also observed that the reloading curve always returns to where the unloading began, and hence the reloading rigidity can be determined by

$$k_1 = F_m / (\alpha_m - \alpha_0)^{3/2} \quad (3-12)$$

In other words, the reloading test is not necessary provided the unloading condition is specified. Some reloading curves obtained following Equations (3-11) and (3-12), and the experimental data are presented in Figures 3.15-3.18.

### 3.3 Discussion

As mentioned before, due to creep the loading rate may affect the contact law (i.e. the value of  $k$ ). A series of tests with different loading rates was performed to examine this point. The maximum loading rate the test equipment can apply without exceeding its capacity is about 50 lb/sec.. It was found that in the range of 5 lb/sec. to 50 lb/sec., the values of  $k$  showed very little scatter, and the effect

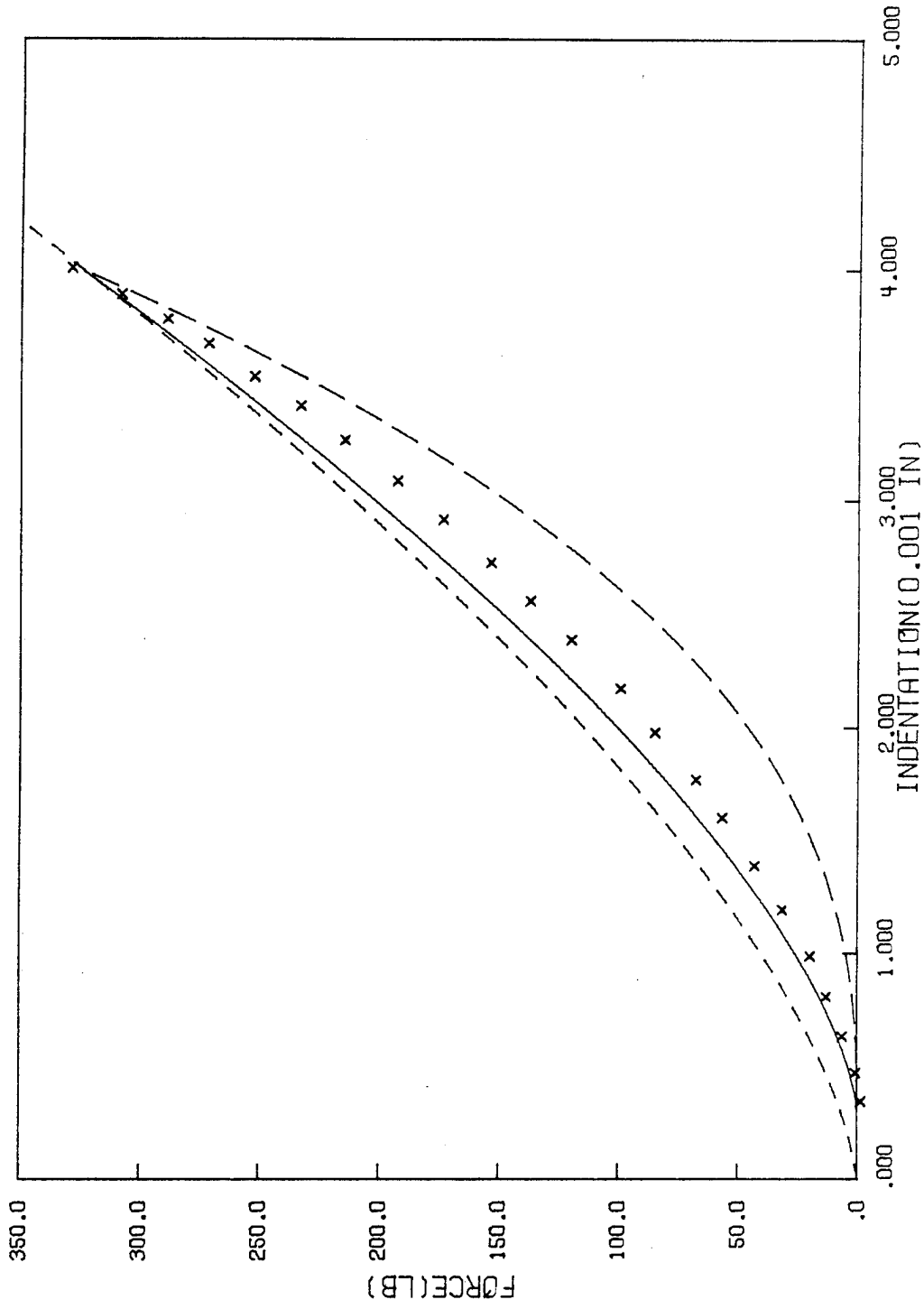


Figure 3.15 Reloading curve of  $[0^\circ/45^\circ/0^\circ/-45^\circ/0^\circ]_{2s}$  specimens with 0.5 inch indenter ( $p=1.5$ )

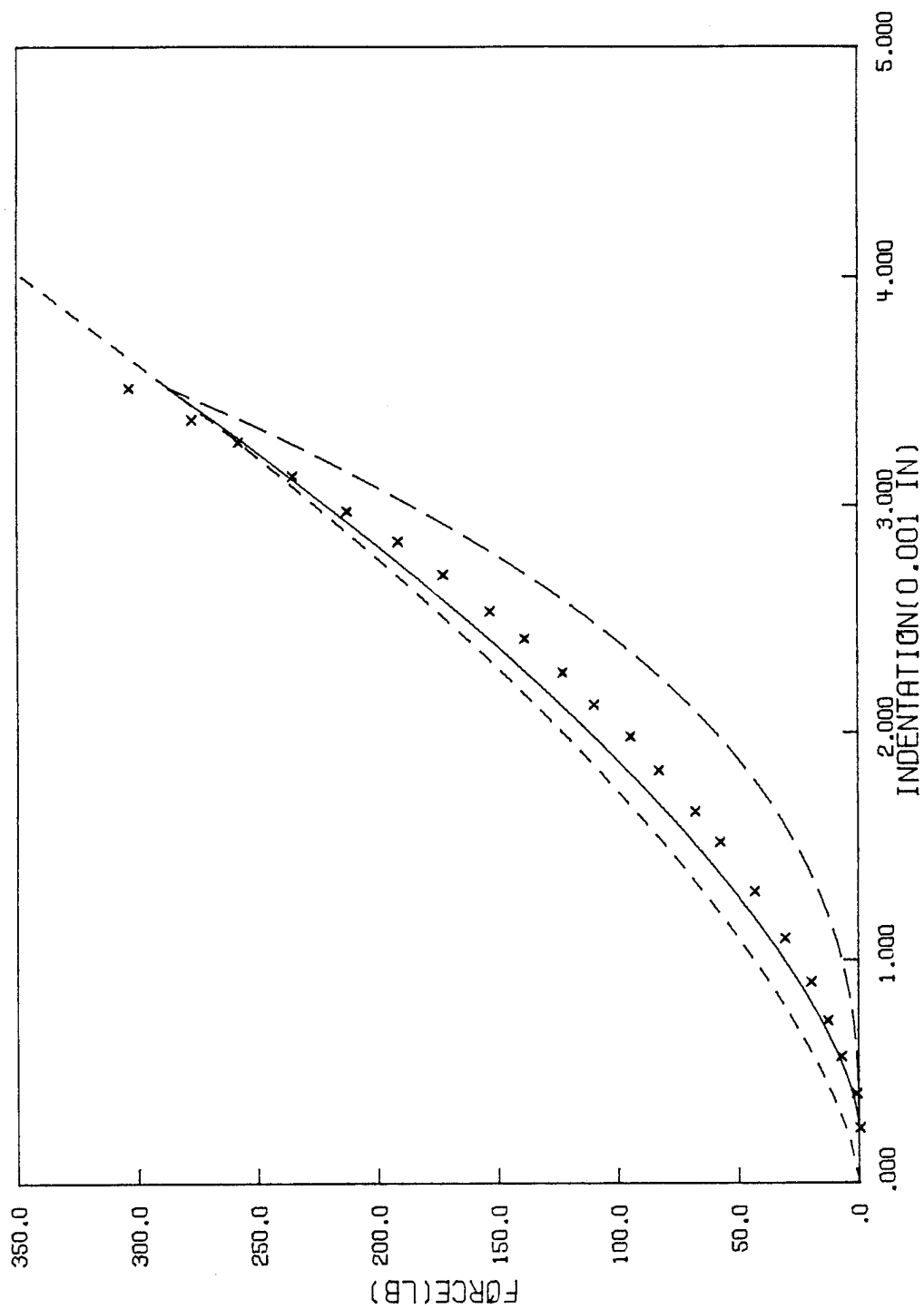


Figure 3.16 Reloading curve of  $[90^\circ/45^\circ/90^\circ/-45^\circ/90^\circ]_2$  specimens with 0.5 inch indenter ( $p=1.5$ )

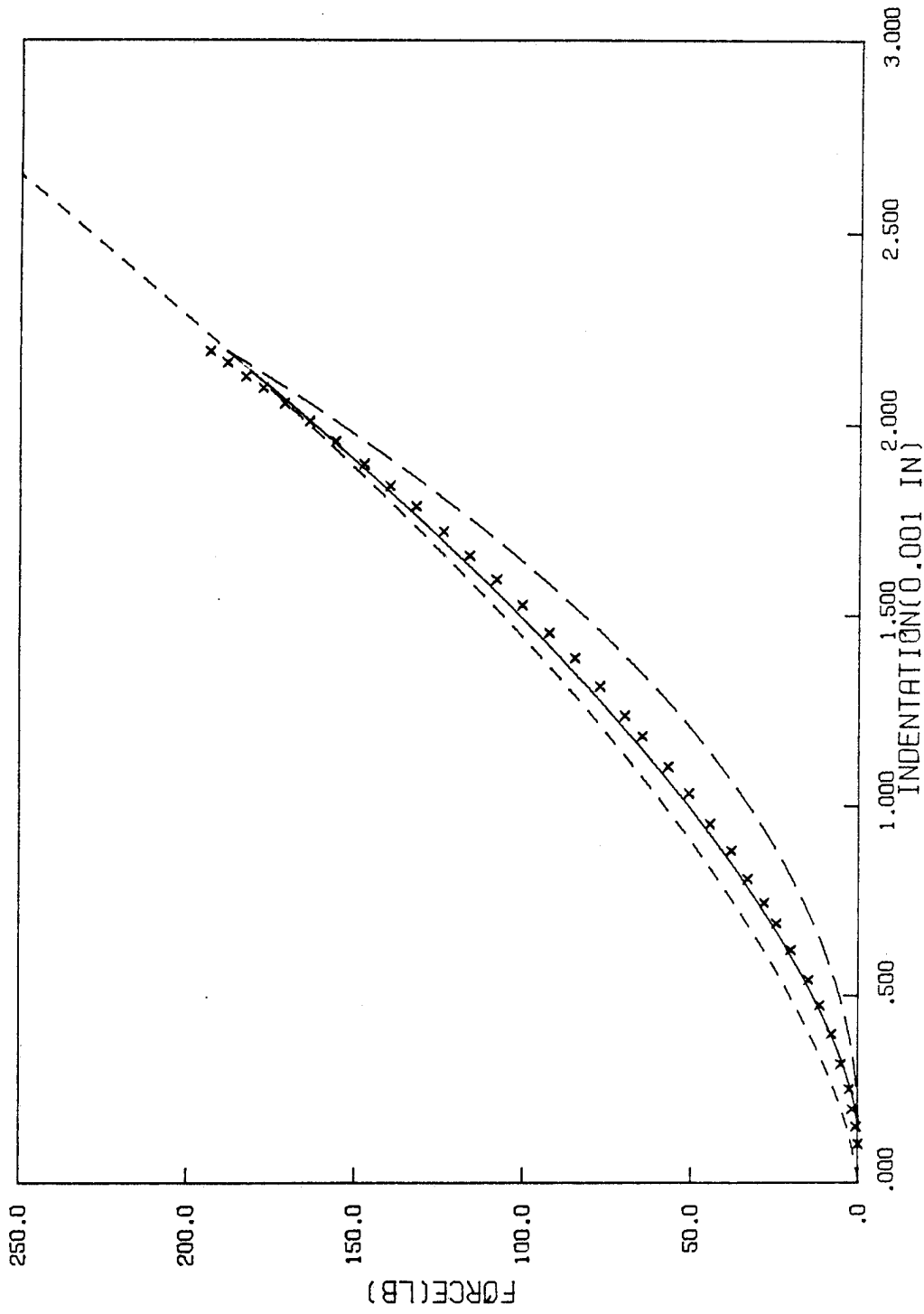


Figure 3.17 Reloading curve of  $[0^\circ/45^\circ/0^\circ/-45^\circ/0^\circ]_{2s}$  specimens with 0.75 inch indenter ( $p=1.5$ )

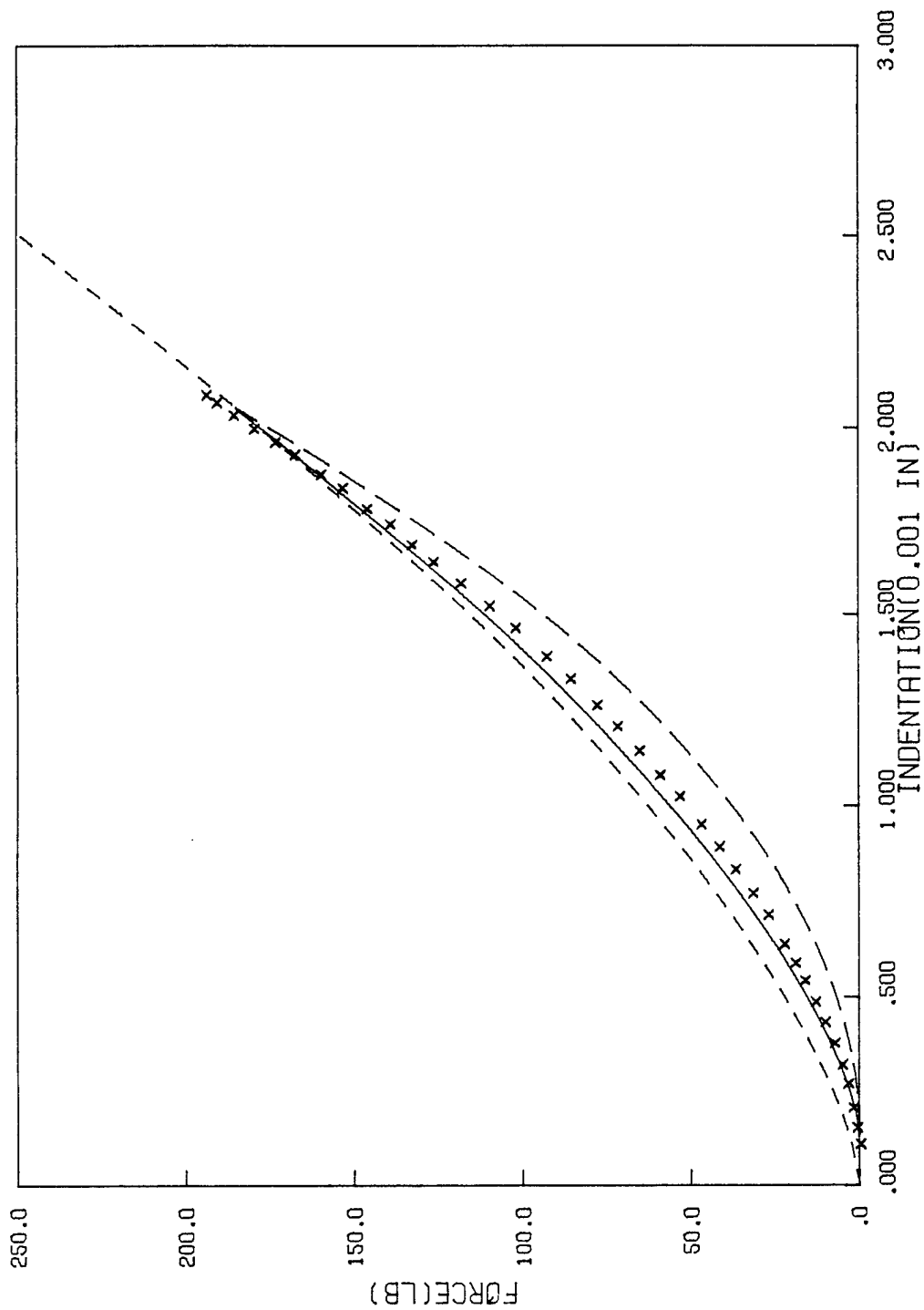


Figure 3.18 Reloading curve of  $[90^\circ/45^\circ/90^\circ/-45^\circ/90^\circ]_{2s}$  specimens with 0.75 inch indenter ( $p=1.5$ )

due to local material nonhomogeneity in the composite may be even greater than the one due to the loading rate. However, an appreciable decrease of the value  $k$  was observed when the loading rate was lower than 1 lb/sec.. In some extreme cases where loadings were applied as slow as 10 lb/min., the average value of  $k$  for 0.5 in. indenter was very close to the one obtained previously by Yang [14] using dial gage to measure the indentation. In this study, the loading rates for all tests were approximately equal to 10 lb/sec..

Unlike the exponent  $n$  of the loading law for which the value of  $3/2$  seems to yield good agreement with all experimental data, the exponent  $q$  of the unloading law (Equation 3-3 or 3-4) reveals much wider deviation for different sizes of indenter. Value of  $q = 3/2$  corresponding to an elastic recovery according to the Hertzian theory was previously used by Crook [28] in a study of impacts between metal bodies. The experimental results from [14] and present study show that the value of  $q$  varies from 1.5 to 2.5. Local plastic deformation, anisotropic properties of composite material and unloading rate are all possible causes for this deviation. Obviously, an analytical study to determine the value of  $q$  as function of aforementioned factors is impracticable. Since the purpose of this study is to establish a contact law that can be used in the analysis of impact, the validity of this law must be verified from impact experiment. This will be investigated

in the next chapter.

From Equation (3-3) or (3-4), it can be seen that  $\alpha_0$  plays an essential role in the unloading law and hence the value of it must be estimated accurately. Both of Equation (3-7) used by Yang [14] and Equation (3-8) used in this study for calculating  $\alpha_0$  were obtained experimentally, in which  $\alpha_{c,r}$  and  $\alpha_p$  are considered to be material constants and were determined using  $\alpha_0$  and  $\alpha_m$  from test data. However, it was pointed out in [14] that the values of  $\alpha_0$  might not be the true permanent indentations. They were the values which could make the power law given by Equation (3-4) fit the total data under the unloading path. In fact, the load corresponding to the value of  $\alpha_{c,r} = 3.16 \times 10^{-3}$  in. obtained in [14] is about 200 lb. for 0.5 in. indenter, which is apparently too high. The value of  $\alpha_p = 6.564 \times 10^{-4}$  in. obtained in this study, which corresponds to about 20 lb of loading, seems more reasonable as a critical value in indentation. For comparison, the relations between unloading rigidity  $s$  and maximum indentation  $\alpha_m$  using Equation (3-7) with  $\alpha_{c,r} = 3.16 \times 10^{-3}$  in. and Equation (3-8) with  $\alpha_p = 6.564 \times 10^{-4}$  in., respectively, are plotted in Figure 3.19. It is interesting to see that these two equations give almost the same values of  $s$  up to  $\alpha_m = 4 \times 10^{-3}$  in. which is approximately the maximum indentation before failure could occur to the specimen. The advantage of using Equation (3-7) for the formulation of the unloading law is

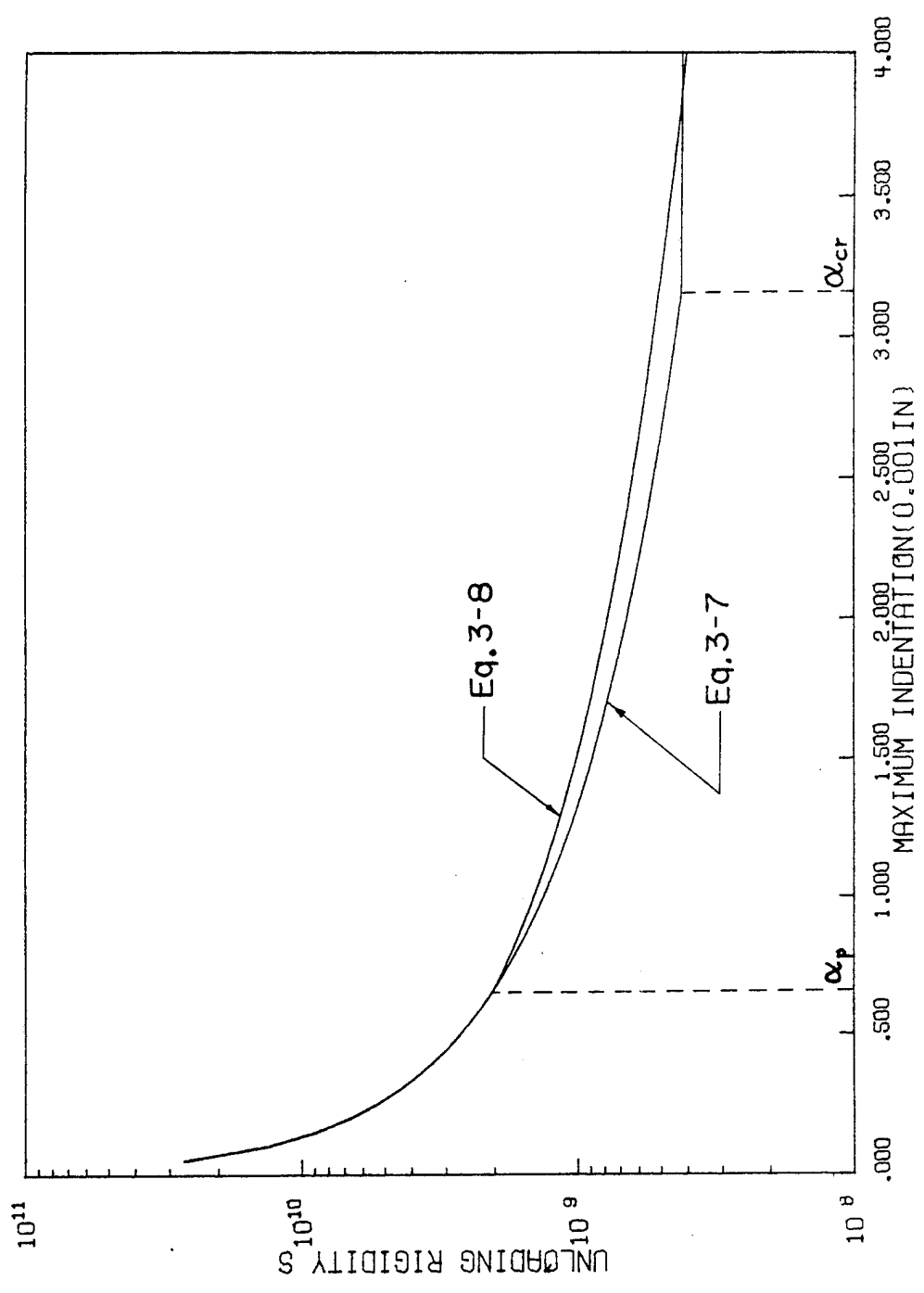


Figure 3.19 Unloading rigidity  $s$  as function of maximum indentation

that the value of  $s$  is constant for any  $\alpha_m$  once the indentation passes  $\alpha_{c,r}$ , and only one unloading test is necessary to determine  $\alpha_{c,r}$  provided the load is high enough to produce permanent indentations. The use of Equation (3-9) needs performing many tests to obtain a proper relation between  $\alpha_0$  and  $\alpha_m$  according to Equation (3-8). However, it should be noted that Equation (3-7) is valid only if  $q = 5/2$  is used in the unloading equation (3-4), while Equation (3-8) has no such restriction.

## CHAPTER 4

### IMPACT EXPERIMENTS

High velocity impacts usually result in very small contact time and the material under impact loadings may behave differently from static contact due to the strain rate effect. The statically determined contact laws presented in the previous chapter thus must be verified experimentally before it can be applied to the impact analysis. Wang [15] has conducted many impact experiments on laminated composite beams and plates using spherical steel balls as impacters. The strain response histories at various points on the specimens were recorded and compared with the finite element analysis with which the contact laws obtained by Yang [14] was incorporated. The results showed that the test data agreed with the predictions using the statical indentation laws quite well. In this chapter, an attempt was made to measure the contact force directly so that the applicability of statical contact laws in impact analysis can be further evaluated.

#### 4.1 Experimental Procedure

A 6 in. by 4 in. laminated plate cut from a  $[0^\circ/45^\circ/0^\circ/-45^\circ/0^\circ]_{2s}$  graphite/epoxy panel was used as the impact target. The  $0^\circ$ -direction was arranged to parallel the long side of the plate. Seven strain gages (Micro Measurement Company TYPE EA-13-062 AQ 350) were placed at different locations as shown in Figure 4.1 to record the dynamic strain histories. One of the gages was placed on the surface directly opposite to the impact point to trigger the oscilloscope. This plate was hung with two strings at two corners to achieve the free boundary condition.

The projectile was made of an impact-force transducer with a spherical steel cap of 0.75 inch in diameter glued on the impact side and a steel rod of 5/8 inch in diameter glued on the other side as shown in Figure 4.2. It was then attached to a thin rod to form a pendulum which could produce impact velocities up to 150 in/sec. The total mass of the projectile is 0.000181 lb-sec<sup>2</sup>/in.

The schematic diagram for this impact experimental set-up is shown in Figure 4.3. Signals from gages and transducer were amplified by a 3A9 Textronix amplifier and displayed on the screen of an oscilloscope.

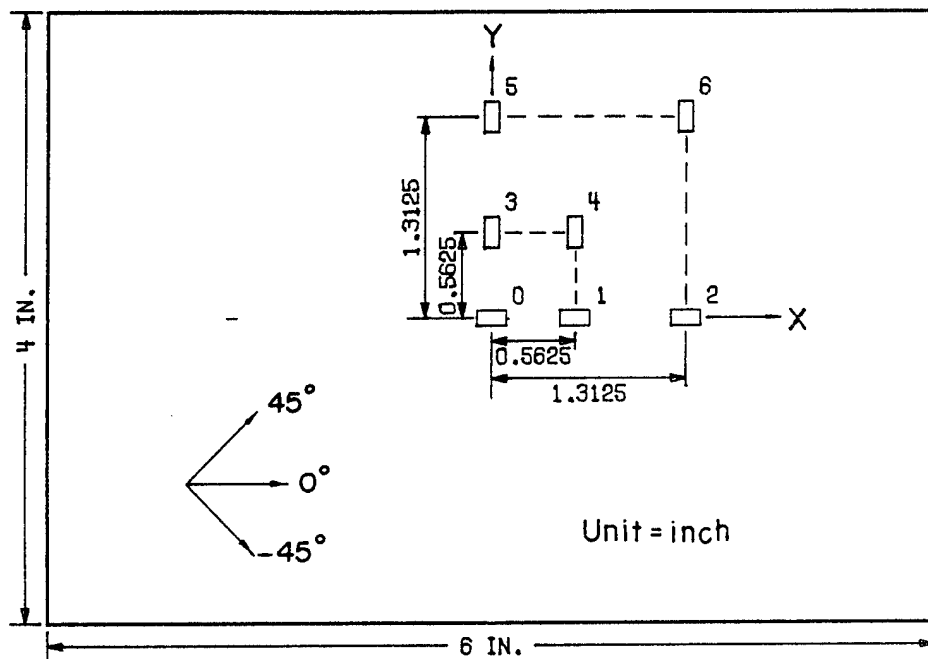


Figure 4.1 Laminate dimension and strain gage locations

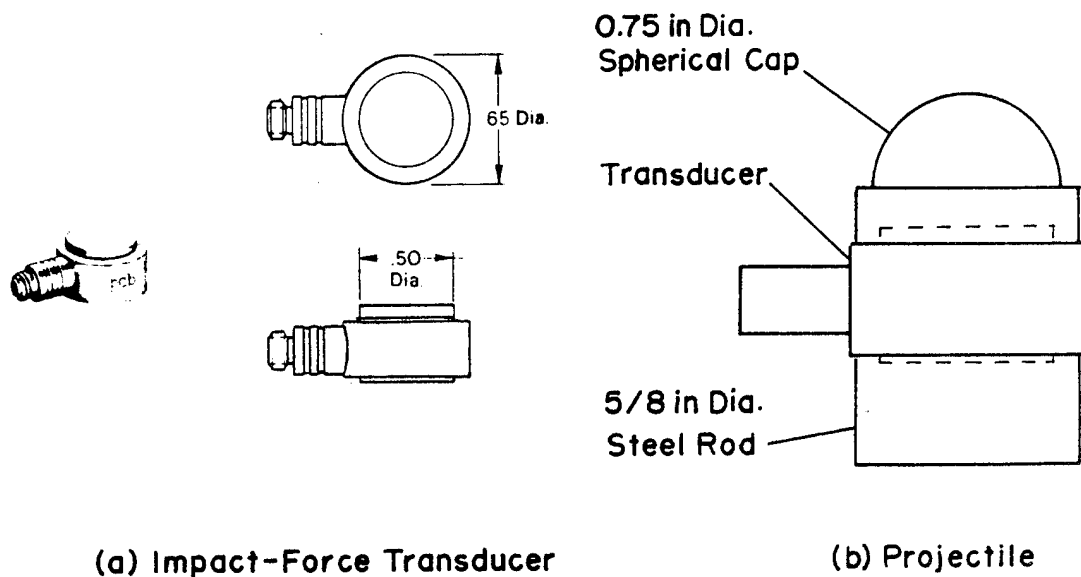


Figure 4.2 Graphical illustration of impact projectile

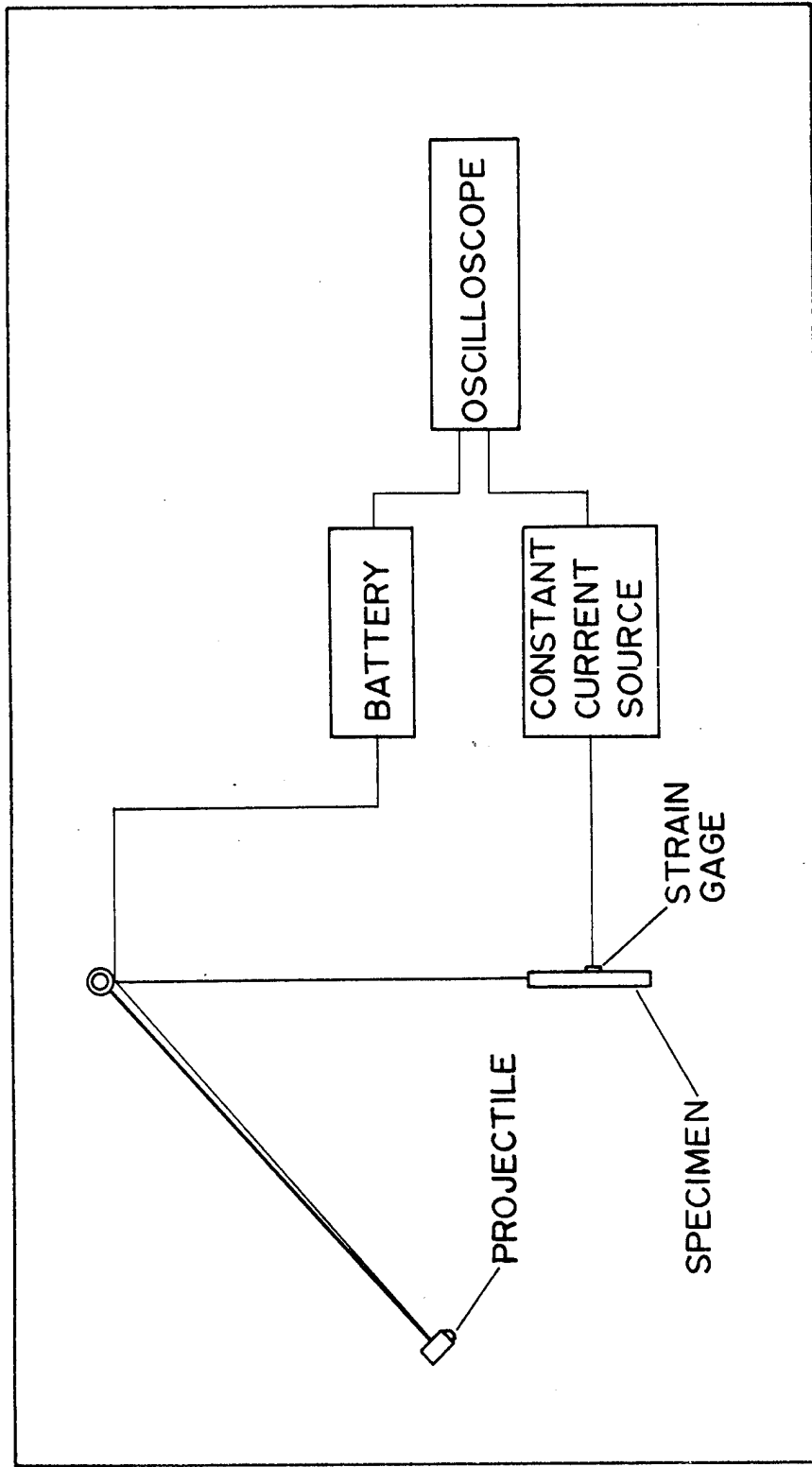


Figure 4.3 Schematic diagram for the impact experimental set-up

## 4.2 Calibration of Impact-Force Transducer

The impact-force transducer used was Modal 200A05 marketed by PCB Piezotronics Inc. Some of its specifications are shown in Table 4.1 [30]. The structure of this transducer contains two thin quartz disks operating in a thickness compression mode and sandwiched between hardened steel cylindrical members. A built-in amplifier can reduce the high impedance of the voltage from the quartz element and provides an output voltage which can be read out on oscilloscope, recorder, etc.. The impact force is then computed using the equation,

$$F = V_F/c_F \quad (4-1)$$

where  $V_F$  is the output voltage and  $c_F$  is the sensitivity of the transducer. Since the value of  $c_F$  in Table 4.1 was obtained under quasi-static condition [30], it must be verified under impact condition first so that later the results from impact experiment can be correctly interpreted.

A circular cylindrical steel rod of 2 inch in diameter and 1.19 inch long hung on strings was used as the impact target to calibrate the transducer. The acceleration of the rod was measured by using a Model 302A accelerometer which was mounted on the end of the rod opposite to the impacted end as shown in Figure 4.4. The total weight of the target is 1.105 lb.

Table 4.1  
Specifications for Model 200A05 Impact-Force Transducer

Range, Compression (5V output)	lb.	5,000
Maximum Compression	lb.	10,000
Resolution (200 $\mu$ V p-p noise)	lb.	0.2
Stiffness	lb/ $\mu$ in	100
Sensitivity	mV/lb	1.0
Resonant Frequency (no load)	Hz	70,000
Rise Time	$\mu$ sec	10
Discharge Time Constant (T.C.)	sec	2,000
Low-Frequency (-5%)	Hz	0.0003
Linearity, B.F.S.L.	%	1
Output Impedance	ohms	100
Excitation (thru C.C.diode)	VDC/mA	+18 to 24/2 to 20
Temperature Coefficient	%/ $^{\circ}$ F	0.03
Temperature Range	$^{\circ}$ F	-100 to +250
Shock (no load)	g	10,000

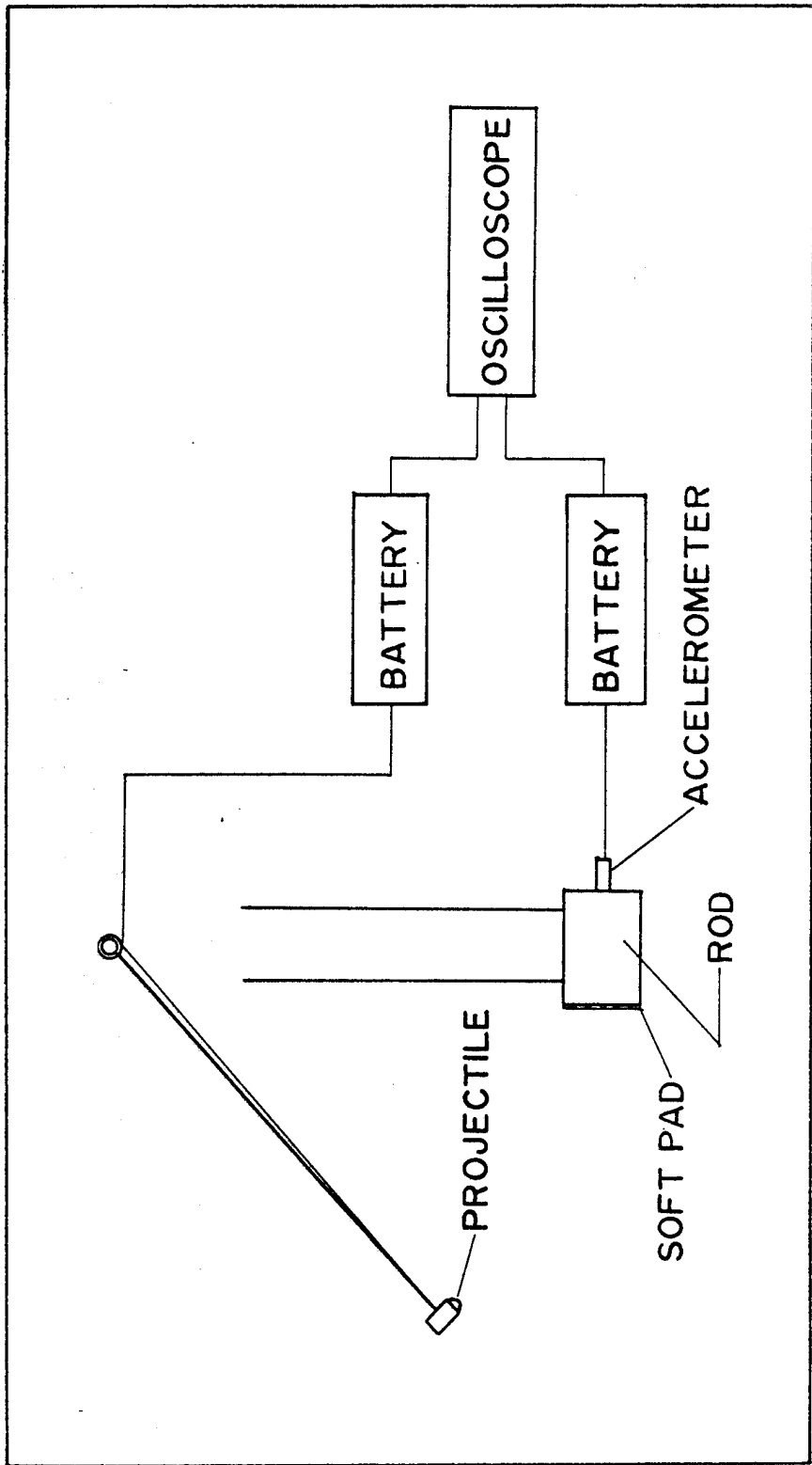


Figure 4.4 Experimental set-up for the calibration of impact-force transducer

Using Equation (4-1) and

$$a = V_a/c_a \quad (4-2)$$

$$F = ma \quad (4-3)$$

we obtain

$$c_F = (c_a/m)(V_F/V_a) \quad (4-4)$$

where  $V_a$  and  $c_a$  are the output voltage and the sensitivity of the accelerometer, respectively,  $a$  is acceleration of the target, and  $m$  is the mass of the target.

When impacting a metal projectile on a metal target with no pad on the impact surface, a high frequency ringing can be seen at the output of the transducer. In order to obtain smooth output curves, a soft pad was placed on the impact region of the target to eliminate the high frequency ringing. The cause of this ringing phenomenon will be discussed later. Typical output voltages of transducer and accelerometer read from the oscilloscope are shown in Figure 4.5. Values of  $V_F$  were plotted vs the corresponding values of  $V_a$  taken from these two curves at several discrete points in time and then fitted into a straight line as shown in Figure 4.6. The slope of this line represents the ratio of  $V_F/V_a$  which is then substituted in Equation (4-4) to calculate the sensitivity  $c_F$ .

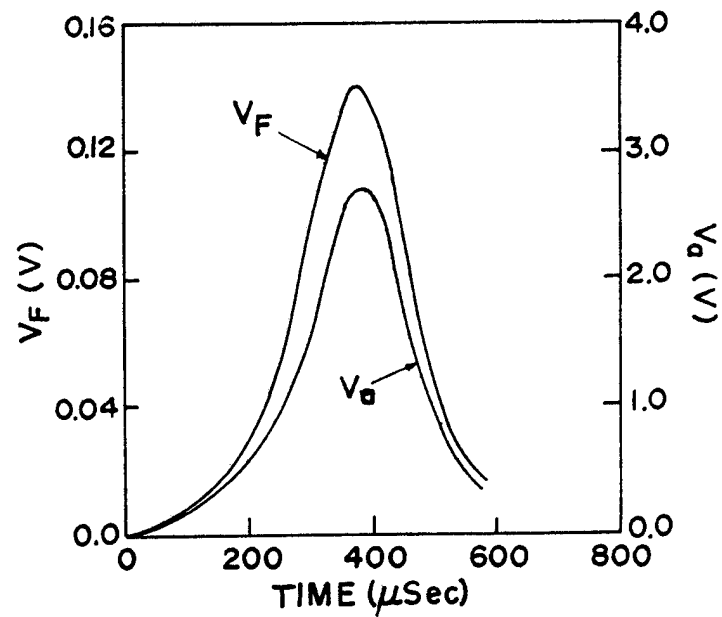


Figure 4.5 Typical output voltages from transducer and accelerometer

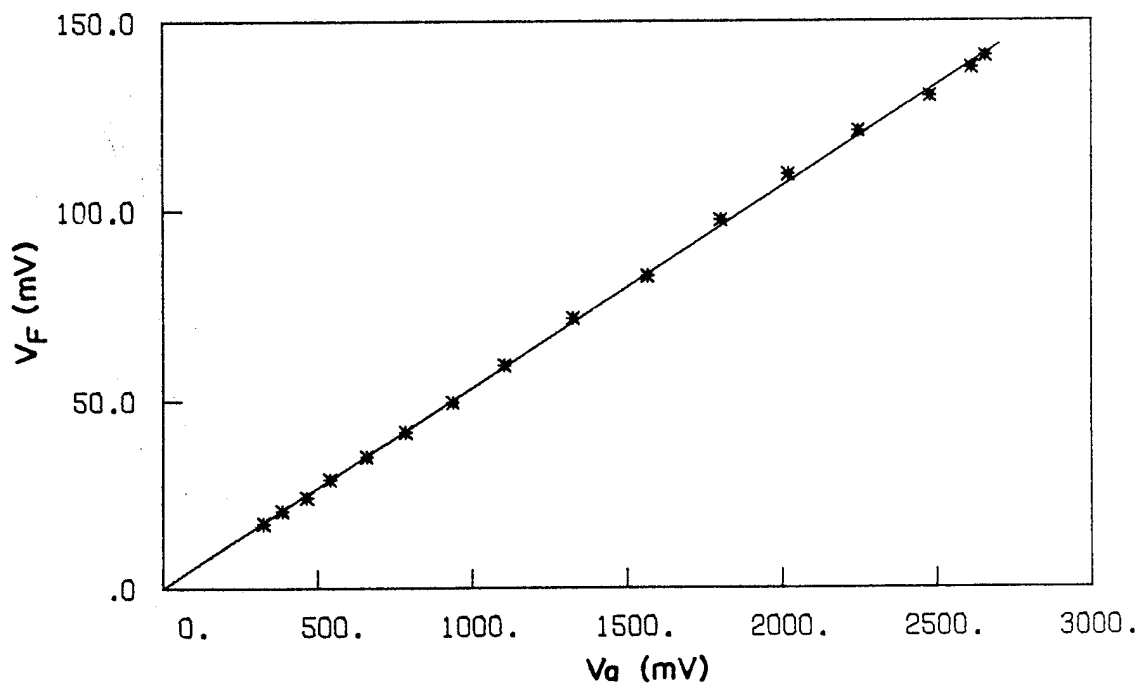


Figure 4.6 Relation between V<sub>F</sub> and V<sub>a</sub>

Assuming the sensitivity of the accelerometer  $c_a$  is correct, and using Equation (4-4) and the test data, the average value of  $c_f$  calculated was 0.494 mV/lb.. A comparison with the value of 1.0 mV/lb from Table 4.1 shows that the test result has more than 50% error. However, since the quartz elements are located at the center of the projectile while the impact force is applied at the end, we were not certain that the force history picked up by the quartz elements did represent the real history of the impact force. The following simple analysis was performed to examine this uncertainty.

Consider a 1 in. long steel rod with free-free boundary conditions. For a impulse loading given by

$$F(t) = F_0 \text{EXP}[-(t-\tau)^2/4b^2] \quad (4-5)$$

at one end, the force history at the midpoint of the rod,  $F_m(t)$ , was computed and plotted in Figure 4.7 together with the applied force history. It should be noted that the values of  $F_0 = 1000$  lb.,  $\tau = 200 \times 10^{-6}$  sec. and  $b = 40 \times 10^{-6}$  sec. were chosen in Equation (4-5) so that the applied force history is similar to the experimental loading history. From Figure 4.7, it can be seen that  $F_m(t)$  is only about half of the applied force  $F(t)$ . The average ratio of  $F_m(t)/F(t)$  was obtained to be 0.498, which is very close to the value of  $c_f$  obtained previously. The accelerations at the two ends and the midpoint of the rod were also

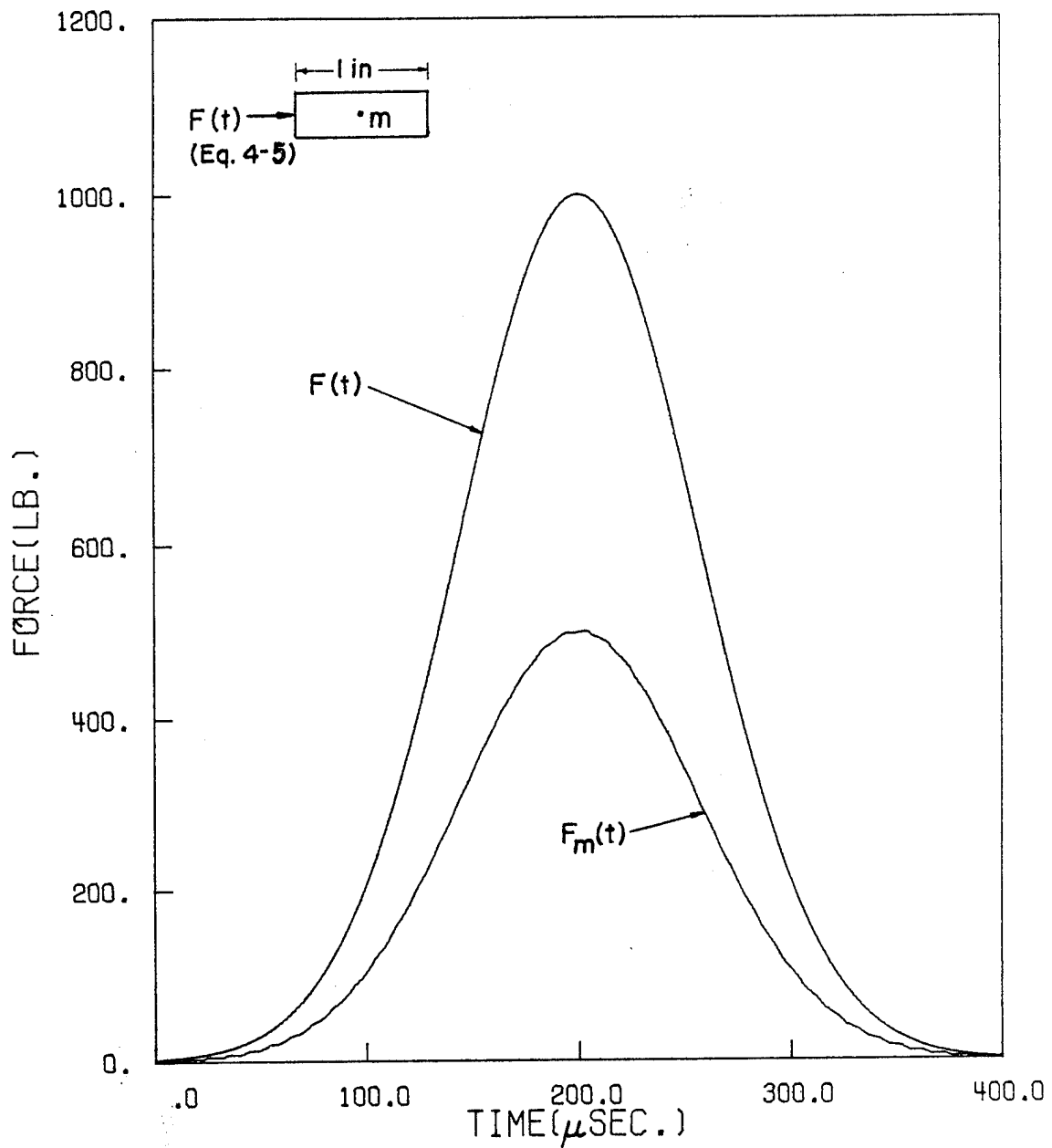


Figure 4.7 Assumed exponential impulsive loading and the response history at the midpoint of the rod

calculated and plotted in Figure 4.8. It shows that the magnitudes of acceleration at any position of the rod have virtually no difference. This indicates that the accelerometer did measure the real acceleration of the target while the impact-force transducer only picked up the force history at the point of it's own position. In other words, the wave motion in the projectile can not be neglected, hence it must be treated as an elastic body.

Repeating the previous analysis by changing the impulse loading of Equation (4-5) to

$$F(t) = F_0 \sin(\pi t/b) \quad (4-6)$$

and letting  $F_0 = 1000$  lb. and  $b = 400 \times 10^{-6}$  sec., we obtain the force history at the midpoint of the rod as shown in Figure 4.9. Comparing Figure 4.9 with Figure 4.8, it is clear that the initial slope of the impulse forcing function would affect the amplitude of ringing. The steeper the initial slope is, the higher the amplitude of ringing will be. When impacting the steel projectile on graphite/epoxy surface, this ringing phenomenon was also observed.

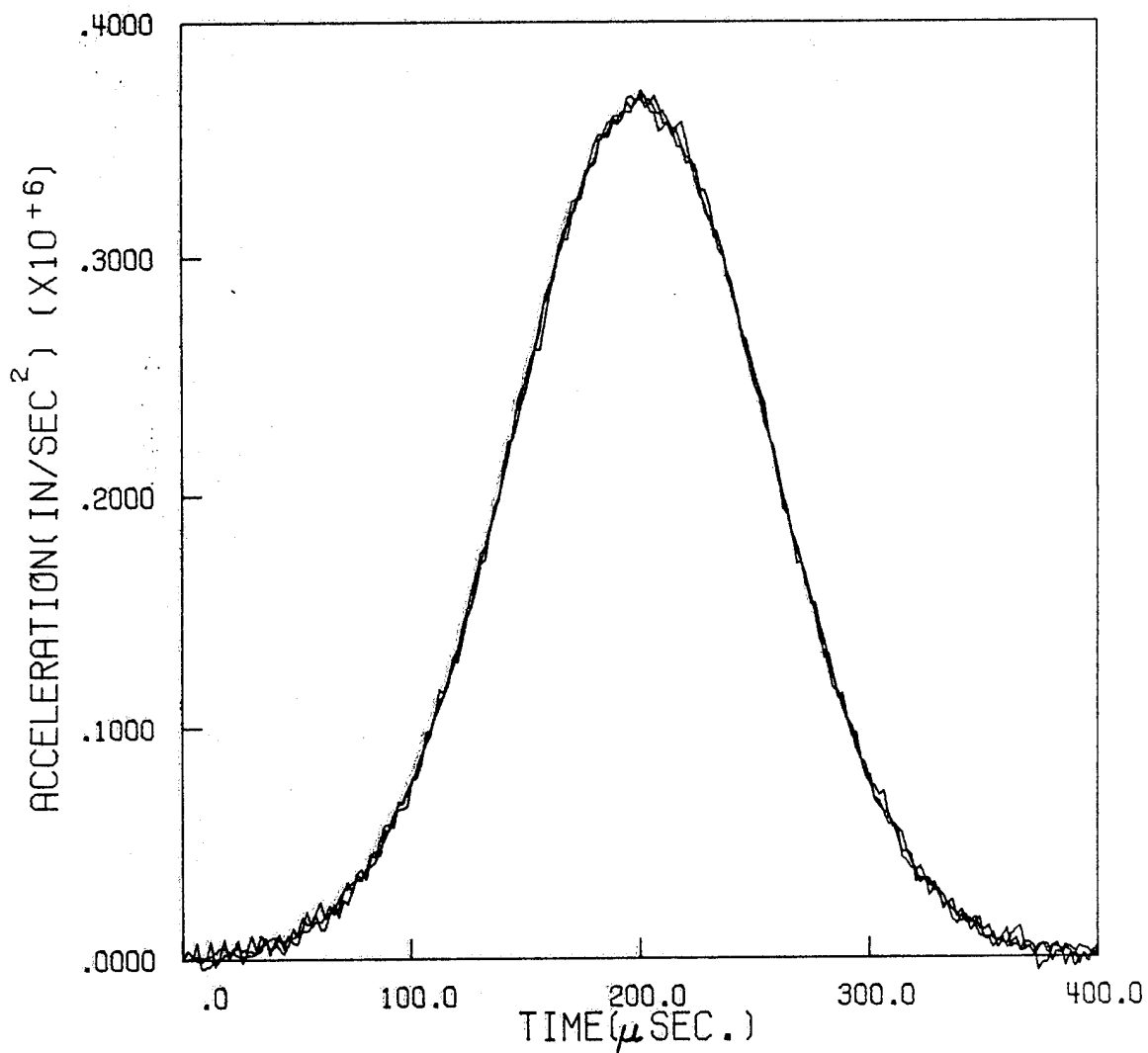


Figure 4.8 Accelerations of rod for assumed exponential impulsive loading

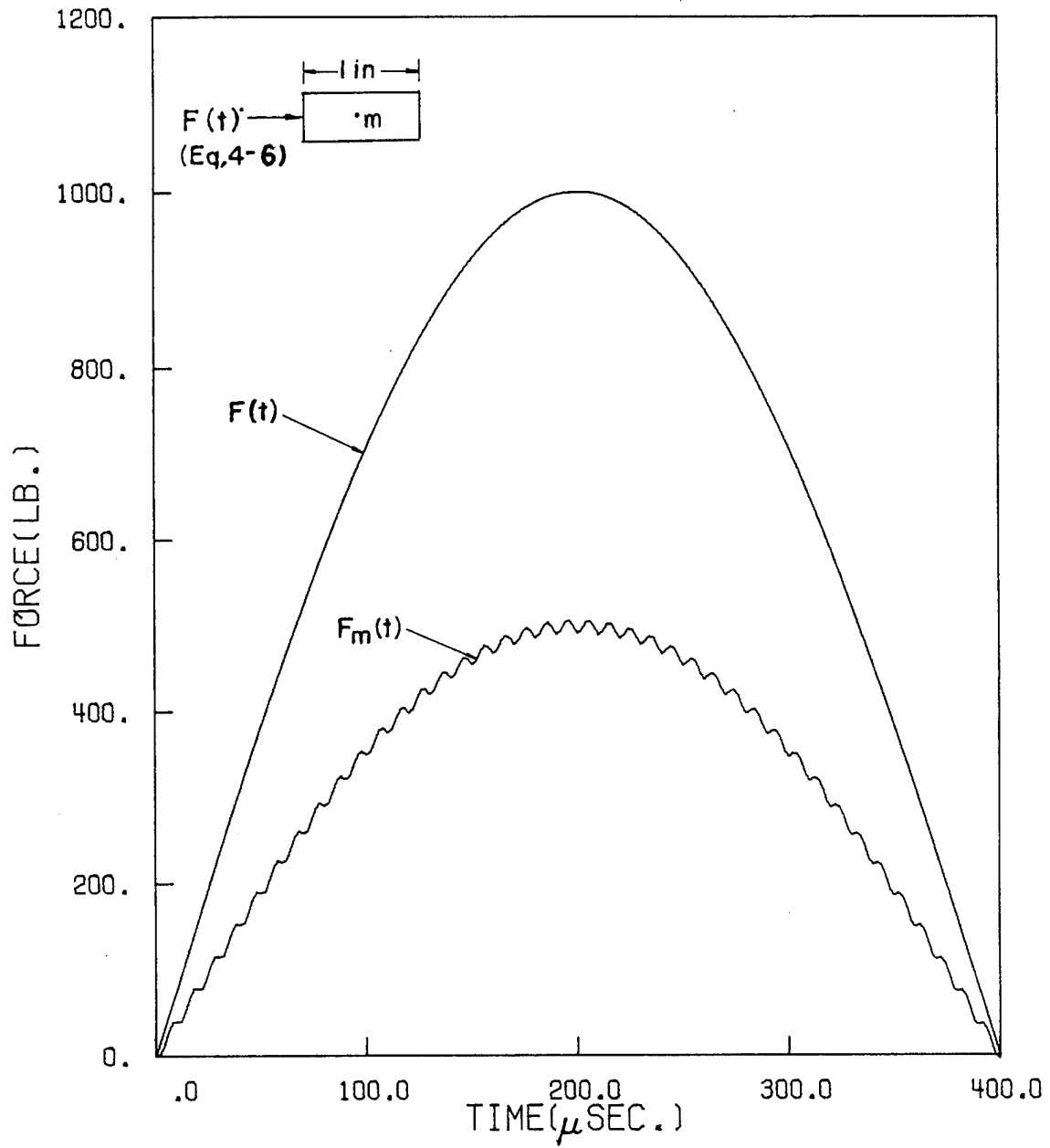


Figure 4.9 Assumed sine-function impulsive loading and the response history at the midpoint of the rod

### 4.3 Finite Element Analysis

#### 4.3.1 Plate Finite Element

A 9-node isoparametric plate finite element (see Figure 4.10) developed by Yang [31] based upon the laminate theory of Whitney and Pagano [18] was used to model the dynamic motion of the laminated plate. At each node there are five degrees of freedom. Among them,  $u^0$ ,  $v^0$  and  $w$  are displacement components of mid-plane in the  $x$ -,  $y$ - and  $z$ -direction, respectively, and  $\phi_x$  and  $\phi_y$  are rotations of the cross-sections perpendicular to the  $x$ - and  $y$ -axis, respectively. For symmetric laminates, the flexural deformation is uncoupled from the in-plane extensional and shear deformations, and hence, the degrees of freedom corresponding to  $u^0$  and  $v^0$  can be neglected in the transverse impact problem.

The isoparametric plate finite element is developed using the following shape functions:

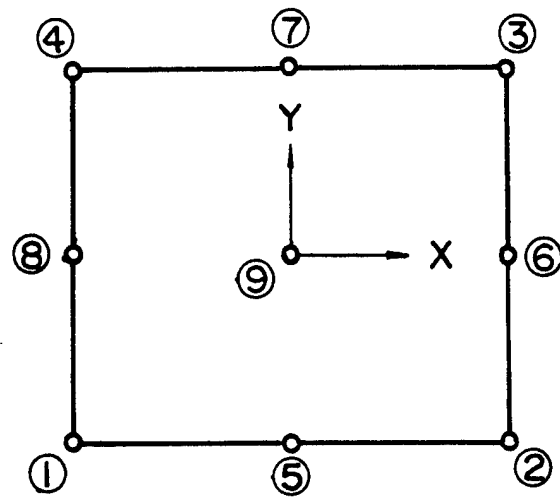
For corner nodes:

$$S_i = (1/4)(1 + \xi_0)(1 + \eta_0)(\xi_0 + \eta_0 - 1) + (1/4)(1 - \xi^2)(1 - \eta^2) \quad (4-7)$$

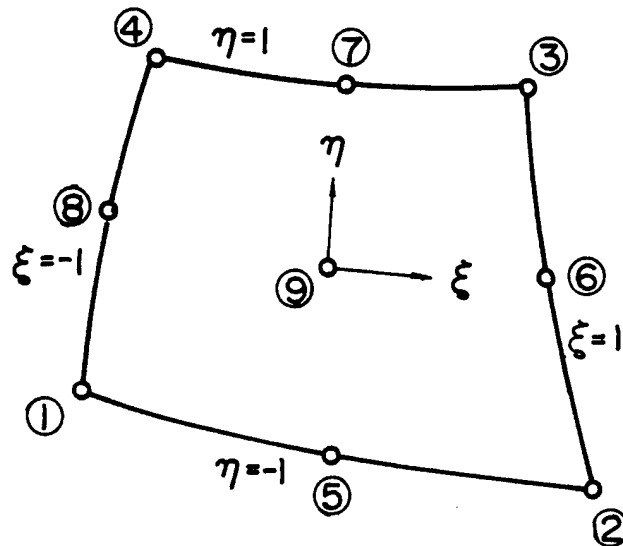
For nodes at  $\xi = 0$  and  $\eta = \pm 1$ :

$$S_i = (1/2)(1 - \xi^2)(\eta_0 + \eta^2) \quad (4-8)$$

For nodes at  $\xi = \pm 1$  and  $\eta = 0$ :



(a) PARENT ELEMENT



(b) DISTORTED ELEMENT

Figure 4.10 9-node isoparametric plate element

$$S_i = (1/2)(\xi_0 + \xi^2)(1 - \eta^2) \quad (4-9)$$

For the center node:

$$S_i = (1/2)(1 - \xi^2)(1 - \eta^2) \quad (4-10)$$

In the above shape functions,  $\xi$  and  $\eta$  are normalized local coordinates, and

$$\xi_0 = \xi \xi_i, \quad \eta_0 = \eta \eta_i \quad (4-11)$$

where  $\xi_i$  and  $\eta_i$  are the natural coordinates of node  $i$  (Figure 4.10).

Using the shape functions, the plate displacements  $w$ ,  $\phi_x$  and  $\phi_y$  are approximated by

$$\begin{Bmatrix} w \\ \phi_x \\ \phi_y \end{Bmatrix} = \sum_{i=1}^9 [S_i] \{q_p\}_i \quad (4-12)$$

where  $\{q_p\}_i$  is the nodal displacement vector at node  $i$  and

$$[S]_i = S_i [I] \quad (4-13)$$

The stiffness and mass matrices are obtained by numerical integration using Gauss quadrature. Following standard finite element procedures, the system stiffness matrix  $[K_p]$  and mass matrix  $[M_p]$  are assembled from the element matrices. The equations of motion are expressed in matrix

form as

$$[M_p]\{\ddot{q}_p\} + [K_p]\{q_p\} = \{P_p\} \quad (4-14)$$

where

$$\{P_p\}^T = \{0, \dots, F, \dots, 0\} \quad (4-15)$$

is the force vector in which  $F$  is the contact force associated with the degree of freedom corresponding to the  $w$ -displacement at the impact point. The subscript  $p$  in Equations (4-12) through (4-15) denotes those are quantities corresponding to laminated plate.

#### 4.3.2 Modeling of Projectile

In Section 4.2 we showed that in order to interpret the experimental transducer response, it is necessary to treat the projectile as an elastic body. A higher order rod finite element developed by Yang and Sun [32] was used to model the projectile. This element has two degrees of freedom at each node, namely the axial displacement  $u$  and its first derivative  $\partial u / \partial x$ . It has been shown that this higher order element is far more superior than the elements with less degrees of freedom in the analysis of dynamic problems. The displacement function is taken as

$$u = a_1 + a_2x + a_3x^2 + a_4x^3 \quad (4-16)$$

where  $a_i$  are constant coefficients. Solving these coefficients in terms of the nodal degrees of freedom and substituting into Equation (4-16), we obtain

$$u = \{N\}^T \{q_r\}_e \quad (4-17)$$

where

$$\{q_r\}_e^T = \{(u)_1, (\partial u / \partial x)_1, (u)_2, (\partial u / \partial x)_2\} \quad (4-18)$$

is the vector of element nodal degrees of freedom, and

$$\{N\}^T = \{f_1(x), f_2(x), f_3(x), f_4(x)\} \quad (4-19)$$

in which

$$f_1(x) = (1 - x/L)^2(1 + 2x/L)$$

$$f_2(x) = x(1 - x/L)^2$$

$$f_3(x) = x^2/L^2(3 - 2x/L)$$

$$f_4(x) = x^2/L(x/L - 1)$$

are shape functions. The subscript  $r$  in Equation (4-17) denotes quantities corresponding to the rod.

Using variational principle, the equations of motion for one element are obtained as

$$[m_r]\{\ddot{q}_r\}_e + [k_r]\{q_r\}_e = \{p_r\}_e \quad (4-20)$$

where  $\{p_r\}_e$  is the vector of the generalized forces associated with the nodal degrees of freedom  $\{q_r\}_e$ ,  $[m_r]$  is the element mass matrix whose entries are given by

$$(m_r)_{ij} = \rho A \int_0^L f_i f_j dx \quad i, j = 1, 2, 3, 4 \quad (4-21)$$

and  $[k_r]$  is the element stiffness matrix whose entries are given by \*

$$(k_r)_{ij} = EA \int_0^L f_i' f_j' dx \quad i, j = 1, 2, 3, 4 \quad (4-22)$$

In Equations (4-21) and (4-22),  $\rho$ ,  $E$  and  $A$  are mass density, Young's modulus and cross-sectional area of the projectile, respectively, and  $L$  is the length of the element. The explicit forms of  $[k_r]$  and  $[m_r]$  are given by

$$[k_r] = \frac{EA}{30L} \begin{bmatrix} 36 & 3L & -36 & 3L \\ 3L & 4L^2 & -3L & -L^2 \\ -36 & -3L & 36 & -3L \\ 3L & -L^2 & -3L & 4L^2 \end{bmatrix} \quad (4-23)$$

and

$$[m_r] = \frac{\rho AL}{420} \begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix} \quad (4-24)$$

Following the usual manner, the system stiffness and mass matrices are assembled from the element stiffness and mass matrices, and the system equations of motion are expressed as

$$[M_r]\{\ddot{q}_r\} + [K_r]\{q_r\} = \{P_r\} \quad (4-25)$$

where

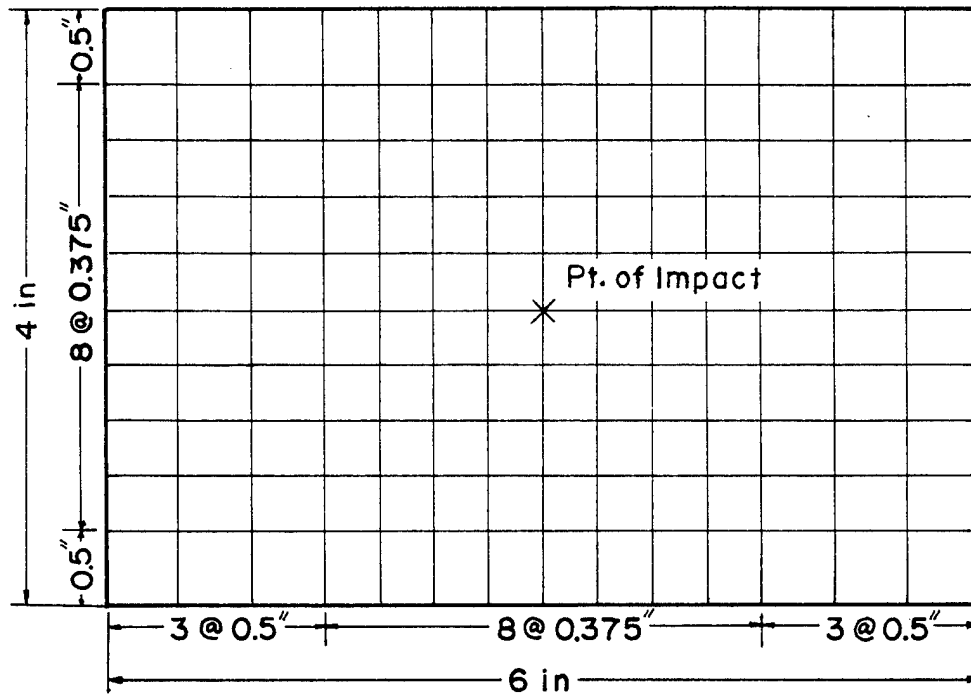
$$\{P_r\}^T = \{F, 0, \dots, 0\} \quad (4-26)$$

in which  $F$  is the contact force applied at the impacting end of the projectile.

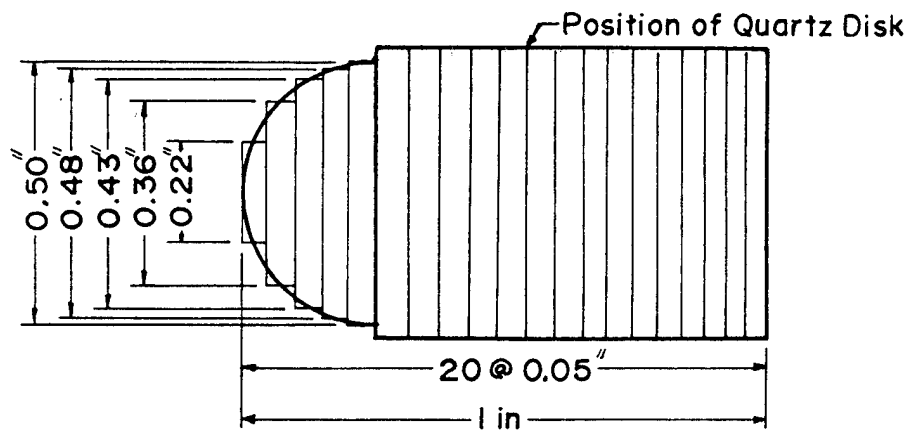
#### 4.4. Results and Discussion

The 6 in. by 4 in. graphite/epoxy laminate was modeled by 140 (14 x 10 mesh) plate elements while the projectile was modeled by 20 rod elements (see Figure 4.11). The two sets of equations (4-14) and (4-25) along with the contact laws given by Equations (3-1), (3-3) and (3-11) were solved simultaneously. The finite difference method with  $\Delta t = 0.2 \mu\text{sec.}$  was used to integrate the time variable. A coarser finite element mesh for plate was used and it was found that the present mesh yielded converged solutions. A 3-Dimensional analysis using 112 axisymmetric finite elements to model the projectile was also performed, and the results showed the the response at the midpoint of the projectile to have no significant difference comparing with the one obtained by using rod elements.

An impact velocity of 115 in/sec was used in the experiment. Figures 4.12-4.17 show the strain response histories at the six locations picked up by the strain



(a) Plate



(b) Projectile

Figure 4.11 Finite element mesh for laminated plate and projectile

gages. The results obtained using the finite element methods and the contact laws are also shown in these figures. It is evident that the finite element solutions agree with the experimental data very well.

In Figure 4.18, the experimental transducer responses and the computed transducer responses using finite element are plotted against time as curve I and curve II, respectively. The computed contact force history is also plotted as curve III. It can be seen that the magnitudes of curve I and curve II agree fairly well. The frequencies of ringing for these two curves, however, are quite different. For the finite element results, the time interval between two consecutive peaks of ringing is approximately equal to the time that the longitudinal stress wave needed to travel the distance between two ends of the projectile. This indicates that the ringing is simply caused by the transient wave travelling back and forth in the projectile.

From Figure 4.18 we can see that curve I has exact 9 peaks in 180 microseconds, and the time interval between two consecutive peaks is about 20 microseconds. It is noted that this transducer has a rise time of 10 microseconds (see Table 4.1), which is the time it needs to reach the maximum response. Any input signal with period smaller than twice of this value will be smoothed out by the transducer, and the output signal may appear to have lower frequency. In other words, the period of the output signal will be at

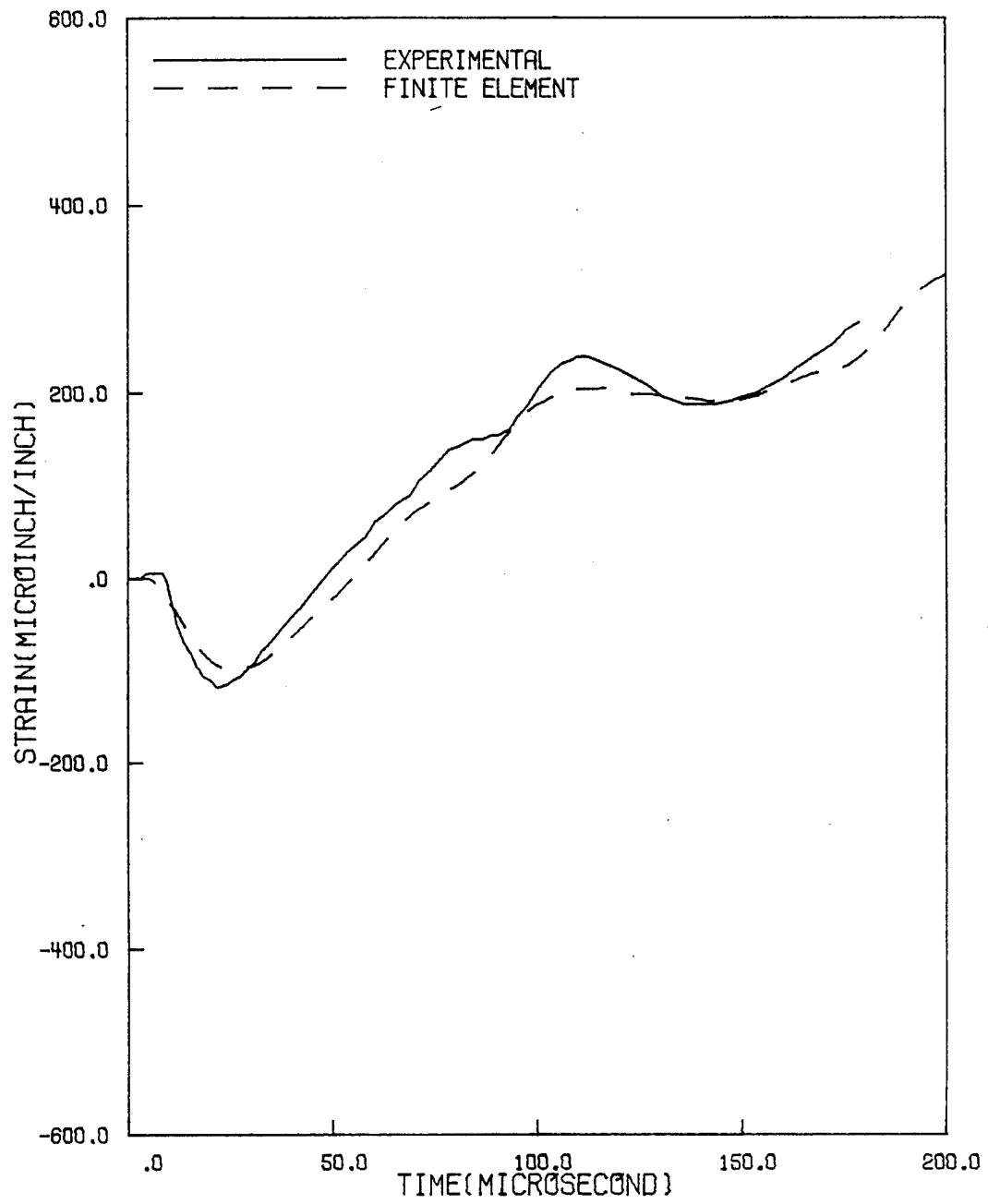


Figure 4.12 Strain response history at gage No.1

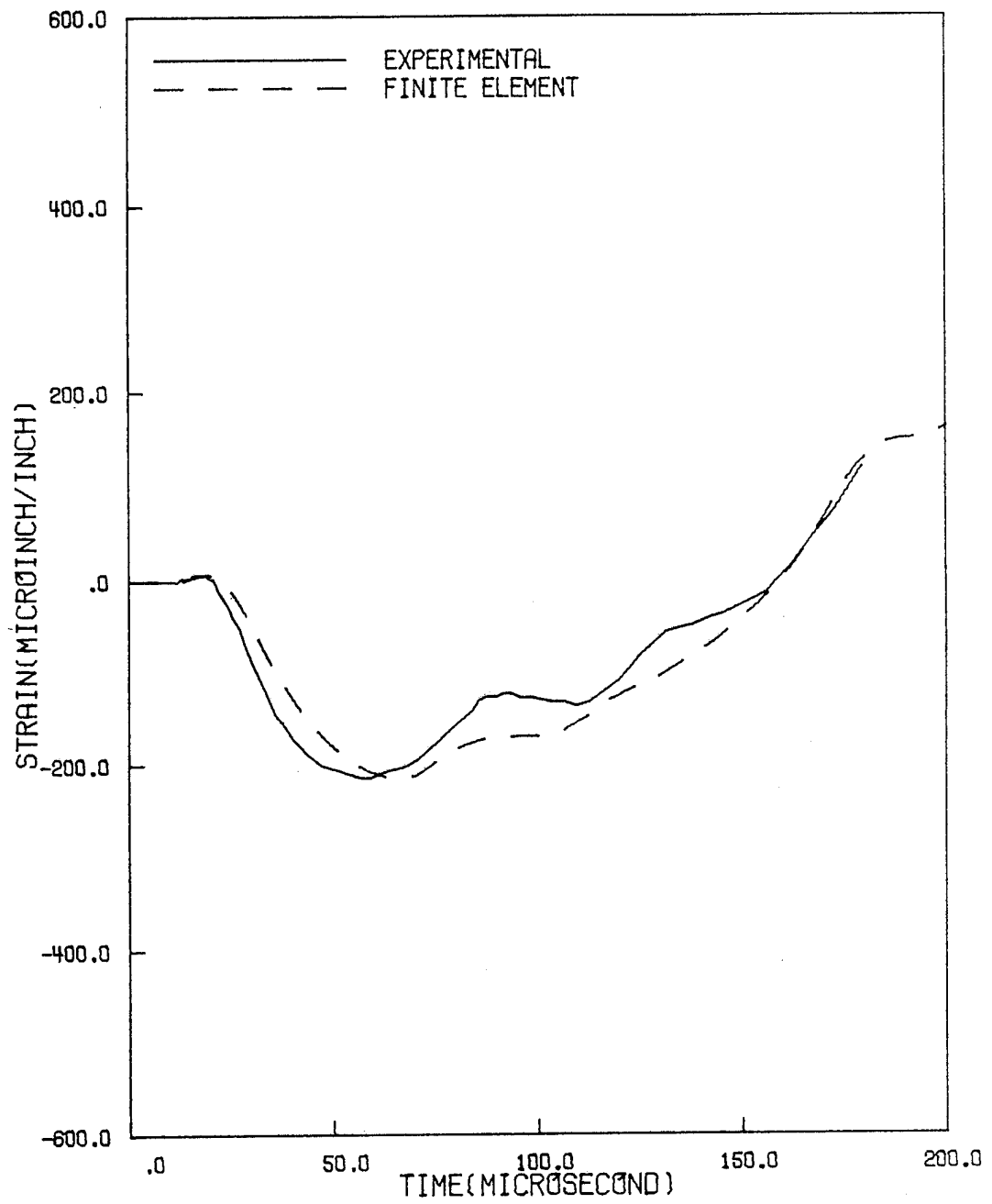


Figure 4.13 Strain response history at gage No.2

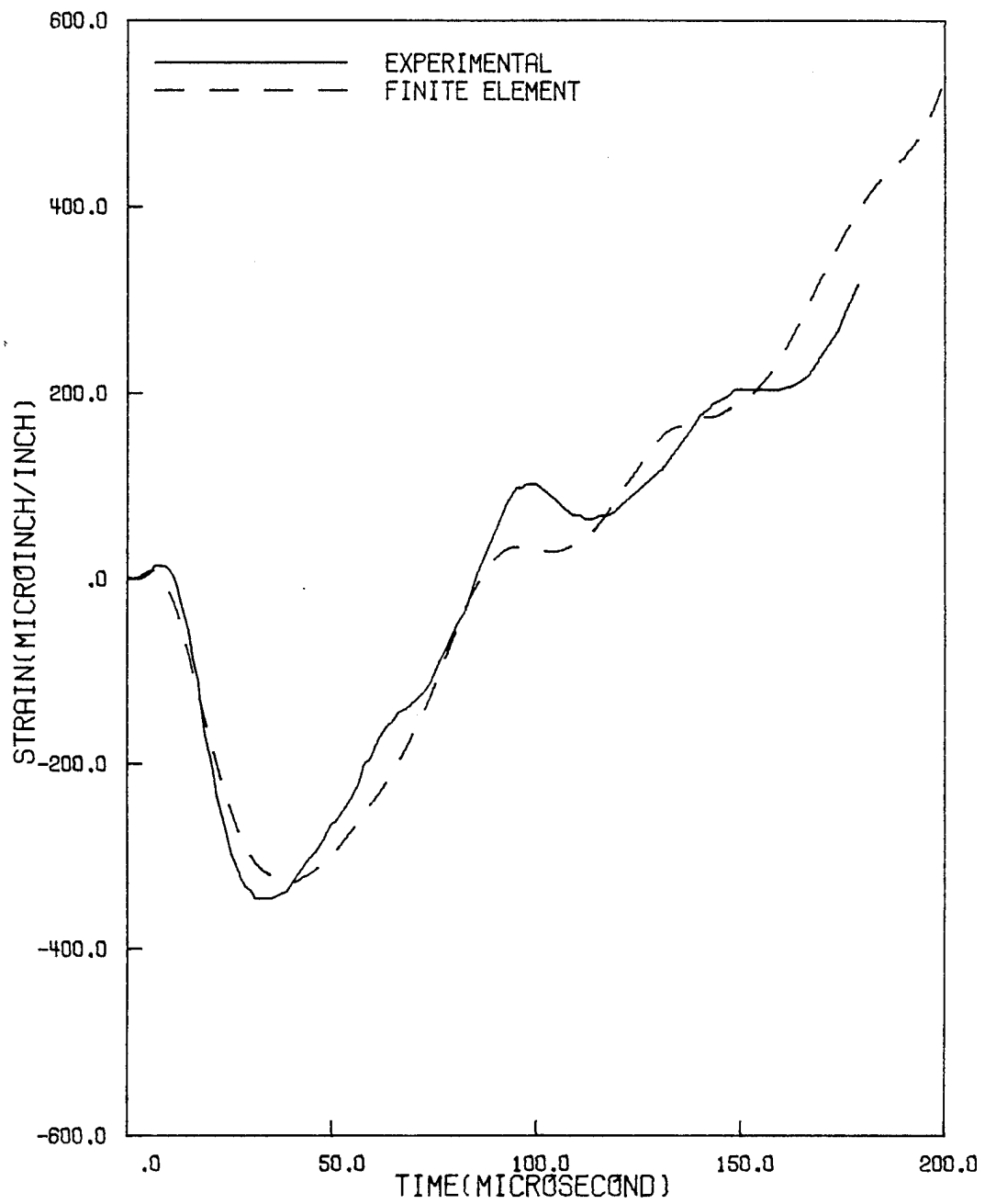


Figure 4.14 Strain response history at gage No.3

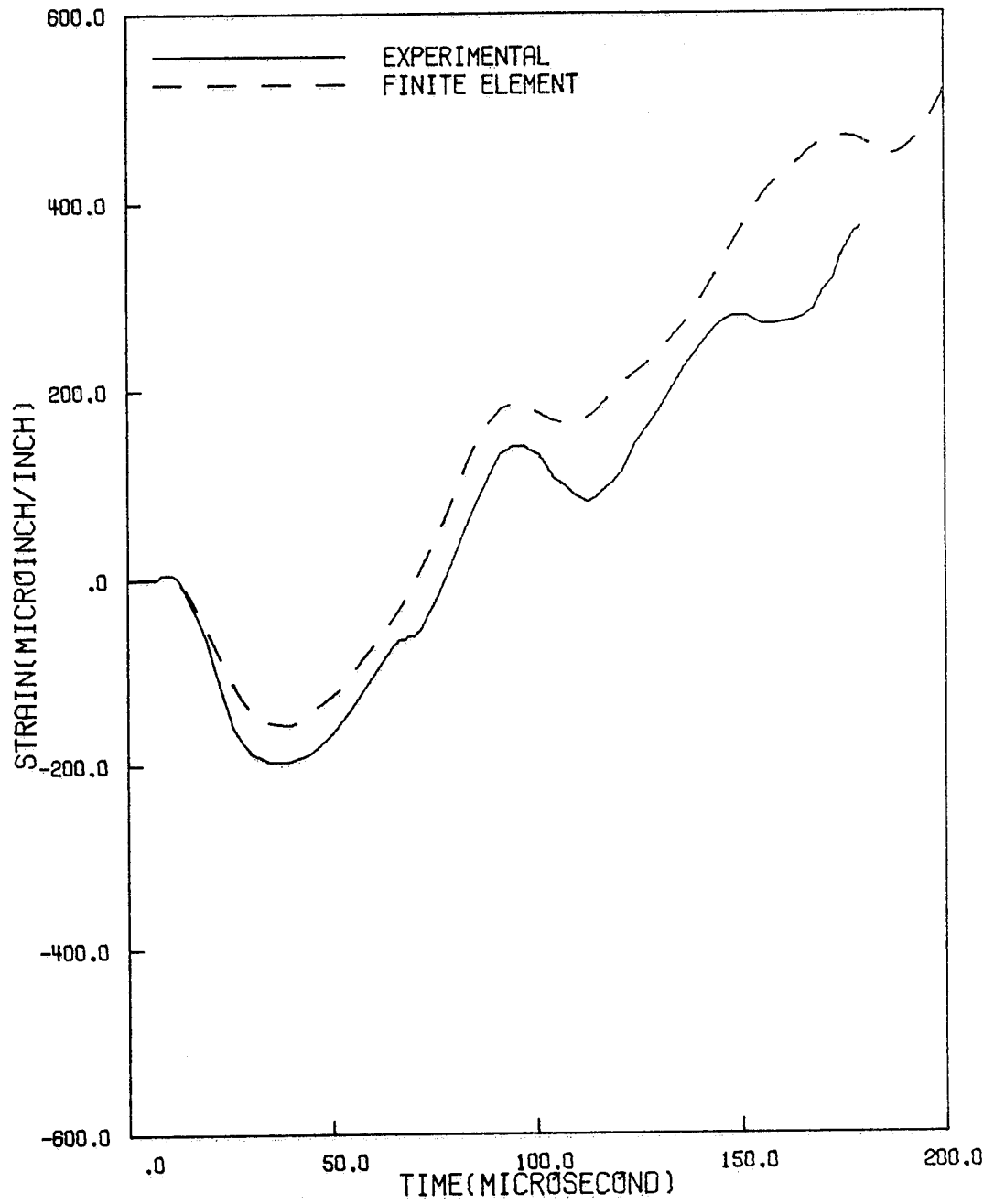


Figure 4.15 Strain response history at gage No.4

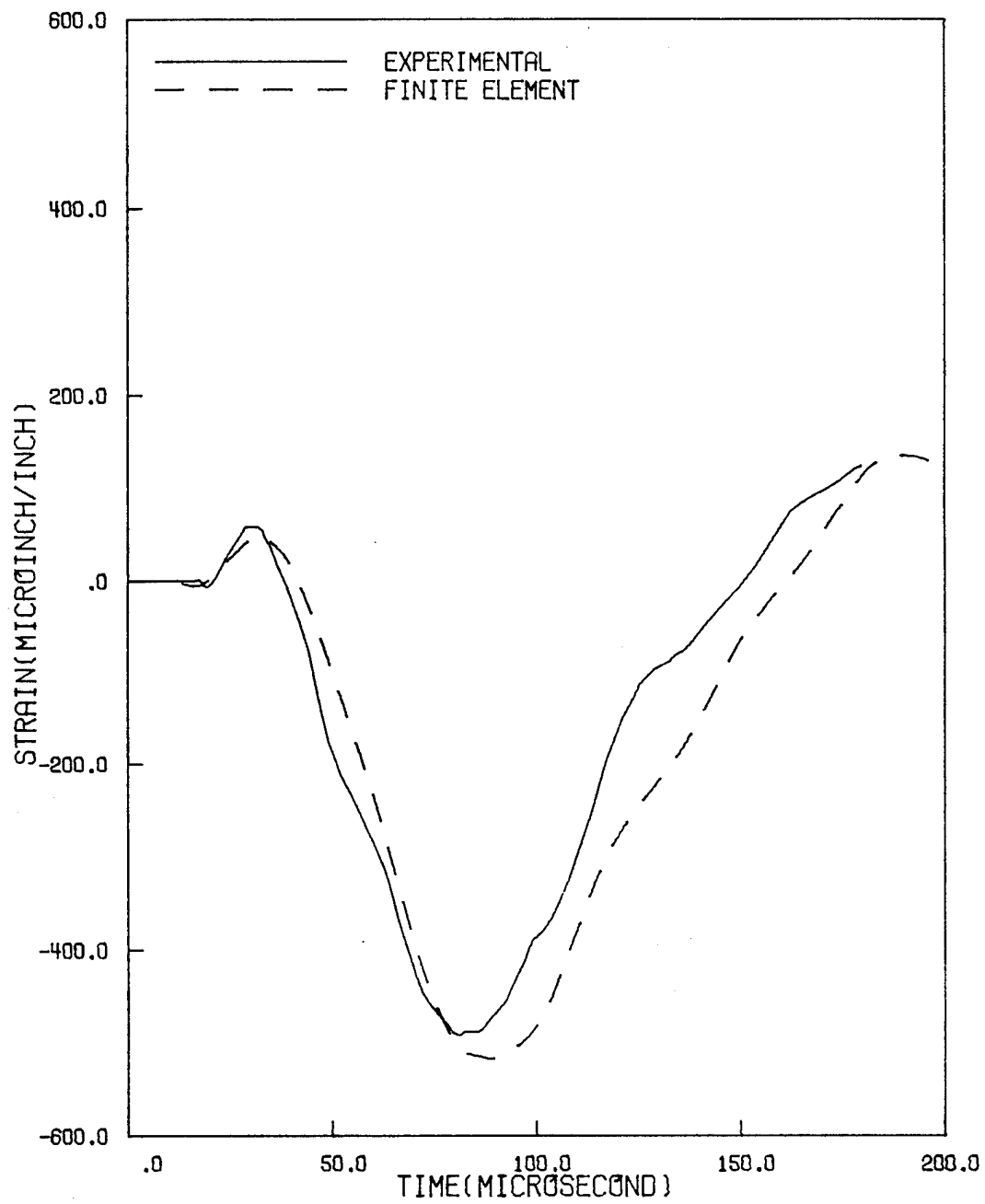


Figure 4.16 Strain response history at gage No.5

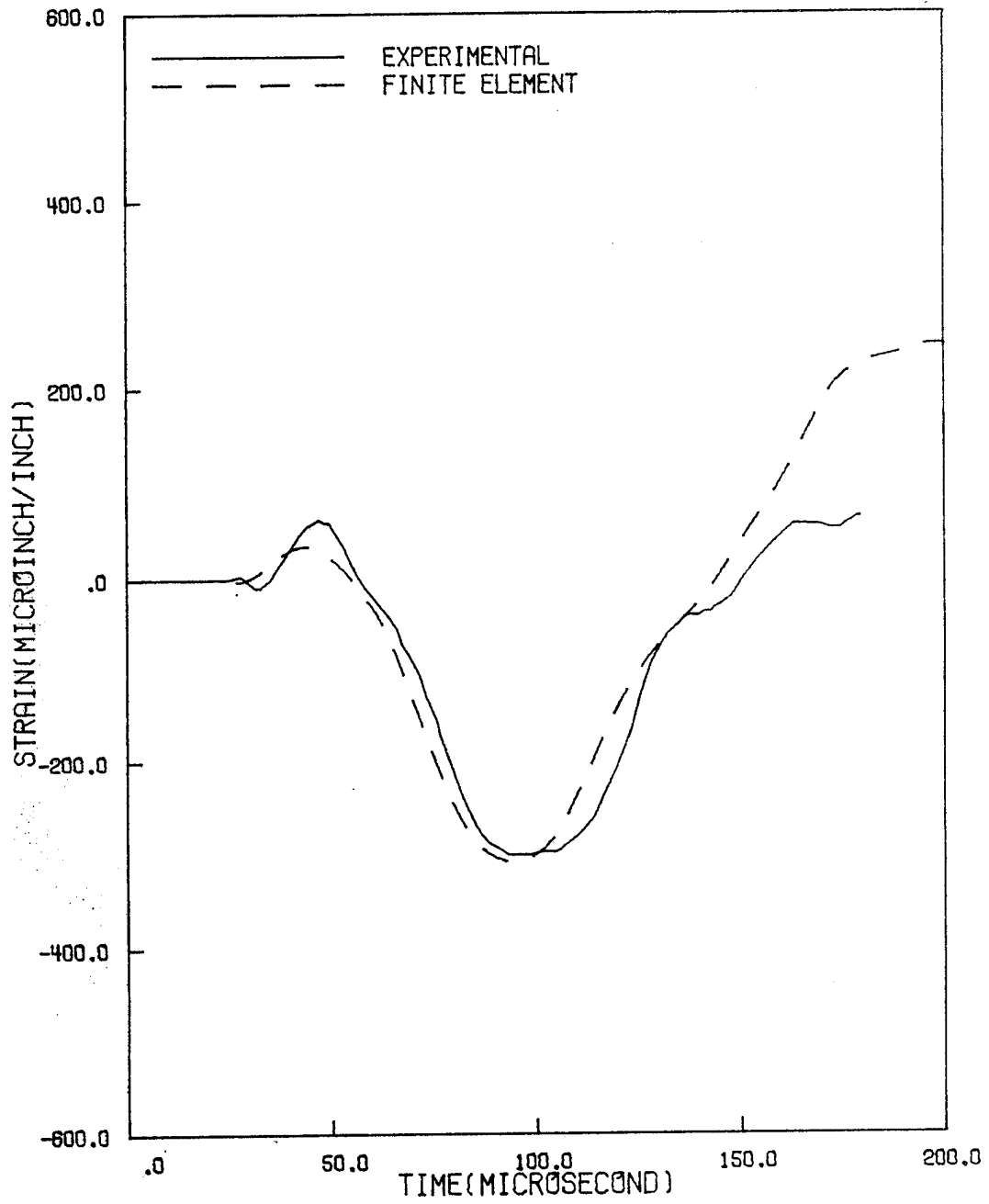


Figure 4.17 Strain response history at gage No.6

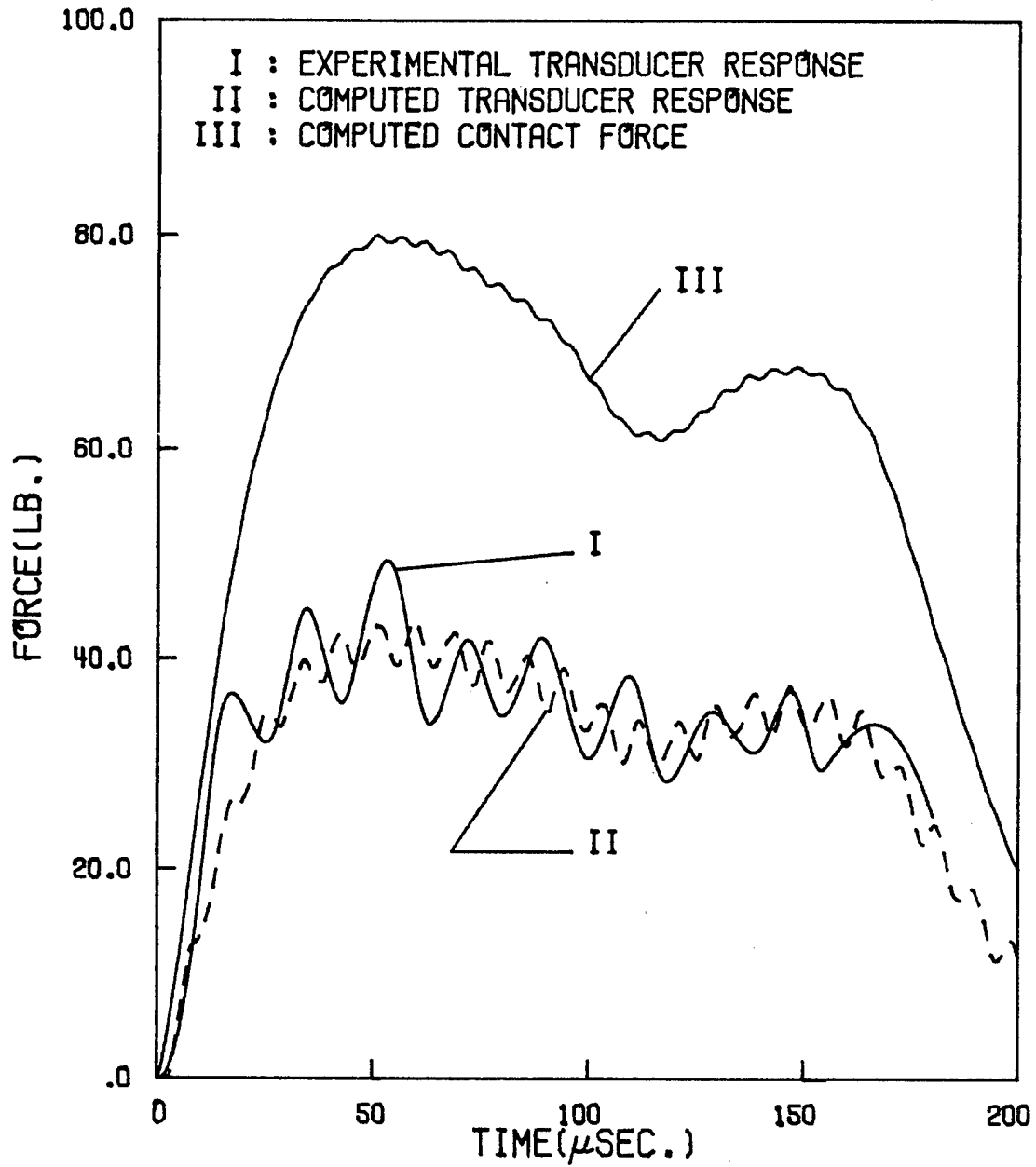


Figure 4.18 Transducer response and contact force histories from experimental and finite element results

least 20 microseconds. This might explain the lower frequency of ringing in the output voltage from the transducer.

The total duration of contact for this impact test is about 800 microseconds, and multiple contact is also observed from the test data. Figure 4.19 shows the experimental transducer responses and the computed transducer responses up to 800 microseconds. Although these two results do not match very well after the end of the first contact, it is evident that the finite element analysis does predict the multiple contact phenomenon, and the calculated total duration of contact is also approximately the same as the test result.

Figure 4.20 presents a number of deformed configurations of the laminated plate after impact. It is seen that at the point of impact, there is a strong discontinuity in slope of the transverse displacement indicating the presence of a significant transverse shear deformation.

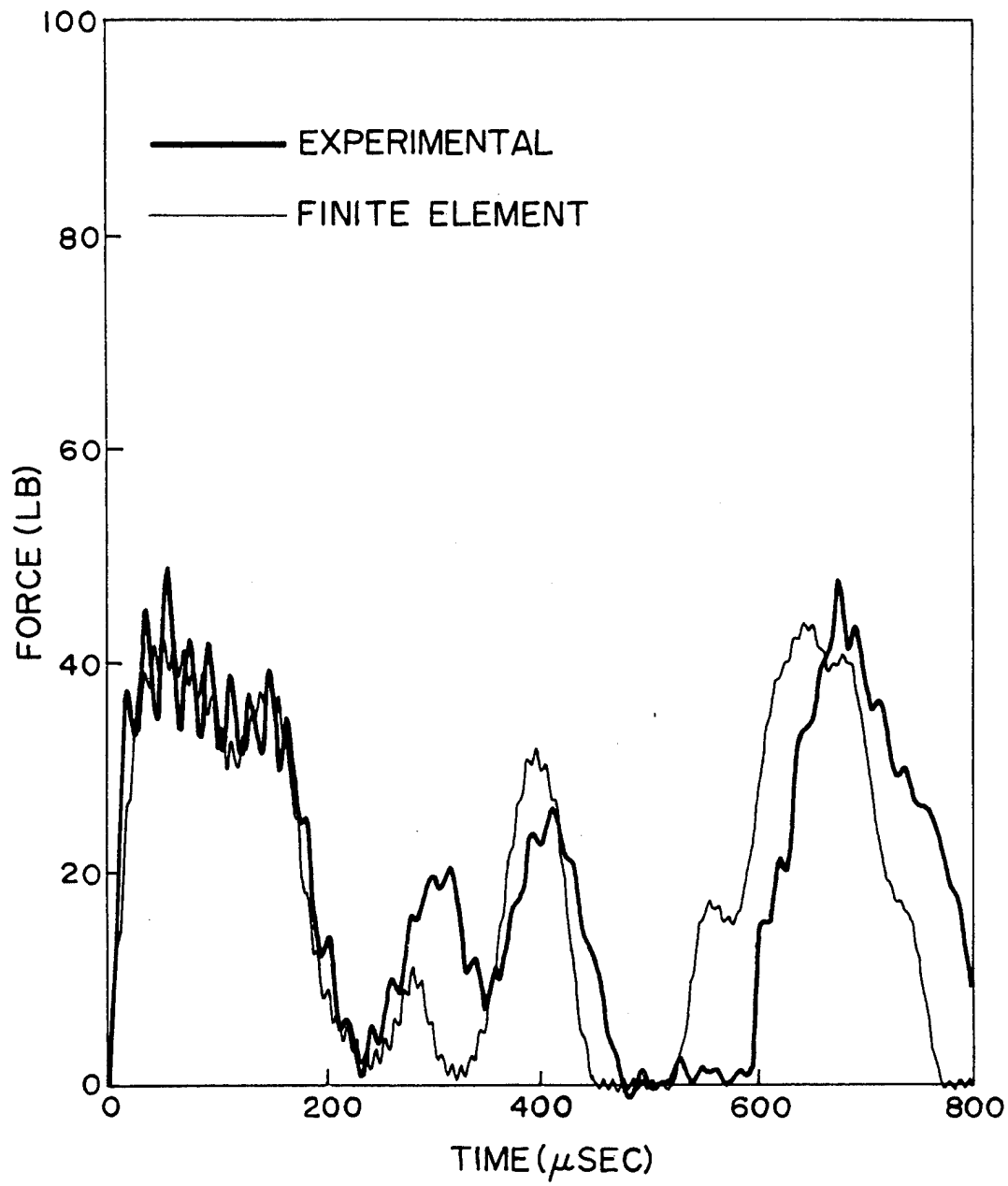


Figure 4.19 Transducer response histories from experimental and finite element results up to 800 microseconds

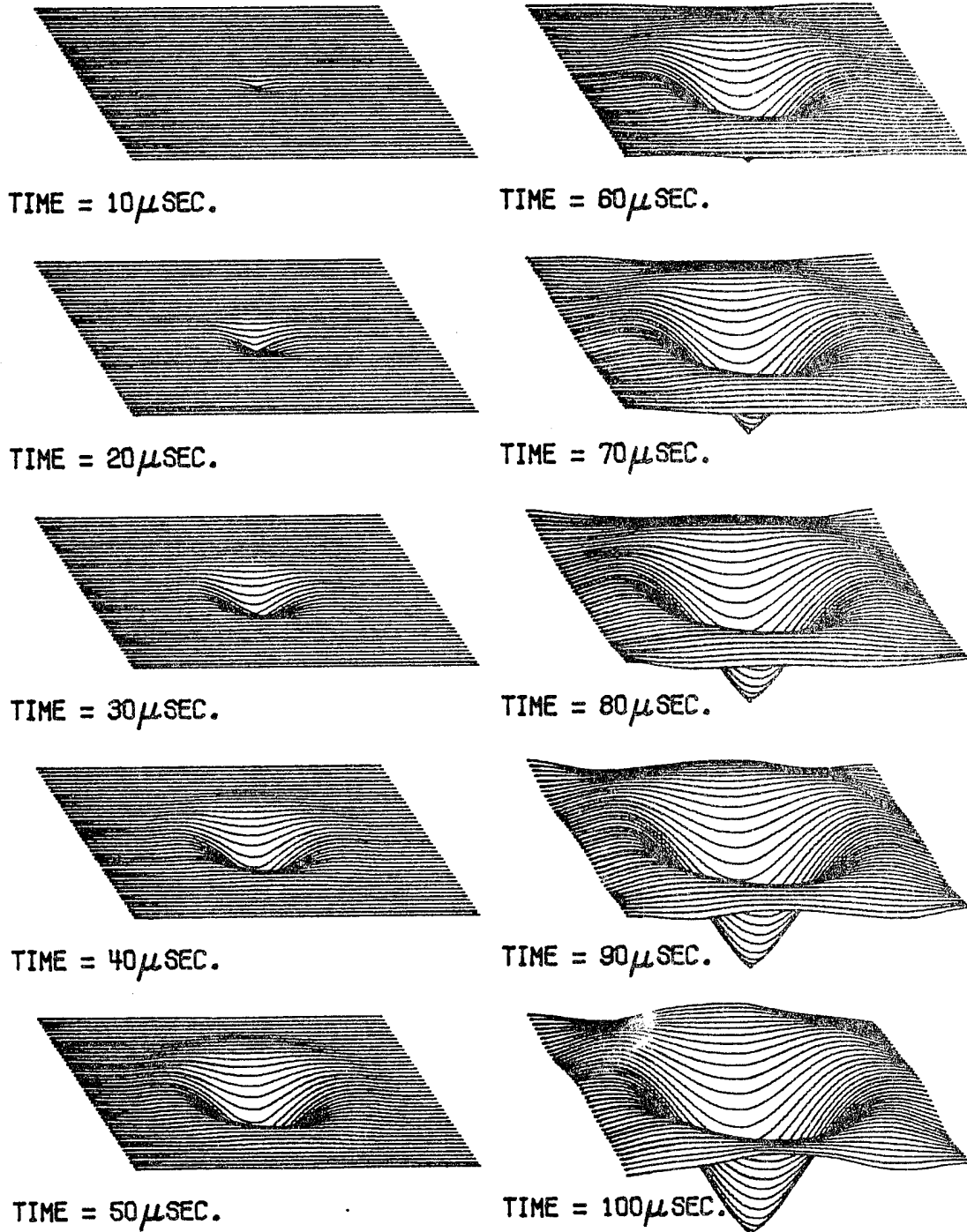


Figure 4.20 Deformed configurations of laminated plate after impact

## CHAPTER 5

### SUMMARY AND CONCLUSION

The laminate theory developed by Whitney and Pagano was employed for studies of harmonic wave and propagation of wave front in a  $[0^\circ/45^\circ/0^\circ/-45^\circ/0^\circ]_{2s}$  graphite/epoxy laminate. The dispersion properties of flexural waves were investigated. The wave front surface was constructed using ray theory. It was shown that due to the anisotropic properties of composite laminate, the transient wave would propagate with different velocities in different directions. The growth and decay of the wave front strength were also discussed.

The contact laws between 0.5 inch and 0.75 inch spherical steel indenters and the graphite/epoxy laminate were determined experimentally by means of a statical indentation test. Loading, unloading and reloading curves were fitted into power equations. Linear relation was found between the permanent indentation and the maximum indentation at unloading, which is seen to be independent of the size of indenters. This relation was then used to determine the coefficient of the unloading law. It was demonstrated that there was no need to perform reloading experiments once the loading and unloading laws were established. Test results

showed loading and reloading curves followed the power laws with power indices of 1.5 very well, while the power indices for unloading curves varied from 1.5 to 2.5.

The statically determined contact laws were incorporated into an existing 9-node isoparametric plate finite element program to study the dynamic response of a graphite/epoxy laminated plate subjected to impact of a hard object. An impact experiment was conducted to verify the validity of statical contact laws in the dynamical impact analysis. It was shown that the strain responses predicted using the finite element method agreed with the test results very well. The contact force history of the impact test was measured by an impact-force transducer, which was also seen to match the finite element result in magnitude as well as contact duration.

The indentation tests have been used ever since the beginning of the century to determine the static and dynamic hardnesses of metals in terms of the applied loading, the size of the indenter, and the chordal diameter of the permanent indentation [33]. If similar systematic indentation tests are performed on the laminated composite materials, then the relations between contact coefficients and the sizes of the indenters could be determined more rigorously, and the usefulness of the contact laws could be further extended.

As the verification of the contact laws has been limited to low velocity impacts in this study, their accuracy under high velocity impact conditions is not clear. Besides the contact behavior which may be significantly different from the static one, the damage induced by waves could be quite extensive which needs to be included in the analysis. While the present study tried to establish experimentally contact laws which can be used in the analysis of low velocity impact, the damage of laminate due to impact loading has not been discussed. It is apparent that more work needs to be done so that the failure mechanism in laminated composites due to impact can be better understood. Stress waves propagating in thickness direction, which may be responsible for the delamination of laminates, is one of the important subjects that should be investigated. Strength and fatigue life degradations of laminates after impact, which have been examined briefly by Wang [15], also need more extensive study.

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APPENDIX  
COMPUTER PROGRAM AND USER INSTRUCTIONS

The computer program used in this research was written following the program by Professor R. L. Taylor [34] with some necessary modification in order to solve the impact problems of laminated plates. A brief instruction of the input data for solving the impact problem specified in Chapter 4 of this report is given in this appendix. The detailed descriptions of data input as well as the macro instructions for solving various types of problems can be found in [34]. The listing of input is shown at the end of this appendix, followed by the listing of program.

I. Title and control information:

1. Title card-Format(20A4)

<u>Columns</u>	<u>Description</u>
----------------	--------------------

1-4	Must contain FECM
-----	-------------------

5-80	Alphanumeric information to be printed with output as page header.
------	---

2. Control information card-Format(6I5)

<u>Columns</u>	<u>Description</u>
----------------	--------------------

1-5	Number of nodes (NUMNP)
-----	-------------------------

6-10	Number of elements (NUMEL)
------	----------------------------

11-15	Number of layers (LAYER)
16-20	Spatial dimension (NDM)
21-25	Number of unknowns per node (NDF)
26-30	Number of nodes per element (NEN)

## II. Mesh and initial information:

The input of each segment in this part of data is controlled by the alphanumeric value of macros, which must be followed immediately by the appropriate data. Except for the END card which must be the last card of this part, the data segments can be in any order. Each segment is terminated with blank card(s). The meaning of each macro is given by the following:

<u>Macro</u>	<u>Data to be input</u>
COOR	Coordinate data
ELEM	Element data
BOUN	Boundary condition data
MATE	Material data
ROD	Initial condition of the projectile
EXPE	Experimental indentation laws data
END	Must be the last card of this part, terminates mesh and initial information input.

### 1. Coordinate data-Format(2I5,2F10.0)

<u>Columns</u>	<u>Description</u>
1-5	Nodal number
6-10	Generation increment

11-20 X-coordinate

21-30 Y-coordinate

2. Element data-Format(11I5)

Columns Description

1-5 Element number

6-10 Node 1 number

11-15 Node 2 number

etc.

46-50 Node 9 number

51-55 Generation increment

3. Boundary condition data-Format(7I5)

Columns Description

1-5 Node number

6-10 Generation increment

11-15 DOF 1 boundary code

16-20 DOF 2 boundary code

21-25 DOF 3 boundary code

26-30 DOF 4 boundary code

31-35 DOF 5 boundary code

4. Initial condition of the projectile-Format(2I5,F10.0)

Columns Description

1-5 The node at which the projectile hits

6-10 DOF corresponding to the direction of impact

11-20 Initial impact velocity

## 5. Experimental indentation laws data-Format(4F10.0)

Columns Description

1-10	Contact coefficient $k$
11-20	Critical indentation $\alpha_p$
21-30	Constant $s_p$ of Equation 3-9
31-40	Power index $q$ of the unloading law

## 6. Material data

Card 1-format(3I5,F10.0)

Columns Description

1-5	Order of Gauss quadrature for the numerical integration of the bending energy
6-10	Order of Gauss quadrature for the numerical integration of the transverse shear energy
11-15	Order of Gauss quadrature for strain outputs at Gauss points if $>0$ at nodal points if $<0$
16-25	Total thickness of the laminate

Card 2-Format(7F10.0)

Columns Description

1-10	Mass density
11-20	Poisson's ratio $\nu_{12}$
21-30	Longitudinal Young's modulus $E_1$
31-40	Transverse Young's modulus $E_2$
41-50	Shear modulus $G_{12}$
11-20	Shear modulus $G_{13}$
11-20	Shear modulus $G_{23}$

Card 3,4,... Format(I5,F5.0,F10.0)

Columns Description

1-5      Layer number  
 6-10     Fiber angle  
 11-20    Thickness of the layer

III. Macro instructions:

The first instruction must be a card with MACR in columns 1 to 4. The macro instructions needed to solve the problem specified in Chapter 4 of this report are shown in the listing of input. Cards must be input in the precise order. The following is the explanation of each macro:

<u>Columns</u> 1-4	<u>Columns</u> 5-10	<u>Columns</u> 11-15	<u>Description</u>
LMAS			Lumped mass formulation
DT		V	Set time increment to value V
LOOP		N	Execute N times the instructions between this macro and macro NEXT
TIME			Advance time by DT value
RODP		N	Integration of the equations of motion using the finite difference method. Contact force, indentation and element strain will be stored stored every N steps in loop
DISP		N	Nodal displacements will be stored every N steps in loop
NEXT			End of loop instructions



- 37-48 Bending strain  $\kappa_{xy}$
- 49-60 Transverse shearing strain  $\gamma_{yz}$
- 49-60 Transverse shearing strain  $\gamma_{xz}$

Tape9: Contact force, indentation and the response of the  
projectile - Format(6E12.4)

The following information is saved on tape9 at each  
requested output time step:

Columns Data saved

- 1-12 Contact force
- 13-24 Indentation
- 25-36 'Transducer' response (see Chapter 4)
- 37-48 Displacement of the projectile at the impacted end
- 37-48 Velocity of the projectile at the impacted end
- 37-48 Acceleration of the projectile at the impacted end

## LISTING OF INPUT DATA

FECM \*\*LOW VELOCITY IMPACT OF LAMINATED PLATE\*\*  
 609 140 20 2 5 9

COORD			
1	1	0.0	0.0000
7	1	1.5	0.0000
23	1	4.5	0.0000
29	0	6.0	0.0000
30	1	0.0	0.2500
36	1	1.5	0.2500
52	1	4.5	0.2500
58	0	6.0	0.2500
59	1	0.0	0.5000
65	1	1.5	0.5000
81	1	4.5	0.5000
87	0	6.0	0.5000
88	1	0.0	0.6875
94	1	1.5	0.6875
110	1	4.5	0.6875
116	0	6.0	0.6875
117	1	0.0	0.8750
123	1	1.5	0.8750
139	1	4.5	0.8750
145	0	6.0	0.8750
146	1	0.0	1.0625
152	1	1.5	1.0625
168	1	4.5	1.0625
174	0	6.0	1.0625
175	1	0.0	1.2500
181	1	1.5	1.2500
197	1	4.5	1.2500
203	0	6.0	1.2500
204	1	0.0	1.4375
210	1	1.5	1.4375
226	1	4.5	1.4375
232	0	6.0	1.4375
233	1	0.0	1.6250
239	1	1.5	1.6250
255	1	4.5	1.6250
261	0	6.0	1.6250
262	1	0.0	1.8125
268	1	1.5	1.8125
284	1	4.5	1.8125
290	0	6.0	1.8125
291	1	0.0	2.0000
297	1	1.5	2.0000
313	1	4.5	2.0000
319	0	6.0	2.0000
320	1	0.0	2.1875
326	1	1.5	2.1875
342	1	4.5	2.1875
348	0	6.0	2.1875
349	1	0.0	2.3750
355	1	1.5	2.3750
371	1	4.5	2.3750
377	0	6.0	2.3750
378	1	0.0	2.5625
384	1	1.5	2.5625
400	1	4.5	2.5625
406	0	6.0	2.5625
407	1	0.0	2.7500
413	1	1.5	2.7500
429	1	4.5	2.7500
435	0	6.0	2.7500
436	1	0.0	2.9375
442	1	1.5	2.9375

458	1	4.5	2.9375
464	0	6.0	2.9375
465	1	0.0	3.1250
471	1	1.5	3.1250
487	1	4.5	3.1250
493	0	6.0	3.1250
494	1	0.0	3.3125
500	1	1.5	3.3125
516	1	4.5	3.3125
522	0	6.0	3.3125
523	1	0.0	3.5000
529	1	1.5	3.5000
545	1	4.5	3.5000
551	0	6.0	3.5000
552	1	0.0	3.7500
558	1	1.5	3.7500
574	1	4.5	3.7500
580	0	6.0	3.7500
581	1	0.0	4.0000
587	1	1.5	4.0000
603	1	4.5	4.0000
609	0	6.0	4.0000

## ELEM

1	1	3	61	59	2	32	60	30	31	2
15	59	61	119	117	60	90	118	88	89	2
29	117	119	177	175	118	148	176	146	147	2
43	175	177	235	233	176	206	234	204	205	2
57	233	235	293	291	234	264	292	262	263	2
71	291	293	351	349	292	322	350	320	321	2
85	349	351	409	407	350	380	408	378	379	2
99	407	409	467	465	408	438	466	436	437	2
113	465	467	525	523	466	496	524	494	495	2
127	523	525	583	581	524	554	582	552	553	2

## BOUN

1	1	-1	-1	0	0	0
609	0	1	1	0	0	0

## ROD

305	3	115.0
-----	---	-------

## EXPE

1912000.	0.0006564	0.094	2.0
----------	-----------	-------	-----

## MATE

3	3	-3	.106					
0.000148		0.3	17500000.	1150000.	800000.	800000.	800000.	
1	0.	0.0053						
2	45.	0.0053						
3	0.	0.0053						
4	-45.	0.0053						
5	0.	0.0053						
6	0.	0.0053						
7	45.	0.0053						
8	0.	0.0053						
9	-45.	0.0053						
10	0.	0.0053						
11	0.	0.0053						
12	-45.	0.0053						
13	0.	0.0053						
14	45.	0.0053						
15	0.	0.0053						
16	0.	0.0053						
17	-45.	0.0053						
18	0.	0.0053						
19	45.	0.0053						
20	0.	0.0053						

END

MACR	
LMAS	
DT	.2E-6
LOOP	10
TIME	
RODP	5
DISP	5
NEXT	
END	
STOP	

## LISTING OF PROGRAM

```

PROGRAM MAIN(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE2,TAPE3,
1          TAPE8,TAPE9)
C**** MAIN PROGRAM
LOGICAL PCOMP
COMMON /PRSIZE/ MAX
COMMON /CTDATA/ O,HEAD(20),NUMNP,NUMEL,LAYER,NEQ,IPR
COMMON /LABELS/ PDIS(6),A(6),BC(2),DI(6),CD(3),FD(3)
COMMON /LODATA/ NDF,NDM,NEN,NST,NKM
COMMON /PARATS/ NPAR(14),NEND
DIMENSION TITL(20),WD(3)
COMMON G(39000)
DIMENSION M(39000)
EQUIVALENCE (G(1),M(1))
MAX=39000
WD(1)=4HFECM
WD(2)=4HMACR
WD(3)=4HSTOP
999 READ(5,1000) TITL
IF(PCOMP(TITL(1),WD(1))) GO TO 100
IF(PCOMP(TITL(1),WD(2))) GO TO 200
IF(PCOMP(TITL(1),WD(3))) STOP
GO TO 999
100 DO 101 I=1,20
101 HEAD(I)=TITL(I)
READ(5,1001) NUMNP,NUMEL,LAYER,NDM,NDF,NEN
WRITE(6,2000) HEAD,NUMNP,NUMEL,LAYER,NDM,NDF,NEN
PDIS(2)=A(NDM)
NST=NEN*NDF
DO 110 I=1,14
110 NPAR(I)=1
NPAR(1)=1
NPAR(2)=NPAR(1)+3*NST*IPR
NPAR(3)=NPAR(2)+NDM*NEN*IPR
NPAR(4)=NPAR(3)+NST
NPAR(5)=NPAR(4)+NST*IPR
NPAR(6)=NPAR(5)+NEN*NUMEL
NPAR(7)=NPAR(6)+NDF*NUMNP
NPAR(8)=NPAR(7)+NDM*NUMNP*IPR
NPAR(9)=NPAR(8)+NDF*NUMNP*IPR
NPAR(10)=NPAR(9)+NDF*NUMNP
CALL SETMEM(NPAR(9))
CALL PZERO(G(1),NPAR(9))
CALL PMESH(M(NPAR(3)),G(NPAR(2)),M(NPAR(5)),M(NPAR(6)),
1 G(NPAR(7)),G(NPAR(8)),M(NPAR(9)),NDF,NDM,NEN,NKM)
NPAR(10)=NPAR(9)+NEQ
NPAR(11)=NPAR(10)+NDF*NUMNP*IPR
NEND=NPAR(11)+NEQ*IPR
NE=NEND
CALL SETMEM(NE)
CALL PZERO(G(NPAR(10)),NE-NPAR(10))
GO TO 999
200 CALL PMACR(G(NPAR(1)),G(NPAR(2)),M(NPAR(3)),G(NPAR(4)),
1 M(NPAR(5)),M(NPAR(6)),G(NPAR(7)),G(NPAR(8)),M(NPAR(9)),
2 G(NPAR(10)),G(NPAR(11)),G(NE),NDF,NDM,NEN,NST)
CALL PZERO(G,MAX)
GO TO 999
1000 FORMAT(20A4)
1001 FORMAT(16I5)
2000 FORMAT(1H1,20A4//
1 5X,=CONTROL INFORMATION S//
2 10X,35HNUMBER OF NODAL POINTS =,I6/
3 10X,35HNUMBER OF ELEMENTS =,I6/
4 10X,35HNUMBER OF MATERIAL LAYERS =,I6/
5 10X,35HDIMENSION OF COORDINATE SPACE =,I6/
6 10X,35HDEGREES OF FREEDOM FOR EACH NODE =,I6/
7 10X,35HNODES PER ELEMENT (MAXIMUM) =,I6/
END

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C	BLOCK DATA	BLOC 1
C****	BLOCK DATA	BLOC 2
	COMMON /CTDATA/ O,HEAD(20),NUMNP,NUMEL,LAYER,NEQ,IPR	BLOC 3
	COMMON /LABELS/ PDIS(6),A(6),BC(2),DI(6),CD(3),FD(3)	BLOC 4
	DATA O/1H1/,IPR/1/	BLOC 5
	DATA PDIS/4H(I10,2H, ,4HF13.,4H4, ,4H6E13,4H.4) /	BLOC 6
	DATA A/2H,1,2H,2,2H,3,2H,4,2H,5,2H,6/	BLOC 7
	DATA BC/4H B.C,2H. /	BLOC 8
	DATA DI/4H DIS,2HPL,4H VEL,2HOC,4H ACC,2HEL/	BLOC 9
	DATA CD/4H COO,4HRDIN,4HATES/	BLOC 10
	DATA FD/4H FOR,4HCE/D,4HISPL/	BLOC 11
	END	BLOC 12
C	SUBROUTINE PMACR(UL,XL,LD,P,IX,ID,X,F,JDIAG,DR,B,CT,NDF,NDM, NEN,NST)	PMAC 1
C****	MACRO INSTRUCTION ROUTINE	PMAC 2
	LOGICAL PCOMP	PMAC 3
	COMMON G(1)	PMAC 4
	DIMENSION M(1)	PMAC 5
	EQUIVALENCE (G(1),M(1))	PMAC 6
	COMMON /CTDATA/ O,HEAD(20),NUMNP,NUMEL,LAYER,NEQ,IPR	PMAC 7
	COMMON /PROLOD/ PROP	PMAC 8
	COMMON /TMDATA/ TIME,DT,DDT,FORCE,ALPHA	PMAC 9
	COMMON /ISWIDX/ ISW	PMAC 10
	COMMON /PARATS/ NPAR(14),NEND	PMAC 11
	COMMON /RODATA/ UR,IQ,NDS	PMAC 12
	DIMENSION UL(1),XL(1),LD(1),P(1),IX(1),ID(1),X(1),F(1), ^  JDIAG(1),DR(1),B(1)	PMAC 13
	DIMENSION WD(9),CT(4,16),LVE(9)	PMAC 14
	DATA WD/4HLOOP,4HNEXT,4HDT ,4HPROP,4HLMAS,4HRODP, 1  4HSTRE,4HDISP,4HCEC/	PMAC 15
	DATA NWD/9/,ENDM/4HEND /	PMAC 16
C....	INITIALIZATION	PMAC 17
	DT = 0.0	PMAC 18
	PROP = 1.0	PMAC 19
	TIME = 0.0	PMAC 20
	NNEQ = NDF*NUMNP	PMAC 21
	NPLD = 0	PMAC 22
	FORCE= 0.	PMAC 23
	ALPHA= 0.	PMAC 24
	WRITE(6,2001) O,HEAD	PMAC 25
	LL = 1	PMAC 26
	LMAX = 16	PMAC 27
	CALL SETMEM(NEND+LMAX*4*IPR)	PMAC 28
	CT(1,1) = WD(1)	PMAC 29
	CT(3,1) = 1.0	PMAC 30
100	LL = LL + 1	PMAC 31
	IF(LL.LT.LMAX) GO TO 110	PMAC 32
	LMAX = LMAX + 16	PMAC 33
	CALL SETMEM(NEND+LMAX*4*IPR)	PMAC 34
110	READ(5,1000) (CT(J,LL),J=1,4)	PMAC 35
	WRITE(6,2000) (CT(J,LL),J=1,4)	PMAC 36
	IF(.NOT.PCOMP(CT(1,LL),ENDM)) GO TO 100	PMAC 37
	CT(1,LL) = WD(2)	PMAC 38
	NEND = NEND +LMAX*4*IPR	PMAC 39
	LX = LL - 1	PMAC 40
	DO 230 L=1,LX	PMAC 41
	IF(.NOT.PCOMP(CT(1,L),WD(1))) GO TO 230	PMAC 42
	J = 1	PMAC 43
	K = L + 1	PMAC 44
	DO 210 I=K,LL	PMAC 45
	IF(PCOMP(CT(1,I),WD(1))) J = J + 1	PMAC 46
	IF(J .GT. 9) GO TO 401	PMAC 47
	IF(PCOMP(CT(1,I),WD(2))) J = J - 1	PMAC 48
210	IF(J.EQ.0) GO TO 220	PMAC 49
	GO TO 400	PMAC 50
220	CT(4,I) = L	PMAC 51
	CT(4,L) = I	PMAC 52
230	CONTINUE	PMAC 53
		PMAC 54
		PMAC 55
		PMAC 56

J = 0	PMAC 57
DO 240 L=1,LL	PMAC 58
IF(PCOMP(CT(1,L),WD(1))) J = J + 1	PMAC 59
240 IF(PCOMP(CT(1,L),WD(2))) J = J - 1	PMAC 60
IF(J.NE.0) GO TO 400	PMAC 61
LV = 0	PMAC 62
L = 1	PMAC 63
299 DO 300 J=1,NWD	PMAC 64
300 IF(PCOMP(CT(1,L),WD(J))) GO TO 310	PMAC 65
GO TO 330	PMAC 66
310 I = L - 1	PMAC 67
GO TO (1,2,3,4,5,6,7,8,9),J	PMAC 68
C.... SET LOOP START INDICATORS	PMAC 69
1 LV = LV + 1	PMAC 70
LX = CT(4,L)	PMAC 71
LVE(LV) = LX	PMAC 72
CT(3,LX) = 1.	PMAC 73
GO TO 330	PMAC 74
C.... LOOP TERMINATOR CONTROL	PMAC 75
2 N = CT(4,L)	PMAC 76
CT(3,L) = CT(3,L) + 1.0	PMAC 77
IF(CT(3,L).GT.CT(3,N)) LV = LV - 1	PMAC 78
IF(CT(3,L).LE.CT(3,N)) L = N	PMAC 79
GO TO 330	PMAC 80
C.... SET TIME INCREMENT	PMAC 81
3 DT = CT(3,L)	PMAC 82
DDT= DT*DT	PMAC 83
GO TO 330	PMAC 84
C.... INPUT PROPORTIONAL LOAD TABLE	PMAC 85
4 NPLD = CT(3,L)	PMAC 86
PROP = PROPLD(0.,NPLD)	PMAC 87
GO TO 330	PMAC 88
C.... FORM LUMPED MASS MATRIX	PMAC 89
5 ISW=3	PMAC 90
CALL KMLIB	PMAC 91
GO TO 330	PMAC 92
C.... IMPACT	PMAC 93
6 NDS=CT(3,L)	PMAC 94
IF(NDS.EQ.0) NDS=1	PMAC 95
CALL RODIPCT	PMAC 96
GO TO 330	PMAC 97
C.... PRINT STRESS/STRAIN VALUE	PMAC 98
7 ISW=4	PMAC 99
LX = LVE(LV)	PMAC100
IF(AMOD(CT(3,LX),AMAX1(CT(3,L),1.))) 330,71,330	PMAC101
71 CALL FSTREA(UL,XL,LD,P,IX,ID,X,F,JDIAG,DR,B,NDF,NDM,NEN,NST,NNEQ)	PMAC102
GO TO 330	PMAC103
C.... PRINT DISPLACEMENTS	PMAC104
8 LX = LVE(LV)	PMAC105
IF(AMOD(CT(3,LX),AMAX1(CT(3,L),1.))) 330,81,330	PMAC106
81 CALL PRDIS(UL, ID, X, B, F, DR, NDM, NDF)	PMAC107
GO TO 330	PMAC108
C.... CHECK	PMAC109
9 WRITE(6,5001) NEND,JDIAG(NEQ)	PMAC110
RETURN	PMAC111
330 L=L+1	PMAC112
IF(L.GT.LL) RETURN	PMAC113
GO TO 299	PMAC114
C.... PRINT ERROR FORMATS	PMAC115
400 WRITE(6,4000)	PMAC116
RETURN	PMAC117
401 WRITE(6,4001)	PMAC118
RETURN	PMAC119
C.... INPUT/OUTPUT FORMATS	PMAC120
1000 FORMAT(A4,1X,A4,1X,2F5.0)	PMAC121
2000 FORMAT(10X,A4,1X,A4,1X,2G15.5)	PMAC122
2001 FORMAT(A1,20A4//,5X,18HMACRO INSTRUCTIONS//5X,15HMACRO STATEMENT	PMAC123
^ ,5X,10HVARIABLE 1,5X,10HVARIABLE 2)	PMAC124
4000 FORMAT(5X,46H**PMACR ERROR 01** UNBALANCED LOOP/NEXT MACROS )	PMAC125
4001 FORMAT(5X,45H**PMACR ERROE 02** LOOPS NESTED DEEPER THAN 8)	PMAC126

5001	FORMAT(1H1,////5X,32HCHECK MESH DATA AND MEMORY SPACE//	PMAC127
^	10X,12H            NEND =,I10//10X,12HJDIAG(NEQ) =,I10)	PMAC128
	END	PMAC129
C		
	SUBROUTINE PZERO(U,NN)	PZER 1
C****	ZERO REAL ARRAY	PZER 2
	DIMENSION U(NN)	PZER 3
	DO 100 N=1,NN	PZER 4
100	U(N) = 0.0	PZER 5
	RETURN	PZER 6
	END	PZER 7
C		
	SUBROUTINE SETMEM(J)	SETM 1
C****	MONITOR AVAILABLE MEMORY IN BLANK COMMON	SETM 2
	COMMON /PRSIZE/ MAX	SETM 3
	K = J	SETM 4
	IF(K.LE.MAX) RETURN	SETM 5
	WRITE(6,1000) K,MAX	SETM 6
	STOP	SETM 7
1000	FORMAT(5X,49H**SETMEM ERROR 01** INSUFFICIENT STORAGE IN BLANK,	SETM 8
^	SH COMMON //17X,11HREQUIRED =,I8/17X,11HAVAILABLE =,I8)	SETM 9
	END	SETM 10
C		
	LOGICAL FUNCTION PCOMP(A,B)	PCOM 1
C****	LOGICAL COMPARISON	PCOM 2
	IF(A-B) 10,20,10	PCOM 3
10	PCOMP = .FALSE.	PCOM 4
	RETURN	PCOM 5
20	PCOMP = .TRUE.	PCOM 6
	RETURN	PCOM 7
	END	PCOM 8
C		
	SUBROUTINE ACTCOL(A,B,JDIAG,NEQ,AFAC,BACK,ISS)	ACTC 1
C****	ACTIVE COLUMN PROFILE SYMMETRIC EQUATION SOLVER	ACTC 2
	LOGICAL AFAC,BACK,FLAG	ACTC 3
	DIMENSION A(1),B(1),JDIAG(1)	ACTC 4
C....	FACTOR A TO UT*D*U, REDUCE B	ACTC 5
	FLAG=.FALSE.	ACTC 6
	JR = 0	ACTC 7
	DO 600 J=1,NEQ	ACTC 8
	JD = JDIAG(J)	ACTC 9
	JH = JD - JR	ACTC 10
	IS = J - JH + 2	ACTC 11
	IF(JH-2) 600,300,100	ACTC 12
100	IF(.NOT.AFAC) GO TO 500	ACTC 13
	IE = J - 1	ACTC 14
	K = JR + 2	ACTC 15
	ID = JDIAG(IS-1)	ACTC 16
C....	REDUCE ALL EQUATIONS EXCEPT DIAGONAL	ACTC 17
	DO 200 I=IS,IE	ACTC 18
	IR = ID	ACTC 19
	ID = JDIAG(I)	ACTC 20
	IH = MINO(ID-IR-1,I-IS+1)	ACTC 21
	IF(IH.GT.0) A(K)=A(K)-DOT(A(K-IH),A(ID-IH),IH)	ACTC 22
200	K = K + 1	ACTC 23
C....	REDUCE DIGONAL TERM	ACTC 24
300	IF(.NOT.AFAC) GO TO 500	ACTC 25
	IR = JR + 1	ACTC 26
	IE = JD - 1	ACTC 27
	K = J - JD	ACTC 28
	DO 400 I=IR,IE	ACTC 29
	ID = JDIAG(K+I)	ACTC 30
	IF(A(ID)) 301,400,301	ACTC 31
301	D = A(I)	ACTC 32
	A(I) = A(I)/A(ID)	ACTC 33
	A(JD) = A(JD) - D*A(I)	ACTC 34
400	CONTINUE	ACTC 35
	IF(A(JD))450,450,500	ACTC 36
450	IF(ISS.NE.0) GO TO 500	ACTC 37
	IF(FLAG) GO TO 465	ACTC 38

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        WRITE(6,460)
460  FORMAT(/50H**ACTCOL ERROR 01** STIFFNESS MATRIX NOT POSITIVE ,
1     8HDEFINITE)
        FLAG=.TRUE.
465  WRITE(6,466) J,A(JD)
466  FORMAT(32H NONPOSITIVE PIVOT FOR EQUATION ,I4,5X,7HPQVIT =,
^     E20.10)
C....  REDUCE RHS
500  IF(BACK) B(J) = B(J) - DOT(A(JR+1),B(IS-1),JH-1)
600  JR = JD
        IF(FLAG) STOP
        IF(.NOT.BACK) RETURN
C....  DIVIDED BY DIAGONAL PIVOTS
        DO 700 I=1,NEQ
        ID = JDIAG(I)
        IF(A(ID)) 650,700,650
650  B(I) = B(I)/A(ID)
700  CONTINUE
C....  BACK SUBSTITUTE
        J = NEQ
        JD = JDIAG(J)
800  D = B(J)
        J = J - 1
        IF(J.LE.0) RETURN
        JR = JDIAG(J)
        IF(JD-JR.LE.1) GO TO 1000
        IS = J - JD + JR + 2
        K = JR - IS + 1
        DO 900 I=IS,J
900  B(I) = B(I) - A(I+K)*D
1000 JD = JR
        GO TO 800
        END

C
SUBROUTINE ADDSTF(A,S,P,JDIAG,LD,NST,NEL,FLG)
C****  ASSEMBLE GLOBAL ARRAYS
        LOGICAL FLG
        DIMENSION A(1),S(NST,1),P(1),JDIAG(1),LD(1)
        DO 200 J=1,NEL
        K = LD(J)
        IF(K.EQ.0) GO TO 200
        IF(FLG) GO TO 50
        A(K)=A(K)+P(J)
        GO TO 200
50  L = JDIAG(K) - K
        DO 100 I=1,NEL
        M = LD(I)
        IF(M.GT.K .OR. M.EQ.0) GO TO 100
        M = L + M
        A(M)=A(M)+S(I,J)
100  CONTINUE
200  CONTINUE
        RETURN
        END

C
FUNCTION DOT(A,B,N)
C****  VECTOR DOT PRODUCT
        DIMENSION A(1),B(1)
        DOT = 0.0
        DO 100 I=1,N
100  DOT = DOT + A(I)*B(I)
        RETURN
        END

C
SUBROUTINE PLOAD(ID,F,B,NN,P)
C****  FORM LOAD VECTOR IN COMPACT FORM
        DIMENSION ID(1),F(1),B(1)
        DO 100 N=1,NN
        J=ID(N)
100  IF(J.GT.0) B(J)=F(N)*P

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ADDS 20

DOT 1  
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DOT 4  
DOT 5  
DOT 6  
DOT 7  
DOT 8

PLOA 1  
PLOA 2  
PLOA 3  
PLOA 4  
PLOA 5  
PLOA 6

RETURN	PLOA	7
END	PLOA	8
C		
FUNCTION PROPLD(T,J)	PROP	1
C**** PROPORTIONAL LOAD TABLE (ONE LOAD CARD ONLY)	PROP	2
COMMON /CTDATA/ O,HEAD(20),NUMNP,NUMEL,LAYER,NEQ,IPR	PROP	3
DIMENSION A(5)	PROP	4
IF (J.LE. 0) GO TO 200	PROP	5
C.... INPUT TABLE OF PROPORTIONAL LOADS	PROP	6
I=1	PROP	7
READ(5,1000) K,L,TMIN,TMAX,(A(KKK),KKK=1,5)	PROP	8
WRITE(6,2000) O,HEAD,I,K,L,TMIN,TMAX,(A(KKK),KKK=1,5)	PROP	9
RETURN	PROP	10
C.... COMPUTE VALUE AT TIME T	PROP	11
200 PROPLD = 0.0	PROP	12
IF(T.LT.TMIN .OR. T.GT.TMAX) RETURN	PROP	13
L = MAX0(L,1)	PROP	14
PROPLD = A(1)+A(2)*T+A(3)*(SIN(A(4)*T+A(5)))*L	PROP	15
RETURN	PROP	16
1000 FORMAT(2I5,7F10.0)	PROP	17
2000 FORMAT(A1,20A4//5X,23HPROPORTIONAL LOAD TABLE//11H NUMBER ,	PROP	18
1 43H TYPE EXP. MINIMUM TIME MAXIMUM TIME,13X,2HA1,13X,	PROP	19
2 2HA2,13X,2HA3,13X,2HA4,13X,2HA5/(3I8,7G15.5))	PROP	20
END	PROP	21
C		
SUBROUTINE PRDIS(UL, ID, X, B, F, T, NDM, NDF)	PRTD	1
C**** OUTPUT NODAL VALUES	PRTD	2
LOGICAL PCOMP	PRTD	3
COMMON /PROLOD/ PROP	PRTD	4
COMMON /CTDATA/ O,HEAD(20),NUMNP,NUMEL,LAYER,NEQ,IPR	PRTD	5
COMMON /LABELS/ PDIS(6),A(6),BC(2),DI(6),CD(3),FD(3)	PRTD	6
COMMON /TMDATA/ TIME,DT,DDT,FORCE,ALPHA	PRTD	7
DIMENSION X(NDM,1),B(1),UL(6),ID(NDF,1),F(NDF,1),T(1)	PRTD	8
DATA BL/4HBLAN/	PRTD	9
DO 102 N=1,NUMNP	PRTD	10
IF(PCOMP(X(1,N),BL)) GO TO 101	PRTD	11
DO 100 I=1,NDF	PRTD	12
UL(I) = F(I,N)*PROP	PRTD	13
K = IABS(ID(I,N))	PRTD	14
100 IF(K.GT.0) UL(I)=B(K)	PRTD	15
T(N)=UL(3)	PRTD	16
101 CONTINUE	PRTD	17
102 CONTINUE	PRTD	18
WRITE(3,2001) (T(I),I=1,NUMNP)	PRTD	19
RETURN	PRTD	20
2001 FORMAT(6E12.4)	PRTD	21
END	PRTD	22
C		
SUBROUTINE FSTREA(UL,XL,LD,P,IX,ID,X,F,JDIAG,DR,B,NDF,NDM,NEN,	FSTR	1
^ NST,NNEQ)	FSTR	2
C**** ELEMENT ROUTINE	FSTR	3
COMMON /CTDATA/ O,HEAD(20),NUMNP,NUMEL,LAYER,NEQ,IPR	FSTR	4
COMMON /ELDATA/ N,NEL,MCT	FSTR	5
COMMON /ISWIDX/ ISW	FSTR	6
COMMON /PROLOD/ PROP	FSTR	7
DIMENSION UL(NDF,1),XL(NDM,1),LD(NDF,1),P(1),IX(NEN,1),	FSTR	8
1 ID(NDF,1),X(NDM,1),F(NDF,1),JDIAG(1),DR(1),B(1),S(1)	FSTR	9
IF(ISW.EQ.5) CALL PLOAD(ID,F,DR,NNEQ,PROP)	FSTR	10
MCT=0	FSTR	11
DO 110 N=1,NUMEL	FSTR	12
CALL PFORM(UL,XL,LD,IX,ID,X,F,B,NDF,NDM,NEN,ISW)	FSTR	13
CALL ELMT01(UL,XL,IX(1,N),P,NDF,NDM,NST,ISW)	FSTR	14
IF(ISW.NE.4) CALL ADDSTF(DR,S,P,JDIAG,LD,1,NEL*NDF,.FALSE.)	FSTR	15
110 CONTINUE	FSTR	16
RETURN	FSTR	17
END	FSTR	18
C		
SUBROUTINE PFORM(UL,XL,LD,IX,ID,X,F,U,NDM,NEN,ISW)	PFOR	1
C**** FORM LOCAL ARRAYS	PFOR	2
COMMON /ELDATA/ N,NEL,MCT	PFOR	3

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COMMON /PROLOD/ PROP
DIMENSION UL(NDF,1),XL(NDM,1),LD(NDF,1),IX(NEN,1),ID(NDF,1),
^ X(NDM,1),F(NDF,1),U(1)
DO 108 I=1,NEN
  II = IX(I,N)
  IF(II .NE. 0) GO TO 105
  DO 103 J=1,NDM
103 XL(J,I) = 0.
  DO 104 J=1,NDF
    UL(J,I) = 0.
104 LD(J,I) = 0
    GO TO 108
105 IID = II*NDF - NDF
    NEL = I
    DO 106 J=1,NDM
106 XL(J,I) = X(J,II)
    DO 107 J=1,NDF
      K = IABS(ID(J,II))
      UL(J,I) = F(J,II)*PROP
      IF(K.GT.0) UL(J,I)=U(K)
      IF(ISW.EQ.6) K=IID+J
107 LD(J,I) = K
108 CONTINUE
  RETURN
END
C
SUBROUTINE ELMT01(UL,XL,IX,P,NDF,NDM,NST,ISW)
C**** LINEAR ELASTIC IN-PLANE ^ BENDING ELEMENT ROUTINE
LOGICAL TAN
COMMON /ELDATA/ N,NEL,MCT
COMMON /MTDATA/ RHO,UU12,E1,E2,G12,G13,G23,THK,WIDTH
COMMON /COMPST/ ABD(6,6),DS(2,2),QBR(3,3,25),QBS(2,2,25),
^ TH(25),ZK(25)
COMMON /DMATIX/ D(10),DB(6,6),LINT
COMMON /TMDATA/ TIME,DT,DDT,FORCE,ALPHA
COMMON /GAUSSP/ SG(16),TG(16),WG(16)
COMMON /EXTRAS/ TAN
DIMENSION UL(NDF,1),XL(NDM,1),IX(1),P(1),SHP(3,12),
1 SIGT(3),SIGB(3),SIGS(2),EPT(3),EPB(3),EPS(2)
C
DO 20 L=1,NST
20 P(L) = 0.0
C.... COMPUTE NEUTRAL STRAINS AND STRESS RESULTANTS
L = D(1)
IF(ISW.EQ.4) L=D(3)
CALL PGAUSS(L,LINT)
DO 600 L=1,LINT
C .. COMPUTE ELEMENT SHAPE FUNCTIONS
CALL SHAPE(SG(L),TG(L),XL,SHP,XSJ,NDM,NEL,IX,.FALSE.)
C .. COMPUTE STRAINS AND COORDINATES
DO 410 I=1,3
  EPT(I) = 0.0
410 EPB(I) = 0.0
DO 420 I=1,2
420 EPS(I) = 0.0
  XX = 0.0
  YY = 0.0
  DO 430 J=1,NEL
    XX = XX + SHP(3,J)*XL(1,J)
    YY = YY + SHP(3,J)*XL(2,J)
C .. IN-PLANE STRAINS
  EPT(1) = EPT(1) + SHP(1,J)*UL(1,J)
  EPT(2) = EPT(2) + SHP(2,J)*UL(2,J)
  EPT(3) = EPT(3) + SHP(1,J)*UL(2,J) + SHP(2,J)*UL(1,J)
C .. BENDING CURVATURES
  EPB(1) = EPB(1) - SHP(1,J)*UL(4,J)
  EPB(2) = EPB(2) - SHP(2,J)*UL(5,J)
  EPB(3) = EPB(3) - SHP(1,J)*UL(5,J) - SHP(2,J)*UL(4,J)
C .. SHEARING STRAINS
  EPS(1) = EPS(1) + SHP(1,J)*UL(3,J) - SHP(3,J)*UL(4,J)

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430 EPS(2) = EPS(2) + SHP(2,J)*UL(3,J) - SHP(3,J)*UL(5,J)
IF(ISW.EQ.5.AND.TAN)
^ WRITE(9,9001) N,L,(EPB(II),II=1,3),(EPS(II),I=1,2)
9001 FORMAT(2I6,5E12.4)
C .. COMPUTE STRESS RESULTANTS
DO 440 I=1,3
SIGT(I) = 0.
SIGB(I) = 0.
DO 440 J=1,3
SIGT(I) = SIGT(I) + ABD(I,J)*EPT(J) + ABD(I,J+3)*EPB(J)
440 SIGB(I) = SIGB(I) + ABD(I+3,J)*EPT(J) + ABD(I+3,J+3)*EPB(J)
DO 450 I=1,2
SIGS(I) = 0.
DO 450 J=1,2
450 SIGS(I) = SIGS(I) + DS(I,J)*EPS(J)
IF(ISW.GT.4) GO TO 620
C .. OUTPUT STRESS RESULTANTS AND STRAINS
MCT = MCT - 2
IF(MCT.GT.0) GO TO 470
WRITE(6,2001) TIME
MCT = 50
470 WRITE(6,2002) N,XX,YY,EPT,EPB,EPS,SIGT,SIGB,SIGS
GO TO 600
C.... COMPUTE INTERNAL FORCES
620 DV = XSJ*WG(L)
J1 = 1
DO 610 J=1,NEL
P(J1) = P(J1) - (SHP(1,J)*SIGT(1)+SHP(2,J)*SIGT(3))*DV
P(J1+1) = P(J1+1) - (SHP(2,J)*SIGT(2)+SHP(1,J)*SIGT(3))*DV
P(J1+2) = P(J1+2) - (SHP(1,J)*SIGS(1)+SHP(2,J)*SIGS(2))*DV
P(J1+3) = P(J1+3) + (SHP(1,J)*SIGB(1)+SHP(2,J)*SIGB(3)+SHP(3,J)
^ *SIGS(1))*DV
^ P(J1+4) = P(J1+4) + (SHP(2,J)*SIGB(2)+SHP(1,J)*SIGB(3)+SHP(3,J)
^ *SIGS(2))*DV
610 J1 = J1 + NDF
600 CONTINUE
RETURN
C
2001 FORMAT(1H1//
^ 5X,6HTIME =,E12.3//5X,33HELEMENT STRAINS/STRESS RESULTANTS//
1 8H ELEMENT,3X,7H1-COORD,3X,7H2-COORD,4X,9HXX-STRAIN,4X,
2 9HY-Y-STRAIN,4X,9HXY-STRAIN,3X,10HKXX-STRAIN,3X,
3 10HKYY-STRAIN,3X,10HKXY-STRAIN,4X,9HSX-STRAIN,4X,
4 9HSY-STRAIN/28X,8(GX,7H-STRESS)//)
2002 FORMAT(I8,2F10.4,8E13.4/28X,8E13.4)
END
C
SUBROUTINE PGAUSS(LL,LINT)
C**** GAUSSIAN POINTS AND WEIGHTS FOR TWO DIMENSIONS
COMMON /GAUSSP/ SG(16),TG(16),WG(16)
DIMENSION LR(9),LZ(9),LW(9),WR(2),GR(2),GC(2)
DATA LR/-1,1,1,-1,0,1,0,-1,0/,LZ/-1,-1,1,1,-1,0,1,0,0/
DATA LW/4*25,4*40,64/
DATA GR/0.861136311594053,0.339981043584856/
DATA GC/1.0,0.3333333333/
DATA WR/0.347854845137454,0.652145154862546/
LINT = LL*LL
L=IABS(LL)
GO TO (1,2,3,4),L
C.... 1X1 INTEGRATION
1 SG(1) = 0.
TG(1) = 0.
WG(1) = 4.
RETURN
C.... 2X2 INTEGRATION
2 G = 1./SQRT(3.)
IF(LL.LT.0) G=1.
DO 21 I=1,4
SG(I) = G*LR(I)
TG(I) = G*LZ(I)

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PGAU 23

21	WG(I) = 1.	PGAU 24
	RETURN	PGAU 25
C....	3X3 INTEGRATION	PGAU 26
3	G = SQRT(0.6)	PGAU 27
	IF(LL.LT.0) G=1.	PGAU 28
	H = 1./81.	PGAU 29
	DO 31 I=1,9	PGAU 30
	SG(I) = G*LR(I)	PGAU 31
	TG(I) = G*LZ(I)	PGAU 32
31	WG(I) = H*LW(I)	PGAU 33
	RETURN	PGAU 34
C....	4X4 INTEGRATION	PGAU 35
4	DO 41 I=1,4	PGAU 36
	I1 = 1+MOD(I+1,2)	PGAU 37
	I2 = 1	PGAU 38
	IF(I.GT.2) I2 = 2	PGAU 39
	DO 41 J=1,4	PGAU 40
	JJ = (I-1)*4+J	PGAU 41
	SG(JJ) = LR(J)*GR(I1)	PGAU 42
	IF(LL.LT.0) SG(JJ) = LR(J)*GC(I1)	PGAU 43
	TG(JJ) = LZ(J)*GR(I2)	PGAU 44
	IF(LL.LT.0) TG(JJ) = LZ(J)*GC(I2)	PGAU 45
41	WG(JJ) = WR(I1)*WR(I2)	PGAU 46
	RETURN	PGAU 47
	END	PGAU 48
C		
	SUBROUTINE SHAPE(SS,TT,X,SHP,XSJ,NDM,NEL,IX,FLG)	SHAP 1
C****	SHAPE FUNCTION ROUTINE FOR TWO DIMENSIONAL ELEMENTS	SHAP 2
	LOGICAL FLG	SHAP 3
	DIMENSION SHP(3,4),X(NDM,1),S(4),T(4),XS(2,2),SX(2,2),IX(9)	SHAP 4
	DATA S/-0.5,0.5,0.5,-0.5/,T/-0.5,-0.5,0.5,0.5/	SHAP 5
C....	FORM 4-NODE QUADRILATERAL SHAPE FUNCTIONS	SHAP 6
	DO 100 I=1,4	SHAP 7
	SHP(3,I) = (0.5+S(I)*SS)*(0.5+T(I)*TT)	SHAP 8
	SHP(1,I) = S(I)*(0.5+T(I)*TT)	SHAP 9
100	SHP(2,I) = T(I)*(0.5+S(I)*SS)	SHAP 10
	IF(NEL.GE.4) GO TO 120	SHAP 11
C....	FORM TRIANGLE BY ADDING THIRD AND FOURTH TOGETHER	SHAP 12
	DO 110 I=1,3	SHAP 13
110	SHP(I,3) = SHP(I,3)+SHP(I,4)	SHAP 14
C....	ADD QUADRATIC TERMS IF NECESSARY	SHAP 15
120	IF(NEL.GT.4 .AND. NEL.LT.10) CALL SHAP2(SS,TT,SHP,IX,NEL)	SHAP 16
C....	ADD CUBIC TERMS IF NECESSARY	SHAP 17
	IF(NEL.GT.9) CALL SHAP3(SS,TT,SHP,IX,NEL)	SHAP 18
C....	CONSTRUCT JACOBIAN AND ITS INVERSE	SHAP 19
	DO 130 I=1,NDM	SHAP 20
	DO 130 J=1,2	SHAP 21
	XS(I,J) = 0.0	SHAP 22
	DO 130 K=1,NEL	SHAP 23
130	XS(I,J) = XS(I,J)+ X(J,K)*SHP(I,K)	SHAP 24
	XSJ = XS(1,1)*XS(2,2)-XS(1,2)*XS(2,1)	SHAP 25
	IF(XSJ .GT. 0.0000001) GO TO 135	SHAP 26
	WRITE(6,2000) IX	SHAP 27
C	STOP	SHAP 28
135	IF(FLG) RETURN	SHAP 29
	SX(1,1) = XS(2,2)/XSJ	SHAP 30
	SX(2,2) = XS(1,1)/XSJ	SHAP 31
	SX(1,2) = -XS(1,2)/XSJ	SHAP 32
	SX(2,1) = -XS(2,1)/XSJ	SHAP 33
C....	FORM GLOBAL DERIVATIVES	SHAP 34
	DO 140 I=1,NEL	SHAP 35
	TP = SHP(1,I)*SX(1,1)+SHP(2,I)*SX(2,1)	SHAP 36
	SHP(2,I) = SHP(1,I)*SX(1,2)+SHP(2,I)*SX(2,2)	SHAP 37
140	SHP(1,I) = TP	SHAP 38
	RETURN	SHAP 39
2000	FORMAT(SX,67H**SHAPE ERROR 01** ZERO OR NEGATIVE JACOBIAN DET. FOR	SHAP 40
	^ELEMENT NODES:/20X,i2i4)	SHAP 41
	END	SHAP 42
C		
	SUBROUTINE SHAP2(S,T,SHP,IX,NEL)	SHAP 1

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C**** ADD QUADRATIC FUNCTIONS AS NECESSARY
      DIMENSION IX(9),SHP(3,12)
      S2 = (1.-S*S)/2.
      T2 = (1.-T*T)/2.
      DO 100 I=5,NEL
      DO 100 J=1,3
100 SHP(J,I) = 0.0
C.... MIDSIDE NODES (SERENDIPITY)
      IF(IX(5).EQ.0) GO TO 101
      SHP(1,5) = -S*(1.-T)
      SHP(2,5) = -S2
      SHP(3,5) = S2*(1.-T)
101 IF(NEL.LT.6) GO TO 107
      IF(IX(6).EQ.0) GO TO 102
      SHP(1,6) = T2
      SHP(2,6) = -T*(1.+S)
      SHP(3,6) = T2*(1.+S)
102 IF(NEL.LT.7) GO TO 107
      IF(IX(7).EQ.0) GO TO 103
      SHP(1,7) = -S*(1.+T)
      SHP(2,7) = S2
      SHP(3,7) = S2*(1.+T)
103 IF(NEL.LT.8) GO TO 107
      IF(IX(8).EQ.0) GO TO 104
      SHP(1,8) = -T2
      SHP(2,8) = -T*(1.-S)
      SHP(3,8) = T2*(1.-S)
C.... INTERIOR NODE (LAGRANGIAN)
104 IF(NEL.LT.9) GO TO 107
      IF(IX(9).EQ.0) GO TO 107
      SHP(1,9) = -4.*S*T2
      SHP(2,9) = -4.*T*S2
      SHP(3,9) = 4.*S2*T2
C.... CORRECT EDGE NODES FOR INTERIOR NODE(LAGRANGIAN)
      DO 106 J=1,3
      DO 105 I=1,4
105 SHP(J,I) = SHP(J,I) - 0.25*SHP(J,9)
      DO 106 I=5,8
106 IF(IX(I).NE.0) SHP(J,I) = SHP(J,I) -0.5*SHP(J,9)
C.... CORRECT CORNER NODES FOR PRESENCE OF MIDSIDE NODES
107 K = 8
      DO 109 I=1,4
      L = I + 4
      DO 108 J=1,3
108 SHP(J,I) = SHP(J,I) - 0.5*(SHP(J,K)+SHP(J,L))
109 K = L
      RETURN
      END

C
      SUBROUTINE SHAP3(S,T,SHP,IX,NEL)
C**** ADD CUBIC FUNCTION AS NECESSARY (SERENDIPITY)
      DIMENSION IX(12),SHP(3,12)
      DO 100 I=5,NEL
      DO 100 J=1,3
100 SHP(J,I)=0.0
      IF(IX(5).EQ.0) GO TO 101
      S1=-1./3.
      T1=-1.
      CALL CSHAPE(S,T,S1,T1,SHP,1,5)
101 IF(IX(6).EQ.0) GO TO 102
      S1=1.
      T1=-1./3.
      CALL CSHAPE(S,T,S1,T1,SHP,2,6)
102 IF(IX(7).EQ.0) GO TO 103
      S1=1./3.
      T1=1.
      CALL CSHAPE(S,T,S1,T1,SHP,1,7)
103 IF(IX(8).EQ.0) GO TO 104
      S1=-1.
      T1=1./3.

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CALL CSHAPE(S,T,S1,T1,SHP,2,8)
104 IF(IX(9).EQ.0) GO TO 105
    S1=-1.
    T1=-1./3.
    CALL CSHAPE(S,T,S1,T1,SHP,2,9)
105 IF(NEL.LT.10) GO TO 200
    IF(IX(10).EQ.0) GO TO 106
    S1=1./3.
    T1=-1.
    CALL CSHAPE(S,T,S1,T1,SHP,1,10)
106 IF(NEL.LT.11) GO TO 200
    IF(IX(11).EQ.0) GO TO 107
    S1=1.
    T1=1./3.
    CALL CSHAPE(S,T,S1,T1,SHP,2,11)
107 IF(NEL.LT.12) GO TO 200
    IF(IX(12).EQ.0) GO TO 200
    S1=-1./3.
    T1=1.
    CALL CSHAPE(S,T,S1,T1,SHP,1,12)
C.... CORRECT CORNER NODES
200 DO 210 I=1,4
    I1=I+4
    I2=I+8
    IF(I.EQ.1) I3=I+7
    IF(I.GT.1) I3=I+3
    IF(I.LT.4) I4=I+9
    IF(I.EQ.4) I4=I+5
    DO 210 J=1,3
210 SHP(J,I)=SHP(J,I)-2./3.*(SHP(J,I1)+SHP(J,I2))-1./3.*(SHP(J,I3)
    ^ +SHP(J,I4))
    RETURN
    END

C
SUBROUTINE CSHAPE(S,T,S1,T1,SHP,K,L)
C**** SUPPLEMENTAL ROUTINE FOR THE SHAPE FUNCTIONS
DIMENSION SHP(3,12)
C=9./32.
GO TO (1,2),K
1 SHP(1,L)=C*(1.+T1*T)*(9.*S1-2.*S-27.*S1*S*S)
  SHP(2,L)=C*T1*(1.-S*S)*(1.+9.*S1*S)
  SHP(3,L)=C*(1.+T1*T)*(1.-S*S)*(1.+9.*S1*S)
  RETURN
2 SHP(1,L)=C*S1*(1.-T*T)*(1.+9.*T1*T)
  SHP(2,L)=C*(1.+S1*S)*(9.*T1-2.*T-27.*T1*T*T)
  SHP(3,L)=C*(1.+S1*S)*(1.-T*T)*(1.+9.*T1*T)
  RETURN
  END

C
SUBROUTINE PMESH(IDL,XL,IX,ID,X,F,JDIAG,NDF,NDM,NEN,NKM)
C**** INPUT MESH DATA
LOGICAL PRT,ERR,PCOMP
COMMON /CTDATA/ O,HEAD(20),NUMNP,NUMEL,LAYER,NEQ,IPR
COMMON /MTDATA/ RHO,UU12,E1,E2,G12,G13,G23,THK,WIDTH
COMMON /LABELS/ PDIS(6),A(6),BC(2),DI(6),CD(3),FD(3)
COMMON /EXDATA/ QLAW(4)
COMMON /RODATA/ UR,IQ,NDS
DIMENSION IDL(6),XL(7),IX(NEN,1),ID(NDF,1),X(NDM,1),
    ^ F(NDF,1),DUM(1),WD(13),JDIAG(1)
DATA WD/4HCOOR,4HELEM,4HMATE,4HBOUN,4HFORC,4HROD ,
    ^ 4HEND ,4HPRIN,4HNOPR,4HPAGE,4HEXPE/
DATA BL/4HBLAN/,LIST/11/,PRT/.TRUE./
C.... INITIALIZE ARRAYS
ERR = .FALSE.
DO 501 I=1,4
501 QLAW(I)=0.
DO 502 N=1,NUMNP
DO 502 I=1,NDF
ID(I,N)=0
F(I,N)=0.

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502 CONTINUE	PMES 22
C.... READ A CARD AND COMPARE WITH MACRO LIST	PMES 23
10 READ(5,1000) CC	PMES 24
DO 20 I=1,LIST	PMES 25
20 IF(PCOMP(CC,WD(I))) GO TO 30	PMES 26
GO TO 10	PMES 27
30 GO TO (1,2,3,4,5,6,7,8,9,11,12),I	PMES 28
C.... NODAL COORDINATE DATA INPUT	PMES 29
1 DO 102 N=1,NUMNP	PMES 30
102 X(1,N)= BL	PMES 31
CALL GENVEC(NDM,XL,X,CD,PRT,ERR)	PMES 32
GO TO 10	PMES 33
C.... ELEMENT DATA INPUT	PMES 34
2 L=0	PMES 35
DO 206 I=1,NUMEL,50	PMES 36
IF(PRT) WRITE(6,2001) O,HEAD, (K,K=1,NEN)	PMES 37
J = MINO(NUMEL,I+49)	PMES 38
DO 206 N=I,J	PMES 39
IF(L-N) 200,202,203	PMES 40
200 READ(5,1001) L,(IDL(K),K=1,NEN),LX	PMES 41
IF(L.EQ.0) L=NUMEL+1	PMES 42
IF(LX.EQ.0) LX=1	PMES 43
IF(L-N) 201,202,203	PMES 44
201 WRITE(6,3001) L,N	PMES 45
ERR = .TRUE.	PMES 46
GO TO 206	PMES 47
202 NX = LX	PMES 48
DO 207 K=1,NEN	PMES 49
207 IX(K,L) = IDL(K)	PMES 50
GO TO 205	PMES 51
203 IX(NEN,N) = IX(NEN,N-1)	PMES 52
DO 204 K=1,NEN	PMES 53
IX(K,N) = IX(K,N-1) + NX	PMES 54
204 IF(IX(K,N-1).EQ.0) IX(K,N) = 0	PMES 55
205 IF(PRT) WRITE(6,2002) N,(IX(K,N),K=1,NEN)	PMES 56
206 CONTINUE	PMES 57
GO TO 10	PMES 58
C.... MATERIAL DATA INPUT	PMES 59
3 WRITE(6,2004) O,HEAD	PMES 60
CALL MATLIB	PMES 61
GO TO 10	PMES 62
C.... READ IN THE RESTRAINT CONDITIONS FOR EACH NODE	PMES 63
4 IF(PRT) WRITE(6,2000) O,HEAD,(I,BC,I=1,NDF)	PMES 64
N = 0	PMES 65
NG = 0	PMES 66
420 L = N	PMES 67
LG = NG	PMES 68
READ(5,1001) N,NG,IDL	PMES 69
IF(N.LE.0 .OR. N.GT.NUMNP) GO TO 50	PMES 70
DO 41 I=1,NDF	PMES 71
ID(I,N) = IDL(I)	PMES 72
41 IF(L.NE.0 .AND. IDL(I).EQ.0 .AND. ID(I,L).LT.0) ID(I,N)=-1	PMES 73
LG = ISIGN(LG,N-L)	PMES 74
42 L = L+LG	PMES 75
IF((N-L)*LG .LE. 0) GO TO 420	PMES 76
DO 43 I=1,NDF	PMES 77
43 IF(ID(I,L-LG) .LT. 0) ID(I,L) = -1	PMES 78
GO TO 42	PMES 79
50 DO 48 N=1,NUMNP	PMES 80
DO 46 I=1,NDF	PMES 81
46 IF(ID(I,N) .NE. 0) GO TO 47	PMES 82
GO TO 48	PMES 83
47 IF(PRT) WRITE(6,2007) N,(ID(I,N),I=1,NDF)	PMES 84
48 CONTINUE	PMES 85
GO TO 10	PMES 86
C.... FORCE/DISPL DATA INPUT	PMES 87
5 CALL GENVEC(NDF,XL,F,FD,PRT,ERR)	PMES 88
GO TO 10	PMES 89
C.... END OF MESH DATA INPUT	PMES 90
C.... COMPUTE THE PROFILE OF GLOBLE ARRAYS	PMES 91

```

7 IF(ERR) STOP
CALL PROFIL(JDIAG, ID, IX, NDF, NEN, NKM, PRT)
RETURN
C.... PRINT OPTION
8 PRT = .TRUE.
GO TO 10
C.... NOPRINT OPTION
9 PRT = .FALSE.
GO TO 10
C.... READ IN PAPER EJECTION OPTION
11 READ(5,1000) O
GO TO 10
C.... INPUT EXPERIMENTAL INDENTATION LAW
12 READ(5,1007) (QLAW(I),I=1,4)
WRITE(6,2008) O,HEAD,(QLAW(I),I=1,4)
GO TO 10
C.... INPUT INITIAL IMPACT CONDITION
6 WRITE(6,2009) O,HEAD
READ(5,1002) NQ, INDF, UR
WRITE(6,2010) NQ, INDF, UR
F(INDF, NQ)=1.0
IQ=ID(INDF, NQ)
GO TO 10
C.... INPUT/OUTPUT FORMATS
1000 FORMAT(A4,75X,A1)
1001 FORMAT(16I5)
1002 FORMAT(2I5,F10.0)
1007 FORMAT(4F10.0)
2000 FORMAT(A1,20A4//5X,10HNODAL B.C.,7X//6X,5HNODE ,9(I7,A4,A2)/1X)
2001 FORMAT(A1,20A4//5X,8HELEMENTS//3X,7HELEMENT,
^ 14(I3,5H NODE)/(20X,14(I3,5H NODE)))
2002 FORMAT(I10,14I8/(10X,14I8))
2004 FORMAT(A1,20A4//5X,19HMATERIAL PROPERTIES)
2007 FORMAT(I10,9I13)
2008 FORMAT(A1,20A4//5X,≠EXPERIMENTAL INDENTATION LAW≠//
1 10X,≠CONTACT COEFFICIENT: ≠,E12.4/
2 10X,≠CRITICAL INDENTATION: ≠ E12.4/
3 10X,≠CONSTANT S: ≠ E12.4/
4 10X,≠POWER INDEX OF UNLOADING LAW:≠ F12.3)
3001 FORMAT(5X,26H**PMESH ERROR 01** ELEMENT, I5,
^ 22H APPEARS AFTER ELEMENT, I5)
2009 FORMAT(A1,20A4, //5X,≠IMPACT OF LAMINATED PLATE≠)
2010 FORMAT(//10X,≠IMPACT NODAL POINT: ≠, I10/
^ 10X,≠IMPACT D.O.F.: ≠, I10/
^ 10X,≠INITIAL IMPACT VELOCITY:≠,E12.4)
END
C
SUBROUTINE GENUVEC(NDM,XL,X,CD,PRT,ERR)
C**** GENERATE REAL DATA ARRAYS BY LINEAR INTERPOLATION
LOGICAL PRT,ERR,PCOMP
COMMON /CTDATA/ O,HEAD(20),NUMNP,NUMEL,LAYER,NEQ,IPR
DIMENSION X(NDM,1),XL(7),CD(3)
DATA BL/4HBLAN/
N=0
NG=0
102 L=N
LG=NG
READ(5,1000) N,NG,XL
IF(N.LE.0 .OR. N.GT.NUMNP) GO TO 108
DO 103 I=1,NDM
103 X(I,N)=XL(I)
IF(LG) 104,102,104
104 LG=ISIGN(LG,N-L)
LI=(IABS(N-L+LG)-1)/IABS(LG)
DO 105 I=1,NDM
105 XL(I)=(X(I,N)-X(I,L))/LI
106 L=L+LG
IF((N-L)*LG .LE. 0) GO TO 102
IF(L.LE.0 .OR. L.GT.NUMNP) GO TO 110
DO 107 I=1,NDM

```

PMES 92  
PMES 93  
PMES 94  
PMES 95  
PMES 96  
PMES 97  
PMES 98  
PMES 99  
PMES100  
PMES101  
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PMES137

GENU 1  
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GENU 10  
GENU 11  
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GENU 16  
GENU 17  
GENU 18  
GENU 19  
GENU 20  
GENU 21  
GENU 22  
GENU 23

107	X(I,L)=X(I,L-LG)+XL(I)	GENU 24
	GO TO 106	GENU 25
110	WRITE(6,3000) L,(CD(I),I=1,3)	GENU 26
	ERR = .TRUE.	GENU 27
	GO TO 102	GENU 28
108	DO 109 I=1,NUMNP,50	GENU 29
	IF(PRT) WRITE(6,2000) O,HEAD,(CD(L),L=1,3),(L,CD(1),CD(2),L=1,NDM)	GENU 30
	N = MINO(NUMNP,I+49)	GENU 31
	DO 109 J=I,N	GENU 32
	IF(PCOMP(X(1,J),BL) .AND. PRT) WRITE(6,2008) N	GENU 33
109	IF(.NOT.PCOMP(X(1,J),BL).AND.PRT) WRITE(6,2009) J,(X(L,J),L=1,NDM)	GENU 34
	RETURN	GENU 35
1000	FORMAT(2I5,7F10.0)	GENU 36
2000	FORMAT(A1,20A4//5X, 5HNODAL,3A4//6X,4HNODE,9(I7,A4,A2))	GENU 37
2008	FORMAT(5X,21H**GENUEC WARNING 01**,I10,	GENU 38
	^ 32H HAS NOT BEEN INPUT OR GENERATED)	GENU 39
2009	FORMAT(I10,9F13.4)	GENU 40
3000	FORMAT(5X,44H**GENUEC ERROR 01**ATTEMPT TO GENETATE NODE,I5,	GENU 41
	1 3H IN,3A4)	GENU 42
	END	GENU 43
C	SUBROUTINE PROFIL(JDIAG, ID, IX, NDF, NEN, NKM, PRT)	PROF 1
C****	COMPUTE PROFILE OF GLOBAL ARRAYS	PROF 2
	LOGICAL PRT	PROF 3
	COMMON /CTDATA/ O,HEAD(20),NUMNP,NUMEL,LAYER,NEQ,IPR	PROF 4
	DIMENSION JDIAG(1),ID(NDF,1),IX(NEN,1),EQ(2)	PROF 5
	DATA EQ/4H DOF,2H. /	PROF 6
C....	SET UP THE EQUATION NUMBERS	PROF 7
	NEQ = 0	PROF 8
	DO 50 N=1,NUMNP	PROF 9
	DO 40 I=1,NDF	PROF 10
	J = ID(I,N)	PROF 11
	IF(J) 30,20,30	PROF 12
20	NEQ = NEQ + 1	PROF 13
	ID(I,N) = NEQ	PROF 14
	JDIAG(NEQ) = 0	PROF 15
	GO TO 40	PROF 16
30	ID(I,N) = 0	PROF 17
40	CONTINUE	PROF 18
50	CONTINUE	PROF 19
	IF(.NOT.PRT) GO TO 70	PROF 20
	WRITE(6,2000) O,HEAD,(I,EQ,I=1,NDF)	PROF 21
	DO 60 I=1,NUMNP	PROF 22
60	WRITE(6,2001) I,(ID(K,I),K=1,NDF)	PROF 23
C....	COMPUTE COLUMN HEIGHTS	PROF 24
70	DO 500 N=1,NUMEL	PROF 25
	DO 400 I=1,NEN	PROF 26
	II = IX(I,N)	PROF 27
	IF(II .EQ. 0) GO TO 400	PROF 28
	DO 300 K=1,NDF	PROF 29
	KK = ID(K,II)	PROF 30
	IF(KK.EQ.0) GO TO 300	PROF 31
	DO 200 J=I,NEN	PROF 32
	JJ = IX(J,N)	PROF 33
	IF(JJ.EQ.0) GO TO 200	PROF 34
	DO 100 L=1,NDF	PROF 35
	LL = ID(L,JJ)	PROF 36
	IF(LL.EQ.0) GO TO 100	PROF 37
	M = MAXO(KK,LL)	PROF 38
	JDIAG(M) = MAXO(JDIAG(M),IABS(KK-LL))	PROF 39
100	CONTINUE	PROF 40
200	CONTINUE	PROF 41
300	CONTINUE	PROF 42
400	CONTINUE	PROF 43
500	CONTINUE	PROF 44
C....	COMPUTE DIAGONAL POINTERS FOR PROFILE	PROF 45
	NKM = 1	PROF 46
	JDIAG(1) = 1	PROF 47
	IF(NEQ.EQ.1) RETURN	PROF 48
	DO 600 N=2,NEQ	PROF 49

```

600 JDIAG(N) = JDIAG(N) + JDIAG(N-1) + 1
      NKM = JDIAG(NEQ)
2000 FORMAT(A1,20A4//5X,16SEQUATION NUMBERS//6X,5HNODE ,
      ^ 9(I5,A4,A2)/1X)
2001 FORMAT(I10,9I11)
      RETURN
      END
C
C***** SUBROUTINE MATLIB
      MATERIAL PROPERTIES ROUTINE
      COMMON /CTDATA/ O,HEAD(20),NUMNP,NUMEL,LAYER,NEQ,IPR
      COMMON /MTDATA/ RHO,UU12,E1,E2,G12,G13,G23,THK,WIDTH
      COMMON /COMPST/ ABD(6,6),DS(2,2),QBR(3,3,25),QBS(2,2,25),
      ^ TH(25),ZK(25)
      COMMON /DMATIX/ D(10),DB(6,6),LINT
      DIMENSION WD(5)
      DATA WD/6H 1SD-,6H ORTHO,6HTROPIC,6H COMP,6HOSITE /
C.... INPUT MATERIAL PROPERTIES
      READ(5,1000) L1,L2,K,THK,WIDTH
      READ(5,1001) RHO,UU12,E1,E2,G12,G13,G23
      DO 150 J=1,3
      DO 150 I=1,3
      IF(I.EQ.3 .OR. J.EQ.3) GO TO 150
      DS(J,I) = 0.
150 ABD(J,I) = ABD(J+3,I) = ABD(J,I+3) = ABD(J+3,I+3) = 0.
      L1 = MIN0(4,MAX0(1,L1))
      D(1) = L1
      L2 = MIN0(4,MAX0(1,L2))
      D(2) = L2
      D(3) = K
      LINT=0
      IF(E1-E2) 120,110,120
110 G12=E1/(2.*(1.+UU12))
      J1=1 $ J2=3
      GO TO 200
120 J1=4 $ J2=5
      IF(LAYER.EQ.1) J1=2 $ J2=3
200 WRITE(6,2000) LAYER,WD(J1),WD(J2),THK,E1,E2,G12,G13,G23,UU12,
      ^ RHO,L1,L2,K
      CALL CMPD
      RETURN
C.... FORMAT FOR INPUT-OUTPUT
1000 FORMAT(3I5,2F10.0)
1001 FORMAT(7F10.0)
2000 FORMAT(/5X,I2,12H LAYER(S) OF,2AG,21H PLATE WITH THICKNESS,
1 F10.4//10X,15HYOUNG=S MODULUS,10X,≠E1=≠,E10.4,10X,≠E2=≠,E10.4/
2 10X,15HSHEAR MODULUS,9X,≠G12=≠,E10.4,9X,≠G13=≠,E10.4,9X,
3 ≠G23=≠,E10.4/10X,15HPOISSON RATIO,8X,≠UU12=≠,F5.3/10X,
4 7HDENSITY,17X,≠RHO=≠,E10.4/10X,13HGAUSS PTS/DIR,12X,≠L1=≠,I5,
5 5X,≠L2=≠,I5/10X,12HSTRESS POINT,14X,≠K=≠,I5/)
      END
C
C***** SUBROUTINE CMPD
      COMPUTE ≠ABD≠ MATRIX AND ≠DS≠ MATRIX
      COMMON /CTDATA/ O,HEAD(20),NUMNP,NUMEL,LAYER,NEQ,IPR
      COMMON /MTDATA/ RHO,UU12,E1,E2,G12,G13,G23,THK,WIDTH
      COMMON /COMPST/ ABD(6,6),DS(2,2),QBR(3,3,25),QBS(2,2,25),
      ^ TH(25),ZK(25)
      DIMENSION Q(3,3),OS(2,2),TK(25)
      LL=LAYER
      MM=LL+1
      READ(5,1000) (L,TH(L),TK(L),I=1,LL)
      ZK(1)=TTK=0.0
      DO 15 I=1,LL
      TTK=TTK+TK(I)
      ZK(I+1)=TK(I)+ZK(I)
15 CONTINUE
      DO 25 I=1,MM
      ZK(I)=ZK(I)-TTK/2.
25 CONTINUE

```

PROF 50  
PROF 51  
PROF 52  
PROF 53  
PROF 54  
PROF 55  
PROF 56

MATL 1  
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MATL 41  
MATL 42  
MATL 43

CMPD 1  
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CMPD 4  
CMPD 5  
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CMPD 14  
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CMPD 16  
CMPD 17  
CMPD 18

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DEL=4.*ATAN(1.)/180.
DEN = 1. - E2*UU12**2/E1
Q(1,1) = E1/DEN
Q(2,2) = E2/DEN
Q(1,2) = Q(2,1) = UU12*Q(2,2)
Q(3,3) = G12
Q(1,3) = Q(2,3) = Q(3,1) = Q(3,2) = 0.0
QS(1,1) = G13
QS(2,2) = G23
QS(1,2) = QS(2,1) = 0.0
DO 40 I=1,LL
ANGL=TH(I)*DEL
C=COS(ANGL)
W=SIN(ANGL)
QBR(1,1,I)=Q(1,1)*C**4+2.*(Q(1,2)+2.*Q(3,3))*(C*W)**2+Q(2,2)*W**4
QBR(1,2,I)=QBR(2,1,I)=(Q(1,1)+Q(2,2)-4.*Q(3,3))*(C*W)**2
$ +Q(1,2)*(W**4 +C**4 )
QBR(2,2,I)=Q(1,1)*W**4+2.*(Q(1,2)+2.*Q(3,3))*(C*W)**2+Q(2,2)*C**4
QBR(1,3,I)=QBR(3,1,I)=(Q(1,1)-Q(1,2)-2.*Q(3,3))*W**3 +
$ (Q(1,2)-Q(2,2)+2.*Q(3,3))*C*W**3
QBR(2,3,I)=QBR(3,2,I)=(Q(1,1)-Q(1,2)-2.*Q(3,3))*W**3 *C+
$ (Q(1,2)-Q(2,2)+2.*Q(3,3))*W**3 *C
QBR(3,3,I)=(Q(1,1)+Q(2,2)-2.*Q(1,2)-2.*Q(3,3))*(W**2+
$ Q(3,3)*(W**4 +C**4 )
QBS(1,1,I) = QS(1,1)*C**2 + QS(2,2)*W**2
QBS(2,2,I) = QS(1,1)*W**2 + QS(2,2)*C**2
QBS(1,2,I) = QBS(2,1,I) = (QS(1,1)-QS(2,2))*C*W
40 CONTINUE
DO 50 J=1,3
DO 50 K=1,3
DO 50 I=1,LL
ABD(J ,K )= ABD(J ,K )+QBR(J,K,I)*(ZK(I+1)-ZK(I))
ABD(J+3,K )= ABD(J ,K+3)= ABD(J+3,K)+QBR(J,K,I)*
$ (ZK(I+1)**2-ZK(I)**2)/2.
ABD(J+3,K+3)= ABD(J+3,K+3)+QBR(J,K,I)*(ZK(I+1)**3-ZK(I)**3)/3.
50 CONTINUE
DO 55 I=1,6
DO 55 J=1,6
IF(1.GE.3 .OR. J.GE.3) GO TO 55
IF(ABS(DS(I,J)) .LT. 1.E-06) DS(I,J)=0.0
55 IF(ABS(ABD(I,J)) .LT. 1.E-06) ABD(I,J)=0.0
WRITE(6,2001) ((ABD(I,J),J=1,6),I=1,6)
DO 60 J=1,2
DO 60 K=1,2
DO 60 I=1,LL
60 DS(J,K) = DS(J,K) + QBS(J,K,I)*(ZK(I+1)-ZK(I))
WRITE(6,2002) ((DS(I,J),J=1,2),I=1,2)
1000 FORMAT(15,F5.0,F10.0)
2001 FORMAT(/,1X,10HABD MATRIX//6(2X,6E13.4/))
2002 FORMAT(/,1X,9HDS MATRIX//2(2X,2E13.4/))
RETURN
END
C
SUBROUTINE KMLIB
C**** ASSEMBLE GLOBLE ARRAY
COMMON G(1)
DIMENSION M(1)
EQUIVALENCE (G(1),M(1))
COMMON /ISWIDX/ ISW
COMMON /CTDATA/ O,HEAD(20),NUMNP,NUMEL,LAYER,NEQ,IPR
COMMON /LODATA/ NDF,NDM,NEN,NST,NKM
COMMON /PARATS/ NPAR(14),NEND
N1=NEND
N2=N1+NST*NST*IPR
IF(ISW.LE.2) NE=N2*NKM*IPR
IF(ISW.GT.2) NE=N2*NEQ*IPR
CALL SETMEM(NE)
CALL PZERO(G(NEND),NE-NEND)
CALL MASSO1(G(NPAR(1)),G(NPAR(2)),M(NPAR(3)),G(NPAR(4)),
1 M(NPAR(5)),M(NPAR(6)),G(NPAR(7)),G(NPAR(8)),M(NPAR(9)),
KMLI 19
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KMLI 17

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2   G(NPAR(11)),G(N1),G(N2),NDF,NDM,NEN,NST,NKM)
RETURN
END
KMLI 18
KMLI 19
KMLI 20

C
SUBROUTINE MASS01(UL,XL,LD,P,IX,ID,X,F,JDIAG,B,S,A,NDF,NDM,NEN,
^  NST,NKM)
C**** FORM MASS MATRIX
COMMON /CTDATA/ O,HEAD(20),NUMNP,NUMEL,LAYER,NEQ,IPR
COMMON /MTDATA/ RHO,UJ12,E1,E2,G12,G13,G23,THK,WIDTH
COMMON /DMATIX/ D(10),DB(6,6),LINT
COMMON /ELDATA/ N,NEL,MCT
COMMON /ISWIDX/ ISW
COMMON /GAUSSP/ SG(16),TG(16),WG(16)
DIMENSION UL(1),XL(NDM,1),LD(NDF,1),P(1),IX(NEN,1),ID(NDF,1),
1   X(NDM,1),F(1),JDIAG(1),B(1),S(NST,1),A(1),SHP(3,12)
C.... LOOP ON ELEMENTS
DO 110 N=1,NUMEL
DO 10 I=1,NST
DO 10 J=1,NST
10 S(I,J)=0.
C.... SET UP LOCAL ARRAYS
CALL PFORM(UL,XL,LD,IX,ID,X,F,B,NDF,NDM,NEN,ISW)
C.... COMPUTE CONSISTENT MASS MATRIX
L = D(1)
CALL PGAUSS(L,LINT)
DO 500 L=1,LINT
C .. COMPUTE SHAPE FUNCTIONS
CALL SHAPE(SG(L),TG(L),XL,SHP,XSJ,NDM,NEL,IX,.FALSE.)
DU = WG(L)*XSJ*RHO*THK
C .. FOR EACH NODE J COMPUTE DB=RHO*SHAPE*DU
K1 = 1
DO 500 J=1,NEL
W11 = SHP(3,J)*DU
W33 = W11*THK**2/12.
C .. FOR EACH NODE K COMPUTE MASS MATRIX (UPPER TRIANGULAR PART)
J1 = K1
DO 510 K=J,NEL
S(J1 ,K1 ) = S(J1 ,K1 ) + SHP(3,K)*W11
S(J1+3,K1+3) = S(J1+3,K1+3) + SHP(3,K)*W33
510 J1 = J1 + NDF
500 K1 = K1 + NDF
C .. COMPUTE MISSING PARTS AND LOWER PART BY SYMMETRY
NSL = NEL*NDF
DO 530 K=1,NSL,NDF
DO 520 J=K,NSL,NDF
S(J+2,K+2) = S(J+1,K+1) = S(J ,K )
S(J+4,K+4) = S(J+3,K+3)
S(K ,J ) = S(J ,K )
S(K+3,J+3) = S(J+3,K+3)
S(K+2,J+2) = S(K+1,J+1) = S(J ,K )
520 S(K+4,J+4) = S(J+3,K+3)
530 CONTINUE
IF(ISW.EQ.2) GO TO 100
C.... LUMPED MASS MATRIX
SUM1 = 0.0
SUM2 = 0.0
SUMD1 = 0.0
SUMD2 = 0.0
DO 540 I=1,NSL,NDF
SUMD1 = SUMD1 + S(I,I)
SUMD2 = SUMD2 + S(I+3,I+3)
DO 540 J=1,NSL,NDF
SUM1 = SUM1 + S(I,J)
540 SUM2 = SUM2 + S(I+3,J+3)
DO 550 I=1,NSL,NDF
P(I) = S(I,I)*SUM1/SUMD1
P(I+2) = P(I+1) = P(I)
P(I+3) = S(I+3,I+3)*SUM2/SUMD2
550 P(I+4) = P(I+3)
C.... ADD TO TOTAL ARRAY
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MASS 66

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```

100 CALL ADDSTF(A,S,P,JDIAG,LD,NST,NEL*NDF,.FALSE.)
110 CONTINUE
    REWIND 2
    IF(ISW.EQ.2) WRITE(2) (A(I),I=1,NKM)
    IF(ISW.EQ.3) WRITE(2) (A(I),I=1,NEQ)
    RETURN
    END
C
SUBROUTINE RODIPCT
C****
LOGICAL FLAG
COMMON G(1)
DIMENSION M(1)
EQUIVALENCE (G(1),M(1))
COMMON /CTDATA/ O,HEAD(20),NUMNP,NUMEL,LAYER,NEQ,IPR
COMMON /LODATA/ NDF,NDM,NEN,NST,NKM
COMMON /PARATS/ NPAR(14),NEND
COMMON /RODATA/ UR,IQ,NDS
COMMON /ROELEM/ NER,NEQR,ER
DATA FLAG/.FALSE./,NER/20/,ER/30000000./
IF(FLAG) GO TO 50
NEQR=2*(NER+1)
NKMR=7*NER+3
N1=NEND
N2=N1+NEQ*IPR
N3=N2+NEQ*IPR
N4=N3+NEQ*IPR
N5=N4+NKMR*IPR
N6=N5+NEQR*IPR
N7=N6+NEQR
N8=N7+NEQR*IPR
N9=N8+NEQR*IPR
N10=N9+NEQR*IPR
N11=N10+NEQR*IPR
NE=N11+NEQR*IPR
CALL SETMEM(NE)
CALL PZERO(G(NEND),NE-NEND)
FLAG=.TRUE.
50 CALL WIMPCT(G(NPAR(1)),G(NPAR(2)),M(NPAR(3)),G(NPAR(4)),
1      M(NPAR(5)),M(NPAR(6)),G(NPAR(7)),G(NPAR(8)),
2      M(NPAR(9)),G(NPAR(10)),G(NPAR(11)),G(N1),G(N2),
3      G(N3),G(N4),G(N5),M(N6),G(N7),G(N8),G(N9),G(N10),
4      G(N11))
    RETURN
    END
C
SUBROUTINE WIMPCT(UL,XL,LD,P,IX,ID,X,F,JDIAG,DR,U,B,U,A,RK,RM,
^      JDR,RU,RV,RA,RB,FR)
C****
SOLVE IMPACT PROBLEM
LOGICAL FLAG,TAN
COMMON G(1)
DIMENSION M(1)
EQUIVALENCE (G(1),M(1))
COMMON /CTDATA/ O,HEAD(20),NUMNP,NUMEL,LAYER,NEQ,IPR
COMMON /TMDATA/ TIME,DT,DDT,FORCE,ALPHA
COMMON /LODATA/ NDF,NDM,NEN,NST,NKM
COMMON /NITERS/ ITR
COMMON /PARATS/ NPAR(14),NEND
COMMON /RODATA/ UR,IQ,NDS
COMMON /ROELEM/ NER,NEQR,ER
COMMON /CONSTS/ A0,A2,A4,A5,A6,A7,A8,AREA
COMMON /PROLOD/ PROP
COMMON /ISWIDX/ ISW
COMMON /EXTRAS/ TAN
DIMENSION UL(1),XL(1),LD(1),P(1),IX(1),ID(1),X(1),F(1),JDIAG(1),
1      DR(1),U(1),B(1),U(1),A(1),RK(1),RM(1),JDR(1),RU(1),
2      RU(1),RA(1),RB(1),FR(1),Q(3),OP(3)
DATA ITR/5/,FLAG/.FALSE./,WIL/1.4/,INTE/24/
IF(FLAG) GO TO 50
DO 1 I=1,3

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Q(I)=0.0	WIMP 25
QP(I)=0.0	WIMP 26
1 CONTINUE	WIMP 27
IDS=1	WIMP 28
TAN=.FALSE.	WIMP 29
REWIND 2	WIMP 30
READ(2) (B(I),I=1,NEQ)	WIMP 31
FORCE=0.0	WIMP 32
ALPHA=0.0	WIMP 33
PROP=0.0	WIMP 34
NNEQ=NDF*NUMNP	WIMP 35
A0=6./(WIL*DT)**2	WIMP 36
A2=6./(WIL*DT)	WIMP 37
A4=A0/WIL	WIMP 38
A5=-A2/WIL	WIMP 39
A6=1.-3./WIL	WIMP 40
A7=DT/2.	WIMP 41
A8=DDT/6.	WIMP 42
CALL FORMROD(RK, RM, JDR)	WIMP 43
DO 10 I=1, NEQR	WIMP 44
10 RU(I)=-UR	WIMP 45
Q(2)=-UR	WIMP 46
FLAG=.TRUE.	WIMP 47
50 ISW=5	WIMP 48
IF (IDS.EQ.NDS) TAN=.TRUE.	WIMP 49
CALL FSTREA(UL, XL, LD, P, IX, ID, X, F, JDIAG, DR, U, NDF, NDM, NEN, NST, NNEQ)	WIMP 50
DO 20 I=1, NEQ	WIMP 51
A(I)=DR(I)/B(I)	WIMP 52
V(I)=U(I)+DT*A(I)	WIMP 53
U(I)=U(I)+DT*V(I)	WIMP 54
20 CONTINUE	WIMP 55
QP(1)=U(IQ)	WIMP 56
QP(2)=U(IQ)	WIMP 57
QP(3)=A(IQ)	WIMP 58
DO 30 I=1, NEQR	WIMP 59
RB(I)=RM(I)*(A0*RU(I)+A2*RU(I)+2.*RA(I))	WIMP 60
30 CONTINUE	WIMP 61
RBIQ=RU(1)+DT*RU(1)+DDT/3.*RA(1)	WIMP 62
ROT=0.000001	WIMP 63
ICOV=0	WIMP 64
DO 100 IT=1, ITR	WIMP 65
RUT=RBIQ+Q(3)*DDT/6.	WIMP 66
AF=-RUT-QP(1)	WIMP 67
CALL RODLOAD(FIQ, AF)	WIMP 68
DO 110 I=1, NEQR	WIMP 69
FR(I)=RB(I)	WIMP 70
110 CONTINUE	WIMP 71
FR(1)=FR(1)+(1.-WIL)*FORCE+WIL*FIQ	WIMP 72
CALL ACTCOL(RK, FR, JDR, NEQR, .FALSE., .TRUE., 0)	WIMP 73
Q(3)=A4*(FR(1)-RU(1))+A5*RU(1)+A6*RA(1)	WIMP 74
RUTT=RBIQ+Q(3)*DDT/6.	WIMP 75
ROTR=ABS((RUTT-RUT)/RUTT)	WIMP 76
IF (ROTR.LT.ROT) ICOV=1	WIMP 77
IF (ICOV.GT.0) GO TO 200	WIMP 78
100 CONTINUE	WIMP 79
200 DO 210 I=1, NEQR	WIMP 80
FR(I)=A4*(FR(I)-RU(I))+A5*RU(I)+A6*RA(I)	WIMP 81
RU(I)=RU(I)+DT*RU(I)+A8*(FR(I)+2.*RA(I))	WIMP 82
RU(I)=RU(I)+A7*(FR(I)+RA(I))	WIMP 83
RA(I)=FR(I)	WIMP 84
210 CONTINUE	WIMP 85
Q(1)=RU(1)	WIMP 86
Q(2)=RU(1)	WIMP 87
Q(3)=RA(1)	WIMP 88
FORCE=FIQ	WIMP 89
PROP=FORCE	WIMP 90
ALPHA=-Q(1)-QP(1)	WIMP 91
RODFR=RU(INTE)*AREA*ER	WIMP 92
WRITE(8, 8001) FORCE, ALPHA, RODFR, (Q(I), I=1, 3)	WIMP 93
8001 FORMAT(6E12.4)	WIMP 94

	IDS=IDS+1	WIMP 95
	IF(IDS.GT.NDS) IDS=1	WIMP 96
	TAN=.FALSE.	WIMP 97
	RETURN	WIMP 98
	END	WIMP 99
C		
	SUBROUTINE FORMROD(RK, RM, JDR)	FORM 1
C****	FORM STIFFNESS AND MASS MATRICES. OF ROD	FORM 2
	COMMON /RODATA/ UR, IQ, NDS	FORM 3
	COMMON /ROELEM/ NER, NEQR, ER	FORM 4
	COMMON /CONSTS/ A0, A2, A4, A5, A6, A7, A8, AREA	FORM 5
	DIMENSION RK(1), RM(1), JDR(2), D(6)	FORM 6
	DATA RHOR/.0003225/, RL/1.0/	FORM 7
	DATA D/.22, .36, .43, .48, .50, .625/	FORM 8
	EL=RL/NER	FORM 9
	PAI=4.*ATAN(1.)	FORM 10
	JDR(1)=1	FORM 11
	JDR(2)=3	FORM 12
	DO 100 I=1, NER	FORM 13
	IF(I.LT.6) A=PAI*(D(I)/2.):**2	FORM 14
	IF(I.GE.6) A=PAI*(D(6)/2.):**2	FORM 15
	TT=A*ER/30./EL	FORM 16
	J1=2*(I+1)-1	FORM 17
	J2=J1+1	FORM 18
	J1M1=J1-1	FORM 19
	J1M2=J1-2	FORM 20
	JDR(J1)=JDR(J1M1)+3	FORM 21
	JDR(J2)=JDR(J1)+4	FORM 22
	K1=JDR(J1M2)	FORM 23
	K2=JDR(J1M1)-1	FORM 24
	RK(K1 )=RK(K1 )+TT*36.	FORM 25
	RK(K2 )=RK(K2 )+TT*3.*EL	FORM 26
	RK(K2+1)=RK(K2+1)+TT*4.*EL**2	FORM 27
	RK(K2+2)=RK(K2+2)-TT*36.	FORM 28
	RK(K2+3)=RK(K2+3)-TT*3.*EL	FORM 29
	RK(K2+4)=RK(K2+4)+TT*36.	FORM 30
	RK(K2+5)=RK(K2+5)+TT*3.*EL	FORM 31
	RK(K2+6)=RK(K2+6)-TT*EL**2	FORM 32
	RK(K2+7)=RK(K2+7)-TT*3.*EL	FORM 33
	RK(K2+8)=RK(K2+8)+TT*4.*EL**2	FORM 34
	TT=RHOR*A*EL	FORM 35
	L1=2*I-1	FORM 36
	RM(L1 )=RM(L1 )+TT/2.	FORM 37
	RM(L1+1)=RM(L1+1)+TT*EL**2/420.	FORM 38
	RM(L1+2)=RM(L1+2)+TT/2.	FORM 39
	RM(L1+3)=RM(L1+3)+TT*EL**2/420.	FORM 40
100	CONTINUE	FORM 41
	AREA=A	FORM 42
	DO 20 I=1, NEQR	FORM 43
	J=JDR(I)	FORM 44
20	RK(J)=RK(J)+A0*RM(I)	FORM 45
	CALL ACTCOL(RK, RM, JDR, NEQR, .TRUE., .FALSE., 0)	FORM 46
	RETURN	FORM 47
	END	FORM 48
C		
	SUBROUTINE RODLOAD(F, AF)	RODL 1
C****	COMPUTE CONTACT LOADING	RODL 2
	LOGICAL RELD, UNLD, PIL	RODL 3
	COMMON /TMDATA/ TIME, DT, DDT, FORCE, ALPHA	RODL 4
	COMMON /EXDATA/ Q(4)	RODL 5
	DATA UNLD/.FALSE./, PIL/.FALSE./, RELD/.FALSE./	RODL 6
	IF(PIL) GO TO 10	RODL 7
	AMAX=AMIN=FMAX=0.0	RODL 8
	PIL=.TRUE.	RODL 9
10	IF(RELD) GO TO 50	RODL 10
	IF(UNLD) GO TO 20	RODL 11
	F=Q(1)*AF**1.5	RODL 12
	IF(F.GE.FORCE) RETURN	RODL 13
	UNLD=.TRUE.	RODL 14
	AMAX=ALPHA	RODL 15

```
FMAX=FORCE
IF(AMAX.GT.Q(2)) UK=FMAX/((1.-Q(3))*AMAX+Q(2)*Q(3)**Q(4)
IF(AMAX.LE.Q(2)) UK=FMAX/AMAX**Q(4)
AMIN=Q(3)*(AMAX-Q(2))
IF(AMIN.LT.0.) AMIN=0.0
20 IF(AF.LE.AMIN) GO TO 30
F=UK*(AF-AMIN)**Q(4)
IF(F.LT.FORCE) RETURN
RELD=.TRUE.
RK=FMAX/(AMAX-AMIN)**1.5
50 IF(AF.LE.AMIN) GO TO 30
F=RK*(AF-AMIN)**1.5
RETURN
30 F=0.0
RETURN
END
```

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RODL 31

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