ARMY RESEARCH LABORATORY

Vibratory vs. Electromagnetic Waves as Used in an Analysis of a Gas Turbine Engine

Thomas A. Korjack

ARL-TR-988 March 1996

NOTICES

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	March 1996	Final, 30 Jun 95 - 1 Jan 96

4. TITLE AND SUBTITLE
Vibratory vs. Electromagnetic Waves as Used in an Analysis of a Gas Turbine Engine

5. FUNDING NUMBERS
4B010503350000

6. AUTHOR(S)
Thomas A. Korjack

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
U.S. Army Research Laboratory
ATTN: AMSRL-SC-II
Aberdeen Proving Ground, MD 21005-5067

8. PERFORMING ORGANIZATION REPORT NUMBER
ARL-TR-988

9. SPONSORING/MONITORING AGENCY NAMES(S) AND ADDRESS(ES)

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release, distribution is unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

The kinematics and energy balances of wave propagation occurring in a gas turbine engine are investigated to show a similarity of mathematical relationship between sonic and electromagnetic waves. Transverse electric and magnetic waves are shown to be similar in nature to viscoelastic SH waves with respect to a plane of symmetry in a monoclinic medium. The Maxwell constitutive model is related to the Maxwell equations in electromagnetics. It was confirmed that a direct similarity exists with particle velocity and magnetic field, electric field with stresses, permittivity and compliance as well as conductivity with viscosity and permeability with material density.

14. SUBJECT TERMS
sonics, electromagnetics, Maxwell's equations, waves

15. NUMBER OF PAGES
14

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

INTENTIONALLY LEFT BLANK.

TABLE OF CONTENTS

INTENTIONALLY LEFT BLANK.

1. INTRODUCTION

It is very much possible to reshape the viscoelastodynamic equations into a form that resembles or parallels Maxwell's equation to achieve an analogy or a complete mathematical equivalence; hence, a mathematical parallelism can be generated between electromagnetics and vibratory/acoustic energy waves. This work will show that the wave equation in a Maxwell anisotropic-viscoelastic solid is analogous to the two-dimensional (2-D) Maxwell equations describing propagation of the transverse electromagnetic mode in anisotropic media. This type of analogy, i.e., between the process of conduction-static induction through dielectrics and viscosity - elasticity, was probably assumed initially by Maxwell himself (Everitt 1975).

This phenomenological analogy can be utilized in many ways. Many viscoelastodynamic modeling codes can be changed to simulate electromagnetic wave propagation. In addition, many sets of solutions of the viscoelastic problem can be used to test electromagnetic codes. Furthermore, plane propagation wave theory of harmonic waves especially involving anisotropic-viscoelastic media apply analogously to electromagnetic anisotropic wave propagation. It is well known that velocity and attenuation anisotropy of vibratory waves are important in frequency-amplitude vs. time analyses conducted on the M1 Abrams tank gas turbine engine (Korjack 1995).

2. MAXWELL'S EQUATIONS

The Maxwell equations can be written as (Chew 1990 and Jordan 1950):

$$\nabla \times E = -\frac{\partial B}{\partial t} + M, \tag{1}$$

$$\nabla \times H = \frac{\partial D}{\partial t} + J, \tag{2}$$

where E, B, H, and D are the electric field intensity, the magnetic flux density, the magnetic field intensity and the electric flux density, respectively, and J and M are the electric and magnetic current densities, respectively. Equations (1) and (2) constitute six scalar equations with 12 scalar unknowns, since M is assumed to be given and J is a known function of the electric field as stated explicitly by Equation (5) below. The six additional scalar equations are the constitutive relations, which for isotropic media can be written as

$$D = \varepsilon \cdot E, \tag{3}$$

$$B = \mu \cdot H, \tag{4}$$

where ε and μ are the permittivity and permeability tensors, respectively. The product appearing in Equations (3) and (4) is simply Cayley multiplication. Moreover, the current density is

$$J = \sigma \cdot E + J_s, \tag{5}$$

where σ is the conductivity tensor and J_s is the given contribution of the sources. Substituting the constitutive relations and the current density into Equations (1) and (2) gives

$$\nabla \times E = -\mu \cdot \frac{\partial H}{\partial t} + M, \tag{6}$$

$$\nabla \times H = \sigma \cdot E + \varepsilon \cdot \frac{\partial E}{\partial t} + J_s. \tag{7}$$

3. THE SONIC-WAVE FIELD EQUATIONS

The elementary equations of acoustics, as written in terms of particle velocity and stress, can be expressed in terms of first order time derivatives. Cauchy's equations can be written as (Auld 1990),

$$\nabla \cdot T = \rho \frac{\partial v}{\partial t} - F, \tag{8}$$

where

$$T = [\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{yz}, \sigma_{xz}, \sigma_{xy}]^T \tag{9}$$

is the stress tensor, v is the particle velocity tensor, ρ is the density, F is the body force tensor. The DEL (∇) operator can be simply expressed for rectangular cartesian coordinates as,

$$\nabla = \begin{bmatrix} \partial/\partial x & 0 & 0 & 0 & \partial/\partial z & \partial/\partial y \\ 0 & \partial/\partial y & 0 & \partial/\partial z & 0 & \partial/\partial x \\ 0 & 0 & \partial/\partial z & \partial/\partial y & \partial/\partial x & 0 \end{bmatrix}. \tag{10}$$

The strain can be given in terms of the displacement $u = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix}$ by,

$$S = [\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}, 2\varepsilon_{yz}, 2\varepsilon_{xz}, 2\varepsilon_{xy}]^T, \tag{11}$$

such that $\varepsilon_{xx} = \partial u_x/\partial x$, $\varepsilon_{xy} = (\partial u_x/\partial y + \partial u_y/\partial x)/2$, etc. Strain and particle velocity can be related as,

$$\nabla^T \cdot v = \frac{\partial S}{\partial t}. \tag{12}$$

The sonic-electromagnetic analogy was first established by Auld (1990) by using a 3-D Kelvin-Voigt Model,

$$T = c_k \cdot S + \eta_k \cdot \frac{\partial S}{\partial t}, \tag{13}$$

where c_k and η_k are the (Kelvin-Voigt) elasticity and viscosity tensors, respectively. If we use the time derivative of the strain by using Equation (12), and defining the tensor,

$$\tau_k = c_k^{-1} \cdot \eta_k,$$

it can be deduced that,

$$\nabla^T \cdot V + \tau_k \cdot \nabla^T \frac{\partial v}{\partial t} = c_k^{-1} \cdot \frac{\partial T}{\partial t}. \tag{14}$$

By introducing the 3-D Maxwell constitutive relation (Casula and Carcione 1992), we have,

$$\frac{\partial S}{\partial t} = c_M^{-1} \cdot \frac{\partial T}{\partial t} + \eta_M^{-1} \cdot T, \tag{15}$$

such that c_M and η_M are the elasticity and the viscosity tensors, respectively. This relation compares very similarly to the 1-D Maxwell stress-strain relation (Ben-Menahem and Singh 1981). If we drop out the strain tensor by invoking Equation (12), we end up with a relation similar to Greenfield and Wu (1991) as,

$$\nabla^T \cdot v = \eta_M^{-1} \cdot T + c_M^{-1} \cdot \frac{\partial T}{\partial t}. \tag{16}$$

If a compliance tensor can be defined as

$$s_M = c_M^{-1}, \tag{17}$$

along with the complementary compliance tensor of

$$\tau_M = \eta_M^{-1}, \tag{18}$$

then our Equation (16) simply reverts to the expression,

$$\nabla^T \cdot v = \tau_M \cdot T + s_M \cdot \frac{\partial T}{\partial t}. \tag{19}$$

Generally, the analogy does not mean that sonic-wave and electromagnetic equations illustrate the exact same mathematical problem. In actuality, T is a 6-D vector and E is a 3-D vector. In addition, acoustics involve at least order 6 matrices for material properties and electromagnetism involves merely order 3 matrices. However, a version of complete equivalence can be adequately established in the 2-D case by using the Maxwell model.

4. SONIC-WAVE AND ELECTROMAGNETIC ANALOGY

By considering anisotropic permittivity and conductivity tensors with symmetry, we may imagine a medium such that

$$\varepsilon = \begin{bmatrix} \varepsilon_{11} & 0 & \varepsilon_{13} \\ 0 & \varepsilon_{22} & 0 \\ \varepsilon_{13} & 0 & \varepsilon_{33} \end{bmatrix} \tag{20}$$

and

$$\sigma = \begin{bmatrix} \sigma_{11} & 0 & \sigma_{13} \\ 0 & \sigma_{22} & 0 \\ \sigma_{13} & 0 & \sigma_{33} \end{bmatrix}. \tag{21}$$

These previously mentioned tensors are for a medium having the vertical axis at 90° to the symmetric plane. It is well known that a coordinate transformation exists that fully diagonalizes these symmetric tensors, which is commonly referred to as the principal system of the particular medium in question; hence, the three principal components of the tensors become immediately apparent. The permeability tensor is usually isotropic. Here, it is assumed that $\mu = \mu \cdot I$ such that μ is the permeability and I is the identity tensor of order 3.

If we allow the material properties to not change in the y-direction and only allow propagation to occur in the x-z plane, then it follows that E_x, H_x, and H_z are the only components under consideration;

therefore, if there are no electric sources within the media, it follows that the field equations for the transverse electric and magnetic (TEM) field take on the form,

$$\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} = \mu \frac{\partial H_y}{\partial t} - M_y , \tag{22}$$

$$-\frac{\partial H_y}{\partial z} = \sigma_{11} E_x + \sigma_{13} E_z + \varepsilon_{11} \frac{\partial E_x}{\partial t} + \varepsilon_{13} \frac{\partial E_z}{\partial t} , \tag{23}$$

$$\frac{\partial H_y}{\partial x} = \sigma_{13} E_x + \sigma_{33} E_z + \varepsilon_{13} \frac{\partial E_x}{\partial t} + \varepsilon_{33} \frac{\partial E_z}{\partial t} . \tag{24}$$

According to Greenfield and Wu (1991), Equations (22) through (24) conceptualize an isotropic model. In addition, if we have uniform properties in the y-direction as in the field of sonics, then we have the situation that one of the shear waves has its own differential equation, which is decoupled; this kind of equation is called the SH wave equation (Virieux 1984), which represents a veracity in the plane of symmetry (mirror) of a monoclinic medium. If propagation exists in this plane of symmetry, then we have anti-plane strain motion which represents the most general situation under which pure shear waves can exist at all angles of propagation. However, hexagonal media possessing pure shear wave propagation are deemed to be part of a degenerate situation.

If we consider a set of parallel fractures contained integrally within a transversely isotropic media, then we have the representation of a monoclinic medium; here, the pure anti-plane strain waves are SH waves when the plane of symmetry of this medium is vertical.

In addition, it can be said that other cases containing symmetry occur in monoclinic media such as orthorhombic media, strong trigonal media, and weak tetragonal media.

If we consider a monoclinic media, Virieux (1984) suggests that the elasticity and viscosity tensors and their inverses are:

$$
\begin{bmatrix}
a_{11} & a_{12} & a_{13} & 0 & a_{15} & 0 \\
a_{12} & a_{22} & a_{23} & 0 & a_{25} & 0 \\
a_{13} & a_{23} & a_{33} & 0 & a_{35} & 0 \\
0 & 0 & 0 & a_{44} & 0 & a_{46} \\
a_{15} & a_{25} & a_{35} & 0 & a_{55} & 0 \\
0 & 0 & 0 & a_{46} & 0 & a_{66}
\end{bmatrix} . \tag{25}
$$

Now, according to Neumann's Principle (Neumann 1885), any kind of symmetry possessed by the attenuation flows in conjunction with the symmetry of the crystallographic network of the substance or material.

Hence, the significant elements or components depicting the movement of the SH wave(s) are:

$$
\begin{bmatrix}
a_{44} & a_{46} \\
a_{46} & a_{66}
\end{bmatrix} . \tag{26}
$$

Thus, it can be readily deduced that the partial differential equations can be extracted from the second row of tensor equation (8) and the fourth and sixth rows of tensor equation (19) as:

$$
\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial z} = \rho \frac{\partial v_y}{\partial t} - F_y , \tag{27}
$$

$$
-\frac{\partial v_y}{\partial z} = -\tau_{44}\sigma_{yz} - \tau_{46}\sigma_{xy} - s_{44}\frac{\partial \sigma_{yz}}{\partial t} - s_{46}\frac{\partial \sigma_{xy}}{\partial t} , \tag{28}
$$

$$
\frac{\partial v_y}{\partial x} = \tau_{46}\sigma_{yz} + \tau_{66}\sigma_{xy} + s_{46}\frac{\partial \sigma_{yz}}{\partial t} + s_{66}\frac{\partial \sigma_{xy}}{\partial t} , \tag{29}
$$

where

$$\tau_{44} = \eta_{66}/\bar{\eta}\,, \quad \tau_{46} = -\eta_{46}/\bar{\eta}\,, \quad \bar{\eta} = \eta_{44}\eta_{66} - \eta_{46}^2\,, \tag{30}$$

and

$$s_{44} = c_{66}/c\,, \quad s_{66} = c_{44}/c\,, \quad s_{46} = -c_{46}/c\,, \quad c = c_{44}c_{66} - c_{46}^2\,, \tag{31}$$

such that the stiffness C_{ij} and the viscosities η_{ij} (where i,j = 4,6) are the (i^{th}, j^{th}) components of the tensors C_M, η_M, respectively. According to Carcione and Cavallini (1993), Equations (22)–(24) can be easily converted into Equations (27)–(29) reversibly as long as the following relations are taken into account (noting that s and τ can be introduced as simple order 2 tensors):

$$V \equiv \begin{bmatrix} v_y \\ \sigma_{yz} \\ \sigma_{xy} \end{bmatrix} \Leftrightarrow \begin{bmatrix} H_y \\ -E_x \\ E_z \end{bmatrix}, \tag{32}$$

$$F_y \Leftrightarrow M_y\,, \tag{33}$$

$$S \equiv \begin{bmatrix} s_{44} & s_{46} \\ s_{46} & s_{66} \end{bmatrix} \Leftrightarrow \begin{bmatrix} \varepsilon_{11} & -\varepsilon_{13} \\ -\varepsilon_{13} & \varepsilon_{33} \end{bmatrix} \equiv \varepsilon'\,, \tag{34}$$

$$\tau \equiv \begin{bmatrix} \tau_{44} & \tau_{46} \\ \tau_{46} & \tau_{66} \end{bmatrix} \Leftrightarrow \begin{bmatrix} \sigma_{11} & -\sigma_{13} \\ -\sigma_{13} & \sigma_{33} \end{bmatrix} \equiv \sigma'\,, \tag{35}$$

$$\rho \Leftrightarrow \mu\,, \tag{36}$$

Similarly, if we allow the stiffness and viscosity tensors to be of order 2, then we have,

$$c = \begin{bmatrix} c_{44} & c_{46} \\ c_{46} & c_{66} \end{bmatrix}, \qquad \eta = \begin{bmatrix} \eta_{44} & \eta_{46} \\ \eta_{46} & \eta_{66} \end{bmatrix}. \tag{37}$$

Hence, we can arrive at the 2-D identities

$$S = C^{-1} \text{ and } \tau = \eta^{-1}, \tag{38}$$

typifying the 3-D Equations (17) and (18), respectively. Therefore, it follows logically in conclusion that the anisotropic SH wave relation premised upon a Maxwell rheology is equivalent to the anisotropic Maxwell equations in a mathematical analogy such that the forcing function expression would be the magnetic current.

INTENTIONALLY LEFT BLANK.

5. REFERENCES

Auld, B. A. Acoustic Fields and Waves in Solids. Malabar, FL: Robert E. Krieger Publishing Co., vol. 1, 1990.

Ben-Menahem, A., and S. G. Singh. Seismic Waves and Sources. New York: Springer-Verlag, 1981.

Carcione, J. M., and F. Cavallini. "Energy Balance and Fundamental Relations in Anisotropic Viscoelastic Media." Wave Motion, vol. 18, pp. 11–20, 1993.

Casula, G., and J. M. Carcione. "Generalized Mechanical Model Analogies of Linear Viscoelastic Behavior." Boll. Geofis. Teor. Appl., vol. 3, pp. 235–256, 1992.

Chew, W. C. Waves and Fields in Inhomogeneous Media. New York: Van Nostrand Reinhold, 1990.

Everitt, C. W. F. James Clerk Maxwell, Physicist and Natural Philosopher, New York: Charles Scribner's Sons, 1975.

Greenfield, R. J., and T. Wu. "Electromagnetic Wave Propagation in Disrupted Coal Seams." Geophysics, vol. 56, pp. 1571–1577, 1991.

Jordan, E. C. Electromagnetic Waves and Radiating Systems. New Jersey: Prentice-Hall, 1950.

Korjack, T. A. "Vibration Diagnostics for the M1 Abrams Tank Gas Turbine Engine." ARL-TR-805, U.S. Army Research Laboratory, Aberdeen Proving Ground, MD, 1995.

Neumann, F. E. Vorlesungen uber die Theorie der Elasticitat. Leipzig, 1885.

Virieux, J. SH-Wave Propagation in Heterogeneous Media: Velocity-Stress Finite-Difference Method. Geophysics, vol. 49, pp. 1933–1957, 1984.

INTENTIONALLY LEFT BLANK.

NO. OF COPIES	ORGANIZATION
2	DEFENSE TECHNICAL INFO CTR ATTN DTIC DDA 8725 JOHN J KINGMAN RD STE 0944 FT BELVOIR VA 22060-6218
1	DIRECTOR US ARMY RESEARCH LAB ATTN AMSRL OP SD TA 2800 POWDER MILL RD ADELPHI MD 20783-1145
3	DIRECTOR US ARMY RESEARCH LAB ATTN AMSRL OP SD TL 2800 POWDER MILL RD ADELPHI MD 20783-1145
1	DIRECTOR US ARMY RESEARCH LAB ATTN AMSRL OP SD TP 2800 POWDER MILL RD ADELPHI MD 20783-1145
	ABERDEEN PROVING GROUND
5	DIR USARL ATTN AMSRL OP AP L (305)

NO. OF COPIES	ORGANIZATION
	ABERDEEN PROVING GROUND
8	DIR, USARL ATTN: AMSRL-SC-II, T. KORJACK (4 CP) R. HELFMAN J. DUMER D. ULERY AMSRL-SC-I, J. GANTT

USER EVALUATION SHEET/CHANGE OF ADDRESS

This Laboratory undertakes a continuing effort to improve the quality of the reports it publishes. Your comments/answers to the items/questions below will aid us in our efforts.

1. ARL Report Number ARL-TR-988 Date of Report March 1996

2. Date Report Received ______

3. Does this report satisfy a need? (Comment on purpose, related project, or other area of interest for which the report will be used.) ______

4. Specifically, how is the report being used? (Information source, design data, procedure, source of ideas, etc.) ______

5. Has the information in this report led to any quantitative savings as far as man-hours or dollars saved, operating costs avoided, or efficiencies achieved, etc? If so, please elaborate. ______

6. General Comments. What do you think should be changed to improve future reports? (Indicate changes to organization, technical content, format, etc.) ______

CURRENT ADDRESS

Organization

Name

Street or P.O. Box No.

City, State, Zip Code

7. If indicating a Change of Address or Address Correction, please provide the Current or Correct address above and the Old or Incorrect address below.

OLD ADDRESS

Organization

Name

Street or P.O. Box No.

City, State, Zip Code

(Remove this sheet, fold as indicated, tape closed, and mail.)

(DO NOT STAPLE)

DEPARTMENT OF THE ARMY

OFFICIAL BUSINESS

NO POSTAGE NECESSARY IF MAILED IN THE UNITED STATES

BUSINESS REPLY MAIL
FIRST CLASS PERMIT NO 0001,APG,MD

POSTAGE WILL BE PAID BY ADDRESSEE

DIRECTOR
U.S. ARMY RESEARCH LABORATORY
ATTN: AMSRL-SC-II
ABERDEEN PROVING GROUND, MD 21005-5067