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The Free Vibration Behaviour of Ring Stiffened Cylinders – A Critical Review of the Unclassified Literature

C. Norwood

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ABSTRACT

The free vibration characteristics of a submarine hull have an important influence on the noise signature. A submarine hull, or portion of one, can frequently be idealised as a ring stiffened cylinder subjected to external loading from the surrounding water, for the purposes of vibration analysis. The modal behaviour of ring stiffened cylinders is reviewed, including the effect of external pressure loading and added mass effects from surrounding fluid. The existing unclassified literature is inadequate in its coverage of the problem and these shortcomings are discussed, in order to identify the requirements for futher work in order to be able to satisfactorily analyse a submarine hull structure.

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Executive Summary

The determination of the natural frequencies of vibration is of particular interest to the submarine community. The whole body vibrational modes provide the opportunity for detection and classification through techniques such as low frequency active sonar. There are a number of factors which influence these low frequency vibrational modes. The ring stiffeners, end conditions, deep frames and bulkheads, external hydrostatic pressure and the mass of the surrounding water will all have an effect on the vibrational behaviour. The existing literature contains numerous papers which consider the problem, and the influence of one or other of the factors listed above. However, none of the papers covers all the factors. This paper reviews the available unclassified literature and discusses the influence that each of the factors has on the vibrational modes.

The ring stiffeners can be treated analytically by smearing their effect to produce an equivalent uniform shell. The treatment will provide accurate estimates of the whole body modes, but will not provide any insight on interstiffener behaviour. The boundary conditions assumed during the analysis have a large influence on the result. It is possible to analyse a long cylinder that is subdivided by bulkheads and deep frames in its individual sections. The bulkheads can be replaced by shear diaphragm boundary conditions and the deep frames by built in boundary conditions. The external hydrostatic pressure will place the shell in compression and hence reduce the modal frequencies. There is a correlation between the model frequency behaviour and the elastic instability (buckling) behaviour of the shell. A plot of frequency squared vs pressure produces a straight line, with the zero frequency point being the buckling pressure. The mass of the external water will also reduce the modal frequencies. Analytical formulae to estimate the reduction in modal frequency are available, as well as the use of numerical techniques such as finite elements. The combined effect of external pressure and water mass loading may be accounted for using superposition of the two different effects of the unpressurised, invacuo modal frequencies.

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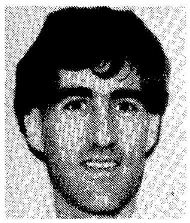


Chris Norwood graduated with a degree in Mechanical Engineering in 1975 and was awarded a MEngSci in 1977 from Melbourne University. After working for a year in industry and four years as a research engineer at MRL and Loughborough University (UK), from where he received an MSc in 1980, he taught at Victoria University of Technology for eight years. He joined MRL in 1990 and has worked on the reduction of ship and submarine noise and vibration. His current position is Senior Research Engineer in the Ship Structures and Materials Division.

Contents

1. Introduction

A problem of particular interest in submarine signature management is the determination of the natural frequencies and mode shapes of the submerged structure. The acoustic signature is strongly influenced by hull vibrations, particularly at low to medium frequencies, [1][2]. These lower frequencies are also the most sensitive to a variation in external environment. For instance an increase in external pressure, due to an increase in operating depth, will have the greatest affect on the lower frequencies of the submarine hull. The control of these low frequency modes is assuming greater significance in acoustic signature management as sonar systems are improved.

The free vibration behaviour of ring stiffened cylinders has been studied since the mid-1950's, [3][4][5]. However, few papers specifically deal with the effects of boundary conditions, external hydrostatic pressure and mass loading due to water. These topics are of crucial importance to the modal behaviour of submarine hulls.

In treating a portion of the hull as a ring stiffened cylinder it is necessary to have a knowledge of the appropriate boundary conditions that are required to be applied, and a knowledge of the effect of boundary conditions on the vibrational behaviour. These boundary conditions may be dictated by deep-ring stiffening frames, bulkheads or more complex structural components such as end caps.

Through the 1970's a large amount of work on the effects of external loadings on the behaviour of stiffened cylinders was undertaken at the Israel Institute of Technology, [6]-[9]. This work showed the correlation between cylinder vibrational behaviour and buckling behaviour. In addition, the vibrational behaviour for various boundary conditions was investigated. The research led to methods of experimentally determining cylinder boundary conditions and a method of predicting buckling loads from a knowledge of the cylinder's natural frequencies. The work gives an insight into the effects of external loading and boundary conditions on the free vibration behaviour of cylinders. It, however, focuses on aeronautical structures, with most attention being given to axial stiffening, rather than ring stiffening. Additionally, the external loads considered were mostly axial rather than hydrostatic. So while this work is of general interest it does not provide much insight into submarine structures.

To the author's knowledge there are no reviews which deal comprehensively with the modal behaviour of ring stiffened cylinders. In 1992 Mukhopadhyay and Sinha, [10], found that there was no comprehensive review dealing with stiffened shells in general. Leissa's review of shell vibration only deals briefly with stiffened shells, [11]. This report is intended to provide a review of the vibrational behaviour of ring stiffened cylinders and to provide a background of knowledge in this topic. Of particular interest are those areas which are of importance to submarine type structures. It is intended to highlight those previously mentioned topics which have not been dealt with thoroughly, and thereby identify what further work is required in this area. In the report the finite element method is used extensively to calculate frequencies and mode shapes for comparison with other theoretical or empirical formulae, in order to assess the suitability of the method for further numerical studies.

2. Review of Modal Behaviour

It is convenient to describe cylinders with the cylindrical coordinate system shown in Figure 1. The coordinate system has its origin at the mid-surface of the cylinder shell. Here, *u, v, w* describe the axial, circumferential and radial deformations respectively.

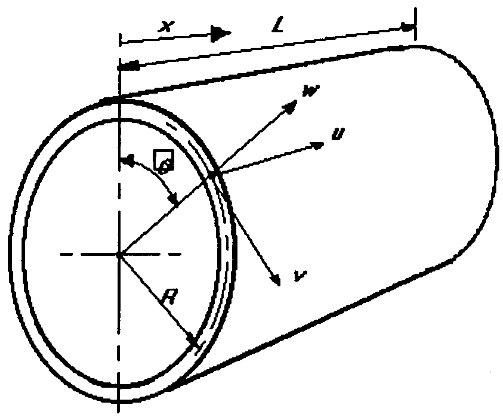


Figure 1 Coordinate system for cylinders

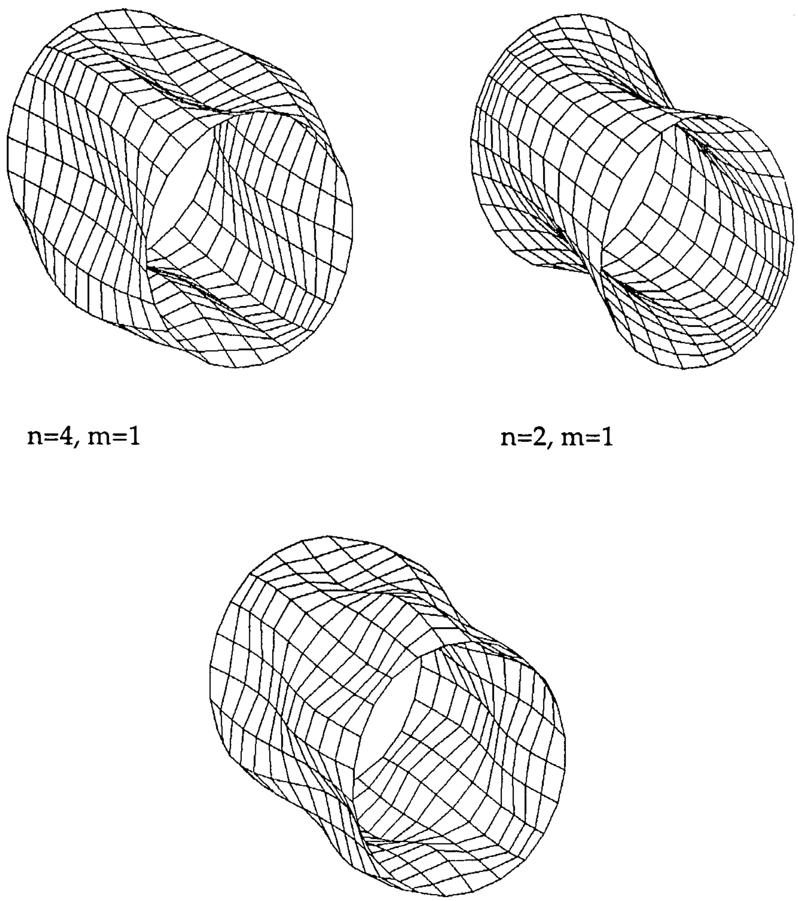
It is instructive to first consider the matter of boundary conditions. The term "simply supported" is often used to describe boundary conditions at the cylinder ends. It is generally taken to mean

$$\left.\begin{array}{l} u \neq 0 \\ v = w = 0 \\ w' = \partial w / \partial z \neq 0 \end{array}\right\} \; x = 0, L \qquad (1)$$

This terminology has been borrowed from beam and flat plate theory. It is however inadequate for fully describing the boundary conditions of a cylinder. While it indicates hinging around the circumference at the cylinder ends, the terminology does not impart any information about the behaviour of u, v or w. Another term frequently used to describe the above boundary conditions is "freely supported".

Leissa [11] in his comprehensive coverage of shell vibrations preferred to adopt the term "shear diaphragm" boundary conditions. This is because a cylinder enclosed at its ends by thin, flat plates will exhibit behaviour very similar to one with the boundary conditions described in (1). The flat plate has substantial stiffness in its own plane, but has low resistance in the out of plane direction, due to comparatively low bending stiffness. As such, v and w will be restrained, while u and w' will be unrestrained.

In characterising the various vibration modes of a cylinder it is useful to introduce n and m wave numbers. The n and m numbers give the number of circumferential waves and the number of axial waves, respectively, for a particular mode of vibration, Figure 2. The mode shapes were determined using finite element analysis, with shear diaphragm boundary conditions assumed.



n=4, m=1

n=2, m=1

n=5, m=2

Figure 2 Modal patterns for a freely vibrating cylinder

The n = 0 modes correspond with breathing behaviour, where the cross-section at any axial location remains circular and there is no circumferential bending of the shell involved. An n number of 1 corresponds to beam-type bending vibrations (not illustrated here). In this type of mode the cross-section remains undeformed. These bending modes will be more significant for longer, more slender cylinders, which more closely approximate a beam or column. The fundamental bending mode is n = 1, m = 1.

Unlike many other structures, such as flat plates, the simplest modes of vibration (ie lowest n and m numbers) do not have the lowest natural frequencies. The lowest natural frequency will generally occur for a mode with n greater than one. This is due to the sharing of the bending and stretching strain energies within the cylinder shell, [12]. The actual value of n for the lowest mode depends upon the geometry of the cylinder in question. A long slender cylinder may have a fundamental mode with n =

1, m = 1, as it largely behaves as a beam or column. More typical behaviour is shown in Figures 3 and 4, for two different cylinder geometries. In both cases the cylinders have a radius of 2 m and a shell thickness of 20 mm, (ρ = 7800 kg/m^3, E = 200 GPa and μ = 0.3). The cylinder in Figure 3 is 6m long, while the cylinder in Figure 4 is 3m long and shear diaphragm boundary conditions have been assumed.

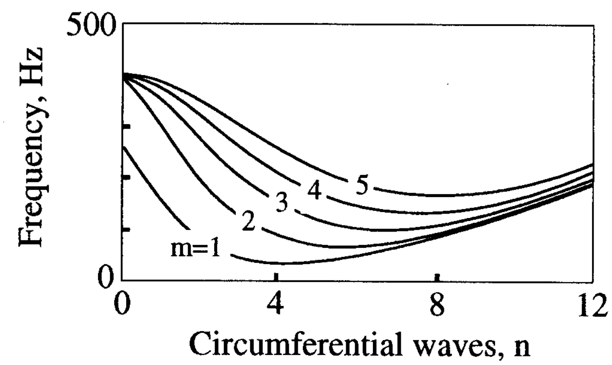


Figure 3: Natural frequencies of an unstiffened cylindrical steel shell, shear diaphragm boundary conditions, geometry: R = 2 m, L = 6 m, h = .02 m.

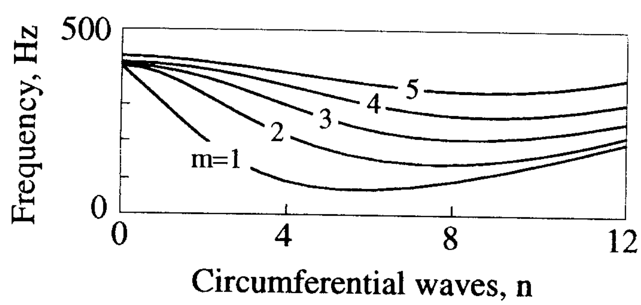


Figure 4: Natural frequencies of an unstiffened cylindrical steel shell, shear diaphragm boundary conditions, geometry: R = 2 m, L = 3 m, h = .02 m.

These results were derived analytically from the Donnell-Mushtari equations,[11]. The Donnell-Mushtari equations are the simplest of a range of eighth order differential equations which have been used by different researchers to describe the dynamic behaviour of unstiffened cylindrical shells. The natural frequencies of the system are found by substituting appropriate boundary conditions and solving for the eigenvalues of the system of equations. For shear diaphragm boundary conditions the displacements are assumed to have the following form, which satisfy the boundary conditions,

$$u = A\cos\frac{m\pi x}{L}\cos n\theta\cos\omega t$$
$$v = B\sin\frac{m\pi x}{L}\sin n\theta\cos\omega t \qquad (2)$$
$$w = C\sin\frac{m\pi x}{L}\cos n\theta\cos\omega t$$

where A, B and C are unknown constants and ω is the frequency (in radians per second) of the vibration. Substitution into the Donnell-Mushtari equations yields a sixth order polynomial (characteristic equation) of the form

$$\omega^6 - \alpha_1\omega^4 + \alpha_2\omega^2 - \alpha_3 = 0 \qquad (3)$$

where the coefficients α_1, α_2 and α_3 are functions of m, n, the cylinder geometry and the material properties. Thus for a known cylinder and chosen values of m and n the equation can be solved, yielding three solutions for ω. Thus for any m and n combination there are three natural frequencies, each of which corresponds to a distinct mode of vibration, despite all three having the same number of circumferential and longitudinal waves. Mathematically this means that these three modes will each be characterised by a unique set of values for A, B and C in equations (2).

The lowest value of the three roots is generally much smaller than the other two. This lowest frequency mode is primarily radial. The two higher modes tend to be either primarily longitudinal or primarily circumferential. In Figure 5 all three frequencies have been plotted for m=1 for the 6m long cylinder considered above. The roots have been derived from the Donnell-Mushtari equations, but this time all three roots have been obtained.

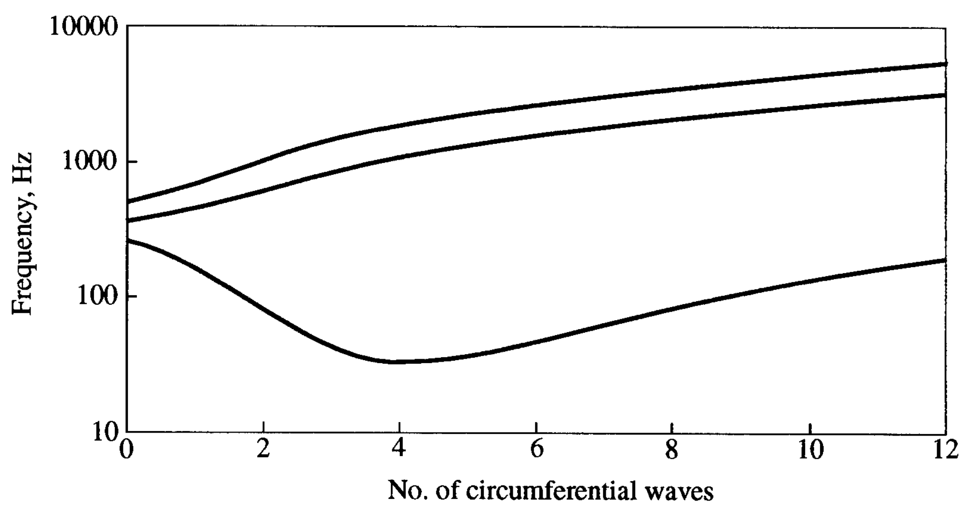


Figure 5: All three natural frequencies of a cylindrical shell, as given by solving equation (3). Results are for m = 1 only. Cylinder is that of Figure 3.

The higher modes, being primarily longitudinal and circumferential involve motion tangential to any surrounding fluid and will not be efficient sound radiators. The lowest mode, however, is primarily radial and will be the strongest sound radiator. It has also been found that the two higher modes are not significantly affected by external hydrostatic pressure. This is further discussed in section 6. In practice the two higher frequencies are usually neglected, and only the lowest frequency is generally reported in the literature, as it is in this report.

3. Effect of Ring Stiffeners

The free vibration modes of ring stiffened cylinders are often very similar in appearance to those of an unstiffened cylinder. When the rings are uniform and closely spaced, as is the case with a submarine hull, the behaviour can be identified with m and n numbers and the displacements can be described by equations (2). It should be noted here that if the rings are spaced unevenly, or if they have non-uniform properties or if they are heavy and sparse, the vibrational behaviour can be very complex and is beyond the scope of the present report.

The full analytical treatment of the problem is much more complex. However, the problem can be significantly simplified by averaging the effect of the stiffeners over the skin of the cylinder. In this way the stiffened cylinder can be treated as an unstiffened cylinder with orthotropic shell properties. The circumferential shell properties will now be different from the longitudinal shell properties. This method of analysis is often referred to as "smeared stiffener theory", [3]. The method is of course limited to frequencies where the longitudinal wavelength is greater than the interstiffener spacing and it can not predict interstiffener motion, [10].

The frequencies for an unstiffened cylinder compared with those for a stiffened cylinder of the same radius, length and shell thickness, are shown in Figure 6. The stiffeners are rectangular cross section steel rings (depth = 100 mm, width = 40 mm, ρ = 7800, E = 200 GPa) on the inside of the shell spaced 250 mm apart, Figure 7. The results for the unstiffened cylinder were derived analytically using the Donnell-Mushtari equations while the stiffened cylinder results were calculated using finite element analysis.

There is a noticeably sharper rise in the frequencies of the stiffened cylinder with increasing n number than for the unstiffened cylinder, Figure 6. This is to be expected because as the n number increases the rings will contribute proportionally more stiffness, thus increasing the frequency. Also apparent is a fall in frequency for the stiffened cylinder compared to the unstiffened cylinder for low n numbers. This is because there is little skin bending occurring in the circumferential direction at low n numbers, so the stiffeners are not contributing to the stiffness. They are however adding to the inertia of the cylinder thus causing the lower frequency, [11].

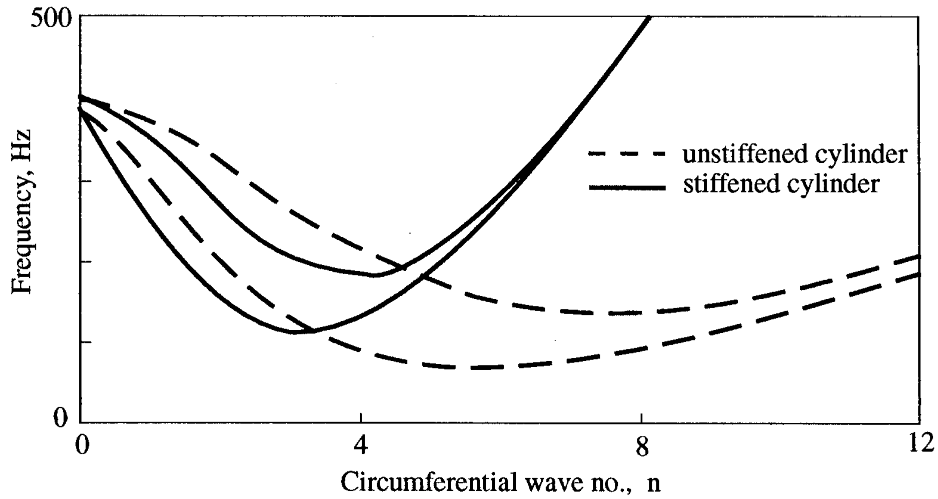


Figure 6: Effect of ring stiffeners on the natural frequencies of cylinders. Plots are for m = 1 and m = 2. Both cylinders are steel with overall geometry: R = 2 m, L = 3 m, h = .02 m, and shear diaphragm boundary conditions. Extra details for the stiffened cylinder given in Figure 7.

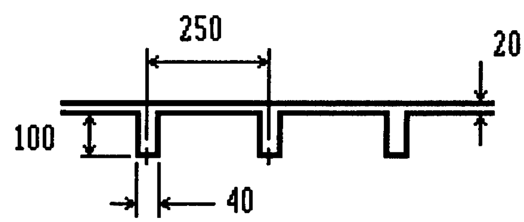


Figure 7: Ring stiffener details for the stiffened cylinder of Figure 6. Dimensions are in millimetres.

4. Boundary Conditions

The shear diaphragm boundary conditions introduced in section 2 have special significance. It was shown by Leissa [11] that an infinitely long cylindrical shell oscillating with a finite axial wavelength, l, will behave identically to a finite cylinder of length $l/2$, with shear boundary conditions and m = 1. This means that the modal behaviour of a cylindrical shell, of length L, with shear diaphragm boundary conditions, and m > 1, can be described by another cylinder of the same radius and wall thickness, but of length L/m and m = 1. This characteristic also applies to uniformly stiffened cylinders provided the behaviour is not affected by the discreteness of the stiffeners. This depends upon the ratio of stiffener spacing to axial wavelength.

This is illustrated in Figure 8. The axial mode shape of a 6m long ring stiffened cylinder, m = 2 and n = 4, is compared with that of a 3m long ring stiffened cylinder, m = 1 and n = 4. The two cylinders have the same radius, shell thickness and stiffener properties. The two cylinders were analysed using finite element methods. The frequencies for the two cylinders were the same, and it is apparent that the mode shape of the shorter cylinder is identical to the half mode shape for the longer cylinder.

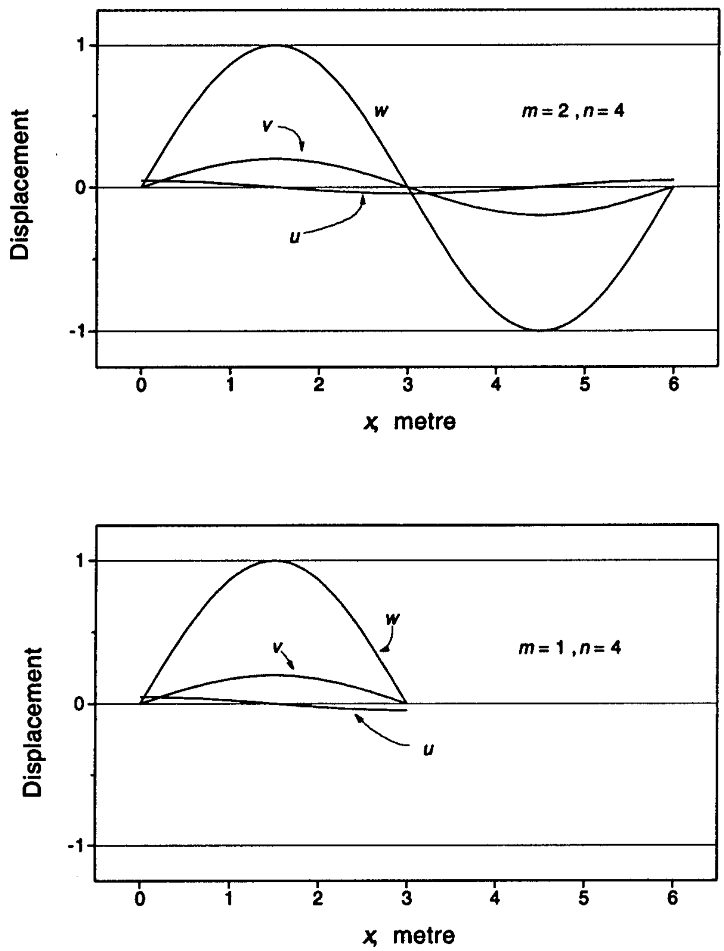


Figure 8: Comparison of mode shapes of two different ring stiffened cylinders. Both cylinders are identical in every respect except length. The top cylinder has L = 6 m while the bottom cylinder has L = 3 m. The mode of the top cylinder is that described by m = 2, n = 4. The other cylinder has m = 1, n = 4. In each case the mode shape has been normalised against the largest value of w.

More generally the modal behaviour of a cylinder, length L_1,with arbitrary boundary conditions and many axial waves (eg $m_1 > 5$) can be approximated by a shorter cylinder, length L2, with shear diaphragm boundary conditions and m2 axial waves, provided that L1/m1 = L2/m2. As an example the mode shape for a 9m long ring stiffened cylinder with totally constrained boundary conditions (u = v = w = w' = 0) and m=3, n=4 is shown in Figure 9. It is apparent that the deflected shape of the centre section is the same as the previously considered 3m long cylinder with shear diaphragm boundary conditions (Figure 8). It is apparent that the importance of the boundary conditions is significantly reduced at only half an axial wavelength from the constrained edge.

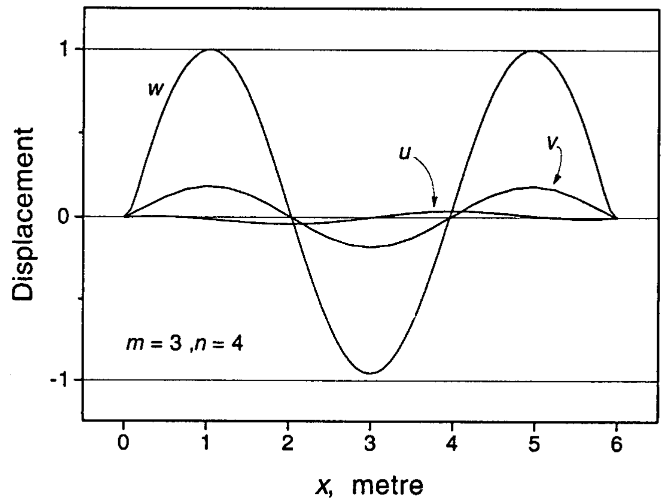


Figure 9: Axial mode shape of a ring stiffened cylinder with totally constrained boundary conditions, $u = v = w = w' = 0$ at both ends, oscillating with $m = 3$, $n = 4$.

The influence of boundary conditions on the modal behaviour of unstiffened cylindrical shells has been studied comprehensively by Forsberg, [13], who analysed the effect of constraining different combinations of u, v, w and w' at the two ends of a cylinder with linear, isotropic material properties. Based on these findings the following two observations can be made. Boundary conditions have decreasing importance as the values of L/R and m increase. The effect of axial restraint (u = 0) at the cylinder ends is generally far more significant than the effect of restraining rotation (w' = 0) at the cylinder ends.

For a cylinder with clamped ends (u ≠ 0, v = w = w' = 0), R/h = 20 and L/R = 1, the natural frequency will be up to seven percent higher than an identical cylinder with shear diaphragm boundary conditions. The difference is only three percent for L/R = 2, and continues to decrease as L/R increases. The effect of clamping also reduces as the shell becomes thinner. For R/h = 100 and L/R > 2 the effect of clamping is insignificant, while for R/h = 500 the effect of clamping becomes insignificent for values of L/R as low as 0.5.

The case where the cylinder is simply supported with axial restraint, (u = v = w = 0 and w' ≠ 0) is very different. A cylinder with these boundary conditions and R/h = 20 will have natural frequencies up to 50 percent higher than the shear diaphragm case for values of L/R as high as 10. Additionally a thinner shell will not necessarily be less sensitive to the boundary conditions. For a cylinder with R/h = 500 the frequency can be nearly 100 percent higher than the shear diaphragm case, and it is not until L/R approaches 100 that the difference becomes negligible.

The above clearly indicates that the out of plane stiffness of end plates or closures will generally be far more important than the degree of rotational restraint that is applied to the cylinder, [13].

5. End Closures

A recent analysis of the behaviour of an unstiffened cylinder closed at one end by a plate and subjected to shear diaphragm boundary conditions at the other has been made by Cheng and Nicolas [14]. Both the cylinder and the plate had R/h = 30 and the same material properties. The study concluded that analysing an open cylinder with enforced boundary conditions generally gave results comparable with the behaviour of the cylinder plate structure. Replacing the plate with clamped boundary conditions ($u \neq 0$, $v = w = w' = 0$) gave better results than assuming shear diaphragm conditions at both ends, because of the relatively thick end plate. Nevertheless, the differences between the clamped case and the shear diaphragm case were small for L/R greater than two, especially for higher axial wave numbers ($m > 1$).

A cylinder-plate structure will have modes where the plate vibrations predominate, and others where the cylinder modes are secondary. However, it is usually the case that these modes are quite uncoupled from the cylinder. Conversely the cylinder modes are often uncoupled from the plate. It is for this reason that the cylinder behaviour for the cylinder-plate structure can frequently be modelled by a cylinder in isolation with appropriate boundary conditions, when these conditions occur. The study by Cheng and Nicolas [14], however, does show that there are instances where strong coupling between the plate and cylinder occur. This happens when the frequency of the plate in isolation, for a particular value of n, is the same as the frequency of the cylinder in isolation, for the same value of n. In such instances the modal characteristics of the cylinder in isolation will be less accurate. The study suggested, however, that the frequency of the structure will still be close to that predicted by the cylinder in isolation.

For longer cylinders where the cylinder frequencies tend to be lower than most of the plate frequencies the significance of plate/cylinder coupling is less. Also, for longer cylinders the interaction occurs over a much narrower range of L/R.

In the case of submarines and aircraft, end closures are usually stiffened and may be hemispherical, significantly raising their frequencies. Galletly and Mistry, [15], used a variational finite difference method to study the modal behaviour of cylinders closed at one end by various shells of revolution (cones, hemispheres, ellipsoids, etc.) and totally fixed at the other. The cylindrical section had an L/R of 1 and R/h = 50. The boundary conditions at the fixed end were u = v = w = w' = 0.

They found that for n > 3 and m = 1 most of the modal displacement occurred in the cylinder. This suggests an ability to study these structures, for m = 1, as isolated cylinders with specified boundary conditions. Correspondingly it was found that the lower n modes were most affected by the change in geometry of the shells of revolution. However, the frequencies of the combined shell-cylinder structure did not vary greatly for the different end closures. For the lowest four natural frequencies of the combined structure, over the entire range of end closures studied, the maximum difference in frequency prediction was 70 percent. For longer cylinders the effect of end shape would be further reduced.

The effect of not constraining v was also investigated. This was found to cause substantial reductions in the frequencies of the n = 1 modes for all the structures studied. In many cases it resulted in the n = 1 mode becoming the fundamental frequency for the structure. It was also found that as the value of R/h was increased , while holding L/R constant, this effect was reduced. For R/h = 500 and L/R = 1 the effect of freeing v was very small. The importance of constraining v on the n = 1 modes was also observed by Forsberg, [13], for isolated cylindrical shells.

The discussions above concerning the effect of end closures have referred to unstiffened cylindrical shells. There is little literature available specifically dealing with the influence of boundary conditions and end closures on the modal behaviour of ring stiffened shells. Rosen and Singer, [6] [16], discuss the behaviour of stiffened cylinders under axial loading, but concentrate on axially stiffened cylinders rather than ring stiffened ones. Additionally, the behaviour of axially loaded cylinders is generally quite different from that of unloaded or hydrostatically loaded cylinders.

There is a strong correlation between the vibrational behaviour and the linear buckling of cylindrical shells, with the correlation between the two types of behaviour strongest at lower frequencies, where the vibration modes closely resemble the cylinder instability modes. The influence of boundary conditions on the natural frequencies of stiffened cylinders is very similar to the influence on the buckling behaviour of these cylinders, [8].

It is therefore useful to consider the effect of boundary conditions on the buckling behaviour. For axially loaded cylindrical shells, the influence of boundary conditions is very similar for both ring stiffened and unstiffened shells, [17][18]. This can be contrasted with the situation for axially stiffened shells, where the behaviour differs quite markedly from that of unstiffened shells, [17]. In the case of hydrostatic pressure the degree of similarity between ring stiffened and unstiffened shells depends on the relative bending stiffness of the rings. Cylindrical shells with relatively light rings will behave similarly to unstiffened shells, but as the ring stiffener becomes heavier their behaviour will diverge.

In a submarine hull the ring stiffened cylindrical shell is separated into compartments by bulkheads or deep frames. The modes of a long segment of ring stiffened shell with a double bulkhead at each end and a deep frame stiffener were calculated using finite element methods. The bulkheads and deep frame were then replaced by differing boundary conditions and the modes recalculated for comparison. Boundary conditions considered were shear diaphragm and built-in, but with $v \neq 0$.

For all modes considered , except n = 2 m = 1, the two shell sections (divided by the deep frame) act independently, and can be analysed separately. Replacing the double bulkhead with a pair of shear diaphragm constraints gave modal frequencies less than 1% different from those originally calculated. It was found that the built-in boundary conditions more accurately replicated the deep frame, rather than shear diaphragm constraint. This is because the heavy frame has a large rotational inertia and therefore offers considerable bending reinforcement to the shell. From the modelling it was concluded that the separate sections of the hull could satisfactorily be analysed independently, with the double bulkhead and deep frame being replaced by suitable boundary conditions. Only calculation of the n = 2 m = 1 mode required the entire shell to be analysed.

The above suggests that the boundary effects on the modal behaviour of ring stiffened shells will often be similar to that for unstiffened shells, particularly for relatively light stiffening ribs. However, care should be taken when extending the results of unstiffened shells to stiffened shells with heavy ribs and more work in this area would be useful.

6. External Hydrostatic Pressure

An increase in external hydrostatic pressure acting on a cylindrical shell will decrease the values of the resonant frequencies. As the cylinder buckling pressure is approached, the resonant frequency approaches zero, [19]. An alternate statement of this, is that as the instability pressure corresponding to a particular combination of m and n is approached, the frequency of that m-n mode will approach zero. This similarity between the vibration modes and the instability modes, particularly for the lower frequencies, was noted by Singer, [7].

A ring stiffened cylinder under hydrostatic pressure can fail in a number of different ways. These include local modes, such as inter-stiffener buckling where lobes develop between the rings, yielding of the shell between the rings and stiffener tripping where the rings buckle first,[20], as well as general instability mode of the shell. It is usually the general instability mode that is of most interest when considering cylinder vibrations. This mode of failure can occur between bulkheads or deep frames, [21], and is caused by elastic buckling of the frame and shell in unison. These general instability modes occur at low m numbers and have the an appearance similar to the vibration modes in Figure 2. However, sometimes large m number modes can also be of interest when considering hydrostatic pressure. The elastic instability of these high m number modes can correspond to relatively low pressures. These modes will tend to correspond with the inter-stiffener buckling mode. As such, they can be sensitive to hydrostatic pressure

A plot of frequency squared versus external pressure for three different modes is shown in Figure 10. This is for the 6m long ring stiffened cylinder previously considered, with shear diaphragm boundary conditions and shows a linear relationship between the square of the frequency and the external pressure. The frequencies were determined using finite element analysis. The instability pressure for each of the modes under investigation is also shown in Figure 10. These were determined using a method developed by Kendrick, [22], for the prediction of buckling. The equations were originally developed to determine instability pressures in two axial modes, either m=1, or an m number corresponding to inter-stiffener buckling, for an arbitrary n number. One of these two modes will provide the minimum instability pressure. The equations may be extended to the case of arbitrary m number with relative ease.

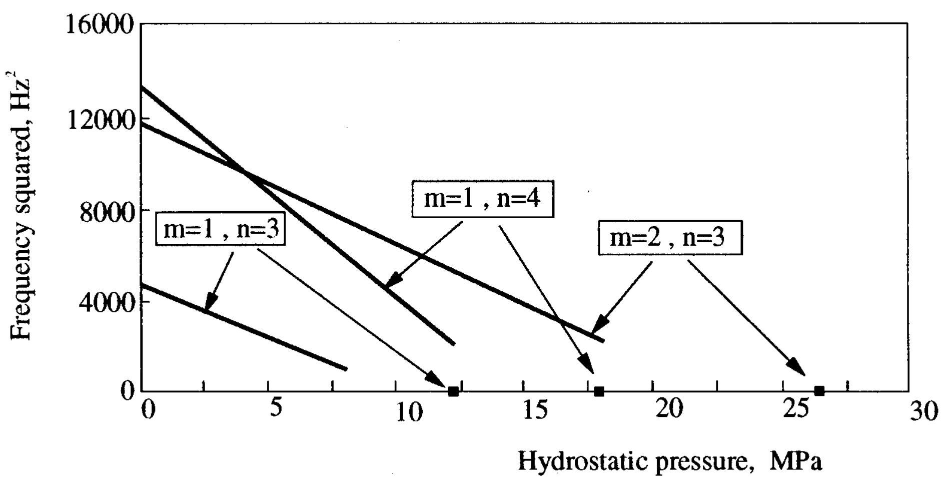


Figure 10: The effect of external hydrostatic pressure on the square of the frequency for three particular modes of vibration of a ring stiffened cylinder with shear diaphragm supports. Also, plotted along the horizontal axis are the three corresponding Kendrick (1953) instability pressures.

To determine which modes will be most sensitive to hydrostatic pressure it is instructive to review the instability analysis of cylinders more closely. In doing so it must be remembered that we are not interested in buckling as such, since buckling analysis is concerned with the first failure mode, and this often occurs at a lower pressure than that given by the elastic stability analysis. This is due to a degree of nonlinear behaviour occurring prior to buckling. However, as the buckling pressure is higher than the working pressure this nonlinearity can be ignored over the range of practical working pressures, [23]. Since the modal vibrations are small in magnitude and linear it is valid to consider the linear-elastic instability analysis for comparison.

The instability pressure as a function of circumferential wave number, n, is plotted for a range of m numbers in Figure 11. The curves show that the modes which are most sensitive to external pressure occur for m=1 and n in the range of 2 to 4. These are the general instability modes and correspond to the lowest frequencies of vibration. If an instability mode were possible for m=2 it would occur at a much higher pressure.

As the m number increases to 4 the modes become less sensitive to hydrostatic pressures. A further increase in m number to 6 reveals another dip in the instability pressures, implying greater sensitivity to hydrostatic pressure. This dip reaches a minimum for m ≈ 12, and values of n between 5 and 6. There is not a significant increase in pressure in this region until m is greater than 20.

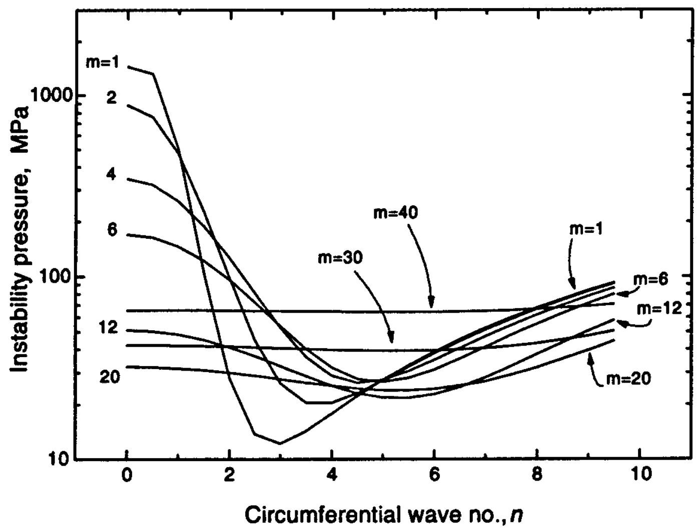


Figure 11: Instability pressures for a ring stiffened cylinder under hydrostatic load according to Kendrick's (1953) smeared stiffener equations.

The important observation here is that for ring stiffened cylinders at large m numbers there is a second region which is relatively sensitive to external hydrostatic pressure. This region is caused by the axial component of the pressure loading. Ignoring this axial component in the cylinder loading eliminates the second dip and results in an increase in instability pressure with increasing m number.

The appearance of the second dip is not due to an inter-stiffener effect, as similar results are obtained using smeared stiffener theory. For modes with axial wavelengths equal to or smaller than the stiffener spacing the instability pressures can be approximated by analysing the cylinder without stiffeners, and sufficiently high m number. The instability pressures for an unstiffened cylinder, but with the same overall geometric and material properties, are plotted in Figure 12. The first inter-stiffener instability mode, m=24, occurs at a pressure of approximately 30 Mpa, which is close to the values at the second dip predicted by smeared stiffener theory, Figure 11.

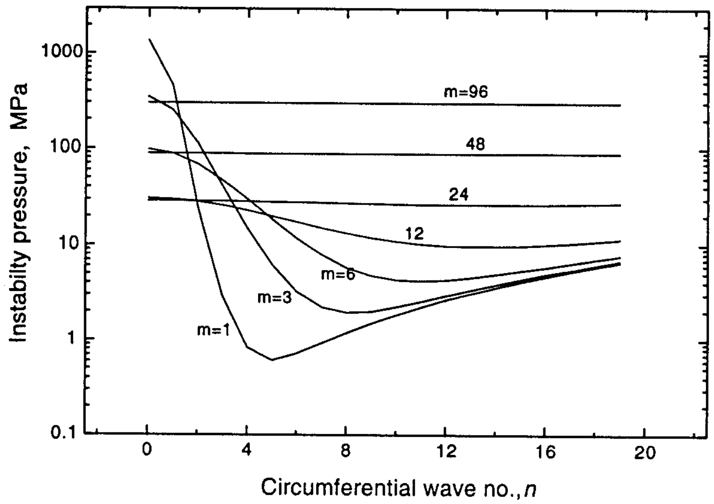


Figure 12: Instability pressures for an unstiffened cylinder under hydrostatic load. Cylinder has same overall geometry and material properties as the ring stiffened cylinder of Figure 11.

7. Added Mass of Water Effect

The analysis considered so far has assumed that there is negligible effect on the vibrations of the structure from the surrounding fluid. This assumption is acceptable when the density of the fluid is much less that that of the structure, for instance metallic structures in air. However when the density of the fluid is approaches that of the structure, as occurs with submarines etc, then the added mass effect from the surrounding fluid must be considered.

The surrounding body of fluid will reduce the modal frequencies, compared with those in a vacuum, while the mode shapes remain similar to those in a vacuum. The extra damping due to the water will have a negligible effect on the modal frequencies, [19].

When a structure is oscillating in a fluid its motion is not only resisted by its own inertia but also by the inertia of fluid that is entrapped by the structural motion and moves with the structure. This results in an apparent increase in the magnitude of the mass of the structure, which is commonly referred to as the added mass. The added mass depends upon the direction of motion of the structure and the frequency. As the frequency increases the added mass effect diminishes, [24]. The effect of the added mass on the modal frequencies is given by the expression

$$\frac{f_w}{f} = \frac{1}{(1 + M_a / M)^{1/2}} \qquad (4)$$

where f_w is the submerged frequency, f is the *in vacuo* frequency, M_a is the added mass of fluid and M is the structural mass.

The added mass depends upon the geometry of the structure, the nature of the motion and the frequency. For the simple case of rigid body motion the added mass may be determined from experiment or by the application of potential flow theory, [25]. For nonrigid oscillations however the determination of the added mass is far more complex. There is no closed form solution for the calculation of the added mass acting on a finite cylinder, [24] [25]. For a cylinder oscillating as a flexible body, the determination of added mass must rely on approximate methods. For a plate or cylindrical shell the ratio of added mass of water to structural mass is given by, [24] [25], [26].

$$\frac{M_a}{M} = \alpha \frac{\rho_w R}{\rho h} \qquad (5)$$

where α is a non-dimensional constant which is a function of the mode shape, ρ_w is the density of the fluid and ρ is the mean structural density. The structural mass per unit area of a plate or shell is given by ρh. For a ribbed stiffened shell the mass per unit surface area must include the mass of the stiffeners.

For a cylinder with shear diaphragm boundary conditions a first order estimate of α is given by $\alpha \leq 1/n$, [25], where n is the circumferential mode number. This will give a lower bound for the submerged frequency. A more accurate expression, including the effect of the longitudinal wave number, m, is given by, [26]

$$\alpha = n^2/((n^2 + 1)\,(n^2 \quad + (m\pi R/L)^2\,)^{1/2}\,) \qquad (6)$$

This expression was derived using theory related to an infinite cylindrical shell for n>0. As discussed above in section 4, the mode shapes of a finite cylinder with shear diaphragm boundary conditions are identical to those of an infinite cylinder over a finite axial length. Using this reasoning equation (6) may be used to calculated the approximate added mass on a finite cylinder. There will however be other effects associated with the finite shell, such as boundary effects, not accounted for in the infinite cylinder formulation, [26]. The mode shapes of finite shells with boundary conditions other than shear diaphragm are frequently similar to shells with shear diaphragm boundary conditions, especially for long slender cylinders and at points away from the ends, (section 4). It is therefore reasonable to extend the application of equation (6) to determine the approximate added mass for finite cylinders with other boundary conditions.

For more accurate determination of submerged frequencies and the effect of the fluid added mass it is necessary to use numerical methods such as finite element and boundary element techniques. Everstine [24] used finite element and boundary element techniques to determine the frequencies of a submerged cylindrical shell with flat end closures. To determine the effect of the surrounding water, the frequencies for the cylinder were determined both *in vacuo* and in water. The cylinder studied had a radius of 5 m, length of 60 m and shell thickness of 0.05 m. The ratio of submerged frequencies to *in vacuo* frequencies is shown in Figure 13. These show excellent agreement with the theoretical ratios predicted by equations (4) to (6).

Gilroy, [29], modelled the Everstine cylinder using the COUPLE/VAST,[30], suite of computer programs. The *in vacuo* modal frequencies were within a few percent of the results obtained by Everstine, however the results for the submerged cylinder differed significantly. Gilroy concluded that without experimental validation it was not possible to determine the cause of the differences. This does however highlight the problems of trying to calculate the submerged modes using finite element analysis, and the care that is required to obtain meaningful results.

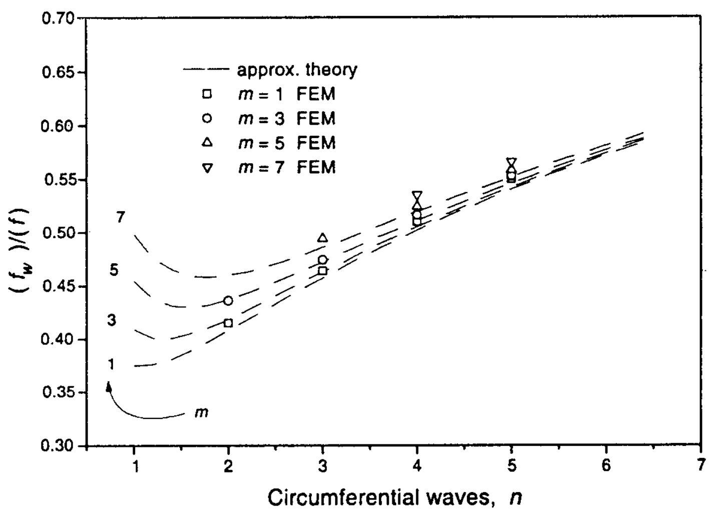


Figure 13: Ratio of submerged frequencies to in vacuo frequencies for a cylindrical shell. Finite element results from Everstine (1991).

Ross and Johns [27] and Ross and Richards [28] compared experimentally measured frequencies to FEA predictions for the modal frequencies of both circular cylinders and ring stiffened cones under external water pressure. His results show excellent agreement between the two. However, the tests and analysis were performed for the specimens submerged in a small pressurised tank and therefore cannot be compared to ratios predicted by the equations of Junger and Feit since there was not sufficient water loading.

8. Conclusions

The free vibrational characteristics of cylindrical shells used for underwater applications are affected by ring stiffeners, external pressure loadings and the added mass of the surrounding water. The external pressure and surrounding mass of water both act to reduce the modal frequencies, while ring stiffeners will increase the modal frequencies when compared to unstiffened shells.

When analysing shells that are divided by deep ring stiffeners or bulkheads it is possible to treat the subsections individually, provided appropriate boundary conditions are applied, thus reducing the size of the problem that must be modelled. Numerical analysis, such as the finite element technique can be used to calculate the expected vibrational behaviour of shells with good accuracy. The effects of external pressure may be included with FE analysis, and the water mass loading may be included using either specially formulated fluid elements or by the using boundary elements to model the external water continuum.

Additional work is required in several areas. The first is to investigate the behaviour of cylinders with internal bulkheads and deep frame stiffeners, as occur in submarines. In particular the way in which the modal behaviour of the individual sections can be treated seperately, without reference to the other sections. The second, concerns the effect of the external water loading. The accuracy of commercial FEA and boundary element packages in predicting the effects of water loading on the modes of submerged cylinders should be considered. Many of these packages contain fluid elements, which have the ability to interact with structures, and the accuracy of these elements in modelling the modal behaviour of submerged cylinders compared to results obtained with special element formulations, such as those used by Everstine (26), should be examined.

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The Free Vibration Behaviour of Ring Stiffened Cylinders - A Critical Review of the Unclassified Literature

C. Norwood

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20. ABSTRACT

The free vibration characteristics of a submarine hull have an important influence on the noise signature. A submarine hull, or portion of one, can frequently be idealised as a ring stiffened cylinder subjected to external loading from the surrounding water, for the purposes of vibration analysis. The modal behaviour of ring stiffened cylinders is reviewed, including the effect of external pressure loading and added mass effects from surrounding fluid. The existing unclassified literature is inadequate in its coverage of the problem and these shortcomings are discussed, in order to identify the requiremnets for futher work in order to be able to satisfactorily analyse a submarine hull structure.