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Stability of Honeycomb Sandwich Cylinders

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This paper treats the instability of a honeycomb-sandwich cylinder under axial compression, torsion, and external lateral pressure. The simply supported honeycomb-sandwich cylinder considered here has face sheets of equal thickness and an isotropic honeycomb core. Curves are presented for the cases of axial compression and lateral pressure only, as well as torsion only. The combined case is handled by means of interaction equations based on the curves shown.

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Stability of Honeycomb Sandwich Cylinders

MEHMET O. KICIMAN **DON Y. KONISHI**

NOMENCLATURE

b = circumferential dimension (= $2\pi R$)
= core thickness
D = flexural rigidity of panel $\left(= \frac{E_f\ I}{1-u^2}\right)$
E_f = Young's Modulus for the face sheet
f = actual stress
F = allowable stress
$G_c^{\prime}$ = core shear modulus
J = a parameter representing ratio of shear to flexual stiffness $\left(= \frac{L^2 u}{\pi^2 D}\right)$
K = buckling coefficient $\left(= N \frac{L^2}{\pi^2 D}\right)$
L = length of cylinder
m, n = integers (1,2,3, and so on)
N = load (lb/in.)
p = external lateral pressure load (psi)
R = radius of cylinder to mid-point of face sheets or ratio of actual stress to theoretical critical stress
t_F = face sheet thickness
u,v,w = displacement of point on shell median surface in X, Y, and Z-directions respectively
U = shear rigidity of panel $\left[= G_C^{\prime}(C+2t_F)\right]$
X = axial co-ordinate of panel
Y = circumferential co-ordinate of panel
Z = radial co-ordinate of panel

$$Z_1^2 = \frac{2Et_f}{R^2}\ \frac{L^4}{\pi^4 D}$$

$$\beta = \frac{nL}{b}$$

η = plasticity correction
μ = Poisson's ratio (=0.3 for calculations)
$\lambda = N_{xy}/N_x$
$\gamma = N_y/N_x$
$\gamma' = 1/\gamma$
$\theta = b/R$

$$\theta_{b_K} = \theta \cdot \frac{b}{C+t_F} \cdot \left(\frac{L}{b}\right)^2 = \frac{L^2}{R(C+t_F)}$$

$$\nabla^2 = \frac{\partial^2}{\partial_x^2} + \frac{\partial^2}{\partial_y^2}$$

$$\nabla^4 = \frac{\partial^4}{\partial_x^4} + \frac{2\,\partial^4}{\partial_x^2\,\partial_y^2} + \frac{\partial^4}{\partial_y^4}$$

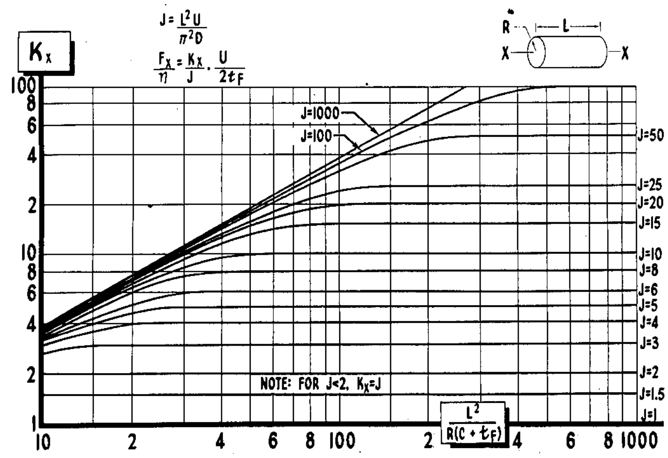

Fig. 1 Buckling of S S sandwich cylinder loaded in axial compression

∇^{-4} = inverse of ∇^4

Subscripts

s = shear
x = in x-direction
y = in y-direction
xy = in xy-direction (torsion)

INTRODUCTION

For the case of thin-walled cylinders, the critical combination of torsion and axial load has been treated previously (3),[1] as has been the combination of torsion and external lateral pressure (10). Theoretical equations for sandwich cylinders under axial compression only (12), torsion only (6), and lateral pressure only (7) have also been treated. Therefore, the scope of this paper is to extend the presently known formulas to the case studied, and to present the results in the form of easily used graphs.

DISCUSSION

The mathematical derivations of the equations, as well as the assumptions, are presented in the Appendix.

The critical combination of loads that causes the general instability failure of a simply supported, isotropic honeycomb sandwich cylinder under axial compression, external lateral pressure, and torsion can be obtained from the equations

[1] Underlined numbers in parentheses designate References at the end of the paper.

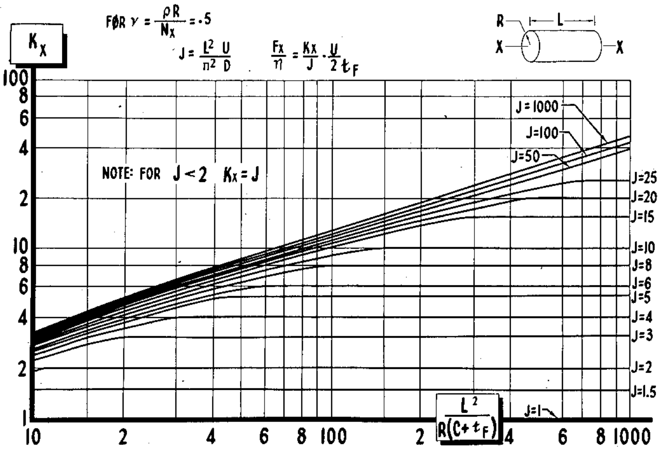

Fig. 2 Buckling of S S sandwich cylinder loaded in axial compression and lateral pressure

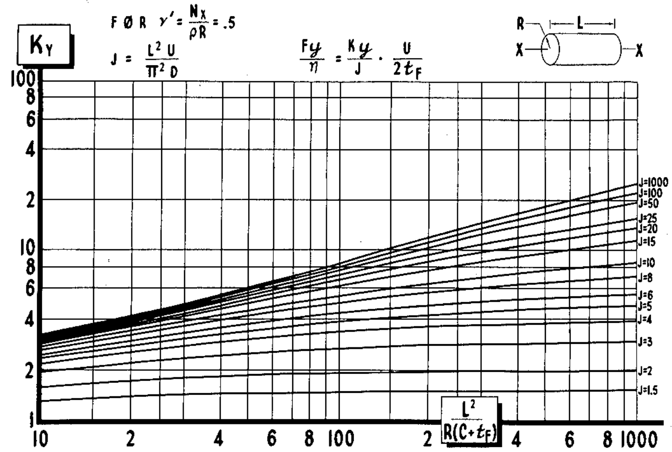

Fig. 4 Buckling of S S sandwich cylinder loaded in axial compression and lateral pressure

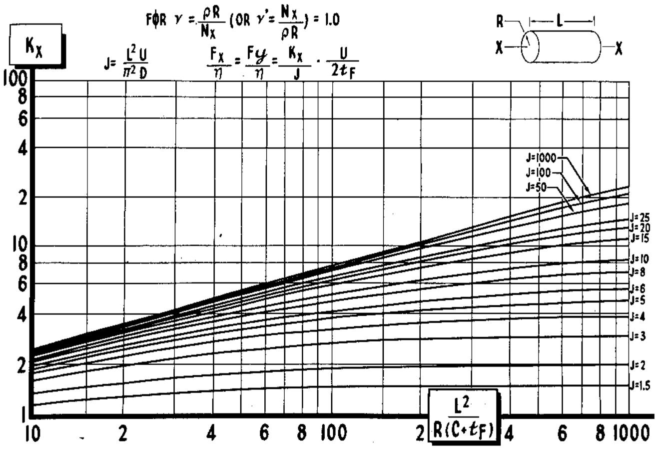

Fig. 3 Buckling of S S sandwich cylinder loaded in axial compression and lateral pressure

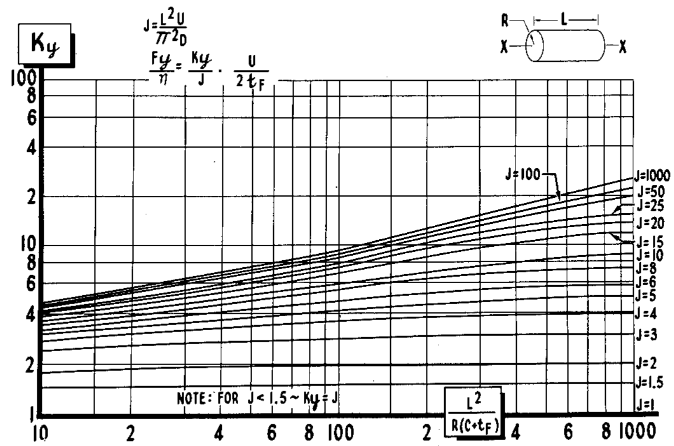

Fig. 5 Buckling of S S sandwich cylinder loaded in lateral pressure

$$\frac{F_x}{\eta} = \frac{K_x' \pi^2 D}{2L^2 t_F} \text{ or } \frac{K_x'}{J} \cdot \frac{U}{2t_F} \tag{1}$$

or

$$\frac{F_y}{\eta} = \frac{K_y'}{J} \cdot \frac{U}{2t_F} \tag{2}$$

and

$$\frac{F_s}{\eta} = \frac{K_{xy}'}{J} \cdot \frac{U}{2t_F} \tag{3}$$

when the stress coefficients K_x' or K_y' and K_{xy}' are known. The curves required to obtain these coefficients are shown in Figs.1 through 11. The equations used to obtain the curves are equations (A12) and (A13) (Appendix). Since the radius of the cylinder is assumed to be large compared to the thickness of the sandwich, $N_y = pR$.

When a cylinder is loaded under axial compression only, axial compression and lateral pressure only, lateral pressure only, or torsion only, Figs.1, 2 through 4, 5, or 6, respectively, can be used to calculate K_x, K_y, or K_{xy}. However, when the combined case is required, use of the interaction curves, Figs.7 through 11, will be required. The use of these curves is as follows:

1 Find K_x or K_y and K_{xy} for the uncombined cases, Figs.1 through 6.

2 Calculate $R_x/R_{xy} = (f_x/f_s)(K_{xy}/K_x)$ or $R_y/R_{xy} = (f_y/f_s)(K_{xy}/K_y)$

3 Using the appropriate interaction curve, Figs.7 through 11, K_x' or K_y' and K_{xy}' can now be

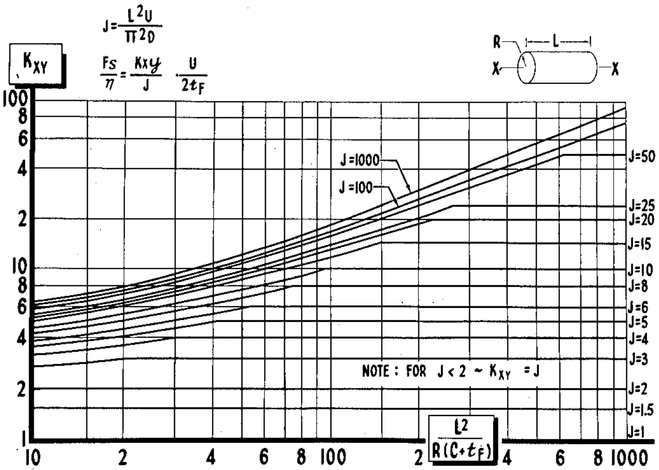

Fig. 6 Buckling of S S sandwich cylinder loaded in torsion

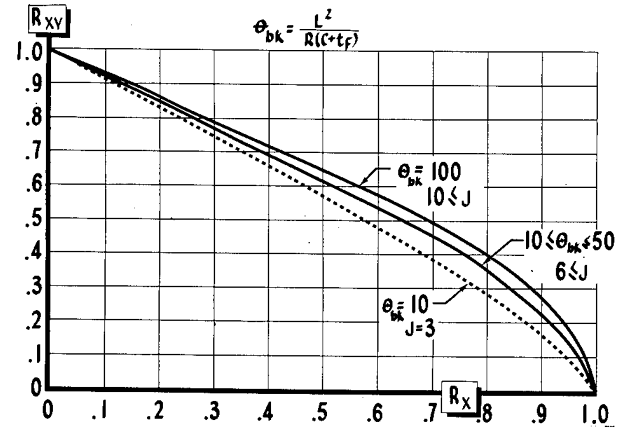

Fig. 8 Interaction curves for axial compression and external lateral pressure and torsion for $\nu = 1/2$

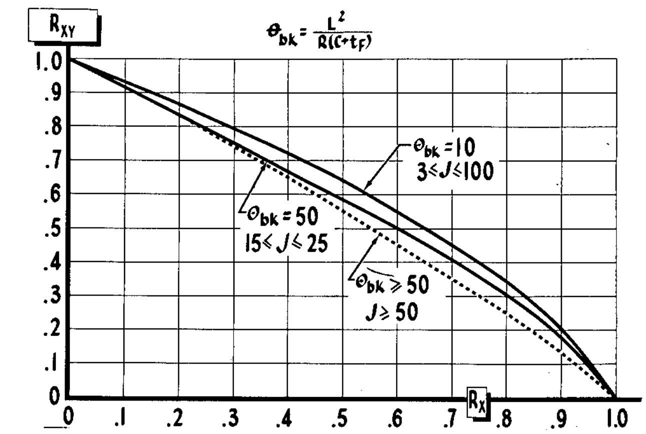

Fig. 7 Interaction curves for axial compression and torsion

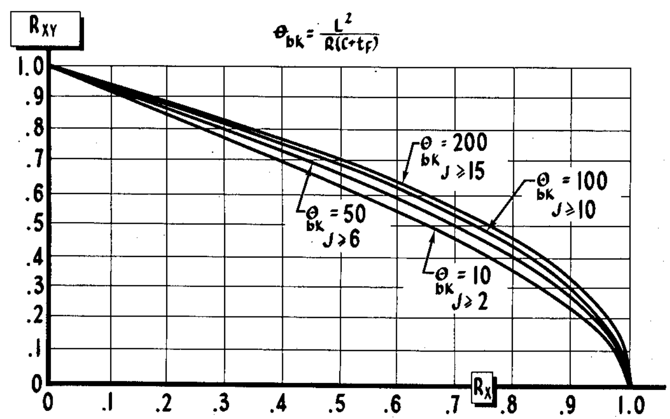

Fig. 9 Interaction curves for axial compression and lateral external pressure and torsion for $\nu = 1$

found from the relationships $K_x' = R_x K_x$ or $K_y' = R_y K_y$ and $K_{xy}' = R_{xy} K_{xy}$.

4 F_x or F_y and F_s can now be obtained from equation (1) or (2) and equation (3).

In order to facilitate further the calculations, J can be obtained from Fig.12.

In order to obtain the curves, Figs.1 through 11, equations (A12) and (A13) (Appendix) were solved for various J, θ_{bK}, γ, and R_x or R_y. This was done by means of the IBM 7090 high-speed digital computer. Since it was impossible to plot all the interaction curves which resulted, the curves shown represent typical results and will produce a maximum of a 5 per cent error from the theoretical in the range given. This is well within engineering, accuracy for a problem of this type. When J and θ_{bK} are chosen so that $K_{xy} = J$, then the interaction equation R_x (or R_y) $+ R_{xy}^2 = 1$ should be used. It should also be noted that when $L/R > 10$, the values obtained from the curves will be conservative, since n in equation (A13) must be an integer.

It has previously been observed that test results for axially loaded, thin-walled cylinders did not agree with the theoretical values obtained from the small-deflection theory. However, because of the increased thickness inherent in honeycomb-sandwich construction, this problem should not be encountered. Test results are available for axial compresssion only (11, 12), and torsion only (11). The tests for axial compression were in the region of $K_x = J$, and there is reasonably good agreement with theory. The torsion tests were inconclusive.

According to (12) $K_x = J$ for $\gamma = 0$ when $\theta_{bK} \geq 5.18$ J, and $K_{xy} = J$ when $\theta_{bK} \geq 10.36$ J. The curves for Figs.1 ($\gamma = 0$) and 6 seem to verify these relationships. However, there is a slight difference in the case where $K_{xy} = J$, since an insufficient number of terms of the ma-

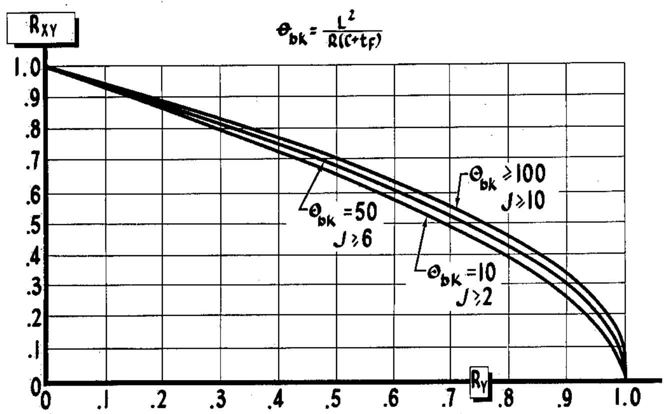

Fig. 10 Interaction curves for axial compression and external pressure and tension for ν' 1/2

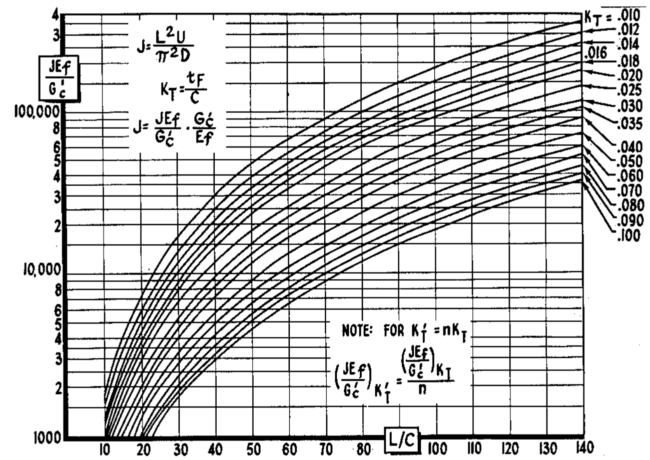

Fig. 12 Determination of J

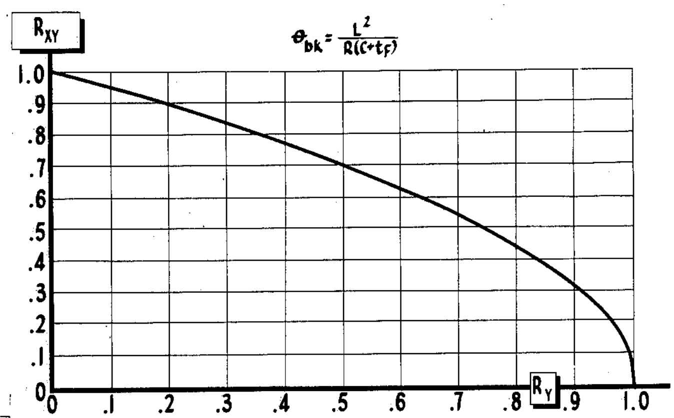

Fig. 11 Interaction curve for external lateral pressure and torsion $\nu' = 0$

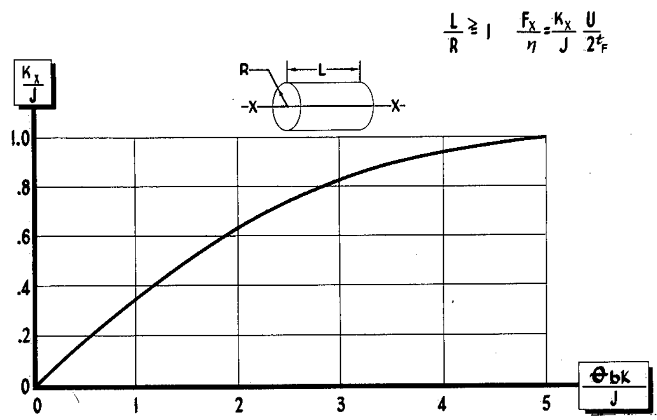

Fig. 13 Compression buckling of simply supported sandwich cylinders loaded in the axial direction only

trix, equation (A12) were taken. The curves for $\nu' = 0$, Fig.1, can be replaced by the single curve in Fig.13, by changing the parameters to those shown.

CONCLUSION

A theoretical solution for the general instability of a simply supported, isotropic honeycomb sandwich cylinder under axial compression, external lateral pressure, and torsion is presented in the Appendix.

The resulting curves were drawn and shown in 11 Figs. ~~1 through 11.~~ The use of these curves should produce an error of less than ± 5 per cent from the theoretical equations.

The very limited test results available showed reasonably good agreement with the theory.

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APPENDIX A

THEORETICAL SOLUTION FOR A SANDWICH CYLINDER

The following assumptions were made to derive the equations in this appendix:

1. Face plates are of identical material and thicknesses.
2. Core is isotropic.
3. Core does not carry any bending stresses.
4. Face sheets do not carry any normal shear stresses.
5. Radius of the cylinder or curved plate is large compared to the thickness of the sandwich.
6. Axial compression is uniform.
7. Lateral external pressure produces uniform compression constant throughout the perimeter.
8. Radial deflection of the cylinder before buckling is small compared to the sandwich thickness.

The equation of equilibrium for a honeycomb sandwich cylinder in terms of the transverse deflections of the median surface was formulated by Wang and De Santo (Reference 12). This modified form of Donnell's equation is shown below.

$$Q_1(w) = 0 \qquad \text{(A1)}$$

Where (Q_1) is the operator defined by

$$Q_1 = \nabla^4 + \left(\frac{1}{D} - \frac{\nabla^2}{u}\right)\left[\nabla^{-4}\frac{2Et_F}{R^2}\frac{\partial^4}{\partial x^4} - \left(N_x\frac{\partial^2}{\partial x^2} + N_y\frac{\partial^2}{\partial y^2} + 2N_{xy}\frac{\partial^2}{\partial x \partial y}\right)\right] \qquad \text{(A2)}$$

Here the implied boundary conditions are

$$w(0,y) = w(0,L) = \frac{\partial^2 w}{\partial x^2}(0,y) = \frac{\partial^2 w}{\partial x^2}(L,y) = 0 \qquad \text{(A3)}$$

Using the Galerkin method and defining w as shown below in order to satisfy the boundary conditions

$$w = \sin\frac{n\pi y}{b}\sum_{m=1}^{\infty} a_m \sin\frac{m\pi x}{L} + \cos\frac{n\pi y}{b}\sum_{m=1}^{\infty} b_m \sin\frac{m\pi x}{L} \tag{A4}$$

Following Galerkin's method, two sets of n simultaneous equations are obtained.

$$\iint V_p Q(w)\,dx\,dy = 0 \tag{A5a}$$

$$\iint W_p Q(w)\,dx\,dy = 0 \tag{A5b}$$

where

$$V_p = \sin\frac{n\pi y}{b}\sin\frac{p\pi x}{L} \tag{A6a}$$

$$W_p = \cos\frac{n\pi y}{b}\sin\frac{p\pi x}{L} \tag{A6b}$$

Letting $Q = Q_1$ and substituting equations A2, A4, and A6a into equation A5a, we obtain

$$0 = \int_0^L\int_0^b \sin\frac{n\pi y}{b}\sin\frac{p\pi x}{L}\left\{\left[\nabla^4 + \left(\frac{1}{D} - \frac{\nabla^2}{u}\right)\left[\nabla^{-4}\frac{2Et_F}{R^2}\frac{\partial^4}{\partial x^4} - \left(N_x\frac{\partial^2}{\partial x^2} + N_y\frac{\partial^2}{\partial y^2} + 2N_{xy}\frac{\partial^2}{\partial x\partial y}\right)\right]\right]\right.$$
$$\left.\left[\sin\frac{n\pi y}{b}\sum_{m=1}^{\infty} a_m \sin\frac{m\pi x}{L} + \cos\frac{n\pi y}{b}\sum_{m=1}^{\infty} b_m \sin\frac{m\pi x}{L}\right]\right\}dx\,dy \tag{A7}$$

Also letting $Q = Q_1$ and substituting equations A2, A4, and A6b into equation A5b, we obtain the second set of equations.

$$0 = \int_0^L\int_0^b \cos\frac{n\pi y}{b}\sin\frac{p\pi x}{L}\left\{\left[\nabla^4 + \left(\frac{1}{D} - \frac{\nabla^2}{u}\right)\left[\nabla^{-4}\frac{2Et_F}{R^2}\frac{\partial^4}{\partial x^4} - \left(N_x\frac{\partial^2}{\partial x^2} + N_y\frac{\partial^2}{\partial y^2} + 2N_{xy}\frac{\partial^2}{\partial x\partial y}\right)\right]\right]\right.$$
$$\left.\left[\sin\frac{n\pi y}{b}\sum_{m=1}^{\infty} a_m \sin\frac{m\pi x}{L} + \cos\frac{n\pi y}{b}\sum_{m=1}^{\infty} b_m \sin\frac{m\pi x}{L}\right]\right\}dx\,dy \tag{A8}$$

Performing the integration over the limits gives,

$$a_m\left\{(m^2+\beta^2)^2 + \frac{m^4 Z_1^2}{(m^2+\beta^2)^2}\left[1+\frac{m^2+\beta^2}{J}\right] + K_x(m^2+\gamma\beta^2)\left[1+\frac{m^2+\beta^2}{J}\right]\right\} - 8K_{xy}\frac{\beta}{\pi}\sum_{p=1}^{\infty} b_p\frac{mp}{m^2-p^2}\left[1+\frac{p^2+\beta^2}{J}\right] = 0 \tag{A9a}$$

$$b_m\left\{(m^2+\beta^2)^2 + \frac{m^4 Z_1^2}{(m^2+\beta^2)^2}\left[1+\frac{m^2+\beta^2}{J}\right] + K_x(m^2+\gamma\beta^2)\left[1+\frac{m^2+\beta^2}{J}\right]\right\} + 8K_{xy}\frac{\beta}{\pi}\sum_{p=1}^{\infty} a_p\frac{mp}{m^2-p^2}\left[1+\frac{p^2+\beta^2}{J}\right] = 0 \tag{A9b}$$

Where:

tension is (+), $m \mp p$ = odd integers,

$N_{xy} = K_{xy}\frac{\pi^2 D}{L^2}$ and $N_x = K_x\frac{\pi^2 D}{L^2}$

It is possible to obtain a nontrivial solution for equations A9 and A10 by setting their determinant equal to zero

$$
\begin{array}{c|cccccc c|}
 & a_1 & b_2 & a_3 & b_4 & a_5 & b_6 & \\
\hline
m=1 & M_1/K_{xy} & \frac{2}{3}\left(1+\frac{4+\beta^2}{J}\right) & 0 & \frac{4}{15}\left(1+\frac{16+\beta^2}{J}\right) & 0 & \frac{6}{35}\left(1+\frac{36+\beta^2}{J}\right) & \cdots \\
m=2 & \frac{2}{3}\left(1+\frac{1+\beta^2}{J}\right) & M_2/K_{xy} & -\frac{6}{5}\left(1+\frac{9+\beta^2}{J}\right) & 0 & \frac{-10}{21}\left(1+\frac{25+\beta^2}{J}\right) & 0 & \cdots \\
m=3 & 0 & -\frac{6}{5}\left(1+\frac{4+\beta^2}{J}\right) & M_3/K_{xy} & \frac{12}{7}\left(1+\frac{16+\beta^2}{J}\right) & 0 & \frac{2}{3}\left(1+\frac{36+\beta^2}{J}\right) & \cdots \\
m=4 & \frac{4}{15}\left(1+\frac{1+\beta^2}{J}\right) & 0 & \frac{12}{7}\left(1+\frac{9+\beta^2}{J}\right) & M_4/K_{xy} & -\frac{20}{9}\left(1+\frac{25+\beta^2}{J}\right) & 0 & \cdots \\
m=5 & 0 & \frac{-10}{21}\left(1+\frac{4+\beta^2}{J}\right) & 0 & -\frac{20}{9}\left(1+\frac{16+\beta^2}{J}\right) & M_5/K_{xy} & \frac{30}{11}\left(1+\frac{36+\beta^2}{J}\right) & \cdots \\
m=6 & \frac{6}{35}\left(1+\frac{1+\beta^2}{J}\right) & 0 & \frac{2}{3}\left(1+\frac{9+\beta^2}{J}\right) & 0 & \frac{30}{11}\left(1+\frac{25+\beta^2}{J}\right) & M_6/K_{xy} & \cdots \\
 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots &
\end{array} = 0 \qquad \text{(A 10)}
$$

Here $$M_i = \frac{\pi}{8\beta}\left\{(i^2+\beta^2)^2 + \frac{i^4 Z_1^2}{(i^2+\beta^2)^2}\left(1+\frac{i^2+\beta^2}{J}\right) + K_x\left(i^2+\gamma\beta^2\right)\left(1+\frac{i^2+\beta^2}{J}\right)\right\} \qquad \text{(A 11)}$$

Factoring each column with the proper $\left(1+\frac{i^2+\beta^2}{J}\right)$ and calling

$$N_i = M_i \Big/ \left(1+\frac{i^2+\beta^2}{J}\right)$$

We obtain the following determinant:

$$
\begin{array}{c|cccccc c|}
 & a_1 & b_2 & a_3 & b_4 & a_5 & b_6 & \\
\hline
m=1 & N_1/K_{xy} & \frac{2}{3} & 0 & \frac{4}{15} & 0 & \frac{6}{35} & \cdots \\
m=2 & \frac{2}{3} & N_2/K_{xy} & -\frac{6}{5} & 0 & \frac{-10}{21} & 0 & \cdots \\
m=3 & 0 & -\frac{6}{5} & N_3/K_{xy} & \frac{12}{7} & 0 & \frac{2}{3} & \cdots \\
m=4 & \frac{4}{15} & 0 & \frac{12}{7} & N_4/K_{xy} & -\frac{20}{9} & 0 & \cdots \\
m=5 & 0 & -\frac{10}{21} & 0 & -\frac{20}{9} & N_5/K_{xy} & \frac{30}{11} & \cdots \\
m=6 & \frac{6}{35} & 0 & \frac{2}{3} & 0 & \frac{30}{11} & N_6/K_{xy} & \cdots \\
 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots &
\end{array} = 0
$$

(A12)

Here $$N_i = \frac{\pi}{8\beta}\left[\frac{(i^2+\beta^2)^2}{\left(1+\frac{i^2+\beta^2}{J}\right)} + \frac{\frac{4(1-\mu^2)}{\pi^4}\,\theta_{bK}^2\,K\,i^4}{(i^2+\beta^2)^2} + K_x\left(i^2+\gamma\beta^2\right)\right] \qquad (A13)$$

Where $$\theta_{bK} = \frac{L^2}{R(c+t_F)} \qquad (A14)$$

Using a third-order determinant and solving for K_{xy} we obtain

$$K_{xy} = \sqrt{\frac{N_1 N_2 N_3}{\left(\frac{6}{5}\right)^2 N_1 + \left(\frac{2}{3}\right)^2 N_3}} \qquad (A15)$$

Knowing K_x, γ, θ_{bk} we can plot K_{xy} against β and find the minimum value of K_{xy}.

A better approximation can be obtained from the fourth order determinant

$$K_{xy}^2 = \frac{\left(\frac{12}{7}\right)^2 N_1N_2 + \left(\frac{6}{5}\right)^2 N_1N_4 + \left(\frac{4}{15}\right)^2 N_2N_3 + \left(\frac{2}{3}\right)^2 N_3N_4 - \sqrt{\left[\left(\frac{12}{7}\right)^2 N_1N_2 + \left(\frac{6}{5}\right)^2 N_1N_4 + \left(\frac{4}{15}\right)^2 N_2N_3 + \left(\frac{2}{3}\right)^2 N_3N_4\right]^2 - 4N_1N_2N_3N_4\left(\frac{8}{7}+\frac{8}{25}\right)^2}}{2\left(\frac{8}{7}+\frac{8}{25}\right)^2} \qquad (A16)$$

PARTICULAR CASES

Case I - axial compression only

Let N_i and γ (in equation A13) $= 0$
$K_x = (-)$

Then $$K_x = \frac{(i^2+\beta^2)^2}{i^2\left(1+\frac{i^2+\beta^2}{J}\right)} + \frac{4(1-\mu^2)\,\theta_{bK}^2\,i^2}{\pi^4(i^2+\beta^2)^2}$$

Case II - Axial compression + lateral pressure

Let N_i (in equation A13) $= 0$
$K_x = (-)$

Then $$K_x = \frac{(i^2+\beta^2)^2}{\left(1+\frac{i^2+\beta^2}{J}\right)(i^2+\gamma\beta^2)} + \frac{4(1-\mu^2)\,\theta_{bK}^2\,i^4}{\pi^4(i^2+\beta^2)^2(i^2+\gamma\beta^2)}$$

Case III - Lateral pressure only

Let $K_x(i^2+\gamma\beta^2) = -K_y\beta^2$ (in equation A13)

and let $N_i = 0$

Then $$K_y = \frac{(i^2+\beta^2)^2}{\beta^2\left(1+\frac{i^2+\beta^2}{J}\right)} + \frac{4(1-\mu^2)\,\theta_{bK}^2\,i^4}{\pi^4\beta^2(i^2+\beta^2)^2}$$

Case IV - Torsion only

Let K_x (in equation A13) $= 0$

Then $$N_i = \frac{\pi}{8\beta}\left[\frac{(i^2+\beta^2)^2}{\left(1+\frac{i^2+\beta^2}{J}\right)} + \frac{4(1-\mu^2)\,\theta_{bK}^2\,i^4}{\pi^4(i^2+\beta^2)^2}\right]$$

Case V - Small deflection theory equation for a thin-walled cylinder

Let $J \to \infty$ and $\theta_{bK}^2 = \frac{3L^4}{R^2t^2}$

in equation A13 and cases I, II, III, and IV