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Robust Control Theory and Applications
Final Report, AFOSR F49620-92-J-0026

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Abstract

This is the final report for AFOSR F49620-92-J-0026, which began in 1992 and ended in 1995. The research described here includes robust control, integrated modeling, identification, and analysis, nonlinear control, implicit systems, and experimental testbeds for evaluating theoretical contributions. During the period of this program, Caltech established a new graduate department in Control and Dynamical Systems (CDS) which will also be briefly described.

1 Introduction

Our research program is guided by two observations: 1) Of the multiple technological problems usually tackled by control engineers, only a small fraction reduces to the design of a feedback system for a given plant. 2) The prospect of useful, general nonlinear theory resembling existing useful, general linear theory is hopeless. It is essential to understand these limitations and the opportunities they implicitly create. First, we must view control in a much broader context, because it is intimately and inextricably intertwined with all aspects of total system design. Secondly, we must be realistic about nonlinear systems, pursue multiple and complementary research paths, and do quite fundamental theoretical research in concert with very engineering-oriented applications.

1.1 Control and Dynamical Systems at Caltech

In 1993 Caltech established a graduate option in Control and Dynamical Systems (CDS) with a core faculty of John Doyle from EE, Richard Murray from ME, Stephen Wiggins from Applied Mechanics, and Manfred Morari from ChE. At Caltech, CDS is in the Division of Engineering and Applied Sciences, and is organizationally parallel to traditional options such as EE, ME, Aeronautics and Applied Math. In 1995 CDS added Professor Jerry Marsden to create a unique blend of mathematics and engineering.

A specific focus of CDS is on developing general methods for the analysis and design of complex uncertain nonlinear systems, and particularly the issue of reliable prediction of the performance of complex systems. All of science and engineering deals with these issues, but typically in a domain-specific, "hand-crafted" manner. Increasingly, complex systems' performance depends on the interconnection of heterogeneous components with interacting fluid, structural, material, chemical, electromagnetic, and electronic subsystems, whose modeling and analysis does not fit neatly within existing engineering domains. The fields of control and dynamical systems have the right foundation to treat complex systems in a more systematic and general way, but both areas must greatly expand their view of their problem domains. We believe that control theorist must get beyond the plant/controller paradigm and consider more general interconnections of components,

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must view modeling, model validation with data, and analysis of models in a more unified manner, and more effectively apply robustness analysis methods to nonlinear models. Dynamical systems must also expand its vision, going beyond chaos and the celebration of "complexity" to the more difficult questions of how to design and verify systems that behave predictably.

Making contact with engineering is made both more challenging and more exciting because the interdisciplinary nature of CDS applications means that CDS no longer has a natural home within any one traditional engineering discipline. While this has always been true to some extent, changing technology has made advanced CDS relevant to a much broader class of application areas. For example, active applications areas in CDS at Caltech include nonlinear flight dynamics for highly maneuverable aircraft; turbomachines and complex combustion systems; large flexible structures; global bifurcation analysis and control of nonlinear systems, including strongly nonlinear and chaotic dynamics; vortex structures in complex fluid flows; mixing and transport processes in fluids; robotic manipulation; real-time control structures for mechanical systems; computer vision; mechanics of rigid body systems, fluids, elasticity, plasma physics, general field theory; mechanical systems and systems with symmetry; chemical process control; refining operations, in particular fluid catalytic cracking and distillation; and the design of autonomous systems.

1.2 The system design process

Control theory must play a broader role in technology. There is less need for design of controllers for a given plant than there is for modeling, system identification, and design and simulation of more general interconnected systems, and we must modify and expand control theory to include these broader issues. Typically, the natural forms of first-principles engineering models for components of large complex systems are incompatible with the models used in either system identification from data or in control design. A major thrust of our program is the further development of a new unified framework for modeling, ID, analysis, and system design.

No matter how obtained, if a model has realistic representations of uncertainties and nonlinearities, it can be too complex to be studied directly. The full design problem must be broken into manageable pieces using a hierarchy of simplified models. Performance evaluation typically involves Monte Carlo simulation with nonlinearities but limited uncertainty combined with more detailed analysis of structured uncertainty using linearized models.

A major goal of our research is to seek a more natural blending of methods to deal with nonlinearities and uncertainties. Essentially, the greatest obstacles to practical application of advanced control theory have traditionally been the presence of nonlinearities, operating constraints, and model uncertainty. In the last decade or so, great research progress has been made in addressing these issues *separately* in a fundamental manner. For example, robust control has focused on the issue of model uncertainty, with tremendous success by almost any measure. Model Predictive Control (MPC) has focused on otherwise linear systems with constraints and has found substantial applications, particularly in process control, but available techniques for robustness analysis are limited. Similarly, nonlinear control and particularly dynamical systems theory have yielded deep insights about nonlinear systems, but with limited treatment of uncertainty and constraints.

Successful domain-specific approaches have evolved through trial and error, with simple schemes that rely heavily on engineering intuition. For example, for PID controllers, we know how to tune them to be insensitive to process nonlinearities and model uncertainty,introduce anti-windup to handle actuator saturation, and add simple nonlinearities when needed. Unfortunately, these simple, practical schemes for PID controllers can be overwhelmed by complex, multivariable systems, although we have had success in extending similar schemes using combinations of feedback linearization and dynamic inversion with multivariable robust control. This is currently our baseline approach to nonlinear design, although we recognize its limitations.

Current feedback linearization and dynamic inversion methods rely heavily on linear techniques and aim to produce linear-looking results. More generally, the engineer's main tools for the analysis of uncertain nonlinear systems are still simulation coupled with robustness analysis of linearizations at equilibria. Exploring regions of uncertain parameter values for nonlinear systems by repeated simulation involves prohibitive computation growth rates. Given the reliance on such simulation throughout the design of engineering systems, and not just aerospace, this is a severe technological problem and one of the major challenges to engineering theory. Addressing this problem is perhaps the central research focus of this program, and a major motivation behind the CITCDS combination of robust and nonlinear control with dynamical systems.

2 Overview of the Research Accomplishments

2.1 Robust control and extensions.

To date, the most successful applications of robust control techniques such as μ analysis and synthesis have occurred in problem domains (flexible structures, flight control, distillation) where there may be substantial uncertainty in the available models, and the degrees of freedom and the dimension of the input, output, and state may be high, but the basic structure of the system is understood, the uncertainty can be quantified. Nonlinearities are bounded and treated as perturbations on a nominal linear model, or handled by gain-scheduling linear point designs. Needless to say, robust control is a major Caltech CDS research area, and we are continuing to develop improvements in analysis and synthesis techniques with particular emphasis on extensions to nonlinear systems. For example, one extension to so-called Linear Parameter Varying (LPV) system design was initiated with our generalization to LFT (Linear Fractional Transformation) models of the Youla parametrization of all stabilizing controllers (Lu *et al.* 1991) . This has rapidly become an active research area, with an LPV/LFT version of H_∞ by Packard and coworkers. We are using these methods in a variety of experiments and applications, to do extended "gain-scheduling" with rapidly varying parameters and nonlinear systems and they will continue to form an important part of our research program.

Another example of a popular attempt to develop robust nonlinear methods has been the generalization to nonlinear systems of state-space H_∞ results, under the misnomer "Nonlinear H_∞." Unfortunately, these methods have limited applicability because they are only very marginally less local than linear H_∞, but computationally prohibitive. A potentially promising alternative formulation is in terms of NLMIs, which are nonlinear generalizations of linear matrix inequalities (LMIs) (Lu and Doyle 1994). These are convex, but infinite dimensional, and practical finite-dimensional approximations schemes have not been developed. This lack of tractable computation is a failing of many nonlinear methods, including all that rely on Hamilton-Jacobi equations or Lyapunov functions and their various extensions.

2.2 Integrated modeling, ID, analysis, and design.

Modeling, system identification, and design and simulation of more general interconnected systems, are currently performed using a variety of mathematical machinery. We have recently developed a promising unified framework for these various aspects of system design (Doyle *et al.* 1994, Newlin 1995, Paganini 1995*c*, Newlin and Smith 1995, Morris and Newlin 1995, Dullerud and Smith 1995*a*, Dullerud and Smith 1995*b*, Dullerud and Smith 1995*c*, Smith and Dullerud 1995). In addition to providing an "interface" between system ID and control, this framework allows advances in different directions to combine readily. For example, it is clear how to combine progress in linear system ID with progress in nonlinear robustness analysis to produce nonlinear system ID methods. Our framework also clarifies many of the computational issues and what basic algorithms must be developed. Conventional system ID allows only very limited uncertainty descriptions and does not efficiently find global solutions except for very simple problem structures. Our approach will certainly remedy the former problem, and experience with robustness analysis suggests that there may be hope for the latter as well.

Implicit models play a key role in this unified framework. We have extended robustness analysis techniques to systems described in implicit form (Paganini and Doyle 1994*b*, Paganini and Doyle 1994*a*), developing new tools for analysis of systems under a combination of time-invariant/time-varying perturbations (Paganini 1995*a*) and exact conditions for robust H_2 performance analysis (Paganini 1995*b*). There is strong engineering motivation for this extension. In fact, the standard control theory I/O formulation is only adequate for systems which are deliberately built to match the "signal flow" conception, and it appears awkward when modeling physical systems from first principles, where physical laws such as mass, momentum, or energy balances or physical laws such as Newton's second law, Ohm's law, and so on are more naturally thought of as relations between variables than as I/O maps. The important uncertainty modeling machinery from robust control can not only be generalized appropriately to implicit systems, but also greatly extended to treat entirely new problems (Paganini 1995*c*).

Efficient computation is perhaps the major issue in robustness analysis, and in the our unified approach to modeling, analysis, ID, and design. Robustness analysis with real parametric uncertainty is NP hard, generally viewed as implying worst-case intractability. We have demonstrated that branch-and-bound can be successfully used to overcome the intractability of mixed μ, and developed power algorithms for the lower bounds which are much faster and produce better bounds than conventional local optimization (Braatz *et al.* 1993, Newlin and Glavaski 1995, Young *et al.* 1995*a*, Young *et al.* 1995*b*, Young *et al.* 1995*c*, Newlin 1995).

2.3 Robustness analysis and nonlinear extensions.

Robustness for nonlinear systems can be proven equivalent to the existence of solutions to Hamilton-Jacobi equations or nonlinear matrix inequalities (Lu and Doyle 1994). However, computational methods to establish the existence of these solutions have not been developed to a level comparable to their linear counterpart (i.e., existence of solutions of Riccati equations and linear matrix inequalities), and are theoretically intractable, even for the cases which are easy for linear systems.

The state of the art in industry still consists in obtaining lower bounds to the performance indices through extensive simulation or local optimization techniques. However, these methods require large amounts of computation; standard optimization techniques fail even for small problems, and a search over parameter space exhibits exponential growth with the number of parameters. The methods actually used in industry share two main characteristics: performance specifications are made over a finite time horizon, and the interface between the analysis method and the system is a simulation.

In recent work at Caltech, we have begun to extend the robustness analysis techniques of linear systems, and in particular the associated computational methods, to nonlinear systems. In (Tierno *et al.* 1995) we presented a power algorithm to compute a lower bound on the performance index associated with the robust trajectory tracking problem (i.e., the distance from the actual to the nominal trajectory). This algorithm is similar in nature to the one developed for the structured singular value (Packard and Doyle 1993, Young *et al.* 1995*b*), and has similar behavior. We carry out several different tests on two different platforms: the Caltech ducted fan experiment and a simplified model of an F-16 jet fighter. The results of these tests are reported in (Tierno *et al.* 1995, Tierno 1995). These results indicate that without significant additional computation, and avoiding computationally expensive parameter searches, a lower bound on the given performance index can be computed that gives more information on the worst case behavior of the system than the standard Monte Carlo procedures.

We have also begun to develop computable upper bounds for the trajectory generation problem by using rational approximations to nonlinear systems and restricting the types of uncertainties which can enter into the dynamics (Tierno and Murray 1995, Tierno 1995). At present, these results are still far from being practical, but they are a starting point in developing computational machinery for performing robust modeling of nonlinear systems.

2.4 Nonlinear dynamic inversion and feedback linearization

Advances in nonlinear control theory have resulted in a much more detailed understanding of the geometry of nonlinear control systems and the interaction between geometric properties and control design. One well-known approach for nonlinear control is to feedback transform the nonlinear system into a linear one and then apply linear techniques. For some classes of systems, such as robot manipulators, the transformation is defined everywhere and allows for global stabilization. More generally, there are well-known geometric conditions for feedback linearization and various extensions and applications of these techniques have appeared.

Researchers at Caltech and as well as our collaborators at Honeywell have extensive experience combining recent techniques in feedback linearization and dynamic inversion with robust control to design controllers for a variety of applications, primarily in flight control. At its best, dynamic inversion extends robust control methods to nonlinear systems. Unfortunately, a serious drawback of all feedback linearization techniques is the failure to account directly for uncertainties, and apparently reasonable designs can be extremely non-robust. We have developed substantial insight into the nature of these problems, and are developing techniques to avoid them. Although it yields a system which appears to be easily analyzed, inverting out the natural dynamics can lead to a nonlinear analog of ill-conditioning in multivariable linear systems, with the resulting severe robustness problems with respect to uncertainties internal to the plant, which is still nonlinear.

Very simple examples can illustrate the potential robustness problems of inversion methods. Consider the disturbance rejection problem for the scalar nonlinear system $\dot{x} = e^{x}(u+d)$ with output $y = x + n$ and control input u, actuator noise d, and sensor noise n. For the system linearized at $x = 0$, $\dot{x} = (u+d)$ and the optimal feedback $u = -y$ minimizes the H_∞ gain between the disturbance vector (d, n) and (y, u) with optimal norm $\sqrt{2}$. In fact, for this particular example, it can be shown, using either operator theoretic or nonlinear H_∞ techniques, that the optimal L_2 (H_∞) gain for the full nonlinear system is $\sqrt{2}$ and that this can be also achieved with $u = -y$. Thus the linear controller designed for the linearized plant just happens to also be the globally optimal nonlinear controller. On the other hand, the precompensator $u = e^{-x}v$ feedback linearizes the system when the disturbances are ignored. The standard approach is to now design a controller for the linearized system and the obvious choice is the linear feedback $v = -x$, yielding the globally stabilizing feedback linearizing controller $u = -e^{-x}x$ with closed loop $\dot{x} = -x$. If, however, we consider a nonzero disturbance d the system becomes $\dot{x} = -x + e^{x}d$ which has finite escape time for a disturbance $d > e^{-1}$. Thus the feedback linearizing controller actually makes the nonlinear system extremely nonrobust. Obviously, the resulting L_2 gain of the system is infinite.

This example is constructed to be extreme, and caused substantial controversy when it was introduced several years ago, but it is now widely recognized that there are potential robustness problems with inversion methods. In the context of flight control systems, however, judicious use of inversion methods can be quite successful, since in many cases the vehicle is actually designed with control effectors that provide adequate control authority in the degrees of freedom where manuevers are performed. For example, aircraft ailerons and rudders are sized by airframe designers to provide adequate coordinated turn capability throughout the envelope. In such a case it makes sense to invert for coordinated roll-rate (using aileron and rudder) as part of the control strategy.

An issue related to ill-conditioning is that it is rarely desirable for systems which have large operating envelopes to be inverted to a single linear system. Thus even when inversion is possible, the desirable full-envelope dynamics are themselves typically nonlinear. This interacts with the robustness issue, and there is currently no systematic way to select among the numerous choices available. While we are pursuing a number of research directions to address these issues, we believe that one of the most promising long-term directions is to draw more on techniques from dynamical systems. The main idea here is to provide a richer class of normal forms than just linear systems.

3 Experimental Validation

In order to validate and motivate new results in robust nonlinear control theory, we have built several control experiments over the past several years which exhibit a range of linear and nonlinear behaviors and provide a testbed for new theory and software. In this section we briefly described two of those experiments as well as the common software infrastructure which is used at Caltech to control them. Both of these experiments as well as the software infrastructure have been developed primarily in Richard Murray's group at Caltech.

3.1 The Caltech Ducted Fan

We have constructed a small flight control experiment whose dynamics are representative of either a Harrier in hover mode or a thrust vectored aircraft (such as the F18-HARV or X-31) in forward flight. A picture of our experimental system is shown in Figure 1 It consists of a high-efficiency electric motor with a 6-inch diameter blade, capable of generating up to 9 Newtons of thrust. Flaps on the fan allow the thrust to be vectored from side to side and even reversed. The engine is mounted on a three degree of freedom stand which allows horizontal and vertical translation as well as unrestricted pitch angle.

This system has been used for a number of studies and papers. A description of the overall design and control considerations is given in (Choi *et al.* 1994). A comparison of several different linear and nonlinear controllers was performed by Kantner et al. (1995) and a more focused comparison on LPV controllers has been presented (Bodenheimer *et al.* 1996). The application of differential flatness based controllers is reported in (van Nieuwstadt and Murray 1996).

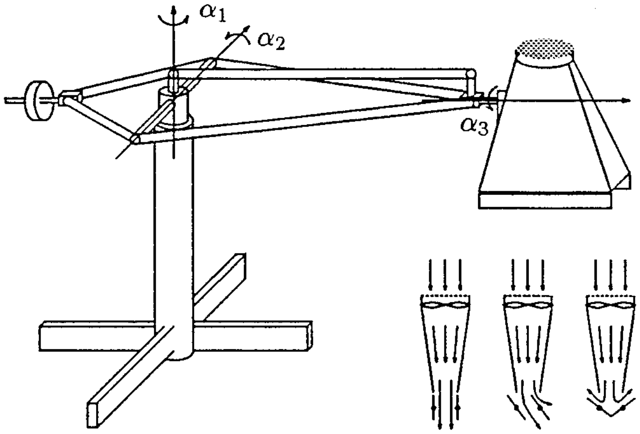

Figure 1: The Caltech Ducted Fan. The inset shows some of the possible thrust modes.

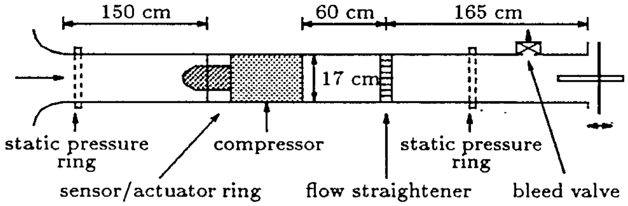

Figure 2: Caltech compressor rig.

More information on the Caltech ducted fan is available via the Ducted Fan Homepage:

`http://avalon.caltech.edu/~dfan`

3.2 The Caltech Low Speed, Axial Flow Compressor Rig

We have built a low-speed, axial flow compressor facility designed for use in validation of active control techniques. While this compressor is substantially simpler than a typical compressor in a gas turbine engine, it has many of the essential operating characteristics of high speed compressors and is ideally suited for implementation of active control techniques due to its size and ease of use.

The entire experimental setup is shown in Figure 2 and was designed and constructed in accordance with standards for measurement and calibration of compressors of this type. The compressor is a 17 cm diameter, single stage, axial flow compressor. In addition to the compressor unit, the system consists of an inlet nozzle, adjustable downstream throttle, and an optional plenum. Sensors include a pair of static pressure rings on the inlet and outlet sides, a pitot measuring plane near the outlet, and an array of six static pressure transducers located in front of the compressor face. Actuation is achieved with a low-speed, electrically driven throttle at the outlet as well as a high response bleed which can be located either before or after the plenum and a set of three air injectors at the compressor face (described in more detail below).

Current work on this system has focused on the use of pulsed air injection for elimination of the hysteresis loop associated with rotating stall. Descriptions of this work can be found in (Behnken *et al.* 1995) and (D'Andrea *et al.* 1995). More information on the Caltech ducted fan is available via the Compressor Homepage:

`http://avalon.caltech.edu/~compress`

3.3 Hardware and Software Infrastructure

All of the control experiments at Caltech are interfaced to PC-based data acquisition and control systems and controlled using the Sparrow Real-Time Kernel. Many university experiments are of sufficiently low bandwidth that they can be effectively controlled with standard PC-based hardware. Currently, the most computational demanding controls experiment at Caltech is the compressor rig, for which we are able to obtain servo rates of up to 4000 Hz using a 100 MHz Pentium-based system and standard data acquisition cards.

All experiments at Caltech use the Sparrow Real-Time Kernel to control the hardware. Sparrow is a collection of programs and a library of C functions intended to aid in the implementation of real-time controllers on PC-based data acquisition and control systems. It contains functions for executing control algorithms at a fixed rate, communicating with hardware interface cards, and displaying data in real-time. More information on Sparrow and the hardware infrastructure used to support it is available via the Sparrow Homepage:

`http://avalon.caltech.edu/~murray/sparrow`

Sparrow was developed with the support of the National Science Foundation and NASA and is currently being expanded with support from AFOSR. It is available for free and can be obtained via the URL listed above.

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