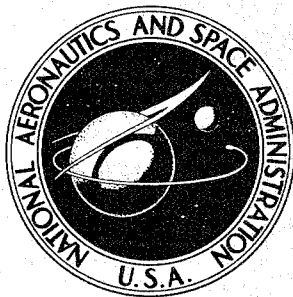
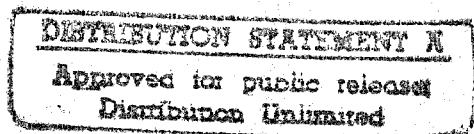


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STRENGTH CHARACTERISTICS
OF COMPOSITE MATERIALS

by Stephen W. Tsai

Prepared under Contract No. NAS 7-215 by

PHILCO CORPORATION

Newport Beach, Calif.

for

DTIC QUALITY INSPECTED 1

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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STRENGTH CHARACTERISTICS OF COMPOSITE MATERIALS

By Stephen W. Tsai

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ABSTRACT

The strength characteristics of quasi-homogeneous, nonisotropic materials are derived from a generalized distortional work criterion. For unidirectional composites, the strength is governed by the axial, transverse, and shear strengths, and the angle of fiber orientation.

The strength of a laminated composite consisting of layers of unidirectional composites depends on the strength, thickness, and orientation of each constituent layer and the temperature at which the laminate is cured. In the process of lamination, thermal and mechanical interactions are induced which affect the residual stress and the subsequent stress distribution under external load.

A method of strength analysis of laminated composites is delineated using glass-epoxy composites as examples. The validity of the method is demonstrated by appropriate experiments.

Commonly encountered material constants and coefficients for stress and strength analyses for glass-epoxy composites are listed in the Appendix.

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NOMENCLATURE

- A_{ij} = A = In-plane stiffness matrix, lb/in.
 A_{ij}^* = A^* = Intermediate in-plane matrix, in./lb
 A_{ij}' = A' = In-plane compliance matrix, in./lb
 B_{ij} = B = Stiffness coupling matrix, lb
 B_{ij}^* = B^* = Intermediate coupling matrix, in.
 B_{ij}' = B' = Compliance coupling matrix, 1/lb
 C_{ij} = Anisotropic stiffness matrix, psi
 D_{ij} = D = Flexural stiffness matrix, lb-in.
 D_{ij}^* = D^* = Intermediate flexural matrix, lb-in.
 D_{ij}' = D' = Flexural compliance matrix, 1/lb-in.
 E = Young's modulus, psi
 E_{11} = Axial stiffness, psi
 H_{ij}^* = H^* = Intermediate coupling matrix, in.
 h = Plate thickness, in.
 M_i = M = Distributed bending (and twisting) moments, lb
 M_i^T = M^T = Thermal moment, lb
 $\overline{M}_i$ = $\overline{M}$ = Effective moment = $M_i + M_i^T$
 m = $\cos \theta$, or
 = cross-ply ratio (total thickness of odd layers over that of even layers)

NOMENCLATURE (Continued)

N_i	= N	= Stress resultant, lb/in.
N_i^T	= N^T	= Thermal forces, lb/in.
$\bar{N}_i$	= $\bar{N}$	= Effective stress resultant = $N_i + N_i^T$
n	= $\sin \theta$, or	
		= total number of layers
p	=	Ratio of normal stresses = σ_2 / σ_1
q	=	Ratio of shear stress = σ_6 / σ_1
r	=	Ratio of normal strengths = X/Y
S	=	Shear strength of unidirectional composite, psi
s	=	Shear strength ratio = X/s
S_{ij}	=	Anisotropic compliance matrix, 1/psi
T	=	Temperature, degree F
T^+	=	Coordinate transformation with positive rotation
T^-	=	Coordinate transformation with negative rotation
X	=	Axial strength of unidirectional composite, psi
Y	=	Transverse strength of unidirectional composite, psi
α_i	=	Thermal expansion matrix, in./in./degree F
ϵ_i	=	Strain component, in./in.
ϵ_i^o	=	In-plane strain, component, in./in.

NOMENCLATURE (Continued)

θ = Fiber orientation or lamination angle, degree

κ_i = Curvature, 1/in.

λ = $1 - \nu_{12} \nu_{21}$

σ_i = Stress components, psi

τ_{ij} = Shear stress, psi

ν = Poisson's ratio

ν_{12} = Major Poisson's ratio

ν_{21} = Minor Poisson's ratio

SUPERSCRIPTS

+

= Positive rotation or tensile property

-

= Negative rotation or compressive property

k

= k-th layer in a laminated composite

-1

= Inverse matrix

SUBSCRIPTS

i, j

= 1, 2, ... 6 or x, y, z in 3-dimensional space, or

= 1, 2, 6 or x, y, s in 2-dimensional space

SECTION 1

INTRODUCTION

Structural Behavior of Composite Materials

The purpose of the present investigation is to establish a rational basis of the designs of composite materials for structural applications. Ultimately, materials design can be integrated into structural design as an added dimension. Higher performance and lower cost in materials and structures applications can therefore be expected.

Following the research method outlined previously,^{1*} the present program combines two traditional areas of research — materials and structures. These two areas are linked by a mechanical constitutive equation, the simplest form of which is the generalized Hooke's law. The materials research is concerned with the influences of the constituent materials on the coefficients of the constitutive equation, which in this case, are the elastic moduli. The structures research, on the other hand, is concerned with the gross behavior of an anisotropic medium. An integrated structural design takes into account, in addition to the traditional variations in thicknesses and shapes, the controllable magnitude and direction of material properties through the selection of proper constituent materials and their geometric arrangement.

*References are listed at the end of this report.

Following the framework just described, the elastic moduli of anisotropic laminated composites were reported previously.^{2,3} The appropriate constitutive equation was:

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \epsilon^0 \\ \kappa \end{bmatrix} \quad (1)$$

This equation, of course, included the quasi-homogeneous orthotropic composite, which represented a unidirectional composite, as a special case. The material coefficients A, B, and D were expressed in terms of material and geometric parameters associated with the constituent materials and the method of lamination. This information provided a rational basis for the design of elastic stiffnesses of an anisotropic laminated composite. Thus, the investigation reported in References 2 and 3 involved both structures research, in the establishment of Equation (1) as an appropriate constitutive equation, and materials research, in the establishment of the parameters that govern the material coefficients of Equation (1).

The present report covers the strength characteristic of anisotropic laminated composites, which again includes the quasi-homogeneous composite, as a special case. Unlike the case of the elastic moduli, the present report covers only the structures aspect of strengths; the materials aspect is to be investigated in the future. The appropriate constitutive equation for the strength characteristics is established in this report. Only when this information is available, can the area of research from the materials standpoint be delineated. Guidelines for the design of composites from the strength consideration can be derived.

Scope of Present Investigation

The present investigation is concerned with the structures aspect of the strength characteristics of composite materials. The strength of a quasi-homogeneous anisotropic composite is first established. Then the strength of a laminated composite consisting of layers of quasi-homogeneous

composites bonded together is investigated. The validity of the theoretical predictions is demonstrated by using glass-epoxy resin composites as test specimens.

The main result of this investigation is that a more realistic method of strength analysis than the prevailing netting analysis is obtained. The structural behavior of composite materials is now better understood, and one can use these materials with higher precision and greater confidence. A stride is made toward the rational design of composite materials. Although much more analyses and data generation still remain to be done, the present knowledge of stiffnesses and strengths of composite materials, as reported in References 2 and 3, and in this report, respectively, is approaching the level of knowledge presently available in the use of isotropic homogeneous materials.

SECTION 2

STRENGTH OF ANISOTROPIC MATERIALS

Mathematical Theory

Several strength theories of anisotropic materials are frequently encountered in the study of composite materials. Hill postulated a theory in 1948⁴ and later repeated it in his plasticity book.⁵ Using his notation, it is assumed that the yield condition is a quadratic function of the stress components

$$\begin{aligned} 2f(\sigma_{ij}) = & F(\sigma_y - \sigma_z)^2 + G(\sigma_z - \sigma_x)^2 + H(\sigma_x - \sigma_y)^2 \\ & + 2L\tau_{yz}^2 + 2M\tau_{zx}^2 + 2N\tau_{xy}^2 = 1 \end{aligned} \quad (2)$$

where F, G, H, L, M, N are material coefficients characteristic of the state of anisotropy, and x, y, z are the axes of material symmetry which are assumed to exist. This yield condition is a generalization of von Mises' condition proposed in 1913 for isotropic materials. Note that Equation (2) reduces to von Mises' condition when the material coefficients are equal. Beyond this necessary condition, there seems to be no further rationale. Nevertheless, this yield condition has the advantages of being reasonable and readily usable in a mathematical theory of strength because it is a continuous function in the stress space. For identification purposes, this condition will be called the distortional energy condition.

Marin proposed⁶ a strength theory equivalent to Equation (2), except the principal stress components were used instead of the general stress components. The use of principal stresses is, in fact, more difficult to apply to anisotropic materials, since the axes of material symmetry, the principal stress, and the principal strain are, in general, not coincident. Thus, principal stresses per se do not have much physical significance.

Another strength theory of anisotropic material is called the "interaction formula," as described by a series of reports by the Forest Products Laboratory^{7, 8, 9} and apparently independently by Ashkenazi.¹⁰ The interaction formula in Hill's notation^{*} takes the following form:

$$\begin{aligned} \left(\frac{\sigma_x}{X}\right)^2 - \frac{\sigma_x \sigma_y}{X Y} + \left(\frac{\sigma_y}{Y}\right)^2 + \left(\frac{\tau_{xy}}{S}\right)^2 &= 1 \\ \left(\frac{\sigma_y}{Y}\right)^2 - \frac{\sigma_y \sigma_z}{Y Z} + \left(\frac{\sigma_z}{Z}\right)^2 + \left(\frac{\tau_{yz}}{Q}\right)^2 &= 1 \\ \left(\frac{\sigma_z}{Z}\right)^2 - \frac{\sigma_z \sigma_x}{Z X} + \left(\frac{\sigma_x}{X}\right)^2 + \left(\frac{\tau_{zx}}{R}\right)^2 &= 1 \end{aligned} \quad (3)$$

Since the composite material of interests now is in the form of thin plates, a state of plane stress is assumed. Then Equations (2) and (3) can be reduced, respectively:

$$\left(\frac{\sigma_x}{X}\right)^2 - \frac{1}{r} \frac{\sigma_x \sigma_y}{X Y} + \left(\frac{\sigma_y}{Y}\right)^2 + \left(\frac{\tau_{xy}}{S}\right)^2 = 1 \quad (4)$$

*The shear strengths used here are Q, R, S rather than R, S, T, in order to spare T for temperature.

$$\left. \begin{aligned} \left(\frac{\sigma_x}{X} \right)^2 - \frac{\sigma_x \sigma_y}{X Y} + \left(\frac{\sigma_y}{Y} \right)^2 + \left(\frac{\tau_{xy}}{S} \right)^2 &= 1 \\ \left(\frac{\sigma_y}{Y} \right)^2 &= 1 \\ \left(\frac{\sigma_x}{X} \right)^2 &= 1 \end{aligned} \right\} \quad (5)$$

The difference between the yield condition of distortional energy, the interaction formula, and von Mises is shown in Figure 1, assuming tensile and compressive strengths of the materials are equal.

For the present program, it is assumed that the distortional energy condition is valid. This, of course, will be substantiated experimentally later in this report. It is also assumed, for the present, that failure by yielding and by ultimate strength are synonymous. This will be shown to be reasonable for glass-epoxy composites, which exhibit linearly elastic behavior up to failure stress with little or no yielding. The work contained in the Forest Product reports^{7, 8, 9} and Askenazi¹⁰ had two restrictions: (1) no differentiation was made between the homogeneous and laminated composite, (2) shear strength was not treated as an independent strength property. In the present investigation, both these restrictions are removed.

Quasi-homogeneous Composites

The strength of quasi-homogeneous anisotropy composites was reported by Azzi and Tsai.¹¹ For the sake of completeness, the essential points of this reference are repeated here.

It is the purpose of this section to demonstrate how the distortional energy condition can be applied to a quasi-homogeneous anisotropy composite subjected to combined stresses. One of the basic assumptions of this condition is that there exist three mutually perpendicular planes of symmetry within the anisotropy body. This means that the body is really orthotropic rather than generally anisotropic from the point of view of strength. Under

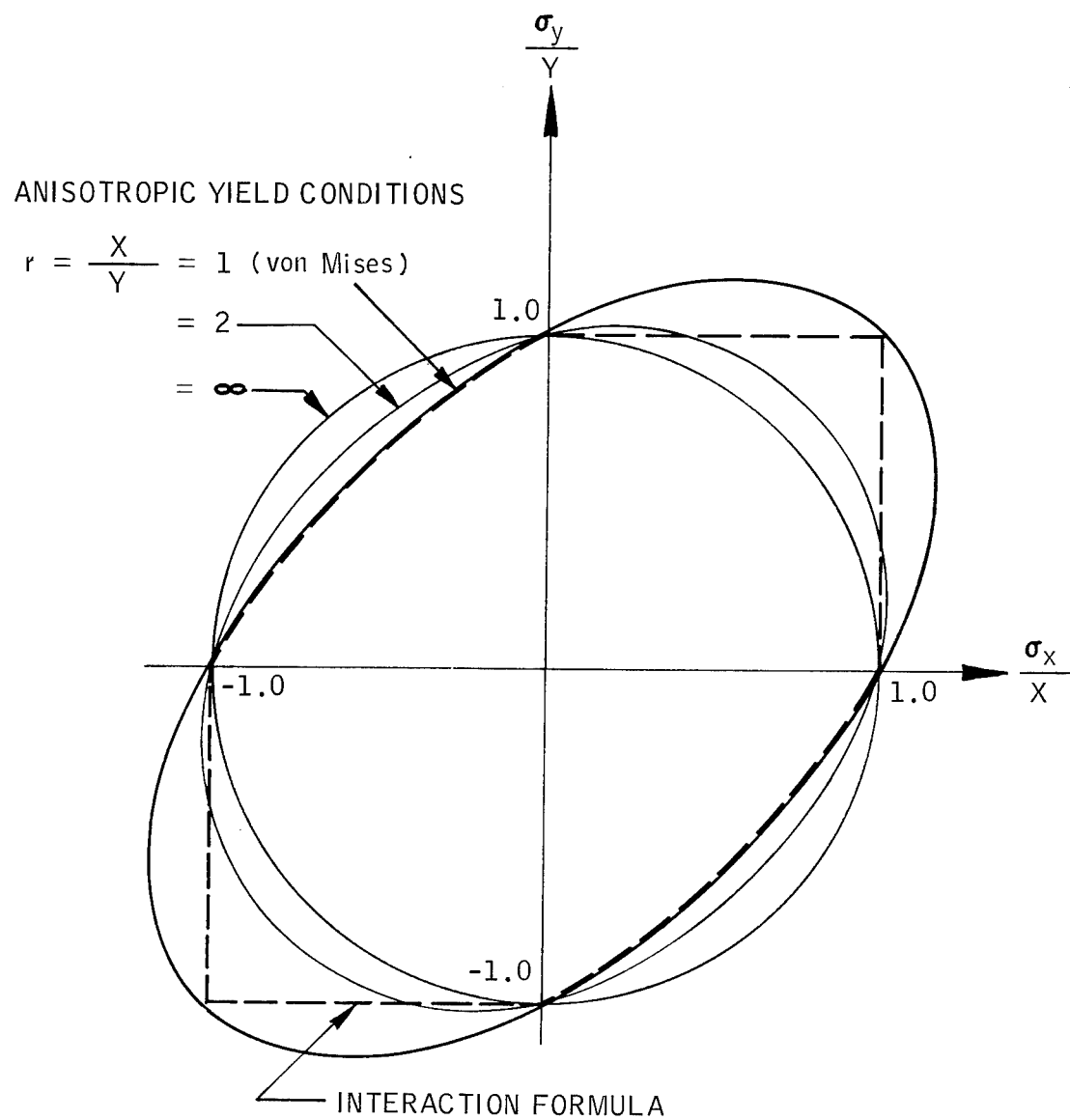


Figure 1. Comparative Yield Surfaces

this assumption, the yield condition must be applied to the state of stress expressed in the coordinate system coincident with that of the material symmetry. Thus, the state of stress imposed on a body must be transformed to the coordinate system of material symmetry and then the yield condition applied. Let x-y be the material symmetry axes, and 1-2, the reference coordinate axes of the externally applied stresses, the usual transformation equation¹² in matrix form is:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_s \end{bmatrix} = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{bmatrix} \quad (6)$$

where $m = \cos \theta$, $n = \sin \theta$, and positive θ is shown in Figure 2.

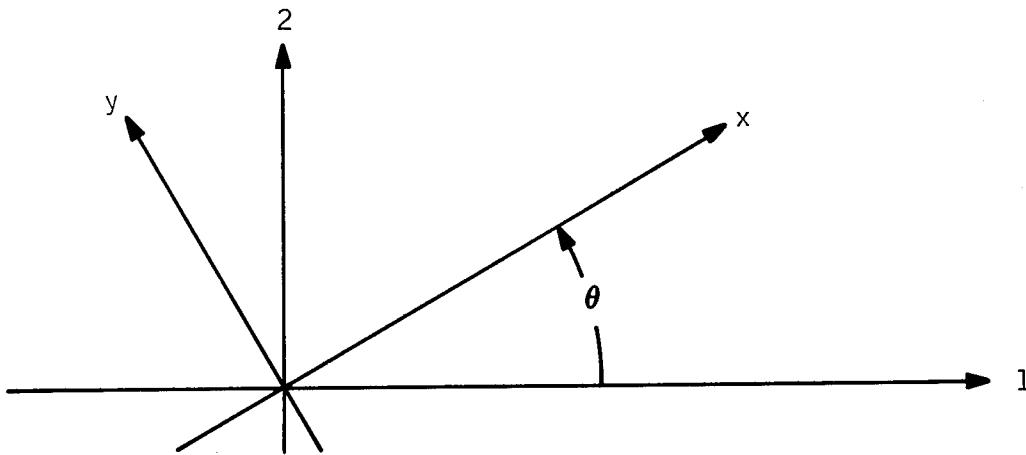


Figure 2. Coordinate Transformation of Stress

For convenience, the following notations are used:

$$p = \sigma_2 / \sigma_1, \quad q = \sigma_6 / \sigma_1, \quad r = X/Y, \quad s = X/S \quad (7)$$

Substituting the notations in Equations (6) and (7) into the yield condition in the form of Equation (4), one obtains:

$$\begin{aligned} & \left[1 - p + p^2 r^2 + q^2 s^2 \right] m^4 + 2 q \left[3 - p - 2 p r^2 + (p - 1) s^2 \right] m^3 n \\ & + \left[8 q^2 + 2 (p + 2 q^2) r^2 + (p - 1)^2 (s^2 - 1) - 2 q^2 s^2 \right] m^2 n^2 \\ & + 2 q \left[3 p - 1 - 2 r^2 - (p - 1) s^2 \right] m n^3 + \left[p^2 - p + r^2 + q^2 s^2 \right] n^4 \\ & = (X / \sigma_1)^2 = (r Y / \sigma_1)^2 = (s S / \sigma_1)^2 \end{aligned} \quad (8)$$

This result may be summarized as follows: For a given anisotropic body in reference coordinates 1-2, specified by X, Y (or r), and S (or s), with a given orientation of the material symmetry axes, θ , and subjected to combined stresses σ_1 , σ_2 (or p) and σ_6 (or q), the magnitude of the applied stress σ_1 , at failure, can be determined by solving Equation (8) for σ_1 . Alternatively, Equation (8) may be regarded as the transformation equation for the strength of a quasi-homogeneous anisotropic material subjected to combined stresses; i. e., the strength characteristics as a function of the orientation of the symmetry axes, θ .

For uniaxial tension, $p = q = 0$, the failure condition is

$$m^4 + (s^2 - 1) m^2 n^2 + r^2 n^4 = (X / \sigma_1)^2 \quad (9)$$

or

$$\sigma_1 = X / \left[m^4 + (s^2 - 1) m^2 n^2 + r^2 n^4 \right]^{1/2} \quad (10)$$

Thus, by performing uniaxial tension tests on specimens with different orientations of the material symmetry axes; i.e., different values of θ , one finds directly the transformation property of strength. What is equally important is that the strength characteristics of a quasi-homogeneous anisotropic material under combined stresses are simultaneously verified. By a simple substitution of Equation (6) into (9), while maintaining $p = q = 0$, one recovers, as expected, the original yield condition shown in Equation (4).

Equation (8) can be reduced to other simple cases in a straightforward manner. For example, the case of hydrostatic pressure requires $p = 1$, $q = 0$, from which one can show that the maximum pressure is equal to the transverse strength, Y , and is independent of the orientation, θ .

The case of an internally pressurized cylindrical shell is described by $p = 2$, $q = 0$, from which Equation (8) reduces to

$$(4r^2 - 1)m^4 + (4r^2 - 1 + s^2)m^2n^2 + (r^2 + 2)n^4 = (X/\sigma_1)^2 \quad (11)$$

For isotropic material, it can be shown that

$$r = 1, \quad s = \sqrt{3}$$

which agrees with von Mises' condition.⁵ Equation (11) then reduces to

$$\sigma_1 = X/\sqrt{3}$$

and

(12)

$$\sigma_2 = 2X/\sqrt{3} = 1.155X$$

which is the well-known result between the maximum hoop stress σ_2 and the uniaxial strength X .⁵

The case of pure shear can be derived by letting $\sigma_1 = \sigma_2 = 0$ in Equation (6), and then by substituting it into Equation (4),* one obtains

$$4 m^2 n^2 (r^2 + 2)/s^2 + (m^2 - n^2)^2 = (S/\sigma_6)^2 \quad (13)$$

or

$$\sigma_6 = S / \left[4 m^2 n^2 (r^2 + 2)/s^2 + (m^2 - n^2)^2 \right]^{1/2} \quad (14)$$

It is interesting to note that:

$$\text{when } \theta = 0^\circ \text{ or } 90^\circ, \quad \sigma_6 = S \quad (15)$$

$$\text{when } \theta = \pm 45^\circ, \quad \sigma_6 = X / \left[r^2 + 2 \right]^{1/2} \quad **$$

$$\equiv Y, \text{ if } r \gg 1 \quad (16)$$

$$= X/\sqrt{3}, \text{ if } r = 1 \text{ (isotropy)}$$

In conclusion, it is seen that the distortional energy condition can be easily applied to cases frequently encountered in the design and use of anisotropic composites. The strength characteristics involve the axial, transverse and shear strengths, X , Y , and S , respectively, and the orientation of the material symmetry axes, θ . This strength theory is quite different from the netting analysis, which is still used extensively in the filament-winding industry. The inaccuracy of netting analysis as a theory or design criterion is far less damaging per se than the influence of its erroneous implications on many recent and even current research programs on filament-winding.

*Equation (8) cannot be used directly for this case because σ_1 is equal to zero.

**This is the shear strength used in Marin's theory.⁶ It is a derived quantity, as opposed to X , Y , and S , which are the "principal strengths."

Experimental Results

In the preceding subsection, the utility from the mathematical standpoint of yield condition as applied to a quasi-homogeneous anisotropic composite has been outlined. In this subsection, experimental results which demonstrate the validity of the proposed theory of strength will be reported.

The specimens used were made of unidirectional glass-filaments preimpregnated with epoxy resin. This material is supplied by the U.S. Polymeric Company with a designation of E-787-NUF.* The curing cycle involved no preheat, 50 psi pressure, and 300°F temperature for 2 hours followed by slow cooling. Tensile test specimens were cut from the cured panels using a wet-bladed masonry saw. As it was found that specimens of uniform cross section had a tendency to fail under the grips at low angles of fiber orientation, a diamond-coated router was used to shape specimens with a reduced test section, in "dog-bone" fashion. Approximate specimen dimensions were (in inches): overall length, 8.00; overall width, 0.450; length of test section, 2.50; width of test section, 0.180; thickness, 0.125. A 3-inch-radius circular arc, tangent to the test section, connected the test section to the maximum end section. Additionally, aluminum tabs (a catalogue item) were bonded to the ends of the specimens to distribute the loads imposed by the grips. A special fixture was devised: (1) to align the tabs with the specimens to ensure application of pure axial load, and (2) to be capable of making up to 20 individual specimens simultaneously. Sample specimens, before and after test, are shown in Figure 3.

The values of the axial and transverse normal strengths X and Y for the material employed were determined from simple tension tests of specimens having fiber orientations of 0 and $\pi/2$ to the direction of applied stress, respectively. The shear strength S was determined from the simple torsion test of a filament-wound thin-walled torsion tube having all circumferential windings.

*The same material was used to make test specimens reported in Reference 2.

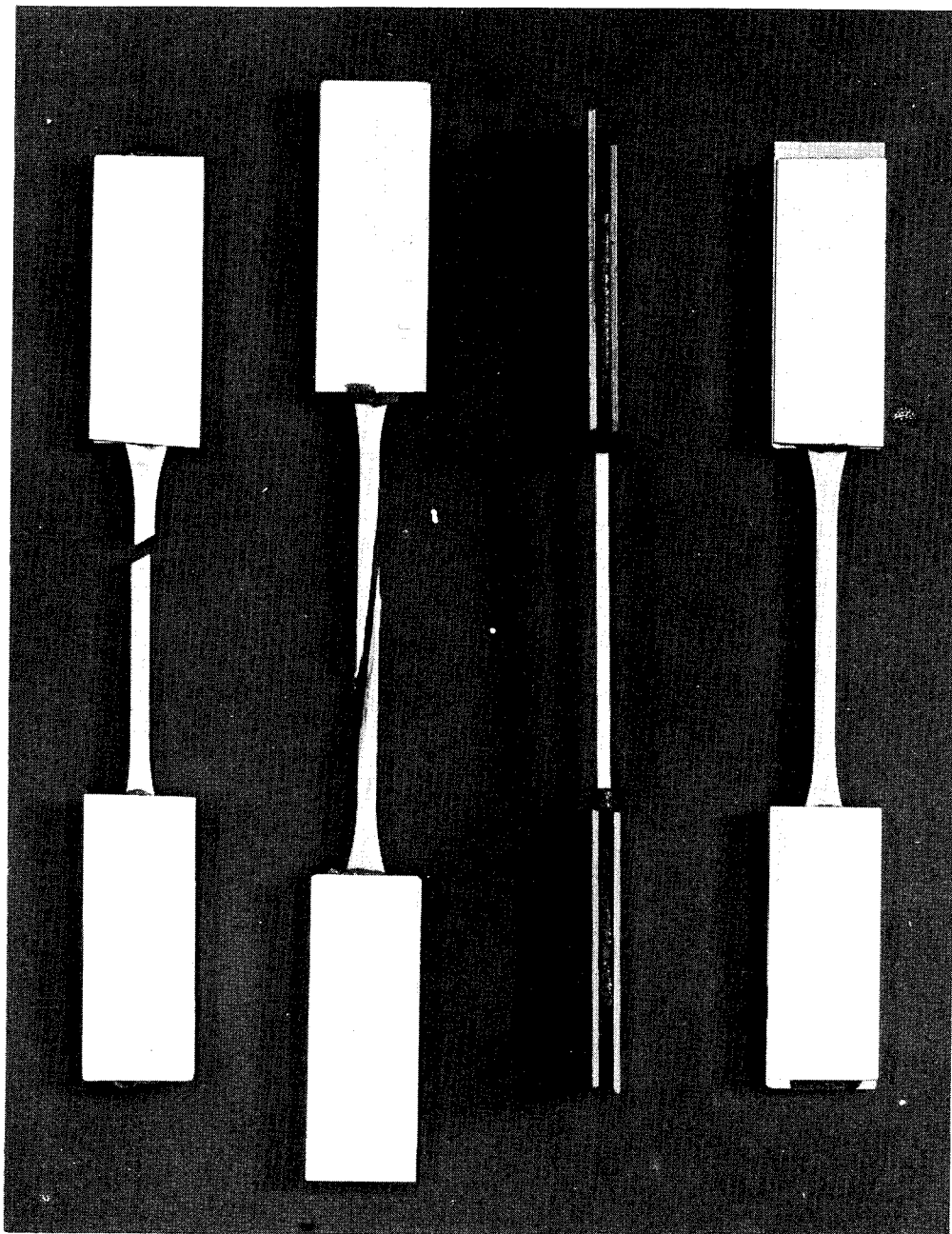


Figure 3. Tensile Test Specimens

To verify the theoretical results, specimens were cut at 5-degree increments in the lower angle ranges where strength variation is greatest, and at 15-degree increments for higher angles. The strengths measured for these specimens were then compared with results obtained from the theory evaluated with the corresponding values for X, Y, and S. The theoretical prediction using Equation (10), and experimental results are shown in Figure 4. The results indicate that the validity of the proposed theory of strength is demonstrated, as most measured strength values are in agreement with theoretical predictions. The values for X, Y, and S for the case illustrated were 150, 4 and 6 ksi. The lack of excellent agreement at some of the higher values of θ may be caused by increased sensitivity of the specimen edges to the shaping operation and the minute crazing that it sometimes induces. This sensitivity increases with the fiber orientation θ ; hence, great care must be exercised in the preparation of specimens.

Also shown in Figure 4 is the theoretically predicted stiffness as a function of fiber orientation, together with experimental measurements. The theoretical curve, shown as the solid line, is computed using the usual transformation equation of the stiffness matrix:

$$C'_{11} = m^4 C_{11} + 2 m^2 n^2 C_{22} + n^4 C_{22} + 4 m^2 n^2 C_{66}$$

where the following moduli, same as those in Reference 2, are used:

$$C_{11} = 7.97 \times 10^6 \text{ psi}$$

$$C_{12} = 0.66 \times 10^6 \text{ psi}$$

$$C_{22} = 2.66 \times 10^6 \text{ psi}$$

$$C_{16} = C_{26} = 0$$

$$C_{66} = 1.25 \times 10^6 \text{ psi}$$

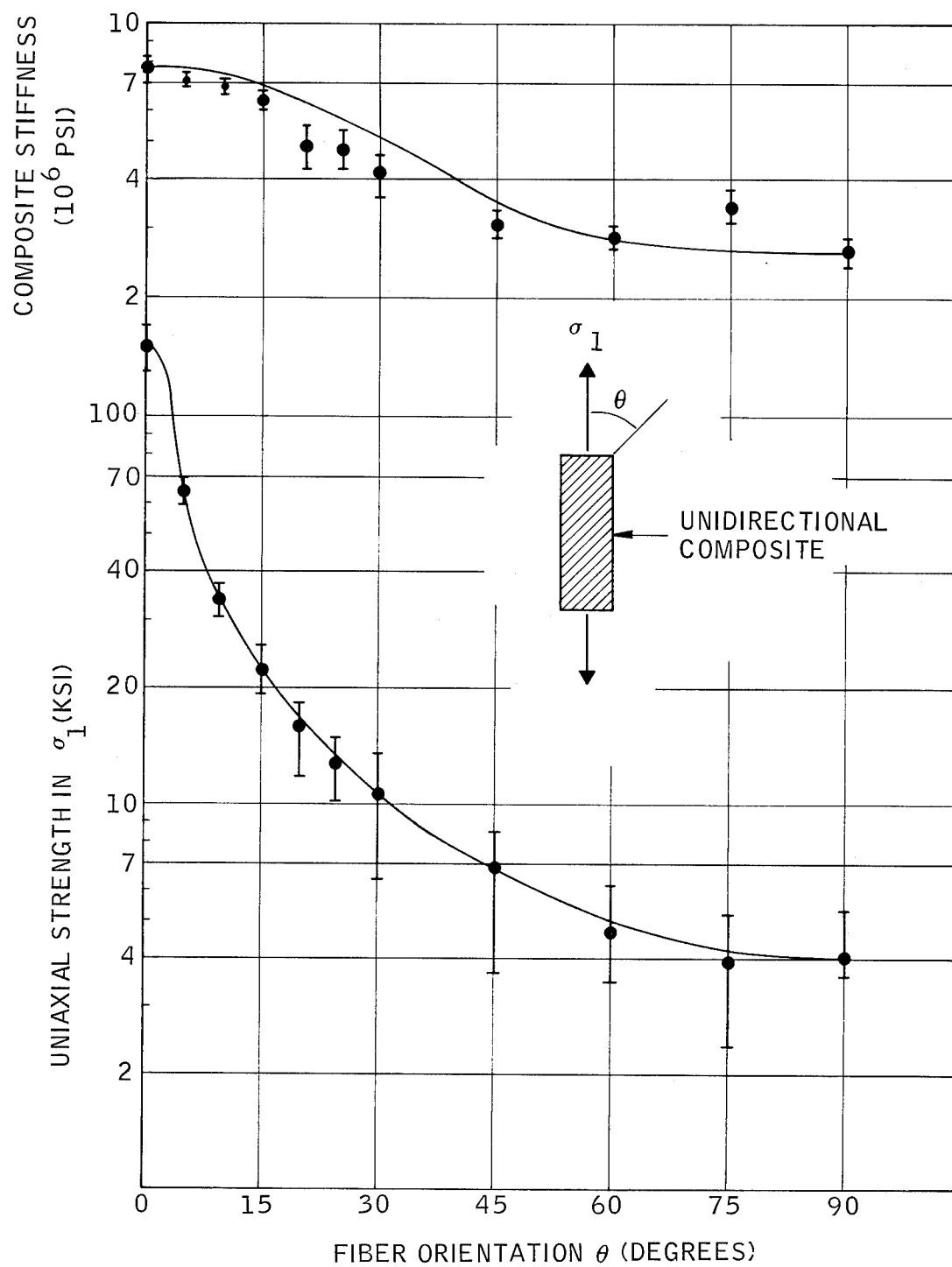


Figure 4. Strength of Unidirectional Composites

From Equation (10), one can examine the variation of the transformation property of composite strength with the basic strength characteristics X , Y , and S . The effect of Y is significant for large angles of orientation, and the effect of axial strength, X , is significant for small angles. Further, the shear strength, S , becomes the dominant strength characteristic for intermediate angles of orientation. These influences of each strength characteristic must be taken into consideration in any attempt to improve the strength of composite materials having arbitrary fiber orientations to the applied load.

It is reasonable to conclude that the present investigation of the strength of a quasi-homogeneous anisotropic composite under any state of combined stresses can be predicted with accuracy. The theory has been developed for the most general case of plane stress and discussed in detail. Although the experiment confirmation was limited to uniaxial tension, a state of combined stresses is actually induced in the coordinate system representing the material symmetry. It is assumed that the tensile and compressive strength characteristics are equal. If they are not equal, one can easily introduce say X^+ , X^- , Y^+ , Y^- , where the plus and minus superscripts denote tensile and compressive strengths, respectively. No conceptual difficulty is expected for this modification, as indicated for example in References 6 and 7.

For the particular specimens, the shear strength, S , falls between the two normal strengths, X and Y . The ratio of the shear strength over the transverse strength are 1.5 for the specimens. This value is not much different from $\sqrt{3}$ which is the ratio for isotropic materials or a composite material reinforced by spherical inclusions. The present specimen has a lower transverse strength than shear strength. This implies that the shear strength is at a minimum for a 45-degree fiber orientation, as can be seen from Equations (14) and (16) (assuming $Y^+ = Y^-$). This is particularly interesting in view of the fact that the shear modulus of common orthotropic materials, which include the present specimens, is at a maximum at 45-degree orientation. The behavior of a laminated composite, on the other hand, will be quite different from a quasi-homogeneous composite, as will be reported in the next sections.

SECTION 3

STRENGTH OF LAMINATED COMPOSITES

Mathematical Theory

The strength of laminated anisotropic composites is dependent on the thermomechanical properties of the constituent layers and the method of lamination, which include the thickness and orientation of each layer, the stacking sequence, cross-ply ratio, helical angle, the laminating temperature, etc. In the process of lamination, two sources of interaction are induced. First, there is a mechanical interaction caused by the transverse heterogeneity of the composite; i. e., material properties vary across the thickness of the composite, and the cross-coupling of the "16" and "26" components of the stiffness matrix. As a result, the stress across the composite is not uniform and is distributed according to the relative stiffnesses of the constituent layers. Second, there is a thermal interaction caused by the differential thermal expansion (or contraction) between constituent layers. Since most composites are laminated at elevated temperatures, initial stresses are induced if the service temperature of the composite is different from the laminating temperature. Taking into account both mechanical and thermal interactions, the strength of a laminated composite can be described by a piecewise linear stress-strain relation. Discontinuous slopes in this curve occur when one or more of the constituent layers have failed. The ultimate strength of the composite is reached when all the constituent layers have failed. Throughout this section, it is assumed, as before, that the tensile and compressive properties are equal, and yielding and strength are synonymous.

The strength analysis for the present investigation is based on the strength-of-materials' approach. The general thermoelastic analysis of laminated anisotropic composites is outlined first. Only the problem of shrinkage stress is treated here, although the analysis is applicable to thermal stress problems in general.

For the sake of completeness, the basic constitutive equation of thermoelasticity and the essential points of Reference 13 are repeated here.

It is assumed that each constituent layer of the laminated composite is quasi-homogeneous and orthotropic, and is in the state of plane stress. Using the usual contracted notations,¹² the three-dimensional generalized Hooke's law for any constituent layer is:

$$\epsilon_i = S_{ij} \sigma_j + \alpha_i T, \quad i, j = 1, 2, \dots, 6 \quad (17)$$

This equation states that the total strain is the sum of mechanical strain (the first term) and free thermal strain (the second term). One can invert Equation (17) and obtain

$$\sigma_i = C_{ij} (\epsilon_j - \alpha_j T) \quad (18)$$

For an orthotropic layer, the stiffness¹² and thermal expansion¹⁴ matrices are:

$$C_{ij} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ & C_{22} & C_{23} & 0 & 0 & 0 \\ & & C_{33} & 0 & 0 & 0 \\ & & & C_{44} & 0 & 0 \\ & & & & C_{55} & 0 \\ & & & & & C_{66} \end{bmatrix} \quad (19)$$

$$a_i = \begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix} \quad (20)$$

For a state of plane stress, it is assumed that:

$$\sigma_3 = \sigma_4 = \sigma_5 = 0 \quad (21)$$

Substituting Equations (19), (20), and (21) into (18),

$$\epsilon_4 = \epsilon_5 = 0, \text{ and} \quad (22)$$

$$\epsilon_3 - a_3 T = -\frac{C_{31}}{C_{33}} (\epsilon_1 - a_1 T) - \frac{C_{32}}{C_{33}} (\epsilon_2 - a_2 T) \quad (23)$$

Substituting Equation (23) into (18),

$$\sigma_1 = (C_{11} - \frac{C_{13}^2}{C_{33}}) (\epsilon_1 - a_1 T) + (C_{12} - \frac{C_{13}C_{32}}{C_{33}}) (\epsilon_2 - a_2 T) \quad (24)$$

$$\sigma_2 = (C_{21} - \frac{C_{23}C_{13}}{C_{33}}) (\epsilon_1 - a_1 T) + (C_{22} - \frac{C_{32}}{C_{33}}) (\epsilon_2 - a_2 T) \quad (25)$$

$$\sigma_6 = C_{66} \epsilon_6 \quad (26)$$

In terms of engineering constants,¹⁵

$$\begin{aligned} C_{11} - \frac{C_{13}^2}{C_{33}} &= E_1 / \lambda \\ C_{22} - \frac{C_{23}^2}{C_{33}} &= E_2 / \lambda \\ C_{12} - \frac{C_{13}C_{23}}{C_{33}} &= \nu_{21}E_1 / \lambda = \nu_{12}E_2 / \lambda \end{aligned} \quad (27)$$

where $\lambda = 1 - \nu_{12} \nu_{21}$

The equivalent constitutive equation for a laminated anisotropic composite can be derived using the basic assumption of the nondeformable normals of the strength of materials. It is assumed that

$$\epsilon_i = \epsilon_i^0 + z \kappa_i \quad (28)$$

where, following the notations in Reference 2, $i = 1, 2$, and 6 .

Equation (18), when integrated across the thickness of the laminated composite, becomes:

$$\bar{N}_i = N_i + N_i^T = A_{ij} \epsilon_j^0 + B_{ij} \kappa_j \quad (29)$$

$$\bar{M}_i = M_i + M_i^T = B_{ij} \epsilon_j^0 + D_{ij} \kappa_j \quad (30)$$

where

$$(N_i, M_i) = \int_{-h/2}^{h/2} \sigma_i (1, z) dz \quad (31)$$

$$(N_i^T, M_i^T) = \int_{-h/2}^{h/2} C_{ij} a_j^T (1, z) dz \quad (32)$$

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-h/2}^{h/2} C_{ij} (1, z, z^2) dz \quad (33)$$

Equations (29) and (30) are the basic constitutive equations for a laminated anisotropic composite, taking into account equivalent thermal loadings.

The stress at any location across the thickness of the composite can be determined as follows:²

$$\begin{bmatrix} \bar{N} \\ \bar{M} \end{bmatrix} = \begin{bmatrix} A & | & B \\ \hline B & | & D \end{bmatrix} \begin{bmatrix} \epsilon^0 \\ \kappa \end{bmatrix} \quad (34)$$

Then, by matrix inversion,

$$\begin{bmatrix} \epsilon^0 \\ \bar{M} \end{bmatrix} = \begin{bmatrix} A^* & | & B^* \\ \hline H^* & | & D^* \end{bmatrix} \begin{bmatrix} \bar{N} \\ \kappa \end{bmatrix} \quad (35)$$

$$\begin{bmatrix} \epsilon^0 \\ \kappa \end{bmatrix} = \begin{bmatrix} A' & | & B' \\ \hline H' & | & D' \end{bmatrix} \begin{bmatrix} \bar{N} \\ \bar{M} \end{bmatrix} \quad (36)$$

$$\begin{aligned}
\text{where } A^* &= A^{-1} \\
B^* &= -A^{-1}B \\
H^* &= BA^{-1} \\
D^* &= D - BA^{-1}B \\
A' &= A^* - B^* D^{*-1} H^* \\
B' &= H' = B^* D^{*-1} \\
D' &= D^{*-1}
\end{aligned} \tag{37}$$

Substituting Equation (36) into (28),

$$\begin{aligned}
\epsilon_i &= \epsilon_i^0 + z\kappa_i \\
&= (A'_{ij} + zB'_{ij}) \bar{N}_j + (B'_{ij} + zD'_{ij}) \bar{M}_j
\end{aligned} \tag{38}$$

from Equation (18), the stress components for the k-th layer are:

$$\begin{aligned}
\sigma_i^{(k)} &= C_{ij}^{(k)} (\epsilon_j - \alpha_j^{(k)} T) \\
&= C_{ij}^{(k)} \left[(A'_{jk} + zB'_{jk}) \bar{N}_k + (B'_{jk} + zD'_{jk}) \bar{M}_k - \alpha_j^{(k)} T \right]
\end{aligned} \tag{39}$$

This is the most general expression of stresses as functions of stress resultants, bending moments, and temperature. The same material coefficients A' , B' and D' , as reported in Reference 2 and also tabulated in the Appendix of this report, can be used for the thermal stress analysis. This single link between the isothermal and nonisothermal analyses is achieved by treating thermal effects as equivalent mechanical loads; e. g., N_i^T and M_i^T in Equation (32).

It can be shown that for quasi-homogeneous plates, $B' = H' = 0$; i. e., no cross-coupling exists. In addition,

$$\begin{aligned}
A_{ij} &= C_{ij} h \\
D_{ij} &= C_{ij} h^3 / 12 = A_{ij} h^2 / 12
\end{aligned} \tag{40}$$

Equation (39) can be reduced to:

$$\begin{aligned}\sigma_i &= \frac{A_{ij}}{h} \left[A'_{jk} (\bar{N}_k + \frac{12}{h^2} z \bar{M}_k) - a_j T \right] \\ &= \frac{1}{h} (\bar{N}_i + \frac{12}{h^2} z \bar{M}_i) - C_{ij} a_j T\end{aligned}\tag{41}$$

using the relationship of A' being the inverse of A for quasi-homogeneous plates. If the plate is also isotropic,

$$\begin{aligned}C_{ij} a_j T &= (C_{11} a_1 + C_{12} a_2) T = \frac{E a T}{1 - \nu} \\ \bar{N}_i &= N_i + N_i^T = N_i + \frac{E a}{1 - \nu} \int_{-h/2}^{h/2} T dz \\ \bar{M}_i &= M_i + M_i^T = M_i + \frac{E a}{1 - \nu} \int_{-h/2}^{h/2} T z dz\end{aligned}\tag{42}$$

Substituting Equations (42) into (41), we obtain the same result as Equation (12.2.7) of Reference 16.

As stated before, thermal stresses are induced when the operating temperature of the composite differs from its laminating temperature. As a typical example, it is assumed that the laminating temperature is T degrees above the operating temperature which is assumed to be ambient. It is further assumed that the zero-stress state exists at the laminating temperature which is now set as the datum temperature. The operation temperature is then $-T$. For a traction-free condition,

$$(\bar{N}_i, \bar{M}_i) = (N_i^T, M_i^T) = -T \int_{-h/2}^{h/2} C_{ij} a_j (1, z) dz\tag{43}$$

From Equation (39)

$$\sigma_i^{(k)} = C_{ij}^{(k)} \left[(A_{jk}^i + zB_{jk}^i) N_k^T + (B_{jk}^i + zD_{jk}^i) M_k^T + \alpha_j^{(k)} T \right] \quad (44)$$

For an isotropic quasi-homogeneous plate under uniform temperature,

$$N_i^T = -\frac{\alpha E T h}{1 - \nu}, \quad M_i^T = 0 \quad (45)$$

$$B_{jk}^i = 0, \quad C_{ij} = A_{ij} / h$$

Substituting Equation (45) in (41) and (38), one obtains, as expected

$$\sigma_i = \frac{1}{h} N_i^T - C_{ij} \alpha_j T = 0 \quad (46)$$

$$\epsilon_i = A_{ij}^i N_j = -\alpha T$$

If the temperature is linear across the thickness of the isotropic quasi-homogeneous plate; i. e.,

$$T = \alpha z \quad (47)$$

then by substituting Equation (47) into (32), one obtains

$$N_i^T = 0, \quad M_i^T = -\frac{E \alpha a h^3}{12(1 - \nu)} \quad (48)$$

Hence, from Equations (41) and (38), one obtains, again as expected,

$$\sigma_i = \frac{1}{h} \frac{12z}{h^2} M_i^T - C_{ij} \alpha_j T = 0 \quad (49)$$

$$\epsilon_i = z D_{ij}^i M_j^T = -\alpha a z$$

The results of Equations (46) and (49) agree with the elementary theory; e. g., Equation (9.5.66) of Reference 16.

The strength analysis of a laminated anisotropic composite is accomplished by substituting the stress components of the k -th constituent layer, calculated from Equation (39), into the general yield condition of Equation (8), or its equivalent equation, when σ_1 is equal to zero, e.g., Equation (13). From Equation (8), the maximum σ_1 , in combination with the particular p and q that each constituent layer can sustain, can be obtained. When this maximum is reached, failure in the particular layer or layers is considered to have occurred. After this failure, the remaining layers, which have not failed, will have to carry additional loads. This shifting of loads is accompanied by a partial or complete uncoupling of the mechanical and thermal interactions mentioned above. The net result is that a new effective stiffness of the laminated composite is now in operation. This new stiffness, as reflected in new values of A , B , and D matrices of Equation (34), will cause a change in the distribution of stresses in each of the constituent layers still intact. The effective stress-strain relation of the composite is changed and a "knee" is exhibited as the slope of the stress-strain relation becomes discontinuous. New values of A' , B' , and D' matrices which are computed from the revised A , B , and D , must now be used in Equation (39) for the computation of the stresses. These new stresses will again be substituted into the yield condition of Equation (8), from which the next layer or layers that would fail can be determined. This process is repeated until all the layers have failed.

The mathematical description of the uncoupling of the mechanical and thermal interactions is not easy to ascertain. As one possibility, cracks transverse to the fibers will develop, which cause a degradation of the effective stiffness and a change in the stress distribution in the composite. Another possibility is a complete delamination of the laminate, thereby uncoupling the thermal and mechanical interactions. The exact description of the degradation process must be treated for particular laminates, as will be shown later.

The important point intended for this section is to illustrate the existence of mechanical and thermal interactions as a direct consequence of lamination. Internal stresses are induced. These stresses exist in addition to

the externally imposed stresses. Unlike the work of References 7 through 10, the present investigation makes the necessary distinction between quasi-homogeneous and laminated composites.

Cross-ply Composites

The general equations for the analysis of strength can be considerably simplified if the laminated composite is a cross-ply composite, which consists of constituent layers oriented alternately at 0 and 90 degrees. All odd layers have one thickness. All even layers also have one thickness but are, in general, different from the odd layers. The lamination parameters, following the notations of Reference 2, include the total number of layers, n , and the cross-ply ratio, m , which is the ratio of the total thickness of the odd layers over that of the even layers. For the present work, as in Reference 2, the odd layers are oriented at 0 degree.

As an illustration of how the strength analysis may be carried out, a particular case of $n = 3$, $m = 0.2$ will be shown in detail. Only uniaxial tension will be considered, i. e., only N_1 is nonzero. Since the laminated composite is symmetrical with respect to the centroidal axis by virtue of having $n = 3$, and only symmetrical loading (i. e., all bending moments are zero) is considered, the stress distribution in the first and third layers will be identical. Thus, only two layers have to be considered in the strength analysis: the inner layer (layer 2) and the outer layer (layer 1 or 3). From Equation (39), for the outer layer,

$$\begin{aligned}
 \sigma_1^{(1)} &= C_{11}^{(1)}(A'_{11}\bar{N}_1 + A'_{12}\bar{N}_2 - \alpha_1^{(1)}T) + C_{12}^{(2)}(A'_{21}\bar{N}_1 + A'_{22}\bar{N}_2 - \alpha_2^{(1)}T) \\
 &= (C_{11}^{(1)}A'_{11} + C_{12}^{(1)}A'_{21})N_1 \\
 &\quad + \left[(C_{11}^{(1)}A'_{11} - C_{12}^{(1)}A'_{21})N_1^T + (C_{11}^{(1)}A'_{12} + C_{12}^{(1)}A'_{22})N_2^T \right. \\
 &\quad \left. - (C_{11}^{(1)}\alpha_1^{(1)} + C_{12}^{(1)}\alpha_2^{(1)})T \right]
 \end{aligned} \tag{50}$$

$$\begin{aligned}
\sigma_2^{(1)} = & (C_{21}^{(1)} A'_{11} + C_{22}^{(1)} A'_{21}) N_1 \\
& + \left[(C_{21}^{(1)} A'_{11} + C_{22}^{(1)} A'_{21}) N_1^T + (C_{21}^{(1)} A'_{12} + C_{22}^{(1)} A'_{22}) N_2^T \right. \\
& \left. - (C_{21}^{(1)} \alpha_1^{(1)} + C_{22}^{(1)} \alpha_2^{(1)}) T \right]
\end{aligned} \tag{51}$$

$$\sigma_6^{(1)} = 0 \tag{52}$$

In the above, Equation (29) was used; i. e.,

$$\bar{N}_1 = N_1 + N_1^T, \bar{N}_2 = N_2^T, \bar{N}_6 = 0 \tag{53}$$

for the inner layer,

$$\sigma_i^{(2)} = C_{ij}^{(2)} (A'_{jk} \bar{N}_k - \alpha_j^{(2)} T) \tag{54}$$

This equation, when expanded, will be the same as Equations (50) through (52), except that superscript (1) will be replaced by superscript (2).

Using the following experimentally determined material properties which represent typical unidirectional glass filament-epoxy resin composites,* one can evaluate the stress components for the inner and outer layers in terms of the axial stress resultant N_1 and the lamination temperature T .

*The same composite which was reported in Section 2.

$$\begin{aligned}
C_{11}^{(1)} &= C_{22}^{(2)} = 7.97 \times 10^6 \text{ psi} \\
C_{12}^{(1)} &= C_{12}^{(2)} = 0.66 \times 10^6 \text{ psi} \\
C_{22}^{(1)} &= C_{11}^{(2)} = 2.66 \times 10^6 \text{ psi} \\
C_{66}^{(1)} &= C_{66}^{(2)} = 1.25 \times 10^6 \text{ psi} \\
C_{16}^{(1)} &= C_{26}^{(1)} = C_{16}^{(2)} = C_{26}^{(2)} = 0 \\
a_1^{(1)} &= a_2^{(2)} = 3.5 \times 10^{-6} / ^\circ\text{F} \\
a_2^{(1)} &= a_1^{(2)} = 11.4 \times 10^{-6} / ^\circ\text{F} \\
a_6^{(1)} &= a_6^{(2)} = 0
\end{aligned} \tag{55}$$

In a three-layer ($n = 3$) and $m = 0.2$ cross-ply composite, one can compute the following quantities which are needed for substitution into Equations (50) through (53). From Equations (33) and (37),*

$$\begin{aligned}
A'_{11} &= 0.29 \times 10^{-6} \text{ in. / lb} \\
A'_{12} &= -0.03 \times 10^{-6} \text{ in. / lb} \\
A'_{22} &= 0.14 \times 10^{-6} \text{ in. / lb}
\end{aligned} \tag{56}$$

*The detailed calculation and some typical data for glass-epoxy composites are shown in the Appendix.

From Equation (32), assuming a constant lamination temperature T , one can compute the equivalent thermal forces and moments:

$$\begin{aligned} N_1^T &= 33.1 T \quad \text{lb-in.} \\ N_2^T &= 35.0 T \quad \text{lb-in.} \\ N_6^T &= M_1^T = 0, \text{ as expected for three-layer cross-ply} \end{aligned} \tag{57}$$

Substituting the computed values in Equations (56) and (57) into the equations for the stress components (50) through (55), for the outer layers,

$$\begin{aligned} \sigma_1^{(1)} &= 2.27 N_1 + 35.5 T \\ \sigma_2^{(1)} &= 0.12 N_1 - 16.0 T \\ \sigma_6^{(1)} &= 0 \end{aligned} \tag{58}$$

and for the inner layer,

$$\begin{aligned} \sigma_1^{(2)} &= 0.75 N_1 - 7.1 T \\ \sigma_2^{(2)} &= 0.02 N_1 + 3.2 T \\ \sigma_6^{(2)} &= 0 \end{aligned} \tag{59}$$

The yield condition governing the initial failure is determined in terms of the maximum axial stress resultant N_1 by substituting Equations (58) and (59) into the general yield condition Equation (8) for $\theta = 0$ and 90 degrees, respectively.

Equation (8) for the case of $q = 0$ (zero shear) becomes, for $\theta = 0$ degree (outer layer),

$$1 - p + p^2 r^2 = (X/\sigma_1)^2$$

$$\text{or } \sigma_1^2 - \sigma_1 \sigma_2 + r^2 \sigma_2^2 = X^2$$
(60)

for $\theta = 90$ degrees (inner layer),

$$p^2 - p + r^2 = (X/\sigma_1)^2$$

$$\text{or } r^2 \sigma_1^2 - \sigma_1 \sigma_2 + \sigma_2^2 = X^2$$
(61)

Using the following experimentally determined strength values which represent a typical unidirectional glass filament-epoxy resin composite,*

Axial Strength	=	X	=	150 ksi	
Transverse Strength	=	Y	=	4 ksi	(62)
Shear Strength	=	S	=	6 ksi	

from which, one obtains

$$r = X/Y = 37.5$$

$$s = X/S = 25.0$$
(63)

Substituting Equations (63) and (59) into (61), and solving the resulting quadratic equation for N_1 , one obtains the stress resultant that causes failure in the inner layer:

$$N_1 \cong 9.6 T + 1.33 Y$$
(64)

*The same composite as reported in Section 2.

For a composite laminate at 270°F, or $T = -200$,

$$N_1 = 3400 \text{ psi} \quad (65)$$

For that laminated at room temperature, or $T = 0$,

$$N_1 = 5320 \text{ psi} \quad (66)$$

Similarly, substituting Equations (63) and (58) into (60), one obtains the stress resultant that causes failure in the outer layer:

$$N_1 = 110 T + (57.5 Y^2 - 3000 T^2)^{1/2} \quad (67)$$

For a composite laminated at 270°F, or $T = -200$,

$$N_1 = 6300 \text{ psi} \quad (68)$$

For that laminated at room temperature, or $T = 0$,

$$N_1 = 30,400 \text{ psi} \quad (69)$$

Comparing the results above, one can see that the inner layer will fail before the outer layer. It is also shown that the first failure would occur at a higher stress if the lamination temperature is ambient. From Equation (59) it can be seen that an elevated lamination temperature ($T = \text{negative}$) causes a pre-tension in σ_1 which is the normal stress transverse to the fibers. This will reduce the maximum N_1 at the "knee."

The effective stiffness of the laminated composite up to the "knee" is simply the reciprocal of A'_{11} (for unity thickness); i. e., from Equation (56) the effective stiffness is 3.4×10^6 psi. Thus, the in-plane strain at the "knee" is, using $N_1 = 3400$ from Equation (65),

$$\epsilon_1^0 = 3400 / 3.4 \times 10^6 = 0.1\% \quad (70)$$

The behavior of the cross-ply composite after the "knee" depends on the degree of uncoupling of the mechanical and thermal interactions. An immediate possibility is that cracks transverse to the fibers are developed in the inner layer. This can be described by letting $C_{22}^{(2)}$ of the inner layer remain constant while the remaining components are "degraded" to a very small fraction of their intact values, as listed in Equation (55). The resulting material properties of this partially degraded composite (inner layer degraded) become in place of Equation (56), (58) and (59),

$$\begin{aligned} A'_{11} &= 0.75 \times 10^{-6} \text{ in. / lb} \\ A'_{12} &= 0.01 \times 10^{-6} \text{ in. / lb} \end{aligned} \quad (71)$$

$$A'_{22} = 0.14 \times 10^{-6} \text{ in. / lb}$$

$$\begin{aligned} \sigma_1^{(1)} &= 6.00 N_1 \\ \sigma_2^{(1)} &= 0.47 N_1 - 19.3 T \\ \sigma_6^{(1)} &= 0 \end{aligned} \quad (72)$$

and

$$\begin{aligned} \sigma_1^{(2)} &= \sigma_6^{(2)} = 0 \\ \sigma_2^{(2)} &= -0.09 N_1 + 3.9 T \end{aligned} \quad (73)$$

Note that the thermal coupling in the 1-direction is reduced to zero. But the thermal coupling in the 2-direction, as shown in Equation (72), is increased after the degradation. In fact, the increase is so high (equal to 19.3 T) that the outer layers cannot remain intact after the initial degradation. What this means is that the outer layers will also degrade immediately, thus causing a complete uncoupling between the layers. Thereafter, only the uncoupled outer layers can carry the load. One can easily solve for the axial load that a partially degraded cross-ply can carry by substituting the stress components of Equation (72), into the yield condition of Equation (60). The maximum N_1 turns out to be considerably lower than the existing stress of 3400 psi.

After two successive failures, which occur almost simultaneously, the laminated composite becomes completely uncoupled both mechanically and thermally. Actual separation among constituent layers has been observed. In order to characterize this completely degraded composite, it is assumed that only the stiffness parallel to the fibers remain; i. e., $C_{11}^{(1)}$ and $C_{22}^{(2)}$ are the only nonzero components. (In order to avoid computational difficulties in the matrix inversion, the other components are assumed to be vanishingly small but not zero.) The resulting material properties of this completely degraded composite become in place of Equations (56), (58) and (59),

$$\begin{aligned} A'_{11} &= 0.77 \times 10^{-6} \text{ in./lb} \\ A'_{12} &= 0 \\ A'_{22} &= 0.15 \times 10^{-6} \text{ in./lb} \end{aligned} \tag{74}$$

The only nonzero stress components due to N_1 is:

$$\sigma_1^{(1)} = 6.00 N_1 \tag{75}$$

Thus, the effective stiffness of the composite after the "knee" is $1/hA'_{11} = 1.3 \times 10^6$ psi. The ultimate strength can be computed as follows. The stress in the outer layers immediately before the degradation of the inner layer is computed from Equation (58) using $N_1 = 3400$ and $T = -200$,

$$\sigma_1^{(1)} = 618 \text{ psi} \approx 600 \text{ psi} \quad (76)$$

Since the maximum stress $\sigma_1^{(1)}$ can reach is equal to the axial strength, 150,000 psi, the outer layers can be stressed an additional amount of $150,000 - 600 = 149,400$ psi. Using Equation (75), this additional stress beyond the "knee" represents a stress resultant of $149,400/6.00 = 24,900$ psi. Then the ultimate stress resultant N_1 is the sum of 24,900 and 3,400, which is 28,300 psi. The experimental measurement of the effective stress-strain relation of a three-layer cross-ply composite is shown in Figure 5. The agreement with the theoretical prediction is excellent for this case.

It can be stated that a "knee" does exist and its existence can be explained in terms of the uncoupling of the mechanical and thermal interactions. If the lamination temperature is ambient, then the "knee" would occur, from Equation (66), at N_1 equal to 5320 psi, instead of 3400 psi. The resultant ultimate strength of the composite, however, turns out to be practically the same as that laminated at 270°F .

The conventional netting analysis predicts the following stiffness and strength, based on two-thirds of glass by volume, with glass stiffness and strength of 10.6×10^6 psi and 400,000 psi, respectively,

$$\begin{aligned} E_{11} &= 10.6 \times 10^6 \times 2/3 \times 2/12 = 1.18 \times 10^6 \text{ psi} \\ \sigma_1 &= 400,000 \times 2/3 \times 2/12 = 44,000 \text{ psi} \end{aligned} \quad (77)$$

These data are also shown in Figure 5. It is interesting to note that the measured strength is only 68 percent of that predicted by netting analysis.

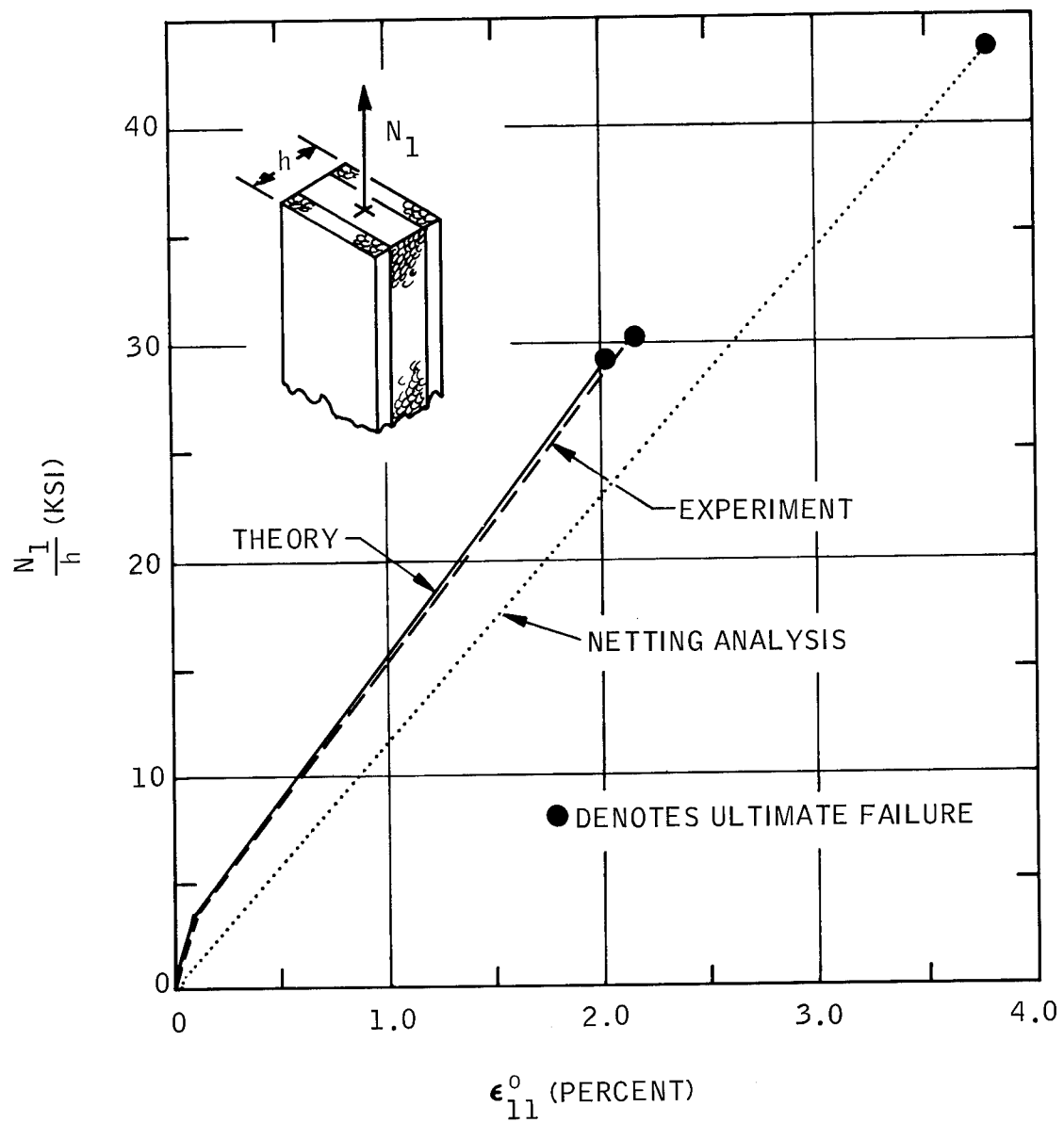


Figure 5. Strength of a Typical Cross-Ply Composite

For the purpose of more extensive experimental confirmation, three-layer cross-ply composites with different cross-ply ratios were made and tested. The theoretical predictions and the experimental results for both the effective initial and final stiffnesses (before and after the "knee," respectively), and the stress levels at the "knee" and the ultimate load are shown in Figure 6. It is fair to state that the present theory is reasonably confirmed experimentally. The scatter of data can be traced partly to the difficulty in making cross-ply tensile specimens. In the process of shaping the specimens by a router, the layer oriented transversely to the axis of the dog-bone specimens is often damaged.

The present theory involves lengthy arithmetic operations. Part of this burden can be relieved by using the tables listed in the Appendix. The input data are those listed in Equation (55). The composite moduli and the equations for the stress components and the thermal forces and moments are computed for two- and three-layer composites with cross-ply ratios varying from 0.2 to 4.0.* For each cross-ply composite, two cases will be listed: Case 1 represents all layers intact; and Case 2, all layers completely "degraded." With the aid of these tables, the data as shown in Equations (56) through (59), and (74) and (75) can be read directly.

In order to demonstrate the existence of thermal forces and moments, a two-layer cross-ply with two equal constituent layers ($m = 1$) was laminated at 270°F . At temperatures lower than the lamination temperature, the laminated plate becomes a saddle-shaped surface. For a square plate with length ℓ , thickness h , clamped at one edge ($y = 0$), as shown in Figure 7,

*As shown in Reference 2, two- and three-layer laminated composites represent two extreme cases, with all composites having larger numbers of layers falling in between the extremes.

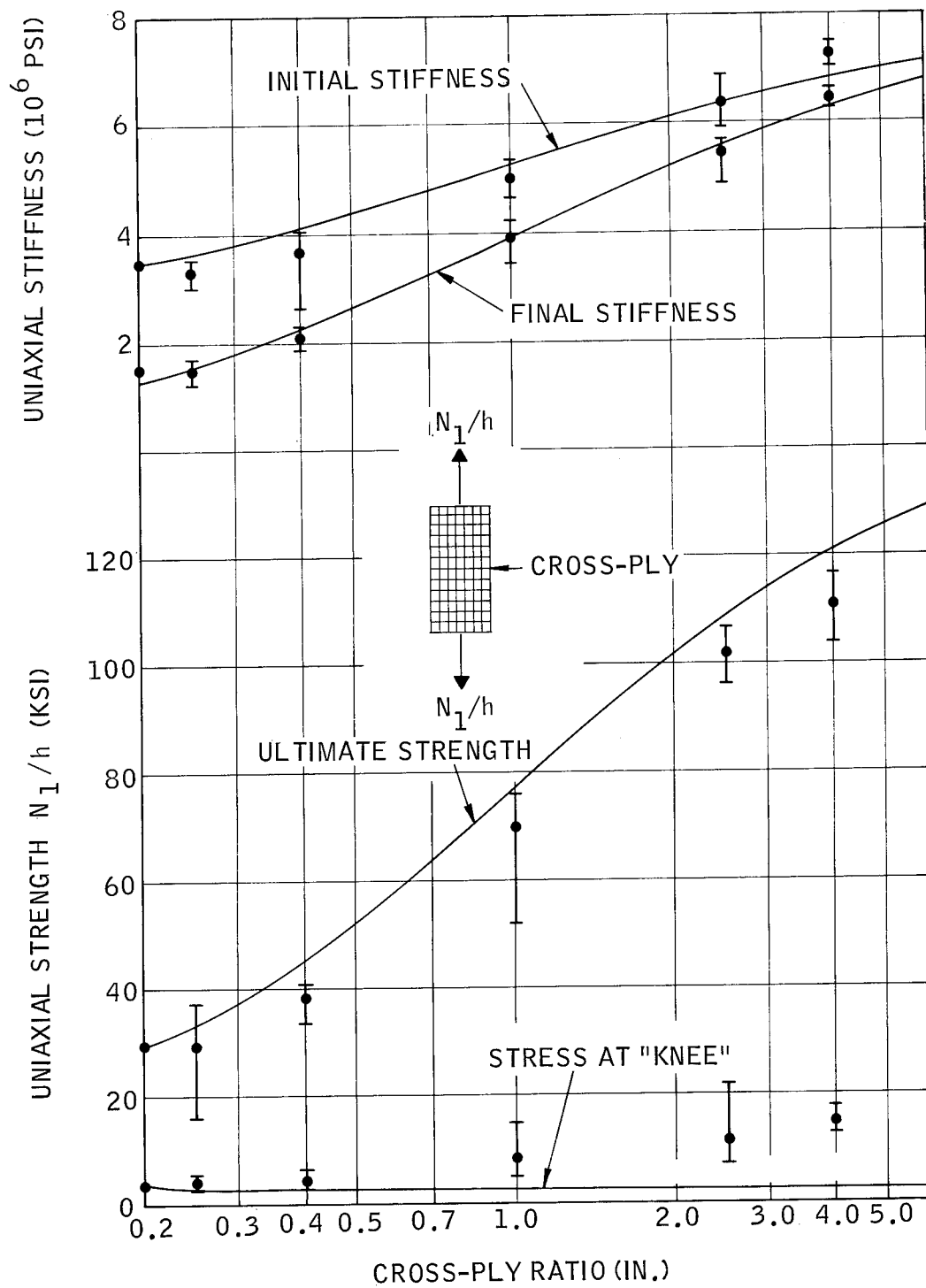


Figure 6. Strength of Cross-Ply Composites

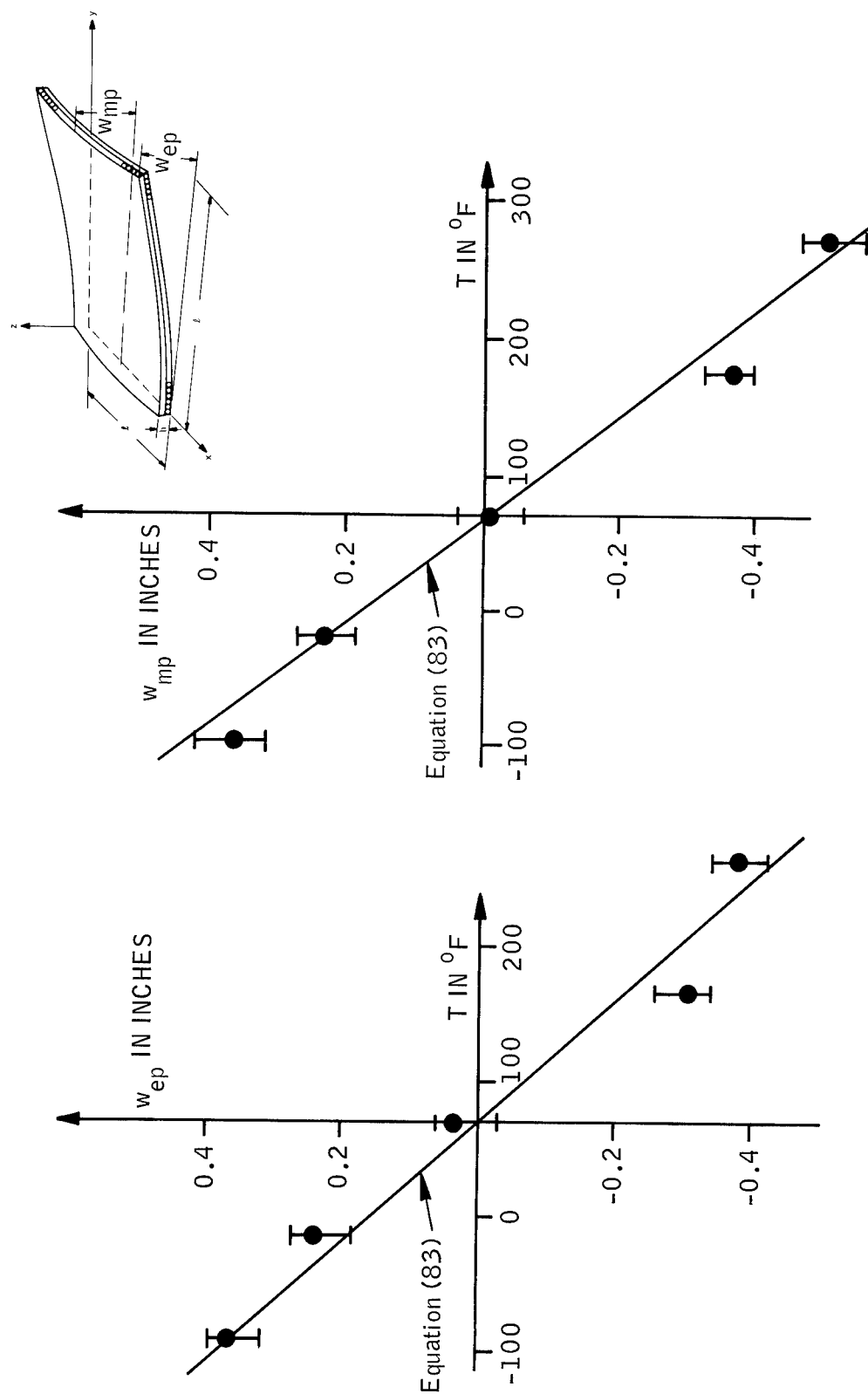


Figure 7. Thermal Warping of a Two-Layer Composite

the deflected surface due to homogeneous stress resultants and bending moments can be shown to be a quadratic surface,

$$w = \frac{1}{2} \kappa_1 x^2 + \frac{1}{2} \kappa_2 y^2 + \frac{1}{2} \kappa_6 xy + ax + by + c \quad (78)$$

Where kappas are the curvatures, constants a, b, c are determined from boundary conditions, as follows:

$$(1) \text{ When } x = y = 0, w = 0$$

$$(2) \text{ When } x = \ell, y = 0, w = 0 \quad (79)$$

$$(3) \text{ When } y = 0, \frac{\partial w}{\partial y} = 0$$

From the above, the displacements at the midpoint ($x = \ell/2, y = \ell$) and the end-point ($x = y = \ell$) as shown in Figure 7 are:

$$w_{mp} = \frac{5}{8} \kappa \ell^2 \quad (80)$$

$$w_{ep} = \frac{1}{2} \kappa \ell^2$$

where $\kappa_6 = 0$, and $\kappa = \kappa_1 = -\kappa_2$. (The last equality is true by virtue of the cross-ply ratio being one.)

Since the warping of the laminated composite is caused by the thermal coupling with no externally imposed loads, one can apply the basic material properties in Equation (55) to Equation (43), and obtain

$$N_1^T = N_2^T = 34.0 \text{ hT} \quad (81)$$

$$M_1^T = -M_2^T = -0.36 \text{ h}^2 \text{T}$$

Substituting these thermal forces and moments into Equation (36), one can establish the curvature

$$\begin{aligned}
 \kappa_1 &= B'_{11} N_1^T + (D'_{11} - D'_{12}) M_1^T \\
 &= \frac{0.35 \times 10^{-6}}{h^2} \times 34.0 hT - \frac{(2.84 + 0.35) \times 10^{-6}}{h^3} \times 0.36 h^2 T \\
 &= 10.75 \times 10^{-6} T/h
 \end{aligned} \tag{82}$$

For the particular test specimen, width $\ell = 8.5$ inches and thickness $h = 0.18$ inch; by substituting these data into Equation (80), one finds

$$\begin{aligned}
 w_{mp} &= 0.0027 T \\
 w_{ep} &= 0.0022 T
 \end{aligned} \tag{83}$$

In Figure 7, Equation (83) and appropriate experimental measurements are shown. A good agreement between theory and experiment is seen. This further substantiates the effect of the thermal coupling as a direct result of lamination.

In this section, the analysis of strength of cross-ply composites is shown. The effect of thermal and mechanical coupling is outlined. It is seen that the effective stress-strain relation has a "knee" resulting from the degradation of the constituent layers. After the "knee," the laminated composite becomes thermally and mechanically uncoupled but can carry an additional load before the ultimate strength is reached. A method is outlined in this section whereby the entire behavior of the cross-ply composite can be determined. Although the method and the experimental confirmation are limited to uniaxial tension, the method can be extended to more general types of loading, in terms of all six stress resultants and bending moments and arbitrary temperature, in a straightforward manner. This will be described further in Section 4.

Angle-ply Composites

The angle-ply composite consists of n constituent layers of an orthotropic material, as represented by a quasi-homogeneous unidirectional composite, with alternating angles of orientation between layers. The odd layers are oriented with an angle $-\theta$ from the 1-axis of the reference coordinate, and the even layers, $+\theta$. All layers have the same thickness. The lamination parameters for the angle-ply composite, as in Reference 2, are the total number of layers n , and the lamination angle θ .

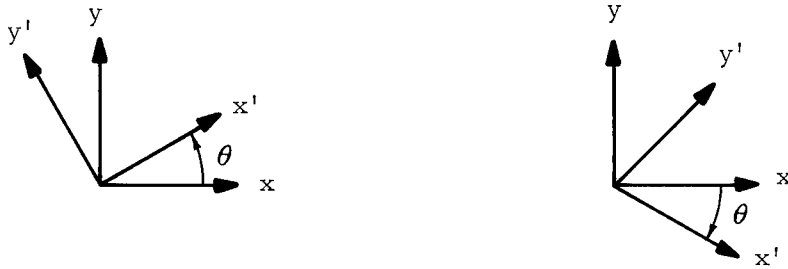
The effective stiffnesses of angle-ply composites made of glass filament and epoxy resin were accurately predicted by using the strength-of-materials approach.² Using those stiffnesses, one can obtain the stress distribution in each constituent layer from Equation (39) as functions of stress resultants, bending moments, and lamination temperature. Similar to the method described for the cross-ply, the general yield condition of Equation (8), can then be applied to each layer. The ultimate strength of the angle-ply can then be calculated. In the case of the angle-ply under uniaxial tension, unlike the cross-ply, there is no "knee" in the effective stress-strain relation. This is explained by the fact that after the layers with positive or negative orientation have failed, the remaining layers alone, although still intact, cannot carry the existing load. Thus, failure of the entire laminated composite occurs immediately after the initial failure of the positively or negatively oriented layers. This is a peculiar behavior of angle-ply composites under uniaxial loading.

Since the strength analysis of angle-ply composites requires the knowledge of the coordinate transformation and its effect on material properties and stress components, the standard coordinate transformation is repeated here and its relevance to angle-ply composites is indicated.

There are positive and negative rotations for coordinate transformation about the z-axis; they are represented symbolically by:

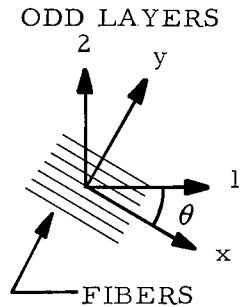
$$T^+ = \begin{pmatrix} m & n & 0 \\ -n & m & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad T^- = \begin{pmatrix} m & -n & 0 \\ n & m & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (84)$$

or graphically:



Equation (6) and Figure 2 correspond to the positive rotation T^+ . The x-y coordinates represent the original axes, and x'-y' the transformed axes. Since all odd layers of an angle-ply composite are oriented with a negative angle, the necessary transformation of the mechanical and thermal properties of this system of layers into the reference coordinates 1-2 requires a positive rotation T^+ ; conversely, all even layers where the orientation is positive requires a negative rotation T^- .

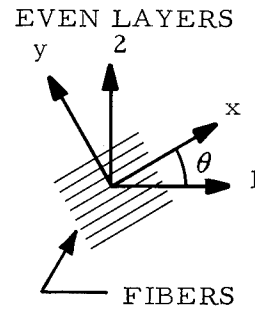
To summarize the results of the transformation for angle-ply composites:



Using T^+ operation on C_{ij} and a_j , where $i, j = x, y, s$ results in: $-C_{16}, -C_{26}, +a_6$.

When using the yield condition, Equation (8), θ is negative; i. e., $n = \text{negative}$.

Stress transformation from 1-2 to x-y systems requires a T^- operation; this corresponds to a counterclockwise rotation of 2θ in Mohr's Circle.



Using T^- operation on C_{ij} and a_i , where $i, j = x, y, s$ results in: $+C_{16}, +C_{26}, -a_6$.

When using the yield condition, Equation (8), θ is positive; i. e., $n = \text{positive}$.

Stress transformation from 1-2 to x-y systems requires a T^+ operation; this corresponds to a clockwise rotation of 2θ in Mohr's Circle, as in Figure 2.

For the purpose of illustrating how a strength analysis of an angle-ply can be carried out, a special case of a three-layer ($n = 3$) composite with a lamination angle of 15 degrees is outlined in the following.

Using the basic material data listed in Equation (55), which represents a typical unidirectional glass filament-epoxy resin composite, one can

obtain for the 15-degree lamination angle the following transformed data for the constituent layers using the proper transformation listed previously, *

$$\begin{aligned}
 C_{11}^{(1)} &= C_{11}^{(2)} = 7.342 \times 10^6 \text{ psi} \\
 C_{12}^{(1)} &= C_{12}^{(2)} = 0.932 \times 10^6 \text{ psi} \\
 C_{22}^{(1)} &= C_{22}^{(2)} = 2.763 \times 10^6 \text{ psi} \\
 C_{16}^{(1)} &= -C_{16}^{(2)} = -1.129 \times 10^6 \text{ psi} \\
 C_{26}^{(1)} &= -C_{26}^{(2)} = -0.199 \times 10^6 \text{ psi} \\
 C_{66}^{(1)} &= C_{66}^{(2)} = 1.519 \times 10^6 \text{ psi} \\
 \alpha_1^{(1)} &= \alpha_1^{(2)} = 4.029 \times 10^{-6}/^\circ\text{F} \\
 \alpha_2^{(1)} &= \alpha_2^{(2)} = 10.870 \times 10^{-6}/^\circ\text{F} \\
 \alpha_6^{(1)} &= -\alpha_6^{(2)} = 1.975 \times 10^{-6}/^\circ\text{F}
 \end{aligned} \tag{85}$$

where superscripts 1 and 2 represent odd and even layers, respectively. Depending on the directions of the rotation, C_{16} , C_{26} , and α_6 have different signs, while the remaining material constants are all positive.

*The transformation equation for C_{ij} , which is a fourth rank tensor, can be found, for example, on Page 12 of R. F. S. Herrman, An Introduction of Applied Anisotropic Elasticity, Oxford University Press, 1961. The transformations listed in this table correspond to a T^+ operation. The transformation equation for α_i is listed in Equation (6) of this report.

Using Equations (33) and (37) one can obtain, *

$$\begin{aligned}
 A'_{11} &= 0.14 \times 10^{-6} \text{ in./lb} \\
 A'_{12} &= -0.05 \times 10^{-6} \text{ in./lb} \\
 A'_{22} &= 0.38 \times 10^{-6} \text{ in./lb} \\
 A'_{16} &= 0.03 \times 10^{-6} \text{ in./lb} \\
 A'_{26} &= 0.005 \times 10^{-6} \text{ in./lb} \\
 A'_{66} &= 0.67 \times 10^{-6} \text{ in./lb}
 \end{aligned} \tag{86}$$

From Equation (32), one can compute the equivalent thermal forces and moments by assuming a constant lamination temperature T .

$$\begin{aligned}
 N_1^T &= 37.5 T \text{ lb/in.} \\
 N_2^T &= 33.2 T \text{ lb/in.} \\
 N_6^T &= -1.2 T \text{ lb/in.} \\
 M_i^T &= 0, \text{ as expected for } n = 3.
 \end{aligned} \tag{87}$$

*The detail calculation and some typical data for glass-epoxy composites are shown in the Appendix.

Substituting the values in Equations (85) and (87) into Equation (39) and letting N_1 be the only nonzero load, one obtains,

$$\begin{aligned}\sigma_1^{(1)} &= 0.97 N_1 - 0.44 T \\ \sigma_2^{(1)} &= -0.08 T\end{aligned}\tag{88}$$

$$\sigma_6^{(1)} = -0.10 N_1 - 1.79 T^*$$

and

$$\begin{aligned}\sigma_1^{(2)} &= 1.05 N_1 + 0.89 T \\ \sigma_2^{(2)} &= 0.01 N_1 + 0.16 T\end{aligned}\tag{89}$$

$$\sigma_6^{(2)} = 0.20 N_1 + 3.58 T^*$$

The yield condition of Equation (8) can be considerably simplified for this particular angle-ply by letting $p = 0$ because the σ_2 in both Equations (88) and (89) is small in comparison with σ_6 . Also using the strength values listed in Equations (62) and (63), one obtained a simplified form for Equation (8) as

$$A \sigma_1^2 + B \sigma_1 \sigma_6 + C \sigma_6^2 = X^2$$

where

$$A = m^4 + 624 m^2 n^2 + 1406 n^4\tag{90}$$

$$B = -(1244 m^3 n + 4386 m n^3)$$

$$C = 625 m^4 + 4382 m^2 n^2 + 625 n^4$$

*These shear stresses can properly be designated as the interlaminar shear stresses which are induced by axial stress resultant N_1 and lamination temperature T . The common usage of the interlaminar shear in the filament winding industry referring to a particular test method is entirely different from the shear stresses above.

For $\theta = -15^\circ$ (this applies to the odd layers),

$$A = 46.20, \quad B = 363.91, \quad C = 821.00 \quad (91)$$

For $\theta = +15^\circ$ (this applies to the even layers),

$$A = 46.20, \quad B = -363.91, \quad C = 821.00 \quad (92)$$

Substituting Equations (91) and (88) into (90), one can solve for the maximum N_1 for the outer layers,

$$16.12 N_1^2 - 359.3 N_1 T + 2938 T^2 - X^2 = 0 \quad (93)$$

or

$$N_1 = 11.14 T + 37,400 \quad (94)$$

For a lamination temperature at 270°F , $T = -200^\circ\text{F}$,

$$N_1 = 35,200 \text{ psi} \quad (95)$$

Similarly, substituting Equations (92) and (89) into (90), one can solve for the maximum N_1 for the inner layers,

$$7.52 N_1^2 - 148.3 N_1 T + 9429 T^2 - X^2 = 0 \quad (96)$$

$$N_1 = 9.87 T + 54,600$$

for $T = -200$,

$$N_1 = 52,600 \text{ psi} \quad (97)$$

Thus, the outer layers will fail first for having a lower N_1 , and in fact, the ultimate load of this composite will be 35,200 psi because the inner layer cannot carry the load alone after the outer layers have failed.

Similar calculations, as described from Equations (93) through (97), are repeated for other lamination angles and the theoretical predictions together with the measured data are shown in Figure 8. Also included in Figure 8 is the initial effective stiffness of the angle-ply composite. For both the strength and the stiffness, excellent agreement exists between the theory and experimental observation. For intermediate lamination angles, nonlinear stress-strain relation is observed. The actual ultimate strain at the failure stress is about 2 to 3 times larger than that computed from the tangent modulus. It is interesting to compare the strength of unidirectional composites, as shown in Figure 4, with the angle-ply, in Figure 8. Up to 45 degrees, the angle-ply has up to 50 percent higher strength than the unidirectional. For angles larger than 45 degrees, the angle-ply becomes weaker than the unidirectional. These differences in strength can be traced directly to the mechanical and thermal interactions, because of the non-vanishing C_{16} and C_{26} , and T , respectively.

In order to facilitate the strength analysis of glass-epoxy angle-ply composites, composite moduli and coefficients for stress components are listed in the Appendix for $n = 2$ and 3 and $\theta = 5, 10, 15, 30, 45, 60$, and 75 degrees.

In conclusion, a method for determining the strength of angle-ply composites has been formulated. This method can be extended to the most complicated types of loading with all six components of stress resultants and bending moments and arbitrary temperature distribution across the thickness of the composite. Differing from the case of cross-ply composites, the angle-ply cannot carry additional uniaxial load after failure has initiated in one system of layers. Consequently, no discontinuity in the slope of the effective stress-strain relation is predicted by the present strength analysis, nor observed experimentally. For this reason, no subsequent degradation of the constituent layers has been investigated.

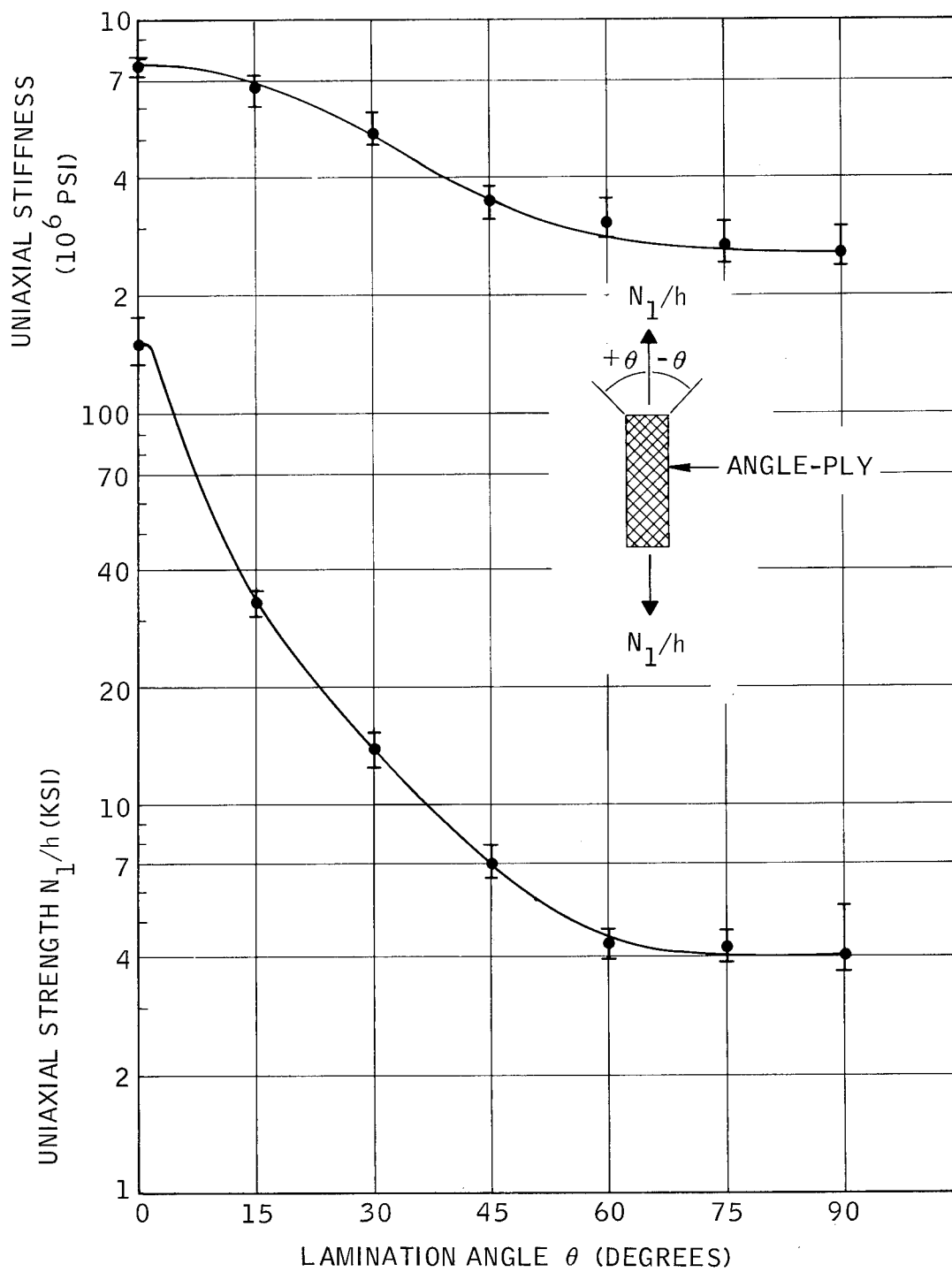


Figure 8. Strength of Angle-Ply Composites

SECTION 4

CONCLUSIONS

The present report outlines a method of strength analysis for both quasi-homogeneous and laminated composites. This method requires the experimental determination of some basic material properties, like those listed in Equations (55) and (62). As stated in the Introduction (Section 1), a clear distinction is made between the structures and materials research on composite materials. The present report only covers the structures aspect of strength. The materials aspect, on the other hand, is to be investigated in the future.

It is important to recognize two aspects of the results of the present investigation: (1) the strength of a nonisotropic material requires three strength characteristics, X , Y , and S ; (2) for fiber-reinforced composites such as the glass-epoxy composite, the strength values thus far must be experimentally determined. Even the case of the axial strength X cannot be predicted from the constituent properties; e. g., the fiber strength and volume ratio, with confidence. The fundamental data of X being 150 ksi for unidirectional glass-epoxy composites, together with Y and S listed in Equation (62), has been shown to be significant in the transformation of strength of a quasi-homogeneous composite (Figure 4), and the strength characteristics of cross-ply and angle-ply composites (Figures 6 and 8, respectively). Insofar as the structures aspect of strength is concerned, it is more important to know the correct value of the axial strength of 150 ksi than to be obsessed by the apparent loss of the theoretical strength. The latter strength, based on netting analysis, is predicted by using the virgin strength of glass (400 ksi) corrected by its volume ratio (66 percent), the result being 266 ksi. Whatever the reason or reasons for the loss of the theoretical strength may be, it

is more important to recognize that only a strength of 150 ksi has been realized under a highly idealized condition, such as the test method used for the present program; in all probability a greater loss of strength will exist in actual structures. Since the application of composite materials is primarily in structures, it is more significant to know what one has at his disposal (that $X = 150$ ksi) than what he does not have (that X should have been 266 ksi).

The present investigation also shows the importance of the transverse and shear strengths, Y and S , respectively. So long as structures are, in general, subjected to more complex loading than uniaxial, loaded along the fiber axis, Y and S should be treated with equal respect as the axial strength X . In fact, the relatively low value of the transverse strength is directly responsible for the "knee" in the cross-ply composite, the presence of which is detrimental to the structure for being less stiff for load beyond the "knee" and for being porous resulting from cracks transverse to the fibers. Thus, the improvement of fiber-reinforced composites may very well depend more on the upgrading of the transverse and shear strengths than the axial strength.

The method of strength analysis outlined in this report can be generalized to loadings other than uniaxial tension. The coefficients for the stress components in terms of all the stress resultants and bending moments, together with the lamination temperature, are listed in the Appendix for typical glass-epoxy composites. For any given combination of N_i , M_i , and T , one can determine the stress components within each constituent layer. One can go to the tables in the Appendix and obtain directly the coefficients for each N_i and M_i and T , derived from the expanded form of Equation (39). The effects of thermal forces N_i^T and thermal moments M_i^T are lumped in the "coefficients of temperature."

There are numerous limitations to the present theory of strength, the most important ones are listed as follows:

- (1) It is assumed that the tensile and compressive stiffnesses and strengths are equal. The present theory can be modified to take into account different tensile and compressive properties by following, for example, the method described in References 6 and 7.

- (2) The composite material is assumed to be linear elastic up to the ultimate failure. For glass-epoxy composites, this assumption has been found to be reasonable with the exception of the unidirectional and angle-ply composites with intermediate angles of fiber orientations, say between 30 and 60 degrees.
- (3) In the case of cross-ply composites, the piece-wise linear stress-strain relation is intended to describe the loading condition only. The behavior of the laminated composite during unloading and reloading has not been investigated.
- (4) The degradation of angle-ply composites because of cracks transverse to the fibers has not been investigated. It is quite conceivable that the composite can carry additional load after initial degradation under more complex loading such as the biaxial stress.

Recommendations for future work include the following:

- (a) The contribution of the constituents' properties to the basic strength characteristics X , Y , and S . This will provide a basis to establish guidelines for the rational design of composite materials.
- (b) More extensive experimental verification of the strength of unidirectional and laminated composites under loading conditions other than uniaxial tension. The test materials should include other combinations of constituents than glass-epoxy.
- (c) The present framework of research (combined structures and materials research) should be extended to include critical problems of nonelastic behavior, creep, and fatigue of composite materials.

It is believed that, with the foregoing information of the strength characteristics of composite materials, an improvement has been made in the basic understanding of the structural behavior of composites. This added knowledge will provide a better basis of design and utilization of composites. It is hoped that additional researchers with interests in structures and materials will begin to contribute to this new area of research. With rapidly advancing technology of new constituent materials and manufacturing processes, a rational basis of materials design is urgently needed. This report may be considered as a typical example of the work still remaining in the field of composite materials.

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APPENDIX

MATERIAL COEFFICIENTS OF GLASS-EPOXY COMPOSITES

The purpose of this Appendix is to show the method of stress analysis of a laminated composite, and to list material coefficients of a typical glass-epoxy composite. The coefficients are intended to reduce the burden of computation in the analyses of stress, strain, and strength. Since most mathematical relations required for the present work have already been covered in this report, they will only be cited by their equation numbers here in the Appendix.

In a laminated composite, the variables of interests are, under the strength-of-materials approach, the stress resultants N , bending moments M , in-plane strains ϵ^0 and curvatures κ . In place of the stress-strain relation, these four quantities are linked by relations shown in Equations (34), (35), and (36). (Thermal forces and moments are automatically included here.) As mentioned in Reference 2, a laminated composite is described by, at most, 18 independent elastic moduli, six each in the A , B , and D matrices, which reduce to two independent moduli for quasi-homogeneous isotropic material. Thus, knowing the 18 moduli for a given laminated composite, one can solve for two of the unknown variables if the other two are given. In general, N and M are given, then using Equation (36) and A' , B' and D' matrices, one can find the in-plane strain and curvature. In special cases, such as a pressurized cylindrical shell, in addition to the known stress resultants which are the membrane stresses, the curvature by virtue of symmetry must be zero. Thus, Equation (35) is the appropriate relation. Figure 17 in Reference 2, for example, reflects the use of A^* , B^* , and H^* and D^* matrices.

The stress in each layer is determined from knowing the in-plane strain and curvature for a laminated composite and the stiffness matrix C_{ij} of the particular layer. Equations (38) and (39) show the precise relations. As governed by the original assumption of the nondeformable normals, the strain is linear, and the stress, piece-wise linear, across the thickness of the laminated composite.

Unfortunately the computation of the A, B, and D matrices and their inversions is difficult for hand computation. The stress equation, such as Equation (39), involves not only the prime matrices A', B' and D', but also much arithmetic operation. A digital program has been prepared to compute the following quantities for a general laminated composite:

- (1) Composite moduli A, B, D, A*, B*, H*, D*, A', B', and D'.
- (2) Thermal forces and moments per Equation (32) for a constant temperature T across the laminated composite.
- (3) Coefficients for each N_i , M_i , and T in the stress relation, Equation (39). Since temperature is assumed to be constant, the contributions of N_i^T and M_i^T and α_i^T to the stress component are lumped into one term designated as "the coefficients of temperature."

The coefficients at the top and bottom of each constituent layer are shown. The stress at any location within a layer can be obtained by a simple linear interpolation.

The information just described is computed and tabulated for typical glass-epoxy cross-ply and angle-ply composites. Also included is the degraded case of cross-ply composites. The exact nature of the degradation, as explained in the Subsection entitled Cross-ply Composites, consists of having cracks developed transverse to the fibers in all constituent layers.

The tables are arranged as follows:

- (1) Cross-ply Composites
Case 1 (all layers intact) pp. 62-71
- (2) Cross-ply Composites
Case 2 (all layers degraded) pp. 72-81
- (3) Angle-ply Composites
Case 1 (all layers intact) pp. 82-95

All material coefficients are computed per unit thickness of the laminate. Let h be the actual thickness of the laminate; the material coefficients as listed in the table must be corrected as follows:

$$hA, h^2B, h^3D$$

$$A^*/h, hB^*, hH^*, h^3D^*$$

$$A'/h, B'/h^2, D'/h^3$$

CROSS-PLY M=0.2 CASE 1 (ALL LAYERS INTACT)
2 LAYERS (N=2)

```

-- ODD LAYERS --
STIFFNESS MATRIX (C)
(10+6 LB./IN.SQ.)
7.9660 0.6638 0.
0.6638 2.6550 0.
0. 0. 1.2500

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)
ALPHA 1 = 3.5000
ALPHA 2 = 11.4000
ALPHA 6 = 0.

-- EVEN LAYERS --
STIFFNESS MATRIX (C)
(10+6 LB./IN.SQ.)
2.6550 0.6638 0.
0.6638 7.9660 0.
0. 0. 1.2500

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)
ALPHA 1 = 11.4000
ALPHA 2 = 3.5000
ALPHA 6 = 0.

```

```

A
(10+6 LB./IN.)
3.5403 0.6638 0.
0.6638 7.0807 0.
0. 0. 1.2500

A*
(10-6 IN./LB.)
0.2875 -0.0270 0.
-0.0270 0.1438 0.
0. 0. 0.8000

A PRIME
(10-6 IN./LB.)
0.3209 -0.0294 0.
-0.0294 0.1496 0.
0. 0. 0.8000

THERMAL FORCE
(LB./IN./DEG.F.)
N1-T = 33.0667
N2-T = 34.9719
N6-T = 0.

B
(10+6 IN.)
-0.3689 0. 0.
0. 0. 0.
0. 0. 0.

B*
(10+0 IN.)
0.1061 0.0099 0.
-0.0099 -0.0530 0.
0. 0. 0.

B PRIME
(10-6 1/LB.)
0.3159 -0.0130 0.
-0.0130 -0.1072 0.
0. 0. 0.

THERMAL MOMENT
(LB./DEG.F.)
M1-T = -0.1985
M2-T = 0.1985
M6-T = 0.

H*
(10+0 IN.)
-0.1061 0.0099 0.
-0.0099 0.0530 0.
0. 0. 0.

C
(10+6 LB.IN.)
0.3770 0.0553 0.
0.0553 0.5081 0.
0. 0. 0.1042

D*
(10+6 LB.IN.)
0.3379 0.0516 0.
0.0516 0.4885 0.
0. 0. 0.1042

D PRIME
(10-6 1/LB.IN.)
3.0083 -0.3181 0.
-0.3181 2.0806 0.
0. 0. 9.6000

Z
(IN.)
-0.5000
-0.3333
-0.3333
0.5000

STRESS COMPONENT
SIGMA 1
2
6

COEF. OF N1
(1/IN.)
1.2828
0.0474
0.
1.7008
0.0766
0.
0.5558
-0.0565
0.
1.2474
0.0317
0.

COEF. OF N2
(1/IN.)
-0.0474
0.5242
0.
-0.0766
0.4753
0.
0.0565
1.4594
0.
-0.0317
0.7408
0.

COEF. OF N6
(1/IN.)
0.
0.
1.0000
0.
0.
1.0000
0.
0.
1.0000
0.
0.
1.0000

COEF. OF M1
(1/IN.SQ.)
-9.3689
-0.4012
0.
-5.4092
-0.2091
0.
-1.7617
0.2846
0.
4.7180
-0.1626
0.

COEF. OF M2
(1/IN.SQ.)
0.4012
-2.9496
0.
0.2091
-2.0640
0.
-0.2846
-6.3161
0.
0.1626
7.3191
0.

COEF. OF M6
(1/IN.SQ.)
0.
-6.0000
0.
0.
-3.9996
0.
0.
-3.9996
0.
0.
6.0000
0.

COEF. OF TEMP.
(LB./IN.SQ./F.)
7.2505
-13.1967
0.
19.2290
-13.8033
0.
-11.9442
12.4112
0.
6.6470
-7.0099
0.

-- LAYER 1 --
-- LAYER 2 --

```

CROSS-PLY M=0.2 CASE 1 (ALL LAYERS INTACT)
3 LAYERS (N=3)

-- ODD LAYERS --

STIFFNESS MATRIX (C)
(10+6 LB./IN.SQ.)

7.9660 0.6638 0.
0.6638 2.6550 0.
0. 0. 1.2500

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)

ALPHA 1 = 3.5000
ALPHA 2 = 11.4000
ALPHA 6 = 0.

-- EVEN LAYERS --

STIFFNESS MATRIX (C)
(10+6 LB./IN.SQ.)

2.6550 0.6638 0.
0.6638 7.9660 0.
0. 0. 1.2500

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)

ALPHA 1 = 11.4000
ALPHA 2 = 3.5000
ALPHA 6 = 0.

A
(10+6 LB./IN.)
3.5401 0.6638 0.
0.6638 7.0809 0.
0. 0. 1.2500

A*
(10-6 IN./LB.)
0.2875 -0.0270 0.
-0.0270 0.1438 0.
0. 0. 0.8000

A PRIME
(10-6 IN./LB.)
0.2875 -0.0270 0.
-0.0270 0.1438 0.
0. 0. 0.8000

THERMAL FORCE
(LB./IN./DEG.F.)

N1-T = 33.0666
N2-T = 34.9720
N6-T = 0.

B
(10+6 IN.)
0. 0. 0.
0. 0. 0.
0. 0. 0.

B*
(10+0 IN.)
0. 0. 0.
0. 0. 0.
0. 0. 0.

B PRIME
(10-6 1/LB.)
0. 0. 0.
0. 0. 0.
0. 0. 0.

THERMAL MOMENT
(LB./DEG.F.)

M1-T = 0.
M2-T = 0.
M6-T = 0.

H*
(10+0 IN.)

0. 0. 0.
0. 0. 0.
0. 0. 0.

D
(10+6 LB.IN.)
0.4077 0.0553 0.
0.0553 0.4774 0.
0. 0. 0.1042

D*
(10+6 LB.IN.)
0.4077 0.0553 0.
0.0553 0.4774 0.
0. 0. 0.1042

D PRIME
(10-6 1/LB.IN.)
2.4919 -0.2888 0.
-0.2888 2.1282 0.
0. 0. 9.6000

Z (IN.) STRESS COMPONENT COEF. OF N1 (1/IN.) COEF. OF N2 (1/IN.) COEF. OF N6 (1/IN.) COEF. OF M1 (1/IN.SQ.) COEF. OF M2 (1/IN.SQ.) COEF. OF M6 (1/IN.SQ.) COEF. OF TET (LB/IN.SQ/F)

-- LAYER 1 --

-0.5000 SIGMA 1 2.2726 -0.1193 0. -9.8296 0.4438 0. 35.5257
2 0.1193 0.3638 0. -0.4438 -2.7294 0. -15.9238
6 0. 0. 1.0000 0. 0. -6.0000 0.

-0.4167 SIGMA 1 2.2726 -0.1193 0. -8.1914 0.3698 0. 35.5257
2 0.1193 0.3638 0. -0.3698 -2.2745 0. -15.9238
6 0. 0. 1.0000 0. 0. -5.0000 0.

-- LAYER 2 --

-0.4167 SIGMA 1 0.7455 0.0239 0. -2.6769 -0.2692 0. -7.1048
2 -0.0239 1.1272 0. 0.2692 -6.9841 0. 3.1846
6 0. 0. 1.0000 0. 0. -5.0000 0.

0.4167 SIGMA 1 0.7455 0.0239 0. 2.6769 0.2692 0. -7.1048
2 -0.0239 1.1272 0. 0. -6.9841 0. 3.1846
6 0. 0. 1.0000 0. 0. 5.0000 0.

-- LAYER 3 --

0.4167 SIGMA 1 2.2726 -0.1193 0. 8.1914 -0.3698 0. 35.5257
2 0.1193 0.3638 0. 0.3698 2.2745 0. -15.9238
6 0. 0. 1.0000 0. 0. 5.0000 0.

0.5000 SIGMA 1 2.2726 -0.1193 0. 9.8296 -0.4438 0. 35.5257
2 0.1193 0.3638 0. 0.4438 2.7294 0. -15.9238
6 0. 0. 1.0000 0. 0. 6.0000 0.

CROSS-PLY M=0.4 CASE 1 (ALL LAYERS INTACT)
2 LAYERS (N=2)

STIFFNESS MATRIX (C)
(10+6 LB./IN.SQ.)

7.9660 0.6638 0.
0.6638 2.6550 0.
0. 0. 1.2500

-- ODD LAYERS --

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)

ALPHA 1 = 3.5000
ALPHA 2 = 11.4000
ALPHA 6 = 0.

STIFFNESS MATRIX (C)
(10+6 LB./IN.SQ.)

2.6550 0.6638 0.
0.6638 7.9660 0.
0. 0. 1.2500

-- EVEN LAYERS --

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)

ALPHA 1 = 11.4000
ALPHA 2 = 3.5000
ALPHA 6 = 0.

A (10+6 LB./IN.)			A* (10-6 IN./LB.)			A PRIME (10-6 IN./LB.)			THERMAL FORCE (LB./IN./DEG.F.)
4.1724	0.6638	0.	0.2437	-0.0251	0.	0.2930	-0.0293	0.	N1-T = 33.4068
0.6638	6.4486	0.	-0.0251	0.1577	0.	-0.0293	0.1753	0.	N2-T = 34.6318
0.	0.	1.2500	0.	0.	0.8000	0.	0.	0.8000	N6-T = 0.
B (10+6 IN.)			B* (10+0 IN.)			B PRIME (10-6 1/LB.)			THERMAL MOMENT (LB./DEG.F.)
-0.5419	0.	0.	0.1320	0.0136	0.	0.3749	-0.0106	0.	M1-T = -0.2916
0.	0.5419	0.	-0.0136	-0.0854	0.	-0.0106	-0.2053	0.	M2-T = 0.2916
0.	0.	0.	0.	0.	0.	0.	0.	0.	M6-T = 0.
D (10+6 LB.IN.)			D* (10+6 LB.IN.)			D PRIME (10-6 1/LB.IN.)			
0.4251	0.0553	0.	0.3536	0.0480	0.	2.8736	-0.3331	0.	
0.0553	0.4600	0.	0.0480	0.4137	0.	-0.3331	2.4560	0.	
0.	0.	0.1042	0.	0.	0.1042	0.	0.	9.6000	
Z (IN.)	STRESS COMPONENT	COEF. OF N1 (1/IN.)	COEF. OF N2 (1/IN.)	COEF. OF N6 (1/IN.)	COEF. OF M1 (1/IN.SQ.)	COEF. OF M2 (1/IN.SQ.)	COEF. OF M6 (1/IN.SQ.)	COEF. OF TEMP. (LB/IN.SQ/F.)	
-- LAYER 1 --									
-0.5000	SIGMA 1	0.8250	-0.0064	0.	-8.3553	0.2908	0.	-5.5876	
	2	0.0064	0.7222	0.	-0.2908	-3.7019	0.	-8.3609	
	6	0.	0.	1.0000	0.	0.	-6.0000	0.	
-0.2143	SIGMA 1	1.6763	-0.0695	0.	-1.8786	-0.0015	0.	18.6922	
	2	0.0695	0.5644	0.	0.0015	-1.9021	0.	-11.2774	
	6	0.	0.	1.0000	0.	0.	-2.5716	0.	
-- LAYER 2 --									
-0.2143	SIGMA 1	0.5467	0.0739	0.	-0.5992	-0.3243	0.	-11.6855	
	2	-0.0739	1.7293	0.	0.3243	-5.7878	0.	20.1888	
	6	0.	0.	1.0000	0.	0.	-2.5716	0.	
0.5000	SIGMA 1	1.2527	-0.0435	0.	4.6925	0.2085	0.	6.4440	
	2	0.0435	0.5561	0.	-0.2085	8.0293	0.	-12.3340	
	6	0.	0.	1.0000	0.	0.	6.0000	0.	

CROSS-PLY M=0.4 CASE 1 (ALL LAYERS INTACT)
3 LAYERS (N=3)

STIFFNESS MATRIX (C) (10+6 LB./IN.SQ.)			-- ODD LAYERS --	THERMAL EXPANSION MATRIX (ALPHA) (IN./IN./DEG.F.)		
7.9660	0.6638	0.		ALPHA 1 =	3.5000	
0.6638	2.6550	0.		ALPHA 2 =	11.4000	
0.	0.	1.2500		ALPHA 6 =	0.	
STIFFNESS MATRIX (C) (10+6 LB./IN.SQ.)			-- EVEN LAYERS --	THERMAL EXPANSION MATRIX (ALPHA) (IN./IN./DEG.F.)		
2.6550	0.6638	0.		ALPHA 1 =	11.4000	
0.6638	7.9660	0.		ALPHA 2 =	3.5000	
0.	0.	1.2500		ALPHA 6 =	0.	

A (10+6 LB./IN.)			A* (10-6 IN./LB.)			A PRIME (10-6 IN./LB.)			THERMAL FORCE (LB./IN./DEG.F.)		
4.1725	0.6638	0.	0.2437	-0.0251	0.	0.2437	-0.0251	0.	M1-T =	33.4069	
0.6638	6.4485	0.	-0.0251	0.1577	0.	-0.0251	0.1577	0.	M2-T =	34.6317	
0.	0.	1.2500	0.	0.	0.8000	0.	0.	0.8000	M6-T =	0.	
B (10+6 IN.)			B* (10+0 IN.)			B PRIME (10-6 1/LB.)			THERMAL MOMENT (LB./DEG.F.)		
0.	0.	0.	0.	0.	0.	0.	0.	0.	M1-T =	0.	
0.	0.	0.	0.	0.	0.	0.	0.	0.	M2-T =	0.	
0.	0.	0.	0.	0.	0.	0.	0.	0.	M6-T =	0.	
H* (10+0 IN.)			0.	0.	0.						
			0.	0.	0.						
			0.	0.	0.						

C (10+6 LB.IN.)			D* (10+6 LB.IN.)			D PRIME (10-6 1/LB.IN.)		
0.5025	0.0553	0.	0.5025	0.0553	0.	2.0221	-0.2924	0.
0.0553	0.3825	0.	0.0553	0.3825	0.	-0.2924	2.6564	0.
0.	0.	0.1042	0.	0.	0.1042	0.	0.	9.6000

Z (IN.)	STRESS COMPONENT	COEF. OF M1 (1/IN.)	COEF. OF M2 (1/IN.)	COEF. OF M6 (1/IN.)	COEF. OF M1 (1/IN.SQ.)	COEF. OF M2 (1/IN.SQ.)	COEF. OF M6 (1/IN.SQ.)	COEF. OF TEMP. (LB/IN.SQ/F.)
-- LAYER 1 --								
-0.5000	SIGMA 1	1.9243	-0.0951	0.	-7.9568	0.2830	0.	25.5422
	2	0.0951	0.4019	0.	-0.2830	-3.4293	0.	-15.4923
	6	0.	0.	1.0000	0.	0.	-6.0000	0.
-0.3571	SIGMA 1	1.9243	-0.0951	0.	-5.6834	0.2021	0.	25.5422
	2	0.0951	0.4019	0.	-0.2021	-2.4495	0.	-15.4923
	6	0.	0.	1.0000	0.	0.	-4.2857	0.
-- LAYER 2 --								
-0.3571	SIGMA 1	0.6303	0.0381	0.	-1.8480	-0.3525	0.	-10.2172
	2	-0.0381	1.2392	0.	0.3525	-7.4881	0.	6.1971
	6	0.	0.	1.0000	0.	0.	-4.2857	0.
0.3571	SIGMA 1	0.6303	0.0381	0.	1.8480	0.3525	0.	-10.2172
	2	-0.0381	1.2392	0.	-0.3525	7.4881	0.	6.1971
	6	0.	0.	1.0000	0.	0.	4.2857	0.
-- LAYER 3 --								
0.3571	SIGMA 1	1.9243	-0.0951	0.	5.6834	-0.2021	0.	25.5422
	2	0.0951	0.4019	0.	0.2021	2.4495	0.	-15.4923
	6	0.	0.	1.0000	0.	0.	4.2857	0.
0.5000	SIGMA 1	1.9243	-0.0951	0.	7.9568	-0.2830	0.	25.5422
	2	0.0951	0.4019	0.	0.2830	3.4293	0.	-15.4923
	6	0.	0.	1.0000	0.	0.	6.0000	0.

CROSS-PLY M=1.0 CASE 1 (ALL LAYERS INTACT)
2 LAYERS (N=2)

STIFFNESS MATRIX (C)
(10+6 LB./IN.SQ.)

7.9660 0.6638 0.
0.6638 2.6550 0.
0. 0. 1.2500

-- ODD LAYERS --

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)

ALPHA 1 = 3.5000
ALPHA 2 = 11.4000
ALPHA 6 = 0.

STIFFNESS MATRIX (C)
(10+6 LB./IN.SQ.)

2.6550 0.6638 0.
0.6638 7.9660 0.
0. 0. 1.2500

-- EVEN LAYERS --

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)

ALPHA 1 = 11.4000
ALPHA 2 = 3.5000
ALPHA 6 = 0.

A
(10+6 LB./IN.)

5.3105 0.6638 0.
0.6638 5.3105 0.
0. 0. 1.2500

A*
(10-6 IN./LB.)

0.1913 -0.0239 0.
-0.0239 0.1913 0.
0. 0. 0.8000

A PRIME
(10-6 IN./LB.)

0.2363 -0.0295 0.
-0.0295 0.2363 0.
0. 0. 0.8000

THERMAL FORCE
(LB./IN./DEG.F.)

N1-T = 34.0193
N2-T = 34.0193
N6-T = 0.

B
(10+6 IN.)

-0.6639 0. 0.
0. 0.6639 0.
0. 0. 0.

B*
(10+6 IN.)

0.1270 0.0159 0.
-0.0159 -0.1270 0.
0. 0. 0.

B PRIME
(10-6 1/LB.)

0.3545 0.0000 0.
0.0000 -0.3545 0.
0. 0. 0.

THERMAL MOMENT
(LB./DEG.F.)

M1-T = -0.3573
M2-T = -0.3573
M6-T = 0.

H*
(10+0 IN.)

-0.1270 0.0159 0.
-0.0159 0.1270 0.
0. 0. 0.

C
(10+6 LB./IN.)

0.4425 0.0553 0.
0.0553 0.4425 0.
0. 0. 0.1042

D*
(10+6 LB./IN.)

0.3582 0.0448 0.
0.0448 0.3582 0.
0. 0. 0.1042

D PRIME
(10-6 1/LB./IN.)

2.8358 -0.3545 0.
-0.3545 2.8358 0.
0. 0. 9.6000

Z
(IN.)

STRESS
COMPONENT

COEF. OF N1
(1/IN.)

COEF. OF N2
(1/IN.)

COEF. OF N6
(1/IN.)

COEF. OF M1
(1/IN.SQ.)

COEF. OF M2
(1/IN.SQ.)

COEF. OF M6
(1/IN.SQ.)

COEF. OF TEMP.
(LB./IN.SQ/F.)

-- LAYER 1 --

-0.5000 SIGMA 1 0.4509 0.0392 0. -8.3533 0.2353 0. -15.7070
2 -0.0392 1.0784 0. -0.2353 -4.5881 0. 1.2075
6 0. 0. 1.0000 0. 0. -6.0000 0.

0. SIGMA 1 1.8629 -0.0784 0. 2.8240 -0.2353 0. 24.1643
2 0.0784 0.6078 0. 0.2353 -0.9412 0. -9.6648
6 0. 0. 1.0000 0. 0. 0.

-- LAYER 2 --

0. SIGMA 1 0.6078 0.0784 0. 0.9412 -0.2353 0. -9.6648
2 -0.0784 1.8629 0. 0.2353 -2.8240 0. 24.1643
6 0. 0. 1.0000 0. 0. 0.

0.5000 SIGMA 1 1.0784 -0.0392 0. 4.5881 0.2353 0. 1.2075
2 0.0392 0.4509 0. -0.2353 8.3533 0. -15.7070
6 0. 0. 1.0000 0. 0. 6.0000 0.

CROSS-PLY M=1.0 CASE 1 (ALL LAYERS INTACT)
3 LAYERS (N=3)

STIFFNESS MATRIX (C)
(10+6 LB./IN.SQ.)

7.9660 0.6638 0.
0.6638 2.6550 0.
0. 0. 1.2500

-- ODD LAYERS --

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)

ALPHA 1 = 3.5000
ALPHA 2 = 11.4000
ALPHA 6 = 0.

STIFFNESS MATRIX (C)
(10+6 LB./IN.SQ.)

2.6550 0.6638 0.
0.6638 7.9660 0.
0. 0. 1.2500

-- EVEN LAYERS --

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)

ALPHA 1 = 11.4000
ALPHA 2 = 3.5000
ALPHA 6 = 0.

A
(10+6 LB./IN.)

5.3105 0.6638 0.
0.6638 5.3105 0.
0. 0. 1.2500

A*
(10-6 IN./LB.)

0.1913 -0.0239 0.
-0.0239 0.1913 0.
0. 0. 0.8000

A PRIME
(10-6 IN./LB.)

0.1913 -0.0239 0.
-0.0239 0.1913 0.
0. 0. 0.8000

THERMAL FORCE
(LB./IN./DEG.F.)

N1-T = 34.0193
N2-T = 34.0193
N6-T = 0.

B
(10+6 IN.)

0. 0. 0.
0. 0. 0.
0. 0. 0.

B*
(10+0 IN.)

0. 0. 0.
0. 0. 0.
0. 0. 0.

B PRIME
(10-6 1/LB.)

0. 0. 0.
0. 0. 0.
0. 0. 0.

THERMAL MCMENT
(LB./DEG.F.)

M1-T = 0.
M2-T = 0.
M6-T = 0.

H*
(10+0 IN.)

0. 0. 0.
0. 0. 0.
0. 0. 0.

C
(10+6 LB.IN.)

0.6085 0.0553 0.
0.0553 0.2766 0.
0. 0. 0.1042

D*
(10+6 LB.IN.)

0.6085 0.0553 0.
0.0553 0.2766 0.
0. 0. 0.1042

D PRIME
(10-6 1/LB.IN.)

1.6738 -0.3348 0.
-0.3348 3.6826 0.
0. 0. 9.6000

Z
(IN.)

STRESS
COMPONENT

COEF. OF N1
(1/IN.)

COEF. OF N2
(1/IN.)

COEF. OF N6
(1/IN.)

COEF. OF M1
(1/IN.SQ.)

COEF. OF M2
(1/IN.SQ.)

COEF. OF M6
(1/IN.SQ.)

COEF. OF TEMP.
(LB/IN.SQ/F.)

-- LAYER 1 --

-0.5000 SIGMA 1 1.5080 -0.0635 0. -6.5556 0.1111 0. 13.6921
2 0.0635 0.4920 0. -0.1111 -4.7776 0. -13.6921
6 0. 0. 1.0000 0. 0. -6.0000 0.

-0.2500 SIGMA 1 1.5080 -0.0635 0. -3.2778 0.0556 0. 13.6921
2 0.0635 0.4920 0. -0.0556 -2.3888 0. -13.6921
6 0. 0. 1.0000 0. 0. -3.0000 0.

-- LAYER 2 --

-0.2500 SIGMA 1 0.4920 0.0635 0. -1.0556 -0.3889 0. -13.6921
2 -0.0635 1.5080 0. 0.3889 -7.2784 0. -13.6921
6 0. 0. 1.0000 0. 0. -3.0000 0.

0.2500 SIGMA 1 0.4920 0.0635 0. 1.0556 0.3889 0. -13.6921
2 -0.0635 1.5080 0. -0.3889 7.2784 0. 13.6921
6 0. 0. 1.0000 0. 0. 3.0000 0.

-- LAYER 3 --

0.2500 SIGMA 1 1.5080 -0.0635 0. 3.2778 -0.0556 0. 13.6921
2 0.0635 0.4920 0. 0.0556 2.3888 0. -13.6921
6 0. 0. 1.0000 0. 0. 3.0000 0.

0.5000 SIGMA 1 1.5080 -0.0635 0. 6.5556 -0.1111 0. 13.6921
2 0.0635 0.4920 0. 0.1111 4.7776 0. -13.6921
6 0. 0. 1.0000 0. 0. 6.0000 0.

CROSS-PLY M=2.0 CASE 1 (ALL LAYERS INTACT)
2 LAYERS (N=2)

STIFFNESS MATRIX (C)
(10+6 LB./IN.SQ.)

7.9660 0.6638 0.
0.6638 2.6550 0.
0. 0. 1.2500

-- ODD LAYERS --

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)

ALPHA 1 = 3.5000
ALPHA 2 = 11.4000
ALPHA 6 = 0.

STIFFNESS MATRIX (C)
(10+6 LB./IN.SQ.)

2.6550 0.6638 0.
0.6638 7.9660 0.
0. 0. 1.2500

-- EVEN LAYERS --

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)

ALPHA 1 = 11.4000
ALPHA 2 = 3.5000
ALPHA 6 = 0.

A
(10+6 LB./IN.)

6.1958 0.6638 0.
0.6638 4.4252 0.
0. 0. 1.2500

A*
(10-6 IN./LB.)

0.1640 -0.0246 0.
-0.0246 0.2297 0.
0. 0. 0.8000

A PRIME
(10-6 IN./LB.)

0.1879 -0.0293 0.
-0.0293 0.2815 0.
0. 0. 0.8000

THERMAL FORCE
(LB./IN./DEG.F.)

N1-T = 34.4957
N2-T = 33.5429
N6-T = 0.

B
(10+6 IN.)

-0.5901 0. 0.
0. 0.5901 0.
0. 0. 0.

B*
(10+0 IN.)

0.0968 0.0145 0.
-0.0145 -0.1355 0.
0. 0. 0.

B PRIME
(10-6 1/LB.)

0.2449 0.0087 0.
0.0087 -0.3836 0.
0. 0. 0.

THERMAL MOMENT
(LB./DEG.F.)

M1-T = -0.3175
M2-T = 0.3175
M6-T = 0.

H*
(10+0 IN.)

-0.0968 0.0145 0.
-0.0145 0.1355 0.
0. 0. 0.

D
(10+6 LB.IN.)

0.4507 0.0553 0.
0.0553 0.4343 0.
0. 0. 0.1042

D*
(10+6 LB.IN.)

0.3936 0.0467 0.
0.0467 0.3544 0.
0. 0. 0.1042

D PRIME
(10-6 1/LB.IN.)

2.5809 -0.3405 0.
-0.3405 2.8668 0.
0. 0. 9.6000

Z
(IN.)

STRESS
COMPONENT

COEF. OF N1
(1/IN.)

COEF. OF N2
(1/IN.)

COEF. OF N6
(1/IN.)

COEF. OF M1
(1/IN.SQ.)

COEF. OF M2
(1/IN.SQ.)

COEF. OF M6
(1/IN.SQ.)

COEF. OF TEMP.
(LB/IN.SQ/F.)

-- LAYER 1 --

-0.5000	SIGMA 1	0.4988	0.0460	0.	-8.2104	0.2191	0.	-14.0221
	2	-0.0460	1.2343	0.	-0.2191	-4.7054	0.	5.8021
	6	0.	0.	1.0000	0.	0.	-6.0000	0.
0.1667	SIGMA 1	1.8032	-0.0777	0.	5.3460	-0.3205	0.	22.3463
	2	0.0777	0.5592	0.	0.3205	0.2185	0.	-11.1842
	6	0.	0.	1.0000	0.	0.	2.0004	0.

-- LAYER 2 --

0.1667	SIGMA 1	0.5886	0.0704	0.	1.7605	-0.0651	0.	-10.5036
	2	-0.0704	1.7148	0.	0.0651	0.7194	0.	19.8505
	6	0.	0.	1.0000	0.	0.	2.0004	0.
0.5000	SIGMA 1	0.8072	-0.0068	0.	3.9691	0.2679	0.	-6.1474
	2	0.0068	0.6983	0.	-0.2679	8.2557	0.	-9.0848
	6	0.	0.	1.0000	0.	0.	6.0000	0.

CROSS-PLY M=2.0 CASE 1 (ALL LAYERS INTACT)
3 LAYERS (N=3)

STIFFNESS MATRIX (C)
(10+6 LB./IN.SQ.)

7.9660 0.6638 0.
0.6638 2.6550 0.
0. 0. 1.2500

-- ODD LAYERS --

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)

ALPHA 1 = 3.5000
ALPHA 2 = 11.4000
ALPHA 6 = 0.

STIFFNESS MATRIX (C)
(10+6 LB./IN.SQ.)

2.6550 0.6638 0.
0.6638 7.9660 0.
0. 0. 1.2500

-- EVEN LAYERS --

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)

ALPHA 1 = 11.4000
ALPHA 2 = 3.5000
ALPHA 6 = 0.

A
(10+6 LB./IN.)

6.1956 0.6638 0.
0.6638 4.4254 0.
0. 0. 1.2500

A*
(10-6 IN./LB.)

0.1640 -0.0246 0.
-0.0246 0.2297 0.
0. 0. 0.8000

A PRIME
(10-6 IN./LB.)

0.1640 -0.0246 0.
-0.0246 0.2297 0.
0. 0. 0.8000

THERMAL FORCE
(LB./IN./DEG.F.)

N1-T = 34.4956
N2-T = 33.5430
N6-T = 0.

B
(10+6 IN.)

0. 0. 0.
0. 0. 0.
0. 0. 0.

B*
(10+0 IN.)

0. 0. 0.
0. 0. 0.
0. 0. 0.

B PRIME
(10-6 1/LB.)

0. 0. 0.
0. 0. 0.
0. 0. 0.

THERMAL MOMENT
(LB./DEG.F.)

M1-T = -0.0000
M2-T = -0.0000
M6-T = 0.

H*
(10+0 IN.)

0. 0. 0.
0. 0. 0.
0. 0. 0.

D
(10+6 LB.IN.)

0.6474 0.0553 0.
0.0553 0.2376 0.
0. 0. 0.1042

D*
(10+6 LB.IN.)

0.6474 0.0553 0.
0.0553 0.2376 0.
0. 0. 0.1042

D PRIME
(10-6 1/LB.IN.)

1.5759 -0.3668 0.
-0.3668 4.2934 0.
0. 0. 9.6000

Z (IN.) STRESS COMPONENT COEF. OF N1 (1/IN.) COEF. OF N2 (1/IN.) COEF. OF N6 (1/IN.) COEF. OF M1 (1/IN.SQ.) COEF. OF M2 (1/IN.SQ.) COEF. OF M6 (1/IN.SQ.) COEF. OF TEMP. (LB/IN.SQ/F.)

-- LAYER 1 --

-0.5000 SIGMA 1 1.2904 -0.0436 0. -6.1550 0.0361 0. 7.6041
2 0.0436 0.5934 0. -0.0361 -5.5777 0. -11.1827
6 0. 1.0000 0. 0. 0. -6.0000 0.

-0.1667 SIGMA 1 1.2904 -0.0436 0. -2.0517 0.0120 0. 7.6041
2 0.0436 0.5934 0. -0.0120 -1.8593 0. -11.1827
6 0. 1.0000 0. 0. 0. -2.0000 0.

-- LAYER 2 --

-0.1667 SIGMA 1 0.4192 0.0871 0. -0.6568 -0.3127 0. -15.2077
2 -0.0871 1.8131 0. 0.3127 -5.6597 0. 22.3647
6 0. 1.0000 0. 0. 0. -2.0000 0.

0.1667 SIGMA 1 0.4192 0.0871 0. -0.6568 0.3127 0. -15.2077
2 -0.0871 1.8131 0. -0.3127 5.6597 0. 22.3647
6 0. 1.0000 0. 0. 0. 2.0000 0.

-- LAYER 3 --

0.1667 SIGMA 1 1.2904 -0.0436 0. 2.0517 -0.0120 0. 7.6041
2 0.0436 0.5934 0. 0.0120 1.8593 0. -11.1827
6 0. 1.0000 0. 0. 0. 2.0000 0.

0.5000 SIGMA 1 1.2904 -0.0436 0. 6.1550 -0.0361 0. 7.6041
2 0.0436 0.5934 0. 0.0361 5.5777 0. -11.1827
6 0. 1.0000 0. 0. 0. 6.0000 0.

CROSS-PLY M=4.0 CASE 1 (ALL LAYERS INTACT)
2 LAYERS (N=2)

STIFFNESS MATRIX (C)
(10+6 LB./IN.SQ.)

7.9660	0.6638	0.
0.6638	2.6550	0.
0.	0.	1.2500

-- ODD LAYERS --

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)

ALPHA 1 = 3.5000
ALPHA 2 = 11.4000
ALPHA 6 = 0.

STIFFNESS MATRIX (C)
(10+6 LB./IN.SQ.)

2.6550	0.6638	0.
0.6638	7.9660	0.
0.	0.	1.2500

-- EVEN LAYERS --

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)

ALPHA 1 = 11.4000
ALPHA 2 = 3.5000
ALPHA 6 = 0.

A (10+6 LB./IN.)			A* (10-6 IN./LB.)			A PRIME (10-6 IN./LB.)			THERMAL FORCE (LB./IN./DEG.F.)
6.9038	0.6638	0.	0.1474	-0.0263	0.	0.1559	-0.0293	0.	N1-T = 34.8767
0.6638	3.7172	0.	-0.0263	0.2737	0.	-0.0293	0.3130	0.	N2-T = 33.1619
0.	0.	1.2500	0.	0.	0.8000	0.	0.	0.8000	N6-T = 0.

B (10+6 IN.)			B* (10+0 IN.)			B PRIME (10-6 1/LB.)			THERMAL MOMENT (LB./DEG.F.)
-0.4249	0.	0.	0.0626	0.0112	0.	0.1336	0.0128	0.	M1-T = -0.2286
0.	-0.4249	0.	-0.0112	-0.1163	0.	0.0128	-0.3386	0.	M2-T = 0.2286
0.	0.	0.	0.	0.	0.	0.	0.	0.	M6-T = 0.

H*
(10+0 IN.)

-0.0626	0.0112	0.
-0.0112	0.1163	0.
0.	0.	0.

D (10+6 LB.IN.)			D* (10+6 LB.IN.)			D PRIME (10-6 1/LB.IN.)		
0.4903	0.0553	0.	0.4637	0.0506	0.	2.1914	-0.3209	0.
0.0553	0.3947	0.	0.0506	0.3453	0.	-0.3209	2.9428	0.
0.	0.	0.1042	0.	0.	0.1042	0.	0.	9.6000

Z (IN.)	STRESS COMPONENT	COEF. OF N1 (1/IN.)	COEF. OF N2 (1/IN.)	COEF. OF N6 (1/IN.)	COEF. OF M1 (1/IN.SQ.)	COEF. OF M2 (1/IN.SQ.)	COEF. OF M6 (1/IN.SQ.)	COEF. OF TEMP. (LB/IN.SQ/F.)
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-- LAYER 1 --

-0.5000	SIGMA 1	0.6859	0.0357	0.	-7.5488	0.1786	0.	-8.5780
	2	-0.0357	1.2568	0.	-0.1786	-4.6906	0.	6.8103
	6	0.	0.	1.0000	0.	0.	-6.0000	0.
0.3000	SIGMA 1	1.5443	-0.0625	0.	6.2461	-0.3035	0.	14.8407
	2	0.0625	0.5443	0.	0.3035	1.3894	0.	-12.1131
	6	0.	0.	1.0000	0.	0.	3.6000	0.

-- LAYER 2 --

0.3000	SIGMA 1	0.5034	0.0727	0.	2.0448	0.1397	0.	-13.0568
	2	-0.0727	1.6668	0.	-0.1397	4.2795	0.	18.3017
	6	0.	0.	1.0000	0.	0.	3.6000	0.
0.5000	SIGMA 1	0.5761	0.0346	0.	3.1659	0.3600	0.	-11.9939
	2	-0.0346	1.1290	0.	-0.3600	8.9254	0.	2.9094
	6	0.	0.	1.0000	0.	0.	6.0000	0.

CROSS-PLY M=4.0 CASE 1 (ALL LAYERS INTACT)
3 LAYERS (N=3)

STIFFNESS MATRIX (C)
(10+6 LB./IN.SQ.)

7.9660	0.6638	0.
0.6638	2.6550	0.
0.	0.	1.2500

-- ODD LAYERS --

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)

ALPHA 1 = 3.5000
ALPHA 2 = 11.4000
ALPHA 6 = 0.

STIFFNESS MATRIX (C)
(10+6 LB./IN.SQ.)

2.6550	0.6638	0.
0.6638	7.9660	0.
0.	0.	1.2500

-- EVEN LAYERS --

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)

ALPHA 1 = 11.4000
ALPHA 2 = 3.5000
ALPHA 6 = 0.

A (10+6 LB./IN.)			A* (10-6 IN./LB.)			A PRIME (10-6 IN./LB.)			THERMAL FORCE (LB./IN./DEG.F.)
6.9034	0.6638	0.	0.1474	-0.0263	0.	0.1474	-0.0263	0.	M1-T = 34.8765
0.6638	3.7176	0.	-0.0263	0.2737	0.	-0.0263	0.2737	0.	M2-T = 33.1621
0.	0.	1.2500	0.	0.	0.8000	0.	0.	0.8000	M6-T = 0.

B (10+6 IN.)			B* (10+0 IN.)			B PRIME (10-6 1/LB.)			THERMAL MOMENT (LB./DEG.F.)
-0.0000	0.	0.	0.0000	0.0000	0.	0.0000	0.0000	0.	M1-T = -0.0000
0.	0.0000	0.	-0.0000	-0.0000	0.	0.0000	-0.0000	0.	M2-T = 0.0000
0.	0.	0.	0.	0.	0.	0.	0.	0.	M6-T = 0.

H*
(10+0 IN.)

-0.0000	0.0000	0.
-0.0000	0.0000	0.
0.	0.	0.

D (10+6 LB.IN.)			D* (10+6 LB.IN.)			D PRIME (10-6 1/LB.IN.)		
0.6603	0.0553	0.	0.6603	0.0553	0.	1.5464	-0.3805	0.
0.0553	0.2248	0.	0.0553	0.2248	0.	-0.3805	4.5421	0.
0.	0.	0.1042	0.	0.	0.1042	0.	0.	9.6000

Z (IN.)	STRESS COMPONENT	COEF. OF N1 (1/IN.)	COEF. OF N2 (1/IN.)	COEF. OF N6 (1/IN.)	COEF. OF M1 (1/IN.SQ.)	COEF. OF M2 (1/IN.SQ.)	COEF. OF M6 (1/IN.SQ.)	COEF. OF TEMP. (LB/IN.SQ/F.)
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-- LAYER 1 --

-0.5000	SIGMA 1	1.1566	-0.0280	0.	-6.0328	0.0081	0.	3.9620
	2	0.0280	0.7092	0.	-0.0081	-5.9035	0.	-8.0954
	6	0.	0.	1.0000	0.	0.	-6.0000	0.
-0.1000	SIGMA 1	1.1566	-0.0280	0.	-1.2065	0.0016	0.	3.9626
	2	0.0280	0.7092	0.	-0.0016	-1.1808	0.	-8.0967
	6	0.	0.	1.0000	0.	0.	-1.2000	0.

-- LAYER 2 --

-0.1000	SIGMA 1	0.3738	0.1118	0.	-0.3853	-0.2005	0.	-15.8443
	2	-0.1118	2.1628	0.	0.2005	-3.5934	0.	32.3750
	6	0.	0.	1.0000	0.	0.	-1.2000	0.
0.1001	SIGMA 1	0.3738	0.1118	0.	0.3856	0.2006	0.	-15.8443
	2	-0.1118	2.1627	0.	-0.2006	3.5952	0.	32.3729
	6	0.	0.	1.0000	0.	0.	1.2008	0.

-- LAYER 3 --

0.1001	SIGMA 1	1.1566	-0.0280	0.	1.2075	-0.0016	0.	3.9630
	2	0.0280	0.7092	0.	0.0016	1.1814	0.	-8.0974
	6	0.	0.	1.0000	0.	0.	1.2008	0.
0.5000	SIGMA 1	1.1566	-0.0280	0.	6.0330	-0.0081	0.	3.9637
	2	0.0280	0.7091	0.	0.0081	5.9033	0.	-8.0987
	6	0.	0.	1.0000	0.	0.	6.0000	0.

CROSS-PLY M=0.2 CASE 2 (ALL LAYERS DEGRADED)
2 LAYERS (N=2)

STIFFNESS MATRIX (C)
(10*6 LB./IN.SQ.)

7.8000 0.0000 0.
0.0000 7.8000 0.
0. 0. 0.0000

-- ODD LAYERS --

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)

ALPHA 1 = 3.5000
ALPHA 2 = 11.4000
ALPHA 6 = 0.

STIFFNESS MATRIX (C)
(10*6 LB./IN.SQ.)

0.0000 0.0000 0.
0.0000 7.8000 0.
0. 0. 0.0000

-- EVEN LAYERS --

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)

ALPHA 1 = 11.4000
ALPHA 2 = 3.5000
ALPHA 6 = 0.

A (10*6 LB./IN.)			A* (10*6 IN./LB.)			A PRIME (10*6 IN./LB.)			THERMAL FORCE (LB./IN./DEG.F.)	
1.3003	0.0000	0.	0.7691	-0.0000	0.	58.4172	-0.0000	0.	N1-T =	4.5509
0.0000	6.4997	0.	-0.0000	0.1539	0.	-0.0000	0.1723	0.	N2-T =	22.7491
0.	0.	0.0000	0.	0.	0.0000	0.	0.	0.0000	N6-T =	0.
B (10*6 IN.)			B* (10*0 IN.)			B PRIME (10*6 1/LB.)			THERMAL MOMENT (LB./DEG.F.)	
-0.5418	0.	0.	0.4166	0.0000	0.	138.3611	-0.0000	0.	M1-T =	-1.8961
0.	0.2419	0.	-0.0000	-0.0833	0.	-0.0000	-0.2216	0.	M2-T =	1.8961
0.	0.	0.	0.	0.	0.	0.	0.	0.	M6-T =	0.
H (10*0 IN.)			H* (10*0 IN.)			D PRIME (10*6 1/LB./IN.)				
			-0.4166	0.0000	0.					
			-0.0000	0.0833	0.					
			0.	0.	0.					
U (10*6 LB./IN.)			U* (10*6 LB./IN.)							
0.2287	0.0000	0.	0.0030	0.0000	0.	332.0801	-0.0000	0.		
0.0000	0.4213	0.	0.0000	0.3761	0.	-0.0000	2.6588	0.		
0.	0.	0.0000	0.	0.	0.0000	0.	0.	0.0000		
Z (IN.)	STRESS COMPONENT	COEF. OF N1 (1/IN.)	COEF. OF N2 (1/IN.)	COEF. OF N6 (1/IN.)	COEF. OF M1 (1/IN.SQ.)	COEF. OF M2 (1/IN.SQ.)	COEF. OF M6 (1/IN.SQ.)	COEF. OF TEMP. (LB/IN.SQ/F.)		
-- LAYER 1 --										
-0.5000	SIGMA 1	-83.9542	0.0000	0.	-215.8961	0.0000	0.	-0.0007		
	2	-0.0000	0.0000	0.	-0.0000	-0.0000	0.	-0.0000		
	6	0.	0.	1.0000	0.	0.	-6.0000	0.		
-0.3333	SIGMA 1	95.9511	-0.0000	0.	215.8944	-0.0000	0.	0.0008		
	2	0.0000	0.0000	0.	0.0000	-0.0000	0.	-0.0000		
	6	0.	0.	1.0000	0.	0.	-3.9996	0.		
-- LAYER 2 --										
-0.3333	SIGMA 1	0.0000	0.0000	0.	0.0000	-0.0000	0.	-0.0000		
	2	-0.0000	1.9202	0.	0.0000	-8.6407	0.	0.0000		
	6	0.	0.	1.0000	0.	0.	-3.9996	0.		
0.5000	SIGMA 1	0.0001	0.0000	0.	0.0003	0.0000	0.	-0.0000		
	2	-0.0000	0.4798	0.	-0.0000	8.6407	0.	-0.0000		
	6	0.	0.	1.0000	0.	0.	6.0000	0.		

CROSS-PLY M=0.2 CASE 2 (ALL LAYERS DEGRADED)
3 LAYERS (N=3)

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-- ODD LAYERS --
STIFFNESS MATRIX (C)
(10+6 LB./IN.SQ.)
7.8000 0.0000 0.
0.0000 0.0000 0.
0. 0. 0.0000

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)
ALPHA 1 = 3.5000
ALPHA 2 = 11.4000
ALPHA 6 = 0.

-- EVEN LAYERS --
STIFFNESS MATRIX (C)
(10+6 LB./IN.SQ.)
0.0000 0.0000 0.
0.0000 7.8000 0.
0. 0. 0.0000

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)
ALPHA 1 = 11.4000
ALPHA 2 = 3.5000
ALPHA 6 = 0.

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A (10+6 LB./IN.)
1.2999 0.0000 0.
0.0000 6.5001 0.
0. 0. 0.0000

A* (10+6 IN./LB.)
0.7693 -0.0000 0.
-0.0000 0.1538 0.
0. 0. 0.0000.0000

A PRIME (10+6 IN./LB.)
0.7693 -0.0000 0.
-0.0000 0.1538 0.
0. 0. 0.0000.0000

THERMAL FORCE (LB./IN./DEG.F.)
N1-T = 4.5498
N2-T = 22.7502
N6-T = 0.

B (10+6 IN.)
0. 0. 0.
0. 0. 0.
0. 0. 0.

B* (10+6 IN.)
0. 0. 0.
0. 0. 0.
0. 0. 0.

B PRIME (10+6 1/LB.)
0. 0. 0.
0. 0. 0.
0. 0. 0.

THERMAL MOMENT (LB./DEG.F.)
M1-T = 0.
M2-T = 0.
M6-T = 0.

H* (10+6 IN.)
0. 0. 0.
0. 0. 0.
0. 0. 0.

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D (10+6 LB.IN.)
0.2738 0.0000 0.
0.0000 0.3762 0.
0. 0. 0.0000

D* (10+6 LB.IN.)
0.2738 0.0000 0.
0.0000 0.3762 0.
0. 0. 0.0000

D PRIME (10+6 1/LB.IN.)
3.6519 -0.0000 0.
-0.0000 2.6584 0.
0. 0. 0.0000.0000

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Z (IN.) STRESS COEF. OF N1 COEF. OF N2 COEF. OF N6 COEF. OF M1 COEF. OF M2 COEF. OF M6 COEF. OF TEMP.
COMPONENT (1/IN.) (1/IN.) (1/IN.SQ.) (1/IN.SQ.) (1/IN.SQ.) (1/IN.SQ.) (1/IN.SQ./F.)

-- LAYER 1 --
-0.5000 SIGMA 1 6.0002 -0.0000 0. -14.2422 0.0000 0. 0.0000
2 0.0000 0.0000 0. -0.0000 0.0000 0. -0.0000
6 0. 0. 1.0000 0. -6.0000 0.
-0.4167 SIGMA 1 6.0002 -0.0000 0. -11.8686 0.0000 0. 0.0000
2 0.0000 0.0000 0. -0.0000 0.0000 0. -0.0000
6 0. 0. 1.0000 0. -5.0000 0.

-- LAYER 2 --
-0.4167 SIGMA 1 0.0000 0.0000 0. -0.0000 0.0000 0. -0.0000
2 -0.0000 1.2000 0. 0.0000 -8.6399 0. 0.0000
6 0. 0. 1.0000 0. -5.0000 0.
0.4167 SIGMA 1 0.0000 0.0000 0. 0.0000 0.0000 0. -0.0000
2 -0.0000 1.2000 0. -0.0000 8.6399 0. 0.0000
6 0. 0. 1.0000 0. 5.0000 0.

-- LAYER 3 --
0.4167 SIGMA 1 6.0002 -0.0000 0. 11.8686 -0.0000 0. 0.0000
2 0.0000 0.0000 0. 0.0000 0.0000 0. -0.0000
6 0. 0. 1.0000 0. 5.0000 0.
0.5000 SIGMA 1 6.0002 -0.0000 0. 14.2422 -0.0000 0. 0.0000
2 0.0000 0.0000 0. 0.0000 0.0000 0. -0.0000
6 0. 0. 1.0000 0. 6.0000 0.

```

CROSS-PLY M=0.4 CASE 2 (ALL LAYERS DEGRADED)
2 LAYERS (N=2)

STIFFNESS MATRIX (C) (10*6 LB./IN.SQ.)			THERMAL EXPANSION MATRIX (ALPHA) (IN./IN./DEG.F.)		
7.8000	0.0000	0.	ALPHA 1 = 3.5000		
0.0000	0.0000	0.	ALPHA 2 = 11.4000		
0.	0.	0.0000	ALPHA 6 = 0.		
STIFFNESS MATRIX (C) (10*6 LB./IN.SQ.)			THERMAL EXPANSION MATRIX (ALPHA) (IN./IN./DEG.F.)		
0.0000	0.0000	0.	ALPHA 1 = 11.4000		
0.0000	7.8000	0.	ALPHA 2 = 3.5000		
0.	0.	0.0000	ALPHA 6 = 0.		
A (10*6 LB./IN.)			A PRIME (10*6 IN./LB.)		
2.2285	0.0000	0.	8.8637	-0.0000	0.
0.0000	5.5715	0.	-0.0000	0.2656	0.
0.	0.	0.0000	0.	0.	0.0000,0000
B (10*6 IN.)			B PRIME (10*6 1/LB.)		
-0.7959	0.	0.	23.5614	-0.0000	0.
0.	0.7959	0.	-0.0000	-0.6030	0.
0.	0.	0.	0.	0.	0.
H* (10*6 IN.)			D PRIME (10*6 1/LB.IN.)		
			65.9705	-0.0000	0.
			-0.0000	4.2213	0.
			0.	0.	0.0000,0000
D (10*6 LB.IN.)			D PRIME (10*6 1/LB.IN.)		
0.2994	0.0000	0.	0.0152	0.0000	0.
0.0000	0.3505	0.	0.0000	0.2369	0.
0.	0.	0.0000	0.	0.	0.0000
Z (IN.)			D PRIME (10*6 1/LB.IN.)		
			0.	0.	0.
			0.	0.	0.
			0.	0.	0.
STRESS COMPONENT			D PRIME (10*6 1/LB.IN.)		
			0.	0.	0.
			0.	0.	0.
			0.	0.	0.
COEF. OF M1 (1/IN.)			D PRIME (10*6 1/LB.IN.)		
			0.	0.	0.
			0.	0.	0.
			0.	0.	0.
COEF. OF M2 (1/IN.SQ.)			D PRIME (10*6 1/LB.IN.)		
			0.	0.	0.
			0.	0.	0.
			0.	0.	0.
COEF. OF M6 (1/IN.SQ.)			D PRIME (10*6 1/LB.IN.)		
			0.	0.	0.
			0.	0.	0.
			0.	0.	0.
COEF. OF TEMP. (LB/IN.SQ/F.)			D PRIME (10*6 1/LB.IN.)		
			0.	0.	0.
			0.	0.	0.
			0.	0.	0.

CROSS-PLY M=0.4 CASE 2 (ALL LAYERS DEGRADED)
3 LAYERS (N=3)

STIFFNESS MATRIX (C)
(10⁶ LB./IN.².)

7.8000 0.0000 0.
0.0000 0.0000 0.
0. 0. 0.0000

-- ODD LAYERS --

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)

ALPHA 1 = 3.5000
ALPHA 2 = 11.4000
ALPHA 6 = 0.

STIFFNESS MATRIX (C)
(10⁶ LB./IN.².)

0.0000 0.0000 0.
0.0000 7.8000 0.
0. 0. 0.0000

-- EVEN LAYERS --

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)

ALPHA 1 = 11.4000
ALPHA 2 = 3.5000
ALPHA 6 = 0.

A
(10⁻⁶ LB./IN.)

2.2286 0.0000 0.
0.0000 5.5714 0.
0. 0. 0.0000

A*
(10⁻⁶ IN./LB.)

0.4487 -0.0000 0.
-0.0000 0.1795 0.
0. 0. 0.0000

A PRIME
(10⁻⁶ IN./LB.)

0.4487 -0.0000 0.
-0.0000 0.1795 0.
0. 0. 0.0000

THERMAL FORCE
(LB./IN./DEG.F.)

N1-T = 7.8002
N2-T = 19.4999
N6-T = 0.

B
(10⁻⁶ IN.)

0. 0. 0.
0. 0. 0.
0. 0. 0.

B*
(10⁻⁶ IN.)

0. 0. 0.
0. 0. 0.
0. 0. 0.

B PRIME
(10⁻⁶ 1/LB.)

0. 0. 0.
0. 0. 0.
0. 0. 0.

THERMAL MOMENT
(LB./DEG.F.)

M1-T = 0.
M2-T = 0.
M6-T = 0.

H*
(10⁻⁶ IN.)

0. 0. 0.
0. 0. 0.
0. 0. 0.

D
(10⁻⁶ LB./IN.)

0.4131 0.0000 0.
0.0000 0.2369 0.
0. 0. 0.0000

D*
(10⁻⁶ LB./IN.)

0.4131 0.0000 0.
0.0000 0.2369 0.
0. 0. 0.0000

D PRIME
(10⁻⁶ 1/LB./IN.)

2.4206 -0.0000 0.
-0.0000 4.2216 0.
0. 0. 0.0000

Z
(IN.)

STRESS
COMPONENT

COEF. OF N1
(1/IN.)

COEF. OF N2
(1/IN.)

COEF. OF N6
(1/IN.)

COEF. OF M1
(1/IN.SQ.)

COEF. OF M2
(1/IN.SQ.)

COEF. OF M6
(1/IN.SQ.)

COEF. OF TEMP.
(LB/IN.SQ/F.)

-- LAYER 1 --

-0.5000 SIGMA 1 3.4999 -0.0000 0. -9.4402 0.0000 0. 0.0000
2 0.0000 0.0000 0. -0.0000 -0.0000 0. -0.0000
6 0. 0. 1.0000 0. 0. -6.0000 0.

-0.3571 SIGMA 1 3.4999 -0.0000 0. -6.7430 0.0000 0. 0.0000
2 0.0000 0.0000 0. -0.0000 -0.0000 0. -0.0000
6 0. 0. 1.0000 0. 0. -4.2857 0.

-- LAYER 2 --

-0.3571 SIGMA 1 0.0000 0.0000 0. -0.0000 -0.0000 0. -0.0000
2 -0.0000 1.4000 0. 0.0000 -11.7602 0. 0.0000
6 0. 0. 1.0000 0. 0. -4.2857 0.

0.3571 SIGMA 1 0.0000 0.0000 0. 0.0000 0.0000 0. -0.0000
2 -0.0000 1.4000 0. -0.0000 11.7602 0. 0.0000
6 0. 0. 1.0000 0. 0. 4.2857 0.

-- LAYER 3 --

0.3571 SIGMA 1 3.4999 -0.0000 0. 6.7430 -0.0000 0. 0.0000
2 0.0000 0.0000 0. 0.0000 0.0000 0. -0.0000
6 0. 0. 1.0000 0. 0. 4.2857 0.

0.5000 SIGMA 1 3.4999 -0.0000 0. 9.4402 -0.0000 0. 0.0000
2 0.0000 0.0000 0. 0.0000 0.0000 0. -0.0000
6 0. 0. 1.0000 0. 0. 6.0000 0.

CROSS-PLY M=1.0 CASE 2 (ALL LAYERS DEGRADED)
2 LAYERS (N=2)

-- ODD LAYERS --

STIFFNESS MATRIX (C)
(10+6 LB./IN.SQ.)

7.8000 0.0000 0.
0.0000 0.0000 0.
0. 0. 0.0000

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)

ALPHA 1 = 3.5000
ALPHA 2 = 11.4000
ALPHA 6 = 0.

-- EVEN LAYERS --

STIFFNESS MATRIX (C)
(10+6 LB./IN.SQ.)

0.0000 0.0000 0.
0.0000 7.8000 0.
0. 0. 0.0000

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)

ALPHA 1 = 11.4000
ALPHA 2 = 3.5000
ALPHA 6 = 0.

A (10+6 LB./IN.)			A* (10+0 IN./LB.)			A PRIME (10-6 IN./LB.)			THERMAL FORCE (LB./IN./DEG.F.)		
3.9000	0.0000	0.	0.2564	-0.0000	0.	1.0256	-0.0000	0.	N1-T =	13.6500	
0.0000	3.9000	0.	-0.0000	0.2564	0.	-0.0000	1.0256	0.	N2-T =	13.6500	
0.	0.	0.0000	0.	0.	0.0000.0000	0.	0.	0.0000.0000	N6-T =	0.	

B (10+6 IN.)			B* (10+0 IN.)			B PRIME (10-6 1/LB.)			THERMAL MOMENT (LB./DEG.F.)		
-0.9750	0.	0.	0.2500	0.0000	0.	5.0769	0.0000	0.	M1-T =	-3.4125	
0.	0.9750	0.	-0.0000	-0.2500	0.	-0.0000	-3.0769	0.	M2-T =	3.4125	
0.	0.	0.	0.	0.	0.	0.	0.	0.	M6-T =	0.	

D (10+6 LB.IN.)			D* (10+6 LB.IN.)			D PRIME (10-6 1/LB.IN.)		
0.3250	0.0000	0.	0.0313	0.0000	0.	12.3077	-0.0000	0.
0.0000	0.3250	0.	0.0000	0.0313	0.	-0.0000	12.3077	0.
0.	0.	0.0000	0.	0.	0.0000	0.	0.	0.0000.0000

Z (IN.)	STRESS COMPONENT	COEF. OF M1 (1/IN.)	COEF. OF M2 (1/IN.)	COEF. OF M6 (1/IN.)	COEF. OF M1 (1/IN.SQ.)	COEF. OF M2 (1/IN.SQ.)	COEF. OF M6 (1/IN.SQ.)	COEF. OF TEMP. (LB/IN.SQ/F.)
-- LAYER 1 --								
-0.5000	SIGMA 1	-4.0000	0.0000	0.	-24.0000	0.0000	0.	-0.0000
	2	-0.0000	0.0000	0.	-0.0000	-0.0000	0.	-0.0000
	6	0.	0.	1.0000	0.	0.	-6.0000	0.
0.	SIGMA 1	8.0000	-0.0000	0.	24.0000	0.0000	0.	0.0001
	2	0.0000	0.0000	0.	0.0000	-0.0000	0.	-0.0000
	6	0.	0.	1.0000	0.	0.	0.	0.
-- LAYER 2 --								
0.	SIGMA 1	0.0000	0.0000	0.	0.0000	-0.0000	0.	-0.0000
	2	-0.0000	8.0000	0.	0.0000	-24.0000	0.	0.0001
	6	0.	0.	1.0000	0.	0.	0.	0.
0.5000	SIGMA 1	0.0000	-0.0000	0.	0.0000	0.0000	0.	-0.0000
	2	0.0000	-4.0000	0.	-0.0000	24.0000	0.	-0.0000
	6	0.	0.	1.0000	0.	0.	6.0000	0.

CROSS-PLY M=1.0 CASE 2 (ALL LAYERS DEGRADED)
3 LAYERS (N=3)

STIFFNESS MATRIX (C)
(10+6 LB./IN.SQ.)

7.8000 0.0000 0.
0.0000 0.0000 0.
0. 0. 0.0000

-- ODD LAYERS --

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)

ALPHA 1 = 3.5000
ALPHA 2 = 11.4000
ALPHA 6 = 0.

STIFFNESS MATRIX (C)
(10+6 LB./IN.SQ.)

0.0000 0.0000 0.
0.0000 7.8000 0.
0. 0. 0.0000

-- EVEN LAYERS --

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)

ALPHA 1 = 11.4000
ALPHA 2 = 3.5000
ALPHA 6 = 0.

A
(10+6 LB./IN.)

3.9000 0.0000 0.
0.0000 3.9000 0.
0. 0. 0.0000

A*
(10-6 IN./LB.)

0.2564 -0.0000 0.
-0.0000 0.2564 0.
0. 0.0000 0.0000

A PRIME
(10-6 IN./LB.)

0.2564 -0.0000 0.
-0.0000 0.2564 0.
0. 0.0000 0.0000

THERMAL FORCE
(LB./IN./DEG.F.)

N1-T = 13.6500
N2-T = 13.6500
N6-T = 0.

B
(10+6 IN.)

0. 0. 0.
0. 0. 0.
0. 0. 0.

B*
(10+0 IN.)

0. 0. 0.
0. 0. 0.
0. 0. 0.

B PRIME
(10-6 1/LB.)

0. 0. 0.
0. 0. 0.
0. 0. 0.

THERMAL MOMENT
(LB./DEG.F.)

M1-T = 0.
M2-T = 0.
M6-T = 0.

M*
(10+0 IN.)

0. 0. 0.
0. 0. 0.
0. 0. 0.

D
(10+6 LB.IN.)

0.5588 0.0000 0.
0.0000 0.0813 0.
0. 0. 2.0000

D*
(10+6 LB.IN.)

0.5588 0.0000 0.
0.0000 0.0813 0.
0. 0. 0.0000

D PRIME
(10-6 1/LB.IN.)

1.7582 -0.0000 0.
-0.0000 12.3077 0.
0. 0. 0.0000 0.0000

Z
(IN.)

STRESS
COMPONENT

COEF. OF N1
(1/IN.)

COEF. OF N2
(1/IN.)

COEF. OF N6
(1/IN.)

COEF. OF M1
(1/IN.SQ.)

COEF. OF M2
(1/IN.SQ.)

COEF. OF M6
(1/IN.SQ.)

COEF. OF TEMP.
(LB/IN.SQ/F.)

-- LAYER 1 --

-0.5000 SIGMA 1 2.0000 -0.0000 0. -6.8571 0.0000 0. 0.0000
2 0.0000 0.0000 0. -0.0000 +0.0000 0. -0.0000
6 0. 0. 1.0000 0. -6.0000 0.

-0.2500 SIGMA 1 2.0000 -0.0000 0. -3.4286 0.0000 0. 0.0000
2 0.0000 0.0000 0. -0.0000 +0.0000 0. -0.0000
6 0. 0. 1.0000 0. -3.0000 0.

-- LAYER 2 --

-0.2500 SIGMA 1 0.0000 0.0000 0. -0.0000 +0.0000 0. -0.0000
2 -0.0000 2.0000 0. 0.0000 -24.0000 0. 0.0000
6 0. 0. 1.0000 0. -3.0000 0.

0.2500 SIGMA 1 0.0000 0.0000 0. 0.0000 0.0000 0. -0.0000
2 -0.0000 2.0000 0. -0.0000 24.0000 0. 0.0000
6 0. 0. 1.0000 0. 3.0000 0.

-- LAYER 3 --

0.2500 SIGMA 1 2.0000 -0.0000 0. 3.4286 +0.0000 0. 0.0000
2 0.0000 0.0000 0. 0.0000 0.0000 0. -0.0000
6 0. 0. 1.0000 0. 0. 3.0000 0.

0.5000 SIGMA 1 2.0000 -0.0000 0. 6.8571 +0.0000 0. 0.0000
2 0.0000 0.0000 0. 0.0000 0.0000 0. -0.0000
6 0. 0. 1.0000 0. 0. 6.0000 0.

CROSS-PLY M=2.0 CASE 2 (ALL LAYERS DEGRADED)
3 LAYERS (N=3)

-- ODD LAYERS --

STIFFNESS MATRIX (C)
(10+6 LB./IN.SQ.)

7.8000 0.0000 0.
0.0000 0.0000 0.
0. 0. 0.0000

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)

ALPHA 1 = 3.5000
ALPHA 2 = 11.4000
ALPHA 6 = 0.

-- EVEN LAYERS --

STIFFNESS MATRIX (C)
(10+6 LB./IN.SQ.)

0.0000 0.0000 0.
0.0000 7.8000 0.
0. 0. 0.0000

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)

ALPHA 1 = 11.4000
ALPHA 2 = 3.5000
ALPHA 6 = 0.

A (10+6 LB./IN.)			A* (10+6 IN./LB.)			A PRIME (10+6 IN./LB.)			THERMAL FORCE (LB./IN./DEG.F.)	
5.1999	0.0000	0.	0.1923	-0.0000	0.	0.1923	-0.0000	0.	N1-T = 18.1998	
0.0000	2.6001	0.	-0.0000	0.3846	0.	-0.0000	0.3846	0.	N2-T = 9.1002	
0.	0.	0.0000	0.	0.	0.000,0000	0.	0.	0.000,0000	N6-T = 0.	
B (10+6 IN.)			B* (10+6 IN.)			B PRIME (10+6 1/LB.)			THERMAL MOMENT (LB./DEG.F.)	
0.	0.	0.	0.	0.	0.	0.	0.	0.	M1-T = -0.0000	
0.	0.	0.	0.	0.	0.	0.	0.	0.	M2-T = -0.0000	
0.	0.	0.	0.	0.	0.	0.	0.	0.	M6-T = 0.	
D (10+6 LB.IN.)			D* (10+6 LB.IN.)			D PRIME (10+6 1/LB.IN.)				
0.6259	0.0000	0.	0.6259	0.0000	0.	1.5476	-0.0000	0.		
0.0000	0.0241	0.	0.0000	0.0241	0.	-0.0000	41.5358	0.		
0.	0.	0.0000	0.	0.	0.0000	0.	0.	0.000,0005		
Z (IN.)	STRESS COMPONENT	COEF. OF N1 (1/IN.)	COEF. OF N2 (1/IN.)	COEF. OF N6 (1/IN.)	COEF. OF M1 (1/IN.SQ.)	COEF. OF M2 (1/IN.SQ.)	COEF. OF M6 (1/IN.SQ.)	COEF. OF TEMP. (LB/IN.SQ/F.)		
-- LAYER 1 --										
-0.5000	SIGMA 1	1.5000	-0.0000	0.	-6.2308	0.0000	0.	0.0000		
	2	0.0000	0.0000	0.	-0.0000	-0.0000	0.	-0.0000		
	6	0.	0.	1.0000	0.	0.	-6.0000	0.		
-0.1667	SIGMA 1	1.5000	-0.0000	0.	-2.0770	0.0000	0.	0.0000		
	2	0.0000	0.0000	0.	-0.0000	-0.0000	0.	-0.0000		
	6	0.	0.	1.0000	0.	0.	-2.0000	0.		
-- LAYER 2 --										
-0.1667	SIGMA 1	0.0000	0.0000	0.	-0.0000	-0.0000	0.	-0.0000		
	2	-0.0000	2.9999	0.	0.0000	-53.9977	0.	0.0000		
	6	0.	0.	1.0000	0.	0.	-2.0000	0.		
0.1667	SIGMA 1	0.0000	0.0000	0.	0.0000	0.0000	0.	-0.0000		
	2	-0.0000	2.9999	0.	-0.0000	53.9977	0.	0.0000		
	6	0.	0.	1.0000	0.	0.	2.0000	0.		
-- LAYER 3 --										
0.1667	SIGMA 1	1.5000	-0.0000	0.	2.0770	-0.0000	0.	0.0000		
	2	0.0000	0.0000	0.	0.0000	0.0000	0.	-0.0000		
	6	0.	0.	1.0000	0.	0.	2.0000	0.		
0.5000	SIGMA 1	1.5000	-0.0000	0.	6.2308	-0.0000	0.	0.0000		
	2	0.0000	0.0000	0.	0.0000	0.0000	0.	-0.0000		
	6	0.	0.	1.0000	0.	0.	6.0000	0.		

CROSS-PLY M=4.0 CASE 2 (ALL LAYERS DEGRADED)
2 LAYERS (N=2)

-- ODD LAYERS --

STIFFNESS MATRIX (C)
(10+6 LB./IN.SQ.)

7.8000 0.0000 0.
0.0000 0.0000 0.
0. 0. 0.0000

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)

ALPHA 1 = 3.5000
ALPHA 2 = 11.4000
ALPHA 6 = 0.

-- EVEN LAYERS --

STIFFNESS MATRIX (C)
(10+6 LB./IN.SQ.)

0.0000 0.0000 0.
0.0000 7.8000 0.
0. 0. 0.0000

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)

ALPHA 1 = 11.4000
ALPHA 2 = 3.5000
ALPHA 6 = 0.

A (10+6 LB./IN.)			A* (10+6 IN./LB.)			A PRIME (10+6 IN./LB.)			THERMAL FORCE (LB./IN./DEG.F.)	
6.2400	0.0000	0.	0.1603	-0.0000	0.	0.1903	-0.0000	0.	N1-T =	21.8400
0.0000	1.5600	0.	-0.0000	0.6419	0.	-0.0000	31.4088	0.	N2-T =	5.4500
0.	0.	0.0000	0.	0.	0.0000.0000	0.	0.	0.0000.0000	N6-T =	0.
B (10+6 IN.)			B* (10+0 IN.)			B PRIME (10+6 1/LB.)			THERMAL MOMENT (LB./DEG.F.)	
-0.6240	0.	0.	0.1300	0.0000	0.	0.3005	0.0000	0.	M1-T =	-2.1840
0.	0.6240	0.	-0.0000	-0.4000	0.	0.0000	-76.9194	0.	M2-T =	2.1840
0.	0.	0.	0.	0.	0.	0.	0.	0.	M6-T =	0.
H (10+0 IN.)			H* (10+0 IN.)			D (10+6 LB.IN.)				
			-0.1000	0.0000	0.	0.3752	0.0000	0.		
			-0.0000	0.4000	0.	0.0000	0.2549	0.		
			0.	0.	0.	0.	0.	0.0000		
D (10+6 LB.IN.)			D* (10+6 LB.IN.)			D PRIME (10+6 1/LB.IN.)				
0.3752	0.0000	0.	0.3328	0.0000	0.	3.0048	-0.0000	0.		
0.0000	0.2549	0.	0.0000	0.0052	0.	-0.0000	192.2987	0.		
0.	0.	0.0000	0.	0.	0.0000	0.	0.	0.0000.0005		
Z (IN.)	STRESS COMPONENT	COEF. OF N1 (1/IN.)	COEF. OF N2 (1/IN.)	COEF. OF N6 (1/IN.)	COEF. OF M1 (1/IN.SQ.)	COEF. OF M2 (1/IN.SQ.)	COEF. OF M6 (1/IN.SQ.)	COEF. OF TEMP. (LB/IN.SQ/F.)		
-- LAYER 1 --										
-0.5000	SIGMA 1	0.3125	-0.0000	0.	-9.3750	0.0000	0.	-0.0000		
	2	0.0000	0.0001	0.	-0.0000	-0.0002	0.	-0.0000		
	6	0.	0.	1.0000	0.	0.	-6.0000	0.		
0.3000	SIGMA 1	2.1675	-0.0000	0.	9.3750	0.0000	0.	0.0000		
	2	0.0000	0.0000	0.	0.0000	-0.0000	0.	-0.0000		
	6	0.	0.	1.0000	0.	0.	3.0000	0.		
-- LAYER 2 --										
0.3000	SIGMA 1	0.0000	0.0000	0.	0.0000	-0.0000	0.	-0.0000		
	2	-0.0000	64.9970	0.	0.0000	-149.9926	0.	0.0005		
	6	0.	0.	1.0000	0.	0.	3.0000	0.		
0.5000	SIGMA 1	0.0000	-0.0000	0.	0.0000	0.0000	0.	-0.0000		
	2	0.0000	-54.9973	0.	-0.0000	149.9934	0.	-0.0004		
	6	0.	0.	1.0000	0.	0.	6.0000	0.		

CROSS-PLY M=4.0 CASE 2 (ALL LAYERS DEGRADED)
3 LAYERS (N=3)

STIFFNESS MATRIX (C)
(10*6 LB./IN.SQ.)

7.8000	0.0000	0.
0.0000	0.0000	0.
0.	0.	0.0000

-- ODD LAYERS --

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)

ALPHA 1 = 3.5000
ALPHA 2 = 11.4000
ALPHA 6 = 0.

STIFFNESS MATRIX (C)
(10*6 LB./IN.SQ.)

0.0000	0.0000	0.
0.0000	7.8000	0.
0.	0.	0.0000

-- EVEN LAYERS --

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)

ALPHA 1 = 11.4000
ALPHA 2 = 3.5000
ALPHA 6 = 0.

A (10*6 LB./IN.)	A* (10*6 IN./LB.)	A PRIME (10*6 IN./LB.)	THERMAL FORCE (LB./IN./DEG.F.)
6.2395 0.0000 0.	0.1503 -0.0000 0.	0.1603 -0.0000 0.	N1-T = 21.8381
0.0000 1.5605 0.	-0.0000 0.6408 0.	-0.0000 0.6408 0.	N2-T = 5.4619
0. 0. 0.0000	0. 0. 0.0000.0000	0. 0. 0.0000.0000	N6-T = 0.
B (10*6 IN.)	B* (10*0 IN.)	B PRIME (10*6 1/LB.)	THERMAL MOMENT (LB./DEG.F.)
-0.0001 0. 0.	0.0000 0.0000 0.	0.0000 0.0000 0.	M1-T = -0.0002
0. 0.0001 0.	-0.0000 -0.0000 0.	0.0000 -0.0067 0.	M2-T = 0.0002
0. 0. 0.	0. 0. 0.	0. 0. 0.	M6-T = 0.
D (10*6 LB.IN.)	D* (10*6 LB.IN.)	D PRIME (10*6 1/LB.IN.)	
0.6448 0.0000 0.	0.6448 0.0000 0.	1.5509 -0.0000 0.	
0.0000 0.0052 0.	0.0000 0.0052 0.	-0.0000 192.1029 0.	
0. 0. 0.0000	0. 0. 0.0000	0. 0. 0.0000.0005	

Z (IN.)	STRESS COMPONENT	COEF. OF N1 (1/IN.)	COEF. OF N2 (1/IN.)	COEF. OF N6 (1/IN.)	COEF. OF M1 (1/IN.SQ.)	COEF. OF M2 (1/IN.SQ.)	COEF. OF M6 (1/IN.SQ.)	COEF. OF TEMP. (LB/IN.SQ/F.)
-- LAYER 1 --								
-0.5000	SIGMA 1	1.2501	-0.0000	0.	-6.0483	0.0000	0.	0.0000
	2	0.0000	0.0000	0.	-0.0000	-0.0001	0.	-0.0000
	6	0.	0.	1.0000	0.	0.	-6.0000	0.
-0.1000	SIGMA 1	1.2501	-0.0000	0.	-1.2096	0.0000	0.	0.0000
	2	0.0000	0.0000	0.	-0.0000	-0.0000	0.	-0.0000
	6	0.	0.	1.0000	0.	0.	-1.2000	0.
-- LAYER 2 --								
-0.1000	SIGMA 1	0.0000	0.0000	0.	-0.0000	-0.0000	0.	-0.0000
	2	-0.0000	5.0035	0.	0.0000	-149.8927	0.	0.0000
	6	0.	0.	1.0000	0.	0.	-1.2000	0.
0.1001	SIGMA 1	0.0000	0.0000	0.	0.0000	0.0000	0.	-0.0000
	2	-0.0000	4.9930	0.	-0.0000	149.8927	0.	0.0000
	6	0.	0.	1.0000	0.	0.	1.2000	0.
-- LAYER 3 --								
0.1001	SIGMA 1	1.2501	-0.0000	0.	1.2106	-0.0000	0.	0.0000
	2	0.0000	0.0000	0.	0.0000	0.0000	0.	-0.0000
	6	0.	0.	1.0000	0.	0.	1.2000	0.
0.5000	SIGMA 1	1.2502	-0.0000	0.	6.0485	-0.0000	0.	0.0000
	2	0.0000	0.0000	0.	0.0000	0.0001	0.	-0.0000
	6	0.	0.	1.0000	0.	0.	6.0000	0.

ANGLE-PLY THETA = 5 DEGREES CASE 1 (ALL LAYERS INTACT)
2 LAYERS (N=2)

```

-- ODD LAYERS --
STIFFNESS MATRIX (C)
(10+6 LB./IN.SQ.)
7.8930 0.6962 -0.4140
0.6962 2.6630 -0.0471
-0.4140 -0.0471 1.2830

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)
ALPHA 1 = 3.5600
ALPHA 2 = 11.3400
ALPHA 6 = 0.6859

-- EVEN LAYERS --
STIFFNESS MATRIX (C)
(10+6 LB./IN.SQ.)
7.8930 0.6962 0.4140
0.6962 2.6630 0.0471
0.4140 0.0471 1.2830

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)
ALPHA 1 = 3.5600
ALPHA 2 = 11.3400
ALPHA 6 = -0.6859

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```

A
(10+6 LB./IN.)
7.8930 0.6962 0.
0.6962 2.6630 0.
0. 0. 1.2830

A*
(10-6 IN./LB.)
-0.1297 -0.0339 0.
-0.0339 0.3844 0.
0. 0. 0.7794

A PRIME
(10-6 IN./LB.)
0.1313 -0.0338 0.
-0.0338 0.3844 0.
0. 0. 0.7895

THERMAL FORCE
(LB./IN./DEG.F.)
N1-T = 35.7100
N2-T = 32.6446
N6-T = 0.0000

B
(10+6 IN.)
0. 0. 0.1035
0. 0. 0.0118
0.1035 0.0118 0.

B*
(10+6 IN.)
0. 0. -0.0130
0. 0. -0.0010
-0.0807 -0.0092 0.

B PRIME
(10-6 1/LB.)
0. 0. -0.1234
0. 0. -0.0096
-0.1234 -0.0096 0.

THERMAL MOMENT
(LB./DEG.F.)
M1-T = 0.
M2-T = 0.
M6-T = 0.2820

H*
(10+6 IN.)
0. 0. 0.0807
0. 0. 0.0092
0.0130 0.0010 0.

C
(10+6 LB./IN.)
0.6577 0.0580 0.
0.0580 0.2219 0.
0. 0. 0.1069

D*
(10+6 LB./IN.)
0.6494 0.0571 0.
0.0571 0.2218 0.
0. 0. 0.1056

D PRIME
(10-6 1/LB./IN.)
1.5755 -0.4053 0.
-0.4053 4.6127 0.
0. 0. 9.4736

Z
(IN.)
-0.5000
0.
0.
0.5000

STRESS COMPONENT
SIGMA 1
SIGMA 2
SIGMA 6

COEF. OF N1
(1/IN.)
0.9872
-0.0015
0.0264

COEF. OF N2
(1/IN.)
-0.0010
0.9999
0.0021

COEF. OF N6
(1/IN.)
0.1634
0.0186
0.9871

COEF. OF M1
(1/IN.SQ.)
-6.0255
-0.0029
0.1563

COEF. OF M2
(1/IN.SQ.)
-0.0020
-6.0002
0.0124

COEF. OF M6
(1/IN.SQ.)
0.9805
0.1116
-6.0258

COEF. OF TEMP.
(LB./IN.SQ./F.)
-0.2121
-0.0241
0.4381

-- LAYER 1 --
-0.5000 SIGMA 1 0.9872 -0.0010 0.1634 -6.0255 -0.0020 0.9805 -0.2121
2 -0.0015 0.9999 0.0186 -0.0029 -6.0002 0.1116 -0.0241
6 0.0264 0.0021 0.9871 0.1563 0.0124 -6.0258 0.4381

0. SIGMA 1 1.0128 0.0010 -0.3268 0.0511 -0.0040 -0.9805 0.2121
2 0.0015 1.0001 -0.0372 0.0058 0.0005 -0.1116 0.0241
6 -0.0528 -0.0041 1.0129 -0.1583 -0.0124 0.0515 -0.8763

-- LAYER 2 --
0. SIGMA 1 1.0128 0.0010 0.3268 -0.0511 -0.0040 -0.9805 0.2121
2 0.0015 1.0001 0.0372 -0.0058 -0.0005 -0.1116 0.0241
6 0.0528 0.0041 1.0129 -0.1583 -0.0124 -0.0515 0.8763

0.5000 SIGMA 1 0.9872 -0.0010 -0.1634 6.0255 0.0020 0.9805 -0.2121
2 -0.0015 0.9999 -0.0186 0.0029 6.0002 0.1116 -0.0241
6 -0.0264 -0.0021 0.9871 0.1583 0.0124 6.0258 -0.4381

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ANGLE-PLY THETA = 5 DEGREES CASE 1 (ALL LAYERS INTACT)
3 LAYERS (N=3)

STIFFNESS MATRIX (C) (10+6 LB./IN.SQ.)			-- ODD LAYERS --	THERMAL EXPANSION MATRIX (ALPHA) (IN./IN./DEG.F.)		
7.8930	0.6962	-0.4140		ALPHA 1 =	3.5600	
0.6962	2.6630	-0.0471		ALPHA 2 =	11.3400	
-0.4140	-0.0471	1.2830		ALPHA 6 =	0.6859	

STIFFNESS MATRIX (C) (10+6 LB./IN.SQ.)			-- EVEN LAYERS --	THERMAL EXPANSION MATRIX (ALPHA) (IN./IN./DEG.F.)		
7.8930	0.6962	0.4140		ALPHA 1 =	3.5600	
0.6962	2.6630	0.0471		ALPHA 2 =	11.3400	
0.4140	0.0471	1.2830		ALPHA 6 =	-0.6859	

A (10+6 LB./IN.)			A* (10-6 IN./LB.)			A PRIME (10-6 IN./LB.)			THERMAL FORCE (LB./IN./DEG.F.)		
7.8930	0.6962	-0.1379	0.1299	-0.0339	0.0136	0.1299	-0.0339	0.0136	N1-T =	35.7100	
0.6962	2.6630	-0.0157	-0.0339	0.3844	0.0011	-0.0339	0.3844	0.0011	N2-T =	32.6446	
-0.1379	-0.0157	1.2830	0.0136	0.0011	0.7809	0.0136	0.0011	0.7809	N6-T =	-0.3758	

B (10+6 IN.)			B* (10+0 IN.)			B PRIME (10-6 1/LB.)			THERMAL MOMENT (LB./DEG.F.)		
0.	0.	0.	0.	0.	0.	0.	0.	0.	M1-T =	-0.0000	
0.	0.	0.	0.	0.	0.	0.	0.	0.	M2-T =	-0.0000	
0.	0.	0.	0.	0.	0.	0.	0.	0.	M6-T =	0.	

H* (10+0 IN.)			C (10+6 LB.IN.)			D* (10+6 LB.IN.)			C PRIME (10-6 1/LB.IN.)		
0.	0.	0.	0.6577	0.0580	-0.0319	0.6577	0.0580	-0.0319	1.5783	-0.4051	0.4578
0.	0.	0.	0.0580	0.2219	-0.0036	0.0580	0.2219	-0.0036	-0.4051	4.6127	0.0357
0.	0.	0.	-0.0319	-0.0036	0.1069	-0.0319	-0.0036	0.1069	0.4578	0.0357	9.4911

Z (IN.)	STRESS COMPONENT	COEF. OF N1 (1/IN.)	COEF. OF N2 (1/IN.)	COEF. OF N6 (1/IN.)	COEF. OF M1 (1/IN.SQ.)	COEF. OF M2 (1/IN.SQ.)	COEF. OF M6 (1/IN.SQ.)	COEF. OF TEMP. (LB/IN.SQ/F.)
-- LAYER 1 --								
-0.5000	SIGMA 1	0.9963	-0.0003	-0.2156	-5.9930	0.0005	0.1456	-0.0621
	2	-0.0004	1.0000	-0.0245	0.0008	-5.9999	0.0166	-0.0071
	6	-0.0348	-0.0027	0.9962	0.0235	0.0018	-5.9929	-0.5780
-0.1667	SIGMA 1	0.9963	-0.0003	-0.2156	-1.9981	0.0002	0.0485	-0.0621
	2	-0.0004	1.0000	-0.0245	0.0003	-2.0004	0.0055	-0.0071
	6	-0.0348	-0.0027	0.9962	0.0078	0.0006	-1.9980	-0.5780
-- LAYER 2 --								
-0.1667	SIGMA 1	1.0075	0.0006	0.4310	-2.0612	-0.0048	-1.2615	0.1242
	2	0.0009	1.0001	0.0490	-0.0069	-2.0009	-0.1435	0.0141
	6	0.0696	0.0054	1.0075	-0.2036	-0.0159	-2.0618	1.1556
0.1667	SIGMA 1	1.0075	0.0006	0.4310	2.0612	0.0048	1.2615	0.1242
	2	0.0009	1.0001	0.0490	0.0069	2.0009	0.1435	0.0141
	6	0.0696	0.0054	1.0075	0.2036	0.0159	2.0618	1.1556
-- LAYER 3 --								
0.1667	SIGMA 1	0.9963	-0.0003	-0.2156	1.9981	-0.0002	-0.0485	-0.0621
	2	-0.0004	1.0000	-0.0245	-0.0003	2.0004	-0.0055	-0.0071
	6	-0.0348	-0.0027	0.9962	-0.0078	-0.0006	1.9980	-0.5780
0.5000	SIGMA 1	0.9963	-0.0003	-0.2156	5.9930	-0.0005	-0.1456	-0.0621
	2	-0.0004	1.0000	-0.0245	-0.0008	5.9999	-0.0166	-0.0071
	6	-0.0348	-0.0027	0.9962	-0.0235	-0.0018	5.9929	-0.5780

ANGLE-PLY THETA = 10 DEGREES CASE 1 (ALL LAYERS INTACT)
2 LAYERS (N=2)

STIFFNESS MATRIX (C) (10+6 LB./IN.SQ.)			-- ODD LAYERS --		THERMAL EXPANSION MATRIX (ALPHA) (IN./IN./DEG.F.)		
7.6800	0.7893	-0.7989			ALPHA 1 =	3.7382	
0.7893	2.6900	-0.1093			ALPHA 2 =	11.1620	
-0.7989	-0.1093	1.3760			ALPHA 6 =	1.3510	
STIFFNESS MATRIX (C) (10+6 LB./IN.SQ.)			-- EVEN LAYERS --		THERMAL EXPANSION MATRIX (ALPHA) (IN./IN./DEG.F.)		
7.6800	0.7893	0.7989			ALPHA 1 =	3.7382	
0.7893	2.6900	0.1093			ALPHA 2 =	11.1620	
0.7989	0.1093	1.3760			ALPHA 6 =	-1.3510	

A (10+6 LB./IN.)			A* (10-6 IN./LB.)			A PRIME (10-6 IN./LB.)			THERMAL FORCE (LB./IN./DEG.F.)	
7.6800	0.7893	0.	0.1343	-0.0394	0.	0.1403	-0.0388	0.	N1-T =	36.4402
0.7893	2.6900	0.	-0.0394	0.3833	0.	-0.0388	0.3834	0.	N2-T =	32.8287
0.	0.	1.3760	0.	0.	0.7267	0.	0.	0.7613	N6-T =	0.0000
B (10+6 IN.)			B* (10+0 IN.)			B PRIME (10-6 1/LB.)			THERMAL MOMENT (LB./DEG.F.)	
0.	0.	0.1997	0.	0.	-0.0257	0.	0.	-0.2351	M1-T =	0.0000
0.	0.	0.0273	0.	0.	-0.0026	0.	0.	-0.0238	M2-T =	0.
0.1997	0.0273	0.	-0.1451	-0.0199	0.	-0.2351	-0.0238	0.	M6-T =	0.5869
C (10+6 LB./IN.)			D* (10+6 LB./IN.)			C PRIME (10-6 1/LB./IN.)				
0.6400	0.0658	0.	0.6110	0.0618	0.	1.6837	-0.4654	0.		
0.0658	0.2242	0.	0.0618	0.2236	0.	-0.4654	4.6004	0.		
0.	0.	0.1147	0.	0.	0.1095	0.	0.	9.1362		
Z (IN.)	STRESS COMPONENT	COEF. OF N1 (1/IN.)	COEF. OF N2 (1/IN.)	COEF. OF N6 (1/IN.)	COEF. OF M1 (1/IN.SQ.)	COEF. OF M2 (1/IN.SQ.)	COEF. OF M6 (1/IN.SQ.)	COEF. OF TEMP. (LB/IN.SQ/F.)		
-- LAYER 1 --										
-0.5000	SIGMA	1	0.9530	-0.0048	0.3041	-6.0939	-0.0095	1.8247	-0.7966	
		2	-0.0064	0.9993	0.0416	-0.0129	-6.0013	0.2496	-0.1090	
		6	0.0539	0.0055	0.9524	0.3236	0.0328	-6.0952	0.9147	
0.	SIGMA	1	1.0470	0.0048	-0.6082	0.1879	0.0190	-1.8247	0.7966	
		2	0.0064	1.0007	-0.0832	0.0257	0.0026	-0.2496	0.1090	
		6	-0.1079	-0.0109	1.0476	-0.3236	-0.0328	0.1905	-1.8295	
-- LAYER 2 --										
0.	SIGMA	1	1.0470	0.0048	0.6082	-0.1879	-0.0190	-1.8247	0.7966	
		2	0.0064	1.0007	0.0832	-0.0257	-0.0026	-0.2496	0.1090	
		6	0.1079	0.0109	1.0476	-0.3236	-0.0328	-0.1905	1.8295	
0.5000	SIGMA	1	0.9530	-0.0048	-0.3041	6.0939	0.0095	1.8247	-0.7966	
		2	-0.0064	0.9993	-0.0416	0.0129	6.0013	0.2496	-0.1090	
		6	-0.0539	-0.0055	0.9524	0.3236	0.0328	6.0952	-0.9147	

ANGLE-PLY THETA = 10 DEGREES CASE 1 (ALL LAYERS INTACT)
3 LAYERS (N=3)

STIFFNESS MATRIX (C)
(10+6 LB./IN.SQ.)

7.6800 0.7893 -0.7989
0.7893 2.6900 -0.1093
-0.7989 -0.1093 1.3760

-- ODD LAYERS --

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)

ALPHA 1 = 3.7382
ALPHA 2 = 11.1620
ALPHA 6 = 1.3510

STIFFNESS MATRIX (C)
(10+6 LB./IN.SQ.)

7.6800 0.7893 0.7989
0.7893 2.6900 0.1093
0.7989 0.1093 1.3760

-- EVEN LAYERS --

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)

ALPHA 1 = 3.7382
ALPHA 2 = 11.1620
ALPHA 6 = -1.3510

A (10+6 LB./IN.)			A* (10-6 IN./LB.)			A PRIME (10-6 IN./LB.)			THERMAL FORCE (LB./IN./DEG.F.)
7.6800	0.7893	-0.2662	0.1351	-0.0393	0.0251	0.1351	-0.0393	0.0251	N1-T = 36.4402
0.7893	2.6900	-0.0364	-0.0393	0.3833	0.0025	-0.0393	0.3833	0.0025	N2-T = 32.8287
-0.2662	-0.0364	1.3760	0.0251	0.0025	0.7317	0.0251	0.0025	0.7317	N6-T = -0.7822
B (10+6 IN.)			B* (10+0 IN.)			B PRIME (10-6 1/LB.)			THERMAL MOMENT (LB./DEG.F.)
0.	0.	0.	0.	0.	0.	0.	0.	0.	M1-T = -0.0000
0.	0.	0.	0.	0.	0.	0.	0.	0.	M2-T = -0.0000
0.	0.	0.	0.	0.	0.	0.	0.	0.	M6-T = 0.0000
C (10+6 LB.IN.)			D* (10+6 LB.IN.)			D PRIME (10-6 1/LB.IN.)			
0.6400	0.0658	-0.0616	0.6400	0.0658	-0.0616	1.6947	-0.4643	0.8768	
0.0658	0.2242	-0.0084	0.0658	0.2242	-0.0084	-0.4643	4.6005	0.0888	
-0.0616	-0.0084	0.1147	-0.0616	-0.0084	0.1147	0.8768	0.0888	9.1988	
Z (IN.)	STRESS COMPONENT	COEF. OF N1 (1/IN.)	COEF. OF N2 (1/IN.)	COEF. OF N6 (1/IN.)	COEF. OF M1 (1/IN.SQ.)	COEF. OF M2 (1/IN.SQ.)	COEF. OF M6 (1/IN.SQ.)	COEF. OF TEMP. (LB/IN.SQ/F.)	
-- LAYER 1 --									
-0.5000	SIGMA 1	0.9866	-0.0014	-0.3898	-5.9740	0.0026	0.2723	-0.2268	
	2	-0.0018	0.9998	-0.0533	0.0036	-5.9996	0.0373	-0.0310	
	6	-0.0691	-0.0070	0.9864	0.0483	0.0049	-5.9737	-1.1723	
-0.1667	SIGMA 1	0.9866	-0.0014	-0.3898	-1.9917	0.0009	0.0908	-0.2268	
	2	-0.0018	0.9998	-0.0533	0.0012	-2.0003	0.0124	-0.0310	
	6	-0.0691	-0.0070	0.9864	0.0161	0.0016	-1.9916	-1.1723	
-- LAYER 2 --									
-0.1667	SIGMA 1	1.0267	0.0027	0.7793	-2.2253	-0.0228	-2.3593	0.4535	
	2	0.0037	1.0004	0.1066	-0.0308	-2.0035	-0.3228	0.0620	
	6	0.1382	0.0140	1.0271	-0.4184	-0.0424	-2.2284	2.3440	
0.1667	SIGMA 1	1.0267	0.0027	0.7793	2.2253	0.0228	2.3593	0.4535	
	2	0.0037	1.0004	0.1066	0.0308	2.0035	0.3228	0.0620	
	6	0.1382	0.0140	1.0271	0.4184	0.0424	2.2284	2.3440	
-- LAYER 3 --									
0.1667	SIGMA 1	0.9866	-0.0014	-0.3898	1.9917	-0.0009	-0.0908	-0.2268	
	2	-0.0018	0.9998	-0.0533	-0.0012	2.0003	-0.0124	-0.0310	
	6	-0.0691	-0.0070	0.9864	-0.0161	-0.0016	1.9916	-1.1723	
0.5000	SIGMA 1	0.9866	-0.0014	-0.3898	5.9740	-0.0026	-0.2723	-0.2268	
	2	-0.0018	0.9998	-0.0533	-0.0036	5.9996	-0.0373	-0.0310	
	6	-0.0691	-0.0070	0.9864	-0.0483	-0.0049	5.9737	-1.1723	

ANGLE-PLY THETA = 15 DEGREES CASE 1 (ALL LAYERS INTACT)
2 LAYERS (N=2)

STIFFNESS MATRIX (C) (10+6 LB./IN.SQ.)	-- ODD LAYERS --	THERMAL EXPANSION MATRIX (ALPHA) (IN./IN./DEG.F.)
7.3420 0.9320 -1.1290		ALPHA 1 = 4.0292
0.9320 2.7430 -0.1993		ALPHA 2 = 10.8700
-1.1290 -0.1993 1.5190		ALPHA 6 = 1.9750

STIFFNESS MATRIX (C) (10+6 LB./IN.SQ.)	-- EVEN LAYERS --	THERMAL EXPANSION MATRIX (ALPHA) (IN./IN./DEG.F.)
7.3420 0.9320 1.1290		ALPHA 1 = 4.0292
0.9320 2.7430 0.1993		ALPHA 2 = 10.8700
1.1290 0.1993 1.5190		ALPHA 6 = -1.9750

A (10+6 LB./IN.)	A*	A PRIME (10-6 IN./LB.)	THERMAL FORCE (LB./IN./DEG.F.)
7.3420 0.9320 0.	0.1423 -0.0484 0.	0.1547 -0.0466 0.	N1-T = 37.4835
0.9320 2.7430 0.	-0.0484 0.3810 0.	-0.0466 0.3812 0.	N2-T = 33.1780
0. 0. 1.5190	0. 0. 0.6583	0. 0. 0.7205	N6-T = 0.

B (10+6 IN.)	B*	B PRIME (10-6 1/LB.)	THERMAL MOMENT (LB./DEG.F.)
0. 0. 0.2822	0. 0. -0.0378	0. 0. -0.3265	M1-T = 0.0000
0. 0. 0.0498	0. 0. -0.0053	0. 0. -0.0461	M2-T = 0.0000
0.2822 0.0498 0.	-0.1856 -0.0328 0.	-0.3265 -0.0461 0.	M6-T = 0.9288

H*
(10+0 IN.)
0. 0. 0.1858
0. 0. 0.0328
0.0378 0.0053 0.

C (10+6 LB./IN.)	C*	C PRIME (10-6 1/LB./IN.)
0.6118 0.0777 0.	0.5594 0.0684 0.	1.8561 -0.5595 0.
0.0777 0.2286 0.	0.0684 0.2269 0.	-0.5595 4.5749 0.
0. 0. 0.1266	0. 0. 0.1157	0. 0. 8.6462

Z (IN.)	STRESS COMPONENT	COEF. OF N1 (1/IN.)	COEF. OF N2 (1/IN.)	COEF. OF N6 (1/IN.)	COEF. OF M1 (1/IN.SQ.)	COEF. OF M2 (1/IN.SQ.)	COEF. OF M6 (1/IN.SQ.)	COEF. OF TEMP. (LB./IN.SQ./F.)
-- LAYER 1 --								
-0.5000	SIGMA 1	0.9078	-0.0130	0.4067	-6.1843	-0.0260	2.4404	-1.6197
	2	-0.0163	0.9977	0.0718	-0.0325	-6.0046	0.4308	-0.2859
	6	0.0827	0.0117	0.9055	0.4960	0.0700	-6.1889	1.4528
0.	SIGMA 1	1.0922	0.0130	-0.8135	0.3687	0.0521	-2.4404	1.6197
	2	0.0163	1.0023	-0.1436	0.0651	0.0092	-0.4308	0.2859
	6	-0.1653	-0.0233	1.0945	-0.4960	-0.0700	0.3778	-2.9056
-- LAYER 2 --								
0.	SIGMA 1	1.0922	0.0130	0.8135	-0.3687	-0.0521	-2.4404	1.6197
	2	0.0163	1.0023	0.1436	-0.0651	-0.0092	-0.4308	0.2859
	6	0.1653	0.0233	1.0945	-0.4960	-0.0700	-0.3778	2.9056
0.5000	SIGMA 1	0.9078	-0.0130	-0.4067	6.1843	0.0260	2.4404	-1.6197
	2	-0.0163	0.9977	-0.0718	0.0325	6.0046	0.4308	-0.2859
	6	-0.0827	-0.0117	0.9055	0.4960	0.0700	6.1889	-1.4528

ANGLE-PLY THETA = 15 DEGREES CASE 1 (ALL LAYERS INTACT)
3 LAYERS (N=3)

STIFFNESS MATRIX (C) (10+6 LB./IN.SQ.)			-- ODD LAYERS --	THERMAL EXPANSION MATRIX (ALPHA) (IN./IN./DEG.F.)		
7.3420	0.9320	-1.1290		ALPHA 1 =	4.0292	
0.9320	2.7430	-0.1993		ALPHA 2 =	10.8700	
-1.1290	-0.1993	1.5190		ALPHA 6 =	1.9750	
STIFFNESS MATRIX (C) (10+6 LB./IN.SQ.)			-- EVEN LAYERS --	THERMAL EXPANSION MATRIX (ALPHA) (IN./IN./DEG.F.)		
7.3420	0.9320	1.1290		ALPHA 1 =	4.0292	
0.9320	2.7430	0.1993		ALPHA 2 =	10.8700	
1.1290	0.1993	1.5190		ALPHA 6 =	-1.9750	

A (10+6 LB./IN.)			A* (10-6 IN./LB.)			A PRIME (10-6 IN./LB.)			THERMAL FORCE (LB./IN./DEG.F.)		
7.3420	0.9320	-0.3762	0.1440	-0.0481	0.0336	0.1440	-0.0481	0.0336	N1-T =	37.4835	
0.9320	2.7430	-0.0664	-0.0481	0.3810	0.0047	-0.0481	0.3810	0.0047	N2-T =	33.1780	
-0.3762	-0.0664	1.5190	0.0336	0.0047	0.6668	0.0336	0.0047	0.6668	N6-T =	-1.2379	
B (10+6 IN.)			B* (10+0 IN.)			B PRIME (10-6 1/LB.)			THERMAL MOMENT (LB./DEG.F.)		
0.	0.	0.	0.	0.	0.	0.	0.	0.	M1-T =	-0.0000	
0.	0.	0.	0.	0.	0.	0.	0.	0.	M2-T =	-0.0000	
0.	0.	0.	0.	0.	0.	0.	0.	0.	M6-T =	0.0000	
C (10+6 LB./IN.)			D* (10+6 LB./IN.)			D PRIME (10-6 1/LB./IN.)					
0.6118	0.0777	-0.0871	0.6118	0.0777	-0.0871	1.8796	-0.5562	1.2259			
0.0777	0.2286	-0.0154	0.0777	0.2286	-0.0154	-0.5562	4.5754	0.1731			
-0.0871	-0.0154	0.1266	-0.0871	-0.0154	0.1266	1.2259	0.1731	8.7646			

Z (IN.)	STRESS COMPONENT	COEF. OF N1 (1/IN.)	COEF. OF N2 (1/IN.)	COEF. OF N6 (1/IN.)	COEF. OF M1 (1/IN.SQ.)	COEF. OF M2 (1/IN.SQ.)	COEF. OF M6 (1/IN.SQ.)	COEF. OF TEMP. (LB./IN.SQ./F.)
-- LAYER 1 --								
-0.5000	SIGMA 1	0.9747	-0.0036	-0.5020	-5.9487	0.0072	0.3667	-0.4441
	2	-0.0045	0.9994	-0.0886	0.0091	-5.9987	0.0647	-0.0784
	6	-0.1020	-0.0144	0.9741	0.0745	0.0105	-5.6474	-1.7931
-0.1667	SIGMA 1	0.9747	-0.0036	-0.5020	-1.9833	0.0024	0.1223	-0.4441
	2	-0.0045	0.9994	-0.0886	0.0030	-2.0000	0.0216	-0.0784
	6	-0.1020	-0.0144	0.9741	0.0248	0.0035	-1.9829	-1.7931
-- LAYER 2 --								
-0.1667	SIGMA 1	1.0505	0.0071	1.0037	-2.4447	-0.0627	-3.1768	0.8879
	2	0.0089	1.0013	0.1772	-0.0784	-2.0115	-0.5608	0.1567
	6	0.2040	0.0288	1.0518	-0.6457	-0.0912	-2.4558	3.5852
0.1667	SIGMA 1	1.0505	0.0071	1.0037	2.4447	0.0627	3.1768	0.8879
	2	0.0089	1.0013	0.1772	0.0784	2.0115	0.5608	0.1567
	6	0.2040	0.0288	1.0518	0.6457	0.0912	2.4558	3.5852
-- LAYER 3 --								
0.1667	SIGMA 1	0.9747	-0.0036	-0.5020	1.9833	-0.0024	-0.1223	-0.4441
	2	-0.0045	0.9994	-0.0886	-0.0030	2.0000	-0.0216	-0.0784
	6	-0.1020	-0.0144	0.9741	-0.0248	-0.0035	1.9829	-1.7931
0.5000	SIGMA 1	0.9747	-0.0036	-0.5020	5.9487	-0.0072	-0.3667	-0.4441
	2	-0.0045	0.9994	-0.0886	-0.0091	5.9987	-0.0647	-0.0784
	6	-0.1020	-0.0144	0.9741	-0.0745	-0.0105	5.6474	-1.7931

ANGLE-PLY THETA = 30 DEGREES CASE 1 (ALL LAYERS INTACT)
2 LAYERS (N=2)

STIFFNESS MATRIX (C)
(10+6 LB./IN.SQ.)

5.8340	1.4690	-1.6150
1.4690	3.1780	-0.6852
-1.6150	-0.6852	2.0550

-- ODD LAYERS --

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)

ALPHA 1 = 5.4750
ALPHA 2 = 9.4250
ALPHA 6 = 3.4208

STIFFNESS MATRIX (C)
(10+6 LB./IN.SQ.)

5.8340	1.4690	1.6150
1.4690	3.1780	0.6852
1.6150	0.6852	2.0550

-- EVEN LAYERS --

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)

ALPHA 1 = 5.4750
ALPHA 2 = 9.4250
ALPHA 6 = -3.4208

A (10+6 LB./IN.)			A* (10-6 IN./LB.)			A PRIME (10-6 IN./LB.)			THERMAL FORCE (LB./IN./DEG.F.)
5.8340	1.4690	0.	0.1940	-0.0897	0.	0.2220	-0.0786	0.	M1-T = 40.2619
1.4690	3.1780	0.	-0.0897	0.3561	0.	-0.0786	0.3605	0.	M2-T = 35.6515
0.	0.	2.0550	0.	0.	0.4866	0.	0.	0.5886	M6-T = 0.0000

B (10+6 IN.)			B* (10+0 IN.)			B PRIME (10-6 1/LB.)			THERMAL MOMENT (LB./DEG.F.)
0.	0.	0.4037	0.	0.	-0.0630	0.	0.	-0.4447	M1-T = 0.0000
0.	0.	0.1713	0.	0.	-0.0248	0.	0.	-0.1752	M2-T = 0.0000
0.4037	0.1713	0.	-0.1965	-0.0834	0.	-0.4447	-0.1752	0.	M6-T = 2.0676

H* (10+0 IN.)		
0.	0.	0.1965
0.	0.	0.0834
0.0630	0.0248	0.

C (10+6 LB.IN.)			D* (10+6 LB.IN.)			D PRIME (10-6 1/LB.IN.)		
0.4862	0.1224	0.	0.4068	0.0888	0.	2.6638	-0.9437	0.
0.1224	0.2648	0.	0.0888	0.2506	0.	-0.9437	4.3255	0.
0.	0.	0.1712	0.	0.	0.1416	0.	0.	7.0631

Z (IN.)	STRESS COMPONENT	COEF. OF N1 (1/IN.)	COEF. OF N2 (1/IN.)	COEF. OF N6 (1/IN.)	COEF. OF M1 (1/IN.SQ.)	COEF. OF M2 (1/IN.SQ.)	COEF. OF M6 (1/IN.SQ.)	COEF. OF TEMP. (LB/IN.SQ/F.)
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-- LAYER 1 --

-0.5000	SIGMA 1	0.8204	-0.0707	0.4753	-6.3591	-0.1414	2.8517	-3.8540
	2	-0.0762	0.9700	0.2017	-0.1524	-0.0600	1.2099	-1.6352
	6	0.1523	0.0600	0.7904	0.9139	0.3599	-6.4191	3.2694
0.	SIGMA 1	1.1796	0.0707	-0.9506	0.7182	0.2829	-2.8517	3.8540
	2	0.0762	1.0300	-0.4033	0.3047	0.1200	-1.2099	1.6352
	6	-0.3046	-0.1200	1.2096	-0.9139	-0.3599	0.8382	-6.5387

-- LAYER 2 --

0.	SIGMA 1	1.1796	0.0707	0.9506	-0.7182	-0.2829	-2.8517	3.8540
	2	0.0762	1.0300	0.4033	-0.3047	-0.1200	-1.2099	1.6352
	6	0.3046	0.1200	1.2096	-0.9139	-0.3599	-0.8382	6.5387
0.5000	SIGMA 1	0.8204	-0.0707	-0.4753	6.3591	0.1414	2.8517	-3.8540
	2	-0.0762	0.9700	-0.2017	0.1524	0.0600	1.2099	-1.6352
	6	-0.1523	-0.0600	0.7904	0.9139	0.3599	6.4191	-3.2694

ANGLE-PLY THETA = 30 DEGREES CASE 1 (ALL LAYERS INTACT)
3 LAYERS (N=3)

STIFFNESS MATRIX (C) (10+6 LB./IN.SQ.)			-- ODD LAYERS --	THERMAL EXPANSION MATRIX (ALPHA) (IN./IN./DEG.F.)		
5.8340	1.4690	-1.6150		ALPHA 1 =	5.4750	
1.4690	3.1780	-0.6852		ALPHA 2 =	9.4250	
-1.6150	-0.6852	2.0550		ALPHA 6 =	3.4208	
STIFFNESS MATRIX (C) (10+6 LB./IN.SQ.)			-- EVEN LAYERS --	THERMAL EXPANSION MATRIX (ALPHA) (IN./IN./DEG.F.)		
5.8340	1.4690	1.6150		ALPHA 1 =	5.4750	
1.4690	3.1780	0.6852		ALPHA 2 =	9.4250	
1.6150	0.6852	2.0550		ALPHA 6 =	-3.4208	

A (10+6 LB./IN.)			A* (10-6 IN./LB.)			A PRIME (10-6 IN./LB.)			THERMAL FORCE (LB./IN./DEG.F.)		
5.8340	1.4690	-0.5381	0.1975	-0.0883	0.0419	0.1975	-0.0883	0.0419	N1-T =	40.2619	
1.4690	3.1780	-0.2283	-0.0883	0.3567	0.0165	-0.0883	0.3567	0.0165	N2-T =	35.6515	
-0.5381	-0.2283	2.0550	0.0419	0.0165	0.4994	0.0419	0.0165	0.4994	N6-T =	-2.7557	
B (10+6 IN.)			B* (10+0 IN.)			B PRIME (10-6 1/LB.)			THERMAL MOMENT (LB./DEG.F.)		
0.	0.	0.	0.	0.	0.	0.	0.	0.	M1-T =	-0.0000	
0.	0.	0.	0.	0.	0.	0.	0.	0.	M2-T =	-0.0000	
0.	0.	0.	0.	0.	0.	0.	0.	0.	M6-T =	0.0000	
C (10+6 LB.IN.)			D* (10+6 LB.IN.)			D PRIME (10-6 1/LB.IN.)					
0.4862	0.1224	-0.1246	0.4862	0.1224	-0.1246	2.7238	-0.9201	1.6979			
0.1224	0.2648	-0.0529	0.1224	0.2648	-0.0529	-0.9201	4.3348	0.6687			
-0.1246	-0.0529	0.1712	-0.1246	-0.0529	0.1712	1.6979	0.6687	7.2813			

Z (IN.)	STRESS COMPONENT	COEF. OF N1 (1/IN.)	COEF. OF N2 (1/IN.)	COEF. OF N6 (1/IN.)	COEF. OF M1 (1/IN.SQ.)	COEF. OF M2 (1/IN.SQ.)	COEF. OF M6 (1/IN.SQ.)	COEF. OF TEM (LB/IN.SQ/F.)
-- LAYER 1 --								
-0.5000	SIGMA 1	0.9549	-0.0178	-0.5378	-5.8984	0.0400	0.4358	-0.9687
	2	-0.0191	0.9925	-0.2282	0.0431	-5.9830	0.1849	-0.4110
	6	-0.1724	-0.0679	0.9473	0.1397	0.0550	-5.8814	-3.6995
-0.1667	SIGMA 1	0.9549	-0.0178	-0.5378	-1.9665	0.0133	0.1453	-0.9687
	2	-0.0191	0.9925	-0.2282	0.0144	-1.9947	0.0616	-0.4110
	6	-0.1724	-0.0679	0.9473	0.0466	0.0183	-1.9609	-3.6995
-- LAYER 2 --								
-0.1667	SIGMA 1	1.0902	0.0355	1.0753	-2.8807	-0.3467	-3.7753	1.9369
	2	0.0383	1.0151	0.4562	-0.3735	-2.1475	-1.6017	0.8218
	6	0.3446	0.1357	1.1053	-1.2098	-0.4765	-3.0278	7.3968
0.1667	SIGMA 1	1.0902	0.0355	1.0753	2.8807	0.3467	3.7753	1.9369
	2	0.0383	1.0151	0.4562	0.3735	2.1475	1.6017	0.8218
	6	0.3446	0.1357	1.1053	1.2098	0.4765	3.0278	7.3968
-- LAYER 3 --								
0.1667	SIGMA 1	0.9549	-0.0178	-0.5378	1.9665	-0.0133	-0.1453	-0.9687
	2	-0.0191	0.9925	-0.2282	-0.0144	1.9947	-0.0616	-0.4110
	6	-0.1724	-0.0679	0.9473	-0.0466	-0.0183	1.9609	-3.6995
0.5000	SIGMA 1	0.9549	-0.0178	-0.5378	5.8984	-0.0400	-0.4358	-0.9687
	2	-0.0191	0.9925	-0.2282	-0.0431	5.9830	-0.1849	-0.4110
	6	-0.1724	-0.0679	0.9473	-0.1397	-0.0550	5.8814	-3.6995

ANGLE-PLY THETA = 45 DEGREES CASE 1 (ALL LAYERS INTACT)
2 LAYERS (N=2)

STIFFNESS MATRIX (C) (10+6 LB./IN.SQ.)			-- ODD LAYERS --	THERMAL EXPANSION MATRIX (ALPHA) (IN./IN./DEG.F.)		
4.2380	1.7370	-1.3280		ALPHA 1 =	7.4500	
1.7370	4.2380	-1.3280		ALPHA 2 =	7.4500	
-1.3280	-1.3280	2.3230		ALPHA 6 =	3.9500	
STIFFNESS MATRIX (C) (10+6 LB./IN.SQ.)			-- EVEN LAYERS --	THERMAL EXPANSION MATRIX (ALPHA) (IN./IN./DEG.F.)		
4.2380	1.7370	1.3280		ALPHA 1 =	7.4500	
1.7370	4.2380	1.3280		ALPHA 2 =	7.4500	
1.3280	1.3280	2.3230		ALPHA 6 =	-3.9500	

A (10+6 LB./IN.)			A* (10-6 IN./LB.)			A PRIME (10-6 IN./LB.)			THERMAL FORCE (LB./IN./DEG.F.)	
4.2380	1.7370	0.	0.2836	-0.1162	0.	0.3033	-0.0965	0.	N1-T =	39.2681
1.7370	4.2380	0.	-0.1162	0.2836	0.	-0.0965	0.3033	0.	N2-T =	39.2681
0.	0.	2.3230	0.	0.	0.4305	0.	0.	0.5318	N6-T =	0.
B (10+6 IN.)			B* (10+0 IN.)			B PRIME (10-6 1/LB.)			THERMAL MOMENT (LB./DEG.F.)	
0.	0.	0.3320	0.	0.	-0.0556	0.	0.	-0.3546	M1-T =	0.0000
0.	0.	0.3320	0.	0.	-0.0556	0.	0.	-0.3546	M2-T =	0.
0.3320	0.3320	0.	-0.1429	-0.1429	0.	-0.3546	-0.3546	0.	M6-T =	2.6528
C (10+6 LB.IN.)			C* (10+0 LB.IN.)			C PRIME (10-6 1/LB.IN.)				
0.3532	0.1447	0.	0.3057	0.0973	0.	3.6397	-1.1584	0.		
0.1447	0.3532	0.	0.0973	0.3057	0.	-1.1584	3.6397	0.		
0.	0.	0.1936	0.	0.	0.1567	0.	0.	6.3821		
Z (IN.)	STRESS COMPONENT	COEF. OF N1 (1/IN.)	COEF. OF N2 (1/IN.)	COEF. OF N6 (1/IN.)	COEF. OF M1 (1/IN.SQ.)	COEF. OF M2 (1/IN.SQ.)	COEF. OF M6 (1/IN.SQ.)	COEF. OF TEMP (LB/IN.SQ/F.)		
-- LAYER 1 --										
-0.5000	SIGMA 1	0.8823	-0.1177	0.3531	-6.2355	-0.2355	2.1189	-3.6254		
	2	-0.1177	0.8823	0.3531	-0.2355	-6.2355	2.1189	-3.6254		
	6	0.1373	0.1373	0.7645	0.8238	0.8238	-6.4709	4.2278		
0.	SIGMA 1	1.1177	0.1177	-0.7063	0.4709	0.4709	-2.1189	3.6254		
	2	0.1177	1.1177	-0.7063	0.4709	0.4709	-2.1189	3.6254		
	6	-0.2746	-0.2746	1.2355	-0.8238	-0.8238	0.9419	-8.4556		
-- LAYER 2 --										
0.	SIGMA 1	1.1177	0.1177	0.7063	-0.4709	-0.4709	-2.1189	3.6254		
	2	0.1177	1.1177	0.7063	-0.4709	-0.4709	-2.1189	3.6254		
	6	0.2746	0.2746	1.2355	-0.8238	-0.8238	-0.9419	8.4556		
0.5000	SIGMA 1	0.8823	-0.1177	-0.3531	6.2355	0.2355	2.1189	-3.6254		
	2	-0.1177	0.8823	-0.3531	0.2355	6.2355	2.1189	-3.6254		
	6	-0.1373	-0.1373	0.7645	0.8238	0.8238	6.4709	-4.2278		

ANGLE-PLY THETA = 45 DEGREES CASE 1 (ALL LAYERS INTACT)
3 LAYERS (N=3)

STIFFNESS MATRIX (C) (10+6 LB./IN.SQ.)			-- ODD LAYERS --	THERMAL EXPANSION MATRIX (ALPHA) (IN./IN./DEG.F.)		
4.2380	1.7370	-1.3280		ALPHA 1 =	7.4500	
1.7370	4.2380	-1.3280		ALPHA 2 =	7.4500	
-1.3280	-1.3280	2.3230		ALPHA 6 =	3.9500	
STIFFNESS MATRIX (C) (10+6 LB./IN.SQ.)			-- EVEN LAYERS --	THERMAL EXPANSION MATRIX (ALPHA) (IN./IN./DEG.F.)		
4.2380	1.7370	1.3280		ALPHA 1 =	7.4500	
1.7370	4.2380	1.3280		ALPHA 2 =	7.4500	
1.3280	1.3280	2.3230		ALPHA 6 =	-3.9500	

A (10+6 LB./IN.)			A* (10-6 IN./LB.)			A PRIME (10-6 IN./LB.)			THERMAL FORCE (LB./IN./DEG.F.)		
4.2380	1.7370	-0.4425	0.2860	-0.1138	0.0328	0.2860	-0.1138	0.0328	N1-T =	39.2681	
1.7370	4.2380	-0.4425	-0.1138	0.2860	0.0328	-0.1138	0.2860	0.0328	N2-T =	39.2681	
-0.4425	-0.4425	2.3230	0.0328	0.0328	0.4430	0.0328	0.0328	0.4430	N6-T =	-3.5357	
B (10+6 IN.)			B* (10+0 IN.)			B PRIME (10-6 1/LB.)			THERMAL MOMENT (LB./DEG.F.)		
0.	0.	0.	0.	0.	0.	0.	0.	0.	M1-T =	-0.0000	
0.	0.	0.	0.	0.	0.	0.	0.	0.	M2-T =	-0.0000	
0.	0.	0.	0.	0.	0.	0.	0.	0.	M6-T =	-0.0000	

H* (10+0 IN.)		
0.	0.	0.
0.	0.	0.
0.	0.	0.

C (10+6 LB.IN.)			D* (10+6 LB.IN.)			D PRIME (10-6 1/LB.IN.)		
0.3532	0.1447	-0.1025	0.3532	0.1447	-0.1025	3.6829	-1.1152	1.3591
0.1447	0.3532	-0.1025	0.1447	0.3532	-0.1025	-1.1152	3.6829	1.3591
-0.1025	-0.1025	0.1936	-0.1025	-0.1025	0.1936	1.3591	1.3591	6.6045

Z (IN.)	STRESS COMPONENT	COEF. OF N1 (1/IN.)	COEF. OF N2 (1/IN.)	COEF. OF N6 (1/IN.)	COEF. OF M1 (1/IN.SQ.)	COEF. OF M2 (1/IN.SQ.)	COEF. OF M6 (1/IN.SQ.)	COEF. OF TEMP. (LB/IN.SQ/°F.)
-- LAYER 1 --								
-0.5000	SIGMA 1	0.9710	-0.0290	-0.3923	-5.9331	0.0669	0.3250	-0.8945
	2	-0.0290	0.9710	-0.3923	0.0669	-5.9331	0.3250	-0.8945
	6	-0.1525	-0.1525	0.9419	0.1264	0.1264	-5.8662	-4.6961
-0.1667	SIGMA 1	0.9710	-0.0290	-0.3923	-1.9781	0.0223	0.1084	-0.8945
	2	-0.0290	0.9710	-0.3923	0.0223	-1.9781	0.1084	-0.8945
	6	-0.1525	-0.1525	0.9419	0.0421	0.0421	-1.9558	-4.6961
-- LAYER 2 --								
-0.1667	SIGMA 1	1.0581	0.0581	0.7843	-2.5799	-0.5795	-2.8158	1.7885
	2	0.0581	1.0581	0.7843	-0.5795	-2.5799	-2.8158	1.7885
	6	0.3049	0.3049	1.1162	-1.0947	-1.0947	-3.1593	9.3894
0.1667	SIGMA 1	1.0581	0.0581	0.7843	2.5799	0.5795	2.8158	1.7885
	2	0.0581	1.0581	0.7843	0.5795	2.5799	2.8158	1.7885
	6	0.3049	0.3049	1.1162	1.0947	1.0947	3.1593	9.3894
-- LAYER 3 --								
0.1667	SIGMA 1	0.9710	-0.0290	-0.3923	1.9781	-0.0223	-0.1084	-0.8945
	2	-0.0290	0.9710	-0.3923	-0.0223	1.9781	-0.1084	-0.8945
	6	-0.1525	-0.1525	0.9419	-0.0421	-0.0421	1.9558	-4.6961
0.5000	SIGMA 1	0.9710	-0.0290	-0.3923	5.9331	-0.0669	-0.3250	-0.8945
	2	-0.0290	0.9710	-0.3923	-0.0669	5.9331	-0.3250	-0.8945
	6	-0.1525	-0.1525	0.9419	-0.1264	-0.1264	5.8662	-4.6961

ANGLE-PLY THETA = 60 DEGREES CASE 1 (ALL LAYERS INTACT)
2 LAYERS (N=2)

STIFFNESS MATRIX (C)
(10+6 LB./IN.SQ.)

3.1780	1.4690	-0.6853
1.4690	5.8330	-1.6150
-0.6853	-1.6150	2.0550

-- ODD LAYERS --

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)

ALPHA 1 =	9.4250
ALPHA 2 =	5.4750
ALPHA 6 =	3.4208

STIFFNESS MATRIX (C)
(10+6 LB./IN.SQ.)

3.1780	1.4690	0.6853
1.4690	5.8330	1.6150
0.6853	1.6150	2.0550

-- EVEN LAYERS --

THERMAL EXPANSION MATRIX (ALPHA)
(IN./IN./DEG.F.)

ALPHA 1 =	9.4250
ALPHA 2 =	5.4750
ALPHA 6 =	-3.4208

A
(10+6 LB./IN.)

3.1780	1.4690	0.	0.3561	-0.0897	0.	0.3605	-0.0787	0.
1.4690	5.8330	0.	-0.0897	0.1940	0.	-0.0787	0.2220	0.
0.	0.	2.0550	0.	0.	0.4866	0.	0.	0.5886

A*

0.3561	-0.0897	0.
-0.0897	0.1940	0.
0.	0.	0.4866

A PRIME
(10-6 IN./LB.)

0.3605	-0.0787	0.
-0.0787	0.2220	0.
0.	0.	0.5886

THERMAL FORCE
(LB./IN./DEG.F.)

M1-T =	35.6512
M2-T =	40.2564
M6-T =	0.0000

B
(10+6 IN.)

0.	0.	0.1713	0.	0.	-0.0248	0.	0.	-0.1752
0.	0.	0.4037	0.	0.	-0.0630	0.	0.	-0.4448
0.1713	0.4037	0.	-0.0834	-0.1965	0.	-0.1752	-0.4448	0.

B*

0.	0.	-0.0248
0.	0.	-0.0630
-0.0834	-0.1965	0.

B PRIME
(10-6 1/LB.)

0.	0.	-0.1752
0.	0.	-0.4448
-0.1752	-0.4448	0.

THERMAL MOMENT
(LB./DEG.F.)

M1-T =	0.
M2-T =	0.0000
M6-T =	2.0678

H*
(10+0 IN.)

0.	0.	0.0834
0.	0.	0.1965
0.0834	0.1965	0.

C
(10+6 LB.IN.)

0.2648	0.1224	0.	0.2505	0.0888	0.	4.3256	-0.9439	0.
0.1224	0.4861	0.	0.0888	0.4068	0.	-0.9439	2.6644	0.
0.	0.	0.1712	0.	0.	0.1416	0.	0.	7.0634

C*

0.2505	0.0888	0.
0.0888	0.4068	0.
0.	0.	0.1416

C PRIME
(10-6 1/LB.IN.)

4.3256	-0.9439	0.
-0.9439	2.6644	0.
0.	0.	7.0634

Z (IN.)	STRESS COMPONENT	COEF. OF N1 (1/IN.)	COEF. OF N2 (1/IN.)	COEF. OF N6 (1/IN.)	COEF. OF P1 (1/IN.SQ.)	COEF. OF P2 (1/IN.SQ.)	COEF. OF P6 (1/IN.SQ.)	COEF. OF TEMP. (LB/IN.SQ/F.)
-- LAYER 1 --								
-0.5000	SIGMA 1	0.9700	-0.0762	0.2017	-6.0600	-0.1524	1.2101	-1.6354
	2	-0.0707	0.8204	0.4753	-0.1415	-6.3592	2.8518	-3.8540
	6	0.0600	0.1523	0.7904	0.3600	0.9141	-6.4192	3.2693
0.	SIGMA 1	1.0300	0.0762	-0.4034	0.1201	0.3048	-1.2101	1.6354
	2	0.0707	1.1796	-0.9506	0.2829	0.7183	-2.8518	3.8540
	6	-0.1200	-0.3047	1.2096	-0.3600	-0.9141	0.8384	-6.5386
-- LAYER 2 --								
0.	SIGMA 1	1.0300	0.0762	0.4034	-0.1201	-0.3048	-1.2101	1.6354
	2	0.0707	1.1796	0.9506	-0.2829	-0.7183	-2.8518	3.8540
	6	0.1200	0.3047	1.2096	-0.3600	-0.9141	-0.8384	6.5386
0.5000	SIGMA 1	0.9700	-0.0762	-0.2017	6.0600	0.1524	1.2101	-1.6354
	2	-0.0707	0.8204	-0.4753	0.1415	6.3592	2.8518	-3.8540
	6	-0.0600	-0.1523	0.7904	0.3600	0.9141	6.4192	-3.2693

ANGLE-PLY THETA = 60 DEGREES CASE 1 (ALL LAYERS INTACT)
3 LAYERS (N=3)

STIFFNESS MATRIX (C) (10+6 LB./IN.SQ.)			-- ODD LAYERS --	THERMAL EXPANSION MATRIX (ALPHA) (IN./IN./DEG.F.)		
3.1780	1.4690	-0.6853		ALPHA 1 =	9.4250	
1.4690	5.8330	-1.6150		ALPHA 2 =	5.4750	
-0.6853	-1.6150	2.0550		ALPHA 6 =	3.4208	
STIFFNESS MATRIX (C) (10+6 LB./IN.SQ.)			-- EVEN LAYERS --	THERMAL EXPANSION MATRIX (ALPHA) (IN./IN./DEG.F.)		
3.1780	1.4690	0.6853		ALPHA 1 =	9.4250	
1.4690	5.8330	1.6150		ALPHA 2 =	5.4750	
0.6853	1.6150	2.0550		ALPHA 6 =	-3.4208	

A (10+6 LB./IN.)			A* (10-6 IN./LB.)			A PRIME (10-6 IN./LB.)			THERMAL FORCE (LB./IN./DEG.F.)	
3.1780	1.4690	-0.2283	0.3567	-0.0883	0.0165	0.3567	-0.0883	0.0165	N1-T =	35.6512
1.4690	5.8330	-0.5381	-0.0883	0.1975	0.0419	-0.0883	0.1975	0.0419	N2-T =	40.2564
-0.2283	-0.5381	2.0550	0.0165	0.0419	0.4994	0.0165	0.0419	0.4994	N6-T =	-2.7560
B (10+6 IN.)			B* (10+0 IN.)			B PRIME (10-6 1/LB.)			THERMAL MOMENT (LB./DEG.F.)	
0.	0.	0.	0.	0.	0.	0.	0.	0.	M1-T =	-0.0000
0.	0.	0.	0.	0.	0.	0.	0.	0.	M2-T =	-0.0000
0.	0.	0.	0.	0.	0.	0.	0.	0.	M6-T =	0.0000
C (10+6 LB./IN.)			C* (10+6 LB./IN.)			C PRIME (10-6 1/LB./IN.)				
0.2648	0.1224	-0.0529	0.2648	0.1224	-0.0529	4.3349	-0.9203	0.6688		
0.1224	0.4861	-0.1246	0.1224	0.4861	-0.1246	-0.9203	2.7244	1.6982		
-0.0529	-0.1246	0.1712	-0.0529	-0.1246	0.1712	0.6688	1.6982	7.2816		

Z (IN.)	STRESS COMPONENT	COEF. OF N1 (1/IN.)	COEF. OF N2 (1/IN.)	COEF. OF N6 (1/IN.)	COEF. OF M1 (1/IN.SQ.)	COEF. OF M2 (1/IN.SQ.)	COEF. OF M6 (1/IN.SQ.)	COEF. OF TE (LB/IN.SQ/F)
-- LAYER 1 --								
-0.5000	SIGMA 1	0.9925	-0.0192	-0.2282	-5.9830	0.0431	0.1849	-0.4111
	2	-0.0178	0.9549	-0.5378	0.0400	-5.8984	0.4358	-0.9687
	6	-0.0679	-0.1724	0.9473	0.0550	0.1397	-5.8814	-3.6993
-0.1667	SIGMA 1	0.9925	-0.0192	-0.2282	-1.9947	0.0144	0.0617	-0.4111
	2	-0.0178	0.9549	-0.5378	0.0133	-1.9665	0.1453	-0.9687
	6	-0.0679	-0.1724	0.9473	0.0183	0.0466	-1.9609	-3.6993
-- LAYER 2 --								
-0.1667	SIGMA 1	1.0151	0.0383	0.4563	-2.1476	-0.3736	-1.6020	0.8219
	2	0.0355	1.0903	1.0753	-0.3468	-2.8809	-3.7754	1.9368
	6	0.1357	0.3447	1.1053	-0.4766	-1.2101	-3.0281	7.3965
0.1667	SIGMA 1	1.0151	0.0383	0.4563	2.1476	0.3736	1.6020	0.8219
	2	0.0355	1.0903	1.0753	0.3468	2.8809	3.7754	1.9368
	6	0.1357	0.3447	1.1053	0.4766	1.2101	3.0281	7.3965
-- LAYER 3 --								
0.1667	SIGMA 1	0.9925	-0.0192	-0.2282	1.9947	-0.0144	-0.0617	-0.4111
	2	-0.0178	0.9549	-0.5378	-0.0133	1.9665	-0.1453	-0.9687
	6	-0.0679	-0.1724	0.9473	-0.0183	-0.0466	1.9609	-3.6993
0.5000	SIGMA 1	0.9925	-0.0192	-0.2282	5.9830	-0.0431	-0.1849	-0.4111
	2	-0.0178	0.9549	-0.5378	-0.0400	5.8984	-0.4358	-0.9687
	6	-0.0679	-0.1724	0.9473	-0.0550	-0.1397	5.8814	-3.6993

ANGLE-PLY THETA = 75 DEGREES CASE 1 (ALL LAYERS INTACT)
2 LAYERS (N=2)

STIFFNESS MATRIX (C) (10+6 LB./IN.SQ.)			-- ODD LAYERS --	THERMAL EXPANSION MATRIX (ALPHA) (IN./IN./DEG.F.)		
2.7430	0.9321	-0.1993		ALPHA 1 =	10.8700	
0.9321	7.3420	-1.1290		ALPHA 2 =	4.0292	
-0.1993	-1.1290	1.5190		ALPHA 6 =	1.9750	

STIFFNESS MATRIX (C) (10+6 LB./IN.SQ.)			-- EVEN LAYERS --	THERMAL EXPANSION MATRIX (ALPHA) (IN./IN./DEG.F.)		
2.7430	0.9321	0.1993		ALPHA 1 =	10.8700	
0.9321	7.3420	1.1290		ALPHA 2 =	4.0292	
0.1993	1.1290	1.5190		ALPHA 6 =	-1.9750	

A (10+6 LB./IN.)			A* (10-6 IN./LB.)			A PRIME (10-6 IN./LB.)			THERMAL FORCE (LB./IN./DEG.F.)
2.7430	0.9321	0.	0.381C	-0.0484	0.	0.3812	-0.0466	0.	N1-T = 33.1784
0.9321	7.3420	0.	-0.0484	0.1423	0.	-0.0466	0.1547	0.	N2-T = 37.4845
0.	0.	1.5190	0.	0.	0.6583	0.	0.	0.7205	N6-T = 0.

B (10+6 IN.)			B* (10+0 IN.)			B PRIME (10-6 1/LB.)			THERMAL MOMENT (LB./DEG.F.)
0.	0.	0.0498	0.	0.	-0.0053	0.	0.	-0.0461	M1-T = 0.0000
0.	0.	0.2822	0.	0.	-0.0378	0.	0.	-0.3265	M2-T = 0.0000
0.0498	0.2822	0.	-0.0328	-0.1858	0.	-0.0461	-0.3265	0.	M6-T = 0.9288

C (10+6 LB.IN.)			D* (10+6 LB.IN.)			D PRIME (10-6 1/LB.IN.)			
0.2286	0.0777	0.	0.2269	0.0684	0.	4.5750	-0.5595	0.	
0.0777	0.6118	0.	0.0684	0.5594	0.	-0.5595	1.8561	0.	
0.	0.	0.1266	0.	0.	0.1157	0.	0.	8.6462	

Z (IN.)	STRESS COMPONENT	COEF. OF N1 (1/IN.)	COEF. OF N2 (1/IN.)	COEF. OF N6 (1/IN.)	COEF. OF M1 (1/IN.SQ.)	COEF. OF M2 (1/IN.SQ.)	COEF. OF M6 (1/IN.SQ.)	COEF. OF TEMP. (LB/IN.SQ/F.)
-- LAYER 1 --								
-0.5000	SIGMA 1	0.9977	-0.0163	0.0718	-6.0046	-0.0325	0.4308	-0.2859
	2	-0.0130	0.9078	0.4067	-0.0260	-6.1843	2.4404	-1.6197
	6	0.0117	0.0827	0.9055	0.0700	0.4960	-6.1889	1.4528
0.	SIGMA 1	1.0023	0.0163	-0.1436	0.0092	0.0651	-0.4308	0.2859
	2	0.0130	1.0922	-0.8135	0.0520	-0.3687	-2.4404	1.6197
	6	-0.0233	-0.1653	1.0945	-0.0700	-0.4960	0.3778	-2.9056
-- LAYER 2 --								
0.	SIGMA 1	1.0023	0.0163	0.1436	-0.0092	-0.0651	-0.4308	0.2859
	2	0.0130	1.0922	0.8135	-0.0520	-0.3687	-2.4404	1.6197
	6	0.0233	0.1653	1.0945	-0.0700	-0.4960	-0.3778	2.9056
0.5000	SIGMA 1	0.9977	-0.0163	-0.0718	6.0046	0.0325	0.4308	-0.2859
	2	-0.0130	0.9078	-0.4067	0.0260	6.1843	2.4404	-1.6197
	6	-0.0117	-0.0827	0.9055	0.0700	0.4960	6.1889	-1.4528

ANGLE-PLY THETA = 75 DEGREES CASE 1 (ALL LAYERS INTACT)
3 LAYERS (N=3)

STIFFNESS MATRIX (C) (10+6 LB./IN.SQ.)			-- ODD LAYERS --	THERMAL EXPANSION MATRIX (ALPHA) (IN./IN./DEG.F.)		
2.7430	0.9321	-0.1993		ALPHA 1 = 10.8700		
0.9321	7.3420	-1.1290		ALPHA 2 = 4.0292		
-0.1993	-1.1290	1.5190		ALPHA 6 = 1.9750		
STIFFNESS MATRIX (C) (10+6 LB./IN.SQ.)			-- EVEN LAYERS --	THERMAL EXPANSION MATRIX (ALPHA) (IN./IN./DEG.F.)		
2.7430	0.9321	0.1993		ALPHA 1 = 10.8700		
0.9321	7.3420	1.1290		ALPHA 2 = 4.0292		
0.1993	1.1290	1.5190		ALPHA 6 = -1.9750		

A (10+6 LB./IN.)			A* (10-6 IN./LB.)			A PRIME (10-6 IN./LB.)			THERMAL FORCE (LB./IN./DEG.F.)	
2.7430	0.9321	-0.0664	0.3810	-0.0481	0.0047	0.3810	-0.0481	0.0047	N1-T = 33.1784	
0.9321	7.3420	-0.3762	-0.0481	0.1440	0.0336	-0.0481	0.1440	0.0336	N2-T = 37.4845	
-0.0664	-0.3762	1.5190	0.0047	0.0336	0.6668	0.0047	0.0336	0.6668	N6-T = -1.2379	
B (10+6 IN.)			B* (10+0 IN.)			B PRIME (10-6 1/LB.)			THERMAL MOMENT (LB./DEG.F.)	
0.	0.	0.	0.	0.	0.	0.	0.	0.	M1-T = -0.0000	
0.	0.	0.	0.	0.	0.	0.	0.	0.	M2-T = -0.0000	
0.	0.	0.	0.	0.	0.	0.	0.	0.	M6-T = 0.0000	
H* (10+0 IN.)										
0.	0.	0.	0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.	0.		

C (10+6 LB.IN.)			C* (10+6 LB.IN.)			C PRIME (10-6 1/LB.IN.)		
0.2286	0.0777	-0.0154	0.2286	0.0777	-0.0154	4.5754	-0.5562	0.1730
0.0777	0.6118	-0.0871	0.0777	0.6118	-0.0871	-0.5562	1.8796	1.2259
-0.0154	-0.0871	0.1266	-0.0154	-0.0871	0.1266	0.1730	1.2259	8.7646

Z (IN.)	STRESS COMPONENT	COEF. OF N1 (1/IN.)	COEF. OF N2 (1/IN.)	COEF. OF N6 (1/IN.)	COEF. OF M1 (1/IN.SQ.)	COEF. OF M2 (1/IN.SQ.)	COEF. OF M6 (1/IN.SQ.)	COEF. OF TEMP. (LB./IN.SQ./F.)
-- LAYER 1 --								
-0.5000	SIGMA 1	0.9994	-0.0045	-0.0886	-5.9987	0.0091	0.0647	-0.0784
	2	-0.0036	0.9747	-0.5020	0.0072	-5.9487	0.3667	-0.4441
	6	-0.0144	-0.1020	0.9741	0.0105	0.0745	-5.9474	-1.7931
-0.1667	SIGMA 1	0.9994	-0.0045	-0.0886	-2.0000	0.0030	0.0216	-0.0784
	2	-0.0036	0.9747	-0.5020	0.0024	-1.9833	0.1223	-0.4441
	6	-0.0144	-0.1020	0.9741	0.0035	0.0248	-1.9829	-1.7931
-- LAYER 2 --								
-0.1667	SIGMA 1	1.0013	0.0089	0.1772	-2.0115	-0.0784	-0.5608	0.1567
	2	0.0071	1.0505	1.0037	-0.0627	-2.4447	-3.1768	0.8879
	6	0.0288	0.2040	1.0518	-0.0911	-0.6457	-2.4558	3.5852
0.1667	SIGMA 1	1.0013	0.0089	0.1772	2.0115	0.0784	0.5608	0.1567
	2	0.0071	1.0505	1.0037	0.0627	2.4447	3.1768	0.8879
	6	0.0288	0.2040	1.0518	0.0911	0.6457	2.4558	3.5852
-- LAYER 3 --								
0.1667	SIGMA 1	0.9994	-0.0045	-0.0886	2.0000	-0.0030	-0.0216	-0.0784
	2	-0.0036	0.9747	-0.5020	-0.0024	1.9833	-0.1223	-0.4441
	6	-0.0144	-0.1020	0.9741	-0.0035	-0.0248	1.9829	-1.7931
0.5000	SIGMA 1	0.9994	-0.0045	-0.0886	5.9987	-0.0091	-0.0647	-0.0784
	2	-0.0036	0.9747	-0.5020	-0.0072	5.9487	-0.3667	-0.4441
	6	-0.0144	-0.1020	0.9741	-0.0105	-0.0745	5.9474	-1.7931