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Modeling and Control of Nonlinear Systems

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We have conducted research on the modeling and control of nonlinear systems. Our efforts have been directed toward understanding the structure and control of *maneuvering* nonlinear systems. The prototypical maneuver is a periodic orbit. We have studied the structure of nonlinear systems in the neighborhood of a periodic orbit and discovered necessary and sufficient conditions for transverse feedback linearization of a system about a periodic orbit. In general, stable maneuvering can be achieved by by providing for the stability of the *transverse* dynamics of the system. We have developed techniques for stabilizing the transverse dynamics of a system based on the transverse linearization. We have also developed methods for converting a stable trajectory tracking control law into a stable maneuvering control law.

Introduction

We have conducted research on the modeling and control of nonlinear systems. Our efforts have been directed toward understanding the structure and control of *maneuvering* nonlinear systems. A number of important nonlinear control system objectives can be accomplished by providing a stable motion along a path through the system state space. This is true for systems ranging from aerospace flight vehicles and robotic manipulators to systems for the manufacture of sophisticated materials.

The idea of maneuver regulation and its importance is easily understood since many of the aggressive tasks that we do each day are accomplished using a maneuver regulation approach. Indeed, suppose that you are driving your car on a curvy mountain road (e.g., to go skiing—another maneuvering task!). Using the recommended speed at each position along the road and a given starting time, one can easily determine a *trajectory* to travel the road. Suppose we now attempt to negotiate the road using a trajectory tracking controller. Unfortunately, our engine is not so powerful (we're spending that money on skiing!) and we get behind during an uphill section of the road. Immediately after the uphill section is a very curvy downhill section that must be negotiated with extreme care. To our dismay, the tracking controller attempts to *catch up* on the downhill section resulting in excessive speed that is dangerous if not fatal. Indeed, it is quite likely that the controller will take the car off-road in its attempt to catch up!

It is clear that this is not the way that a person would negotiate such a road. Indeed, a driver provides safe maneuvering by regulating her lateral position (and velocity) to keep the car in the center of the lane and by regulating the longitudinal velocity to the speed recommended by the longitudinal position on the road (so that she will not have excessive speed around dangerous hairpin curves). We emphasize that longitudinal position (and not time) plays the key role in determining what state the system should be regulated to.

A prototypical maneuver is a periodic orbit. Many important maneuvers may be imbedded in a periodic orbit. We have studied the structure of nonlinear systems in the neighborhood of a periodic orbit and discovered necessary and sufficient conditions for transverse feedback linearization of a system about a periodic orbit. In general, stable maneuvering can be achieved by by providing for the stability of the *transverse* dynamics of the system. We have developed techniques for stabilizing the transverse dynamics of a system based on the transverse linearization. We have also developed methods for converting a stable trajectory tracking control law into a stable maneuvering control law.

Nonlinear Maneuvering

Consider the nonlinear control system

$$\dot{x} = f(x, u) \tag{1}$$

with state $x \in \mathbf{R}^n$ and control $u \in \mathbf{R}^m$.

A *maneuver* is a curve in the state-control space that is consistent with the system dynamics. For example,

$$\bar{\eta} = \{(\alpha(\theta), \mu(\theta)) \in \mathbf{R}^n \times \mathbf{R}^m : \theta \in \mathbf{R}\}$$

is a maneuver of (1) provided that

$$\alpha'(\theta) = f(\alpha(\theta), \mu(\theta)), \quad \theta \in \mathbf{R} \tag{2}$$

where $\alpha'(\theta) := \frac{d\alpha}{d\theta}(\theta)$. We will use η to denote the state curve (a section of $\bar{\eta}$). The specific parametrization used to specify the maneuver is unimportant (though useful in calculations)—the maneuver $\bar{\eta}$ is the curve. Thus consistency with the system dynamics (1) only requires that $f(\alpha(\theta), \mu(\theta)) \in T_{\alpha(\theta)}\eta$ for all θ. For simplicity, in the sequel we will assume that the parametrization of maneuvers is consistent with a time parametrization, i.e., (2) is satisfied. Furthermore, we will assume that all maneuvers are such that $\alpha(\cdot) \in C^2$.

A *trajectory* is a specific *time* parametrized maneuver, for example,

$$\bar{\eta}_{t_0} := \{(\alpha(t - t_0), \mu(t - t_0)) \in \mathbf{R}^n \times \mathbf{R}^m : t \in \mathbf{R}\}.$$

Note that an infinite collection of trajectories give rise to the same maneuver.

The class of periodic orbits forms a special class of maneuvers. The additional (periodic, compact) structure of a periodic orbit makes possible many strong results.

Maneuver related research has been reported in a number of articles including [], [3], [4], [5], [1], [2], and [7].

Transverse Dynamics

Suppose that $\bar{\eta}$ is a maneuver for (1) with a time consistent parametrization. One can define a set of *transverse coordinates* (θ, ρ), valid on a neighborhood of the maneuver (see, e.g., [5]). The tangential coordinate θ is used to define a *transverse foliation* on a neighborhood of η. Then transverse coordinates ρ are (smoothly in θ) defined on the transverse sections. Defining $v := u - \mu(\theta)$ the system (1) in transverse coordinates is given by

$$\begin{aligned} \dot{\theta} &= 1 + f_\theta(\theta, \rho, v) \\ \dot{\rho} &= f_\rho(\theta, \rho, v) \end{aligned} \tag{3}$$

where $f_\theta(\theta, 0, 0) = 0$ and $f_\rho(\theta, 0, 0) = 0$. In these coordinates, the maneuver is defined simply by $\rho = 0$ and $v = 0$.

The $n - 1$ dimensional nonlinear system

$$\dot{\rho} = f_\rho(t, \rho, v) \tag{4}$$

is the *transverse dynamics*. As in the case of a Poincaré map (a discrete time description of the transverse dynamics), the stability properties of the system around the orbit and those of the transverse dynamics are closely related.

An important form involves expanding f_ρ to first order in ρ and v:

$$\begin{aligned} \dot{\theta} &= 1 + f_\theta(\theta, \rho, v) \\ \dot{\rho} &= A(\theta)\rho + B(\theta)v + f_2(\theta, \rho, v) \end{aligned} \tag{5}$$

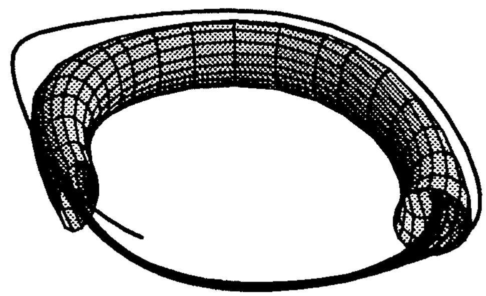
so that f_2 is second order in ρ and v. The *transverse linearization*

$$\dot{\rho} = A(t)\rho + B(t)v \tag{6}$$

can be used to obtain many important results—both theoretical and practical—see, e.g., [5, 3, 4]. For example, if $\rho = 0$ is uniformly asymptotically stable for

$$\dot{\rho} = A(t)\rho$$

then the maneuver $\bar{\eta}$ will also be uniformly asymptotically stable. The (time-varying) controllability of $(A(\cdot), B(\cdot))$ can be exploited to construct a *nonlinear* maneuver regulation feedback. One can also use the transverse linearization to estimate the domain of attraction of an exponentially stable periodic orbit:



We also mention that, in addition to inputs, disturbances are easily incorporated in the maneuvering formulation. Notions of L_2 gains of the operator from disturbances to the deviation from the maneuver have been investigated leading to a nonlinear H_∞ theory for maneuvering systems [3]. We strongly believe these results will be useful in the nonlinear analysis of, e.g., maneuvering aircraft.

Transverse Feedback Linearization

Some of the most important results in nonlinear system theory are those involving the *geometric structure* of the control system. This type of work often investigates invariant manifolds and attempts to determine whether systems are equivalent. The feedback linearization problem is of this type.

With our new understanding of maneuvering systems it was quite natural to ask whether the structure of the system in the neighborhood of a manuever is such that the transverse dynamics can be made linear by an appropriate nonlinear feedback. This is the essence of the *transverse feedback linearization* problem.

Consider the smooth affine control system

$$\dot{x} = f(x) + g(x)u \tag{7}$$

on $\mathbf{R}^n$ and suppose that $\eta \subset \mathbf{R}^n$ is a periodic orbit of (1) with minimal period T when $u \equiv 0$.

The transverse feedback linearization seeks to determine when it is possible to find new coordinates $(\theta, \rho_1, \ldots, \rho_{n-1})$ and control v so that, after change of coordinates and feedback $u = k(x) + l(x)v$, the dynamics of (7) in a neighborhood of the periodic orbit η have the form

$$\begin{aligned}
\dot{\theta} &= 1 + f_1(\theta, \rho) + g_0(\theta, \rho)v \\
\dot{\rho}_1 &= \rho_2 \\
&\vdots \\
\dot{\rho}_{n-2} &= \rho_{n-1} \\
\dot{\rho}_{n-1} &= v,
\end{aligned} \tag{8}$$

where $f_1(\cdot,\cdot)$ satisfies $f_1(\theta,0)=0$. The variable $\theta \in S^1 = [0,T]$ (we identify 0 and T) parametrizes the periodic orbit η and the coordinates $(\rho_1,\ldots,\rho_{n-1})$ parametrize the transverse dynamics. Note that the *transverse dynamics* of this system is simply a *controllable linear* system.

A system (7) which admits such a feedback transformation is *(globally) transversely feedback linearizable along* η. We have found necessary and sufficient conditions for transverse feedback linearizability for affine single-input nonlinear systems [1]. We have also given conditions under which a system (7), although not globally transversely feedback linearizable, is *locally transversely feedback linearizable* in the sense that one can cover a neighborhood of η with a finite number of open neighborhoods such that the dynamics of (7) in every neighborhood has form (8).

The conditions for transverse feedback linearization are similar to those for classical feedback linearization involving a *linear controllability* condition and an *integrability* condition. (Actually, some additional technical conditions are needed.)

Transverse linear controllability requires that

$$\dim \text{span}\ \{f(x), g(x), ad_f g(x), \ldots, ad_f^{n-2} g(x)\} = n \tag{9}$$

as compared to the *classical linear controllability*

$$\dim \text{span}\ \{g(x), ad_f g(x), \ldots, ad_f^{n-1} g(x)\} = n. \tag{10}$$

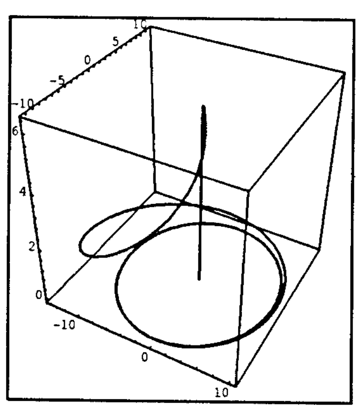
Unlike the classical controllability condition, transverse linear controllability is not generic with respect to *orbits* (though it is generic with respect to points!). In fact, a controllable linear system possessing a periodic orbit will always have two points of transverse controllability loss.

We have studied what can be done when a system exhibits points of transverse controllability loss [2]. In particular, we have found conditions such that it is possible to find new coordinates (θ,ρ_1,ρ_2) and control v so that, after change of coordinates and feedback $u = k(x) + l(x)v$, the dynamics of (1) in a neighborhood of the periodic orbit η have the form

$$\begin{aligned} \dot\theta &= 1 + f_1(\theta,\rho) + g_0(\theta,\rho)v \\ \dot\rho_1 &= a(\theta)\rho_2 \\ \dot\rho_2 &= v, \end{aligned} \tag{11}$$

where $f_1(\cdot,\cdot)$ satisfies $f_1(\theta,0)=0$, $a(\theta)$ is a smooth function periodic in θ with values in the interval $[-1,1]$, $a(\theta)=0$ only on Ω (where (9) fails), $a(\theta)=1$ or $a(\theta)=-1$ except in an arbitrarily small neighborhood of Ω, where $a(\theta)$ changes sign.

In addition to the insights into system structure, transverse feedback linearization results can be used in the design of feedback laws to stabilize a periodic orbit (or maneuver) of interest



From Trajectory Tracking to Nonlinear Maneuvering

Given a nonlinear system (1) and a trajectory $\bar{\eta}_0$, one can, under fairly general conditions, design a trajectory tracking control law

$$u = \beta(x, t) \tag{12}$$

so that the closed loop system (1), (12)

$$\dot{x} = f(x, \beta(x, t))$$

provides uniform asymptotic tracking of η_0 (η_{t_0} with $t_0 = 0$) so that $x(t) \to \alpha(t)$ as $t \to \infty$. For instance, if the (linear) variational system around $\bar{\eta}_0$ is uniformly controllable, a simple (e.g., LQ) linear time-varying controller will be effective. Note that the t in the closed loop system only appears in the feedback.

We have studied the following question. When is it possible to find a mapping

$$\pi : \mathbf{R}^n \to \mathbf{R}$$

that can be used to convert a trajectory tracking control law into a maneuver regulation control law? The mapping $\pi(\cdot)$ is to be thought of as a projection onto the maneuver that selects the appropriate *trajectory time* to be used in regulation. Using $\theta = \pi(x)$ to replace t in the control law, we require that the closed loop maneuvering control system

$$\dot{x} = f(x, \beta(x, \pi(x)))$$

be such that $x(t) \to \eta$ provided $x(0)$ is sufficiently close to η.

We have found that the conversion from trajectory tracking to maneuver regulation can be accomplished under very general conditions [7, 8, 6].

Roughly speaking, all that is needed is a (quadratic type) Lyapunov function that proves stable trajectory tracking. Such will always be available for systems possessing exponentially stable trajectory tracking. With the Lyapunov function in hand, the maneuver projection is chosen according to

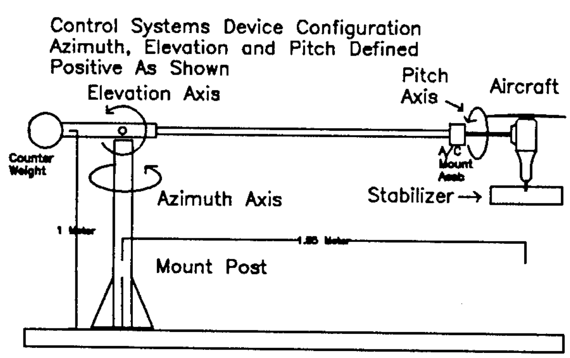
$$\pi(x) := \arg\min_{\theta} V(x, \theta) \tag{13}$$

The maneuver projection operator $\pi(\cdot)$ defines a transverse foliation of a neighborhood of the maneuver. When the system is linear, the Lyapunov function can be chosen to be

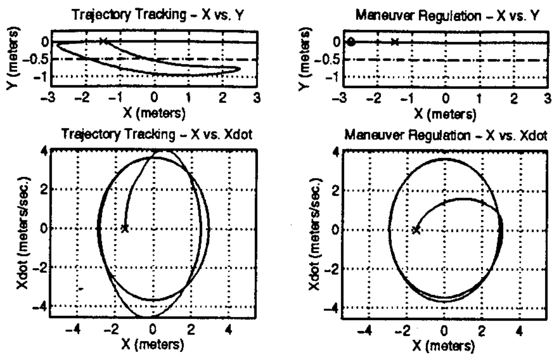
$$V(x, \theta) = ||x - \alpha(\theta)||_P^2$$

where $P > 0$ is a solution to the Lyapunov equation $A^T P + PA + Q = 0$ for some $Q > 0$. The transverse section defined by $\pi(x) = \theta$ is a simple transverse plane.

We have applied these ideas to the design of maneuvering controllers for our tethered aircraft system, the Champagne Flyer [7], picture below



To make the comparison of trajectory tracking and maneuver regulation control laws interesting, we limited the thrust in our model of the system and specified a definite "ground level." Results for a simple maneuver are shown below.



As you can see the initial error plus the thrust limitation resulted in the trajectory tracking system flying *below* ground level while the maneuver regulation system lost very little altitude.

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