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PROFESSOR ALBERTO GRUNBAUM

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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
UNIVERSITY OF CALIFORNIA
336 SPROUL HALL
BERKELEY, CA 94720

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F.A. Grunbaum
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FINAL REPORT

(12/1/91-11/30/95)

During the duration of this contract I have done work on different areas, some involving medical imaging and others involving the development of some mathematical tools to be eventually used in imaging or related problems.

For the last few years my work has centered around certain INVERSE PROBLEMS arising in biomedical imaging, including X-ray and Magnetic Resonance imaging, and more recently X-ray crystallography.I am very interested in exploring ways in which new developments in mathematical physics can yield results of use in some of these medical imaging problems.My efforts of the last few years can be subdivided into three different categories.

Diffuse tomography

The aim here is to obtain images of diagnostic value using probes with very low energy, like an infrared laser.This problem is mathemati cally much harder than the standard ones in either X-ray or Magnetic Resonance imaging.From the very beginning one needs to deal both with attenuation and scattering and the imaging problem is that of finding both distributions.The problem is fully nonlinear.This will certainly not be a substitute for well know existing imaging modalities but rather be of use in specific areas of medical diagnostic like "mammographies" or certain brain blood flow studies (mesuring oxigen content) in a neonatal clinic.

I have carried out three kind of studies: first some preliminary work to get a handle on the relation between microscopic and macroscopic variables, then some numerical studies that show the feasiblity of this project and finally a very detailed study of a very general model in the simplest case of physical interest: an object subdivided into a square array of 4 pixels.The most general mathematical model (ignoring any reasonable physical assumption) leads to a system of 64 NONLINEAR equations in 64 unknowns.

Working with a student (Sarah Patch) we have almost managed to handle these equations all the way to the end.A breakthrough has been the realization that a systematic use of the "Plucker identities" described below allows one to simplify the equations considerably.It is possible to view these identities as part of certain "nonlinear consistency conditions" that the data (i.e. the input-output matrix for the system) ought to satisfy to come from a physical problem.Much simpler, and linear, consistency conditions have turned out to be useful in the design of practical algorithms in both X-ray as well as positron emmission tomography.

The Plucker (or Grassmann) identities date from last century and are a tool in algebraic geometry. In more recent times their use has surfaced again in connection with an explicit description of the "tau function" for a family of nonlinear evolution equations known as the KP equations.Kadomtsev and Petviashvilii after whom the equations are named are two Russian plasma physicists.

This has resulted in several publications mainly in SPIE Proceedings.

The bispectral property

During the past years I have been making reasonable progress on extending to the set-up of tridiagonal matrices work that I had done a while back in connection with Schroedinger type differential operators.In this effort I have been joined by Luc Haine from Louvain-la-Neuve, Belgium.

The paper " Orthogonal polynomials satisfying differential equations : the role of the Darboux process" is about to appear as a publication of the Center for Mathematical Research in Montreal, Canada.

The paper " A theorem of Bochner, revisited" is coming out in a volume edited by T. Fokas and I.M. Gelfand .

The paper "The q-version of a theorem of Bochner" is about to appear in J. of Computational and Applied Mathematics

Finally the manuscipt "Some functions that generalize the Askey-Wilson polynomials" is almost ready and will be submitted to Communications in Mathematical Physics very soon.

All of these papers lay the foundation (both in the case of q-difference equations, as well as in the limiting case when q=1 and one has differential equations) on which one can start looking for "bispectral families of higher order operators" in the context of tridiagonal matrices. We have already found some examples of this situation, but a full theory is still lacking.

This "discrete version" of the problem investigated earlier with J. Duistermaat should be of interest to workers involved with the Toda and similar lattices in mathematical physics.In the continuous case we run into the Korteweg-deVries equation and its master symmetries, the Virasoro flows.Here we run into Toda flows and their master symmetries.Of course the q-versions of these results should be of interest to people working on

Quatum Groups.

This work has already made contact with rather unexpected lines of work, including for instance some recent work on extensions of the Huygens principle.

The last paper of the ones mentioned above uncovers what we are starting to call the Gauss-Askey-Wilson equation.It is one of the possible q-extensions of the famous hypergeometric equation of Gauss, but not the one that people have been studying for the last few years.We expect to see that it will become the center of proper attention.It should have multiple applications.

This last piece of work is coming out as a publication of the Fields Institute in Canada, under the title On a q-analogue of Gauss equation and some q-Riccati equations.

Finally all these ideas are now finding some use in joint work with Jon Mooney, at Hanscom, in the area of multispectral imaging.

The phase problem in X-ray crystallography

The paper The Use of higher order invariants in he determination of generalized Patterson cyclotomic sets has appeared in Acta Crystallographica A, 1995 May 1, vol 51 May 310-323.
It is joint work with Calvin Moore and we hope to make further progress on this subject.

Some of this work has found applications in a different area, as described below.

In his work on laser radar L.Shirley came up with a surprisingly good algorithm for the determination of the scattering function from very limited knowledge, just the support of its correlation function. Previous work of Grunbaum shows that this cannot work in full generality, and that one will at times require higher order information. The two most inmediate problems that they plan to investigate are : obtain an understanding of the mathematical reason behind the remarkable success of Shirley's method and then apply this insights into developing an improved version.

Grunbaum was attracted to this line of work by some previous work of Shirley on "Advanced techniques for target discrimination using laser speckle".Here Shirley uses some higher order correlations of the same type that Grunbaum has used on his work in X-ray crystallography.

Other work

The paper Some variations on the theme "local-vs-global" has appeared in a Special Volume in Honor of R. Ricabarra. Universidad Nacional del Sur, Bahia Blanca Argentina 1995.

The paper "Band-Time-Band limiting integral operators and commuting differential operators" should have appeared in the Proceedings of the Fall Semester at the Euler Institute, St. Petersbourg, Russia with V. Buslaev editor.

Vaughan F.R. Jones
FDF-49620-92-J-0067

FINAL REPORT
(12/1/91-11/30/95)

Work in the program was divided into two parts: knots and combinatorial structures associated with algebras.

In the know work we began investigation of the far field created by a Lissajou knot antenna. Preliminary calculations showed that the field decays far more rapidly than one might expect in the case of even parity. The intriguing observation was also made that Lissajou knots are in fact the same as "three-dimensional billiard knots" - piecewise rectilinear trajectories in a cubic room with ideal reflection off the walls. This opened the possibility of considering billiards in rooms such as the tetrahedron and the dodecahedron. It culminated in the paper "Lissajous knots and billiard knots" by Jones and Przytycki. It seems very difficult to find restrictions on the knot type in the case of more general rooms-the trefoil and the figure-eight knots are readily obtainable and it is possible that the ergodic "argument" that does not work in the Lissajous case may work in a more generic situation to obtain all knot types.

In the comnbinatorial work, the main discovery has been of a family of 2-parameter algebras, generically semisimple in work by Jones and Bisch. These algebras form a colored generalisation of the celebrated Temperley-Lieb algebras. Using results of Popa it is shown that these algebras do occur as the natural invariants of some simple infinite dimensional subalgebras. One intriguing question is the relation of these algebras (whose dimensions are given by the Fuss-Catalan numbers) to the Yang-Baxter equation and knot theory. Bisch and Jones have undertaken a systematic study of all algebras in which one has a Yang-Baxter relation modulo lower order terms. This is the case for the Fuss-Catalan algebras and of course for the Birman-Murakami-Wenzl algebras but, remarkably, there are more such structures as one can see by considering the characters of the finite simple MacLaughlin group and U_3(5).
Bisch and Jones have written two papers on this work:
"Algebras associated to intermediate subfactors" and "A note on free composition of subfactors", both of which are available in preprint form.